Primary Channel Duty Cycle Estimation Under Imperfect Spectrum Sensing Based on Mean Channel Periods

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***Abstract*—The emerged Dynamic Spectrum Access (DSA) concept based on Cognitive Radio (CR) is a promising solution to overcome the problems related to frequency spectrum scarcity. In DSA/CR systems, the inactivity patterns of the licensed frequency channels are exploited in an opportunistic and non-interfering manner by unlicensed users. Therefore, the knowledge of the occupancy rate (i.e., duty cycle) of these licensed channels is crucial for boosting the performance of the DSA/CR system. For example, it can help to select the lowest occupied channel which can offer higher opportunistic spectrum to the unlicensed users. Channel Duty Cycle (DC) is a statistical parameter about the activity of the licensed channel in time-domain, which is initially unknown to the DSA/CR system but can be estimated from the outcomes of spectrum sensing. However, spectrum sensing is imperfect in practice due to sensing errors, which in turn will provide incorrect estimation of the channel DC. In this context, this work successfully finds a novel method to accurately estimate the channel DC even under Imperfect Spectrum Sensing (ISS) without requiring any prior knowledge about the licensed channel activity. This is achieved after accurately analysing the impact of ISS on the estimation of the statistical moment (mean) of the channel activity periods, for which a closed form expression is obtained as a function of the true mean, probability of errors and sensing period. The achieved mathematical expression helps to find a novel method to accurately estimate the true mean of the channel activity periods and subsequently the channel DC based on the outcomes of the ISS.**

***Index Terms*—Cognitive radio, dynamic spectrum access, spectrum sensing, channel duty cycle, primary activity statistics.**

I. Introduction

Frequency spectrum is a precious natural resource that enables communication systems and electronic devices to interconnect wirelessly. However, day by day the spectrum is becoming extremely crowded due to the fast growing wireless communications industry and the emergence of new advanced services. Therefore, continuing adopting frequency allocation policy by regulatory agencies will not be the appropriate strategy to meet the demands for frequency bands in the coming few years. As a result, Dynamic Spectrum Access (DSA) [1] based on the Cognitive Radio (CR) [2] concept is becoming a more accepted solution to overcome the spectrum scarcity problem and to make the use of the spectrum more efficient. In DSA/CR systems, unlicensed (secondary) users (SUs) will be allowed to access a frequency band that is already allocated to the licensed (primary) users (PUs). However, the process of permitting SUs to use the spectrum should be only when PUs are idle (i.e., not using their allocated spectrum), as to ensure no harmful interference could result from this access.

From above, one can understand that primary channels with lower exploitation by their PUs will offer higher opportunities to the SUs in the DSA/CR system. As a result, selecting the lowest occupied primary channel is significantly important to provide higher opportunistic spectrum for the SUs, which in turn will boost the efficiency and the performance of the DSA/CR system. Initially, SUs have no knowledge about the occupancy rate (i.e., DC) of the primary channels, however, such statistical information can be obtained based on the outcomes of the spectrum sensing decisions. Although the main purpose of spectrum sensing is to determine the instantaneous state of the primary channel, the sensing decisions resulting from spectrum sensing can be further exploited to provide a broad range of statistical information about primary channel activity. Such information is significantly important from DSA/CR system perspective to improve its performance and efficiency. For example, it can help to select the most appropriate primary channel (i.e., lowest occupied one) [3]–[5], it can predict the future behaviour of the spectrum occupancy [6] [7], and help to decide future actions to enhance the performance and the efficiency of the system [8]. Therefore, one of the important statistical parameters is the channel DC, which can be estimated from spectrum sensing decisions.

The accuracy of spectrum sensing is essentially dependent on the Signal-to-Noise Ratio (SNR) of the sensed PUs signal at the secondary receiver. Perfect Spectrum Sensing (PSS) can be assumed when the SNR is sufficiently high so that no errors could be produced during the sensing process. However, in practice, DSA/CR terminals are more likely to operate in low SNR environments where sensing errors are not avoidable, which results in Imperfect Spectrum Sensing (ISS). When the DSA/CR system operates under ISS the estimated idle/busy periods of the primary channel can be highly inaccurate and consequently will provide corrupted statistical information about primary channel activity [9].

Several research works in the literature have been conducted to investigate the estimation of channel DC under ISS. For example in [10], the PUs DC is estimated using averaging method. Such work was limited to a typical distribution by assuming that PUs activity periods (idle/busy) follow exponential distribution, which in practical scenarios is not a realistic assumption [11]. In [12], the channel DC is estimated after reconstructing the estimated idle/busy periods of PUs by using reconstruction algorithms. However, these algorithms require a priori knowledge about the minimum time of PUs activity. In addition, no explicit closed-form expression has been provided to express the relationship between the estimated channel DC under ISS and the probability of sensing errors and sensing period. Therefore, the novelty of this work comes from finding a method to accurately estimate the primary channel DC under ISS without requiring any prior knowledge about PUs activity or making any assumption at all. This method is based on estimating the mean of the channel periods accurately, which is achieved after analysing the impact of sensing errors on the estimation of the mean, and finding a closed-form expression to express the estimated mean under ISS as a function of the true mean, probability of errors and sensing period. The obtained results show that the proposed method provides a highly accurate (nearly perfect) estimation of the PU channel DC even under high probability of sensing errors.

The rest of the paper is organised as follows. First, Section II presents the system model. Then the problem of estimating the DC of the primary channel based on spectrum sensing is studied in Section III. Section IV provides the mathematical analysis and a final closed-form expression for the estimated mean of idle/busy periods under ISS as a function of the original mean, probability of errors, and sensing period. Novel methods for the accurate estimation under ISS of the mean period and the channel DC are proposed in Sections V and VI, respectively. The validation and simulation results are shown in Section VII. Finally, Section VIII concludes the paper.

II. System Model

In this work we consider, without loss of generality, a single primary channel which is allocated to a single PU. The activity of the PU within this channel is represented by a sequence of idle/busy periods in the time-domain. The time durations of these periods are continuous random variables, and it is found that, based on the experimental measurements in [11], they are best described by Generalised Pareto (GP) distribution. In this work, however, the distribution of the PU periods is considered to be unknown to the DSA/CR system (i.e., no prior knowledge is required), which makes this work independent and applicable to any distribution type. SUs monitor the activity of the PU using spectrum sensing, by which the instantaneous state of the channel is observed periodically at a constant sensing period denoted as T_s. The outcomes of spectrum sensing are a set of binary decisions, either $\mathcal{H}_0$ to represent the idle state or $\mathcal{H}_1$ to represent the busy state of the channel. These decisions are then processed by the DSA/CR system to estimate the corresponding idle/busy periods of the primary channel. The elapsed time calculated between any two changes in the sensing decisions is considered as an estimation of the original period. As shown in Fig. 1(a), the original idle/busy periods T_i (where i refers to the type of the period, $i = 0$ for idle and $i = 1$ for busy) are estimated as $\widehat{T_i}$ periods under PSS (i.e., without sensing errors). The accuracy of this estimation is only affected by the time resolution of the sensing period T_s where no sensing errors are assumed (the estimation of PU activity statistics under this scenario was investigated in [13]). In practice, however, spectrum sensing is imperfect due to low SNR conditions and sensing errors are likely to occur in the sensing events $\mathcal{H}_i$. Two types of sensing errors can be identified: false alarms (where an idle state of the channel is sensed as a busy state) and missed detections (where a busy state of the channel is sensed as an idle state). These errors are independent and identically distributed (i.i.d.) random variables, and can be modelled with a probability of false alarm P_{fa} or probability of missed detection P_{md}. Fig. 1(b) illustrates how the original idle/busy periods T_i are estimated as $\breve{T}_i$ periods under ISS (with single false alarm).

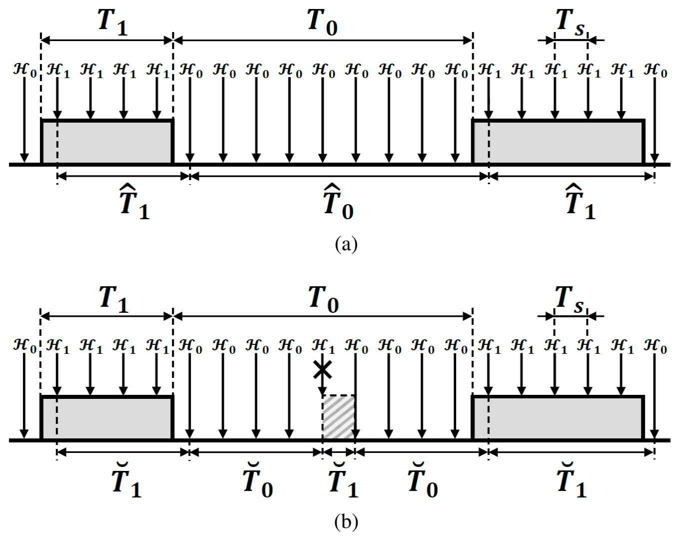


Fig. 1. Estimation of idle/busy periods based on spectrum sensing: (a) perfect spectrum sensing (PSS), (b) imperfect spectrum sensing (ISS) [9].

III. Estimation of the Channel Duty Cycle

The channel DC (also referred to in the literature as the channel load) is conventionally estimated as in [10] by dividing the number of busy sensing events over the total number of sensing events. This is the approach most widely used in the literature but it is very sensitive to the presence of sensing errors. Another method has been also proposed (in the context of PSS) to estimate the DC of the channel by relying on the mean of idle/busy periods as in [13]:

$$\Psi = \frac{\mathbb{E}(T_1)}{\mathbb{E}(T_1) + \mathbb{E}(T_0)} \tag{1}$$

where (Ψ) represents the DC of the channel, $\mathbb{E}(\cdot)$ denotes the mean or expected value, and T_0 and T_1 are the durations of the idle and busy periods of the primary channel, respectively. The estimated idle/busy periods $\widehat{T_i}$ under PSS can serve to obtain an accurate estimation for the mean $\mathbb{E}(\widehat{T_i})$ and therefore an accurate estimation for the DC of the channel as well. However, under ISS the estimated idle/busy periods $\breve{T}_i$ could be highly inaccurate due to the presence of sensing errors (as discussed in Section II) and subsequently the estimated mean of these periods will be inaccurate as well. As a result, the estimation of the channel DC using (1), which relies solely on the mean of idle/busy periods, will be highly inaccurate under ISS. Therefore, this work aims, first, to find a method to accurately estimate the mean $\mathbb{E}(T_i)$ of the idle/busy periods under the presence of sensing errors (i.e., ISS), then to use this method along with (1) in order to obtain a novel estimator for the DC with high level of accuracy even when the probability of errors is high. The achieved DC estimator will then be compared with the conventional method [10], which depends on the sensing events rather than the mean periods.

IV. Mathematical Analysis of the Mean of the Channel Periods Under ISS

Given a set $\{\widehat{T}_{i,n}\}_{n=1}^{N_{pss}}$ of N_{pss} estimated periods under PSS, the mean $\mathbb{E}(\widehat{T}_i)$ of the observed periods can be found based on the corresponding (unbiased) sample mean estimator $\widehat{m}_i$:

$$\mathbb{E}(\widehat{T}_i) \approx \widehat{m}_i = \frac{1}{N_{pss}} \sum_{n=1}^{N_{pss}} \widehat{T}_{i,n} \tag{2}$$

In ISS, when an error occurs in the sensing decisions (i.e., P_{fa} and/or $P_{md} > 0$), the estimated period durations will be divided into shorter periods as shown in Fig. 1(b) and discussed in [9]. As it can be noticed, a single false alarm error could corrupt the estimation of T_0 by producing three new shorter period durations, namely $\breve{T}_0, \breve{T}_1$, and $\breve{T}_0$. The produced duration of $\breve{T}_1$ is equal to the duration of the sensing period T_s, while the two other durations of $\breve{T}_0$ are random, depending on the position of the error itself within T_0. The same principle applies to missed detection errors within busy periods T_1.

In terms of the number of periods, each false alarm will produce an additional estimated idle period $\breve{T}_0$ and an additional estimated busy period $\breve{T}_1$ (except some cases which will be explained later on). As shown in Fig. 1(b), T_0 period is estimated as two $\breve{T}_0$ and one $\breve{T}_1$ periods. If there were 2 false alarms within T_0, they would result in 3 $\breve{T}_0$ and 2 $\breve{T}_1$ periods, and so on. Under ISS, the mean of the observed periods $\breve{T}_i$ may differ significantly from the true mean of the periods T_i (unlike in PSS where there are no additional periods produced during the spectrum sensing process and the accuracy is only affected by the time resolution T_s of the periodic sensing events). In order to find the relationship between the estimated mean under ISS $\mathbb{E}(\breve{T}_i)$ and the true mean $\mathbb{E}(T_i)$ of the channel periods, we first find its relationship with the estimated mean under PSS $\mathbb{E}(\widehat{T}_i)$. Given a set $\{\breve{T}_{i,n}\}_{n=1}^{N_{iss}}$ of N_{iss} estimated periods under ISS, the mean $\mathbb{E}(\breve{T}_i)$ can be found by:

$$\mathbb{E}(\breve{T}_i) \approx \breve{m}_i = \frac{1}{N_{iss}} \sum_{n=1}^{N_{iss}} \breve{T}_{i,n} \tag{3}$$

By considering, without loss of generality, the idle periods in this analysis (i.e., $i = 0$), and by taking the primary channel periods illustrated in Fig. 1(b) as an example, we can write:

$$\breve{m}_0 = \frac{1}{2} \sum_{n=1}^{2} \breve{T}_{0,n} = \frac{\breve{T}_{0,1} + \breve{T}_{0,2}}{2} = \frac{\widehat{T}_0 - \breve{T}_1}{2} = \frac{\widehat{T}_0 - T_s}{2}$$

The summation of both idle periods ($\breve{T}_{0,1}$ and $\breve{T}_{0,2}$) is similar to subtracting $\breve{T}_1$ from the estimated $\widehat{T}_0$ period under PSS, knowing that the produced $\breve{T}_1$ period from the false alarm error is equal to the sensing period T_s. In addition, the denominator 2, which represents the number of the estimated idle periods under ISS (i.e., N_{iss}), can be substituted with the number of estimated idle periods under PSS plus one for the single false alarm (i.e., $N_{iss} = N_{pss} + 1$). Therefore, this analysis of a single false alarm error within a single idle period can be extended to a general form for any arbitrary number of false alarm errors within the whole set of idle periods as:

$$\breve{m}_0 = \frac{\sum_{n=1}^{N_{pss}} \widehat{T}_{0,n} - N_{fa} T_s}{N_{pss} + N_{fa}} \tag{4}$$

where N_{fa} represents the number of false alarm errors in the entire set of observed periods and can be found by multiplying the entire number of $\mathcal{H}_0$ events by the probability of false alarm error P_{fa} as:

$$N_{fa} = \frac{\sum_{n=1}^{N_{pss}} \widehat{T}_{0,n}}{T_s} \cdot P_{fa} \tag{5}$$

The above analysis has assumed no missed detection errors in the spectrum ($P_{md} = 0$). In order to find $\breve{m}_0$ by considering missed detection errors as well, a similar analysis can be applied so that (4) can be rewritten to include both error types:

$$\breve{m}_0 = \frac{\sum_{n=1}^{N_{pss}} \widehat{T}_{0,n} - N_{fa} T_s + N_{md} T_s}{N_{pss} + N_{fa} + N_{md}} \tag{6}$$

where N_{md} represents the number of missed detection errors in the entire set of observed periods and can be found by multiplying the entire number of $\mathcal{H}_1$ events by the probability of missed detection error P_{md} as:

$$N_{md} = \frac{\sum_{n=1}^{N_{pss}} \widehat{T}_{1,n}}{T_s} \cdot P_{md} \tag{7}$$

By substituting (5) and (7) in (6), and by replacing the term $\sum_{n=1}^{N_{pss}} \widehat{T}_{i,n}$ with $N_{pss}\widehat{m}_i$ (from (2)), we can simplify the final expression of $\breve{m}_0$ to:

$$\breve{m}_0 = \frac{\widehat{m}_0 - \widehat{m}_0 P_{fa} + \widehat{m}_1 P_{md}}{1 + \frac{\widehat{m}_0}{T_s} P_{fa} + \frac{\widehat{m}_1}{T_s} P_{md}} \tag{8}$$

This equation is not totally accurate because, as mentioned earlier, there will be some cases where a false alarm will not produce additional estimated idle periods ($\breve{T}_0$) and additional estimated busy periods ($\breve{T}_1$). This will be analysed in the following two particular cases.

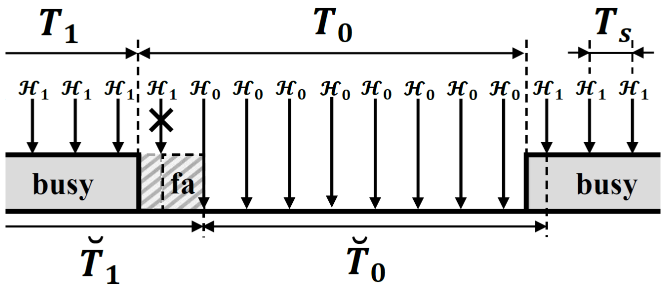


Fig. 2. Case 1: A single sensing error at the edge of a period.

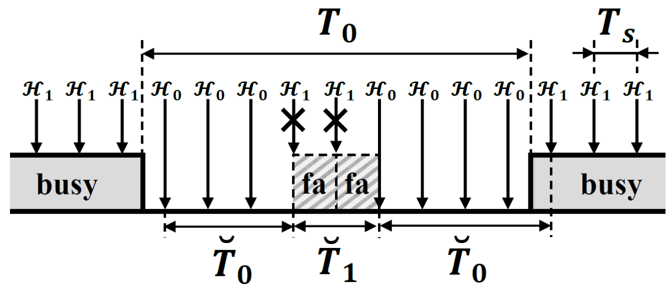


Fig. 3. Case 2: Two consecutive sensing errors in the middle of a period.

A. Case 1

When a false alarm occurs at the edge of the estimated idle period, the period $\breve{T}_1$ resulting from the false alarm itself will be combined with the previous (or next) estimated $\breve{T}_1$ period. As a result, there will be no additional $\breve{T}_0$ or $\breve{T}_1$ periods produced due to such false alarm as shown in Fig.2. Similar phenomenon can also appear with missed detection errors. This case will affect the calculations in (6) and therefore in (8) for finding the sample mean ($\breve{m}_0$), so that the denominator in (6), which represents the number of the estimated idle periods under ISS ($N_{iss} = N_{pss} + N_{fa} + N_{md}$), should not count the cases when the errors appear at the edges (as there will be no additional periods produced by them). This can be achieved by understanding that each estimated idle/busy period will have two edges, and these edges are actually represented by the sensing events $\mathcal{H}_0$/$\mathcal{H}_1$. Therefore, by subtracting 2 from the number of events of a single period (or $2N_{pss}$ from the entire number of the events of N_{pss} periods) will solve the problem in (6) and the resulting (8) regarding this case.

B. Case 2

Another case where a false alarm error will not produce additional periods is when it occurs consecutively to another false alarm error as shown in Fig. 3. Consecutive errors will have the same effect of a single error in terms of the number of produced periods. Therefore, the two consecutive errors in Fig. 3 result in three new shorter periods, $\breve{T}_0$, $\breve{T}_1$, and $\breve{T}_0$ (which are the same as the resulting number of estimated periods when we have a single false alarm). Since the consecutive errors will not produce additional periods, the denominator of (6) should therefore not count these errors and this can be achieved by subtracting the probability of having consecutive false alarm errors from the probability of false alarm error P_{fa} itself as:

$$\grave{P}_{fa} = P_{fa} - \sum_{j=2}^{\infty} P_{fa}^{j} = P_{fa}\left(\frac{1-2P_{fa}}{1-P_{fa}}\right) \tag{9}$$

where $\grave{P}_{fa}$ represents the probability of having false alarms as individual periods (i.e., not in terms of the number of errors). This also applies to the consecutive missed detection errors:

$$\grave{P}_{md} = P_{md} - \sum_{j=2}^{\infty} P_{md}^{j} = P_{md}\left(\frac{1-2P_{md}}{1-P_{md}}\right) \tag{10}$$

Both (9) and (10) will solve the problem in (6) and the resulting (8) regarding this case.

Therefore, after taking these two cases into consideration (i.e., Case 1 and Case 2) and applying the changes to the denominator of (6), the resulting (8) can be rewritten as:

$$\breve{m}_0 = \frac{\widehat{m}_0 - \widehat{m}_0 P_{fa} + \widehat{m}_1 P_{md}}{1 + \left(\frac{\widehat{m}_0}{T_s} - 2\right)\grave{P}_{fa} + \left(\frac{\widehat{m}_1}{T_s} - 2\right)\grave{P}_{md}} \tag{11}$$

This expression provides the relationship between the estimated mean $\breve{m}_0$ under ISS and the estimated means $\widehat{m}_i$ under PSS. Since the analysis in [13] has shown that the estimated mean under PSS is approximately equal to the true mean of the channel periods (i.e., $\widehat{m}_i \approx m_i$), the metrics $\widehat{m}_0$ and $\widehat{m}_1$ in (11) can be substituted with m_0 and m_1, respectively. In addition, since the obtained expression does not depend on the number of periods N_{pss}, the sample mean $\breve{m}_0$ in (11) can be rewritten as population mean $\mathbb{E}(\breve{T}_0)$. As a result, a final closed-form expression for the estimated mean of idle/busy periods $\mathbb{E}(\breve{T}_i)$ under ISS can be written as a function of the true mean $\mathbb{E}(T_i)$, P_{fa}, P_{md}, and T_s:

$$\mathbb{E}(\breve{T}_i) = \frac{\mathbb{E}(T_i) - (-1)^i\,\mathbb{E}(T_0)P_{fa} + (-1)^i\,\mathbb{E}(T_1)P_{md}}{1 + \left(\frac{\mathbb{E}(T_0)}{T_s} - 2\right)\grave{P}_{fa} + \left(\frac{\mathbb{E}(T_1)}{T_s} - 2\right)\grave{P}_{md}} \tag{12}$$

This relationship suggests a new novel method to accurately estimate the true mean of the channel periods and subsequently the channel DC based on the outcomes of the ISS.

V. Novel Method to Estimate the Mean of the Channel Periods Under ISS

In this section we propose a novel method to estimate the true value of the mean of idle/busy periods based on the outcomes of the ISS observations. The achieved mathematical model in (12) gives the following two expressions:

$$\mathbb{E}(\breve{T}_0) = \frac{\mathbb{E}(T_0) - \mathbb{E}(T_0)P_{fa} + \mathbb{E}(T_1)P_{md}}{1 + \left(\frac{\mathbb{E}(T_0)}{T_s} - 2\right)\grave{P}_{fa} + \left(\frac{\mathbb{E}(T_1)}{T_s} - 2\right)\grave{P}_{md}} \tag{13}$$

$$\mathbb{E}(\breve{T}_1) = \frac{\mathbb{E}(T_1) + \mathbb{E}(T_0)P_{fa} - \mathbb{E}(T_1)P_{md}}{1 + \left(\frac{\mathbb{E}(T_0)}{T_s} - 2\right)\grave{P}_{fa} + \left(\frac{\mathbb{E}(T_1)}{T_s} - 2\right)\grave{P}_{md}} \tag{14}$$

We can solve (13) and (14) for the true mean values, $\mathbb{E}(T_0)$ and $\mathbb{E}(T_1)$, as a function of the mean values observed under ISS, $\mathbb{E}(\breve{T}_0)$ and $\mathbb{E}(\breve{T}_1)$, which is shown in (15) and (16), respectively. By substituting (16) in (15), a new expression can be derived for $\mathbb{E}(\widetilde{T}_0)$ to represent the correct estimation

$$\mathbb{E}(T_0) = \mathbb{E}(\breve{T}_0)\frac{1 - 2\grave{P}_{fa} - 2\grave{P}_{md}}{1 - P_{fa} - \frac{\grave{P}_{fa}}{T_s}\mathbb{E}(\breve{T}_0)} + \mathbb{E}(T_1)\frac{\frac{\grave{P}_{md}}{T_s}\mathbb{E}(\breve{T}_0) - P_{md}}{1 - P_{fa} - \frac{\grave{P}_{fa}}{T_s}\mathbb{E}(\breve{T}_0)} \tag{15}$$

$$\mathbb{E}(T_1) = \mathbb{E}(\breve{T}_1)\frac{1 - 2\grave{P}_{fa} - 2\grave{P}_{md}}{1 - P_{md} - \frac{\grave{P}_{md}}{T_s}\mathbb{E}(\breve{T}_1)} + \mathbb{E}(T_0)\frac{\frac{\grave{P}_{fa}}{T_s}\mathbb{E}(\breve{T}_1) - P_{fa}}{1 - P_{md} - \frac{\grave{P}_{md}}{T_s}\mathbb{E}(\breve{T}_1)} \tag{16}$$

$$\mathbb{E}(T_i) \approx \mathbb{E}(\widetilde{T}_i) = \frac{\left(\mathbb{E}(\breve{T}_i)(1 - P_{md}^{1-i}P_{fa}^{i}) - \mathbb{E}(\breve{T}_{1-i})P_{md}^{1-i}P_{fa}^{i}\right)\left(1 - 2\grave{P}_{fa} - 2\grave{P}_{md}\right)}{\left(1 - P_{fa} - \frac{\grave{P}_{fa}}{T_s}\mathbb{E}(\breve{T}_0)\right)\left(1 - P_{md} - \frac{\grave{P}_{md}}{T_s}\mathbb{E}(\breve{T}_1)\right) - \left(\frac{\grave{P}_{fa}}{T_s}\mathbb{E}(\breve{T}_1) - P_{fa}\right)\left(\frac{\grave{P}_{md}}{T_s}\mathbb{E}(\breve{T}_0) - P_{md}\right)} \tag{17}$$

of the true mean $\mathbb{E}(T_0)$ as a function of the estimated mean under ISS (i.e., $\mathbb{E}(\breve{T}_0)$ and $\mathbb{E}(\breve{T}_1)$), probability of errors (i.e., P_{fa} and P_{md}), and sensing period T_s. A similar estimator $\mathbb{E}(\widetilde{T}_1)$ can be derived for the busy periods by substituting (15) in (16). Therefore, a final expression of $\mathbb{E}(\widetilde{T}_i)$, for both idle and busy periods, is given in (17), which represents a novel method to accurately estimate the true mean of the channel periods based on the estimated mean under ISS, probability of errors, and sensing period. Notice that the probability of errors and sensing period are both dependent on the configuration of the spectrum sensing algorithm and are therefore known, thus this method is feasible in practical implementation, as opposed to most of the previous work in the literature.

VI. Novel Method to Estimate the Channel Duty Cycle Under ISS

The calculation of the channel duty cycle given in (1) relies solely on the mean of the idle/busy periods. In ISS, as discussed in Section III, the estimation of the mean $\mathbb{E}(\breve{T}_i)$ is significantly inaccurate due to the sensing errors and consequently will provide an inaccurate estimation for the DC as well. On the other hand, since the analysis of the statistical moment (mean) in Section V has resulted in a novel method to accurately estimate the true value of the mean of the idle/busy periods under ISS, it can also lead to find another novel method to accurately estimate the DC of the channel under ISS by substituting the achieved mean estimator of (17) in (1). The DC can then be estimated as:

$$\widetilde{\Psi} = \frac{\mathbb{E}(\widetilde{T}_1)}{\mathbb{E}(\widetilde{T}_1) + \mathbb{E}(\widetilde{T}_0)} \tag{18}$$

The performance of this novel estimator is assessed below.

VII. Validation and Simulation Results

All mathematical analyses and obtained closed-form expressions in previous sections have been examined to prove their validity by comparing them with simulation results from MATLAB. The idle/busy periods of the primary channel are generated in the simulation (similar to the procedure in [9]) to follow Generalised Pareto distribution, and the true mean of the idle/busy periods are set to 50 time units (t.u.). The simulation results in this section consider the idle periods of the primary channel, however similar results can be obtained for busy periods as well. In Fig. 4, the relative error of calculating the mean of idle periods $\mathbb{E}(\breve{T}_i)$ under ISS matches

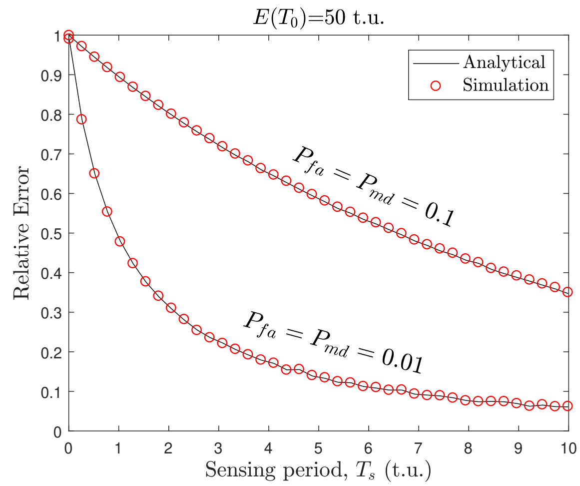


Fig. 4. Relative error of the calculated mean $\mathbb{E}(\breve{T}_0)$ under ISS.

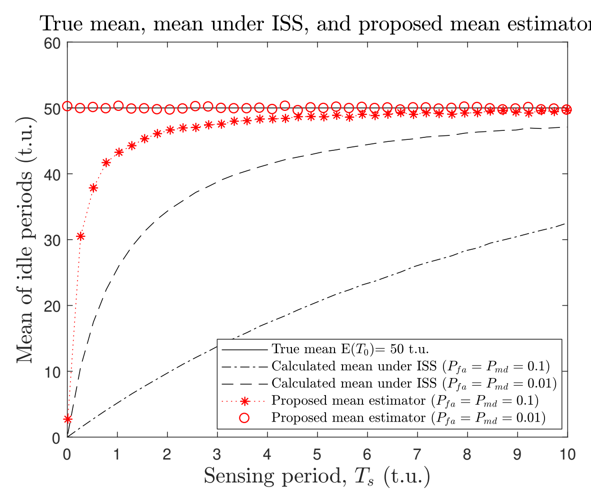
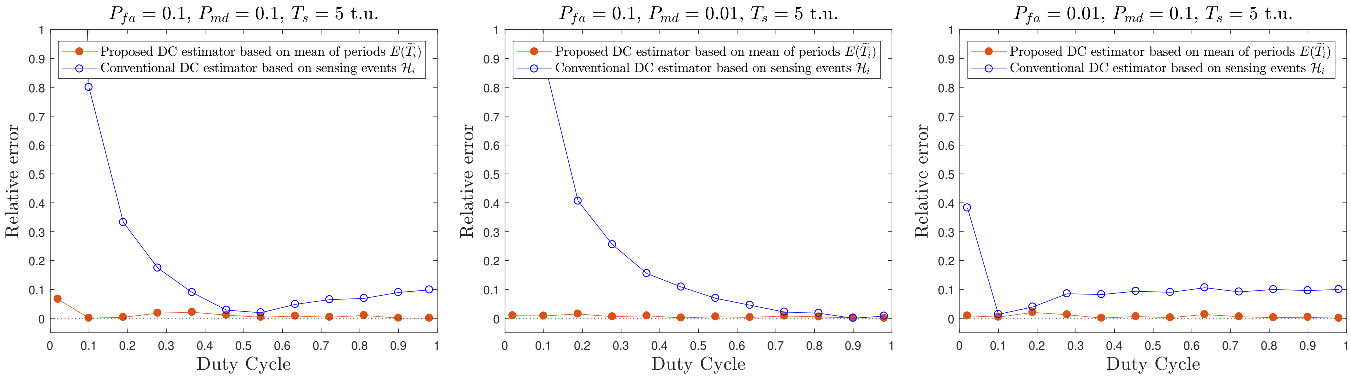


Fig. 5. Accuracy of the proposed mean estimator $\mathbb{E}(\widetilde{T}_i)$ as compared with the true mean $\mathbb{E}(T_i)$ and the calculated mean under ISS $\mathbb{E}(\breve{T}_i)$.

the one which is calculated using the mathematical model in (12) using different values for the parameters P_{fa}, P_{md}, and T_s, which validates the correctness of the analytical results obtained in this work.

The novel method in (17), which represents the proposed estimator $\mathbb{E}(\widetilde{T}_i)$ to accurately estimate the mean of periods under ISS, has also been tested for accuracy validation. As it can be seen in Fig. 5, even when the probability of errors is



(a) (b) (c)

Fig. 6. Relative error of the estimated Duty Cycle under ISS for different values of P_{fa} and P_{md} when $T_s = 5$ t.u..

high ($P_{fa} = P_{md} = 0.1$) the true mean of 50 t.u. can still be estimated with high accuracy, especially when the sensing period T_s is greater than 1 t.u., while it can be perfectly estimated when $P_{fa} = P_{md} = 0.01$.

Finally, the novel DC estimation method in (18), which represents the proposed estimator $\widetilde{\Psi}$ to accurately estimate the channel DC under ISS, shows significant accuracy with almost zero relative error as shown in Fig. 6. In addition, regardless of how severely the estimated periods are affected by sensing errors (i.e., P_{fa} and P_{md}) and what sensing period T_s is used, this method keeps providing almost perfect estimation and it noticeably outperforms the method commonly used in the literature, which relies on the individual sensing events (due to the lack of space, results are shown in Fig. 6 only for $T_s = 5$, however, similar trends were observed for other values of T_s).

VIII. Conclusion

The performance of DSA/CR systems is strongly dependent on how often the spectrum is occupied by the primary users. Therefore, the statistical knowledge about the DC of the primary channels is significantly valuable, so that it can be exploited in several ways to improve the performance of DSA/CR systems. Such information can be estimated based on the outcomes of spectrum sensing decisions. However, in practical operation of the DSA/CR system, errors in the spectrum sensing are not avoidable and may result in inaccurate estimation of the channel statistics. In this context, this work has analysed the impact of sensing errors on the estimation of the statistical moment (mean) by providing a closed-form expression for the estimated mean under ISS as a function of the true mean, probability of errors, and sensing period. It has then proposed novel methods to provide high accuracy estimation for the true value of the mean of the channel periods and the channel DC based on the outcomes of ISS. The validity of the analyses and the novel methods have been proven by means of simulation and the obtained results have shown significant improvement with respect to the conventional method based on individual sensing events, leading to a virtually perfect estimation of the primary channel DC in the presence of sensing errors, regardless of the spectrum sensing configuration and operation conditions.

Acknowledgment

This work was supported by the British Council under UKIERI DST Thematic Partnerships 2016-17 (ref. DST-198/2017). K. Umebayashi would like to thank the support of the JSPS KAKENHI Grant No. JP15K06053 & JP15KK0200.

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