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Adaptive unknown input reconstruction scheme for Hammerstein-Wiener systems

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Abstract

In this paper an adaptive time-varying filter for unknown/unmeasurable input reconstruction is proposed. The algorithm is based on parity-equations and is applicable to Hammerstein-Wiener systems, i.e. systems composed of a linear dynamic part followed and preceded by a memoryless nonlinearity. An error-in-variables case is considered, i.e. known input and output signals are both subjected to measurement uncertainties. The scheme forms an extension to a filter previously proposed by the authors. As the input reconstruction involves transformation of noisy signals through memoryless static functions, measurement noise is either amplified or reduced, depending on the gradient of the nonlinear function. Thus, in the proposed scheme the bandwidth of the filter is adjusted depending on the operating point allowing for a trade-off between noise attenuation and a phase lag.

1 Introduction

Block oriented models are convenient for modelling nonlinear systems. Their relatively simple structure of a linear dynamic block interconnected with nonlinear memoryless function(s) provides a powerful tool for approximation of a large class of nonlinear systems [13, 14]. Block oriented models have been used for modelling such phenomena as for instance: infant EEG seizures [4], a radio frequency amplifier [5], a glucose-insulin process in diabetes type I patient [3], ionospheric dynamics [12] or human operator dynamics [19]. Furthermore, such models are also used for control purposes [1, 6, 7] and fault detection [9, 10].

This paper deals with the problem of unknown/unmeasur-able input reconstruction for Hammerstein-Wiener systems, i.e. where the linear dynamic block is preceded and followed by nonlinear static functions. (In the case of a Hammerstein model a linear block is preceded by a static nonlinear function, whereas in the case of a Wiener model the order of these elements is reversed.) An error-in-variables (EIV) framework [15] is considered, i.e. all the measured signals are affected by white, Gaussian, zero-mean and mutually uncorrelated measurement noise sequences. Up to date, few publications are available on the subject of unknown input reconstruction of Hammertein and Wiener systems. Szabo et al. [18] proposed an inversion of Wiener systems using a geometric method, based on the assumption that the static nonlinearity transforming the output is invertible, whilst Ibnkahla [8] used neural networks for Hammerstein system inversion. In this paper an adaptive time-varying filter for unknown input reconstruction is developed. The proposed scheme forms an extension to the algorithm presented in [17] and allows to adjust the filter bandwidth as the operating point changes leading to improved noise attenuation.

The paper is organised as follows: Section 2 states the unknown input reconstruction problem. In Section 3 the parity equation based unknown input reconstruction method for Hammerstein-Wiener systems (PE-UIO-HW) is presented. The main contribution of this paper is described in Section 4, where the PE-UIO-HW is expanded to an adaptive parity space order case. Then the method is demonstrated using a numerical example in Section 5. Section 6 concludes the paper.

2 Problem statement

It is assumed that a two-input single-output nonlinear system can be described by a Hammerstein-Wiener model. An EIV framework is considered [15], i.e. both measured input and output signals are affected by white, Gaussian, zero-mean and mutually uncorrelated noise sequences, see Fig. 1. Thus, the Hammerstein-Wiener model model is given by the following state-space form:

$$\begin{aligned}
\bar{u}_{0_k} &= g\left(u_{0_k}\right) \\
x_{k+1} &= Ax_k + B\bar{u}_{0_k} + Gv_k \\
\bar{y}_{0_k} &= Cx_k + D\bar{u}_{0_k} + Hv_k \\
y_{0_k} &= f\left(\bar{y}_{0_k}\right) \\
u_k &= u_{0_k} + \tilde{u}_k \\
y_k &= y_{0_k} + \tilde{y}_k
\end{aligned} \tag{1}$$

where $g(\cdot)$ is a static nonlinearity transforming the first system input u_{0_k} into an inaccessible signal $\bar{u}_{0_k}$ which serves as the first input to the linear subsystem. It is assumed that the second input v_k is fed directly (without nonlinear transformation) to the linear block, which is described by the matrices: $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times 1}$, $C \in \mathcal{R}^{1\times n}$, $D \in \mathcal{R}^{1\times 1}$, $G \in \mathcal{R}^{n\times 1}$ and $H \in \mathcal{R}^{1\times 1}$. The term $\bar{y}_{0_k}$ refers to the output of the linear part of the system, which is then transformed by the memoryless function $f(\cdot)$ into the overall system output y_{0_k}. Since the EIV case is considered, all measured variables, which are u_k and y_k, are affected by white, Gaussian, zero-mean and mutually uncorrelated measurement noise sequences denoted by $\tilde{u}_k$ and $\tilde{y}_k$, respectively. Noise sequences are postulated to

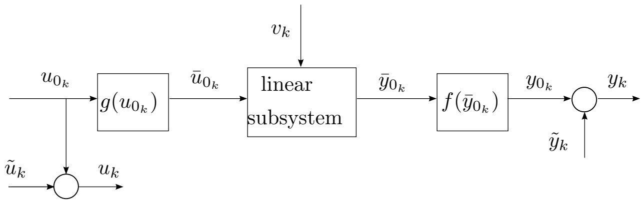

Figure 1: Hammerstein-Wiener system in EIV framework

be uncorrelated with the noise-free but unmeasured system input and output, denoted as u_{0_k} and y_{0_k}, respectively. It is assumed here that $f(\cdot)$ is strictly monotonic, hence its inverse exists. Note that (1) represents a Hammerstein or a Wiener model if, respectively, $f(\cdot)$ or $g(\cdot)$ is an identity function.

The objective of the proposed scheme is to estimate the unknown input v_k, simultaneously minimising the effect of the measurement noise. It is assumed that the model of the system is known.

3 Description of unknown input reconstruction filter

3.1 Parity equations

Consider the system described by (1). The following stacked vector of the unknown input, v_k, is introduced [11]:

$$V_k = \begin{bmatrix} v_{k-s} & v_{k-s+1} & \cdots & v_k \end{bmatrix}^T \tag{2}$$

where the term s denotes the order of the parity space. Analogously, one can build stacked vectors of y_k, y_{0_k}, $\bar{y}_{0_k}$, $\tilde{y}_k$, $\bar{u}_{0_k}$, u_k, u_{0_k} and $\tilde{u}_k$ which are denoted, respectively, as Y_k, Y_{0_k}, $\bar{Y}_{0_k}$, $\tilde{Y}_k$, $\bar{U}_{0_k}$, U_k, U_{0_k} and $\tilde{U}_k$. By making use of this notation the system defined by (1) can be expressed in the form of:

$$\bar{U}_{0_k} = g(U_{0_k}) \tag{3a}$$
$$\bar{Y}_{0_k} = \Gamma x_{k-s} + Q\bar{U}_{0_k} + TV_k \tag{3b}$$
$$Y_{0_k} = f(\bar{Y}_{0_k}) \tag{3c}$$

where $g(U_0)$ is a vector whose elements are $g(u_{0_{k-s}})$, $g(u_{0_{k-s+1}})$, $\cdots$, $g(u_{0_k})$, cf. (2). Analogously, the function $f(\bar{y}_0)$ is defined. The term Γ is an extended observability matrix:

$$\Gamma = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix} \in \mathcal{R}^{(s+1)\times n} \tag{4}$$

and Q is the following block Toeplitz matrix:

$$Q = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}B & CA^{s-2}B & \cdots & D \end{bmatrix} \in \mathcal{R}^{(s+1)\times(s+1)} \quad (5)$$

Analogously, the matrix $T \in \mathcal{R}^{(s+1)\times(s+1)}$ is constructed by replacing B and D in (5) by G and H, respectively.

In order to eliminate the unknown state vector from (3b), a row vector $W \in \mathcal{R}^{1\times(s+1)}$ is defined, which belongs to the left nullspace of Γ, i.e.

$$W\Gamma = 0 \quad (6)$$

Hence (3b) can be transformed to:

$$W\bar{Y}_{0_k} = WTV_k + WQ\bar{U}_{0_k} \quad (7)$$

which, since $f(\cdot)$ is assumed to be invertible, can be reformulated as:

$$Wf^{-1}(Y_{0_k}) = WTV_k + WQg(U_{0_k}) \quad (8)$$

where $f^{-1}(\cdot)$ denotes an inverse of $f(\cdot)$. Due to the fact that y_{0_k} and u_{0_k} are inaccessible, the parity relation (8) can be approximated by the measured values of the input and output:

$$Wf^{-1}(Y_k) = WTV_k + WQg(U_k) + \xi_k \quad (9)$$

where ξ_k accounts for an overall error resulting from the presence of measurement noise. By rearranging the measured (known) variables to the left-hand side and the unknowns to the right-hand side, the following PE is obtained [11]:

$$Wf^{-1}(Y_k) - WQg(U_k) = WTV_k + \xi_k \quad (10)$$

3.2 Unknown input estimation

The unknown input is estimated as follows [17]:

$$\hat{v}_{k-\tau} = Wf^{-1}(Y_k) - WQg(U_k) \quad (11)$$

where τ is the estimation delay and is defined further in this section. In the case of noise-free input and output measurements (11) becomes

$$\hat{v}_{k-\tau} = WTV_k \quad (12)$$

Thus, the unknown input estimate is calculated as a linear combination of the sequence $v_{k-s}, v_{k-s+1}, \cdots, v_k$, i.e.

$$\hat{v}_{k-\tau} = \alpha_0 v_k + \alpha_1 v_{k-1} + \cdots + \alpha_s v_{k-s} \quad (13)$$

where the α parameters are dependent on the choice of the vector W, such that:

$$WT = \left[\begin{array}{cccc} \alpha_s & \alpha_{s-1} & \cdots & \alpha_0 \end{array} \right] \tag{14}$$

One can note that (13) represents a moving average finite impulse response filter with the gain being given by the sum of the α parameters, i.e. the sum of elements of the vector WT. Thus, the vector W is selected in such a way, that this sum is equal to unity, i.e. the gain of the FIR filter (13) is equal to one. Furthermore, the estimation lag τ of the filter (13) is given by a weighted sum of α elements.

$$\tau = \frac{\sum_{i=0}^{s} i\alpha_i}{\sum_{i=0}^{s} \alpha_i} \tag{15}$$

3.3 Selection of optimal W

In the case of noisy input and output measurements the unknown input estimate is affected by the error, cf. (10):

$$\hat{v}_k = WTV_k + \xi_k \tag{16}$$

resulting from both input and output measurement uncertainties, which can be deduced to be given by:

$$\xi_k = W\left(f^{-1}(Y_k) - f^{-1}(Y_{0_k})\right) - WQ\left(g(U_k) - g(U_{0_k})\right) \tag{17}$$

Using the notation

$$\begin{aligned} \tilde{\tilde{Y}}_k &= f^{-1}(Y_k) - f^{-1}(Y_{0_k}) \\ \tilde{\tilde{U}}_k &= g(U_k) - g(U_{0_k}) \end{aligned} \tag{18}$$

Equation (17) can be rewritten as:

$$\xi_k = W\tilde{\tilde{Y}}_k - WQ\tilde{\tilde{U}}_k \tag{19}$$

Since $g(\cdot)$ and $f(\cdot)$ are memoryless, the sequences

$$\tilde{\tilde{u}}_k = g(u_k) - g(u_{0_k}) \tag{20}$$

and

$$\tilde{\tilde{y}}_k = f^{-1}(y_k) - f^{-1}(y_{0_k}) \tag{21}$$

are white and mutually uncorrelated (as $\tilde{u}_k$ and $\tilde{y}_k$ are white and mutually uncorrelated). The variance of $\tilde{\tilde{u}}_k$, further referred to as $\mathrm{var}(\tilde{\tilde{u}}_k)$, is time varying and depends on $g(u_k)$, u_k and the variance of $\tilde{u}_k$ (denoted as $\mathrm{var}(\tilde{u}_k)$). Analogously, the variance of $\tilde{\tilde{y}}_k$, i.e. $\mathrm{var}(\tilde{\tilde{y}}_k)$, is dependent on $\mathrm{var}(\tilde{y}_k)$ and the current values of $f(y_k)$ and y_k. The expression $g(u_{0_k})$ can be approximated using a Taylor expansion at u_k:

$$\begin{aligned} g(u_{0_k}) &\approx g(u_k) + \frac{\partial g(u_k)}{\partial u_k}(u_{0_k} - u_k) \\ &\approx g(u_k) - \frac{\partial g(u_k)}{\partial u_k}\tilde{u}_k \end{aligned} \tag{22}$$

Thus, the dependency between the $\tilde{\tilde{u}}_k$ and $\tilde{u}_k$ can be approximated via, cf. (20)

$$\tilde{\tilde{u}}_k \approx \frac{\partial g(u_k)}{\partial u_k} \tilde{u}_k \tag{23}$$

This means that the ratio between $\tilde{\tilde{u}}_k$ and $\tilde{u}_k$ is proportional to the tangential of $g(u_k)$. Analogously, the ratio between $\tilde{\tilde{y}}_k$ and $\tilde{y}_k$ is proportional to $\frac{\partial f^{-1}(y_k)}{\partial y_k}$. Thus, the variances of $\tilde{\tilde{u}}_k$ and $\tilde{\tilde{y}}_k$ can be approximated, respectively, as:

$$\begin{aligned} \mathrm{var}(\tilde{\tilde{u}}_k) &\approx \left(\frac{\partial g(u_k)}{\partial u_k}\right)^2 \mathrm{var}(\tilde{u}_k) \\ \mathrm{var}(\tilde{\tilde{y}}_k) &\approx \left(\frac{\partial f^{-1}(y_k)}{\partial y_k}\right)^2 \mathrm{var}(\tilde{y}_k) \end{aligned} \tag{24}$$

It should be noted that $\mathrm{var}(\tilde{\tilde{u}}_k)$ and $\mathrm{var}(\tilde{\tilde{y}}_k)$ are, in general, time varying as they depend on the current values of the functions $g(u_k)$ and $f^{-1}(y_k)$.

The PE-UIO-HW algorithm minimises the variance of the error term ξ_k by appropriate selection of W

$$\begin{aligned} \mathrm{var}(\xi_k) &= \mathrm{E}\{(W\tilde{\tilde{Y}}_k - WQ\tilde{\tilde{U}}_k)(W\tilde{\tilde{Y}}_k - WQ\tilde{\tilde{U}}_k)^T\} \\ &= W\Sigma_{\tilde{\tilde{y}}}W^T + WQ\Sigma_{\tilde{\tilde{u}}}Q^T W^T - \\ &\quad W\Sigma^T_{\tilde{\tilde{u}}\tilde{\tilde{y}}}Q^T W^T - WQ\Sigma_{\tilde{\tilde{u}}\tilde{\tilde{y}}}W^T \end{aligned} \tag{25}$$

where $\Sigma_{\tilde{\tilde{u}}} = E\{\tilde{\tilde{U}}_k \tilde{\tilde{U}}_k^T\}$, $\Sigma_{\tilde{\tilde{y}}} = E\{\tilde{\tilde{Y}}_k \tilde{\tilde{Y}}_k^T\}$, $\Sigma_{\tilde{\tilde{u}}\tilde{\tilde{y}}} = E\{\tilde{\tilde{U}}_k \tilde{\tilde{U}}_k^T\}$. The term $\Sigma_{\tilde{\tilde{u}}}$ is calculated via, cf. (24):

$$\Sigma_{\tilde{\tilde{u}}} = \begin{bmatrix} \mathrm{var}(\tilde{\tilde{u}}_{k-s}) & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \mathrm{var}(\tilde{\tilde{u}}_{k-1}) & 0 \\ 0 & \cdots & 0 & \mathrm{var}(\tilde{\tilde{u}}_k) \end{bmatrix} \tag{26}$$

Analogously, the expression $\Sigma_{\tilde{\tilde{y}}}$ is obtained by replacing the terms $\mathrm{var}(\tilde{\tilde{u}}(\cdot))$ from (26) by $\mathrm{var}(\tilde{\tilde{y}}(\cdot))$. Due to the fact that $\tilde{u}_k$ and $\tilde{y}_k$ are mutually uncorrelated, $\Sigma_{\tilde{\tilde{u}}\tilde{\tilde{y}}} = 0$. For the sake of brevity the variable Σ is introduced

$$\Sigma = \Sigma_{\tilde{\tilde{y}}} + Q\Sigma_{\tilde{\tilde{u}}}Q^T \tag{27}$$

Subsequently, the vector W should be selected to minimise the cost function $\phi(W)$:

$$\phi(W) = W\Sigma W^T \tag{28}$$

subject to the following constraints:

1. Sum of elements of WT is equal to 1.

2. $W\Gamma = 0$.

Note that the ratio between $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$ is not constant, i.e. the impact of either input or output measurement noise on the unknown input estimation error can be prevailing, depending on the system operating point. As a result Σ is time-varying. Therefore, the unknown input reconstruction filter should adapt to these changes and the optimal vector W should be updated at each time instance. In order to acknowledge that W is time-varying, the notation W_k is used further in this paper. The PE-UIO-HW algorithm, which minimises cost function (28) has been derived in [17] using the Lagrange multipliers method [2] and is summarised in Algorithm 1.

Algorithm 1 PE-UIO-HW

Select the order of the parity space $s \geq n$ and build matrices Γ, Q and T.
Obtain the matrix spanning the left nullspace of Γ, denoted as $\Gamma^{\perp}$.
for $k = 1$ to N **do**
  Calculate variances of $\tilde{\tilde{u}}_k$ and $\tilde{\tilde{y}}_k$ using (24)
  Compute Σ using (27)
  Calculate the matrix S and the column vector ψ as, respectively[1]

$$S = \Gamma^{\perp}\Sigma(\Gamma^{\perp})^T + (\Gamma^{\perp}\Sigma(\Gamma^{\perp})^T)^T \tag{29}$$

$$\psi = \text{sum}_{row}(\Gamma^{\perp}T) \tag{30}$$

  Obtain the Lagrange multiplier λ as

$$\lambda = \left(\left(S^{-1}\psi\right)^T \psi\right)^{-1} \tag{31}$$

  Calculate the parameter vector P as

$$P = \lambda S^{-1}\psi \tag{32}$$

  Compute the vector W_k as:

$$W_k = P^T\Gamma^{\perp} \tag{33}$$

  if $k = 1$ **then**
    Compute estimation lag τ using (15)
  end if
  Obtain the estimate of $v_{k-\tau}$ via equation (11).
end for

4 Extension to variable parity space order

As it has been discussed in [16] an increase of the parity space order s reduces the bandwidth of the unknown input reconstructor thus improving the noise filtering properties of the filter (i.e. reducing the impact of the noise on the unknown input estimate). However, the reduction of the filter bandwidth results in the input reconstruction filter being sluggish. Due to the fact that $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$ are time varying, the impact of the noise on the unknown input varies. Therefore, it is beneficial to vary the bandwidth of the filter (via changing the value of the parity space order s) as values of $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$ change. In the algorithm proposed in this section the order of the parity space varies according to the changes of $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$. In order to recognise that the order of the parity space is time varying, its value at the time instance k is further denoted as s_k.

4.1 Choice of s_k

It can be observed from (25), that the variance of the error term $\text{var}(\xi_k)$ can be represented as a sum of two terms, each of which depends solely on either the output or the input measurement noise, such as:

$$\text{var}(\xi_k) = \phi_{u_k} + \phi_{y_k} \tag{34}$$

where ϕ_{u_k} and ϕ_{y_k} are defined as:

$$\phi_{u_k} = W_k Q \Sigma_{\tilde{\tilde{u}}} Q^T W_k^T \tag{35a}$$

$$\phi_{y_k} = W_k \Sigma_{\tilde{\tilde{y}}} W_k^T \tag{35b}$$

Therefore, it can be noted that the PE-UIO-HW algorithm minimises the sum of ϕ_{u_k} and ϕ_{y_k}. The accuracy of the unknown input estimation alters over the time, as $\text{var}(\xi_k)$ is changing.

The choice of s_k should depend on both $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$. It is proposed to create a two-dimensional map, which assigns the value of s_k for each couple of $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$. Furthermore, as the values of $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$ are calculated based on the current values of the measured input and output signals (affected by noise), cf. (24), s_k selected based on the current values of $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$ may jitter unnecessarily. In order to avoid this problem, it is proposed to select s_k based on moving averages of $\text{var}(\tilde{\tilde{u}}_k)$ and $\text{var}(\tilde{\tilde{y}}_k)$ defined as:

$$\overline{\text{var}(\tilde{\tilde{u}}_k)} = \frac{1}{k_1 + k_2 + 1} \sum_{i=k-k_1}^{k+k_2} (\text{var}(\tilde{\tilde{u}}_i)) \tag{36a}$$

$$\overline{\text{var}(\tilde{\tilde{y}}_k)} = \frac{1}{k_1 + k_2 + 1} \sum_{i=k-k_1}^{k+k_2} (\text{var}(\tilde{\tilde{y}}_i)) \tag{36b}$$

where k_1 and k_2 are arbitrarily defined by the user.

[1] the operator $\text{sum}_{row}(A)$ denotes a column vector whose elements are sums of the appropriate rows of the matrix A. (In the case of a row vector x, the term $\text{sum}_{row}(x)$ is simply a scalar being the sum of elements of the vector x, whilst, if x is a column vector, $\text{sum}_{row}(x) = x$.)

4.2 Variable estimation lag

At the time instance k, the following delayed unknown input estimate is calculated:

$$\hat{v}_{t-\tau_k} = W_k f^{-1}(Y_k) - W_k Q_k g(U_k) \tag{37}$$

where τ_k is time varying, due to the alternating value of s_k. (Note that the notation τ_k and Q_k has been used instead of τ and Q in order to indicate that the estimation lag τ and the matrix Q as well as the dimensions of Q are time varying.) This would eventually lead to difficulties, such as some time instances of the unknown input would be omitted, and some of them estimated more than once. Therefore, a logic must be implemented, which copes with the variable estimation lag. A difficulty may arise in two situations:

(i) $\tau_k > \tau_{k-1}$

(ii) $\tau_k < \tau_{k-1}$

In the first case a particular time instance of the unknown input estimate is calculated twice. In such a case when two values of the unknown input estimate sample are available, the one should be selected which is less affected by noise. The fact that τ_k increases means an increase of the noise influence, i.e. $\mathrm{var}(\tilde{\bar{u}}_k)$ or $\mathrm{var}(\tilde{\bar{y}}_k)$ has increased. Therefore, the impact of the measurement noise on the unknown input estimate has also increased. Consequently, the value of the unknown input estimate which has been calculated as first is less affected by noise.

In the second case, the situation is opposite, i.e. some time instances of $\hat{v}_k$ will be omitted. It is proposed to use W_{k-1} and Q_{k-1} to calculate the missing values of the unknown input estimate.

Incorporating this logic into Algorithm 1 the adaptive order PE-UIO-HW (AO-PE-UIO-HW) is obtained which is summarised in Algorithm 2.

5 Numerical Example

Consider an examplary system, whose matrices of the linear block are given by:

$$A = \begin{bmatrix} 0 & -0.56 \\ 1 & 1.5 \end{bmatrix} \quad B = \begin{bmatrix} -0.1200 \\ 0.4125 \end{bmatrix} \quad D = 0.125$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad G = \begin{bmatrix} 0.0055 \\ 0.0963 \end{bmatrix} \quad H = 0.025 \tag{40}$$

The memoryless input and output nonlinearities are arbitrarily selected as:

$$\begin{aligned} \bar{u}_{0_k} &= \exp\left(0.165 \cdot 10^{-5} u_{0_k}^3 + u_{0_k}\right) - 1 \\ y_{0_k} &= \exp\left(11 + 0.165 \cdot 10^{-5} \bar{y}_{0_k}^3\right) - 59874 \end{aligned} \tag{41}$$

An output error case is considered for this simulation study, where $\tilde{y}_k$ is a white, Gaussian, zero-mean sequence with the variance equal to 3. It is assumed that there is no noise on the input, i.e. $\tilde{u}_k = 0$.

Algorithm 2 AO-PE-UIO-HW

for $k = 1$ to N **do**
  Calculate $\overline{\mathrm{var}(\tilde{\tilde{u}}_k)}$ and $\overline{\mathrm{var}(\tilde{\tilde{y}}_k)}$
  Using a look-up table select s_k based on $\overline{\mathrm{var}(\tilde{\tilde{u}}_k)}$ and $\overline{\mathrm{var}(\tilde{\tilde{y}}_k)}$
  Obtain W_k, Q_k, and τ_k as in Algorithm 1
  if $\tau_k = \tau_{k-1}$ **then**
    Calculate $\hat{v}(t - \tau_k)$ as:

$$\hat{v}(t - \tau_k) = W_k f^{-1}(Y_k) - W_k Q_k g(U_k) \tag{38}$$

  else if $\tau_k < \tau_{k-1}$ **then**
    for $j = \tau_{k-1}$ to $(\tau_k - 1)$ **do**
      Compute missed samples of unknown input

$$\begin{aligned} \hat{v}_{k-j} &= W_{k-1} f^{-1}(Y_{k-j}) \\ &\quad - W_{k-1} Q_{k-1} g(U_{k-j}) \end{aligned} \tag{39}$$

    end for
    $\hat{v}_{t-\tau_k} = W_k f^{-1}(Y_k) - W_k Q_k g(U_k)$
  else
    do nothing
  end if
end for

Left subfigures of Fig. 2 depict functions $g(\cdot)$ and $f(\cdot)$. As they are both monotonic and with their gradients strictly increasing, the impact of the output measurement noise on the unknown input estimate is expected to be relatively low for high values of y_k (as the gradient of $f^{-1}(y_k)$ is small for large values of y_k). On the other hand, this impact is relatively high for small values of y_k (as the gradient of $f^{-1}(y_k)$ is large for high values of y_k).

The known input and output signals as well as $g(u_k)$ and $f^{-1}(y_k)$ are presented in right subfigures of Fig. 2. For the first 800 samples of the simulation y_k is relatively small and, as the slope of $f(\cdot)$ is less steep for small values of $\bar{y}_k$, the inversion of the noisy measurement y_k leads to amplification of the impact of the output measurement noise in the first 800 samples of the unknown input estimate, which can be seen in the bottom right subfigure of Fig. 2. After 1200 samples both u_k and y_k increase, which results in a reduction of the impact of the output measurement noise on the accuracy of the unknown input estimate, cf. Fig. 2.

Variables k_1 and k_2 have been selected as $2\tau_k$ and 0, respectively. A look-up table has been designed which assigns appropriate value of s_k to $\overline{\mathrm{var}(\tilde{\tilde{y}}_k)}$ and is presented in Fig. 3. The reconstructed input is compared with the original one in Fig. 4. The lower subfigure of Fig. 5 presents s_k and τ_k as functions of time, whilst the upper subfigure of Fig. 5 compares $\mathrm{var}(\tilde{\tilde{y}}_k)$ with ϕ_{y_k}. Note that, although $\tau_k \approx \frac{1}{2}s_k$, a change of s_k does not necessary yield a change in the value of estimation delay τ_k and vice versa. For the first 100 simulation samples,

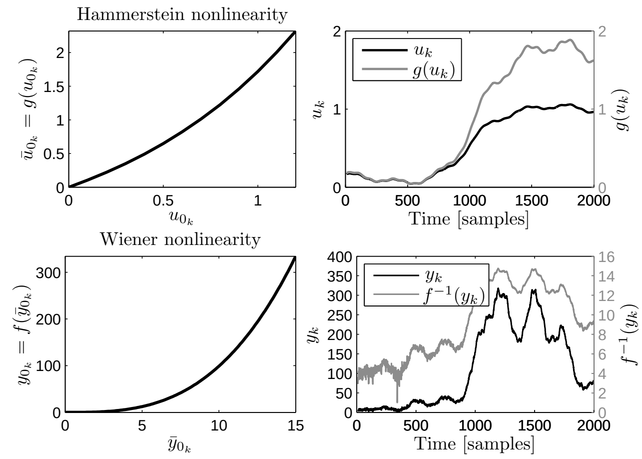

Figure 2: Simulation setup

when s_k remained constant and equal to 25, the value of τ_k changed 16 times (8 times increased from 12 to 13 and 8 times decreased from 13 to 12). This is caused by variations of $\Sigma_{\tilde{\tilde{y}}}$ and, consequently, variations of W_k, cf. equation (35b).On the contrary, between the 800^{th} and 1000^{th} simulation sample, when s_k is decreasing, the rate of change of τ_k is lower than the rate of change of s_k. Furthermore, the value of $\mathrm{var}(\tilde{\tilde{y}}_k)$ varies significantly. However, by altering the order of the parity space, one can reduce variations of ϕ_{y_k}. It is particularly visible between 300^{th} and 400^{th} and between 1000^{th} and 1600^{th} samples. These results have been compared with PE-UIO-HW with a fixed parity space order for two cases, namely $s_k = 14$ and $s_k = 24$. Values of ϕ_{y_k} for the two cases of fixed parity space order are also presented in the upper subfigure of Fig. 5. Fig. 6 compares the unknown input estimation error of AO-PE-UIO-HW with its non-adaptive version. Also Table 1 compares PE-UIO-HW and the AO-PE-UIO-HW in terms of variance of input reconstruction error. Note that, whilst PE-UIO-HW with a large parity space order ($s = 24$) leads to good disturbance attenuation for the first 800 samples of simulation, its bandwidth is to narrow to reconstruct all frequency components of v_k which is visible when the impact of the noise becomes less significant, see Fig. 6. On the other hand, $s_k = 14$ does not allow for optimal noise attenuation during the first 800 samples. For completeness, the frequency responses of the unknown input reconstruction filter defined in equation (13) for different values of s_k are presented in Fig. 7.

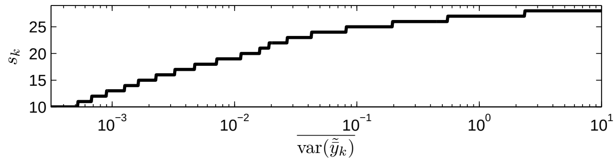

Figure 3: Assignment of parity space order to values of $\overline{\mathrm{var}(\tilde{\bar{y}}_k)}$

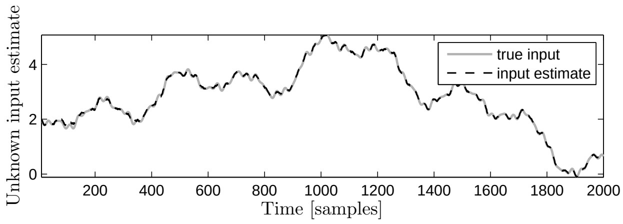

Figure 4: Unknown input vs. its estimate

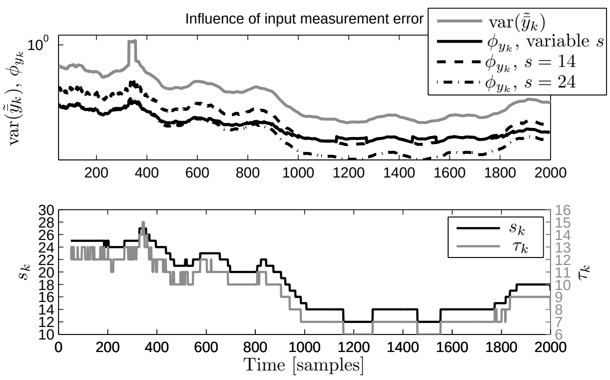

Figure 5: Parity space order and its influence on noise filtering

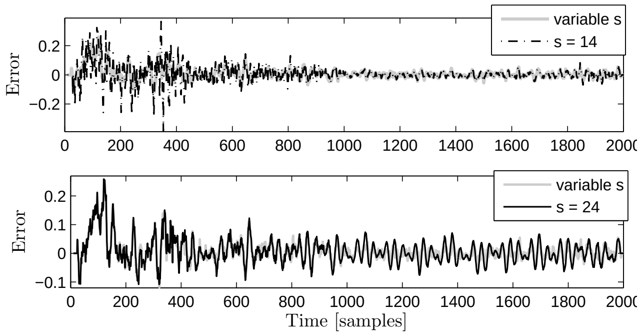

Figure 6: Unknown input estimation error

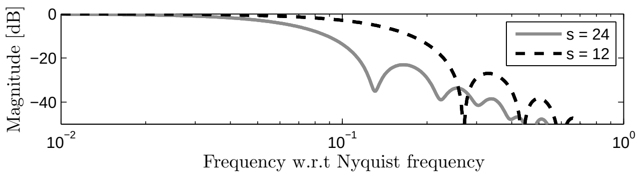

Figure 7: Frequency response of the unknown input reconstruction filter for different values of s_k

Table 1: Comparison of efficacy of the PE-UIO-HW and the AO-PE-UIO-HW in terms of variance of input reconstruction error

	PE-UIO-HW		AO-PE-UIO-HW
sample range	$s = 14$	$s = 24$	variable s
100:2000	0.0031	0.0018	0.0013
100:800	0.0074	0.0030	0.0028
1200:1800	2.9e-4	9.5e-4	2.9e-4

6 Conclusions and further work

In this paper an adaptive time-varying filter scheme for unknown input reconstruction has been developed. The proposed filter adjusts to the operating point of the system resulting in an improved noise attenuation. The scheme, since inherently adaptive, requires at each discrete time step a non negligible computational effort. Therefore, the future work aims towards an optimisation of the computational procedure. It is also intended to extend the algorithm to the multivariable case.

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