

Calculating flux to predict future cave radon concentrations

Rowberry, MD, Marti, X, Frontera, C, Van De Wiel, MJ & Briestenský, M

Author post-print (accepted) deposited by Coventry University's Repository

Original citation & hyperlink:

Rowberry, MD, Marti, X, Frontera, C, Van De Wiel, MJ & Briestenský, M 2016, 'Calculating flux to predict future cave radon concentrations' Journal of Environmental Radioactivity, vol 157, pp. 16-26

<http://dx.doi.org/10.1016/j.jenvrad.2016.02.023>

DOI 10.1016/j.jenvrad.2016.02.023

ISSN 0265-931X

ESSN 1879-1700

Publisher: Elsevier

NOTICE: this is the author's version of a work that was accepted for publication in Journal of Environmental Radioactivity. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Journal of Environmental Radioactivity, [157, (2016)] DOI: 10.1016/j.jenvrad.2016.02.023

© 2016, Elsevier. Licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International

<http://creativecommons.org/licenses/by-nc-nd/4.0/>

Copyright © and Moral Rights are retained by the author(s) and/ or other copyright owners. A copy can be downloaded for personal non-commercial research or study, without prior permission or charge. This item cannot be reproduced or quoted extensively from without first obtaining permission in writing from the copyright holder(s). The content must not be changed in any way or sold commercially in any format or medium without the formal permission of the copyright holders.

This document is the author's post-print version, incorporating any revisions agreed during the peer-review process. Some differences between the published version and this version may remain and you are advised to consult the published version if you wish to cite from it.

Manuscript Number: JENVRAD-D-15-00654R1

Title: Calculating flux to predict future cave radon concentrations

Article Type: Research Paper

Keywords: cave radon concentration; radon flux; cave ventilation;
radioactive decay; fault slip; numerical modelling.

Corresponding Author: Dr. Matt D. Rowberry, Ph.D.

Corresponding Author's Institution: Institute of Rock Structure and
Mechanics

First Author: Matt D. Rowberry, Ph.D.

Order of Authors: Matt D. Rowberry, Ph.D.; Xavi Martí; Carlos Frontera;
Marco J Van De Wiel ; Miloš Briestenský

Abstract: Cave radon concentration measurements reflect the outcome of a perpetual competition which pitches flux against ventilation and radioactive decay. The mass balance equations used to model changes in radon concentration through time routinely treat flux as a constant. This mathematical simplification is acceptable as a first order approximation despite the fact that it sidesteps an intrinsic geological problem: the majority of radon entering a cavity is exhaled as a result of advection along crustal discontinuities whose motions are inhomogeneous in both time and space. In this paper the dynamic nature of flux is investigated and the results are used to predict cave radon concentration for successive iterations. The first part of our numerical modelling procedure focuses on calculating cave air flow velocity while the second part isolates flux in a mass balance equation to simulate real time dependence among the variables. It is then possible to use this information to deliver an expression for computing cave radon concentration for successive iterations. The dynamic variables in the numerical model are represented by the outer temperature, the inner temperature, and the radon concentration while the static variables are represented by the radioactive decay constant and a range of parameters related to geometry of the cavity. Input data were recorded at Driny Cave in the Little Carpathians Mountains of western Slovakia. Here the cave passages have developed along splays of the NE-SW striking Smolenice Fault and a series of transverse faults striking NW-SE. Independent experimental observations of fault slip are provided by three permanently installed mechanical extensometers. Our numerical modelling has revealed four important flux anomalies between January 2010 and August 2011. Each of these flux anomalies was preceded by conspicuous fault slip anomalies. The mathematical procedure outlined in this paper will help to improve our understanding of radon migration along crustal discontinuities and its subsequent exhalation into the atmosphere. Furthermore, as it is possible to supply the model with continuous data, future research will focus on establishing a series of underground monitoring sites with the aim of generating the first real time global radon flux maps.

Calculating flux to predict future cave radon concentrations

Matt D. Rowberry¹, Xavi Martí², Carlos Frontera³, Marco J. Van De Wiel⁴, Miloš Briestenský¹

¹ Institute of Rock Structure & Mechanics, Czech Academy of Sciences, V Holešovičkách 41, 182 09 Prague 8, Czech Republic; ² Institute of Physics, Czech Academy of Sciences, Cukrovarnická 10, 162 53 Prague 6, Czech Republic; ³ Institut de Ciència de Materials de Barcelona, ICMA-B-CSIC, Campus UAB, E-08193 Bellaterra, Barcelona, Spain; ⁴ Centre for Agroecology, Water, and Resilience (CAWR), Coventry University, Priory Street, Coventry, CV1 5FB, United Kingdom.

* corresponding author: rowberry@irsm.cas.cz

Abstract: Cave radon concentration measurements reflect the outcome of a perpetual competition which pitches flux against ventilation and radioactive decay. The mass balance equations used to model changes in radon concentration through time routinely treat flux as a constant. This mathematical simplification is acceptable as a first order approximation despite the fact that it sidesteps an intrinsic geological problem: the majority of radon entering a cavity is exhaled as a result of advection along crustal discontinuities whose motions are inhomogeneous in both time and space. In this paper the dynamic nature of flux is investigated and the results are used to predict cave radon concentration for successive iterations. The first part of our numerical modelling procedure focuses on calculating cave air flow velocity while the second part isolates flux in a mass balance equation to simulate real time dependence among the variables. It is then possible to use this information to deliver an expression for computing cave radon concentration for successive iterations. The dynamic variables in the numerical model are represented by the outer temperature, the inner temperature, and the radon concentration while the static variables are represented by the radioactive decay constant and a range of parameters related to geometry of the cavity. Input data were recorded at Driny Cave in the Little Carpathians Mountains of western Slovakia. Here the cave passages have developed along splays of the NE-SW striking Smolenice Fault and a series of transverse faults striking NW-SE. Independent experimental

observations of fault slip are provided by three permanently installed mechanical extensometers. Our numerical modelling has revealed four important flux anomalies between January 2010 and August 2011. Each of these flux anomalies was preceded by conspicuous fault slip anomalies. The mathematical procedure outlined in this paper will help to improve our understanding of radon migration along crustal discontinuities and its subsequent exhalation into the atmosphere. Furthermore, as it is possible to supply the model with continuous data, future research will focus on establishing a series of underground monitoring sites with the aim of generating the first real time global radon flux maps.

Keywords cave radon concentration; cave radon flux; cave ventilation; radioactive decay; fault slip; numerical modelling.

Highlights

- Cave radon concentrations pitch flux against ventilation and radioactive decay
- Exhalation is dominated by radon advection along discontinuities such as faults
- Our numerical model isolates flux and then predicts future radon concentrations
- Independent observations confirm the close relationship between flux and fault slip

1 1. INTRODUCTION

2 Radon (^{222}Rn) is a radioactive noble gas that results from the decay of solid radium
3 (^{226}Ra). The release of radon is controlled by the alpha particle recoil mechanisms that
4 expel radon from radium. Whether a newly formed radon atom remains in the mineral
5 grain or whether it enters the intergranular pore space is determined by the position of
6 the radium atoms and the direction of radon atom recoil (Appleton 2013). The vast
7 majority of radon atoms remain within the mineral grain only to decay once again into a
8 solid product while the tiny minority that enter the intergranular pore space then begin
9 the process of migration towards the surface. Migration is controlled largely by the
10 water retention and fluid transmission characteristics of the bedrock (Åkerblom &
11 Mellander 1997). The latter include its permeability, its porosity, and its pore size
12 distribution as well as the nature of any crustal discontinuities such as faults, fractures,
13 and joints (Appleton 2013). It is far more common for radon to be emitted into a liquid
14 phase rather than into a gas phase. Radon migration in the liquid phase occurs with the
15 help of carrier fluids and the radon will remain in the liquid phase until a gas phase is
16 introduced. Clearly radon migration in the liquid phase is going to be influenced by
17 factors such as groundwater circulation whereas migration in the gas phase is going to
18 be influenced by factors such as the diffusion characteristics of the gas.

19
20 Radon is generally abundant in confined underground spaces such as caves, tunnels,
21 and mines (Stannard 1988). The numerical model presented in this paper is based on
22 input data recorded in a cave. Measurements of cave radon concentration reflect the
23 outcome of a perpetual competition which pitches flux against ventilation and
24 radioactive decay (Wilkening & Watkins 1976). In the absence of ventilation it is
25 possible for the radon concentration in such settings to approach that characteristic of
26 soil gas (Wilkening 1990). Cave radon clearly accumulates as a result of exhalation from
27 the confining rock mass but it is important to have a basic understanding of the
28 contributions made by diffusive transport and advective transport. The distances over
29 which radon atoms can be transported by diffusion are limited by the short half life of
30 radon ($t_{1/2} = 3.82$ d) while the distances over which they can be transported by
31 advection along structural discontinuities is significantly further, perhaps more than
32 one hundred metres (Appleton 2013). Exhalation by diffusion from solid limestone
33 containing $2.2 \text{ mg kg}^{-1}\text{U}$ may be expected to result in cave radon concentration

34 measurements in the order of 100 Bq/m³ (Appleton 2013). The fact that radon
35 concentration measurements in such settings are generally greater by at least one order
36 of magnitude emphasises the importance of exhalation by advection along structural
37 discontinuities. This importance may be heightened in caves compared to other
38 underground settings as their passages often develop along precisely the same faults
39 and fractures as those used for radon migration.

40

41 Faults and fractures permit the efficient transmission of radon to the surface due to the
42 fact that fluids readily migrate along such crustal discontinuities. Consequently many
43 studies have used high radon concentration measurements to infer the presence of
44 discontinuities under soil or glacial drift. High radon concentration is more likely to be
45 encountered if the discontinuities are active (Swakoń et al. 2005; Ielsch et al. 2010; Neri
46 et al. 2011). Discontinuities in the near surface environment may be thought of as active
47 if they are subjected to thermal expansion as this leads to dilation and constriction
48 whereas those at greater depths tend to be more susceptible to slip caused by either
49 gravitational or tectonic processes. The EU-TecNet fault displacement monitoring
50 network has been making direct experimental observations of fault slip at more than
51 one hundred sites across central Europe. More than a decade of data demonstrate that
52 fault motion in this intracratonic region is commonly characterised by steady
53 progressive creep trends: these may be horizontal (strike-slip), vertical (dip-slip), or a
54 combination of the two (oblique-slip). However, the steady progressive creep trends
55 are sometimes interrupted by short periods of anomalous activity, interpreted to reflect
56 a short term perturbation in the regional stress field (Stemberk et al. 2010; Košťák et al.
57 2011; Briestenský et al. 2015). During these periods the progressive creep trends may
58 be subjected to, for example, a conspicuous reversal; a sudden enduring displacement;
59 or a series of oscillatory displacements. It follows that significant displacements should
60 also be evidenced by radon anomalies especially given that numerous studies have
61 related radon concentration anomalies with other geodynamic phenomena such as
62 earthquakes (Igarashi et al. 1995; Briestenský et al. 2014; Hwa Oh & Kim 2015).

63

64 Once radon has been exhaled into a confined underground space it is then subject to the
65 processes responsible for liberating it into the atmosphere. The most comprehensive
66 recent account of underground meteorology is that of Badino (2010). Air exchange is

67 strongly influenced by convective circulation caused as a result of internal-external
68 buoyancy pressure differences and barometric circulation caused as a result of internal-
69 external pressures differences. The former is particularly important for caves with more
70 than one entrance at different heights whereas the latter is more important for caves
71 with only one entrance or for caves with only extremely small entrances. Diurnal and
72 seasonal circulation changes often result in diurnal and seasonal fluctuations in natural
73 gas and aerosol concentrations (Bezek et al. 2012). The effects of such changes on radon
74 concentration are particularly well known because radon is commonly used as a tracer
75 for cave ventilation modelling (Cunningham & Larock 1991; Hakl et al. 1997; Tanahara
76 et al. 1997; Przylibski 1999; Perrier et al. 2004; Kowalczk & Froelich 2010; Gregorič et
77 al. 2014). However, although the processes governing air exchange are well understood,
78 the mass balance equations used to model changes in radon concentration through time
79 routinely treat flux as a constant. This mathematical simplification sidesteps an intrinsic
80 geological problem: the majority of radon exhalation occurs as a result of advection
81 from crustal discontinuities whose motions are clearly inhomogeneous in both space
82 and time. In this paper the dynamic nature of flux is investigated and the results are
83 used to predict cave radon concentration for successive iterations.

84

85 **2. STUDY AREA**

86 The input data used for our numerical model were recorded at Driny Cave in the
87 geodynamically active Little Carpathian Mountains of western Slovakia (Figure 1). This
88 mountain range trends SW-NE along the southeastern margin of the Bohemian Massif
89 and forms part of the Alpine-Carpathian Orogenic Belt (Lenhardt et al. 2007). It
90 comprises a mesh of clearly defined morphostructural units (Marko et al. 1991)
91 bordered to the northwest by the Vienna Basin and to the southeast by the Pannonian
92 Basin (Plašienka et al. 1997). The range is characterised by moderate seismicity: the
93 strongest earthquake, with a magnitude of $M_s = 5.7$, occurred in the epicentral area of
94 Dobrá Voda on 9 January 1906 and was followed by a large aftershock, with a
95 magnitude of $M_s = 5.3$, on 16 January 1906 (Zsíros 2005). Other events with
96 magnitudes of greater than $M_L = 4.0$ occurred in 1904, 1930, and 1967 (Fojtíková et al.
97 2010). Notable recent earthquakes were recorded close to the town of Vrbové, with a
98 magnitude of $M_L = 3.3$, on 13 March 2006, and close to the town of Studienka, with a
99 magnitude of $M_L = 3.4$, on 5 March 2012. The stress field appears to be dominated by

100 NE-SW horizontal extension (Kováč et al. 2002) although NE-SW horizontal
101 compression has been identified around the epicentral area of Dobrá Voda (Fojtková et
102 al. 2010).

103
104 Driny Cave is located at the contact between the Little Carpathian Mountains and the
105 adjacent Pannonian Basin. The area around the cave hosts a range of slope
106 deformations while the limestone cliffs which host the entrance to the cave are
107 characterised by numerous open fissures (Briestenský et al. 2011a). The cave itself has
108 developed in the lower Cretaceous locally schistose marly limestones of the Hlboč
109 Formation (Michalík et al. 1992) as a result of corrosion by meteoric waters seeping
110 along splays of the NE-SW striking Smolenice Fault and a series of transverse faults
111 striking NW-SE (Droppa 1951). Its main entrance is situated at 399 m asl, its smaller
112 upper entrance is situated at 431 m asl, its chimney descends a total of 36 m, and its
113 passages attain a total length of 680 m (Bella 2006). Faults within the cave often exhibit
114 striations and slickencrysts. These features indicate that the splays of the NE-SW
115 striking Smolenice Fault are primarily affected by dip slip displacements while the
116 transverse faults striking NW-SE are primarily affected by strike slip displacements
117 (Briestenský et al. 2011a). Its walls are commonly covered by flowstones while the
118 ceilings and floors host numerous types of dripstone. The presence of tiny, fresh, cracks
119 in dripstones developing close to fault outcrops suggests that these crustal
120 discontinuities are still active (Briestenský et al. 2011a,b).

121
122 Three mechanical extensometers have been installed to measure fault slip in the cave.
123 The monitoring points Driny 1 and Driny 2 are installed across parallel splays of the
124 Smolenice Fault whereas Driny 3 is installed across one of the transverse faults
125 (Briestenský et al. 2011a,b). Results from Driny 1 and Driny 2 present evidence of
126 sinistral strike slip and uplift of the northwestern block whereas results from Driny 3
127 present evidence of dextral strike slip and uplift of the northeastern block (Table 1). In
128 detail, however, the data are more complicated. At Driny 1, large reversals in the sense
129 of dip slip displacement were recorded towards the end of 2009 and at the beginning of
130 2010 while dextral strike slip dominated between 2010 and 2012. At Driny 2, large
131 reversals in the sense of dip slip displacement were again recorded towards the end of
132 2009 and at the beginning of 2010 while dextral strike slip dominated slightly later,

133 between 2012 and 2013. At Driny 3, subsidence of the northeastern block was recorded
134 between 2011 and 2013 while sinistral strike slip dominated between 2010 and 2012.
135 The anomalous fault slip trends recorded between 2010 and 2013 are interpreted to
136 reflect a short term perturbation in the regional stress field (see, for example, Stemberk
137 et al. 2010; Košťák et al. 2011; Briestenský et al. 2015).

138

139 **3. EXPERIMENTAL METHODS**

140 Measurements of equilibrium equivalent radon concentration, ambient temperature,
141 and barometric pressure were obtained every thirty minutes from January 2010 to
142 August 2011 while measurements of fault slip have been recorded at Driny Cave once
143 every two weeks since January 2006. The measurements of equilibrium equivalent
144 radon concentration were obtained by a radon progeny monitor TS96¹. This was
145 installed in a distal part of the cave in which the host limestone is characterised by low
146 calculated mass activity (Štelcl et al. 2002). These instruments use a 200 mm²
147 semiconductor barrier detector to measure the alpha activity of ²¹⁸Po and ²¹⁴Po with an
148 accuracy of +/-10 %, a resolution of 2 Bq/m³, and a measuring range between 0 Bq/m³
149 and 1999 Bq/m³. EEC_{Rn} measurements were converted to radon concentration using a
150 constant radioactive equilibrium coefficient of $F = 0.5$. It should be noted that the
151 validity of applying a constant coefficient is discussed in Section 6.3. The measurements
152 of ambient temperature were obtained using two Comet System instruments R0110.
153 One was installed outside in a cleft above the lower entrance to the cave and one was
154 installed at the distal end of the cave, approximately five metres from the radon progeny
155 monitor. These instruments have an accuracy of $\pm 0.4^{\circ}\text{C}$, a resolution of 0.1°C , and a
156 measuring range between -40°C and $+80^{\circ}\text{C}$. The measurements of barometric pressure
157 were recorded by an Eijkelkamp BaroDiver. This was installed in a hydrogeological
158 borehole in the nearby epicentral area of Dobrá Voda. These instruments have a
159 resolution of ± 0.1 cm H₂O, a typical accuracy of ± 0.5 cm H₂O, and can withstand a
160 maximum pressure of 15 m H₂O. The readings are converted to give atmospheric
161 pressure in hPa. The measurements of fault slip are recorded using three mechanical
162 extensometers TM-71 (Košťák 1991). Two are installed across splays of the Smolenice
163 Fault (dip→dip direction: $70^{\circ}\rightarrow 290^{\circ}$ and $70^{\circ}\rightarrow 110^{\circ}$) while the third is installed across

¹ The TS96 was developed at the Institute of Particle & Nuclear Physics at Charles University in Prague and approved for equilibrium equivalent radon concentration monitoring by the Czech Metrology Institute.

164 one of the transverse faults (75°→040°). These instruments use the moiré phenomenon
165 of optical interference to record displacements in a three dimensional Cartesian
166 coordinate system with a resolution of ± 0.007 mm (Martí et al. 2013).

167

168 **4. NUMERICAL METHODS**

169 **4.1 Ventilation mechanisms, temperature gradients, and air flow velocities**

170 The presented numerical model assumes a system whose geometry is based on that of
171 Driny Cave (Figure 2A). The first step in the first stage of the modelling procedure
172 specifies the ventilation mechanism. In our model, as at Driny Cave, the air flow
173 direction reflects the presence of two entrances at discordant heights. The specific
174 mechanism is governed by Bernoulli's Principle. When the inner temperature is greater
175 than the outer temperature, the comparatively warm, less dense, air inside the cave is
176 forced out of the upper entrance as a result of the chimney effect. When the outer
177 temperature is greater than the inner temperature, the comparatively cold, denser, air
178 inside the cave is forced out of the lower entrance as a result of Torricelli's Law.

179

180 The second step defines an intermediate parameter, T^* , to account for the difference
181 between the temperature inside the cave, T_i and the temperature outside the cave, T_o ,
182 as illustrated in Eq. 1:

183

$$T^* = \sqrt{\frac{|\Delta T|}{T_j}} \quad (1)$$

184

185 Where: ΔT is the observed difference between the outer temperature and the inner
186 temperature ($\Delta T = T_o - T_i$) and T_j is the temperature corresponding to each ventilation
187 mechanism ($T_j = T_i$ for chimney effect and $T_j = T_o$ for Torricelli's Law). This parameter
188 is needed at various points in the numerical modelling procedure and it allows us to
189 present more concise mathematical formulas in subsequent steps.

190

191 The third step calculates air flow velocity through the system on the basis of Bernoulli's
192 Principle, irrespective of the specific ventilation mechanism, as illustrated in Eq. 2:

193

$$v = T^* \sqrt{2gH} \quad (2)$$

194

195 Where: g is gravity acceleration and H is the height difference between the uppermost
 196 and lowermost points of the cave. The modelled flow velocities for air passing through
 197 the upper entrance, if $T_i > T_o$, or the lower entrance, if $T_o > T_i$ are presented in Figure
 198 2B while an extracted air flow velocity profile is presented in Figure 2C. These
 199 demonstrate, first, that air flow ceases when there is no temperature gradient and,
 200 second, that the air flow velocity is not a linear function of the temperature gradient.

201

202 4.2 Modelling radon flux and predicting radon concentration

203 The first step in the second stage considers the seminal mass balance differential
 204 equation of Wilkening & Watkins (1976), routinely used to model cave radon
 205 concentration changes, as illustrated in Eq. 3:

206

$$\frac{dc}{dt} = \phi - \lambda \cdot c - \frac{Q}{V} \Delta c - \frac{D}{L} \Delta c \quad (3)$$

207

208 Where: ϕ is radon flux; c is radon concentration; λ is the radioactive decay constant; Q is
 209 the displaced volume of air; V is the total volume of air in the cave; D is the radon
 210 diffusion coefficient between two points separated a length L in the absence of
 211 ventilation; Δc is the difference between radon concentration measured inside and
 212 outside the cave. This term can be simplified by assuming that the radon concentration
 213 outside the cave is zero (i.e. $\Delta c = c$) and by neglecting the diffusion term because, in the
 214 absence of ventilation, radon is only transported for short distances compared to the
 215 dimensions of a typical cave.

216

217 The second step consolidates the variables related to cave geometry into one symbol,
 218 allowing us to present more concise mathematical formulas in subsequent steps, as
 219 illustrated in Eq. 4:

220

$$\gamma \equiv \frac{k_j}{l_j} \sqrt{2gH}. \quad (4)$$

221

222 Where: k_j is a coefficient from 0 to 1 that accounts for the proportion of the ceiling or
223 entrance which acts as an aperture for air flow and l_j is a length scale equal to the height
224 of the cave or the magnitude of the length of the cave. The proportion of the ceiling and
225 the height of the cave are used when ventilation is dominated by the chimney effect
226 whereas the proportion of the entrance and the magnitude of the length of the cave are
227 used when ventilation is dominated by Torricelli's Law. The influence that this variable
228 exerts on the outputs of the numerical model is discussed in Section 6.1.

229

230 The third step isolates flux in the mass balance equation in order to simulate real time
231 dependence among the variables, as illustrated in Equation 5:

232

$$\phi(t) = \lambda \cdot c + \gamma T^* \cdot c + \frac{dc}{dt} \quad (5)$$

233

234 This equation connects the perpetual competition pitching flux, the ultimate source of
235 the radon inside the cave, against ventilation. It demonstrates that the retrospective
236 analysis of flux requires only routinely available data: radon concentration, inner
237 temperature, and outer temperature.

238

239 The fourth step delivers an expression to compute radon concentration for successive
240 time instants. It is possible to predict radon concentration, c , for successive time
241 instants, t , using the preceding radon concentration, c_0 , and the ambient temperatures,
242 as illustrated in Equation 6:

243

$$c \approx c_0 + t \cdot (\phi - c_0(\lambda + \gamma T^*)) \quad (6)$$

244

245 As in the mass balance differential equation of Wilkening & Watkins (1976), flux is
246 thought of as favourable, the term preceded by a plus sign, while ventilation and
247 radioactive decay are thought of as unfavourable, the terms preceded by a minus sign.
248 The relevance of this equation is illustrated by a simulation presented in the following
249 section while a flow diagram summarising the outlined numerical modelling procedure
250 is presented in Figure 3.

251

252 **5. MODEL IMPLEMENTATION**

253 An illustrative numerical simulation is presented in Figure 4 to demonstrate the
254 relevance of Equation 6. Figure 4A outlines a hypothetical ventilation scenario, Figure
255 4B outlines a hypothetical flux scenario, and Figure 4C presents iterative estimations of
256 future radon concentration. The first half of the numerical simulation is characterised
257 by an increase and decrease in the air flow velocity while the second half is
258 characterised by a complete absence of ventilation. Flux remains constant throughout
259 this period except at those times when it is interrupted by a spontaneous anomaly.
260 During the first half of the simulation the radon concentration decreases rapidly as
261 ventilation successfully counteracts flux. Furthermore, the first pair of flux anomalies
262 clearly influence the radon concentration. During the second half of the simulation the
263 radon concentration increases steadily as ventilation is no longer able to counteract
264 flux. However, the second pair of flux anomalies do not clearly influence the radon
265 concentration. This simulation suggests that radon concentration is not necessarily a
266 reliable indicator of radon flux.

267
268 Figure 5A-C presents time series of inner temperature, T_i , outer temperature, T_o , and
269 radon concentration, c , recorded at Driny Cave from January 2010 to August 2011. It is,
270 unfortunately, not possible for us to extend this term as it represents the only period in
271 which the input data were recorded simultaneously. Figure 5D maps the relationship
272 between measured radon concentration, c , and the outer temperature, T_o , by plotting
273 the number of events that represent a single pair of the variables. The vertical line to the
274 left denotes the freezing point of water while the vertical line to the right denotes no
275 temperature gradient. This panel reveals a complex, nonlinear, seasonal dependency as
276 higher radon concentration pairs with both lower outer temperatures during the
277 winter, when cave ventilation is dominated by the chimney effect, and higher outer
278 temperatures during the summer, when ventilation follows Torricelli's Law. The
279 assumption that ventilation ceases in the absence of a temperature gradient and,
280 consequently, increases radon concentration is not observed in our model. Far from
281 being a maximum, at $T_o \approx T_i$ the number of events that represent a single pair of the
282 variables appears to be close to a minimum.

283

284 Given that cave radon concentration measurements reflect the outcome of a perpetual
285 competition which pitches flux against ventilation and radioactive decay then the
286 aforementioned contradiction adds further weight to the idea that flux is not a constant.
287 Figure 5E maps the relationship between measured radon concentration, c , and the
288 intermediate parameter, T^* , by plotting the number of events that represent a single
289 pair of the variables. The intermediate parameter, T^* , captures the majority of the
290 experimental data, depicted by warmer colours, in a manner that cannot be attained by
291 linear combinations. This panel, first, demonstrates the close relationship between
292 radon concentration and the temperature gradient and, second, suggests that greater
293 radon concentration measurements pair with faster ventilation rates. The second point
294 is worthy of elaboration. Irrespective of whether radon migrates by diffusion or
295 advection, its ultimate source has to be the confining rock mass, so the suggestion that
296 greater radon concentration measurements pair with faster ventilation rates implies
297 that ventilation is able to act as a pump which draws radon into the cave. It confirms the
298 nonlinear dependence between flux and the temperature gradient.

299

300 Figure 6 interrogates flux modelled as a function of the input data obtained at Driny
301 Cave. Figure 6A maps the relationship between the measurements of radon
302 concentration, c , and the modelled flux, φ , by plotting the number of events that
303 represent a single pair of the variables. This panel demonstrates, first, the close
304 relationship which exists between the measurements of radon concentration and the
305 modelled flux and, second, that the modelled flux is characterised by a generally narrow
306 set of values. The second point is once again worthy of elaboration. There appears to be
307 a reasonable correspondence between the flux values we have obtained through our
308 modelling and those used as constants during cave ventilation modelling (see, for
309 example, Gregorič et al. 2011). It emphasises that this mathematical simplification is
310 acceptable as a first approximation. Figure 6B plots modelled flux as a function of time.
311 This panel shows a series of flux anomalies rising above a seemingly oscillatory
312 background. The four most significant anomalies occurred in January 2010, October
313 2010, November 2010, and March 2011. In the following section the timing of these
314 anomalies is discussed in relation to independent experimental observations of fault
315 slip.

316

317 Figure 7 compares measurements of radon concentration and radon concentration
318 predictions for three different time horizons made using Equation 6. The input data are
319 presented in Figure 5 and the flux is obtained using Equation 5. Figure 7A-C presents
320 data pertaining to a thirty minute time horizon, Figure 7D-F presents data pertaining to
321 an eight hour time horizon, and Figure 7G-I presents data pertaining to a twenty four
322 time horizon. For each time horizon the measurements of radon concentration and
323 radon concentration predictions are compared as a function of time; the relationship
324 between the measurements of radon concentration and radon concentration
325 predictions are mapped; and the difference between the measurements of radon
326 concentration and radon concentration predictions are plotted - positive residuum
327 indicates overestimation of the radon concentration while negative residuum indicates
328 its underestimation. It is clear that there are close relationships between the
329 measurements of radon concentration and radon concentration predictions. With a
330 thirty minute time horizon our radon concentration predictions never deviate from the
331 measured values by more than 100 Bq/m³ while with a twenty four hour time horizon
332 our radon concentration predictions rarely deviate from the measured values by more
333 than 1000 Bq/m³. It is also clear that overestimating the predicted radon concentration
334 is more common than underestimating it.

335

336 **6. DISCUSSION**

337 **6.1 The variable γ**

338 The variable γ was introduced in the second stage of the numerical model to consolidate
339 all the parameters related to cave geometry. It accounts for the proportion of the ceiling
340 or entrance which acts as an aperture for air flow and incorporates a length scale equal
341 to the height of the cave or the magnitude of the length of the cave. Three plots of flux
342 modelled as a function of the input data from Driny Cave are presented in order to
343 interrogate the extent to which our model is sensitive to the parametric inputs used to
344 define the variable γ (Figure 8). In all three cases the height is fixed to forty metres, as
345 at Driny Cave, and the length is fixed to one hundred metres. These parameters have
346 been fixed because they are generally easier to estimate than the proportion of the
347 ceiling or entrance which acts as an aperture for air flow. The first case estimates that
348 the proportion of the ceiling or entrance which acts an aperture for air flow equates to
349 0.1 % (Figure 8A). The second case estimates that the proportion of the ceiling or

350 entrance which acts an aperture for air flow equates to 1 % (Figure 8B). The third case
351 estimates that the proportion of the ceiling or entrance which acts an aperture for air
352 flow equates to 10 % (Figure 8C). It should be noted that the values used for our
353 numerical model are very similar to those presented in the second case (cf. Figure 6B &
354 Figure 8B).

355

356 Figure 8 demonstrates, first, that the flux anomalies occur at precisely the same times
357 irrespective of the proportion of the ceiling or entrance assigned to act as an aperture
358 for air flow and, second, that the magnitude of the flux anomalies remains comparable
359 irrespective of the proportion of the ceiling or entrance assigned to acts as an aperture
360 for air flow. The most conspicuous changes relate to the magnitude of the flux in the
361 oscillatory background but such changes are not relevant in the context of the present
362 study. Inspection of the parameters consolidated by γ reveals that its range must fall
363 between $\gamma = 0$, if air flow is completely impeded, and $\gamma \approx 1$, if the entirety of the
364 entrance or ceiling is completely open (this scenario is clearly a physical impossibility).
365 Typical cave geometries will result in γ being ascribed a value ranging from
366 approximately 0.01 to approximately 0.0001. If the ascribed value is smaller than the
367 actual value then this would underestimate the role played by ventilation whereas if the
368 ascribed value is greater than the actual value then this would underestimate the roles
369 played by flux and radioactive decay. The comparison presented here demonstrates that
370 the use of approximations will not fundamentally change the results of the numerical
371 modelling, as long as completely unrealistic values are not ascribed. The variable γ ,
372 which itself has no physical meaning, simply acts as a scaling coefficient.

373

374 **6.2 Flux anomalies and fault slip**

375 The validation of our numerical model is hindered because flux cannot be measured
376 directly and measurements of radon concentration are needed as inputs in the model.
377 To overcome this problem we compare our flux anomalies to independent experimental
378 observations of fault slip recorded by three mechanical extensometers in Driny Cave.
379 This site is one of more than one hundred which comprise the European fault
380 displacement monitoring network EU-TecNet. The extensometers measure
381 displacement in a three dimensional Cartesian coordinate system: the x -coordinate
382 represents dilation across the fault; the y -coordinate represents strike slip

383 displacements; and the z -coordinate represents dip slip displacement. Typically the
384 strike slip and dip slip displacements at any given monitoring point are characterised by
385 progressive creep trends. These are sometimes interrupted by shorter periods of
386 anomalous activity during which one or both of the slip components may be affected by,
387 for example, conspicuous reversals in the progressive creep trend; a sudden enduring
388 displacement; or a series of oscillatory displacements (see, for example, Briestenský et
389 al. 2015). It is important to note that parallel faults tend to move simultaneously but
390 their sense of movement may differ while perpendicular faults may or may not interact
391 with one another.

392

393 The four most significant flux anomalies occurred in January 2010, October 2010,
394 November 2010, and March 2011. Figure 9A presents the strike slip and dip slip
395 displacements from Driny 1; Figure 9B presents the strike slip and dip slip
396 displacements from Driny 2; Figure 9C presents the strike slip and dip slip
397 displacements from Driny 3. The anomaly in January 2010 is reflected by a significant
398 dip slip displacement at Driny 1 and a significant strike slip displacement at Driny 2.
399 The anomalies in October 2010 and November 2010 are reflected by a series of dip slip
400 displacements, incorporating reversals in the sense of movement, at Driny 3. The
401 anomaly in March 2011 is reflected by a significant dip slip displacements at Driny 3. In
402 all instances the anomalous fault displacements preceded the modelled flux anomaly. It
403 has been proposed that fault slip at Driny Cave reflects both slope deformation and
404 tectonic deformation (Briestenský et al. 2011a). Irrespective of the specific geodynamic
405 process, it is clear that fault slip plays an important role in facilitating radon migration
406 along discontinuities in the crust. The novelty of our experimental design should not be
407 overlooked. It is only possible for us to compare our flux anomalies to experimental
408 observations of fault slip because, first, we monitor faults with different orientations
409 and, second, we are able to record displacements in a three dimensional Cartesian
410 coordinate system.

411

412 **6.3 Using the numerical model**

413 To summarise, the presented numerical model is straightforward to use, as illustrated
414 by Figure 3. The dynamic variables are represented by the outer temperature, the inner
415 temperature, and the radon concentration while the static variables are represented by

416 the radioactive decay constant and a range of parameters related to geometry of the
417 cavity. In Section 3 it was stated that the EEC_{Rn} measurements recorded by the radon
418 progeny monitor had been converted into radon concentration by applying a constant
419 radioactive equilibrium coefficient of $F = 0.5$. It is accepted that the use of a constant
420 equilibrium coefficient does introduce uncertainties into the reported radon
421 concentration. This approach, however, is justified on the following grounds. The focal
422 point of the presented study is the mass balance equation and, in particular, the
423 dynamic nature of the flux term. In this context, the precise calculation of radon
424 concentration is less important than being able to identify the general patterns. If we
425 were to apply a dynamic radioactive equilibrium coefficient then each of the
426 equilibrium equivalent concentration measurements is effectively subjected to a
427 different scaling coefficient. This may serve to modulate the amplitude of a flux anomaly
428 but it cannot change the timing of an anomaly nor can it create or eradicate flux
429 anomalies. In this regard it is important to recall that the four most significant flux
430 anomalies rise above baseline values by a factor of approximately 4. Furthermore, the
431 close relationship between our modelled flux anomalies and independent observations
432 of fault slip also indicates that our use of a constant radioactive equilibrium coefficient
433 is appropriate.

434

435 The presented numerical modelling procedure incorporates a small number of
436 assumptions. First, that air flow velocity is driven solely by the temperature gradient,
437 second, that radon diffusion is negligible and, third, that flux is spatially uniform. It
438 would be possible to develop and tune more sophisticated versions of the numerical
439 model so that it is better able to account for site specific conditions including those
440 encountered in other underground spaces such as tunnels or mines. For example, in
441 certain situations, it may be important to incorporate a time lag between radon
442 concentration measurements and ambient temperature measurements; it may be
443 important to incorporate differences in hydrostatic pressure recorded inside and
444 outside the cave; and it may be important to reintroduce the diffusion term back into
445 the mass balance equation at sites where the distance from any given entrance to the
446 radon monitoring point is small. There are many advantages to being able to produce
447 detailed numerical models that can precisely replicate a wide range of variables. The
448 ultimate aim of this research, however, is to establish a network of monitoring points in

449 order to deliver the first real time global radon flux maps. This turns out to be feasible
450 because we can isolate flux in the mass balance equation and compute radon
451 concentration for successive iterations using only routinely available data: radon
452 concentration, inner temperature, and outer temperature. The synchronised analysis of
453 data obtained from across extensive geographical areas would illuminate our
454 understanding of radon migration along crustal discontinuities and our understanding
455 of its subsequent exhalation into the atmosphere.

456

457 **7. CONCLUSIONS**

458 The dynamic nature of radon flux has been investigated in this paper. The first part of
459 the presented numerical modelling procedure focuses on calculating cave air flow
460 velocity while the second part isolates flux in a mass balance equation to simulate real
461 time dependence among the variables. With this information it was then possible to
462 deliver an expression for computing cave radon concentration for successive iterations.
463 Input data for the model were recorded at Driny Cave in the geodynamically active
464 Little Carpathians Mountains of western Slovakia. Here the cave passages have
465 developed along faults striking broadly NE-SW and NW-SE. Independent experimental
466 observations of fault slip were provided by three permanently installed mechanical
467 extensometers. Our numerical modelling has revealed four important flux anomalies
468 while extensometric measurements demonstrate that each of these was preceded by
469 conspicuous fault slip anomalies. The mathematical procedure outlined in this paper
470 will help to improve our understanding of radon migration along crustal discontinuities
471 and its subsequent exhalation into the atmosphere at the site specific scale.
472 Furthermore, as it is possible to supply the model with continuous data, future research
473 will focus on establishing a series of underground monitoring sites with the aim of
474 generating the first real time global radon flux maps.

475

476 **ACKNOWLEDGEMENTS**

477 The authors would like to thank Peter Zvonár, Sara Argerich-Bergada, Amanda Keen-
478 Zebert, Lenka Thinová, and Petr Otáhal as well as the reviewers whose constructive
479 comments have helped to improve the clarity of the manuscript. This study was
480 conducted with support from the long term conceptual development research
481 organisation RVO: 67985891. It is published in the framework of CzechGeo-EPOS

482 “Distributed system of permanent observatory measurements and temporary
483 monitoring of geophysical fields in the Czech Republic” (MŠMT Project: LM2010008).

484

485 REFERENCES

486 Appleton, J.D., 2013. Radon in Air and Water, in: Selinus, O. (Ed.), *Essentials of Medical*
487 *Geology*. Springer, Dordrecht, pp. 239–277.

488 Åkerblom, G., Mellander, H., 1997. *Geology and Radon*, in: Durrani, S.A., Ilić, R. (Eds.),
489 *Radon Measurements by Etched Track Detectors*. World Scientific, River Edge, pp.
490 21–49.

491 Badino, G., 2012. *Underground meteorology - “What’s the weather underground?”*. *Acta*
492 *Carsologica* 39, 427–448. doi: 10.3986/ac.v39i3.74.

493 Bella, P., 2006. *A contribution to the morphology and genesis of Driny Cave*. *Aragonit* 11,
494 4–9.

495 Bezek, M., Gregorič, A., Kávási, N., Vaupotič, J., 2012. *Diurnal and seasonal variations of*
496 *concentration and size distribution of nano aerosols (10-1100 nm) enclosing*
497 *radon decay products in the Postojna Cave, Slovenia*. *Radiat. Prot. Dosim.* 152,
498 174–178. doi: 10.1093/rpd/ncs218.

499 Briestenský, M., Rowberry, M.D., Stemberk, J., Stefanov, P., Vozár, J., Šebela, S., Petro, L.,
500 Bella, P., Gaal, L., 2015. *Evidence of a plate-wide tectonic pressure pulse provided*
501 *by extensometric monitoring in the Balkan Mountains (Bulgaria)*. *Geol. Carpath.*
502 66, 427–438. doi: 10.1515/geoca-2015-0035.

503 Briestenský, M., Stemberk, J., Michalík, J., Bella, P., Rowberry, M.D., 2011a. *The use of a*
504 *karstic cave system in a study of active tectonics: fault movements recorded at*
505 *Driny Cave, Malé Karpaty Mts (Slovakia)*. *J. Cave Karst Stud.* 73, 114–123. doi:
506 10.4311/jcks2010es0166.

507 Briestenský, M., Thinová, L., Stemberk, J., Rowberry, M.D., 2011b. *The use of caves as*
508 *observatories for recent geodynamic activity and radon gas concentrations in the*
509 *Western Carpathians and Bohemian Massif*. *Radiat. Prot. Dosim.* 145, 166–172.
510 doi: 10.1093/rpd/ncr080.

511 Briestenský, M., Thinová, L., Praksová, R., Stemberk, J., Rowberry, M.D., Knejřlová, Z.,
512 2014. *Radon, carbon dioxide, and fault displacements in central Europe related to*
513 *the Tōhoku Earthquake*. *Radiat. Prot. Dosim.* 160, 78–82. doi:
514 10.1093/rpd/ncu090.

515 Cunningham, K.I., Larock, E.J., 1991. Recognition of microclimate zones through radon
516 mapping, Lechuguilla Cave, Carlsbad Caverns National Park, New Mexico. *Health*
517 *Phys.* 61, 493–500.

518 Droppa, A., 1951. Smolenice Karst in the Little Carpathians. *Zemepisný Sb.* 3, 7–52.

519 Fojtíková, L., Vavryčuk, V., Cipciar, A., Madarás, J., 2010. Focal mechanisms of micro-
520 earthquakes in the Dobrá Voda seismoactive area in the Malé Karpaty Mts. (Little
521 Carpathians), Slovakia. *Tectonophysics* 492, 213–229. doi:
522 10.1016/j.tecto.2010.06.007.

523 Gregorič, A., Vaupotič, J., Šebela, S., 2014. The role of cave ventilation in governing cave
524 air temperature and radon levels (Postojna Cave, Slovenia). *Int. J. Climatol.* 34,
525 1488–1500. doi: 10.1002/joc.3778.

526 Gregorič, A., Zidanšek, A., Vaupotič, J., 2011. Dependence of radon levels in Postojna
527 Cave on outside air temperature. *Nat. Hazards Earth Syst. Sci.* 11, 1523–1528. doi:
528 doi:10.5194/nhess-11-1523-2011.

529 Hakl, J., Hunyadi, I., Csige, I., Géczy, G., Lénárt, L., Várhegyi, A., 1997. Radon transport
530 phenomena studied in karst caves - international experiences on radon levels and
531 exposures. *Radiat. Meas.* 28, 675–684. doi: 10.1016/S1350-4487(97)00163-7.

532 Hwa Oh, Y., Kim, G., 2015. A radon-thoron isotope pair as a reliable earthquake
533 precursor. *Sci. Rep.* 5, 13084. doi: 10.1038/srep13084.

534 Ielsch, G., Cushing, M.E., Combes, P., Cuney, M., 2010. Mapping of the geogenic radon
535 potential in France to improve radon risk management: methodology and first
536 application to region Bourgogne. *J. Environ. Radioact.* 101, 813–820. doi:
537 10.1016/j.jenvrad.2010.04.006.

538 Igarashi, G., Saeki, S., Takahata, N., Sumikawa, K., Tasaka, S., Sasaki, Y., Takahashi, M.,
539 Sano, Y., 1995. Ground-water radon anomaly before the Kobe Earthquake in Japan.
540 *Science* 269, 60–61. doi: 10.1126/science.269.5220.60.

541 Košťák, B., 1991. Combined Indicator Using Moiré Technique, in: Sorum, G. (Ed.), *Field*
542 *Measurements in Geomechanics*. Balkema, Rotterdam, pp. 53–60.

543 Košťák, B., Mrlina, J., Stemberk, J., Chán, B., 2011. Tectonic movements monitored in the
544 Bohemian Massif. *J. Geodyn.* 52, 34–44. doi: 10.1016/j.jog.2010.11.007.

545 Kowalczyk, A.J., Froelich, P.N., 2010. Cave air ventilation and CO₂ outgassing by radon-
546 222 modeling: how fast do caves breathe? *Earth Planet. Sci. Lett.* 289, 209–219.
547 doi: 10.1016/j.epsl.2009.11.010.

548 Kováč, M., Bielik, M., Hók, J., Kronome, B., Labák, P., Moczo, P., Plašienka, D., Šefara, J.,
549 Šujan, M., 2002. Seismic activity and neotectonic evolution of the Western
550 Carpathians (Slovakia). EGU Stephen Mueller Spec. Publ. Ser. 3, 167–184.

551 Lenhardt, W.A., Švancara, J., Melichar, P., Pazdírková, J., Havíř, J., Sýkorová, Z., 2007.
552 Seismic activity of the Alpine-Carpathian-Bohemian Massif region with regard to
553 geological and potential field data. Geol. Carpath. 58, 397–412.

554 Marko, F., Fodor, L., Kováč, M., 1991. Miocene strike-slip faulting and block rotation in
555 Brezovské Karpaty Mts. Miner. Slovaca 23, 189–200.

556 Marti, X., Rowberry, M.D., Blahút, J., 2013. A MATLAB® code for counting the moiré
557 interference fringes recorded by the optical-mechanical crack gauge TM-71.
558 Comput. Geosci. 52, 164–167. doi: 10.1016/j.cageo.2012.09.029.

559 Michalík, J., Reháková, D., Marko, F., 1992. Stratigraphy and tectonics of the Lower
560 Cretaceous limestone sequence in the profile of Driny Cave (Vysoká Unit, Malé
561 Karpaty Mts). Miner. Slovaca 24, 235–243.

562 Neri, M., Giammanco, S., Ferrera, E., Patanè, G., Zanon, V., 2011. Spatial distribution of
563 soil radon as a tool to recognize active faulting on an active volcano: the example
564 of Mt. Etna (Italy). J. Environ. Radioact. 102, 863–870. doi:
565 10.1016/j.jenvrad.2011.05.002.

566 Perrier, F., Richon, P., Crouzeix, C., Morat, P., Le Mouél, J.-L., 2004. Radon-222 signatures
567 of natural ventilation regimes in an underground quarry. J. Environ. Radioact. 71,
568 17–32. doi: 10.1016/S0265-931X(03)00117-6.

569 Plašienka, D., Grecula, P., Putiš, M., Kováč, M., Hovorka, D., 1997. Evolution and Structure
570 of the Western Carpathians: an overview, in: Grecula, P., Hovorka, D., Putiš, M.
571 (Eds.), Geological Evolution of the Western Carpathians. Mineralia Slovaca
572 Monograph, Bratislava, pp. 1–24.

573 Przylibski, T.A., 1999. Radon concentration changes in the air of two caves in Poland. J.
574 Environ. Radioact. 45, 81–94. doi: 10.1016/S0265-931X(98)00081-2.

575 Stannard, J.N., 1988. Radioactivity and Health, Springfield, Pacific Northwest
576 Laboratory.

577 Stemberk, J., Košťák, B., Cacoń, S., 2010. A tectonic pressure pulse and increased
578 geodynamic activity recorded from the long-term monitoring of faults in Europe.
579 Tectonophysics 487, 1–12. doi: 10.1016/j.tecto.2010.03.001.

580 Swakoń, J., Kozak, K., Paszkowski, M., Gradziński, R., Łoskiewicz, J., Mazur, J., Janik, M.,
581 Bogacz, J., Horwacik, T., Olko, P., 2005. Radon concentration in soil gas around
582 local disjunctive tectonic zones in the Krakow area. *J. Environ. Radioact.* 78, 137–
583 149. doi: 10.1016/j.jenvrad.2004.04.004.

584 Štelcl, J., Zimák, J., Zelinka, J., 2002. Rock mass radioactivity in Driny Cave. *Aragonit* 7,
585 25–27.

586 Tanahara, A., Taira, H., Takemura, M., 1997. Radon distribution and the ventilation of a
587 limestone cave on Okinawa. *Geochem. J.* 31, 49–56.

588 Wilkening, M.H., 1990. *Radon in the Environment*, Elsevier, Amsterdam.

589 Wilkening, M.H., Watkins, D.E., 1976. Air exchange and ²²²Rn concentrations in
590 Carlsbad Caverns. *Health Phys.* 31, 139–145.

591 Zsíros, T., 2005. Seismicity of the Western-Carpathians. *Acta Geod. Geophys. Hung.* 40,
592 455–467. doi: 10.1556/AGeod.40.2005.3-4.15.

593 **FIGURE CAPTIONS**

594

595 **Figure 1** The location of Driny Cave in the Little Carpathians Mountains of western
596 Slovakia. This range trends SW-NE along the southeastern margin of the
597 Bohemian Massif and forms part of the Alpine-Carpathian Orogenic Belt. The
598 cave passages have developed along splays of the NE-SW striking Smolenice
599 Fault, which hereabouts marks the contact between the Little Carpathians
600 and the Pannonian Basin, and a series of transverse faults striking NW-SE.
601 Detailed information about the cave and its layout is presented in Michalík et
602 al. (1992) and Bella (2006).

603

604 **Figure 2** (a) A schematic representation of a cave with an upper entrance and a lower
605 entrance modelled with a height, H , a temperature, T_i , a radon concentration,
606 c , the latter provided by spontaneous flux, ϕ . The outer temperature, T_o , and
607 atmospheric pressure, P_{atm} , are also labelled while the arrows denote air
608 flow at velocities V . (b) Plot of the modelled flow velocities for air passing
609 through either the upper entrance, if $T_i > T_o$, or the lower entrance, if $T_o > T_i$.
610 (c) Profile extracted from the modelled air flow velocities which emphasises,
611 first, that air flow ceases when there is no temperature gradient and, second,
612 that the air flow velocity is not a linear function of the temperature gradient.

613

614 **Figure 3** Flow diagram summarising the numerical modelling procedure needed to
615 isolate flux in the mass balance equation and to compute the radon
616 concentration for successive time instants. The dynamic variables in the
617 model are represented by the outer temperature, the inner temperature, and
618 the radon concentration while the static variables are represented by the
619 radioactive decay constant and a range of parameters related to geometry of
620 the cavity.

621

622 **Figure 4** An illustrative numerical simulation demonstrating the relevance of
623 Equation 6. (a) A hypothetical ventilation scenario over an arbitrary period.
624 The first half of this period is characterised by increase and decrease in the
625 air flow velocity while the second half is characterised by a complete absence

626 of ventilation. (b) A hypothetical flux scenario over the same arbitrary
627 period during which constant flux is interrupted by four spontaneous
628 anomalies. (c) The iterative estimations of radon concentration over the
629 same arbitrary period as calculated using Equation 6.

630

631 **Figure 5** (a-c) Time series of inner temperature, T_i , outer temperature, T_o , and radon
632 concentration, c , recorded at Driny Cave for the period January 2010 to
633 August 2011. (d) Plot mapping measurements of radon concentration, c ,
634 against outer temperature, T_o , in which the vertical line on the left denotes
635 the freezing point of water and the vertical line on the right denotes no
636 temperature gradient. (e) Plot mapping measurements of radon
637 concentration, c , against the intermediate parameter, T^* , in which the red
638 dashed line emphasises the strong relationship between the measured radon
639 concentration and the temperature gradient. The colour scale represents the
640 number of times an event reoccurs on the corresponding ordinate and
641 abscissa pairing.

642

643 **Figure 6** (a) Plot mapping radon concentration for successive time instants against
644 flux modelled as a function of the input data recorded at Driny Cave from
645 January 2010 to August 2011. (b) Flux through time modelled as a function
646 of the input data recorded at Driny Cave from January 2010 to August 2011.
647 This plot shows four significant anomalies in January 2010, October 2010,
648 November 2010, and March 2011.

649

650 **Figure 7** (a) Measured radon concentration, indicated by red dots, compared to radon
651 concentration predictions made with a thirty minute time horizon using Eq.
652 6, indicated by a blue line. (b) Plot mapping measured radon concentration
653 against radon concentration predictions made with a thirty minute time
654 horizon. (c) The difference between the radon concentration predictions
655 made with a thirty minute time horizon and the *a posteriori* measured radon
656 concentration. (d) Measured radon concentration, indicated by red dots,
657 compared to radon concentration predictions made with an eight hour time
658 horizon using Eq. 6, indicated by a blue line. (e) Plot mapping measured

659 radon concentration against radon concentration predictions made with an
660 eight hour time horizon. (f) The difference between the radon concentration
661 predictions made with an eight hour time horizon and the *a posteriori*
662 measured radon concentration. (g) Measured radon concentration, indicated
663 by red dots, compared to radon concentration predictions made with a
664 twenty four hour time horizon using Eq. 6, indicated by a blue line. (h) Plot
665 mapping measured radon concentration against radon concentration
666 predictions made with a twenty four time horizon. (i) The difference
667 between the radon concentration predictions made with a twenty four time
668 horizon and the *a posteriori* measured radon concentration.

669

670 **Figure 8** Flux modelled through time as a function of the input data in order to
671 interrogate the extent to which our model is sensitive to the parametric
672 inputs. (a) The proportion of the ceiling or entrance which acts an aperture
673 for air flow equates to 0.1 %. (b) The proportion of the ceiling or entrance
674 which acts an aperture for air flow equates to 1 %. (c) The proportion of the
675 ceiling or entrance which acts an aperture for air flow equates to 10 %. In all
676 three cases the cave height is fixed to 40 m and the cave length is fixed to 100
677 m.

678

679 **Figure 9** Time series of direct experimental observations of fault slip recorded across
680 three faults in Driny Cave. The red lines indicate strike slip displacement; the
681 blue lines indicate dip slip displacement; and the grey areas denote periods
682 of three weeks either side of the modelled flux anomalies. (a) The data
683 recorded at monitoring point Driny 1 show a significant dip slip
684 displacement before the flux anomaly in January 2010. (b) The data
685 recorded at monitoring point Driny 2 show a significant strike slip
686 displacement before the flux anomaly in January 2010. (c) The data recorded
687 at monitoring point Driny 3 show significant strike slip displacements before
688 the flux anomalies in October 2010, November 2010, and March 2011. The
689 radon progeny monitor was located in the immediate vicinity of monitoring
690 points Driny 2 and Driny 3. More detailed information about this research is
691 presented in Briestenský et al. (2011a,b).

692 **TABLES**

693 **Table 1** A summary of the fault slip trends recorded at the three monitoring points in
 694 Driny Cave over a nine year period between January 2006 and January 2015.

695

Monitoring point	Monitored fault	Total strike slip (mm)	Annual slip rate (mm yr ⁻¹)	Sense of movement	Total dip slip (mm)	Annual slip rate (mm yr ⁻¹)	Sense of movement
Driny 1	Smolenice Fault (splay)	0.302	0.034	Sinistral	0.422	0.047	NW block uplift
Driny 2	Smolenice Fault (splay)	0.176	0.019	Sinistral	0.11	0.012	NW block uplift
Driny 3	Transverse Fault (unnamed)	0.223	0.025	Dextral	0.171	0.017	NE block uplift

696

Figure 1
[Click here to download high resolution image](#)

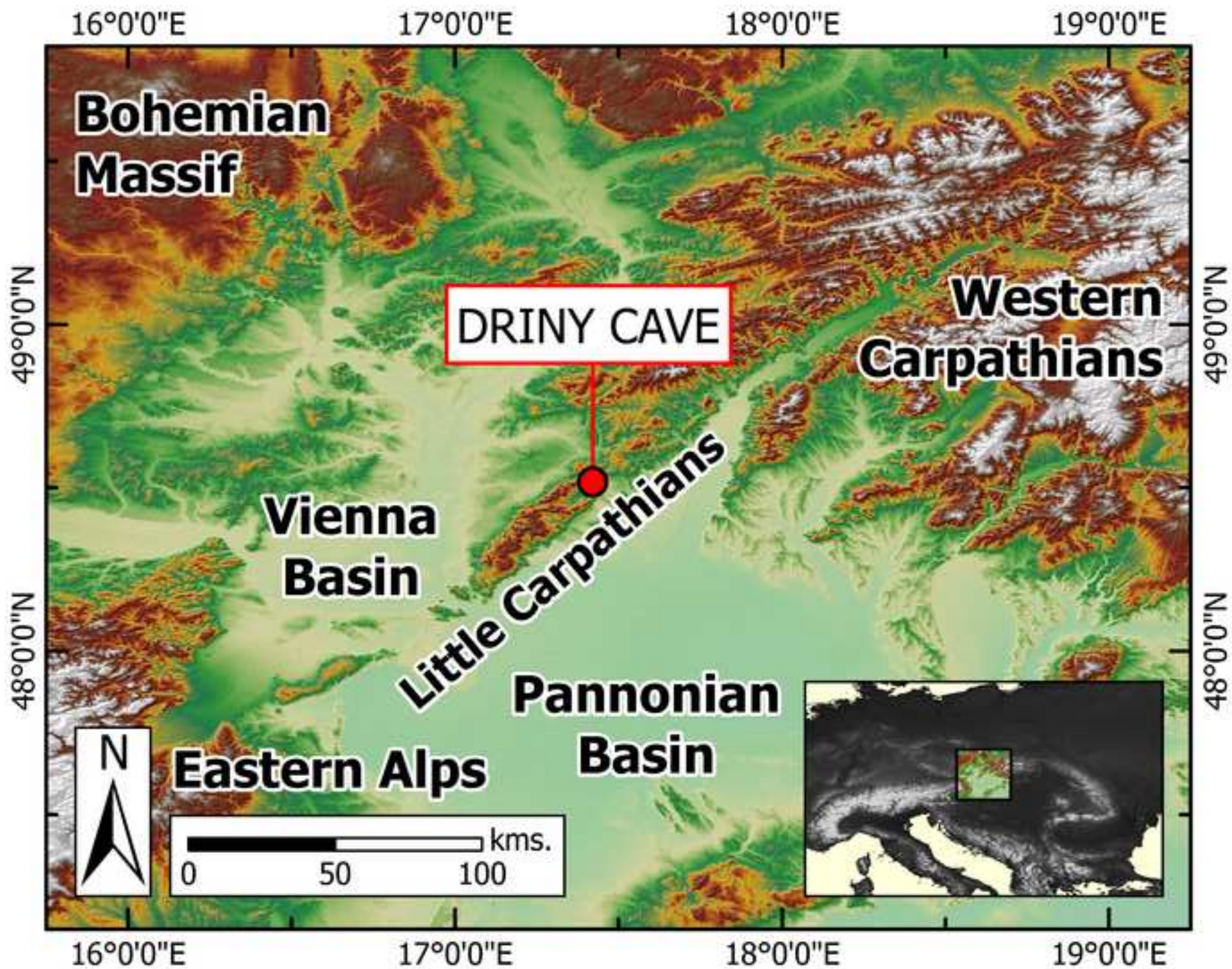


Figure 2

[Click here to download high resolution image](#)

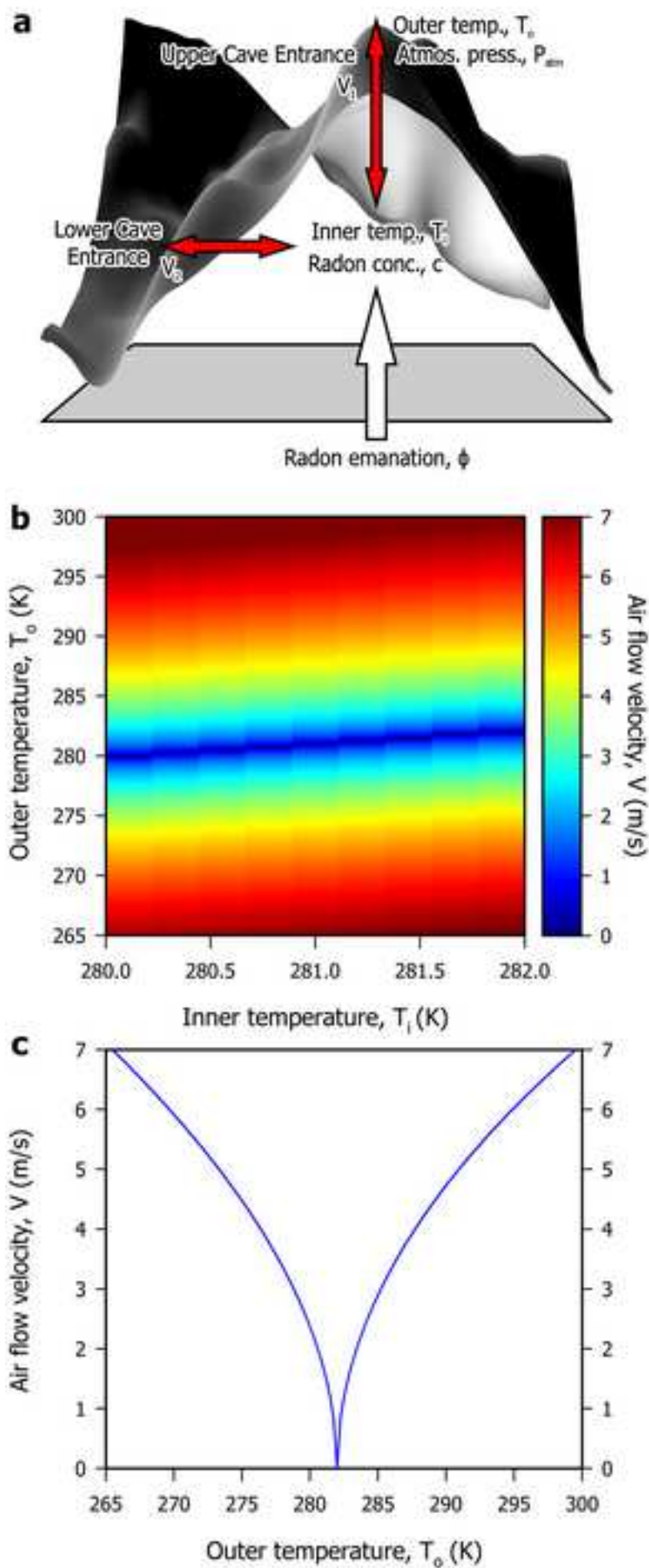


Figure 3
[Click here to download high resolution image](#)

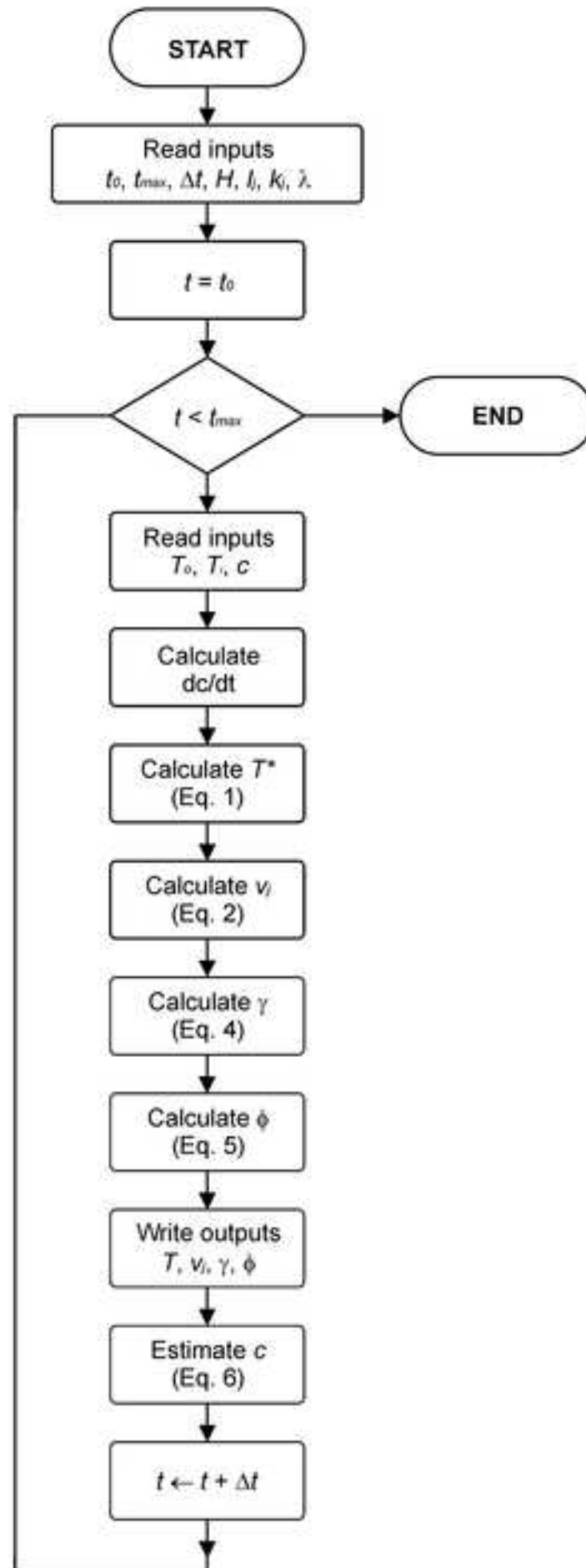


Figure 4
[Click here to download high resolution image](#)

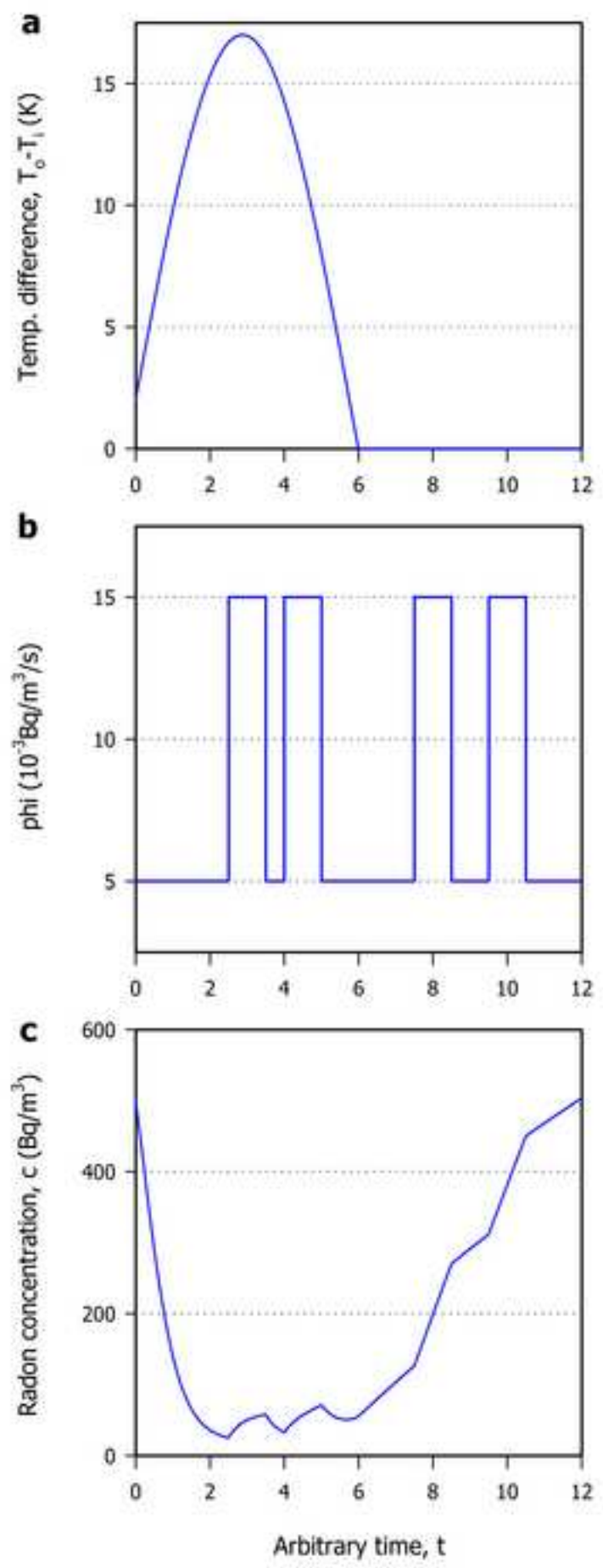


Figure 5
[Click here to download high resolution image](#)

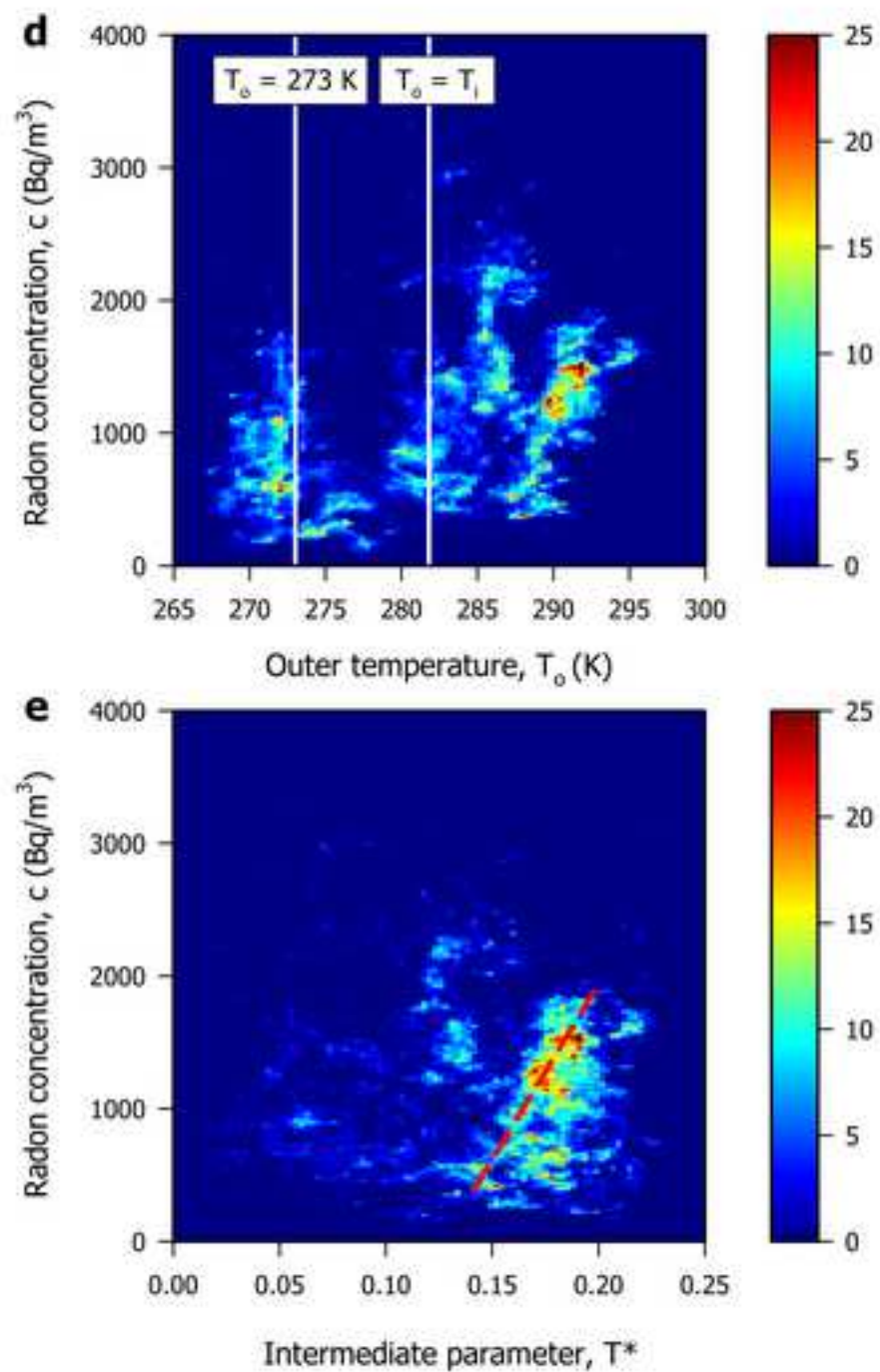
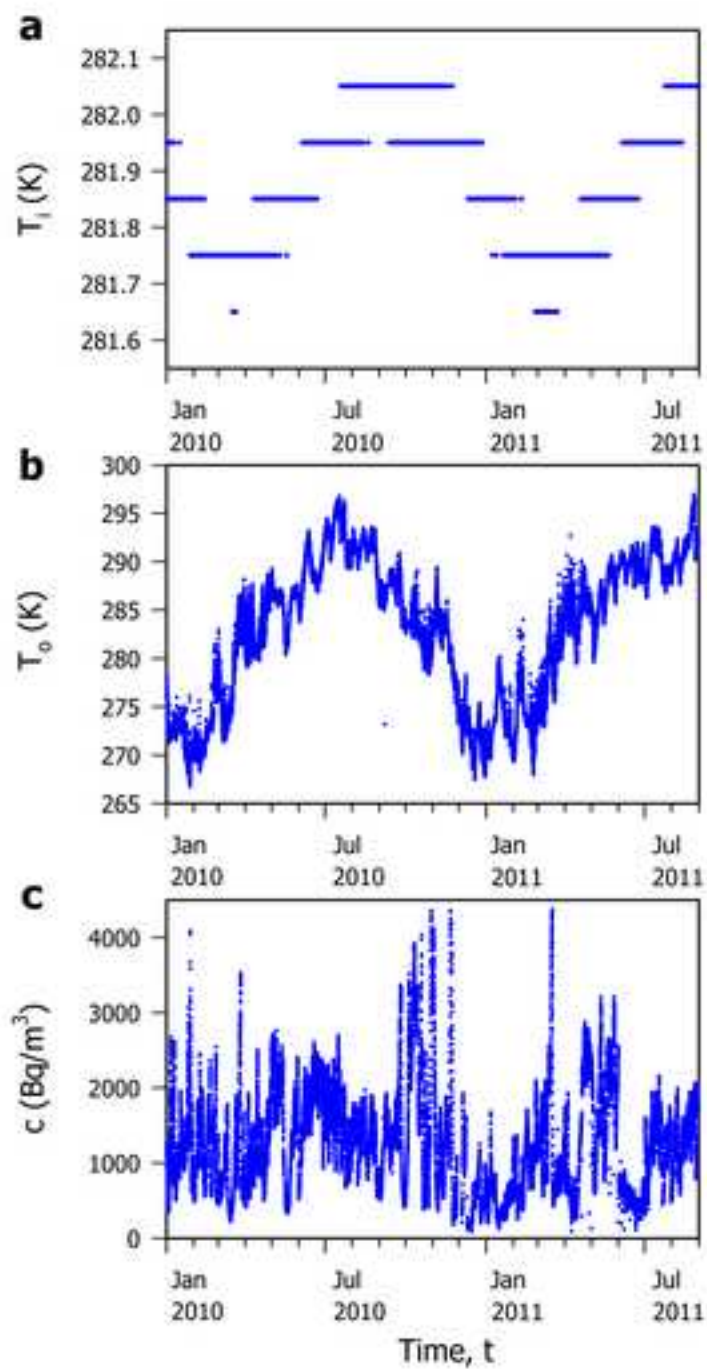


Figure 6
[Click here to download high resolution image](#)

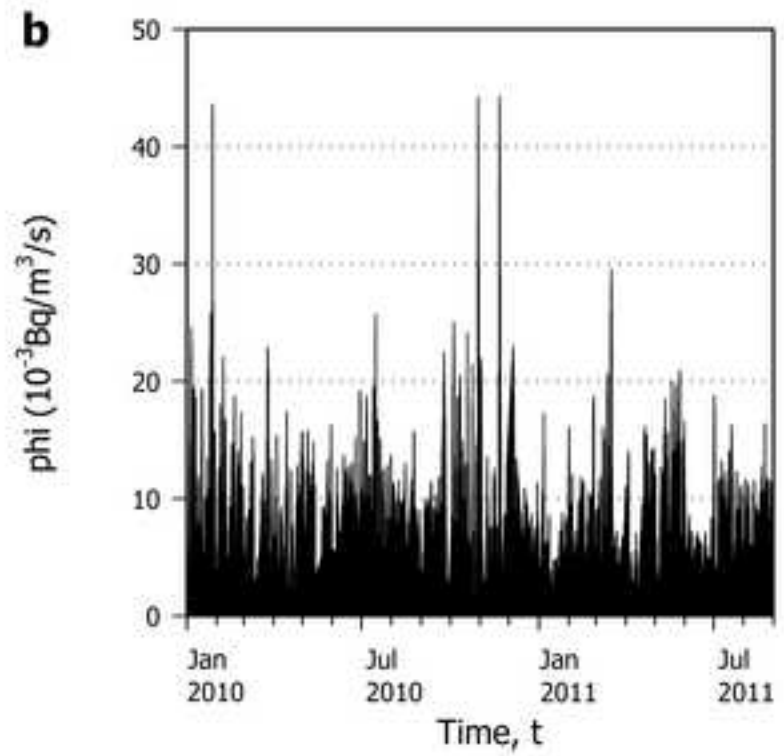
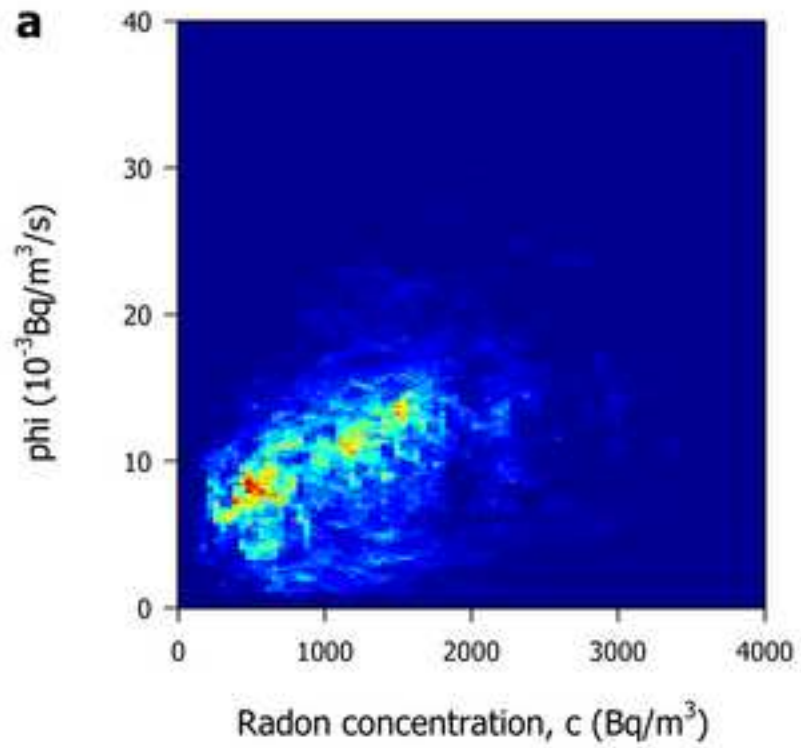


Figure 7

[Click here to download high resolution image](#)

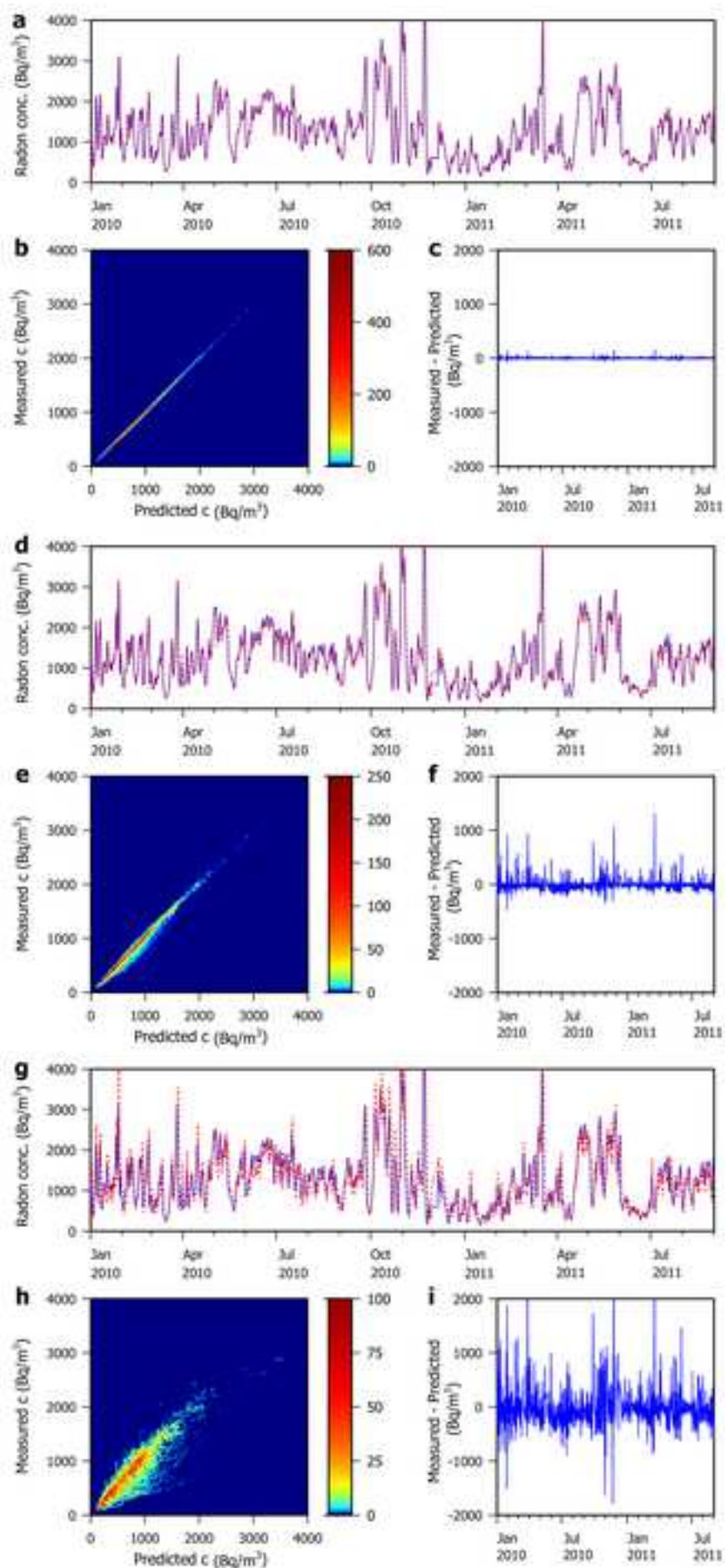


Figure 8

[Click here to download high resolution image](#)

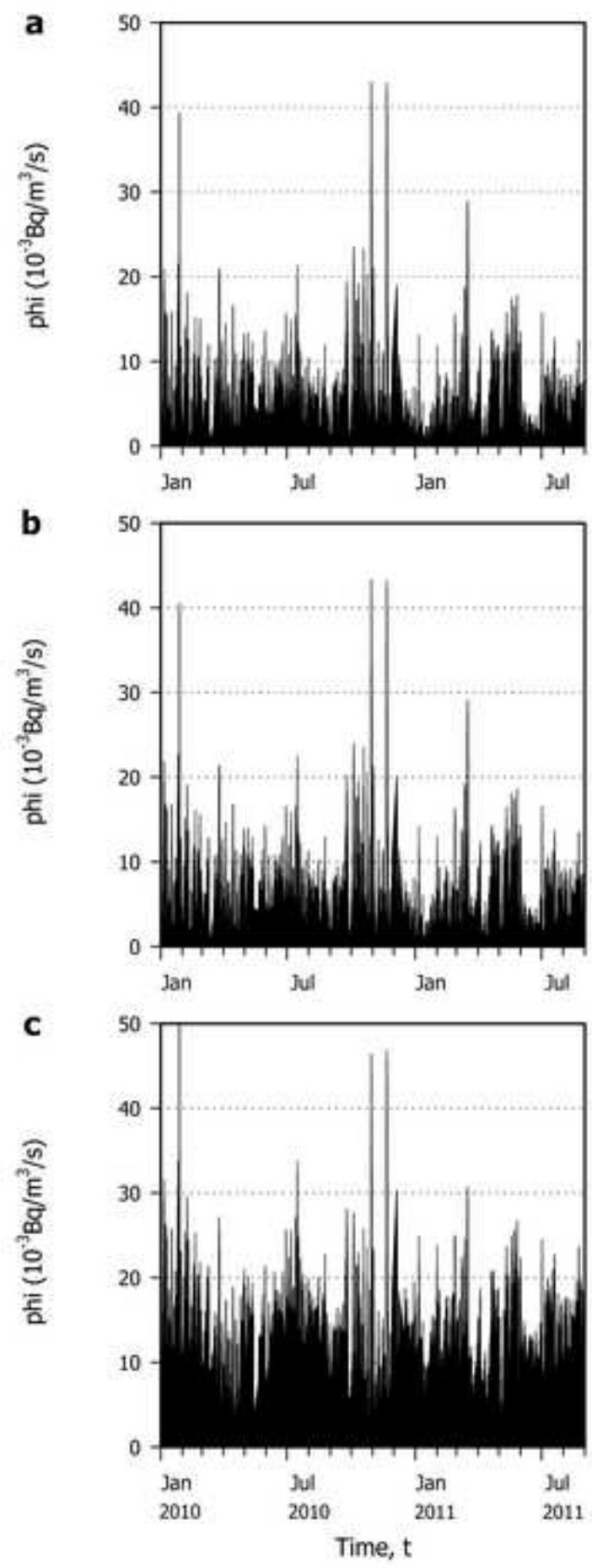


Figure 9

[Click here to download high resolution image](#)

