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Physics Procedia 3 (2010) 1443–1446
Physics Procedia
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Wang-Landau study of the 2d random-bond Blume-Capel model

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We study, via a two-stage Wang-Landau (WL) strategy, the random-bond version of the square lattice ferromagnetic Blume-Capel (BC) model, in both the first- and second-order phase transition regimes of the pure model. The second-order phase transition, emerging under random bonds from the second-order regime of the pure model, has the same values of critical exponents as the 2d Ising universality class, with the effect of the bond disorder on the specific heat being well described by double-logarithmic corrections. On the other hand, the second-order transition, emerging under bond randomness from the first-order regime of the pure model, has a distinctive universality class with $\nu = 1.30(6)$ and $\beta/\nu = 0.128(5)$. These results amount to a strong violation of universality principle of critical phenomena, since these two second-order transitions, with different sets of critical exponents, are between the same ferromagnetic and paramagnetic phases. Furthermore, the latter of these two sets of results supports an extensive but weak universality, since it has the same magnetic critical exponent (but a different thermal critical exponent) as a wide variety of 2d systems with and without quenched disorder.

1. INTRODUCTION

Quenched bond randomness may or may not modify the critical exponents of second-order phase transitions [1], whereas in 2d it always affects first-order phase transitions by conversion to second-order phase transitions even for infinitesimal randomness [2–4]. These predictions have been confirmed by various Monte Carlo simulations and they have been also well verified by the present study of the random-bond 2d BC model using the WL algorithm [5]. Furthermore, we find dramatically different critical behaviors of the second-order phase transitions emerging, due to bond randomness, from the first- and second-order regimes of the pure model, indicating a strong violation of universality with

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doi:10.1016/j.phpro.2010.01.204

different critical exponents along the corresponding two segments of the phase diagram. The random-bond version of the BC model is defined by the Hamiltonian

$$H = -\sum_{<ij>} J_{ij} s_i s_j + \Delta \sum_i s_i^2 \; ; \; P(J_{ij}) = \frac{1}{2}\,[\delta(J_{ij} - J_1) + \delta(J_{ij} - J_2)], \tag{1}$$

where the spin variables s_i take on the values $-1, 0$, or $+1$, $< ij >$ indicates summation over all nearest-neighbor pairs of sites and Δ is the value of the crystal field. The above bimodal distribution describes our choice for the quenched random ferromagnetic exchange interactions where both J_1 and J_2 are taken positive and the temperature scale may be set by fixing $2k_B/(J_1 + J_2) = 1$. It is well-known that the pure square lattice model ($J_1 = J_2$) exhibits a phase diagram with ordered ferromagnetic and disordered paramagnetic phases separated by a transition line that changes from an Ising-like continuous phase transition to a first-order transition at a tricritical point $(T_t, \Delta_t) = (0.609(4), 1.965(5))$ in the temperature - crystal field plane [6,7].

In this study we consider two distinct cases: In the first case we study the conversion to a second-order phase transition, due to the introduction of bond-randomness, of the first-order transition of the pure model at the value $\Delta = 1.975$. In the second case we examine the modifications induced, by bond-randomness, on the critical behavior of the second-order phase transition of the pure model at the value $\Delta = 1$ of the crystal field. Both studies are carried out for the same disorder strength $r = J_2/J_1 = 0.6$ and for the estimation of the properties of a large number (100) of bond disorder realizations (lattice sizes $L = 20 - 100$) of the random-bond versions, we have used our two-stage strategy of a restricted entropic sampling. The first stage is based on a repeated application of a multi-range WL approach and the resulting accurate DOS is then used for a final redefinition of the restricted subspace in which the second stage entropic sampling is applied. The scheme has been described in detail in our recent studies [8,9].

2. RESULTS AND DISCUSSION

We consider first the value $\Delta = 1.975$ [9]. Figure 1(a) contrasts the specific heat results for the pure 2d BC model and two strengths of disorder, $r = 17/23 \simeq 0.74$ and $r = 3/5 = 0.6$. The saturation of the specific heat is clear and signals the conversion of the first-order transition to a second-order transition with a negative critical exponent α. Figures 1(b) - (d) summarize our finite-size scaling analysis (FSS) for the disorder strength $r = 3/5 = 0.6$. The behavior of five pseudocritical temperatures $T_{[Z]^*_{av}}$, $T_{[Z]^*_{av}} = T_c + bL^{-1/\nu}$ (corresponding to the peaks of the following quantities averaged over the disorder realizations: susceptibility, derivative of the absolute order parameter with respect to the inverse temperature ($K = 1/T$) and first-, second-, and fourth-order logarithmic derivatives of the order parameter with respect to the inverse temperature) is presented in figure 1(b) and allow us to estimate both the critical temperature $T_c = 0.626(2)$ and the correlation length exponent $\nu = 1.30(6)$. From the behavior of the peaks of the average susceptibility and the values of the average order parameter at $T_c = 0.626$ we obtain estimates for the exponent ratio γ/ν and β/ν. As shown in the corresponding figures these estimates are very close to $\gamma/\nu = 1.75$ and to $\beta/\nu = 0.125$ which are the exact values of the simple Ising model. It appears that this "weak universality" is well obeyed

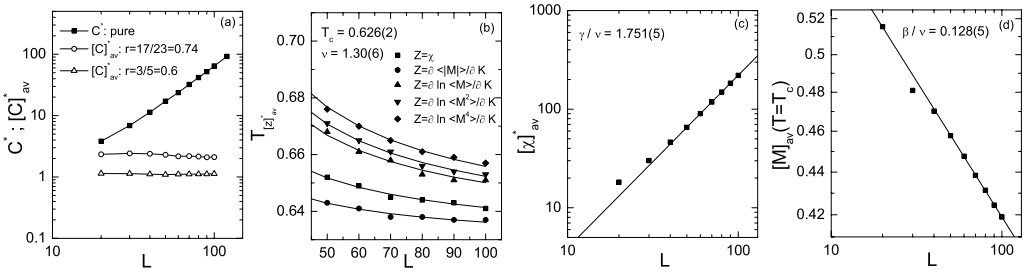

Figure 1. Behavior of the 2d BC model at $\Delta = 1.975$: (a) Illustration of the saturation of the specific heat for the random-bond model (open symbols). FSS behavior, for $r = 0.6$, of (b) pseudocritical temperatures, (c) susceptibility peaks, and (d) order parameter at T_c. Linear fits are applied for $L \geq 50$.

for several pure and disordered models, including the pure and random-bond version of the square Ising model with nearest- and next-nearest-neighbor competing interactions [8]. In conclusion, at $\Delta = 1.975$, the first-order transition of the pure model is converted to second-order, giving a distinctive universality class with $\nu = 1.30(6)$, thus supporting an extensive but weak universality since it has the same (ratios) of magnetic critical exponents as a wide variety of 2d systems without [10] and with [8,11] quenched disorder.

Let us now turn to the random-bond 2d BC model at $\Delta = 1$. At this regime, the random version should be comparable with the random Ising model, a model that has been extensively investigated and debated [8,12–18]. According to these studies, the effect of disorder gives rise only to logarithmic corrections as the critical exponents maintain their 2d Ising values. Fitting our data for the corresponding pseudocritical temperatures in the range $L = 50 - 100$ to the expected power-law behavior mentioned above, we find that the critical temperature is $T_c = 1.3812(4)$ and the shift exponent is $1/\nu = 1.011(22)$. This last estimate is a strong indication that the random-bond 2d BC at $\Delta = 1$ has the same value of the correlation's length critical exponent as the pure version, and therefore, as the 2d Ising model. Furthermore, our data for the specific heat maxima averaged over disorder, $[C]^*_{av}$, showed that the expected double-logarithmic divergence scenario is well obeyed. Finally, the FSS behavior of the susceptibility peaks (giving the estimate 1.749(7) for γ/ν) and the order-parameter values at the estimated critical temperature $T_c = 1.3812$ (giving an estimate $\beta/\nu = 0.126(4)$) are in good agreement with the expected 2d Ising universality class behavior. Thus, at this disorder strength, the random-bond 2d BC model with $\Delta = 1$ belongs to the same universality class as the random Ising model and the effect of the bond disorder on the specific heat is well described by the double logarithmic scenario. In conclusion, our results amount to a strong violation of universality, since the two second-order phase transitions mentioned above, with different sets of critical exponents, are between the same ferromagnetic and paramagnetic phases.

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