Precast concrete terraces under static, incremental loading - a laboratory investigation

Hughes, B.P. and Karadelis, J.N.

PRECAST CONCRETE TERRACES UNDER STATIC, INCREMENTAL LOADING. _A Laboratory Investigation.

B P Hughes
Emeritus Professor, The University of Birmingham & Visiting Professor, University of Coventry.
J N Karadelis
Senior Lecturer, University of Coventry.

University of Coventry, Coventry, United Kingdom

ABSTRACT
This paper describes the experimental investigation of a set of L-shaped precast concrete terrace units subjected to static incremental loading, in order to assess their structural performance and estimate their stiffness, natural frequency and damping ratio. A series of loading-unloading tests were carried out on *uncracked* (as delivered from the factory) and *cracked* (after the first loading-unloading cycle was completed) units. The variation of parameters, such as displacements and strains, with the applied load was recorded and presented in a graphical form. The reduction in stiffness of the units due to cracks was estimated from these graphs.

The predominant mode of failure was found to be cracking initiated at the soffit of the units (tension zone) and mainly around the symmetry line (where maximum bending stresses are formed) and their gradual propagation to the top. The strain distribution across the depth of the vertical part of the terrace unit (*beam*) was found to be predominantly linear, displaying tension at the bottom and compression at the top. However, a large portion of the horizontal part of the unit (*slab*) followed closely the behaviour of the beam, to give tension rather than compression at the top. This could have some implications to the design of the units. The deformed shape of the units was significantly more complex than assumed in their initial design; displacing downwards and rotating about a longitudinal horizontal axis and at the same time, warping at the free slab corners, with further implications in their design.

A series of finite element models were developed, depicting closely the true behaviour of the units and assisting with the recognition and study of one or two structural conditions that otherwise would not be easy to identify (such as, the formation of a 'bowl' at the central region of the units).

It was concluded that the present methods and procedures of evaluating and designing precast concrete terrace units are not integral. Further tests are required, combined with more rigorous analytical work and the establishment of benchmarks, in order to reduce greatly the uncertainties surrounding their performance during their working life.

1. INTRODUCTION

Contrary to the mechanical and aerospace industries, the construction industry has been very slow in incorporating dynamic analysis in design procedures. Hence, the effect some particular types of dynamic loads have on the serviceability and in some cases, safety of a number of structures is only now becoming increasingly more evident and understood and the subject of some concern.

The response of grandstands to vibrations has not, in the past, been significant enough to cause any serious concern and affect drastically their design. The problem is relatively new and made its debut with the need for upper tiers, longer cantilevering ends and long spans without intermediate supports, combined with more slender, aesthetically pleasing structures and ever increasing lively crowds. Hence, the recent problems associated with the Millennium Stadium in Cardiff and the famous Millennium Bridge over the river Thames in London.

The most common construction of sports stadia today is that of a hybrid type where precast concrete terrace units span between inclined (raker) steel beams and rest on each other, thus forming a grandstand (Figure: 1). Accurate analysis and design of these units (elements) as well as the grandstand as a whole requires a good understanding of their behaviour and performance under loading. Optimisation of their structural sections and improving economy, safety and comfort in use, is an

Figure 1 Precast concrete terraces.

on-going engineering challenge with industry and academia working alongside.

Reinforced concrete has by its very nature higher stiffness characteristics and discloses considerably more damping than steel. Therefore, structures made entirely of concrete should not portray serious vibration problems, whereas a hybrid construction can be prone to excessive vibration for several reasons:

a. Structural steel beams are by nature slender and have a low mass to volume ratio. They possess low stiffness and damping properties and are very susceptible to vibrations. The problem is exacerbated when these elements form cantilevers carrying heavy masses at the ends.
b. There is no effective continuity between concrete terrace units themselves so that they can act as one enormous mass and therefore dampen the structure.
c. There are no effective, energy absorbing, connections between the concrete units and the steel raker beams.

2. RELATIONSHIP TO PREVIOUS WORK

It is obvious that loads produced by people jumping or dancing on a structure are significantly larger than the corresponding static loads. When studying the 'dynamic' behaviour of crowd, its effect is significant only when its movements are synchronised (Dynamic Crowd Effect). It has been estimated that, peak loads reached by someone jumping, can be 4.7 times greater than the person's own static weight. Furthermore, displacements reached due to possible resonance of the vibrating structure would be 25 times greater than the corresponding static displacements *[Ellis (1997)]*.

Current British experience recognises that recent trends in design have produced structures which are sensitive to synchronised crowd loads and provides some guidance by considering these loads and the corresponding response of the structure *[BS 6399, (1996)]*. The Standard sets 'boundary values' for natural frequencies as a step forward to a more sophisticated 'dynamic' approach to design and assessment but it does not fully address today's demands.

However, it has recently been appreciated that a much better understanding of the loading histories (harmonic functions) associated with crowd movements is of paramount importance, if successful structural vibrations studies were to be carried out and safety and comfort of the occupants were not to be compromised. Hence, a series of research studies were commenced dealing with the long term monitoring of existing structures during big events or generating purpose made dance and jump type loads for studying. Ji and Ellis presented an experimental and analytical approach of calculating the response of floors to dance type loads. They stressed the importance of selecting an appropriate model (beam or plate) and concluded that the use of these models does not necessarily produce conservative or safe answers. Hence, today, there are no guidelines to suggest a rigorous but simplified procedure for the dynamic approach to design of these structures *[Ellis & Ji, (1994)]*.

Limited research elsewhere has reported that the frequency range for jumping activities usually varies between 1.5 Hz and 3.5 Hz *[Alves et al (1999)]*. As this type of structure is designed for natural frequencies between 1.5 Hz to 9.0 Hz, it is not unusual for resonance to occur.

In addition, stamping frequencies during a rock concert in Canada were noted to vary from 2.03 Hz to nearly 4.0 Hz *[Pernica (1983)]*. During the same concert it was reported that a particular stamping frequency coincided with the fundamental frequency of the stand to produce maximum peak accelerations of the order of 0.3g. to 0.35g. These accelerations would be considered “extremely unpleasant” to “intolerable”, possibly "harmful" by the ISO 2631-1974(E) international standard. Surprisingly, no criteria exist to measure possible accumulated damage and declare the unit fit, or unfit for use, despite the rapid deterioration that can occur in precast concrete construction *[Hughes and Dundar (1986)]*.

Kasperski has recently concluded that the dynamic loading induced from an active audience, becomes decisive for the design of grandstands and that dynamic responses due to resonance with the first and second harmonics of the load may lead to considerable safety problems *[Kasperski, (2002)]*. In the same work he stated that there is no inherent correlation in a series of jumping activities performed by individuals (random process) and stressed the importance of a Monte Carlo simulation of such loads, a significant improvement, as he called it, to the usual Fourier series.

Finally, the effect of excessive vibrations causing enjoyment, discomfort, fear, panic, structural damage, (loading scenarios) as well as the risk of resonance and the ability of a crowd to synchronise its movements (load mechanisms) with, say, music and cause excessive vibrations to a structure (response) is currently the subject of investigation at the Universities of Oxford, Sheffield, Manchester and UMIST. An Interim Guidance on Assessment and Design has been produced as an initial response to the problem *[DTLR et al., 2001]*.

The applied loads and load mechanisms generated by sports and music fans on grandstands and other stadia structures have not yet been fully understood. As the response of these structures depends greatly on these loads, the later have not yet been depicted accurately and therefore it is not surprising that current standards and codes of practice in this country and abroad are not as rigorous and informative, as they should be. Understanding the above would necessitate long-term instrumentation and monitoring of suitable structures and their mathematical 'reproduction' would require complex numerical techniques, based on the experimental findings. This is the subject of on-going research at the Universities mentioned above.

This paper is part of an ongoing research programme at Coventry University, aimed at extending the understanding of structural behaviour of sports stadia assembled from interconnected precast concrete units, by providing all those interested with an accurate interpretation (numerical modelling) of these structures under the above loads.

3. EXPERIMENT PLAN & METHODOLOGY.

A series of comprehensive, laboratory tests have been carefully planned and executed. The aim of this preliminary investigation was twofold: First, to verify the behaviour of a family of RC structures supported at three positions and undergoing static, incremental loading. Second, to estimate the uncracked and fully cracked stiffness of the units. Two tests per unit were carried out for the latter aim. Test 1 assumed the section uncracked as it was transported from the factory and Test 2 considered the same section but this time fully cracked, as received from Test 1.

The L-section terrace units were designed, manufactured and transported to University by BISON Ltd. Due to limited space in the laboratory the smallest actual size units were ordered. They were approximately 5 metres long, encompassing a 700 x 100 mm thick horizontal member (slab) and 150 x 275 mm upstand (Figure 2). Their properties were as follows:

Table 1. Material properties, loading and cover to reinforcement.

Material Properties
Characteristic concrete strength, f_{cu}= 45 Nmm^{-2}
Reinf't (T&C) charact. strength, f_y= 460 Nmm^{-2}
Reinf't (shear) charact. strength, f_{yv}= 460 Nmm^{-2}
Loading
Load Type: Uniformly Distributed Load
Dead Load (self weight) = 3.655 kNm^{-2}
Imposed Load = 4.000 kNm^{-2}
Cover to Reinforcement:
30 mm at sofit and the sides of Upstand and
40 mm at the top of slab.

The design was carried out by BISON, based on BS8110, Part 1, 2000 and produced the following results:

Table 2. Forces and Moments.

Serviceability State:
Reactions: R_1 = R_2 = 12.056 kN
Max Bend. Mom. = 13.563 kNm @ midspan.
Ultimate Limit State:
Reactions R_1 = R_2 = 18.139 kN
Max Bend. Mom. = 20.406 kNm.@ midspan.

Table 3. Output details.

Effective Depth, *d* = 230 mm
K-factor = 0.091
Lever Arm factor = 0.886
Lever Arm, *z* = 205.8.8 mm
Depth to Neutral Axis, *s* = 76.6 mm
Area of Tension Steel Req., A_{st}= 258 mm^2
Tension Steel Provided = 1T20 (314 mm^2)
Area of Compression Steel Req., A_{sc}= 0 mm^2
Compression Steel Provided = 1T12 (113 mm^2)

It is clear from Tables 2 & 3 that, for design purposes, the units were considered simply supported at the ends only and analysed as spanning the long dimension.

Figure 2. A precast concrete terrace unit in the laboratory.

However, this does not depict real conditions. Constructional details show the units supported at the ends of the beam on steel stools and propped along the front edge by the lower unit. Hence, two steel ‘stools’ were placed under the upstand (beam), spanning 4.5 m, simulating raker beam conditions in the laboratory. The front part of the units was propped by a UB-section, simulating the lower unit on site. Mastic was inserted between the two materials to ensure that local damage was minimised.

4. TESTING PROCEDURES.

The heavy structures area of the Civil Engineering laboratories, comprising a strong floor and an array of reusable steel stanchions and beams was utilised. Six concentrated loads simulating a uniformly distributed load (udl) were applied on the slab at 700 mm centres (Figure 2), using hydraulic jacks and spreader beams . The line of action of the udl was parallel to the upstand and at a clear distance of 100 mm from it. The load was applied incrementally and was kept constant during the collection of data. Loading and unloading tests were performed for *'uncracked'* and *'fully cracked'* units. The results of two units are reported in this paper. The following parameters (variables) were measured using appropriate transducers.

a. The maximum displacement was measured at the centre of the unit and at two other symmetrical positions, in order to ensure symmetrical behaviour.
b. The surface strain of the longitudinal tension (bottom) reinforcement of the beam, using electrical resistance strain gauges (ERSG).
c. Also, ERSG were used to measure the strain at the lateral bottom reinforcement of the beam.
d. The concrete surface strain distribution across the depth of the beam section, using four pairs of demec points
e. The concrete surface strain at seven other positions on the unit, using demec points and a set of mechanical (analogue) strain gauge dials.

It was envisaged that the above readings should provide us with a good understanding of the behaviour of the terrace unit.

5. RESULTS AND DISCUSSION.

5.1 Displacements

Figure 3 shows loading and unloading paths of maximum displacement measured at the half span position (LVDT2) for Tests 1 (uncracked) and 2 (fully cracked) units. It is evident that the path described by curve Test 2, is smoother and not characterised by any sudden "strain jumps", until it exceeds the maximum value met in Test 1. Above this load, further cracking occurs, producing another "strain jump" and permanent deformation.

Up to a load of 30 kN the slopes are similar and the two curves show good correlation. However, after the first initial cracking in Test 1 (ie: above 30 kN) the two curves diverge. The displacement of the uncracked section becomes considerably higher compared to that of the cracked. At 60 kN the corresponding displacements for the uncracked and cracked sections were 6.98 and 5.6 mm respectively. At 72 kN (the maximum load reached by the uncracked unit) the corresponding displacements were 10.75 mm and 6.5 mm.

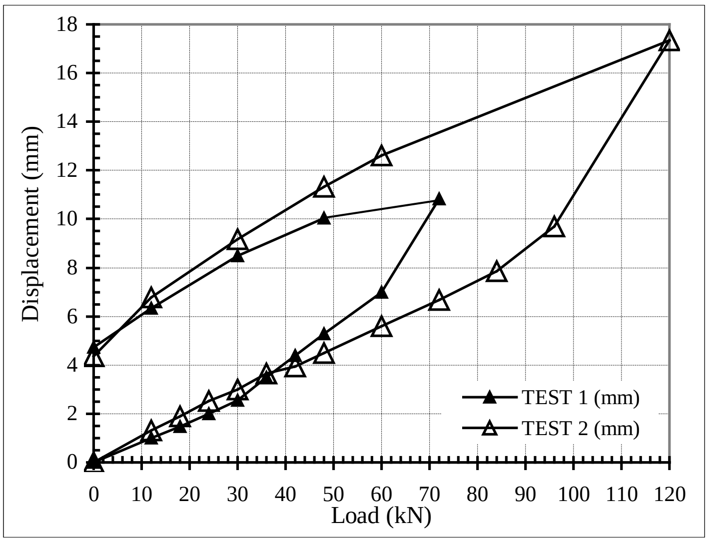

Figure 3. Instrumented terrace unit 1. Tests 1 & 2, uncracked and fully cracked units. Comparison between maximum displacements (LVDT2). Loading-Unloading.

Considering that the fully cracked unit has suffered permanent deformation due to loading at Test 1, it is not surprising that its displacement values are lower than those of the uncracked section.

The maximum displacement reached by the cracked unit was 17.25 mm at 120 kN. The permanent displacements after load removal was found to be approximately 4.5 mm in both cases. Thus the total permanent displacement after Tests 1&2 was the sum of these values, approximately 9 mm.

5.2 Strain distribution across the depth of the up-stand (beam).

Figures 4 & 5 show the distribution of strain per load increment for an initially uncracked (Test 1) and a cracked (Test 2), section respectively. This strain was measured across the vertical symmetry line and at four different levels (D11= 40 mm, D10= 110 mm, D9= 165 mm and D8= 235 mm) above the soffit of the beam.

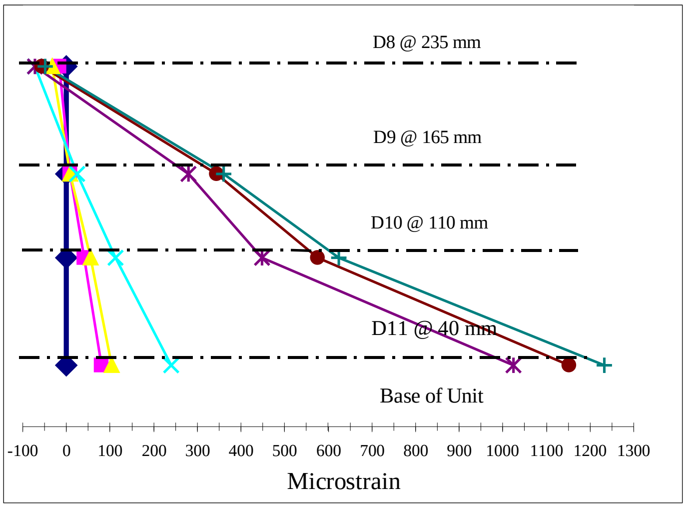

Figure 4. Terrace unit 1. Test 1. Strain distribution per load increment, across the depth of the beam.

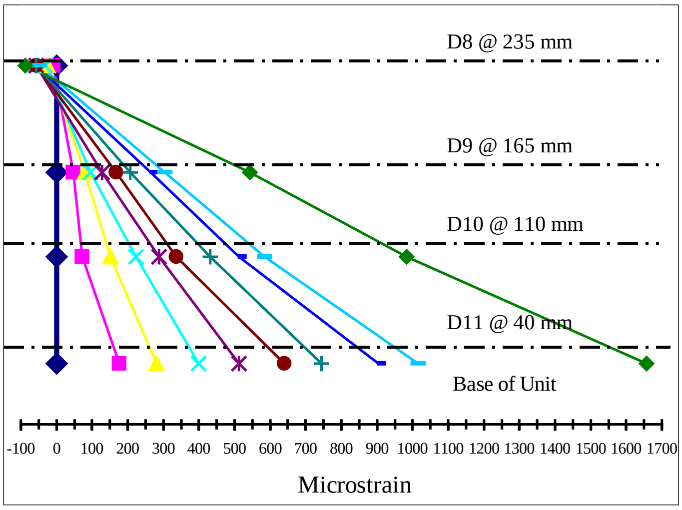

Figure 5. Terrace unit 1. Test 2. Strain distribution per load increment, across the depth of the beam.

The strain diagram in Figure 5 shows perfectly linear behaviour up to and including the load increment of 30 kN. That is, the applied loads are resisted by both concrete and reinforcement. Tension is gradually transferred to the reinforcement as the first cracks at the bottom of the unit appear, characterised by a non-linear distribution of strain for load increments of 48, 60 and 72 kN. Equilibrium of the section is maintained by a gradual movement upwards of the Neutral Axis; that is, by reducing the area of section in compression. Figure 5 presents a similar account; this time the strain was distributed more smoothly. Once the cracks are developed, no sudden changes of strain are present during reloading. As cracks open wider following the load increments, strain readings reach higher values causing the steel reinforcement to undergo local yielding.

5.3 Strain measured at the reinforcement

Electrical resistance strain gauges were attached to the reinforcement as shown in the diagram. SG1 was attached to the transverse reinforcement of the slab and SG2 to the longitudinal tension reinforcement of the beam. Their variation with load increments is shown in Figure 6.

As expected, there was a major difference in the magnitude of strain measured. Readings of strain gauge SG2 were found to be significantly higher than those of SG1, indicating that the main bending took place in the longitudinal direction. Once again, the 30 kN and 48 kN loads were characterised by cracks and crack propagation and de-bonding. Max. values recorded were: $SG1_{max}$= 110 μs and $SG2_{max}$= 2100 μs corresponding to magnitudes of stress of 22 Nmm^{-2} and 420 Nmm^{-2} respectively, assuming a Modulus of Elasticity for steel, E_{steel} = 200 $kNmm^{-2}$ and a linear stress strain relationship.

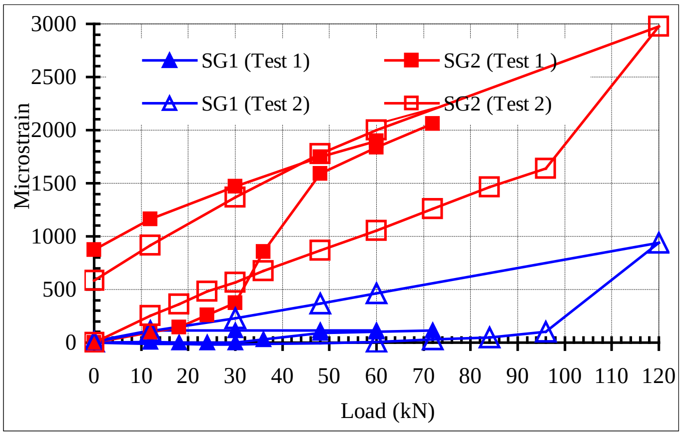

Figure 6. Terrace unit 1. Tests 1 & 2. Comparison between strain gauges SG1 and SG2. Loading-Unloading.

The strain curves for Test 2 (cracked section) are a good deal smoother than those of Test 1. Strain gauge SG1 recorded no strain up to a load of 60 kN. It recorded a strain of 100 μs for 96 kN and a 'strain jump' to 900 μs (180 Nmm^{-2}) for the final load of 120 kN. No residual strain was noticed after unloading, showing that any cracks which formed across the transverse reinforcement must have gradually closed during the unloading procedure. SG2, attached to the main tension reinforcement of the beam, showed a near linear behaviour reaching 1600 μs (320 Nmm^{-2}) for 96 kN, before it finally reaches 3000 μs (600 Nmm^{-2}, well beyond the yield stress of steel) for 120 kN.

5.4 *Strain at SG1, D1, D3 & D4*

Figure 7 shows very similar strain patterns for demec pairs D1, D3 & D4, confirming the validity and accuracy of the readings obtained in Test 1. D1 reached a maximum and levelled at –500 μs, D3 at –250 μs and D4 hovered around zero. There was a noticeable gradual reduction in lateral compressive strain from the extreme support regions to the symmetry line. This indicated an independent behaviour of the slab at the extremes and a similar behaviour with the beam, near the middle. That is, although the slab, as a structural section itself, developed tension and compression on opposite faces near the supports, the entire section was in tension near the centre.

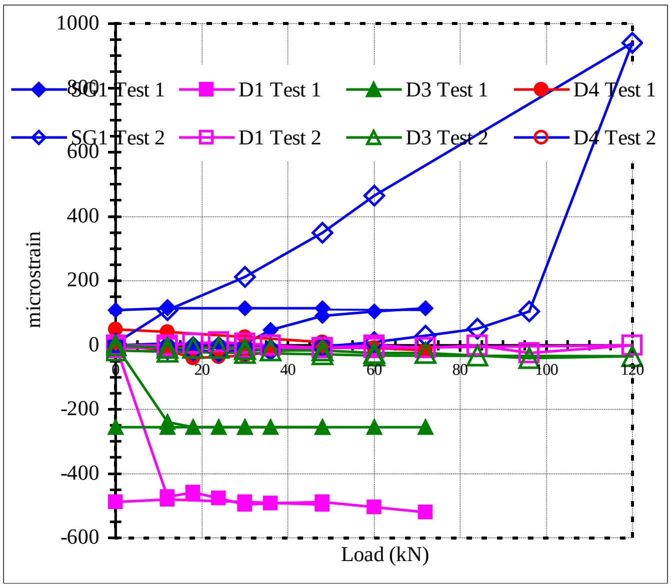

Figure 7. Terrace unit 1. Tests 1 & 2. Comparison between strains @ SG1, D1, D3, D4.

Based on Table 2, the total Serviceability and Ultimate State loads were 24.112 kN and 36.278 kN respectively. The longitudinal strain, measured at D7 and D6, became tensile at 24 kN and 30 kN respectively. Also, the lateral strain at D4 became tensile at 30 kN (Figures 7 & 8). This would indicate that when the unit is about to reach its allowable serviceability load value, part of it does not obey classical RC theory. This has not been taken into account when designing the terrace units and may have significant implications for future designs.

The sensitivity of all three demec pairs was greatly reduced at Test 2, resulting in strain readings very close to zero. It is envisaged that these very low readings are due to the development of a series of cracks outside the effective zone of the demec pairs. SG1, the strain gauge attached to the lateral bottom reinforcement of the slab, recorded tension in both tests. It is clear from Figure 7 that the top part of the slab (compression side) cracks at 12 kN, that is, before the bottom (tension side) develops its first cracks at 30 kN. However, it is important to remember that the top of the slab near the centre behaves in a different manner, with the whole (slab) section being in tension, following the behaviour of the beam. Yet, for cracking in the same direction, the bottom face will still crack in tension before the top face.

5.5 *Strain at D5, D6, D7*

Demec point pairs D5, D6 and D7 were attached on the symmetry line as shown in Appendix A1. Figure 8 shows the variation of strain with load as measured across these points. It is interesting to note that D5 has followed a compressive path, reaching strain of –100 μs at 48 kN and then somehow 'softening', to finish with –70 μs at the maximum load of 72 kN.

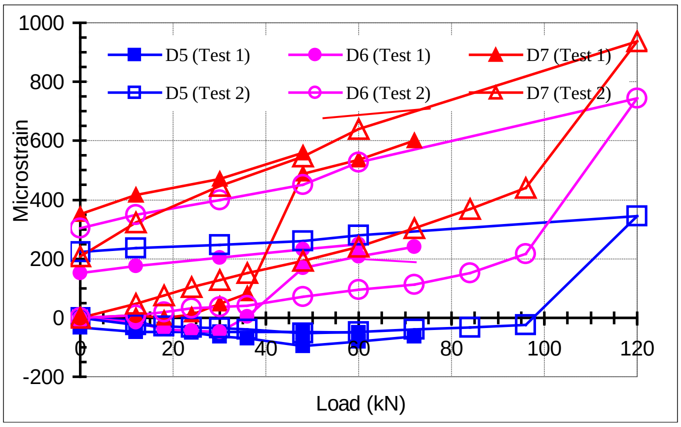

Figure 8. Terrace unit 1. Tests 1 & 2. Comparison between strains @ D5, D6, D7, (on slab).

In contrast, D6 and D7 have recorded considerably larger tensile strains. The tension in this region is somehow surprising, especially when this is developed along the longitudinal direction of the unit. It shows that the slab at the region identified between the beam and demec points D2 and D5 (see Appendix A1), follows the behaviour of the beam; that is, it is in tension and forms a trough. The familiar pattern at 12 kN and 30 kN discussed previously, is repeated here. However, there is an enormous jump of strain between load increments of 30 kN and 50 kN. This is more obvious for demec readings D6 and D7. As strain is recorded tensile, it shows that concrete has yielded locally.

The loading and unloading paths during Test 2 (fully cracked unit) were much smoother than the corresponding ones for the uncracked unit, Test 1. D5 (Figure 8) showed a negative initial tendency with most of the readings appearing below the horizontal axis. This was also the norm of the corresponding demec pair for the uncracked beam in Test 1. New cracks, or further opening of the existing cracks, appeared at 96 kN. The final strain values at 120 kN reached 350, 725 and 925 microstrain for D5, D6 and D7 respectively. Residual strains were 225, 300 and 200 microstrain respectively

5.6 Strains at SG2 & D11

Figure 9 shows a comparison between strains measured by strain gauge SG2, attached to the tension reinforcement of the beam and demec points D11, attached 40 mm above the bottom of the same beam. That is, both SG2 and D11 were at approximately the same level from the soffit of the beam. Initially, and up to the load of 18 kN, both strains showed good correlation, becoming reasonably good up to 30 kN. However, beyond the 30 kN load, SG2 produced considerably higher strain values than D11, although both strain paths were remarkably similar. This indicated that strain resisted by the reinforcement was higher. Maximum values of strain were recorded for the final load of 72 kN and were 2100 μs and 1200 μs for SG2 and D11 respectively.

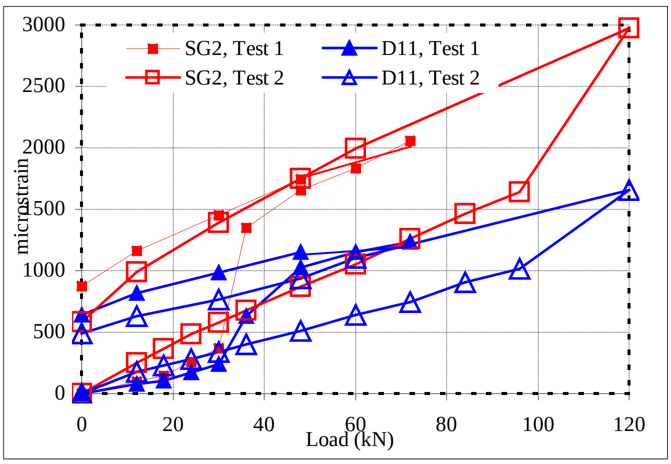

Figure 9. Terrace unit 1. Tests 1 & 2. Comparison between strains @ SG2 & D11

These can be turned into stress values assuming Moduli of Elasticity, E_{steel} = 200 N/mm^2 and (uncracked) $E_{concrete}$ = 35 N/mm^2. Therefore, for SG2, strain = 2100 μs and stress = 420 Nmm^{-2} and for D11, strain = 1200 μs, stress = 42 Nmm^{-2} respectively.

The unloading procedure was carried out without surprises giving residual strain values of 880 μs and 640 μs for SG2 and D11 respectively.

The behaviour of both curves for the fully cracked unit (Test 2) up to the load of 96 kN were smoother and more linear compared with that of the uncracked unit. Although both strain paths are similar, SG2 produced higher strain values for reasons already explained above. Maximum strain was reached at 120 kN with magnitudes equal to 3000 μs (600 Nmm^{-2}) and 1650 μs (57.75 Nmm^{-2}) for SG2 and D11. Both stress values were well beyond the yield stress values of the materials. Finally, unloading produced residual strains of 600 μs and 500 μs for SG2 and D11 respectively.

5.7 Evaluation of Stiffness

Static stiffnesses were estimated from the displacement-load variation. The 'best fit' straight line for each curve was plotted and the slope of the line was calculated based on the relationship,

$$F = k\delta \quad \Rightarrow \quad k = F/\delta \tag{1}$$

It was difficult to estimate the stiffness from Test 1 (uncracked unit) as the slope of the F v δ curve (Figure 20) changed each time a new crack appeared on the unit. Hence, only the first five load increments were used, out of a total of seven outlining the loading path. The magnitudes of the stiffnesses are shown in Table 4 below:

Table 4: Estimated uncracked and cracked stiffnesses for Terrace Unit 1.

Test 1 (uncracked unit)	Test 2 (cracked unit)
11.74 (kN/mm)	9.74 (kN/mm)

Every time a new crack appeared on the unit, its stiffness was reduced. The procedure to estimate the rate of reduction in stiffness with loading and plot the variation of load with stiffness could be useful. This would necessitate a high number of units to be tested and a statistical analysis. However, this is outside the scope of this work and it can only be possible after a large number of different units have been tested.

6. CONCLUSIONS

The following are evident from the incremental, static, loading-unloading tests, carried out on two precast concrete terrace units under laboratory conditions:

a. The predominant mode of failure is the appearance of hair-like cracks at the soffit of the units and around the symmetry line (where bending stresses are maximum) and their gradual propagation upwards.
b. The units are supported at the ends (under the beam) and propped along the front edge of the slab. Hence, they experience a combined bending and torsional effect when loaded near the beam side. This has a knock-on effect on the deformed shape of the units, which is more complex than that assumed in their initial design.
c. As the corners of the slab tend to turn upwards (warping effect) separating themselves from the propping UB-section and the whole unit bends about two different axis (longitudinal and transverse), a 'trough' forms at the central region of the unit. The above leads to the conclusion that a 'plane of inflection' (change from concavity to convexity or vice versa) is present. It has not been possible to define the locus of this plane accurately, with the information obtained from the laboratory. This will be thoroughly examined in Part 2, given the results from a rigorous FE analysis.
d. The maximum displacement of the uncracked unit was found to be higher than the corresponding one for the cracked unit for the same load value. This was because the maximum displacement measured for the cracked unit (Test 2) was relative to the residual displacement from Test 1 and therefore it could be smaller.
e. The strain distribution across the depth of the beam was found to be linear and remarkably similar in both tests. The linearity was more evident in Test 2, as it is not accompanied by any substantial and sudden change in strain due to the formation of additional cracks. When the tension zone was developing its first cracks, equilibrium was maintained by a shift of the Neutral Axis upwards, hence decreasing the sectional area in compression.
f. Strain measured at the longitudinal reinforcement (SG2) is a good indicator of cracks appearing at the tension side of the unit. Strain measured at the lateral reinforcement (SG1) is approximately 21 times smaller than SG2. The 'strain loop' (loading-unloading) for the fully cracked section always enclosed that for the uncracked section (Figures: 21, 21A, 21B).
g. The static stiffness of the uncracked unit was found to be greater than that of the cracked unit as expected.

7. ACKNOWLEDGEMENTS

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