UNIVERSITY OF LIVERPOOL

Distributing Resources: An Exploration of Auctions and Other Allocation Mechanisms

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Abstract

This thesis consists of four largely independent essays on game theory and the allocation of resources. It applies both theoretical and empirical methods, covering both cooperative and non-cooperative game theoretical approaches. The thesis focuses on three different allocation mechanisms: auctions, grandfathering and the Shapley value. The first two mechanisms are applied directly within the fisheries context whilst the Shapley value is discussed in a general setting. Theoretical applications often employ a number of assumptions, which are rarely met in practice, and fail to take into account the contextual setting. This thesis empirically analyses real life examples in order to enhance our understanding of individual players and their interactions. The first chapter on the famous solution concept — the Shapley value — introduces a cooperative game in which some of the information may be missing from the characteristic function. We put forward two ways of extending the Shapley value to the missing information domain. Using our proposals, it is possible to find a 'fair and efficient solution' in situations where coalitions might not have formed or information is withheld. The second chapter presents a theoretical model for fisheries which is capable of exploring the effects of utilising different combinations of allocation mechanisms — grandfathering and auctions — for fishing quota. In the model, the government has applied an environmental tax, which can be reduced through investment, to limit the negative externalities of the fishing activity. This in turn affects the firms' valuations at the auctions, incorporating bidder valuations as an endogenous component within the model. The model demonstrates the effects of changing specific variables within the government's control on revenue, total welfare, level of investment and equilibrium prices at the auction. We conclude by stating that the model is applicable in the fisheries context, but further work is required to improve its relevance in the policy arena. The third chapter is an empirical essay on the ascending and uniform-price multiunit auctions of fishing rights in the Faroe Islands from 2016 to 2018. In the essay, we identify problems with signalling, bidding rings and low-price equilibria. We believe this is the first case of all three phenomena happening in the same bidding environment. We conclude that the underperformance of ascending and uniform-price auctions are not just theoretical curiosities, but rather a pervasive phenomenon in practical auction design. We continue in the empirical domain in the fourth chapter, where we use several methods to analyse price patterns in the auctions at the Faroe Fish Market in 2017. We apply the method employed by Ashenfelter (1989) in his work on the declining price anomaly. We also run regressions to establish how the auction round affects absolute and relative price changes and explore the impact of the number of bids, quantity sold and the price of the first round. Finally, we analyse the average price of cod in every round across the whole period. We find that declining prices are a strong feature of the Faroese fish market, but after considering contributing factors, we conclude that perhaps declining prices should not come as such a surprise after all.

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Introduction

This thesis consists of four largely independent essays on game theory and the allocation of resources. “Game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision makers.” (Myerson, 1991, p. 1). Game theory offers a set of analytical tools designed to understand situations in which a decision maker’s behaviour affects his or her own gains and losses, as well as those of other decision makers (Pham Do, 2009). Since its development in the early 20th century, game theory has been applied in many fields; e.g. economics, political science and biology. The essays in this thesis largely focus on fisheries.

Game theory has been applied extensively to the topic of fisheries. Most of this literature focuses on management of shared fish stocks (e.g. Munro, 1979; Levhari and Mirman, 1980; Lindroos et al., 2007; Pham Do, 2009; Hannesson, 2011), but many also focus on fish market auctions (Armstrong, 2000; Fluvià et al., 2012; Gallegati et al., 2011; Salladarré et al., 2017; Pezanis-Christou, 2000) as well as fishing rights (e.g. Marszalec, 2018a; Anderson et al., 2011; Bromley, 2009). Here the focus is on three different allocation mechanisms: auctions, grandfathering and the Shapley value. The first two mechanisms are applied directly to fisheries whilst the Shapley value is discussed in a general setting.

The thesis applies both theoretical and empirical methods and covers both cooperative and non-cooperative game theoretical approaches. Existing literature on the topic does not typically pay much attention to the specific contextual setting (Hannesson, 2011) and theoretical applications often employ a number of assumptions, which are rarely met in practice. This thesis empirically analyses real-life examples in order to enhance our understanding of the behaviour of individual players and their interactions.

The first two chapters are theoretical essays, the first on the Shapley value, and the second on auctions and grandfathering. The latter two essays are empirical essays on one of the allocation mechanisms discussed: auctions.

The Shapley value discussed in the second chapter is one of the most famous solution concepts in cooperative game theory and rewards players based on their marginal contributions to possible coalitions (Young, 1988). A universal assumption of game theory is that the money worths of the coalitions are known. This work introduces a cooperative game in which some of the information may be missing from the characteristic function. On this domain, two different proposals of how to extend the Shapley value are considered. First, intuitive generalizations of the classical axioms of Shapley (1953) are suggested. Following this, the paper looks for domain restrictions on the set of known coalitions upon which the Shapley axioms characterize a unique solution. Second, a two-stage solution is proposed in which (i) an impartial observer could use the information contained in the characteristic function to fill in the missing information (ii) the Shapley value is then applied to the 'filled in' in game. The paper concludes that despite information being missing from the characteristic function, there are different ways of extending the Shapley value to the missing information domain. Using our proposal, it is therefore possible to find a 'fair and efficient solution' in potential situations where coalitions might not have formed or information is withheld.

In the second chapter, we make a first attempt at developing a theoretical model for fisheries exploring how different combinations of two allocation mechanisms — grandfathering which allocates quota based on historical catches and auctions, which any firm is allowed to participe in — perform in relation to key parameters. The allocation of fishing quota often presents a challenge in fisheries management and it is critically important for a succesful fishery. As another important component of fisheries management is to limit the negative externalities of the activity, this developed model allows the government to incentivize investment into greener and more environmentally friendly fishing technologies. In our setting, the government has applied an environmental tax, which can be reduced through investment, to achieve these environmental objectives. Therefore the firms' level of investment directly affects its valuation of quota sold at auction. This reduces the firms' externalities and decreases their marginal costs, leading to a higher valuation of the fishing quota sold at the auction. This means that bidder valuations are now endogenous within the model, which better captures the complexity related to agents' decisions. The developed model demonstrates how the changing of different government controlled variables (how much is auctioned or the resource fee of grandfathered quota) affect the government revenue, total welfare, the firms' level of investment as well as the equilibrium price at auction. Finally, it considers the applicability of the model to the fisheries context and how the model could be further developed to improve its policy relevance.

The third chapter analyses auctions of fishing rights that took place in the Faroe Islands from 2016 to 2018. Firstly, it describes the setting in which the auctions took place and secondly, it evaluates their performance with respect to collusion, entrants and price equilibria. Two different types of auctions took place: the ascending (or second-price) and uniform-price multiunit auctions. They both have appealing theoretical properties when bidders are symmetric and bid competitively. However, auction designers have long been skeptical about their use in practice (Bulow et al., 1999; Kagel and Levin, 2002). First, asymmetries due to value advantage in ascending (or second-price) auctions with a large common-value component can generate asymmetric equilibria with low revenues (Bikhchandani, 1988). Second, both ascending and uniform-price auctions are susceptible to collusion (Klemperer, 2002b). Sequential ascending auctions make it especially easy to form and coordinate bidding rings (Robinson, 1985). Third, uniform-price auctions are susceptible to low-price equilibria in which bidders can commit to coordinate on high bids for initial units and low bids for final, price-setting units in equilibrium (Hurlbut et al., 2004; Holmberg and Newbery, 2010) — what we in the paper name crank-handle bidding. All three of these patterns have been observed separately in certain settings among sophisticated and experienced bidders (Hurlbut et al., 2004; Holmberg and Newbery, 2010; Klemperer, 2004; Beckmann, 2004).

The essay highlights several problems with the auction design and demonstrates what we believe to be the first case of all three of the above-mentioned phenomena happening in the same bidding environment. Our findings indicate that the underperformance of ascending and uniform-price auctions are not just theoretical curiosities, but a pervasive phenomenon in practical auction design. Finally, we suggest straightforward improvements to the auction design which could have mitigated some of the problems identified in the Faroese auctions.

The fourth chapter analyses the auctions at the Faroese fish market in the period from January to December 2017. It adds to the body of literature on price trends in sequential auctions, which has grown considerably since Ashenfelter's (1989) discovery of the declining price anomaly (or afternoon effect) in wine auctions. This result amongst others has imposed an intriguing puzzle in the literature of sequential auctions because of its violation of famous theoretical works by Milgrom and Weber (1982) and Weber (1983) who predict constant prices in the case of independent private values (IPV) and increasing prices when buyers' valuations are affiliated. Since the discovery of this phenomenon economists have attempted to empirically test for this feature in different settings and markets with contradictory findings (Chanel et al., 1996; Ashenfelter, 1989; Raviv, 2006; Fluvià et al., 2012), although the majority find declining prices (Kells, 2001). However, most of these do not consider homogenous goods. Many attempts have also been made to explain this phenomenon theoretically (Black and de Meza, 1992; McAfee and Vincent, 1993; Engelbrecht-Wiggans and Kahn, 1999; Jeitschko, 1999). Fish markets typically sell lots of homogenous goods in sequential auctions. In this essay, we test for the declining price anomaly in the Faroese fish market. The market is small with many suppliers relative to buyers.

Several methods will be employed to test for declining prices at the Faroe Fish Market. We start by employing the method used by Ashenfelter (1989) in his influential paper on the declining price anomaly. We also run regressions to establish how the auction round affects absolute and relative price changes — both with respect to the first price of the day and to the price in the previous round. Furthermore, we explore the impact that the number of bids, quantity sold and the price of the first round of each species have on price trends. Finally, we apply the method used by Gallegati et al. (2011) by analysing the average price of cod in each round across the whole period. Here we also measure the strength of auction round as a predictor for average prices.

Our conclusions from employing all of the methods demonstrate that declining prices are a strong feature of the Faroese fish market both overall and for major species. We find that in 81 percent of all cases, the prices are declining from one lot to another, with the highest percentage for haddock, where the corresponding figure is 87 percent. We also establish that the auction round is a significant predictor of price trends. Finally, we discuss potential contributing factors to the declining prices and conclude that they should perhaps not come as a surprise considering the format and context of the auctions.

Chapter 1

Information and the Value of a Game

1.1 Introduction

One of the triumphs of cooperative game theory has been to answer the normative question "what money payoffs ought the players to receive in a game?" The answer to the question is most often to reward players by giving them the value proposed by Shapley (1953). The reason the Shapley value is such a compelling solution to the question is because of its simple axiomatic characterization. Furthermore, the Shapley value uses all the information in the characteristic function. However, as the calculation of the Shapley value requires information on the worths of all the coalitions, this is also a weakness. The Shapley value cannot be calculated in cases where information is missing from the characteristic function. Our paper addresses this problem by proposing two solutions for determining how the money worth of the grand coalition should be distributed amongst the players.[1]

[1]This chapter was co-written with Robert R. Routledge.

In their early survey on game theory, Luce and Raiffa (1958, p. 184) noted that the characteristic function "appears to be very well suited to a simple numerical representation of the power of coalitions in human situations." Since then, all the literature on cooperative game theory has taken it as given that a real number can be assigned to each coalition. But if the characteristic function genuinely represents a one-off interaction between the players, then there may be no basis on which to assign worths to the coalitions. A choice problem of this nature, when some relevant parameters are not known, and even probabilities cannot be assigned to them, is commonly referred to, especially in philosophical literature, as choice in the presence of ignorance.[2] An impartial observer may have to accept that some of the information is missing from the game. Even if the players *do* know the worths of the coalitions they are members of, they may not be able to prove what these worths are to the other players, let alone to an impartial observer.

Section 1.2 below takes the first step to address these problems by introducing a cooperative game with missing information. In these games the characteristic function is a mapping from the set of all coalitions into the union of the real line and the empty set. Therefore, the worth of some coalitions may be missing from the game. The only assumptions which are imposed upon the characteristic function are: (i) the worth of the grand coalition of all players is not empty (ii) the worth of the empty set is zero. This space of games includes the standard

[2] See, for example Peterson (2017, Ch. 3). If the characteristic function summarizes many interactions between the players, then perhaps a probabilistic assessment of the worths could be considered. There are many papers which have studied cooperative games in which the worths of the coalitions are uncertain. Recent papers which have studied core-type solutions for uncertain cooperative games include Habis and Herings (2011), studying the weak sequential core, and Routledge (2014) analyzing a variant of the sequential core. Granot (1977) and Suijs (2000) suggested nucleolus-type solutions for stochastic cooperative games.

cooperative game with transferable utility, when the worths of all coalitions are known, as a special case. On this wider domain, the paper tries to answer the question: how should the Shapley value be extended to this domain?

In this paper we suggest two different solutions to this problem, each with their own particular merits. First, the Shapley axioms are extended to the missing information domain. At this point a problem emerges: when information is missing from the characteristic function it may not be possible to reconcile the axioms of null player and efficiency. To remedy this problem, we search for domain restrictions on the set of coalitions whose worths are known which guarantee that there exists a value satisfying the axioms. It is found that there are two domains upon which the Shapley axioms continue to characterize a unique solution. The solution which is characterized by the axioms is an obvious extension of the Shapley value: the value gives each player the average of their marginal contributions to the players preceding them — but only in those marginal vectors with every player's marginal contribution to their predecessors being defined. When all the information is contained in the characteristic function, this value coincides with the Shapley value. These results are quite surprising. It might be thought that if information is missing from the characteristic function then nothing further can be said about what the Shapley value of a game ought to be. The existence of two domains with missing information, upon which there is a unique solution, which is an analogue of the Shapley value, shows that this is not the case.

The second solution which we propose to the problem is not axiomatic. Instead we draw upon the literature on social choice and economic justice to present a two-stage solution to the value problem. The intuition behind the solution is as follows. Suppose an impartial observer were to inspect the characteristic function at the first stage. The impartial observer would ideally like to apply the Shapley value as a solution but knows this is not possible because information is missing from the characteristic function. To solve the problem the observer chooses to assign worths to the unknown coalitions at the first stage. At the second stage the observer applies the Shapley value to the game and rewards each player accordingly. Such a process will inevitably involve moral considerations given that the observer is free to assign worths to the unknown coalitions at the first stage. To see this, consider assigning a worth to some coalition $S \subset N$ in a game. The higher the worth assigned to S, other things equal, the lower is the average marginal contribution of players $i \in N \setminus S$, and the higher is the average marginal contribution of players $i \in S$. Therefore, when assigning worths to unknown coalitions, the trade-offs between different players would have to be taken into consideration.

The suggestion which we propose as to how the observer should assign worths at the first stage is drawn from the literature on social choice. When choosing what worths to assign to the unknown coalitions we suggest that the observer should choose from those feasible vectors which yield lexicographically maximal Shapley value payoff vectors at the second stage. In other words, the observer tries to ensure that the worst-off player receives their highest payoff across all the possible ways of filling in the missing information. This is a normative judgement, although it is one which is frequently used in the literature on economic justice. In outlining a general theory of economic justice, Rawls (1972, pp. 42-45) advanced the lexical procedure in which society's institutions could be organized to maximize the welfare of the worst-off citizen. The Shapley value is often interpreted as simple theory of economic justice, especially in the axiomatizations of Young (1985, 1988). When all the information is contained in the characteristic function, the Shapley value does not involve any special moral judgements, such as the lexical procedure; it rewards players solely on the basis of their marginal contributions. However, when information is missing from the characteristic function it opens up the possibility of additional considerations, such as maximizing the welfare of the least well-off player in the game. In responding to Rawls' theory of justice, Nozick (1974) presented an alternative historical-based theory of economic holdings based upon individuals' rights. The Shapley value is probably closer in spirit to Nozick's conception of justice in rewarding players solely on the basis of their marginal contributions.[3] But, even Nozick (1974, p. 166) conceded that:

> Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice may be made by a social choice mechanism based upon a social ordering; if there are any choices left to make! Rights do not determine a social ordering but instead set the constraints within which a social choice is to be made, by excluding certain alternatives, fixing others, and so on.

In Nozick's language, when all information is contained in the characteristic function, and we wish to reward the players solely on the basis of their marginal con-

[3] An interesting and accessible discussion of different theories of justice, including the Shapley value, is contained in Yaari (1981).

tributions, in addition to satisfying symmetry and efficiency, there are no choices left to make! The Young (1985, 1988) axiomatizations prove that the only payoff vector which meets these conditions is the Shapley value. When information is missing from the characteristic function, however, there may well be additional choices to be made, although the information which is contained in the characteristic function will constrain the payoffs which can be achieved by filling in the information missing from the game. In Section 1.4 it is demonstrated that the two-stage maximization problem has a solution, and that, once the set of vectors from which the impartial observer can choose is fixed, the payoff vector which results is unique.

The two-stage solution is also interesting from another point of view. The two most famous values for cooperative games are Shapley (1953) and the nucleolus of Schmeidler (1969). These two solutions are logically distinct. However, the two-stage solution which we propose brings together ideas from both the Shapley value and the nucleolus.[4] The impartial observer uses a nucleolus-type procedure at the first stage by lexicographically maximizing the ascending payoff vector, and then applies the Shapley value at the second stage.[5]

1.1.1 Related Literature

The literature on axiomatic value theory has its roots in a paper by Shapley (1953). He demonstrated that the axioms of efficiency, symmetry, additivity, along with an axiom relating to carrier games (which is usually restated as null

[4]Many of the results presented in Section 1.4 use similar proof techniques as used in establishing the well-known properties of the nucleolus.

[5]This paper is not the first to consider two-stage solutions to cooperative game. Charnes and Granot (1977) proposed an early two-stage solution to stochastic cooperative games.

player), characterize a unique solution. Since Shapley's brief but striking paper, originally circulated by the RAND Corporation, there has been sustained interest in axiomatic value theory.[6] This review only covers the most influential and recent contributions to the literature.[7] Schmeidler (1969) suggested an alternative value solution. The nucleolus, which was Schmeidler's proposal, minimizes the maximal excess of the coalitions according to the lexicographic order. The nucleolus belongs to the epsilon-core whenever it is non-empty and is continuous on the space of games. Aumann and Dreze (1974) studied cooperative games with coalition structures in which a partition of the players defined which coalitions could form. Myerson (1977) analysed cooperative games played upon graphs and axiomatized a value for such games. In addition, the stability of different graph structures was studied.[8] In a series of papers, Young (1985, 1988) presented a different axiomatization of the Shapley value. Young dispensed with the mathematically convenient axiom of additivity and replaced it with the economically meaningful axiom of marginality.[9] Young's marginal axiomatization proved that the Shapley value is the only solution which rewards players solely on the basis of their own marginal contributions, while satisfying symmetry and efficiency. Hart and Mas-Colell (1989) demonstrated that there is a function, the potential, which assigns a unique number to each cooperative game. Each player's marginal contri-

[6]Shapley himself noted that "It is remarkable that no further conditions are required to determine the value uniquely."

[7]A clear and up-to-date textbook coverage of value theory is contained in Maschler et al. (2013). See especially chapters 18 and 20 on the Shapley value and nucleolus respectively. Mas-Colell et al. (1995, pp. 679-684 and pp. 846-849) also presents a succinct summary of the ideas behind the Shapley value.

[8]The works of Aumann and Dreze (1974), and Myerson (1977), have generated a large literature in which coalitions are restricted from forming. An excellent survey of the literature is Gilles (2010, esp. chapters 3 & 6). Our work is related to this literature, but we take a more fundamental approach in assuming that information is missing from the characteristic function, and we do not impose any structure upon the coalitions which can form.

[9]Young (1985) used the axiom of strong monotonicity. However, strong monotonicity implies marginalism.

bution to the potential is equal to their Shapley value payoff. They also proposed a concept of consistency for single-valued solutions and proved that the Shapley value is consistent. Nowak and Radzik (1994) advanced a value solution which had a similar axiomatization to the Shapley value. They proposed a solidarity value in which each player received the average of the marginal contributions of the players preceding them in each permutation. By replacing null player with an average null player axiom, and by finding a basis different from the usual unanimity game basis, they were able to axiomatize the solidarity value. In van den Brink (2007) the null player axiom was replaced with a nullifying player axiom, and it was demonstrated that the axioms of efficiency, symmetry, additivity and nullifying player characterize the equal division solution. Most closely related to our work is Aguilera et al. (2010), which also considered ways of extending the Shapley value to characteristic functions in which arbitrary information may be missing and there is no graph or coalition structure imposed upon the player set. They proposed two main methods: one based upon graph-theoretic axioms the other an axiomatic system imposed directly upon the space of unanimity games. Our solutions are different and draw more upon the literature from social choice theory as we search for domain restrictions and consider explicit justice considerations. Malawski (2013) studied general procedural values in which the marginal contributions of a player's predecessors can be distributed in different ways. These procedural values include the Shapley value, the solidarity value and the equal division solution as special cases. In Casajus (2015) ideas from value theory were applied to a simple economy. The axioms of efficiency in redistribution, symmetry and monotonicity were shown to characterize overall proportional taxation rules. Routledge (2016) analysed the same model as in this paper and proposed a generalization of the equal division solution. An axiomatic characterization of the value was presented using the nullifying player axiom of van den Brink (2007).

1.1.2 Outline of the Paper

The next section of the paper sets out the model and notation used throughout the paper. In Section 1.3 generalizations of the Shapley axioms to the missing information domain are introduced, and two interesting domain results are presented. In Section 1.4 the two-stage approach to solving a game with missing information is set out. The last section of the paper contains some concluding remarks, and points to some intriguing possibilities for future research.

1.2 The Model

Let $N = \{1, ..., n\}$, $n \geq 2$, be the finite set of players in the game. A **cooperative game with missing information** is (N, v) with v being the characteristic function $v : 2^N \to \mathbb{R} \cup \{\emptyset\}$. A **coalition** is a $S \subseteq N$. The **grand coalition** is N. For any coalition $S \subseteq N$, if $v(S) \neq \emptyset$, then $v(S)$ is the money **worth** which S can obtain independently of the other players. If $v(S) = \emptyset$, then the worth of coalition S is missing from the game.

Assumption 1.2.1 *For every* $v : 2^N \to \mathbb{R} \cup \{\emptyset\}$, $v(N) \neq \emptyset$ *and* $v(\emptyset) = 0$.

Let G denote the space of games satisfying Assumption 1.2.1. A **value** is a function $\varphi : G \to \mathbb{R}^N$. For each $v \in G$ a value assigns a unique payoff vector in

$\mathbb{R}^N$. For any $v \in G$ let the set $K(v)$ be

$$K(v) = \{S \subseteq N : v(S) \neq \emptyset\}.$$

The set $K(v)$ is the set of **known coalitions** in the game v. Two games $v, w \in G$ will be called **comparable** if $K(v) = K(w)$. The **marginal contribution** of player $i \in N$ to $S \subseteq N \setminus \{i\}$ in game v is

$$\Delta_i(S, v) = v(S \cup \{i\}) - v(S).$$

In what follows $\Delta_i(S, v) = \emptyset$ if either $v(S \cup \{i\}) = \emptyset$ or $v(S) = \emptyset$, or both. Let $\pi : N \to N$ be a bijection denoting the permutation of the players in the grandcoalition, and let Π denote the $n!$ permutations of the players. Then, for any $\pi \in \Pi$ the position of player i is given by $\pi(i)$. For any $\pi \in \Pi$, let π_{ij} be the permutation obtained by interchanging players i and j in permutation π. For any $\pi \in \Pi$ the set of **predecessors** of player i in π is

$$P_i(\pi) = \{j \in N : \pi(j) < \pi(i)\}.$$

The marginal contribution of player i to his predecessors in permutation π in game v is

$$\Delta_i^\pi(v) = v(P_i(\pi) \cup \{i\}) - v(P_i(\pi)).$$

Often in the paper we shall restrict attention to games which are subsets of the space G. For any $\Omega \subseteq 2^N$, let G^Ω be the set

$$G^\Omega = \{v \in G : K(v) = \Omega\}.$$

The set $\Omega \subseteq 2^N$ will be called **singleton-defined** if it satisfies the following three conditions: (i) there is a unique $i^* \in N$ such that $\{i^*\} \in \Omega$ and $\{\emptyset\} \in \Omega$ (ii) if $S \cap \{i^*\} \neq \emptyset$ then $S \in \Omega$ (iii) if $S \cap \{i^*\} = \emptyset$ then $S \notin \Omega$.

For $\Omega \subseteq 2^N$, let $\Pi(\Omega) \subseteq \Pi$ be the set

$$\Pi(\Omega) = \{\pi \in \Pi : \text{ for every } v \in G^\Omega \text{ and every } i \in N,\, \Delta_i^\pi(v) \neq \emptyset\}.$$

The set $\Pi(\Omega)$ is those permutations of the players in which every player's marginal contribution to his predecessors is known. If $\Omega = 2^N$, then $\Pi(\Omega) = \Pi$. However, if $\Omega \neq 2^N$, then $\Pi(\Omega)$ will be a strict subset of Π. The set $\Omega \subseteq 2^N$ will be called **permutational** if $\Pi(\Omega) \neq \emptyset$.

1.3 Domain Restrictions and the Shapley Axioms

In this section generalizations of the Shapley axioms to the missing information are advanced. To give the reader a better understanding of the problem being addressed, two example games are presented below. Later in the section it will be shown that the unique payoff vectors suggested as the solutions of the games are, in a special sense, the extension of the usual Shapley value solution.

Example 1.3.1 *Suppose the set of players is $N = \{1, 2, 3\}$. The worth of the grand coalition is $v(N) = 12$. The worths of the smaller coalitions are*

$$v(12) = 6, \quad v(1) = 2, \quad v(\emptyset) = 0$$

and

$$v(S) = \emptyset \quad \textit{for all other coalitions.}$$

How should the money worth of the grand coalition, 12, be distributed amongst the players? The payoff vector which this section proposes to the distribution problem is $(x_1, x_2, x_3) = (2, 4, 6)$.

Example 1.3.2 *Suppose the set of players is $N = \{1, 2, 3, 4\}$. The worth of the grand coalition is $v(N) = 30$. The worths of the smaller coalitions are*

$$v(123) = v(124) = v(134) = 24,$$

$$v(12) = 12, \quad v(13) = v(14) = 18, \quad v(1) = 6, \quad v(\emptyset) = 0$$

and

$$v(S) = \emptyset \quad \textit{for all other coalitions.}$$

Again, how should the money worth of the grand coalition, 30, be distributed amongst the players? The payoff vector which this section proposes to the distribution problem is $(x_1, x_2, x_3, x_4) = (6, 6, 9, 9)$.

1.3.1 The Axioms

How should the axioms of Shapley (1953) be generalized to the domain G? Here four axioms, which are analogues of the Shapley axioms, are presented. When no information is missing from the characteristic function the axioms coincide with the Shapley axioms.

Definition 1.3.1 *A value φ satisfies efficiency if $\sum_{i \in N} \varphi_i(N, v) = v(N)$ for every $v \in G$.*

Efficiency is the simplest of the axioms, and states that the value should distribute all of the money worth of the grand coalition between the players in every game.

In a game $v \in G$ two players $i, j \in N$ are **symmetric** if $v(S \cup \{i\}) = v(S \cup \{j\})$ for every $S \subseteq N \setminus \{i, j\}$. Because information may be missing from the characteristic function it may be that $v(S \cup \{i\}) = v(S \cup \{j\}) = \emptyset$.

Definition 1.3.2 *A value φ satisfies symmetry if, whenever $i, j \in N$ are symmetric in game $v \in G$, then $\varphi_i(N, v) = \varphi_j(N, v)$.*

The axiom of symmetry formalizes the idea that there should be "equal treatment of equals".[10]

Before defining the axiom of additivity, let us define the sum of games on the missing information domain.

Definition 1.3.3 *Suppose $v, w \in G$. If the games v and w are comparable, so $K(v) = K(w)$, the game $u = v + w$ is given by*

$$u(S) = \begin{cases} v(S) + w(S) & \text{if } S \in K(v); \\ \emptyset & \text{if } S \notin K(v). \end{cases} \tag{1.1}$$

Addition of two games is only defined when the games have the same set of known coalitions. The axiom of additivity links the payoffs across different games which can be added together.

Definition 1.3.4 *A value φ satisfies additivity if, whenever $v, w \in G$ are comparable, $\varphi_i(N, v + w) = \varphi_i(N, v) + \varphi_i(N, w)$ for every $i \in N$.*

Finally, the axiom of null player is the only axiom which states what payoff players should receive in games. A player $i \in N$ is a **null player** in game $v \in G$ if $\Delta_i(S, v) \in \{0, \emptyset\}$ for every $S \subseteq N \setminus \{i\}$. A null player in a game is a player whose marginal contributions are always either zero or not known.

[10] There is a subtle point about the axiom of symmetry. In cooperative games, symmetry is often equivalently stated in terms of the coalitional worths or in terms of the marginal contributions of the players. On the missing information domain, symmetry *has* to be stated in terms of the coalitional worths. If $v(S \cup \{i\}) = v(S \cup \{j\})$ for every $S \subseteq N \setminus \{i, j\}$ this implies $\Delta_i(S, v) = \Delta_j(S, v)$ for every $S \subseteq N \setminus \{i, j\}$. However, $\Delta_i(S, v) = \Delta_j(S, v)$ for every $S \subseteq N \setminus \{i, j\}$ does *not* imply $v(S \cup \{i\}) = v(S \cup \{j\})$ for every $S \subseteq N \setminus \{i, j\}$. This is because if $v(S) = \emptyset$ then $\Delta_i(S, v) = \Delta_j(S, v) = \emptyset$ regardless of whether $v(S \cup \{i\}) = v(S \cup \{j\})$.

Definition 1.3.5 *A value φ satisfies null player if, whenever $i \in N$ is a null player in game $v \in G$, then $\varphi_i(N, v) = 0$.*

As noted earlier, these four axioms coincide with the Shapley axioms when no information is missing from the characteristic function. Therefore, the model of a game with missing information, and the proposed axioms, contain Shapley's original characterization as a special case. The remainder of the section demonstrates that the axioms may continue to characterize a unique solution — depending upon the information which is missing from the characteristic function.

1.3.2 An Impossibility Result and Domain Restrictions

Does there exist a value satisfying the four axioms on the domain G? The next proposition shows that it is not possible to reconcile the axioms of efficiency and null player.

Proposition 1.3.1 *On the domain G:*

(a) No value satisfies the axioms of efficiency and null player.

(b) There exists a value satisfying efficiency, symmetry and additivity.

(c) There exists a value satisfying symmetry, additivity and null player.

Proof. (a) Suppose φ satisfies efficiency and null player on G. Consider the game with $N = \{1, 2\}$, and $v(N) = 1$, $v(1) = v(2) = \emptyset$, $v(\emptyset) = 0$. Then null player implies $\varphi_1(N, v) = \varphi_2(N, v) = 0$, and contradicts efficiency which requires $\sum_{i=1}^{2} \varphi_i(N, v) = 1$.

(b) Consider the value on G given by $\varphi_i(N, v) = v(N)/n$ for every $i \in N$. This value, which divides the worth of the grand coalition equally between the players, satisfies efficiency, symmetry and additivity.

(c) Consider the value on G given by $\varphi_i(N, v) = 0$ for every $i \in N$. This value satisfies symmetry, additivity and null player. ■

Part (a) of the previous result demonstrates that it is not possible to satisfy all four axioms on the domain G. However, values on the missing information domain can be further explored. There are some interesting results if we consider $\Omega \subseteq 2^N$ and look at the restricted domain G^Ω. That is, games which have set of known coalitions Ω.

Proposition 1.3.2 *Suppose $\Omega \subseteq 2^N$ is permutational. There exists a value on G^Ω satisfying efficiency, symmetry, additivity and null player.*

Proof. Consider the value given by

$$\varphi_i^*(N, v) = \sum_{\pi \in \Pi(\Omega)} \frac{1}{|\Pi(\Omega)|} \Delta_i^\pi(v) \text{ for every } i \in N. \tag{1.2}$$

As Ω is permutational, the set $\Pi(\Omega)$ is non-empty. The value satisfies all four axioms. ■

An obvious question is whether the value in equation 1.2 is the only value to satisfy the four axioms on G^Ω when Ω is permutational? The following result demonstrates that, without further domain restrictions, this is not always the case.

Let $\Omega \subseteq 2^N$ be permutational. Define the sets $\Pi^{In}(\Omega)$ and $\Pi^i(\Omega)$ to be

$$\Pi^{In}(\Omega) = \{\pi \in \Pi(\Omega) : \text{ there is an } i \text{ and } j \text{ such that } \pi_{ij} \in \Pi(\Omega)\}$$

$$\Pi^i(\Omega) = \{\pi \in \Pi(\Omega) : \pi(i) \leq \pi'(i) \text{ for every } \pi' \in \Pi(\Omega)\}$$

The set $\Pi^{In}(\Omega)$ is those permutations in $\Pi(\Omega)$ which can be obtained by interchanging any two players in another permutation in $\Pi(\Omega)$. The set $\Pi^i(\Omega)$ is those permutations in $\Pi(\Omega)$ in which player $i's$ rank is minimized.

Proposition 1.3.3 *Suppose $\Omega \subseteq 2^N$ is permutational. There may be more than one value on G^Ω satisfying efficiency, symmetry, additivity and null player.*

Proof. The value φ^* in equation 1.2 satisfies the four axioms. Consider the value φ' defined as

$$\varphi_i'(N,v) = \begin{cases} \varphi_i^*(N,v) & \text{if } \Pi^{In}(\Omega) \neq \emptyset; \\ \sum_{\pi \in \Pi^1(\Omega)} \frac{1}{|\Pi^1(\Omega)|} \Delta_i^\pi(v) & \text{if } \Pi^{In}(\Omega) = \emptyset. \end{cases} \tag{1.3}$$

The value φ' also satisfies the four axioms but can yield different payoff vectors to φ^*. ■

The following example demonstrates how the values in equations 1.2 and 1.3 can differ from each other in games.

Example 1.3.3 *Suppose $N = \{1,2,3,4\}$ and $\Omega = \{\emptyset, 1, 4, 12, 34, 123, 234, N\}$. The set $\Pi(\Omega)$ contains two permutations: 1234 and 4321. The set $\Pi^{In}(\Omega)$ is empty. The characteristic function*[11] *has $v(N) = 12$. The worths of the smaller coalitions are*

$$v(123) = 8, \quad v(234) = 12, \quad v(12) = v(34) = 6,$$

$$v(1) = v(4) = 4, \quad v(\emptyset) = 0$$

and

$$v(S) = \emptyset \quad \textit{for all other coalitions.}$$

The value φ^* gives each player their average marginal contribution over the two permutations

$$\varphi^* = (2, 4, 2, 4).$$

[11] Just to make the point made in footnote 10 clear, in this game $\Delta_1(S,v) = \Delta_4(S,v)$ for every $S \subseteq N \setminus \{1,4\}$. However, players 1 and 4 are *not* symmetric because $v(12) = 6 \neq v(24) = \emptyset$.

However, the value φ' gives each player their marginal contribution according to the permutation 1234 because in this permutation player 1's rank is minimal

$$\varphi' = (4, 2, 2, 4).$$

Suppose $\Omega \subseteq 2^N$ is permutational. A game $v \in G^{\Omega}$ can be thought of as a vector in $\mathbb{R}^{\Omega}$. For each $T \in \Omega \setminus \{\emptyset\}$ consider the following generalization of the standard unanimity game:

$$u_{T,\Omega}(S) = \begin{cases} 1 & \text{if } T \subseteq S, \quad S \in \Omega; \\ 0 & \text{if } T \nsubseteq S, \quad S \in \Omega; \\ \emptyset & \text{if } S \notin \Omega. \end{cases} \tag{1.4}$$

For any $\alpha \in \mathbb{R}$ the game $\alpha u_{T,\Omega}$ is given by:

$$\alpha u_{T,\Omega}(S) = \begin{cases} \alpha & \text{if } T \subseteq S, \quad S \in \Omega; \\ 0 & \text{if } T \nsubseteq S, \quad S \in \Omega; \\ \emptyset & \text{if } S \notin \Omega. \end{cases} \tag{1.5}$$

The result below follows from the well-known properties of the unanimity game, with addition defined as in equation 1.1.

Proposition 1.3.4 *Suppose $\Omega \subseteq 2^N$ and $v \in G^{\Omega}$. There exist unique real numbers $\{\alpha_T : T \in \Omega \setminus \{\emptyset\}\}$ such that*

$$v(S) = \sum_{T \in \Omega \setminus \{\emptyset\}} \alpha_T u_{T,\Omega}(S) \text{ for every } S \in \Omega.$$

Using the unanimity game basis, the final results of this section demonstrate that, on two interesting subsets of the permutational domain, the axioms characterize the value in equation 1.2. In the statement of the next result we stick with the convention that when the empty set, $\emptyset$, is an explicit subset of another set, it has cardinality one.

Proposition 1.3.5 *Suppose $\Omega \subseteq 2^N$ is permutational and $|\Omega| = n+1$. Then there is a unique value on G^Ω satisfying efficiency, additivity and null player.*

Proof. The value φ^* in equation 1.2 satisfies the axioms. All that remains to be demonstrated is that the value is uniquely determined by the axioms.

Suppose φ satisfies the axioms. Fix a $v \in G^\Omega$. From Proposition 1.3.4, v can be written as

$$v = \sum_{T \in \Omega \setminus \{\emptyset\}} \alpha_T u_{T,\Omega}.$$

As φ satisfies additivity

$$\varphi(N, v) = \sum_{T \in \Omega \setminus \{\emptyset\}} \varphi(N, \alpha_T u_{T,\Omega}).$$

Consider the game $\alpha_T u_{T,\Omega}$. As $|\Omega| = n+1$, there is a player $i \in T$ such that $\Delta_j(S, \alpha_T u_{T,\Omega}) \in \{\emptyset, 0\}$ for every $j \in N \setminus \{i\}$ and every $S \subseteq N \setminus \{j\}$. Null player implies $\varphi_j(N, \alpha_T u_{T,\Omega}) = 0$ for every $j \in N \setminus \{i\}$. Efficiency implies $\varphi_i(N, \alpha_T u_{T,\Omega}) = \alpha_T$. ■

In the previous result only the axioms of efficiency, additivity and null player were required to characterize the value in equation 1.2. The reason for this is that, when $\Omega \subseteq 2^N$ is permutational and $|\Omega| = n+1$, no players can be symmetric in any $v \in G^\Omega$. The final result of this section considers a richer domain on which the axiom of symmetry is required.

Proposition 1.3.6 *Suppose $\Omega \subseteq 2^N$ is singleton-defined. Then there is a unique value on G^Ω satisfying efficiency, symmetry, additivity and null player.*

Proof. If Ω is singleton-defined then Ω is also permutational, and the value in equation 1.2 satisfies the four axioms. Let i^* be the singleton-defining player. It remains to be demonstrated that the axioms uniquely determine the value.

Suppose φ satisfies the four axioms. Fix a $v \in G^\Omega$. From Proposition 1.3.4, v can be written as

$$v = \sum_{T \in \Omega \setminus \{\emptyset\}} \alpha_T u_{T,\Omega}.$$

As φ satisfies additivity

$$\varphi(N, v) = \sum_{T \in \Omega \setminus \{\emptyset\}} \varphi(N, \alpha_T u_{T,\Omega}).$$

Consider the game $\alpha_T u_{T,\Omega}$ with $T = \{i^*\}$. Every $j \in N \setminus \{i^*\}$ is a null player, so $\varphi_j(N, \alpha_{i^*} u_{i^*,\Omega}) = 0$ for every $j \in N \setminus \{i^*\}$. Efficiency implies $\varphi_{i^*}(N, \alpha_{i^*} u_{i^*,\Omega}) = \alpha_{i^*}$.

Consider the game $\alpha_T u_{T,\Omega}$ with $T \neq \{i^*\}$. Every player $j \in N \setminus T$ is a null player, so $\varphi_j(N, \alpha_T u_{T,\Omega}) = 0$ for every $j \in N \setminus T$. Player i^* is also a null player because $\alpha_T u_{T,\Omega}(i^*) = 0$ and $\Delta_{i^*}(S, \alpha_T u_{T,\Omega}) = \emptyset$ for every non-empty $S \subseteq N \setminus \{i^*\}$. Hence, $\varphi_{i^*}(N, \alpha_T u_{T,\Omega}) = 0$.

If $|T \setminus \{i^*\}| = 1$, efficiency implies $\varphi_j(N, \alpha_T u_{T,\Omega}) = \alpha_T$ for the remaining player $j = T \setminus \{i^*\}$.

If $|T \setminus \{i^*\}| > 1$, the players remaining in $T \setminus \{i^*\}$ are symmetric. To see this, fix $i, j \in T \setminus \{i^*\}$. Consider $\alpha_T u_{T,\Omega}(S \cup \{i\})$ and $\alpha_T u_{T,\Omega}(S \cup \{j\})$ with $S \subseteq N \setminus \{i, j\}$. If $S \cap \{i^*\} \neq \emptyset$, then $\alpha_T u_{T,\Omega}(S \cup \{i\}) = \alpha_T u_{T,\Omega}(S \cup \{j\}) = 0$. If $S \cap \{i^*\} = \emptyset$, then $\alpha_T u_{T,\Omega}(S \cup \{i\}) = \alpha_T u_{T,\Omega}(S \cup \{j\}) = \emptyset$. Efficiency and symmetry imply that $\varphi_j(N, \alpha_T u_{T,\Omega}) = \alpha_T/(|T| - 1)$ for every $j \in T \setminus \{i^*\}$. ■

If the reader reconsiders the two illustrative examples at the beginning of this section, it should be clear why the proposed payoff vectors are extensions of the Shapley value. In the first example, suppose the game had been drawn from G^Ω with $\Omega = \{\emptyset, 1, 12, N\}$. The set Ω is permutational and $|\Omega| = n + 1$. The domain satisfies the conditions in Proposition 1.3.5 and the proposed payoff vector is φ^*.

In the second example, the set of known coalitions Ω is singleton-defined (set $i^* = \{1\}$). Again, the proposed payoff vector is φ^* (the average of the six known marginal vectors).

1.4 A Two-Stage Solution

In the previous section it was assumed that an impartial observer has taken the missing information as given. The solution proposed in this section is not axiomatic, but instead is based upon an interesting thought experiment: suppose an impartial observer looks at the characteristic function and 'fills in' the missing information before applying the Shapley value to the 'filled in' game. Is there a fair way of assigning worths to the unknown coalitions in a game? The idea advanced in this section uses the simple principle, widely used in the literature on economic justice and distribution, that the impartial observer should try to "maximize the minimum" of the players' payoffs. To understand the two-stage solution, we present an example demonstrating how the process is applied to a game.

Example 1.4.1 *The set of players $N = \{1, 2, 3\}$. The worth of the grand coalition is $v(N) = 20$. The worths of the smaller coalitions are*

$$v(13) = 9, \quad v(23) = 8, \quad v(1) = 3, \quad v(2) = 1, \quad v(3) = 4, \quad v(\emptyset) = 0$$

and

$$v(S) = \emptyset \quad \textit{for all other coalitions.}$$

In this game, the worth of coalition 12 is missing from the game. What do the Shapley value payoffs look like as a function of $x = v(12)$? By applying the

Shapley value formula to the game we obtain the following functions.

$$Sh_1(v,x) = \frac{17}{3} + \frac{1}{6}x$$

$$Sh_2(v,x) = \frac{25}{6} + \frac{1}{6}x$$

$$Sh_3(v,x) = \frac{61}{6} - \frac{1}{3}x$$

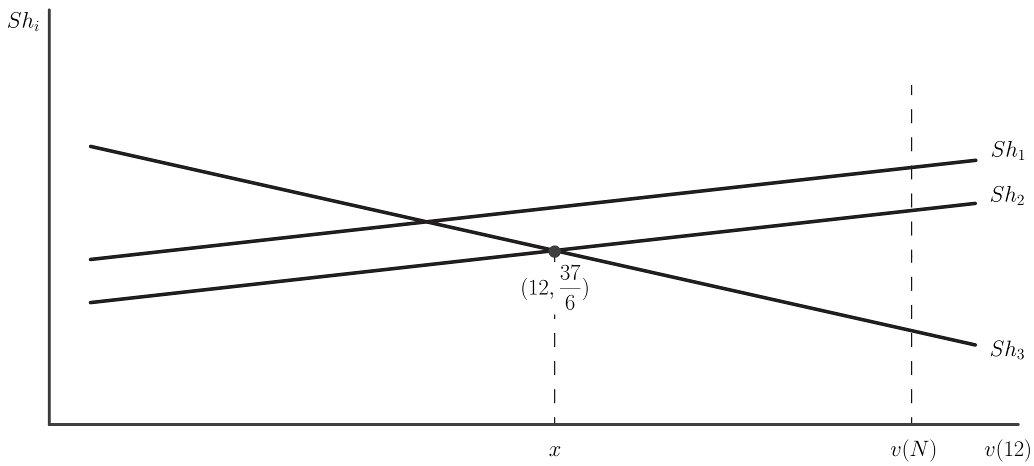


Figure 1.1: Shapley value payoffs and the unknown worth

Now suppose an impartial observer wanted to choose x so that the minimum payoff of the players is maximized. In this particular example, it is clear from looking at the figure above that there is only one point which maximizes the minimum payoff of the three players. The figure plots the Shapley value payoffs against the unknown worth, and the point which maximizes the minimum payoff is $x = 12$.

Replacing $x = 12$ in the Shapley value formula yields the following payoffs

$$\boldsymbol{Sh}(v,x) = (\frac{23}{3}, \frac{37}{6}, \frac{37}{6}).$$

1.4.1 Formalizing the Two-Stage Solution

Let the set $U(v) = 2^N \setminus K(v)$ contain those coalitions whose worths are unknown in game v. For each $v \in G$ let $X(v) \subseteq \mathbb{R}^{U(v)}$ be those vectors from which an impartial observer could choose to fill in the information missing from the game. Note that for each game $v \in G$ the set $X(v)$ could be chosen to be as large as required so as to not exclude relevant worths of the unknown coalitions. Throughout this section we impose the following standard technical assumptions upon the set $X(v)$.[12]

Assumption 1.4.1 *For each $v \in G$ the set $X(v)$ is non-empty, compact and convex.*

The two-stage problem can now be summarized as $(v, X(v))$. For each $v \in G$ and $\boldsymbol{x} \in X(v)$ let $Sh_i(v, \boldsymbol{x})$ be the Shapley value payoff which player i receives when the missing information in the game v is filled in with vector $\boldsymbol{x}$. The vector of all the players' payoffs is

$$\boldsymbol{Sh}(v, \boldsymbol{x}) = (Sh_1(v, \boldsymbol{x}), Sh_2(v, \boldsymbol{x}), ..., Sh_n(v, \boldsymbol{x})).$$

For each $\boldsymbol{x} \in X(v)$ let

$$\theta(\boldsymbol{x}) = (Sh_i(v, \boldsymbol{x}), Sh_j(v, \boldsymbol{x}), ..., Sh_k(v, \boldsymbol{x}))$$

with $Sh_i(v, \boldsymbol{x}) \leq Sh_j(v, \boldsymbol{x}) \leq, ..., \leq Sh_k(v, \boldsymbol{x})$. The vector $\theta(\boldsymbol{x})$ orders the Shapley value payoffs at $\boldsymbol{x}$ in ascending order. Finally, let $\gtrsim$ denote the standard lexicographic vector ordering on the space $\mathbb{R}^n$.

[12]The assumptions imposed upon the set of vectors from which the impartial observer can choose are the same assumptions imposed upon the choice set when one applies the nucleolus solution to a cooperative game. Of course, the two-stage solution, just like the nucleolus, is sensitive to the set from which the observer can choose.

With this notation, the two-stage solution can be stated formally as:

(i) Given a game $v \in G$, with $U(v) \neq \emptyset$, the impartial observer chooses an $\boldsymbol{x} \in X^*(v)$ with

$$X^*(v) = \{\boldsymbol{x} \in X(v) : \theta(\boldsymbol{x}) \gtrsim \theta(\boldsymbol{y}) \textit{ for every } \boldsymbol{y} \in X(v)\}$$

(ii) Each player receives their Shapley value payoff $Sh_i(v, \boldsymbol{x})$ in the filled-in game.

At the first stage, the impartial observer chooses from the set of vectors which lexicographically maximize the Shapley value payoff vector in the filled-in game. At the second stage, after the missing information has been filled in, the Shapley value payoffs are awarded to each player.

The next lemma demonstrates that if there are multiple vectors in $X(v)$ then a particular convex combination of them yields a particular Shapley value payoff vector. This result is used in Proposition 1.4.2 when demonstrating the uniqueness of the two-stage solution.

Lemma 1.4.1 *Fix a $v \in G$, $\boldsymbol{x}, \boldsymbol{y} \in X(v)$ and let $\boldsymbol{z} = \frac{1}{2}(\boldsymbol{x} + \boldsymbol{y})$. Then*

$$\frac{1}{2}[\boldsymbol{Sh}(v, \boldsymbol{x}) + \boldsymbol{Sh}(v, \boldsymbol{y})] = \boldsymbol{Sh}(v, \boldsymbol{z}).$$

Proof. If $\boldsymbol{x}, \boldsymbol{y} \in X(v)$ then for each $i \in N$

$$Sh_i(v, \boldsymbol{x}) = a + \sum_{S \in U(v)} b_S x_S$$

and

$$Sh_i(v, \boldsymbol{y}) = a + \sum_{S \in U(v)} b_S y_S$$

with a and $\{b_S\}_{S \in U(v)}$ being constants determined by the Shapley value expression. Hence

$$\frac{1}{2}[Sh_i(v, \boldsymbol{x}) + Sh_i(v, \boldsymbol{y})] = a + \sum_{S \in U(v)} b_S \frac{1}{2}(x_S + y_S) = Sh_i(v, \boldsymbol{z})$$

with $\boldsymbol{z} = \frac{1}{2}(\boldsymbol{x} + \boldsymbol{y})$. ■

The following proposition proves that a solution to the two-stage maximization problem always exists.

Proposition 1.4.1 *For any $v \in G$, if $U(v) \neq \emptyset$, then $X^*(v) \neq \emptyset$.*

Proof. Let

$$X_1(v) = \{\boldsymbol{x} \in X(v) : \theta_1(\boldsymbol{x}) \gtrsim \theta_1(\boldsymbol{y}) \text{ for every } \boldsymbol{y} \in X(v)\}$$

with $\theta_1(\cdot)$ being the first element in the θ vector. As the Shapley value expression is continuous on $X(v)$, $\theta_1(\cdot)$ is a continuous function on $X(v)$. As $X(v)$ is compact, it follows from the Weierstrass theorem that $X_1(v) \neq \emptyset$.

Let

$$X_2(v) = \{\boldsymbol{x} \in X_1(v) : \theta_2(\boldsymbol{x}) \gtrsim \theta_2(\boldsymbol{y}) \text{ for every } \boldsymbol{y} \in X_1(v)\}$$

with $\theta_2(\cdot)$ being the second element in the θ vector. Again, as the Shapley value expression is continuous on $X_1(v)$, $\theta_2(\cdot)$ is a continuous function on $X_1(v)$. As $X_1(v)$ is compact, it follows that $X_2(v) \neq \emptyset$.

Repeating these steps another $n - 2$ times we obtain $X_n(v) = X^*(v) \neq \emptyset$. ■

Having shown that a vector maximizing the lexicographic order always exists, it just remains to be shown that the resulting Shapley payoff vector is unique. The final proposition states that when $|X^*(v)| > 1$ all vectors in $X^*(v)$ yield the same Shapley value payoff vector in the filled-in game.

Proposition 1.4.2 *If $\boldsymbol{x}, \boldsymbol{y} \in X^*(v)$ then $\boldsymbol{Sh}(v, \boldsymbol{x}) = \boldsymbol{Sh}(v, \boldsymbol{y})$.*

Proof. Suppose $\boldsymbol{x}, \boldsymbol{y} \in X^*(v)$ and $\boldsymbol{Sh}(v, \boldsymbol{x}) \neq \boldsymbol{Sh}(v, \boldsymbol{y})$. Then

$$\theta(\boldsymbol{x}) = (Sh_{x1}(v, \boldsymbol{x}), Sh_{x2}(v, \boldsymbol{x}), ..., Sh_{xn}(v, \boldsymbol{x}))$$

with $x1$, $x2$,..., xn being the ordering of the players at vector $\boldsymbol{x}$ and

$$\theta(\boldsymbol{y}) = (Sh_{y1}(v, \boldsymbol{y}), Sh_{y2}(v, \boldsymbol{y}), ..., Sh_{yn}(v, \boldsymbol{y}))$$

with $y1$, $y2$,..., yn being the ordering of the players at vector $\boldsymbol{y}$.

Let $\boldsymbol{z} = \frac{1}{2}(\boldsymbol{x} + \boldsymbol{y})$. From Lemma 1.4.1, $\boldsymbol{Sh}(v, \boldsymbol{z}) = \frac{1}{2}[\boldsymbol{Sh}(v, \boldsymbol{x}) + \boldsymbol{Sh}(v, \boldsymbol{y})]$. At vector $\boldsymbol{z}$

$$\theta(\boldsymbol{z}) = (Sh_{z1}(v, \boldsymbol{z}), Sh_{z2}(v, \boldsymbol{z}), ..., Sh_{zn}(v, \boldsymbol{z}))$$

with $z1$, $z2$,..., zn being the ordering of the players at vector $\boldsymbol{z}$.

As player $x1$ has the minimal payoff at vector $\boldsymbol{x}$

$$Sh_{z1}(v, \boldsymbol{x}) \geq Sh_{x1}(v, \boldsymbol{x}). \tag{1.6}$$

As player $y1$ has the minimal payoff at vector $\boldsymbol{y}$

$$Sh_{z1}(v, \boldsymbol{y}) \geq Sh_{y1}(v, \boldsymbol{y}). \tag{1.7}$$

As $\boldsymbol{x}, \boldsymbol{y} \in X^*(v)$, $Sh_{x1}(v, \boldsymbol{x}) = Sh_{y1}(v, \boldsymbol{y})$. Equations 1.6 and 1.7 imply

$$Sh_{z1}(v, \boldsymbol{z}) = \frac{1}{2}[Sh_{z1}(v, \boldsymbol{x}) + Sh_{z1}(v, \boldsymbol{y})] \geq Sh_{x1}(v, \boldsymbol{x}). \tag{1.8}$$

If the inequality in equation 1.8 is strict, this contradicts $\boldsymbol{x}, \boldsymbol{y} \in X^*(v)$. Hence, it must be that

$$Sh_{z1}(v, \boldsymbol{x}) = Sh_{x1}(v, \boldsymbol{x})$$

and

$$Sh_{z1}(v, \boldsymbol{y}) = Sh_{y1}(v, \boldsymbol{y}).$$

This means we can interchange players $x1$ and $z1$ in vector $\theta(\boldsymbol{x})$, and we can interchange players $y1$ and $z1$ in vector $\theta(\boldsymbol{y})$. Doing this yields

$$\theta(\boldsymbol{x}) = (Sh_{z1}(v, \boldsymbol{x}), Sh_{x2'}(v, \boldsymbol{x}), ..., Sh_{xn'}(v, \boldsymbol{x}))$$

and

$$\theta(\boldsymbol{y}) = (Sh_{z1}(v, \boldsymbol{y}), Sh_{y2'}(v, \boldsymbol{y}), ..., Sh_{yn'}(v, \boldsymbol{y})).$$

If we repeat these steps for $x2'$ and $y2'$ we obtain the following. As player $x2'$ has the second minimal payoff at vector $\boldsymbol{x}$

$$Sh_{z2}(v, \boldsymbol{x}) \geq Sh_{x2'}(v, \boldsymbol{x}). \tag{1.9}$$

As player $y2'$ has the second minimal payoff at vector $\boldsymbol{y}$

$$Sh_{z2}(v, \boldsymbol{y}) \geq Sh_{y2'}(v, \boldsymbol{y}). \tag{1.10}$$

As $\boldsymbol{x}, \boldsymbol{y} \in X^*(v)$, $Sh_{x2'}(v, \boldsymbol{x}) = Sh_{y2'}(v, \boldsymbol{y})$. Equations 1.9 and 1.10 imply

$$Sh_{z2}(v, \boldsymbol{z}) = \frac{1}{2}[Sh_{z2}(v, \boldsymbol{x}) + Sh_{z2}(v, \boldsymbol{y})] \geq Sh_{x2'}(v, \boldsymbol{x}). \tag{1.11}$$

If the inequality in equation 1.11 is strict, this contradicts $\boldsymbol{x}, \boldsymbol{y} \in X^*(v)$. Hence, it must be that

$$Sh_{z2}(v, \boldsymbol{x}) = Sh_{x2'}(v, \boldsymbol{x})$$

and

$$Sh_{z2}(v, \boldsymbol{y}) = Sh_{y2'}(v, \boldsymbol{y}).$$

We can now interchange players $x2'$ and $z2$ in vector $\theta(\boldsymbol{x})$, and we can interchange players $y2'$ and $z2$ in vector $\theta(\boldsymbol{y})$. This yields

$$\theta(\boldsymbol{x}) = (Sh_{z1}(v,\boldsymbol{x}), Sh_{z2}(v,\boldsymbol{x}), Sh_{x3''}(v,\boldsymbol{x}), ..., Sh_{xn''}(v,\boldsymbol{x}))$$

and

$$\theta(\boldsymbol{y}) = (Sh_{z1}(v,\boldsymbol{y}), Sh_{z2}(v,\boldsymbol{y}), Sh_{y3''}(v,\boldsymbol{y}), ..., Sh_{yn''}(v,\boldsymbol{y})).$$

Repeating this step another $n-2$ times we obtain

$$\theta(\boldsymbol{x}) = (Sh_{z1}(v,\boldsymbol{x}), Sh_{z2}(v,\boldsymbol{x}), ..., Sh_{zn}(v,\boldsymbol{x}))$$

and

$$\theta(\boldsymbol{y}) = (Sh_{z1}(v,\boldsymbol{y}), Sh_{z2}(v,\boldsymbol{y}), ..., Sh_{zn}(v,\boldsymbol{y}))$$

which implies $\boldsymbol{Sh}(v,\boldsymbol{x}) = \boldsymbol{Sh}(v,\boldsymbol{y})$ and contradicts $\boldsymbol{Sh}(v,\boldsymbol{x}) \neq \boldsymbol{Sh}(v,\boldsymbol{y})$.

Hence, if $\boldsymbol{x},\boldsymbol{y} \in X^*(v)$ then $\boldsymbol{Sh}(v,\boldsymbol{x}) = \boldsymbol{Sh}(v,\boldsymbol{y})$. ■

1.5 Concluding Remarks

The aim of this paper has been to take the first steps to understanding how the Shapley value should be extended to games in which some of the information may be missing. The authors hope that this work may stimulate further research on what is an interesting, but difficult problem to address. The results indicate that, if too much information is missing from the characteristic function, then it may not be possible to reconcile the axioms of efficiency and null player.

Each of the solutions presented in the paper have particular merits and weaknesses. If the set of known coalitions is permutational, and there are as many known coalitions as players, or the set of known coalitions is singleton-defined, there is a value which satisfies the Shapley axioms. Somewhat surprisingly, the axioms characterize the value. The value φ^* in equation 1.2 is an obvious extension of the Shapley value on the missing information domain. The value gives each player the average of their marginal contributions to their predecessors in each of the known marginal vectors. If the set of known coalitions is not permutational then φ^* cannot be applied to the game. Even if the set of known coalitions is permutational, when assigning payoffs to the players, φ^* may fail to use all the information contained in the characteristic function. To remedy these defects, the two-stage solution draws on ideas in social choice and economic justice. If an impartial observer were free to fill in the information missing from the characteristic function then one normative principle which could be used is "maximize the minimum payoff" once the Shapley value is applied to the filled in game. This is precisely what the two-stage solution does.

We conclude the paper by presenting two possibilities for additional research which the authors hope to pursue in the future.

- Throughout the paper it has been assumed that the worths of coalitions, when known, are independent of the behaviour of players outside the coalition. In many cases this is too strong an assumption to make. A more realistic case is to make the worths of known coalitions dependent upon the behaviour of players outside the coalition. This generates a game in partition function form. Recently, De Clippel and Serrano (2008), McQuillin (2009) and Borm et al. (2015) have all suggested different extensions of the Shapley value to partition function games. These papers assume that all the information is contained in the game. It would be interesting to see whether there is a way of extending the Shapley value to partition function games with missing information.

- Axiomatic value solutions have been studied in detail for the bankruptcy problem introduced by O'Neill (1982), and recently surveyed by Thomson (2015). A bankruptcy problem is summarized by $(E, (c_i)_{i \in N})$ with $E > 0$ being the money worth of the estate to be divided between the players, and $c_i \geq 0$ being the money claim which player i has upon the worth E. It is assumed that $\sum_{i \in N} c_i > E$ so that all the claims of the players cannot be realized. Many value solutions have been proposed to this type of problem. However, all the existing literature assumes that the money claims of the players are known. In many real world bankruptcy problems this assumption is not satisfied. Instead, it is known that individuals have claims upon the estate, but the exact amount of the claim cannot always be proved. This type of problem could be modelled using the idea of a game with missing information introduced in this paper. If one sets $c_i = \emptyset$ for those players who cannot prove their claims upon the estate, this would then generate a characteristic function game with missing information.

Chapter 2

A Model for Quota Allocation

2.1 Introduction

Fish are a relatively scarce biological resource.[1] Ensuring the biological and economic sustainability of the fisheries requires that access to the fisheries is limited and that allocated quota is within safe biological limits. The allocation of fishing quota often presents challenges for the fisheries management, and the design of the catch share program is critically important for a successful fishery. Traditionally fishing permits have largely been granted based on historical rights (often referred to as grandfathering) (Anderson et al., 2011; Lynham, 2014). In a study on allocation of fishing rights worldwide, Lynham (2014) found that in over 91 percent of fisheries surveyed, the access to the fisheries were grandfathered for some of the catch, whilst grandfathering was used as the only method of allocation in 54 percent of fisheries. Other methods used, included equal sharing rules, vessel and gear based rules and least frequently, auctions,[2] which were used in 3 percent

[1] This chapter was co-written with Olga Gorelkina.

[2] For more information on auctions with regards to the fishing industry, see Anderson et al. (2011); Bromley (2009); Grafton (1995); Morgan (1995); Marszalec (2018a)

of the fisheries (Lynham, 2014).[3]

Governments in a handful of countries have used auctions for allocating fishing quota for certain parts of their fisheries.[4] The Faroe Islands, is the most recent example of the implementation of auctions for allocating fishing quota. Trial auctions were run in 2011, 2016 and 2017 and from 2018 onwards auctions became a part of the new fisheries management system implemented in January 2018.

An important component of fisheries management is limiting the negative externalities of the fishing activity. Ensuring that the activity does not result in irreversible damage to the fish stocks and wider ecosystems is essential (Cochrane et al., 2009). Addressing climate change also requires that carbon emissions from the fishing fleet are reduced. In this context, investment into newer greener vessels, as well as fishing equipment, is paramount.

Therefore, fair allocation methods and encouragement of desirable behaviour of agents are two important policy concerns of managers, both of which this paper addresses by posing the following research questions: (1) How does the price and fraction of grandfathered quota affect investment, total welfare and the auction price? (2) How does the level of investment affect auction prices and total welfare?

To explore these questions, we introduce a theoretical model that allows the government to give firms stronger incentives to invest in greener and more environmentally friendly fishing technologies. In the model, firms have two channels through which they can obtain quota: grandfathering — which allocates quota

[3] Auctions for allocating fishing quota are rare and are currently only used in four places: the Faroe Islands, Chile, Washington DC and New Zealand.

[4] Estonia and Russia used auctions over a short period to allocate fishing rights, but have since reverted back to grandfathering the rights based on historical catches (Anferova et al., 2005; Eero et al., 2005; Vetemaa et al., 2002, 2005).

based on historical catches — and auctions, which any firm is allowed to participate in. The management authority has applied an environmental tax to incentivise environmentally friendly fishing technologies. Therefore, the firm's level of investment directly affects its valuation of the quota sold at auction. We interpret this as an opportunity to make an investment in greener technology which reduces a firm's externality and hence decreases their real marginal cost. This will in turn lead to a higher valuation of the object. Treating bidder valuations endogenously in the model allows the model to better take into account the complexity related to agents' decisions, and little focus has been on this.[5] The majority of papers in the literature on auctions treat bidder valuations as exogenous variables. The benchmark auction model by McAfee and McMillan (1987) including many other influential papers (e.g. Riley and Samuelson (1981); Myerson (1981); Vickrey (1961)) consider values which are independent and identically distributed, drawn from a common distribution $F(v)$ that is strictly increasing and differentiable over the interval of valuations $\left[\underline{v}, \bar{v}\right]$ and satisfies $F(\underline{v}) = 0$ and $F(\bar{v}) = 1$.

Most closely related to our work is Gershkov et al. (2018) who study the allocation of multiple units among symmetric agents with unit demands and convex preferences over the probability of receiving an object. These non-standard preferences are a result of agents taking costly actions unobservable to the designer, and that influence their valuations at subsequent auctions. Because of these ex-ante investments, valuations of the agents become endogenous to the mechanism.

Using our model, it is therefore possible to explore how the allocation mechanism impacts on various parameters, including government revenue from fishing

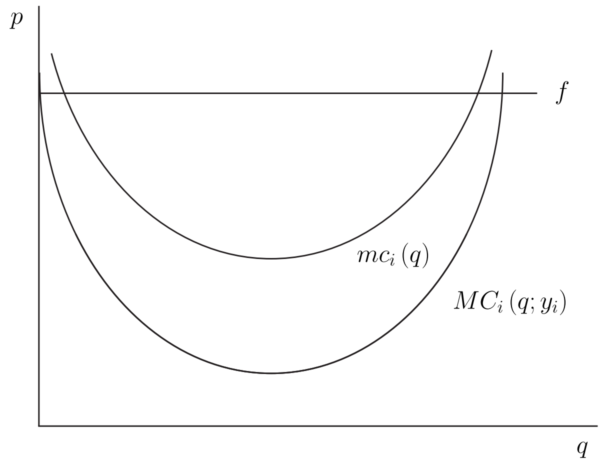
[5]See Gershkov et al. (2018) for an exception.

quota, total welfare, the firms' level of investment and the equilibrium price at the auction. It also considers how the level of resource fee on the grandfathered quota impacts on the level of investment. Surprisingly, we find that the fraction and the price of grandfathered quota does not affect the equilibrium price, investment nor the total welfare. We suggest that these results can be attributed to the design of the model. As expected, we also find that higher investment leads to increased equilibrium prices at auction and that higher prices of grandfathered quota result in lower levels of investment. Finally, we consider the applicability of this model to the fisheries, and suggest how it could be developed in future work.

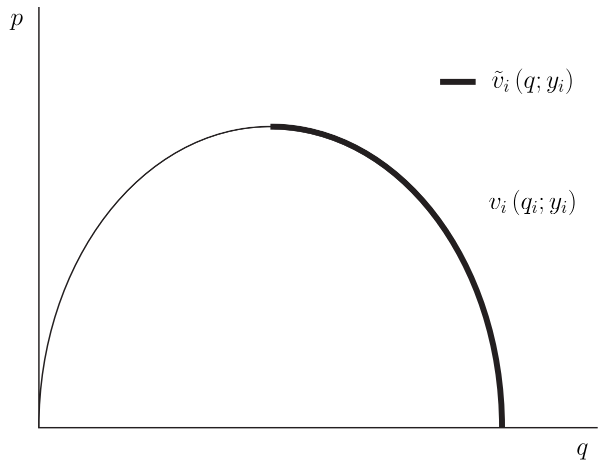
2.2 The Model

Let us consider a fish market in which there are $n \geq 2$ firms and the firms are price takers on the fish market (market for final product). The price of the final product is denoted by f. We assume that all quota is used (i.e. all fish is caught) and therefore quota is equivalent to production.[6] To produce a positive quantity, firm i incurs a fixed cost. Firms also incur a marginal cost of production which is denoted $mc_i(q)$ and satisfies the below assumptions. The fixed cost is the marginal cost at zero units and is demonstrated by the very high initial marginal cost in Figure 2.1a.

[6] The price f is exogenously determined by the world price level, which is a realistic assumption given the size of the fishing industry in the majority of countries.



(a) Marginal cost curves



(b) Value function

Figure 2.1: Marginal cost and value functions for firm i.

Assumption 2.2.1 *The marginal cost of firm i, $mc_i(q)$, satisfies the following conditions:*

1. *there exists $\bar{q}_i \geq 0$ such that $\frac{\partial}{\partial q} mc_i > 0$ for all $q > \bar{q}_i$ (marginal cost starts increasing eventually)*

2. *$\frac{\partial^2}{\partial q^2} mc_i > 0$ (marginal cost is convex)*

This implies that the marginal cost eventually starts to increase and that the marginal cost is a convex function of quota.[7] Fishing activity also produces an

[7] In fisheries it is very often the case that marginal costs are decreasing and that firms never reach full capacity. This is due to limited supply of quota and overcapacity in many fishing

observed externality of $e_i > 0$ per unit, which can be pollution from fuel usage or damage to the wider ecosystem e.g. benthic habitats from bottom trawling. The damage/externality can be reduced if the firm invests into greener technology, and therefore the externality $e_i = e_i(y_i)$, is a decreasing function of the investment y_i ($y_i \geq 0$) and the externality satisfies the following.

Assumption 2.2.2 *The externality $e_i(y_i)$ satisfies the following conditions:*

1. $\frac{\partial}{\partial y} e_i \leq 0$ *(investment reduces externality)*

2. $\frac{\partial^2}{\partial y_i^2} e_i \geq 0$ *(the effect of investment diminishes)*

Given the negative externality, we introduce the social marginal cost of fishing, denoted by MC_i, which is the sum of the production cost and the externality:

$$MC_i(q; y_i) = mc_i(q) + e_i(y_i).$$

To create an incentive for firms to invest in greener vessels or fishing equipment the government applies an environmental tax, which depends on the fuel consumption of the vessel and the type of equipment used. In doing so, the externality of the firm's activity will be internalised. Here the tax will be calculated based on the firm's total quota. An example could be taxation based on the fuel consumption per kilo of fish.

The marginal value of quota at level q_i, $v_i(q_i; y_i)$, equals the price of the final product less the social marginal cost:

$$v_i(q_i; y_i) = [f - MC_i(q_i; y_i)]_+.$$

---

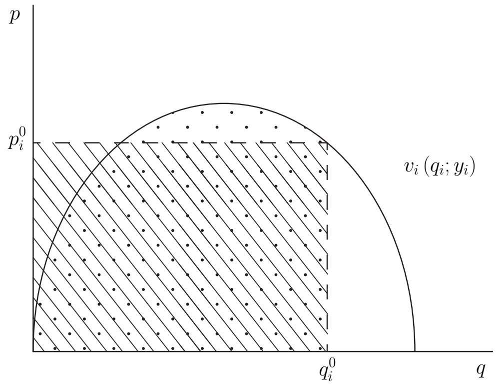
fleets.

The properties of the marginal value function $v_i(q_i; y_i)$ are described by inequalities similar to Assumption 2.2.1 and 2.2.2, where signs are reversed. For a fixed y_i, let $\tilde{v}_i(q; y_i) = v_i(q; y_i)$ be defined in the range where v_i is strictly decreasing.

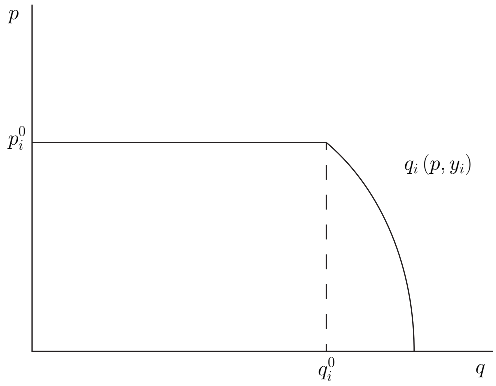
Then the demand at price p per quota (given investment y_i) is given by:

$$q_i(p, y_i) = \begin{cases} \tilde{v}_i^{-1}(p; y_i) & if\, p \leq p_i^0(y_i) \\ \\ 0 & if\, p > p_i^0(y_i) \end{cases},$$

where $p_i^0(y_i) = v_i(q_i^0; y_i)$, and q_i^0 solves $\int_0^{q_i^0} v_i(q; y_i)\, dq = q_i^0 v_i(q_i^0; y_i)$. $p_i^0(y_i)$ is the price beyond which firm i demands zero quota because he would incur a loss if he purchased any quota above this price. See Figure 2.2a for an illustration.



(a) Value function and maximum price



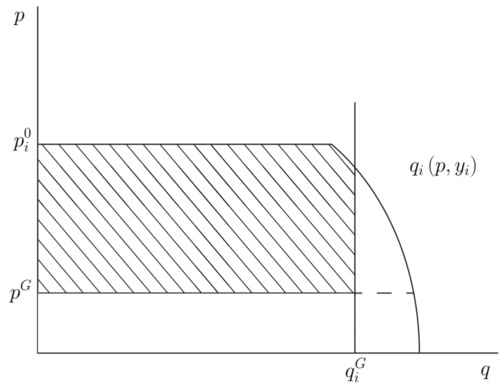
(b) Demand function

Figure 2.2: Value function, demand and maximum price

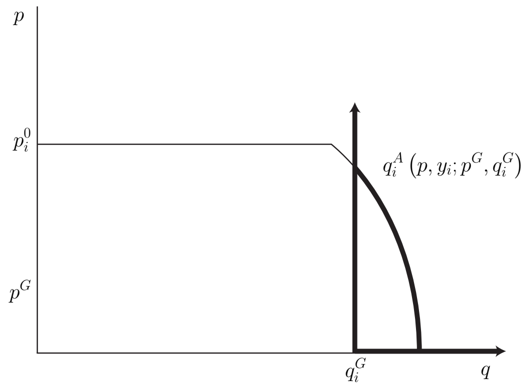
The demand function $q_i(p, y_i)$ is thus the inverse marginal value function 'ironed' appropriately to become monotone decreasing. The function is concave in y_i, i.e. $\frac{\partial^2}{\partial y_i^2} q_i(p, y_i) \leq 0$.[8] See Figure 2.2b.

Allocation of Quota Let q_i^G be the grandfathered quota available to firm i and p^G the (uniform) price per unit of grandfathered quota set by the seller.

[8] *Proof:* $\tilde{v} = p = f - \tilde{mc}(q) - e(y)$, so $q = \tilde{mc}^{-1}(f - p - e(y))$, and $\frac{\partial q}{\partial y} = -\left(\tilde{mc}^{-1}\right)'(f - p - e(y)) * e'(y)$. Hence, $\frac{\partial^2 q}{\partial y^2} = \left(\tilde{mc}^{-1}\right)'' * (e')^2 - \left(\tilde{mc}^{-1}\right)' * e'' \leq 0$, since $\left(\tilde{mc}^{-1}\right)'' \leq 0$, $\left(\tilde{mc}^{-1}\right)' \geq 0$, and $e'' \geq 0$. ■



(a) Surplus from grandfathered quota



(b) Demand at auction

Figure 2.3: Surplus from trade in grandfathered quota and demand at auction

Firm i's demand for grandfathered quota is capped at q_i^G and is therefore given by $\min\left\{q_i(p, y_i), q_i^G\right\}$. Hence, the amount of quota acquired through grandfathering at price p^G is $\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}$.

The firm's surplus from trade in grandfathered quota is given by:

$$\int_{p^G}^{p_i^0(y_i)} \min\left\{q_i(p, y_i), q_i^G\right\} dp.$$

Note that if $p_i^0(y_i) < p^G$ then the term $\int_{p^G}^{p_i^0(y_i)} \min\left\{q_i(p, y_i), q_i^G\right\} dp$ is negative and thus buying grandfathered quota in isolation leads to a loss. For the remainder of this paper we shall therefore assume that $p^G < p_i^0(y_i)$.

Firm i's demand for quota in the auction is therefore their total demand less the quota received through grandfathering:

$$q_i^A\left(p, y_i; p^G, q_i^G\right) = q_i(p, y_i) - \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}.$$

Observe that the demand can be negative at high prices implying that the firm would supply some of its grandfathered quota to the auction.

The price of the grandfathered quota is set by the government and is uniform. For the auction, the Vickrey-Clarke-Groves (VCG) pricing mechanism is used. The VCG auction is a type of sealed bid auction for multiple items where firms are required to submit bids stating the quantity demanded at what price. Firms may submit more than one bid since the willingness to pay for quota will depend on the total quota purchased by the firm. Once all bids are submitted, the firms with winning bids pay the marginal harm their bid has caused to the other firms — which is at most as high as their original bid. Consequently, the VCG auction is an incentive compatible mechanism implying that reporting truthfully is an equilibrium (see, for example, Milgrom (2004)).

The total amount of the available exogenously determined quota, Q, is split between grandfathering and the auction, $Q = Q^A + \sum_i \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}$, where Q^A is the endogenously determined amount of quota sold through the auction.

Now let y be the vector of investment levels, $y = (y_1, y_2, ..y_n)$, and q^G be the vector of grandfathered quotas, $q^G = \left(q_1^G, q_2^G, ..q_n^G\right)$. The price $p^*(y)$ solves $\sum_i q_i^A\left(p, y_i; p^G, q_i^G\right) = Q^A$ in p (which is equivalent to $\sum_i q_i(p, y_i) = Q$).[9] Note that $p^*(y)$ is the highest price paid for one unit of quota sold in the auction and not the unit price paid by all firms. It follows immediately from the definition of the equilibrium price that $p^*(y)$ is not affected by either p^G or q^G. We therefore have the following lemma before having solved our model.

Lemma 2.2.1 *Changing the price p^G or quantity q^G of grandfathered quota has no effect on the equilibrium price $p^*(y)$ at the auction.*

Since the equilibrium price is the highest price paid for a unit of quota sold at the auction, the above result does not imply that the average price for the quota

[9]Proof: $\sum_i q_i^A(p^*, y_i) = Q^A$
$\sum_i (q_i(p^*, y_i) - \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}) = Q^A$
$\sum_i q_i(p^*, y_i) = Q^A + \sum_i \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} = Q$
$\sum_i q_i(p^*, y_i) = Q.$ ■

is unaffected. To illustrate how the model works we present a simple example below demonstrating that although $p^* (y)$ remains unchanged when changing p^G or q^G, the average unit price on the auction does change when changing q^G.[10]

Example 2.2.1 *Consider a setting in which there are two firms who both receive equal fractions of the total allowable catch (TAC) in grandfathered quota. The total amount of quota* $Q = 15$ *and the global price is* $f = 13$*. The price of the grandfathered quota is* $p^G = 3$*. The firms' marginal costs are given by:*

$$MC_1(q, y_1) = 1 + \tfrac{3}{4}q$$

$$MC_2(q, y_2) = 2 + \tfrac{3}{4}q.$$

Given this, their demand functions can be obtained from the value function, given by

$$v_i (q_i; y_i) = [f - MC_i (q_i; y_i)]_+ .$$

The value functions are therefore as follows

$$\begin{aligned} v_1 (q_1; y_1) &= \left[13 - (1 + \frac{3}{4}q)\right]_+ = \left[12 - \frac{3}{4}q\right]_+ \\ v_2 (q_2; y_2) &= \left[13 - (2 + \frac{3}{4}q)\right]_+ = \left[11 - \frac{3}{4}q\right]_+ . \end{aligned}$$

Recalling the assumption of considering only decreasing marginal values and taking the inverse of the value functions the demand functions become

$$q_1 (p, y_1) = 16 - \tfrac{4}{3}p$$

$$q_2 (p, y_2) = \tfrac{44}{3} - \tfrac{4}{3}p.$$

[10]For simplicity, we use linear marginal cost functions, and assume that the firm is at the point where marginal costs are strictly decreasing.

Case 1: Grandfathered quantity is 2 units per firm *Now let us assume that both firms receive 2 units of grandfathered quota, i.e.*

$$q_1^G(p, y_1) = q_2^G(p, y_2) = 2$$

The total demand on auction is

$$q_i^A\left(p, y_i; p^G, q_i^G\right) = q_i(p, y_i) - \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}.$$

With the given demand functions and price p^G the grandfathered quota constraint binds, and hence the demands on auction are given by:

$$q_1^A\left(p, y_1; p^G, q_1^G\right) = (16 - \tfrac{4}{3}p) - 2 = 14 - \tfrac{4}{3}p$$

$$q_2^A\left(p, y_2; p^G, q_2^G\right) = (\tfrac{44}{3} - \tfrac{4}{3}p) - 2 = \tfrac{38}{3} - \tfrac{4}{3}p.$$

The next step is to calculate the supply on the auction and the residual supply facing the firms. Since $Q = Q^A + \sum_i \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}$ we know that $Q^A = 15 - (2 + 2) = 11$.

The residual supply can be obtained from

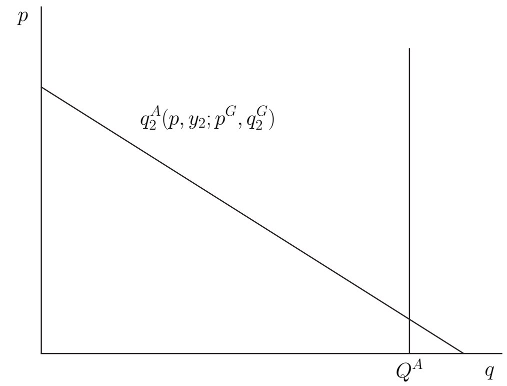
$$s^{-i}(p, y_{j\neq i}; p^G, q_{j\neq i}^G) = max\left\{Q^A - \sum_{j\neq i} q_j^A(p, y_j; p^G, q_j^G), 0\right\}$$

and is defined by the following two functions for $p \geq 0$:

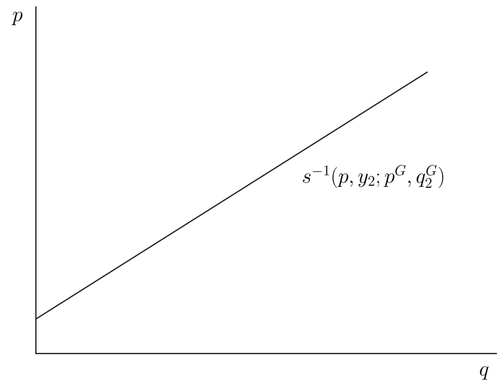
$$s^{-1} = \begin{cases} \frac{4}{3}p - \frac{5}{3}, & if\, p > \frac{5}{4} \\ 0, & if\, 0 \leq p \leq \frac{5}{4} \end{cases}, \quad s^{-2} = \begin{cases} \frac{4}{3}p - 3, & if\, p > \frac{9}{4} \\ 0, & if\, 0 \leq p \leq \frac{9}{4} \end{cases}.$$

To find the equilibrium we set the auction demand and residual supply of the firms equal,

$$s^{-i}(p, y_{j\neq i}; p^G, q_{j\neq i}^G) = q_i^A\left(p, y_i; p^G, q_i^G\right).$$



(a) Demand of firm i and total supply on auction



(b) Residual supply facing firm i

Figure 2.4: How to obtain the residual supply from total supply on auction and demand functions

Firm 1:

$$\begin{aligned} \tfrac{4}{3}p - \tfrac{5}{3} &= 14 - \tfrac{4}{3}p \\ p &= \tfrac{47}{8}. \end{aligned}$$

Firm 2:

$$\begin{aligned} \tfrac{4}{3}p - 3 &= \tfrac{38}{3} - \tfrac{4}{3}p \\ p &= \tfrac{47}{8}. \end{aligned}$$

Note that the equilibrium price $p^*(y) = \frac{47}{8}$ for both firms. However, the total quantity, total payment and price paid per unit are different. The quantities demanded at the auction by the two firms are

$$q_1^A\left(\tfrac{47}{8}\right) = 14 - \tfrac{4}{3} \cdot \tfrac{47}{8} = \tfrac{37}{6} \approx 6.17$$

$$q_2^A\left(\tfrac{47}{8}\right) = \tfrac{38}{3} - \tfrac{4}{3} \cdot \tfrac{47}{8} = \tfrac{29}{6} \approx 4.83.$$

In a VCG auction the price paid in the auction is given by the area below the residual supply curve. In Figure 2.5 we see the total payment of firm 1, where the payment in the auction is given by the striped area to the right of q_1^G. The payments on the auction (PA_i) are therefore given by:

$$PA_1 = \int_{\frac{5}{4}}^{\frac{47}{8}} \tfrac{37}{6} - \left(\tfrac{4}{3}p - \tfrac{5}{3}\right) dp + \tfrac{5}{4} \cdot \tfrac{37}{6} = \tfrac{703}{32} \approx 21.97$$

$$PA_2 = \int_{\frac{9}{4}}^{\frac{47}{8}} \tfrac{29}{6} - \left(\tfrac{4}{3}p - 3\right) dp + \tfrac{9}{4} \cdot \tfrac{29}{6} = \tfrac{1885}{96} \approx 19.64.$$

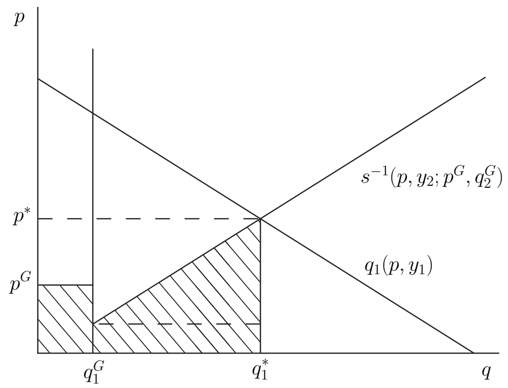


Figure 2.5: Total payment of firm 1

Having calculated the equilibrium quantity and the total payments, we can now calculate the average price per unit purchased through the auction:

Firm 1:

$$\frac{703/32}{37/6} = \tfrac{57}{16} \approx 3.56$$

Firm 2:

$$\frac{1885/96}{29/6} = \tfrac{65}{16} \approx 4.06.$$

Case 2: Grandfathered quantity is 4 units per firm *Now we repeat the above exercise with*

$$q_1^G(p, y_1) = q_2^G(p, y_2) = 4.$$

The quota constraint is still binding and hence auction demands are

$$q_1^A\left(p, y_1; p^G, q_1^G\right) = 12 - \tfrac{4}{3}p$$

$$q_2^A\left(p, y_2; p^G, q_2^G\right) = \tfrac{32}{3} - \tfrac{4}{3}p,$$

and the residual supply of the two firms is given by:

$$s^{-1} = \begin{cases} \frac{4}{3}p - \frac{11}{3}, & if\, p > \frac{11}{4} \\ \\ 0, & if\, 0 \leq p \leq \frac{11}{4} \end{cases}, \quad s^{-2} = \begin{cases} \frac{4}{3}p - 5, & if\, p > \frac{15}{4} \\ \\ 0, & if\, 0 \leq p \leq \frac{15}{4} \end{cases}.$$

Setting demand and residual supply equal for both firms we obtain the following

Firm 1:

$$\begin{aligned} \tfrac{4}{3}p - \tfrac{11}{3} &= 12 - \tfrac{4}{3}p \\ p &= \tfrac{47}{8}. \end{aligned}$$

Firm 2:

$$\begin{aligned} \tfrac{4}{3}p - 5 &= \tfrac{32}{3} - \tfrac{4}{3}p \\ p &= \tfrac{47}{8}. \end{aligned}$$

Hence, the quantities demanded at the equilibrium are

$$q_1^A\left(\tfrac{47}{8}\right) = 12 - \tfrac{4}{3} \cdot \tfrac{47}{8} = \tfrac{25}{6} \approx 4.17$$

$$q_2^A\left(\tfrac{47}{8}\right) = \tfrac{32}{3} - \tfrac{4}{3} \cdot \tfrac{47}{8} = \tfrac{17}{6} \approx 2.83.$$

The total payments for the purchased quota at the auction (PA_i) are therefore

$$PA_1 = \int_{\frac{11}{4}}^{\frac{47}{8}} \tfrac{25}{6} - \left(\tfrac{4}{3}p - \tfrac{11}{3}\right) dp + \tfrac{11}{4} \cdot \tfrac{25}{6} = \tfrac{575}{32} \approx 17.97$$

$$PA_2 = \int_{\frac{15}{4}}^{\frac{47}{8}} \tfrac{17}{6} - \left(\tfrac{4}{3}p - 5\right) dp + \tfrac{15}{4} \cdot \tfrac{17}{6} = \tfrac{1309}{96} \approx 13.64.$$

and the prices per unit of quota for firm 1 and 2, respectively, are

$$\tfrac{575/32}{25/6} = \tfrac{69}{16} \approx 4.31$$

$$\tfrac{1309/96}{17/6} = \tfrac{77}{16} \approx 4.81.$$

Comparing the two cases *If comparing the two cases we see that* $p^*(y) = \frac{47}{8}$ *in both cases for both players as Lemma 2.2.1 shows. As the quantity allocated through grandfathering increases, the average price paid on the auction increases for both players. In this particular example, when* $p^G = 3$ *the total revenue also increases. The revenue would have remained unchanged with* $p^G = 2.5$*. The positive impact on revenue from an increase in grandfathered quota increases with the price of grandfathered quota* p^G*, but only to a certain extent because at too high prices firms would reject the grandfathered quota, and it would be sold on auction instead, resulting in a new equilibrium. The more quota is sold on auction the lower is the average unit price for units purchased through the auction.*

Table 2.1: Effect of changing q_i^G on prices

		Quantity			Average unit price			Payment	
		G	A	Total	G	A	Total	$q_i^G = 2$	$q_i^G = 4$
Firm 1	$q_i^G = 2$	2	6.17	8.17	3	3.56	3.42	27.97	
	$q_i^G = 4$	4	4.17	8.17	3	4.31	3.67		29.97
Firm 2	$q_i^G = 2$	2	4.83	6.83	3	4.06	3.75	25.64	
	$q_i^G = 4$	4	2.83	6.83	3	4.81	3.75		25.64
								53.60	55.60

2.3 Results

To solve the model, we first need to consider the utility of the firms. The utility of firm i is the gain from obtaining the grandfathered quota and the quota bought on auction less the investment made by the firm and is given by:

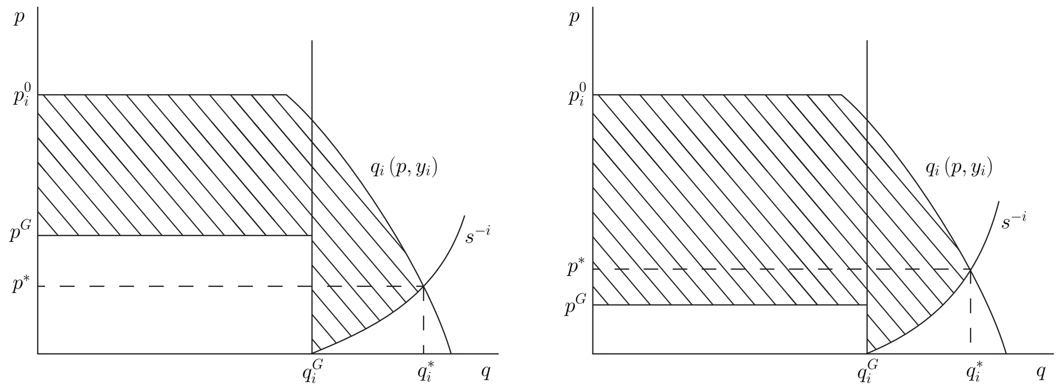
$$\begin{aligned} U_i &= -y_i + \int_{p^G}^{p_i^0(y_i)} \min\left\{q_i\left(p, y_i\right), q_i^G\right\} dp \\ &\quad + \int_{p^*(y)}^{+\infty} q_i^A\left(p, y_i; p^G, q_i^G\right) dp + \int_0^{p^*(y)} \left(Q^A - \sum_{j \neq i} q_j^A\left(p, y_j; p^G, q_j^G\right)\right) dp. \end{aligned}$$

Here, $Q^A - \sum_{j \neq i} q_j^A \left(p, y_j; p^G, q_j^G\right)$ is the residual supply facing firm i in the auction. Recalling that

$$q_i^A \left(p, y_i; p^G, q_i^G\right) = q_i \left(p, y_i\right) - \min \left\{q_i \left(p^G, y_i\right), q_i^G\right\}$$

the utility becomes

$$\begin{aligned} U_i &= -y_i + \int_{p^G}^{p_i^0(y_i)} \min \left\{q_i \left(p, y_i\right), q_i^G\right\} dp \\ &\quad + \int_{p^*(y)}^{+\infty} q_i \left(p, y_i\right) - \min \left\{q_i \left(p^G, y_i\right), q_i^G\right\} dp \\ &\quad + \int_0^{p^*(y)} \left(Q^A - \sum_{j \neq i} q_j^A \left(p, y_j; p^G, q_j^G\right)\right) dp. \end{aligned}$$



(a) Utility when the price of grandfathered quota is higher than the equilibrium price at the auction

(b) Utility when the price of grandfathered quota is lower than the equilibrium price at the auction

Figure 2.6: Utility of firm i

This is equivalent to:

$$\begin{aligned} U_i &= -y_i + \int_{p^*(y)}^{p_i^0(y_i)} q_i \left(p, y_i\right) dp \\ &\quad + \left(p^*(y) - p^G\right) \min \left\{q_i \left(p^G, y_i\right), q_i^G\right\} \\ &\quad + \int_0^{p^*(y)} \left(Q^A - \sum_{j \neq i} q_j^A \left(p, y_j; p^G, q_j^G\right)\right) dp. \end{aligned}$$

To find the level of investment maximising firm i's utility we take the derivative of the utility function with respect to investment. The first order condition then becomes:

$$\begin{aligned}
1 &= \frac{\partial p_i^0}{\partial y_i}(y_i) q_i^0(y_i) + \int_{p^*(y)}^{p_i^0(y_i)} \frac{\partial q_i}{\partial y_i}(p, y_i)\, dp \\
&\quad + \frac{\partial p^*}{\partial y_i}(y) \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} \\
&\quad + \left(p^*(y) - p^G\right) \frac{\partial}{\partial y_i} \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} \\
&\quad + \frac{\partial p^*}{\partial y_i}(y) \left(Q^A - \sum_{j \neq i} q_j^A\left(p^*, y_j; p^G, q_j^G\right)\right).
\end{aligned}$$

Since at equilibrium $Q^A = \sum_i q_i^A\left(p^*, y_i; p^G, q_i^G\right)$, this is equivalent to:

$$\begin{aligned}
1 &= \frac{\partial p_i^0}{\partial y_i}(y_i) q_i^0(y_i) + \int_{p^*(y)}^{p_i^0(y_i)} \frac{\partial q_i}{\partial y_i}(p, y_i)\, dp \\
&\quad + \frac{\partial p^*}{\partial y_i}(y) \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} \\
&\quad + \left(p^*(y) - p^G\right) \frac{\partial}{\partial y_i} \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} \\
&\quad + \frac{\partial p^*}{\partial y_i}(y)\, q_i^A\left(p^*, y_i; p^G, q_i^G\right).
\end{aligned}$$

Summing up the second and fourth line and recalling that $q_i^A\left(p, y_i; p^G, q_i^G\right) = q_i(p, y_i) - \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}$ we obtain

$$\begin{aligned}
1 &= \frac{\partial p_i^0}{\partial y_i}(y_i) q_i^0(y_i) + \int_{p^G}^{p_i^0(y_i)} \frac{\partial q_i}{\partial y_i}(p, y_i)\, dp + \frac{\partial p^*}{\partial y_i}(y)\, q_i(p^*, y_i) \\
&\quad + \left(p^*(y) - p^G\right) \frac{\partial}{\partial y_i} \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}.
\end{aligned}$$

On the left-hand side, we have the marginal cost of investment equal to one. On the right-hand side we have the marginal benefit of investing into greener technology that comes from the fact that the firm's valuation for quota increases as it pays less environmental tax per unit of fish. This increases the firm's participation in the market as it is willing to enter at a higher price $p_i^0(y_i)$ and to bid more aggressively in the auction.

We assume that the solution is interior, therefore the second order condition $S(y) = \frac{\partial^2 U_i}{\partial y_i^2} < 0$, where $S(y)$ is given by:

$$\begin{aligned} S(y) &= \tfrac{\partial p_i^0}{\partial y_i}(y_i)\tfrac{\partial q_i^0}{\partial y_i}(y_i) + \tfrac{\partial^2 p_i^0}{\partial y_i^2}(y_i)\, q_i^0(y_i) \\ &\quad + \tfrac{\partial p_i^0}{\partial y_i}(y_i)\tfrac{\partial q_i}{\partial y_i}\left(p_i^0(y_i), y_i\right) + \int_{p^G}^{p_i^0(y_i)} \tfrac{\partial^2 q_i}{\partial y_i^2}(p, y_i)\, dp + \tfrac{\partial^2 p^*}{\partial y_i^2}(y)\, q_i(p^*, y_i) \\ &\quad + \tfrac{\partial p^*}{\partial y_i}(y)\tfrac{\partial q_i}{\partial y_i}(p^*, y_i) + \tfrac{\partial p^*}{\partial y_i}(y)\tfrac{\partial}{\partial y_i}\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} \\ &\quad + \left(p^*(y) - p^G\right)\tfrac{\partial^2}{\partial y_i^2}\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}. \end{aligned}$$

As we have established, the firm's utility is maximized when the marginal cost of investment equals the marginal benefit from investing. It is also useful to understand how the level of investment influences the equilibrium price, as this impacts on government revenue and the payments of the firms for the quota. We are able to see this from the first order condition. Provided that $\sum_i q_i(p^*, y_i) \equiv Q$

$$\frac{\partial p^*}{\partial y_i}(y) = -\frac{\sum_j \frac{\partial q_j}{\partial y_j}(p^*, y_j)}{\sum_j \frac{\partial q_j}{\partial p}(p^*, y_j)} = -\frac{\frac{\partial q_i}{\partial y_i}}{\sum_j \frac{\partial q_j}{\partial p}(p^*, y_j)} > 0.$$

The numerator is positive since the demand for quota increases as investment increases. The denominator is negative as demand decreases with price and hence the derivative is greater than zero. Therefore, we have the following proposition:

Proposition 2.3.1 *The equilibrium price at the auction increases in the level of investment.*

This seems very plausible, as investment should lead to lower marginal costs. These in turn increase the demand for quota — as the activity becomes more profitable — which increases the equilibrium price at the auction.

Equilibrium Stability To study the equilibrium properties we use the implicit function theorem to show that

$$\frac{dy_i}{dy_j} = -\frac{\frac{\partial^2 p^*}{\partial y_i \partial y_j}(y)\, q_i(p^*, y_i) + \frac{\partial p^*}{\partial y_j}(y)\frac{\partial}{\partial y_i}\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}}{S(y)}.$$

Further, if $\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} = q_i^G$ (i.e. the grandfathered quota constraint is binding) then

$$\frac{dy_i}{dy_j} = -\frac{\frac{\partial^2 p^*}{\partial y_i \partial y_j}(y)\, q_i\left(p^*, y_i\right)}{S(y)} < 0.$$

This implies that optimal investment levels are substitutes: when another firm increases its investment y_j into the value of quota, the residual supply facing i shrinks. There is less return on i's investment and hence their incentive to invest decreases.

Proposition 2.3.2 *If the grandfathered quota constraint is binding, then investment levels are substitutes. An increase in firm j's investment leads to a decrease in firm i's investment levels.*

In reality, this would only be the case where some fraction of the quota was auctioned, and not in systems, where all the quota is grandfathered as is often the case. In such cases, firms would receive a fixed proportion of TAC and therefore potential investments by other firms would not have any implications on the amount of quota a firm would receive, nor the return on their investment.

Moreover, if the setting is symmetric[11] then $\frac{\partial^2 p^*}{\partial y_i \partial y_j}(y) = \frac{\partial^2 p^*}{\partial y_i^2}(y)$ and then $\left|\frac{dy_i}{dy_j}\right| < 1$ for all $i \neq j$. (By continuity this should remain the case in settings close enough to being symmetric). This implies that the best reply correspondence is a contraction mapping, and hence the equilibrium is stable.

Proposition 2.3.3 *When firms are symmetric (or near symmetric) the interior equilibrium is stable.*

[11] By symmetric is meant that players have identical marginal costs and grandfathered quota and hence also the same optimal investment, demand and price.

An equilibrium is considered stable when it is self adjusting — the economy will converge back to a stable equilibrium after potential shocks to the market. In fisheries these could include price shocks, trade wars and resource scarcity. In fisheries, boom and bust cycles are very common for many species where abundance fluctuates dramatically (McClatchie et al., 2017).

The Effect of Grandfathered Quota on Investment Now let us take a look at how the price and quantity of the grandfathered quota affect investments. By considering the first order condition and using the implicit function theorem we find that

$$\frac{dy_i}{dq_i^G} = -\frac{(p^*(y) - p^G)\frac{\partial^2}{\partial y_i \partial q_i^G} \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}}{S(y)}.$$

This derivative is always equal to zero. If $\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\} = q_i^G$, i.e. the grandfathered quota constraint is binding, then the demand of grandfathered quota q_i^G does not depend on y_i. If the constraint does not bind then $q_i\left(p^G, y_i\right)$ is independent of q_i^G. We can therefore conclude that the numerator and hence the derivative is always zero.

The changes in grandfathered quota offered to *another* auction participant do not affect the incentive to invest in environmentally friendly production either $\frac{dy_i}{dq_j^G} = -\frac{0}{S(y)<0}$.

Proposition 2.3.4 *The increase in grandfathered quota affects neither the investment level of the firm concerned by the change, nor that of any other firm in the market.*

It is unsurprising that when the firms are allocated more grandfathered quota than their demand, marginal changes in their allocated grandfathered quota do not have any effect on the firms choices, including the choice to invest. However, when firms demand more quota than what is allocated through grandfathering, which would be the most usual scenario, the reason for the non-effect is not so apparent. If the firms are allocated less grandfathered quota, then the amount on auction will increase by the same quantity. The units that they missed out on would be acquired at the equilibrium price $p^*(y)$ at auction instead. As a result, the price itself is unaffected by the changes in grandfathered quotas because when those are reduced, the auction supply automatically increases by the same amount. Hence, the marginal benefit from investment does not change.

However, the model does not take into account factors such as risk aversion and the importance of secure property rights for their decisions (Anderson et al. (2011); Libecap (2009)). Furthermore, as investments often require a long term perspective, it would be expected to be easier to make an investment based on access to a fixed fraction of the TAC, rather than the firms having to compete for access to the fisheries.

The Effect of the Price of Grandfathered Quota on Investment To find the effect on investment of changing the price of grandfathered quota we once more use the first order condition and apply the implicit function theorem to get:

$$\frac{dy_i}{dp^G} = -\frac{A + B - C}{S(y)} < 0,$$

with $A = -\frac{\partial q_i}{\partial y_i}\left(p^G, y_i\right)$, $B = \left(p^*\left(y\right) - p^G\right)\frac{\partial^2}{\partial y_i \partial p^G}\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}$ and $C = \frac{\partial}{\partial y_i}\min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}$.

In the numerator, the first term is negative as demand is increasing in the level of investment. The second term is zero, if the grandfathered quota constraint binds (which is the much more likely case when overcapacity is common), and the third term is negative. Therefore, we can conclude that $\frac{dy_i}{dp^G} < 0$ and that increasing the price of grandfathered quota will decrease the incentive for firms to invest.

Proposition 2.3.5 *If for firm i, the grandfathered quota constraint binds, then investment is decreasing in the price of grandfathered quota.*

This result seems very plausible as facing an increase in grandfathered quota payment, the firm would likely choose to invest less. In other words, when the price goes up the firm can afford less quota for the same money, and with lower quantities the return on investment would decrease. Furthermore, paying higher prices for the resource would result in having less capital available for investment.

Total Welfare The total welfare, also known as total surplus, is the sum of the surplus of the firms and the government running the auction. Figure 2.7 shows the total welfare, from firm i's operations and therefore the total welfare can be obtained by summing this up for all the firms in the market. We do not count the environmental tax towards this surplus since it will go directly into recovering the damage to the environment, assuming it is correctly computed. In our model, consumers do not need to be taken into account because the quantity and the price of final product, fish, remain constant. Consumer surplus is therefore not affected by the decision how much quota is allocated through the auction given that the country is a small producer that does not have the market power to affect the price of fish. The total welfare is therefore equal to the sum of all the firms' utilities and the revenue of the government. Looking at Figure 2.7 we can

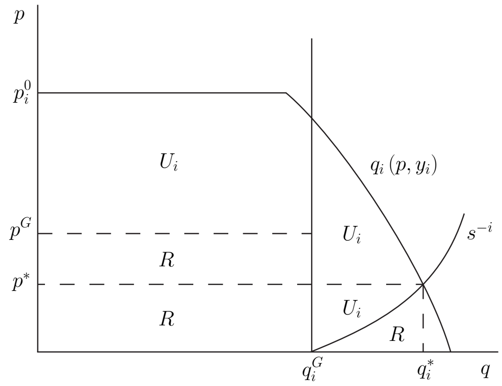


Figure 2.7: Total welfare from firm i's operations

see that the welfare can be expressed by the following equation:

$$\begin{aligned} W &= \sum_i U_i + R \\ &= \int_{p^*(y)}^{+\infty} \sum_i q_i(p, y_i)\, dp + p^*(y)\, Q - \sum_i y_i. \end{aligned}$$

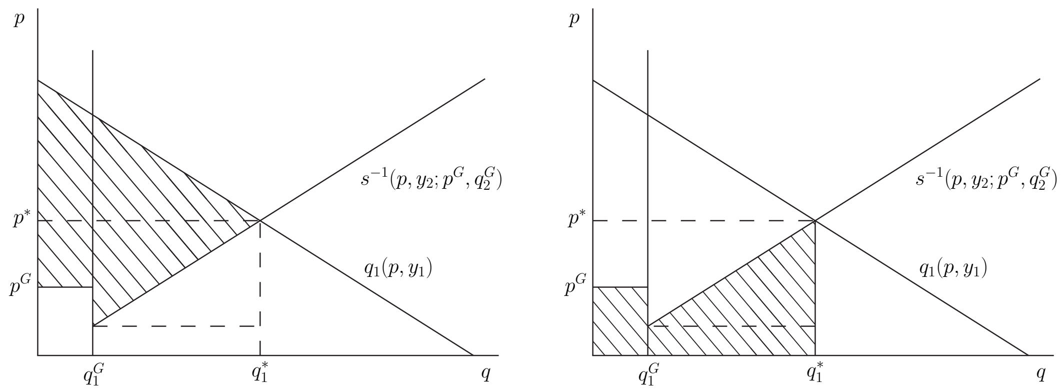
To see the effect of changing the price or quantity of grandfathered quota we can take the derivative of welfare with respect to these variables. It is obvious from the expression that none of the terms depend on either p^G or ${q_i}^G$ and therefore both derivates are zero and the effect absent. Looking at Figure 2.7 it is easy to see that with an equilibrium price that is unaffected by the grandfathered quota variables (see Lemma 2.2.1), the total welfare always remains the same, since investment is not a function of p^G or ${q_i}^G$. The following neutrality result therefore holds.

Proposition 2.3.6 *Total welfare is not affected by changes in the price or quantity of the grandfathered quota.*

Although total welfare is unaffected, it is worthwhile to explore the impact of price and quantity of grandfathered quota on the individual total welfare components, i.e. revenue and utility. To do so, we return to our example on page

52. The example demonstrated the effect on the firms' payments from changing the quantity of grandfathered quota from two to four units. The total payment of firm 1 increased by 2 whilst the payment of firm 2 remained unchanged. The average price paid at the auction increased for both firms.

Let us now calculate the utility of the firms. The utility of the firms from participating in the auctions is the area between the auction demand and residual supply functions and utility from grandfathering is given by the area below the demand function and above the grandfathered quota price (see Figure 2.8 below).



(a) Utility of firm 1

(b) Total payment of firm 1

Figure 2.8: Utility and payments of firm 1

Let us first calculate the utility when grandfathered quota $q_i^G = 2$ for each firm. The utility of the firms is given by:

$$\begin{aligned} U_1 &= \int_{\frac{5}{4}}^{\frac{47}{8}} \left(\tfrac{4}{3}p - \tfrac{5}{3}\right) dp + \int_{\frac{47}{8}}^{\frac{21}{2}} \left(14 - \tfrac{4}{3}p\right) dp \\ &\quad + \int_{3}^{\frac{21}{2}} 2dp + \int_{\frac{21}{2}}^{12} \left(16 - \tfrac{4}{3}p\right) dp = \tfrac{2161}{48} \approx 45.02 \end{aligned}$$

$$\begin{aligned} U_2 &= \int_{\frac{9}{4}}^{\frac{47}{8}} \left(\tfrac{4}{3}p - \tfrac{9}{3}\right) dp + \int_{\frac{47}{8}}^{\frac{19}{2}} \left(\tfrac{38}{3} - \tfrac{4}{3}p\right) dp \\ &\quad \int_{3}^{\frac{19}{2}} 2dp + \int_{\frac{19}{2}}^{11} \left(\tfrac{44}{3} - \tfrac{4}{3}p\right) dp = \tfrac{1537}{48} \approx 32.02. \end{aligned}$$

Doing the same calculations for the case when $q_i^G = 4$ we get the following figures:

$$U_1 = \frac{2065}{48} \approx 43.02$$

$$U_2 = \frac{1537}{48} \approx 32.02.$$

As is evident by the calculations, the utility of firm 2 remains unchanged whilst that of firm 1 decreases by 2. Therefore the total utility decreases by 2 given the fixed grandfathered quota price of $p^G = 3$. The changes in utility depend on the relationship between the grandfathered quota price and the equilibrium price at auction. In this particular case, there would be no effect of changing the grandfathered quota if the price of grandfathered quota were 2.5. Since the total welfare (from firm i's operations) is given by the striped area in figure 2.7 and having demonstrated that the effect of changing the grandfathered quota variables has no effect on total welfare, any decrease in utility would be offset by an equivalent increase in revenue.

In our model, the revenue is given by the striped area in Figure 2.9 and it can be expressed as:

$$\begin{aligned} R &= \sum_i \int_0^{p^*(y)} q_i^A(p^*) - max\left\{Q^A - \textstyle\sum_{j\neq i} q_j^A\left(p, y_i; p^G, q_j^G\right), 0\right\} dp \\ &\quad + p^G \textstyle\sum_i \min\left\{q_i\left(p^G, y_i\right), q_i^G\right\}. \end{aligned}$$

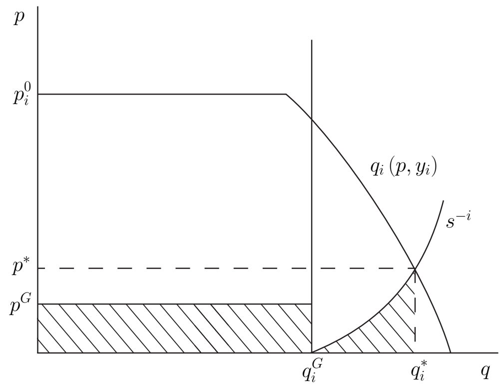


Figure 2.9: Revenue from firm i's purchases of quota

However, since we know that utility decreased by 2, it must be the case that revenue also increases by the same amount. We have already shown this by calculation, since revenue is given by the total amount paid by the two firms through the auction and through the fee on the grandfathered quota. Table 2.2 shows the calculated changes in revenue and utility at the current grandfathered quota price $p^G = 3$. Note that the total quantities purchased remain unchanged in the two cases.

Table 2.2: Utility and revenue when changing q_i^G

		Quantity			Utility		Revenue	
		G	A	Total	$q_i^G = 2$	$q_i^G = 4$	$q_i^G = 2$	$q_i^G = 4$
Firm 1	$q_i^G = 2$	2	6.17	8.17	45.02		27.97	
	$q_i^G = 4$	4	4.17	8.17		43.02		29.97
Firm 2	$q_i^G = 2$	2	4.83	6.83	32.02		25.64	
	$q_i^G = 4$	4	2.83	6.83		32.02		25.64
					77.04	75.04	53.60	55.60

Finally, let us briefly examine what happens to revenue and utility when the price of the grandfathered quota changes. When the grandfathered quota constraint binds, the price change has no effect on the equilibrium at the auction. Therefore, the change in revenue is simply the quantity of grandfathered quota multiplied by the price change. If, on the other hand, the constraint does not bind, the price change will result in lower or higher quantities purchased through grandfathering changing the auction supply. As the above example established, changes in quantities are only affecting the utility and payment of the most efficient firm. Furtermore, changes in utility and revenue only happen, when changing the grandfathered quota available to the most efficient firm. Whether the change in grandfathered quota affects revenue negatively or positively depends on the relationsship between p^G and $p^*(y)$.

2.4 Discussion and Conclusion

The work presented in this paper represents the first attempt at creating a model applicable to fisheries to explore how particular allocation mechanisms impact on various parameters, including government revenue from the fishing quota, total welfare, the firms' level of investment and the price obtained at the auction. It also considers how the level of resource fee on the grandfathered quota impacts the level of investment as well as the auction price. The majority of the results presented in previous sections are plausible, and therefore suggest that the model has relevance in the fisheries context. However, some of the results are less intuitive and somewhat surprising.

The model suggests that the amount of quota allocated to firms through grandfathering has no effect on the equilibrium price on the auction and the willingness to invest. At first glance, the apparent non-effect on the equilibrium price seems surprising, but considering that the equilibrium price is the highest price paid for a unit of quota, this does not imply that there is no effect on average prices, as demonstrated in Example 2.2.1.

More surprising is the lack of effect on the willingness to invest. However, this can be explained by the assumption in the model that the total supply remains unchanged, i.e. if the firms are allocated less grandfathered quota, then the amount on auction will increase by the same quantity. As a result, the price itself is unaffected by the changes in grandfathered quotas because when those are reduced, the auction supply automatically increases by the same amount. Hence, the marginal benefit from investment does not change. Another contributor to this result is the failure of the model to take into account factors, such as risk aversion and the preference for secure rights, which leads to some unrealistic model outcomes.

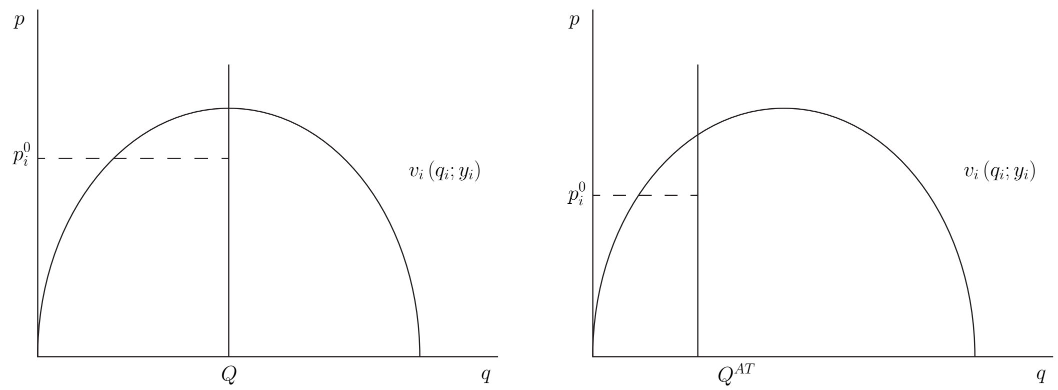
One particular strength of the model is that for the results to hold, the assumption of symmetry is not required. However, there is still considerable room for improvement if the model is to be applied in a decision-making context. In such a scenario, the model would need to be further developed, and several components need to be incorporated and in some cases revised. One of the areas where the model is less realistic with respect to fisheries, concerns how it treats the marginal costs of the firms. The model only considers increasing marginal costs, whereas in fisheries decreasing marginal costs are much more common with increasing quantities, as there is often fleet overcapacity (see García-Enríquez et al. (2016)). In order to get a more realistic picture with respect to the decision making of firms, future work should focus on including decreasing marginal costs. This is important as it is widely assumed that firms can bid to the point where marginal revenue equals marginal cost, but this only holds where fixed costs are already covered within the existing production. In reality, the firm considers its entire costs, and not just marginal ones, when deciding on their bidding strategy. Generally, for marginal units, the firm will demand additional units as long as these increase the total profit of the operation. It can be economically sound for the firm to profit at a lower rate for marginal units, than the average rate of profit per unit for the entire production, which the firm would need to cover its fixed costs. Incorporating the entire cost curve into the model would therefore be a requisite for improving its applicability to the fisheries and would provide a more realistic picture of the key factors involved in the firm's decision-making process with regards to its behaviour at the auctions.

Another area of improvement relates to potential rules at the auction. As currently set up, the model assumes that any firm can purchase the entire quota at the auction. In reality, it is likely that anti-trust rules would be applied at such a quota auction. Therefore anti-trust regulations would need to be incorporated into the model. One particular strength of the model is that it is a generic model, and therefore it is informative in a wide range of contexts, for instance electricity markets. Although, to maximise its potential in the policy arena, it would be essential to configure the model to the specific context in which it should be applied. In what follows, the first ideas for a potential way forward are presented with regards to how the model could be expanded and developed in future work to improve its applicability and relevance in the fisheries context, before final conclusions will be drawn.

2.4.1 Future Model Development

As discussed above, it is necessary to revise how marginal costs are treated in the model to make it more applicable to fisheries. Decreasing marginal costs indicate that there is overcapacity within the individual firm. In such cases, firms would require higher quantities than what was supplied on auction. In the model this would imply that the quota constraint would bind at a point where marginal costs are decreasing and hence the marginal value increasing. In such a case the firm could demand the whole supply and the firm would choose its highest price where the marginal value over all units is equal to the total cost. See Figure 2.10a. The firms' demand would therefore be zero at all prices above the maximum price and Q at all prices below this point. Figure 2.11a demonstrates what such a demand function would look like. If we assumed that each individual firm participating in the auction had enough capacity to fish the total quota auctioned themselves, then the residual supply would be given by two vertical lines, one at zero and one at Q. The intersection of demand and supply would be at the maximum price of the firm with the second highest valuation. See Figure 2.11b for an illustration.

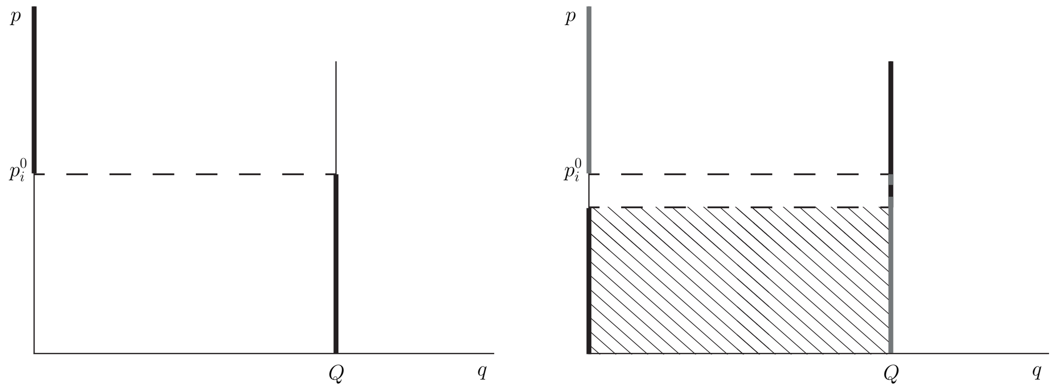
Such a setting would allow one firm to purchase the entire quota available but in reality it would be more likely that antitrust rules would prevent this from happening. If this were to be incorporated into the model you would obtain a new maximum price, which would be lower than without the anti-trust rules. See Figure 2.10b for the new maximum price.



(a) Marginal value when quota constraint is binding

(b) Marginal value with antitrust

Figure 2.10: Marginal values and maximum price on auction



(a) Demand function for bidder i (b) Residual supply facing bidder i

Figure 2.11: Demand and supply on auction

2.4.2 Concluding Remarks

This paper has presented a model for exploring how various allocation mechanisms impact on important concerns in management of fisheries. Applying the model in a theoretical setting, several important results have been obtained, which are very relevant in the fisheries context, including how auction prices (both equilibrium and average) and investment are influenced by both the fraction of grandfathered quota in the system as well as the price (resource fee) paid for this quota. This provides policy-makers with a tool to explore trade-offs between government revenue and investment into environmentally friendly technologies and thereby understand how to incentivise investments in the industry. The preceding sections have also explored how the revenue of the government and utility of the firms is influenced by changing the parameters. This also relates to an important political concern, as this would make evident how particular policy decisions, such as changing the amount or price of grandfathered quota — and thereby also the amount that is auctioned — would impact on industry and the revenue of the government. The majority of the results seem to fit to what one might expect, indicating that the model can be applied in the field of fisheries, although as indicated above, the model and its underlying assumptions need to be refined and developed further to improve its applicability in the policy context. Much work still remains to be done, but here we have taken the first steps in outlining and conceptualising what a more realistic and reliable model would look like in the near future.

Chapter 3

EPIC Fail: How Below-Bid Pricing Backfires in Multiunit Auctions

3.1 Introduction

If a seller wishes to sell a single item, many auction theorists would recommend an ascending auction.[1,2] If the values are affiliated (e.g. common), the symmetric equilibrium in an ascending auction raises greater revenue than the second-price sealed-bid auction and even greater revenue than the first-price sealed-bid or descending auctions (Milgrom and Weber, 1982b). Moreover, when bidders have independent private values, the ascending auction is the unique strategyproof and credible mechanism i.e. it cannot be undetectably manipulated by the seller (Akbarpour and Li, 2018).[3]

[1]This is an extended version of a paper cowritten with Daniel Marszalec and Alexander Teytelboym. See Laksá et al. (2018).

[2]First-price (second-price) sealed bid and descending (ascending) auctions are strategically equivalent from the bidders' point of view (Vickrey, 1961). If bidders' values are independent and private and if bidders are risk-neutral, all standard auctions yield the same revenue (Riley and Samuelson, 1981; Myerson, 1981).

[3]In fact, ascending auctions for a single item are ex-post incentive compatible (EPIC) and obviously strategyproof (Li, 2017).

If a seller wants to sell multiple units of a homogeneous good, many auction theorists would recommend a uniform-price auction.[4] Weber (1981) showed that in the affiliated values model with unit demands, the uniform-price auction yields greater revenue than the discriminatory auction. In a general setting with symmetric bidders, Pycia and Woodward (2016) proved that the unique equilibrium in the discriminatory auction for divisible goods is revenue-equivalent to the seller-optimal uniform-price auction with optimal supply and reserve price.[5] However, Ausubel et al. (2014) pointed out that when bidders are asymmetric uniform- and discriminatory-price auctions cannot be ranked either in terms of revenue or efficiency.[6] Uniform-price auctions appear appealing both from a theoretical and practical perspective. In the early 90s, the United States famously switched from discriminatory- to uniform-price auctions to sell debt (Bikhchandani and Huang, 1993). Today, both auction formats are popular to sell sovereign debt (Brenner et al., 2009). From a structural econometrics perspective, there is a mixed picture on the performance of discriminatory price auctions. Using a private-value model, Hortaçsu and McAdams (2010) find that the two auction formats perform similarly in the Turkish debt market. With a common-value model, Fevrier et al. (2002) and Castellanos and Oviedo (2008) produce contradictory

[4]There are many possible formats here: simultaneous vs. sequential and sealed-bid vs. dynamic, but we will focus on simultaneous, sealed-bid formats. Revenue-equivalence with independent private values holds (in a weaker form) for multiple object settings (Engelbrecht-Wiggans, 1988). While discriminatory-price auctions and uniform-price auctions are considered to be the natural generalizations of first-price and second-price sealed-bid auctions (Weber, 1981), the appropriate strategyproof generalization of the second-price auction is, of course, the Vickrey auction. Vickrey payments can also be obtained by running an English auction i.e. a deferred acceptance algorithm (Gul and Stacchetti, 2000; Ausubel, 2004). Uniform-price auctions are not strategyproof or even EPIC.

[5]Wilson (1979) introduced such "auctions for shares" and analysed the common-value setting. See also Klemperer and Meyer (1989); Green and Newbery (1992); Back and Zender (2001); Wang and Zender (2002). Swinkels (2001) proved revenue-equivalence of uniform- and discriminatory-price auctions in a large market setting.

[6]However, if agents are risk-averse in a common-value model then uniform-price auctions dominate in revenue and efficiency (Wang and Zender, 2002).

rankings for the French and the Mexican debt markets respectively. Using both private and common value models, Marszalec (2017) shows that discriminatory auction revenue-dominates in the Polish sovereign debt market under both sets of assumptions. Both types of auctions are also popular in many liberalised electricity markets (Fabra et al., 2002). Finally, in an experimental setting, Abbink et al. (2006) find that uniform-price auctions outperform discriminatory-price and Spanish auctions. Ausubel et al. (2014, p. 1393) conclude that:

> Uniform pricing has several desirable properties, including: (1) it is easily understood in both static and dynamic forms; (2) it is fair in the sense that the same price is paid by everyone; (3) absent market power it is efficient and strategically simple ("you just bid what you think it's worth"); and (4) the exercise of market power under uniform pricing favours smaller bidders.

But a theorist's preference for ascending and uniform-price auctions often relies on two assumptions rarely met in practice: bidder symmetry and competitiveness of the market.

3.1.1 Consequences of Bidder Asymmetry

It is well-known that when bidders are asymmetric, revenue equivalence of standard auction formats breaks down even in the singe-item, independent value setting: it is no longer possible to universally rank auctions by revenue, but in general stronger bidders prefer the ascending auction and weaker bidders prefer the first-price, sealed-bid auction (Maskin and Riley, 2000).[7]

[7] On the other hand, if either the seller or the bidders are risk-averse, the seller would prefer first-price auction (Maskin and Riley, 1984).

Milgrom and Weber's (1982b) beautiful result on the dominance of ascending auctions depends crucially on the assumption that all bidders are playing a symmetric equilibrium: in asymmetric equilibria their revenue ranking breaks down (Bikhchandani and Riley, 1991). Bulow et al. (1999) analyse such asymmetric equilibria in an *almost*-common-value setting of takeover battles in which one bidder exercises a small advantage by having a slightly larger "toehold" on existing company shares. They show that in this case there is a unique asymmetric equilibrium in the ascending auction in which the weaker bidder, in anticipation of aggression by the stronger bidder (who is interested in bidding more aggressively due to the value advantage), bids a very low amount. With sufficiently asymmetric value advantage, therefore, the stronger bidder can win the auction at a much lower price in an ascending (or second-price) auction than in a first-price auction (Bulow et al., 1999, Proposition 6). One may wonder whether such effects could be strong in practice, but Kagel and Levin (2002) argue that

> it would seem to require very sophisticated bidders for the explosive effect to be realized under these conditions. As such we would expect that bidders outside the laboratory would employ alternative strategies available to them in the less structured environment they operate in to press their private value advantage.

In real-world situations, therefore, bidders with an existing value advantage have an incentive to make announcements that cement their advantage, make other bidders bid even more conservatively or drive them out of the auction altogether. As a result, with strong value advantage asymmetry ascending auctions or second-price auctions could yield low revenue and be highly inefficient.

3.1.2 Consequences of Uncompetitive Bidding

In practice, the competitiveness assumption is often inadequate since bidders often have incentive to collude.[8] Auction design can mitigate some collusive practices (Klemperer, 2002a,b). In general, providing bidders with less information about each other and their bids and using sealed-bid formats is considered to make bidding ring formation more difficult (Robinson, 1985; Marshall and Marx, 2007, 2009, 2012).[9]

Among sealed-bid auction formats, there is little agreement on whether uniform-price auctions are more prone to collusion than discriminatory-price auctions. Friedman (1960) argued that the uniform-price auction "in any of its variants, will make the price the same for all purchasers, reduce the incentive for collusion, and greatly widen the market." Chari and Weber (1992) argued that: "Uniform-price auctions are also likely to be less susceptible to market manipulation."

However, many auction theorists have pointed out that uniform-price auctions are susceptible to coordination on low-price equilibria (Wilson, 1979; Back and Zender, 1993; Noussair, 1995; Engelbrecht-Wiggans and Kahn, 1998; Kremer and Nyborg, 2003, 2004; LiCalzi and Pavan, 2005; McAdams, 2007). In equilibrium, bidders submit high prices for the first few units and very low prices for the final units. The bids are coordinated such that the bids drop sharply as aggregate demand approaches supply. If any bidder wishes to grab more than the equilibrium share, the equilibrium price will jump sharply so he (and others) will have to pay a very high price for every unit of the good. Hence, bidders are deterred

[8]Che and Kim (2009) offer elegant designs of collusion-proof auctions, but they have not yet been used in practice.

[9]Milgrom and Weber's (1982b) "linkage principle" — which states that on average revenues can be increased by providing bidders with more information — fails in multiunit auctions (Perry and Reny, 1999).

from deviating. Indeed, (Milgrom, 2004, p. 264) says that these “extreme price equilibria [are]... of great practical importance”. Such sophisticated strategies have been observed experimentally when bidders have been given access to pre-play communication (Goswami et al., 1996). When these tactics are observed in procurement or supply-function settings they are referred to as “hockey-stick” bidding (since the price offered for the first units is low and price for the final units is very high).[10] Hockey-stick bidding is often found in repeated electricity auctions (Hurlbut et al., 2004; Holmberg and Newbery, 2010). Figure 3.1 illustrates an empirical example of hockey-stick bidding in the Texas electricity market. Such bidding strategies have been discussed in the context of supply function competition (e.g. Bayona et al., 2018; Vives, 2011).[11]

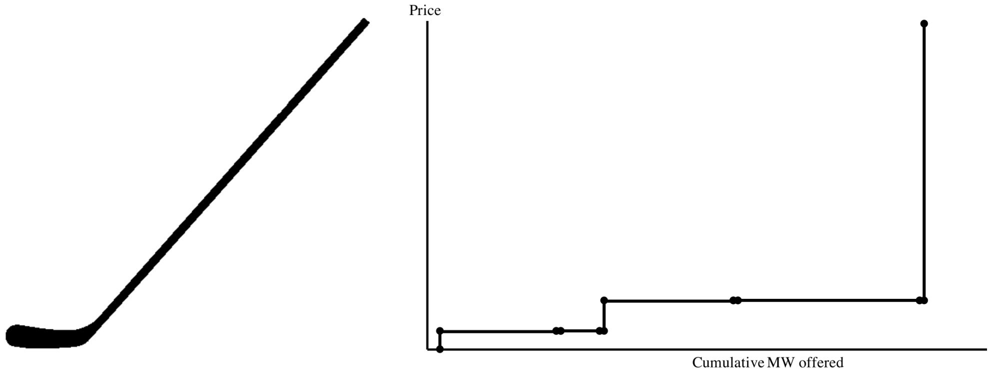


Figure 3.1: Hockey-stick bidding in an electricity market (data taken from Hurlbut et al. 2004)

3.1.3 Contribution of this Paper

In this paper, we present three examples of how inexperienced bidders who are participating in a high-stakes auction are able to exploit bidder asymmetries in an ascending auction as well as successfully coordinate on strategies that generate

[10] Cramton (2003) argues that “such bids are entirely reasonable given reasonable assumptions about demand and supply uncertainty, forward contracts, and marginal cost curves.”

[11] For more information on supply function competition, see the seminal paper by Klemperer and Meyer (1989).

extremely low-price equilibria in uniform-price auctions. We present bidding data from the first three years of auctions for fishing quota in the Faroe Islands which involve overlapping sets of bidders. In one set of sequential ascending auctions the largest quota holder in the relevant fisheries pre-announced that he was willing to pay whatever necessary to obtain all the quota. An entrant, who was the only other bidder, attempted to challenge him in the earlier rounds and, having lost, dropped out in the later rounds leaving the stronger bidder to pick up the remaining quota at the reserve price. We then present an example of coordinated bidding in sequential ascending auctions in which all but two bidders appear to bid as if in a bidding ring. In these auctions, almost all auctions ended at the same price and the winning bidders rotated. In the final set of auctions, we present a set of striking examples of what we call *crank-handle bidding*. We explore how crank-handle bids were successfully coordinated and whether market entry can prevent low-price equilibria. We believe that these are the first real-world examples of exploiting powerful value advantage effects, enforcing nearly perfect bid rotation, and exploiting subtle low-price equilibria among fairly inexperienced bidders in high-stakes auctions.[12]

This paper is organized as follows: First, we briefly introduce the context in which the auctions took place, before describing the design of the auctions in Section 3.3. The following section demonstrates how the ascending auctions were manipulated, before Section 3.5 documents underpricing in the uniform-price auctions. In Section 3.6 we look into common design features that exacerbated the failure of the auctions and suggest improvements before concluding in Section

[12]Our purpose is not to evaluate the suitability of auctions for allocating fishing rights in the the Faroe Islands; rather we want to analyse the design and performance of previous auctions and suggest improvements for the future.

3.7. To protect company identities, we label companies with letters, altering the labeling between sections.

3.2 Fishing Industry in the Faroes

The Faroe Islands is a small country in the North Atlantic which historically has been (and still is) heavily dependent on the fishing industry. In 2017, approximately 13 percent of GDP (Statistics Faroe Islands, 2017b) and 52.5 percent of the country's exports came from the fishing industry (Christiansen and Markná, 2018) and the fisheries sector employed 10.8 percent of the workforce (Statistics Faroe Islands, 2017a). Although the fisheries contribute substantially to the economy, the industry is small in the global context and the number of economic operators is limited. Faroese vessels have traditionally been fishing in many corners of the world, but currently there are three fisheries that have particular commercial significance for the industry: (i) demersal fisheries in Faroese waters; (ii) pelagic fisheries (e.g. mackerel, herring, blue whiting); and (iii) demersal fisheries targeting cod in the Barents Sea. Only the latter two are commercially lucrative. The demersal fisheries in Faroese waters, although currently less commercially significant, is an important economic and social contributor to local fishing communities across the islands.

In 2008, the Faroe Islands passed a Competition Act similar to Denmark's Competition Act. The law, which includes provisions against anti-competitive behavior, abuse of market dominance and merger control, is enforced by the Faroese Competition Authority (Faroese Competition Authority, 2007). Just three companies operate in the Barents Sea (P/F Enniberg, Sp/f Framherji and

P/F JFK Trol). The pelagic fisheries industry is also concentrated: four companies (P/F Christian í Grótinum, Sp/f Framherji, P/F JFK Trol and P/F Varðin) with ten vessels fish approximately 90 percent of the pelagic catch (Faroese Fisheries Inspectorate, 2017). For a complete overview of companies participating in the auctions, see Appendix A.[13] Large fishing companies are fairly homogeneous in costs and share ownership of fish factories and smaller companies in the industry. Compared to the rest of the world, the fishing industry in the Faroes is very small so Faroese firms act as price-takers with respect to global fish prices.

Since the introduction of fishing licenses in 1987, fishing rights in the Faroe Islands have been exclusively allocated based on historical fishing rights (i.e. "grandfathered") with the exception of a number of trial auctions. Meanwhile, all large companies frequently trade fishing quota through bilateral negotiations. During the last two decades, the pelagic and demersal fisheries in the Barents Sea have become increasingly profitable, following a change in legislation allowing increased transferability of fishing rights as well as increases in the total allowable catch (TAC) (Fiskivinnunýskipanarbólkurin, 2016). In 2007, the government cancelled all fishing licenses with a ten-year notice committing its future self to a complete reform of fishing rights by 2018.

In 2016, as part of the ongoing fisheries reform the new government decided to run trial auctions of 10 percent of the TAC for demersal fish in the Barents Sea, blue whiting, herring, and mackerel (Føroya Løgting, 2016b). In 2017, the government auctioned off between 8 and 42 percent of the TAC for the same species (Føroya Løgting, 2016a). In 2018, the Faroe Islands implemented a fisheries

[13]The other 10 percent are caught by around forty other vessels, many of which are owned by Framherji, JFK Trol and Varðin either fully or partially.

reform which introduced auctioning of fishing rights as a permanent feature of the fisheries legislation. According to the new fisheries law, at least 25 percent of the TAC of blue whiting and 15 percent of the TAC of the remaining species would be sold on auction every year, starting in 2018 (Føroya Løgting, 2017).[14] We focus on the design and outcomes of these auctions in this paper. As a fraction of GDP, these are some of the largest auctions ever held.[15]

Globally, auctioning of fishing rights is still comparatively rare. Auctions are currently implemented in some fisheries in New Zealand, Chile and Washington State in the U.S. (Cerda-D' Amico and Urbina-Véliz, 2000; Peña-Torres, 2002; Anderson and Holland, 2006; Lynham, 2014; Washington State Department of Natural Resources, 2018). Russia and Estonia have used auctions in the past, but have since moved away from this method of rights allocation (Vetemaa et al., 2002; Anferova et al., 2005; Eero et al., 2005; Vetemaa et al., 2005). Many economists believe that auction-based quota allocation can significantly improve the efficiency of operations, gives appropriate long-run incentives for innovation investment, and provides opportunities for revenue-recycling (Kominers et al., 2017; Marszalec, 2018a; Teytelboym, 2019). We do not consider broader questions of the desirability of quota auctions in the Faroese context and focus on the performance of the auctions instead.

[14] When TAC for each of the species is below a certain threshold, 15 percent (or 25 percent) will be sold on auction, but all quota above these thresholds will be auctioned off. Therefore, more than 15 or 25 percent of TAC may be sold on auction every year.

[15] Since the sale of the Roman Empire, the "largest ever" auction is often considered to be the 3G auction in the UK held in 2000 — it raised 2.5 percent of GNP by selling 20-year spectrum licenses (Binmore and Klemperer, 2002). By contrast, the Faroese one-year quota auctions held in 2017 raised 0.8 percent of GDP.

3.3 Auction Design

This section briefly explains the rules of the auctions run in the Faroe Islands in 2016–2018. It provides an overview of the setting in which these auctions took place. In following sections we will be using specific examples from these auctions to demonstrate how auctions with below-bid pricing rules may fail in such a setting. An overview of quantities and species sold on auction can be found in Appendix B. Average prices on the auctions and reserve prices can be found in Appendix C and D, respectively.

3.3.1 Trial Auctions

The trial auctions in 2016 and 2017 were designed and run by the Faroese Ministry of Fisheries. In 2016 a total of 10 percent of TAC of the four species (demersal fish, blue whiting, herring and mackerel) was sold on auction in July and September. Auctions were divided into series and two types of auctions — open and closed. The open auctions were run as sequential ascending auctions where each series (i.e. auction of a species in a certain day) was divided into smaller lots and sold sequentially within minutes.

The closed auctions were run as uniform-price sealed-bid auctions where the lowest winning bid would set the price for all winners of the auction. Here participants had to submit quantity-price bids (i.e. how many kilos they would like to purchase at what price) for each auction.

Owners of vessels registered in the Faroe Islands and with a fishing license were allowed to participate in the auction providing they had a letter of credit prior to the auction. Bidders were allowed to submit one bid per vessel in both the open and closed auctions.

The auctions in 2017 were very similar to those in 2016 although significantly larger quantities were sold — over three times more quota was sold (Fiskimálaráðið, 2016, 2017). As percentage of TAC, 8 percent of mackerel, 10 percent of demersal fish, 11 percent of blue whiting and 42 percent of herring were auctioned in 2017. The main difference in the design was that bidders were allowed to submit three bids per vessel in the sealed-bid auctions. These bids counted as separate bids which allowed for several winning bids per bidder. In addition to this a fishing license was not a pre-requisite for participation. Any Faroese company was allowed to contract vessels registered in the Faroe Islands to fish the quota for them as long as this was decided and specified prior to the auction. Some minor changes were also made regarding the publicly available information surrounding the auctions. For a complete description of the design including dates of auctions and quantities sold, see Appendix B.1 and B.2.

3.3.2 Auctions in the New Fisheries Management System

In 2018 between 15 and 29 percent of TAC of the four species was sold on auction. For the first time rights exceeding one year were sold. There were three types of maturities — one, three and eight year rights — and different fishing areas for each species. Companies would no longer submit bids for specific vessels but instead each company was allowed to submit five bids without specifying which vessels would use the quota. All registered companies in the Faroe Islands could participate in the auctions. Furthermore, the owners of the fishing vessel utilizing the quota had to be Faroese nationals, and the vessel had to be registered in the

Faroe Islands and possess a fishing license prior to fishing the quota (Føroya Løgting, 2017). See Appendix B.3 for a complete description of the design, including dates of the auctions and the quantities sold.

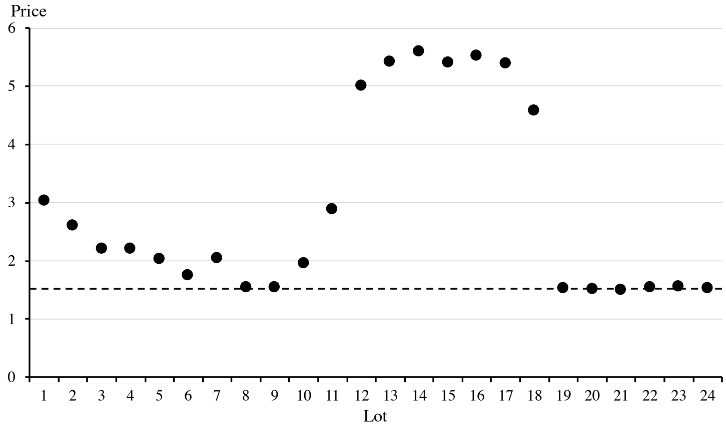
3.4 How Ascending Auctions Fail: Value Advantage, Signalling, and Coordination

3.4.1 Value Advantage and Signalling

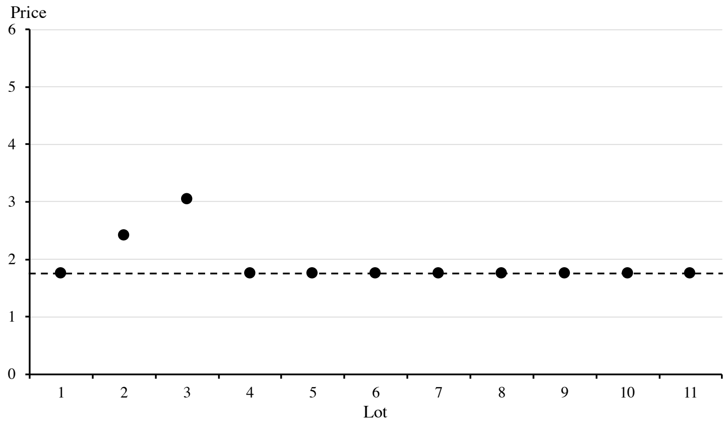
Let us first illustrate how poor auction design can exacerbate one bidder's value advantage in an almost-common-value auction setting. Consider the 2016 and 2017 auctions of quota for demersal fish in the Russian part of the Barents Sea. The auctions were run in sequences of ascending auctions, with similar amounts of quota being auctioned in each round.[16] In 2016, two weeks after the auction design had been announced — and before the auction was run — the largest quota holder made an announcement on national Faroese radio claiming that they were committed to win all the quota at whatever price necessary (Joensen, 2018). Despite several incumbents, all the 24 auctions had the same two bidders: the aforementioned incumbent and one entrant.[17] The outcome of the auctions was that the incumbent won all 24 lots and the entrant was therefore the price-setter in each auction round. Figure 3.2a shows the final price in each auction round. The entrant competed most keenly in rounds 10 to 18, and the last six auction rounds finished marginally above the reserve price. Overall, the average

[16]In 2016, 1200 tonnes were sold in 24 lots, and, in 2017, 1106 tonnes were sold in 11 lots.

[17]There was a third bidder in the 14th out of 24 auctions, but (as per their own explanation) this bidder entered by accident. He stayed in the auction for a very brief time, and was neither a winner, nor a price-setter. We therefore don't read much into this behaviour.



(a) 2016 auction (24 lots)



(b) 2017 auction (11 lots)

Note: Final prices given by dots and reserve prices given by dashed lines.

Figure 3.2: Prices in the auctions for quota for demersal fish in Russian waters

price for rounds 1–18 was approximately 3.4 kr./kg but only 1.5 kr./kg for the last six rounds. It appears that in the middle rounds, the entrant was testing the incumbent's resolve, and perhaps checking whether the incumbent had bought enough quota to be willing to exit the auction. With only six lots left, the entrant stopped competing altogether: one plausible reason for such behaviour is that the remaining amount of quota would not have been sufficient for the entrant to run a profitable operation. With no serious competition on the last six lots the incumbent could buy it marginally above the reserve price. As Figure 3.2b illustrates, the same auction ran in 2017 continued from where the previous year's auction left off: with the incumbent showing a strong commitment to bid whatever necessary to win everything. The entrant only bid in rounds 2 and 3 to test the incumbent's committment, but dropped out thereafter. Consequently, all lots other than 2 and 3 were sold at the reserve price to the incumbent.

Let us return to theory. Clearly, the standard symmetric and competitive bidder assumption does not hold here.[18] In the bidder pool there were three incumbents (one of whom possessed larger quota than the others) and one entrant (who was definitely value-disadvantaged). In the model of Bulow et al. (1999) even a small value advantage between bidders can result in extremely asymmetric bidding strategies in ascending (or second-price) auctions. In particular, the value-advantaged bidder can credibly commit to "stay in the auction indefinitely and win at any price", and in response to this the value-disadvantaged bidder bids very conservatively. As Klemperer (2002b, p. 174) pointed out:

> A strong bidder also has an incentive to create a reputation for aggressiveness that reinforces its advantage. For example, when Glaxo was bidding for Wellcome, it made it clear that it "would almost certainly top a rival bid" [...] Predation may be particularly easy in repeated ascending auctions, such as in a series of spectrum auctions. A bidder who buys assets that are complementary to assets for sale in a future auction or who simply bids very aggressively in early auctions can develop a reputation for aggressiveness (Bikhchandani, 1988). Potential rivals in future auctions will be less willing to participate and will bid less aggressively if they do participate (Klemperer, 2002a).

[18]Indeed, if bidders were competitive and symmetric, the price path should be a martingale and different auction rules should yield revenue equivalence (Milgrom and Weber, 1982b). Figures 3.2a and 3.2b show that the price path was certainly not a martingale in the 2016 and 2017 auctions for quota for demersal fish in the Russian part of the Barents Sea.

The above illustration offers two warnings for the auction practitioner who wishes to implement sequential ascending auctions. First, the main insights of Bulow et al. (1999) can play out powerfully in asymmetric ascending auctions and appear to be strengthened if incumbents can build a reputation over time. Second, the incumbent can block entry by aggressive bidding in earlier rounds and denying the entrant the chance to win enough quota to operate at a minimum feasible scale.

3.4.2 Coordination

Let us now examine how poor auction design can facilitate bidding ring formation. In 2017, the quota for mackerel was sold through two different types of auction: sequential ascending auctions (analogous to the ones we discussed in the previous section) and sealed-bid uniform-price (which we discuss later in Section 3.5). Each type of auction offered 5,447 tonnes of quota.

The sequential ascending auctions took place on 22 August and the 5,447 tonnes were divided into 24 smaller lots ranging from 100 to 467 tonnes each.[19] Seven bidders (we label them A–G in this section) participated in the auction and all bidders except one (Bidder G) won a share of the quota sold. Figure 3.3 shows the winning bidder and the prices paid by the winning bidders for each of the 25 lots. Each letter denotes a different winner. What is striking is the stability of the prices. After three rounds, the prices settled at around 3.10 kr./kg. From the third lot onward, all but two lots were sold at prices between 3.09 or 3.12 kr./kg (the reserve price was 1.25 kr./kg).

[19] Lot 13, originally sized at 100 tonnes, was split in two because a bidder only wanted to buy half, resulting in two lots of 50 tonnes each and 25 lots in total.

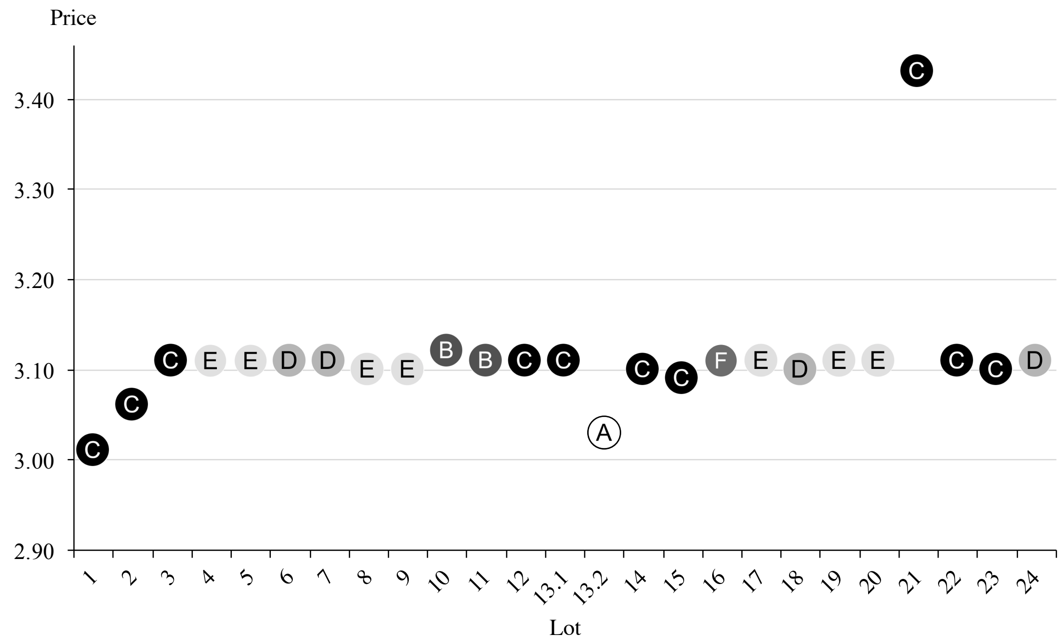


Figure 3.3: Final prices in the 2017 auctions for quota for mackerel (25 lots)

With the exception of lots 13.2 and 21, the prices in all these auctions were set by the same bidder — Bidder A, who exited roughly at the price of 3.10 kr./kg in 22 out of 25 rounds. The identities of the winning bidders rotated across rounds, with five bidders taking turns to outbid Bidder A suggesting that there might have been a bidding ring. In every round, with the exception of rounds 14 and 21, four non-winning ring members either did not participate or dropped out at prices much below 3.00, leaving one designated ring-member bidding against Bidder A.[20]

If the bidding ring were indeed present, then there are at least two explanations for the bidding pattern: Either Bidder A was trying to push up the price paid by all members of the bidding ring or the bidder served to commit members of the bidding ring to pay similar prices. Of course, what we present here is not

[20]In round 14, two non-winning ring bidders dropped out at 3.08 and 3.09 kr./kg respectively, possibly due to confusion caused by the splitting of lot 13 into two lots — 13.1 and 13.2 — just before this. In round 21, it appears that one ring-member (who won in round 20) forgot he was not supposed to win round 21, and stayed in until a price of 3.42 kr./kg. The bidder subsequently quit the auction, and did not participate in subsequent rounds.

conclusive evidence of an explicitly agreed bidding ring: repeated auctions, like any repeated strategic situations, naturally offer opportunities for patient players to tacitly coordinate on outcomes that are better for participants than static, one-shot equilibria (Pesendorfer, 2000; Aoyagi, 2003; Skrzypacz and Hopenhayn, 2004).

Why was bidder coordination so simple in this auction setting? As Robinson (1985) pointed out ascending auctions create ample opportunities for collusion. Moreover, sequential auction designs mean that more bidder signals can be sent (Marshall and Marx, 2007, 2009, 2012). Both observations relate to the two crucial pre-requisites for successful collusion that are detection and punishment: the present auction design easily affords both. Since after each round the winning price is announced, every member of the bidding ring can detect whether a deviation has occurred. If a defection from agreement does occur at any round before the last, remaining ring members can enforce a higher price in subsequent rounds. Since each lot in these auctions was small — and the objective of most ring-members was to win multiple lots — defecting from the agreement to win one additional lot was probably of limited value, relative to the lower prices sustained in later rounds.[21]

[21] An additional reason why we did not observe deviations even towards the end of the auction could be bidder concerns for future auctions. Since the same pool of bidders will be participating in future auctions together, and it is unlikely that the identity of the deviating bidder could be concealed, deviating even in the final rounds may not be profitable for an individual bidder.

3.5 How Uniform-Price Auctions Fail: Crank-Handle Bidding

Let us finally consider the susceptibility of uniform-price auctions to low-revenue equilibria. In 2018, for example, quota for demersal fish in Svalbard was sold in a series of (sealed-bid) uniform-price auctions. In these auctions, bidders were allowed to bid entire demand curves (at most five price-quantity bids that form the individual bid schedule) and the price for all units was set at the quantity where aggregate demand intersected (fixed) aggregate supply. There were three incumbents — let us refer to them as Bidders A, B, and C.

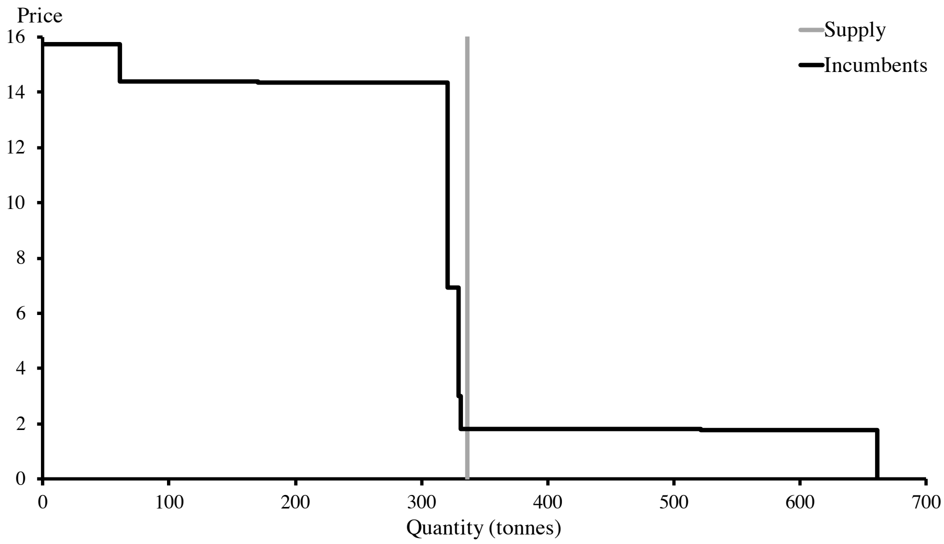
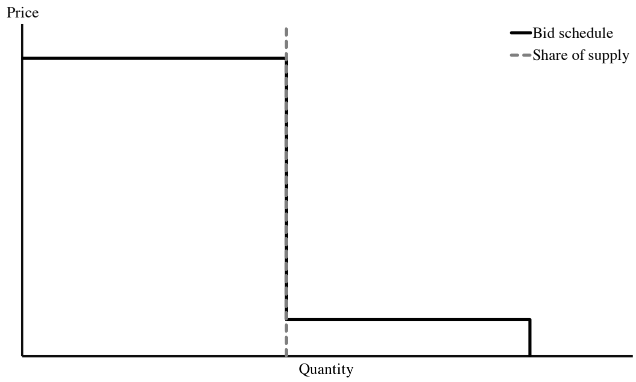
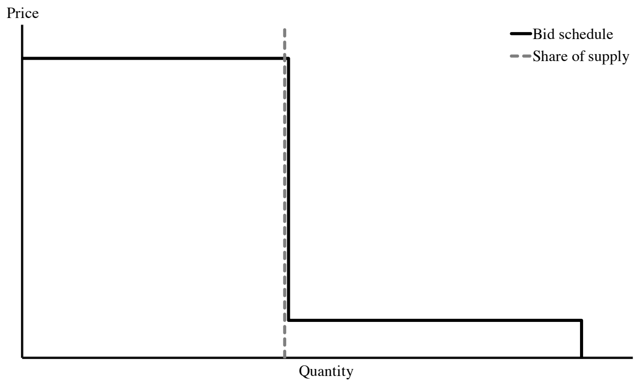


Figure 3.4: Aggregate bid schedule of the incumbents and the supply of quota in the 2018 auction for quota for demersal fish in Svalbard

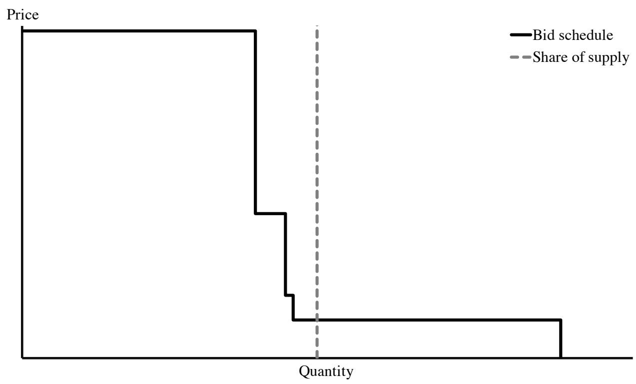
Figure 3.4 shows the final aggregate bid schedule and the supply in the 2018 auction for quota for demersal fish in Svalbard. A striking feature is the sharply declining portion of the aggregate bid schedule that takes place at quantities between 321 and 331 tonnes. The final step decrease occurs less than 1.5 percent from where aggregate demand and supply (of 336 tonnes) intersect. Even more striking are the individual bid schedules presented in Figure 3.5 which are flat over much of the reasonable support for all three bidders before dropping off sharply.



(a) Bidder A



(b) Bidder B



(c) Bidder C

Figure 3.5: Individual bid schedules in the 2018 auction for demersal fish quota in Svalbard and market share of grandfathered quota (prior to the auction)

What explains this unusual bidding pattern? We call these bidding patterns *crank-handle bidding* (see Figure 3.6 for the visual motivation for our nomenclature).[22] While there could be many equilibria in this uniform-price auction, we believe that this is a natural, profitable, low-price equilibrium that incumbent bidders can commit to coordinating on. As we explained in Section 3.1, to successfully implement coordinated bidding in uniform-price auctions, the drop-off in the aggregate bid schedule must happen just before the quantity at which it equals aggregate supply. Suppose that a bidder wants to increase their demand. This would push the vertical portion of the aggregate demand beyond supply and increase the price sharply for all inframarginal units. This would make the deviating bidder worse off and therefore the low-price equilibrium is, in fact, a Nash equilibrium.[23]

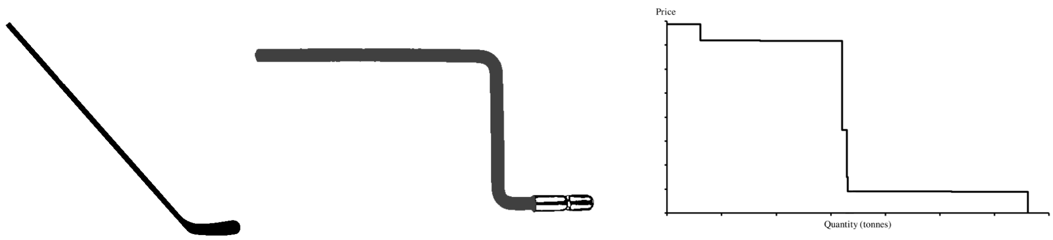


Figure 3.6: A hockey stick, a crank-handle, and a bidding schedule

Without knowing exactly the bidders' motivation for submitting the crank-handle bid schedules, we cannot of course be certain that they are tacitly coordinating on a self-enforcing equilibrium; indeed it might be possible that this is genuinely what their demand functions look like. To cast doubt that these bid schedules are purely coincidental, we note that in all six of the 2018 auctions, the point at which incumbents' bid schedules drop off sharply reflect precisely the incumbents' proportional holdings of the grandfathered quota share held prior

[22]In the procurement context, the "hockey stick" that describes hockey-stick bidding would be horizontally reflected (e.g. Hurlbut et al. (2004) and Holmberg and Newbery (2010)). In our case, the hockey stick presented in Figure 3.6 only describes a portion of the individual and aggregate bid schedules.

[23]An analogous argument also applies for coordination in Vickrey auctions.

to the auction. In Figure 3.5, we illustrate the grandfathered quota share of the supply by the dotted line. It would be a remarkable coincidence that all three bidders' valuations of the quota would change so dramatically at exactly the grandfathered quota shares, in particular since the industry as a whole has considerable spare capacity.

There are two other notable instances of crank-handle bidding in 2018. First, consider the auction for one-year quota for demersal fish in Russian waters. Here, the incumbents submitted crank-handle bids (Figure 3.7). However, the drop-offs in individual bid schedules were not in line with grandfathered quota shares (Figure 3.8). Nevertheless, the aggregate bid schedule still fell off dramatically just before the total quota supply. This indicates that bidders do not necessarily require a clear focal point to coordinate on a low-price equilibrium.

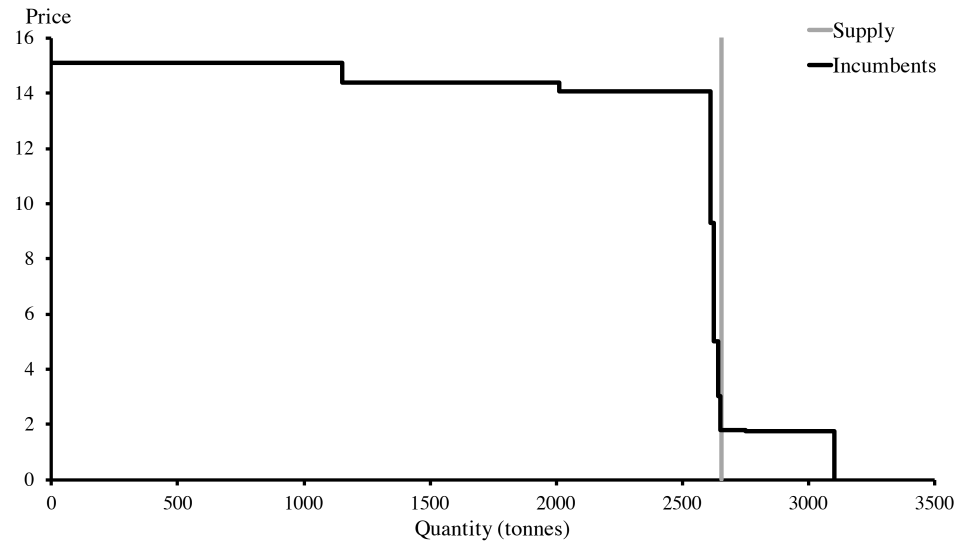
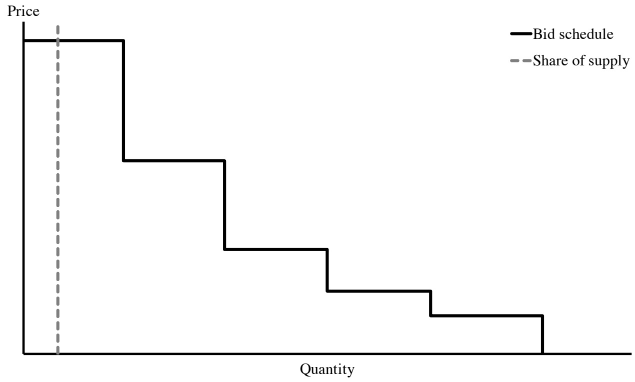
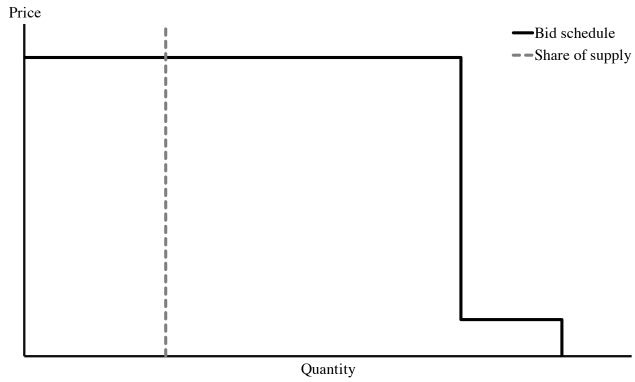


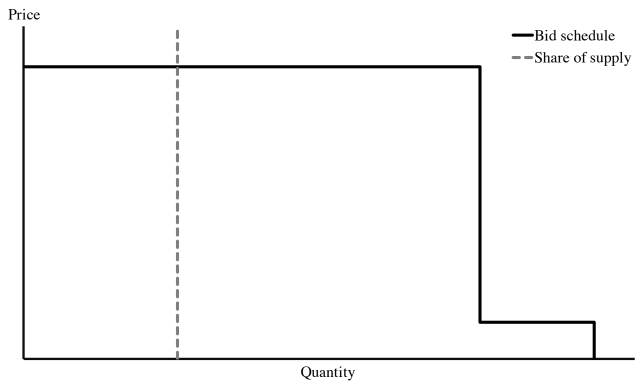
Figure 3.7: Aggregate bid schedule in the 2018 auction for one-year quota for demersal fish in Russian waters



(a) Bidder A



(b) Bidder B



(c) Bidder C

Figure 3.8: Individual bid schedules in the 2018 auction for one-year quota of demersal fish in Russian waters and grandfathered quota share of supply (prior to the auction)

Second, consider the auction for eight-year quota for demersal fish in the Norwegian part of Barents Sea. Once again, all the incumbents submitted crank-handle bids and for two out of the three bidders the drop-off in the bid schedule took place at almost exactly their grandfathered quota shares (Figure 3.10). As before, the drop-off in incumbents' aggregate bid schedule happened just before supply. However, in this case, the bidders did not succeed in coordinating on a low-price equilibrium because an entrant pushed the aggregate bid schedule out and therefore set a higher price (Figure 3.9).

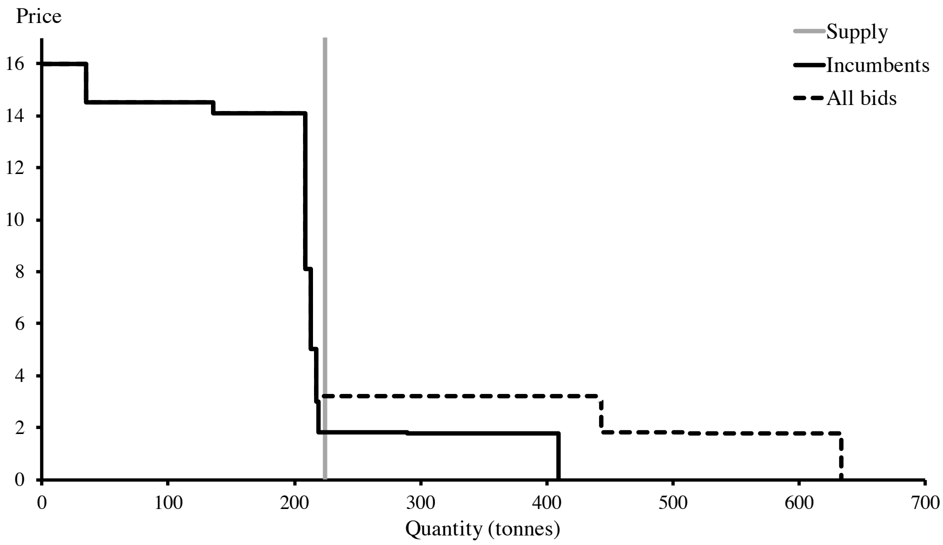
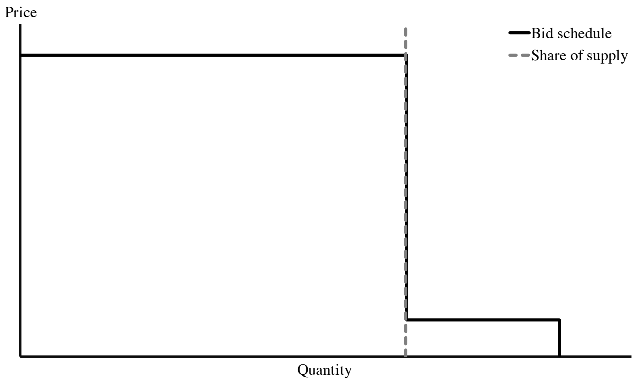


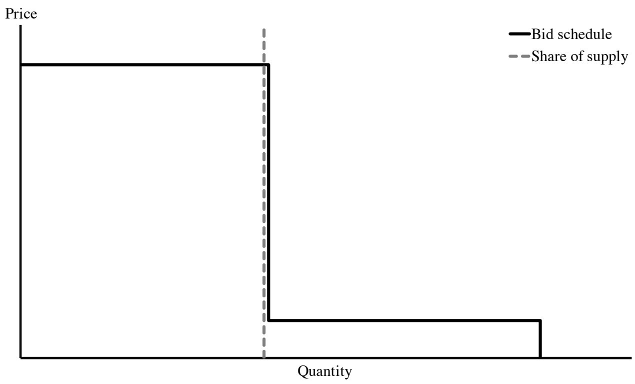
Figure 3.9: Aggregate bid schedule in the 2018 auction for eight-year quota for demersal fish in the Norwegian part of the Barents Sea

Crank-handle bidding was also observed in the three other 2018 auctions for demersal fish (see Appendix E). In each case, we observe attempts at low-price equilibria by the three incumbent bidders. However, all these other attempts were frustrated because of entrants setting a higher price by pushing out the aggregate bid schedule. It is worth noting that the entrants purchased less than one percent of the total quota auctioned. Had no entrants participated in any of the six 2018 auctions for demersal fish, the incumbents' winning shares would have reflected precisely the grandfathered quota shares prior to the auctions.

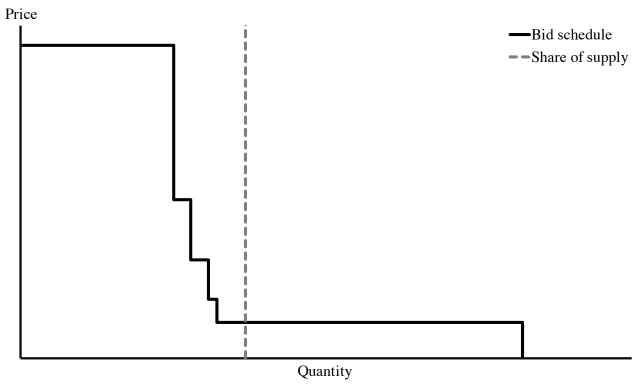
In 2016 and 2017, bidders in demersal fish uniform-price auctions were only allowed to submit one or three bids. Nevertheless, we observe a crank-handle in the aggregate demand schedule for the incumbent bidders which falls off just



(a) Bidder A



(b) Bidder B



(c) Bidder C

Figure 3.10: Individual bid schedules in the 2018 auction for eight-year quota of demersal fish in the Norwegian part of the Barents Sea and grandfathered quota share of supply (prior to the auction)

before supply (see Appendix F). As in our latter examples, whenever an entrant is present, the entrant was the price-setter in the main auction frustrating the incumbents' attempt at a low-price equilibrium. However, in each case, after the auction results were announced, the price-setting entrant declined to purchase any quota since their bid was not accepted in full. This meant that, once again, the incumbents' attempt at coordination on a low price using crank-handle bidding ended successfully, at least in part. Even if the price was higher than they had coordinated on, they still had successfully excluded all entrants.

3.6 Discussion

Let us now review the failures of the three auctions we described and suggest an alternative design that could have ameliorated the problems.

The sequential ascending auctions for quota for demersal fish in the Russian part of the Barents Sea held in 2016 and 2017 failed because the only participating incumbent was able to exercise his value advantage. As Bulow et al. (1999) show, in such a highly asymmetric setting with a common-value component, the first-price auction is expected to perform a lot better than the ascending or second-price auction. Moreover, the sequential nature of the auction allowed the incumbent to signal his commitment early on and drive the entrant out. Finally, because the quota was broken up into small lots, the entrant had little incentive to participate in the latest rounds because he could not achieve minimum viable scale.

The sequential ascending auctions for quota for mackerel in 2017 failed because bidders found a straightforward way to rotate winners and coordinate on a single price. The sequential nature of the auctions meant the bidders had a lot of information between rounds in order to monitor the outcomes of the possible bidding ring.

The uniform-price auctions for demersal fish in 2016–2018 failed because bidders found a low-price equilibrium in which they submitted crank-handle bidding curves. This equilibrium was only possible because in the uniform-price auction, only the marginal unit was setting the price for all quota, therefore the bidders did not need to worry about bids on inframarginal units (except to enforce the equilibrium). As Klemperer (2002b, p. 171) argued:

> Since, with many units, the lowest winning bid in a uniform-price auction is typically not importantly different from the highest losing bid, [the uniform-price] auction is analogous to an ascending auction (in which every winner pays the runner-up's willingness-to-pay). The "threats" that support collusion in a uniform-price auction are likewise analogous to the implicit threats supporting collusion in an ascending auction.

One auction design that could potentially improve all these auctions is the first-price sealed-bid package auction. In this design, bidders could submit bids for packages of quota (divided into sufficiently small lots) in a sealed-bid manner and the winner would pay the price that he bid for the winning packages. This design has several advantages. First, package bidding would allow bidders to express preferences over operational scale which would make it harder to lock out entrants. Second, sealed-bid auctions transmit little information to bidders, making the formation of bidding rings less likely. Third, pay-as-bid auctions reduce the value advantage of incumbent bidders compared to ascending or second-price auctions as well as any incentives for crank-handle bidding in uniform-price auctions. Therefore, pay-as-bid auctions are more likely to generate higher revenue and efficiency compared to ascending or second-price auctions in our setting.

Fourth, although the first-price sealed-bid package auctions have complex Bayes-Nash equilibria, it is often easier to explain a first-price auction to bidders than an auction with more subtle core-selecting pricing rules (for an example, see Prendergast, 2017, footnote 4 and Marszalec, 2018b).

There are two possible variations on the first-price, sealed-bid package auction. First, one could run a single auction for all the species at the same time, allowing bidders to express package bids across different species. However, this is likely to become complicated for the bidders as the bidding space would increase very quickly. Second, the government could also run an auction with uncertain supply of quota, which is most easily achieved by permitting the auctioneer to reduce supply if the stop-out price is too low for their liking. This design mitigates the ease with which bidders could coordinate on a crank-handle equilibrium and is used in practice in many Treasury Bill auctions (Back and Zender, 2001; LiCalzi and Pavan, 2005; McAdams, 2007).[24] However, uncertain supply might not be feasible in the fishing quota context since the annual TAC is usually publicly known once it has been set by the authorities.

3.7 Conclusion

Economic theory based on realistic assumptions about the auction setting can be extremely informative for auction design (Klemperer, 2002b). However, models that do not take into account bidder asymmetry and possibility of collusion could offer very misleading predictions of market behaviour and result in disastrous auc-

[24]Mariño and Marszalec (2018) find that the Department of the Treasury in Philippines uses active supply restriction frequently, and on occasion cancels the entire tender if prices seem too low. As a result, the market clearing price — whenever the auction is completed — is never pathologically low.

tion outcomes. In this paper, we showed that the underperformance of ascending and uniform price auctions, which is predicted by various models of auctions with asymmetric bidders and collusive behaviour, is borne out in practice even among bidders who have had little experience of bidding in such auctions. While fishing companies in Faroese auctions did not (as far as we know) hire dozens of auction consultants, their experience of running profitable businesses in a tightly-knit industry allowed them to easily implement profitable bidding strategies that surprised the policymakers.

We believe that we have documented the first example of such a plethora of interesting bidding strategies occurring simultaneously among a group of overlapping bidders. Our examples of crank-handle bidding are probably the most exciting as we know of no other such clear example from the field (except in electricity markets). However, it is worth noting that the underpricing equilibrium in this case was particularly easy to achieve since the supply of quota was fixed and known. With some uncertainty in supply (as is frequent in electricity and Treasury Bills auctions), such equilibria may be more difficult to achieve.

We therefore caution any auction designer who considers running (sequential) ascending or uniform-price auctions without flexible supply wherever there is serious bidder asymmetry and opportunities for industry coordination. We suggest that first-price sealed-bid package auctions may be more efficient in such settings. Meanwhile, the majesty of ascending and uniform-price auctions walks on theoretical stilts.

3.8 Appendix A: Companies

Table 3.1: Companies operating in pelagic fisheries and demersal fisheries in the Barents Sea

Company	No of vessels	Demersal fish	Blue whiting	Herring	Mackerel
P/F Enniberg	DF: 1	23%			
Sp/f Framherji	DF: 1 PF: 2	32%	20%	14%	12%
P/F Christian í Grótinum	PF: 2		16%	18%	18%
P/F JFK Trol	DF: 2 PF: 4	45%	28%	26%	13%
P/F Varðin	PF: 5		32%	33%	27%
P/F Jókin	DF: 1 PF: 2	0%	2%	3%	5%
P/F Thor Fisheries	DF: 1 PF: 2	0%	1%	4%	3%
P/F Ocean Group Faroes	PF: 1		1%	3%	1%

Note: The percentages are their share of the total catch in 2017 of the respective species with the number of vessels operating in pelagic and demersal fisheries given in the table.

3.9 Appendix B: Design of Auctions

Table 3.2: Total quantities (tonnes) sold on auction in 2016–2018

	2016	2017	2018			
Maturity	1 year	1 year	1 year	3 year	8 year	Total
Demersal fish	3,000	2,827	3,865	227	677	4,769
Blue whiting	20,000	54,594	68,169	7,000	14,000	89,169
Herring	4,000	53,000	10,150	780	1,560	12,490
Mackerel	9,000	10,894	8,283	873	1,745	10,901
Total	36,000	121,315				117,329

B.1 Design of Auctions 2016

Auctions were designed in accordance with Executive Order No. 78 from 29 June 2016 (Fiskimálaráðið, 2016). 10 percent of the TAC of demersal fish in the Barents Sea, blue whiting, herring and mackerel were sold. Auctions were divided into series and two types of auctions — 7 series of "open" and 6 series of "closed" auctions were run in July and September 2016. See Table 3.3 below for dates of auctions and quantity sold in each series. The Minister of Fisheries had the opportunity to set a reserve price and did so in all auctions. All quota bought on auction expired by the end of 2016 and could not be transferred to the next fishing year or to other vessels. The price of the quota had to be paid regardless of usage.

Owners of vessels registered in the Faroe Islands with a fishing license and owners of boats in Group 5,[25] according to the Commercial Fisheries Act from

[25]Group 5 consists of fishing boats under 15 tonnes which do not need a fishing license for fisheries.

1994 (Føroya Løgting, 1994), were allowed to participate in the auctions providing they had a letter of credit prior to the auction.

Open Auctions The open auctions were run as sequential ascending clock auctions (often referred to as Japanese auction). Each series would be split into either 15 or 24 sales lines containing smaller lots of quota. There were 150 lots in total and the reserve price was announced just before the beginning of each series. The procedure for selling each lot was as follows:

- An initial price is displayed (often reserve price) and bidders may enter the auction arena.
- The displayed price increases continuously and bidders may exit the auction at any point.
- No exiting bidders may re-enter the auction arena.
- When there is only one remaining bidder in the arena the auction (and price clock) automatically stops.
- The remaining bidder wins the lot and pays the displayed price — the price is therefore the valuation of the second highest bidder.

During the auctions buyers did not know who or how many bidders they were up against. Bidders were only shown whether there was one or several buyers in the auction arena. After the completion of each series of open auction the following information was publicly available: number of participants, names of winning participants, amounts bought including average prices for the different buyers, and total revenue to the government.

Closed Auctions The closed auctions were in the form of uniform-price sealed-bid auctions and participants had to submit a closed envelope stating how many kilos at what unit price they would like to purchase. Bidders were allowed to submit one bid for each vessel and the lowest winning bid would set the price for all bidders. The procedure for the closed auctions was as follows:

- Bidders submit one quantity-price bid per vessel in a sealed envelope.
- After the submission and before the opening of the envelopes the reserve price is announced.
- All envelopes are opened.
- The bidder with the highest bid is served first, then the bidder with the second highest bid etc. until no quota is left to sell.
- The price of the lowest winning bid sets the price for all winning bidders.

The opening of the envelopes was open to the public so all information regarding bids was publicly available.

Table 3.3: Dates and types of auctions including quantities (tonnes) sold in 2016

Date	Type	DF (NO)	DF (RU)	BW	HE	MA
11.07.2016	Open		1,200			
14.07.2016	Closed		600			
14.07.2016	Open				2,000	3,500
18.07.2016	Closed	600				
21.07.2016	Closed		600			
21.07.2016	Open				2,000	3,500
22.07.2016	Closed					2,000
05.09.2016	Open			-*		
06.09.2016	Open			-*		
08.09.2016	Closed			5,000		
12.09.2016	Open			5,000		
15.09.2016	Closed			5,000		
19.09.2016	Open			5,000*		
		600	2,400	20,000	4,000	9,000

*Note that the auctions of blue whiting quota on the the 5th and 6th of September were both unsuccessful because no buyers entered the auction. The reason for no interest to bid was because of the reserve price which was perceived as too high by the industry. On the 5th the reserve price was 0.30 kr./kg. It was then lowered to 0.18 kr./kg the following day but this proved to be an insufficient drop in reserve price to attract bidders. On the third attempt the reserve price was lowered to 0.12 kr./kg and this time all 5,000 tonnes were sold at an average price of 0.13 kr./kg.

B.2 Design of Auctions 2017

Auctions were designed in accordance with Executive Order No. 101 from 12 July 2017 (Fiskimálaráðið, 2017). As percentage of TAC, 8 percent of mackerel, 10 percent of demersal fish, 11 percent of blue whiting and 42 percent of herring were auctioned on open and closed auctions in August and September 2017. In

2017 there were a total of 17 series — 9 closed auctions and 8 open auctions. As in 2016, all quota bought on auction expired by the end of 2017. It could not be transferred to the next fishing year or between vessels and the price of the quota had to be paid regardless of usage. The Minister of Fisheries also set a reserve price in all auctions in 2017. See Table 3.4 for dates of the two types of auctions and the quantity sold in each series.

In 2017 some significant changes were made to the participation rules. This year a fishing license was not a prerequisite to be able to participate in the auction. It was not necessary for participants to own a vessel either. Any Faroese company was allowed to contract any vessel registered in the Faroe Islands to fish the quota for them. All contracts had to be made three weeks prior to the auctions taking place and only the vessel specified in the contract could fish the quota bought on auction. The main idea behind this change was to attract new bidders and to make it easier for new players to enter the auctions since the auctions in 2016 did not result in any new entrants to the market.

The open auctions were identical to the ones in 2016 apart from the division of the series into smaller lots. In 2017 each series was divided into 11 or 24 lots totalling 263 lots for the year. The exact same information (see appendix B.1) regarding the open auctions was publicly available after each auction.

The closed auctions were similar to those in 2016. Auctions were still run as sealed-bid uniform-price auctions where the lowest winning bid would set the price. The main difference was that bidders were allowed to submit three bids per vessel. The publicly available information after the sealed-bid auctions also changed — in 2017 the only information available to the public were names of winning participants, amounts bought and prices paid.

Table 3.4: Dates and types of auctions including quantities (tonnes) sold in 2017

Date	Type	DF (NO)	DF (RU)	BW	HE	MA
22.08.2017	Open		1,106			5,477
24.08.2017	Closed	614	1,107			5,477
29.08.2017	Open			9,000	9,000	
31.08.2017	Closed			9,000	9,000	
12.09.2017	Open			9,000	9,000	
14.09.2017	Closed			9,000	9,000	
19.09.2017	Open			9,297	8,500	
21.09.2017	Closed			9,297	8,500	
		614	2,213	54,594	53,000	10,894

B.3 Design of Auctions 2018

The auctions in 2018 were the first auctions after the new fisheries legislation passed in the government in December 2017. 15 series of auctions were designed in accordance with Executive Order No. 32 from 17 April 2018 (Fiskimálaráðið, 2018) and between 15 and 29 percent of TAC of the same four species were sold on open and closed auctions. The major difference in the auctions this year was that bids belonged to companies rather than vessels and that quota of different maturities was sold. All registered companies in the Faroe Islands could participate in the auctions. Furthermore, the owners of the fishing vessel utilizing the quota had to be Faroese nationals, and the vessel had to be registered in the Faroe Islands and possess a fishing licence prior to fishing the quota (Føroya Løgting, 2017).

The open and closed auctions were identical to the ones in 2017 with respect to publicly available information after each auction. There were only 3 series of open auctions — 1 series (16 lots) of herring quota and 2 series (25 lots each) of blue whiting quota — which were run identically to the ones in 2016 and 2017. There were 16 series of closed auctions, which also were similar to those in 2017 with the only difference being that bidders could submit 5 bids each. See Table 3.5 for an overview of all the auctions run in 2018. For quota lasting for more than one year the price of the quota in subsequent years would be calculated using the average landing value of the relevant years (Fiskimálaráðið, 2018).

Table 3.5: Dates and types of auctions including quantity, maturity and area of quota sold in 2018

Date	Type	Species	Maturity (years)	Area	Tonnes
24.04.2018	Open	Blue whiting	1	FO/IS/NEAFC	25,000
02.05.2018	Open	Blue whiting	1	FO/IS/NEAFC	25,000
06.06.2018	Closed	Blue whiting	1	FO/IS/NEAFC	18,169
06.06.2018	Closed	Blue whiting	3	FO/IS/NEAFC	7,000
06.06.2018	Closed	Blue whiting	8	FO/IS/NEAFC	14,000
13.06.2018	Closed	Demersal fish	1	NO	874
13.06.2018	Closed	Demersal fish	1	RU	2,655
13.06.2018	Closed	Demersal fish	1	SV	336
13.06.2018	Closed	Demersal fish	3	RU	227
13.06.2018	Closed	Demersal fish	8	NO	224
13.06.2018	Closed	Demersal fish	8	RU	453
18.06.2018	Open	Herring	1	FO/IS/NEAFC	5,000
21.08.2018	Closed	Mackerel	1	FO/EU/NEAFC	4,226
21.08.2018	Closed	Mackerel	1	FO/NO/NEAFC	4,057
21.08.2018	Closed	Mackerel	3	FO/NO/NEAFC	873
21.08.2018	Closed	Mackerel	8	FO/NEAFC	1,745
23.08.2018	Closed	Herring	1	FO/IS/NEAFC	5,150
23.08.2018	Closed	Herring	3	FO/IS/NEAFC	780
23.08.2018	Closed	Herring	8	FO/IS/NEAFC	1,560

3.10 Appendix C: Average Prices

Table 3.6: Weighted average prices in 2016

	DF (NO)	DF (RU)	BW	HE	MA
Open	-	2.98	0.13	3.57	3.63
	-	1,200	10,000	4,000	7,000
Closed	4.50	3.33	0.10	-	3.60
	600	1,200	10,000	-	2,000
Combined	4.50	3.15	0.11	3.57	3.66
	600	2,400	20,000	4,000	9,000

Table 3.7: Weighted average prices in 2017

	DF (NO)	DF (RU)	BW	HE	MA
Open	-	1.93	0.24	1.55	3.12
	-	1,106	27,297	26,500	5,477
Closed	4.50	3.01	0.24	1.47	3.25
	614	1,107	27,297	26,500	5,447
Combined	4.50	2.47	0.24	1.51	3.19
	614	2,213	54,594	53,000	10,894

Table 3.8: Weighted average prices in 2018

	Maturity (years)	DF (NO)	DF (RU)	DF (SV)	BW	HE	MA
Open	1	-	-	-	0.46	1.96	-
		-	-	-	50,000	5,000	-
	3	-	-	-	-	-	-
		-	-	-	-	-	-
	8	-	-	-	-	-	-
		-	-	-	-	-	-
Closed	1	3.10	1.82	1.83	0.40	2.50	5.26
		874	2,655	336	18,169	5,150	8,283
	3	-	3.20	-	0.60	2.75	5.11
		-	227	-	7,000	780	873
	8	3.20	3.20	-	0.66	2.95	6.10
		224	453	-	14,000	1,560	1,745
Combined		3.12	2.10	1.83	0.49	2.36	5.38
		1,098	3,335	336	89,169	12,490	10,901

3.11 Appendix D: Reserve Prices

Table 3.9: Reserve prices in 2016

	DF (NO)	DF (RU)	BW	HE	MA
Open	-	1.50	0.12*	1.00	1.25
Closed	1.75	1.75	0.10	1.00	1.25

*Reserve price was initially set at 0.30kr./kg, then at 0.18 kr./kg, but no bidders entered at those prices.

Table 3.10: Reserve prices in 2017

	DF (NO)	DF (RU)	BW	HE	MA
Open	-	1.75	0.10	1.00	1.25
Closed	1.75	1.75	0.10	1.00	1.25

Table 3.11: Reserve prices in 2018

	Maturity (years)	DF (NO)	DF (RU)	DF (SV)	BW	HE	MA
	1	-	-	-	0.10	1.00	-
Open	3	-	-	-	-	-	-
	8	-	-	-	-	-	-
	1	1.75	1.75	1.75	0.10	1.00	1.25
Closed	1.75	1.75	0.10	1.00	1.25	1.00	1.25
	8	1.75	1.75	-	0.10	1.00	1.25

3.12 Appendix E: Other Demersal Fish Quota Auctions in 2018

E.1 Three-Year Quota for Demersal Fish in the Russian Part of the Barents Sea

In this auction nine bids were submitted by three incumbents (Bidders A, B and C) and one entrant (Bidder D). If looking at the bids of the incumbents only, the sharply declining portion of the aggregate bid schedule takes place at quantities between 212 to 224 tonnes, with an aggregate supply of 227 tonnes. The final step occurs 1.3 percent from where aggregate demand and supply would have intersected with only the incumbents' bids. The price without the entrant would have been 1.82 kr./kg. However, with the entrant's bid the price was pushed up to 3.20 kr./kg. The entrant bought the remaining 7 tonnes even though it was significantly less than the 227 tonnes demanded. The individual bid schedules for all incumbents drop off sharply very close to the intersection of the bid schedule and the share of supply — for Bidder A and C the drop off happens just before, and for bidder B just after.

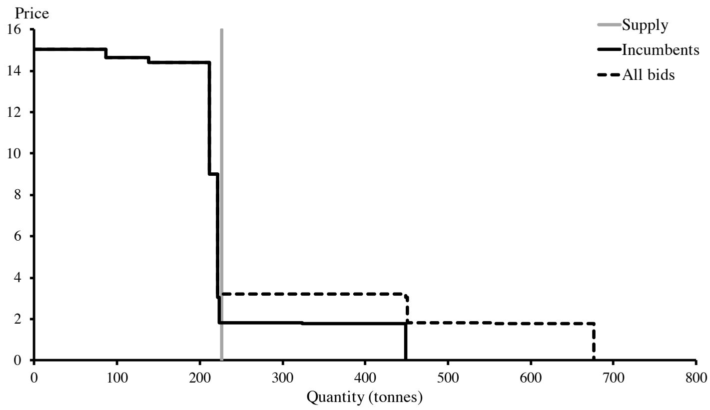
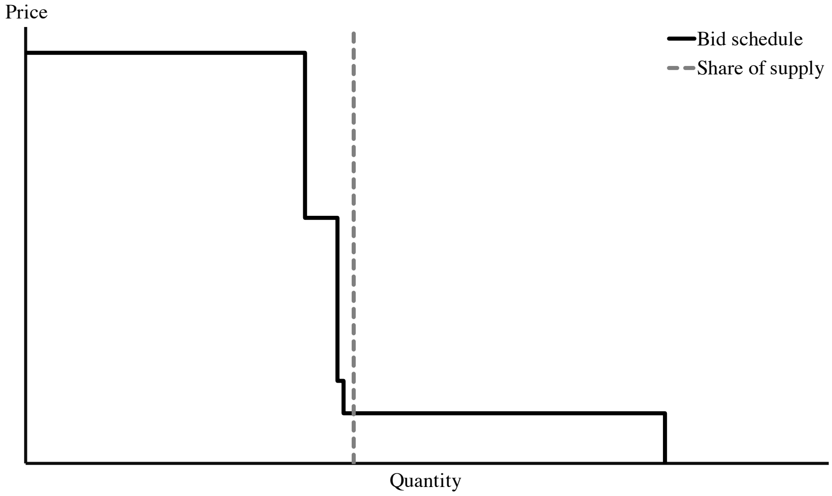
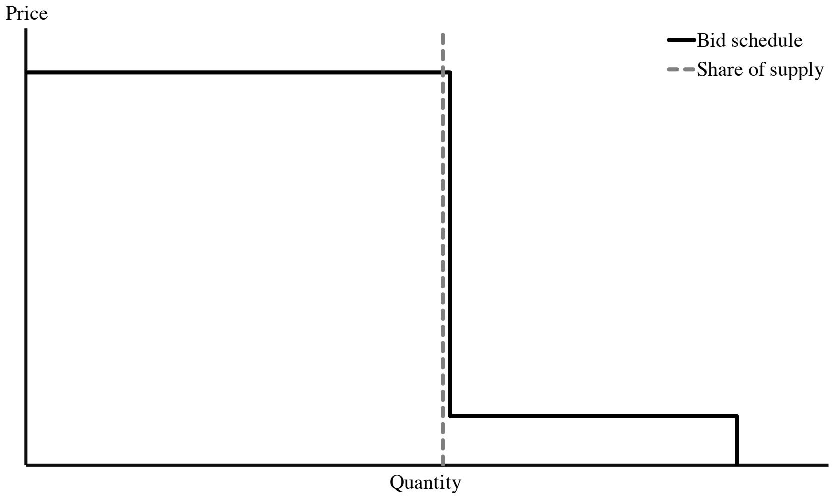


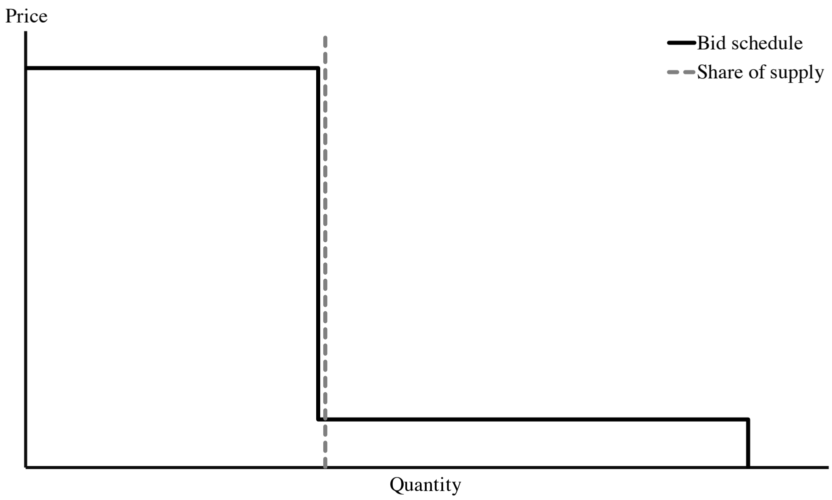
Figure 3.11: Aggregate bid schedule in the 2018 auction for three-year quota for demersal fish in Russian waters



(a) Bidder A



(b) Bidder B



(c) Bidder C

Figure 3.12: Individual bid schedules in the 2018 auction for three-year quota for demersal fish in Russian waters and grandfathered quota share of supply (prior to the auction)

E.2 Eight-Year Quota for Demersal Fish in the Russian Part of the Barents Sea

In this auction there was a total of ten bids submitted by the three incumbents (Bidders A, B and C) and one entrant (Bidder D). If looking at the bids of the incumbents only, the sharply declining portion of the aggregate bid schedule takes place at quantities between 433 to 448 tonnes, with an aggregate supply of 453 tonnes. The final step occurs at 1.1 percent from where aggregate demand and supply would have intersected with only the incumbents' bids. The price without the participation of the entrant would have been 1.85 kr./kg. However, with the entrant's bid the price for the quota settled at 3.20 kr./kg. Once more, Bidder D bought the remaining 10 tonnes, although significantly less than the 453 tonnes demanded. Like in the previous example, the individual bid schedules for all three incumbents drop off sharply very close to the intersection of their individual demands and their share of the supply — the drop off for bidders B and C is just before the intersection of the bid schedule and share of supply whilst for bidder A the drop off happens 150 kg after this intersection.

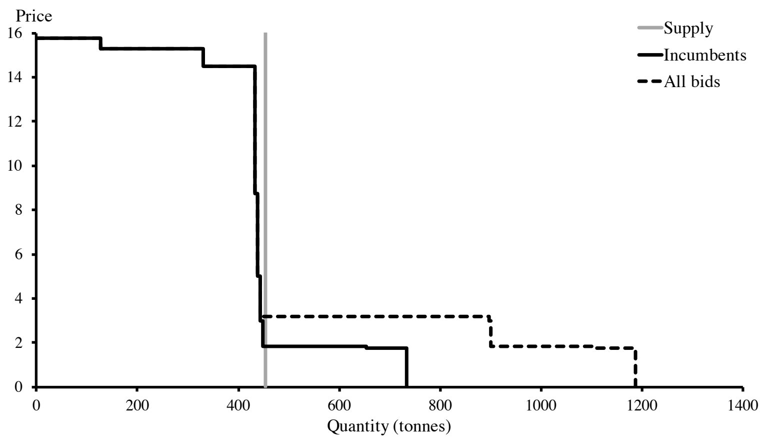
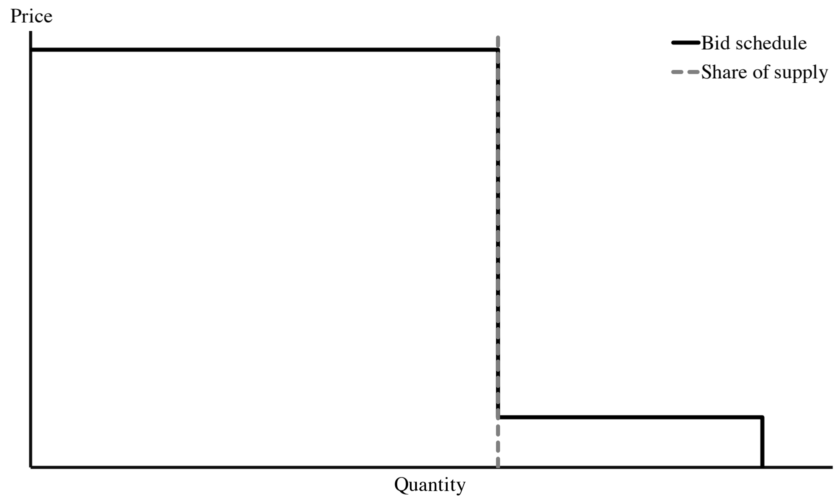
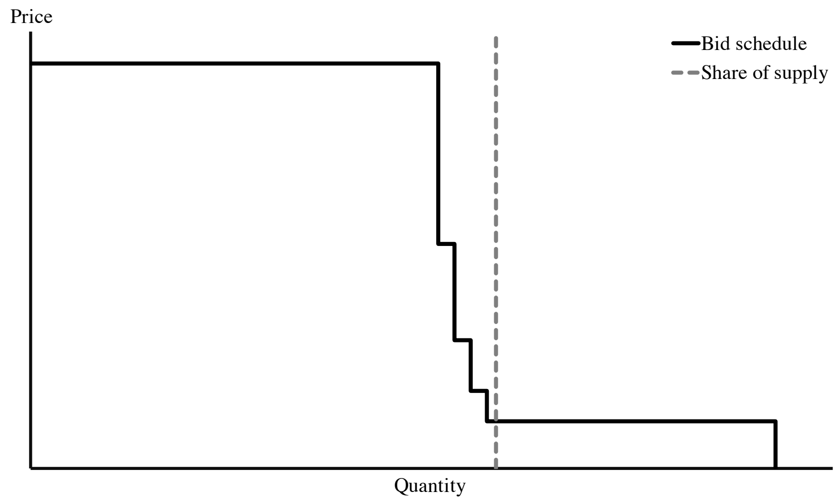


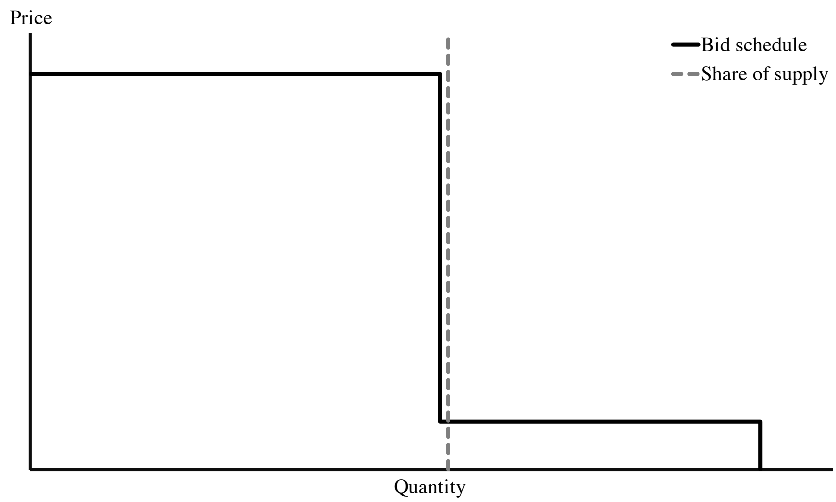
Figure 3.13: Aggregate bid schedule in the 2018 auction for eight-year quota for demersal fish in Russian waters



(a) Bidder A



(b) Bidder B



(c) Bidder C

Figure 3.14: Individual bid schedules in the 2018 auction for eight-year quota for demersal fish in Russian waters and grandfathered quota share of supply (prior to the auction)

E.3 One-Year Quota for Demersal Fish in the Norwegian Part of the Barents Sea

In this auction there was a total of nine bids submitted by the same three incumbents (A, B and C) and one entrant (Bidder E). If looking at the bids of the incumbents only, the sharply declining portion of the aggregate bid schedule takes place at quantities between 844 to 869 tonnes, with an aggregate supply of 874 tonnes. The final step occurs less than 0.6 percent from where aggregate demand and supply would have intersected with only the incumbents' bids. The price without the participation of the entrant would have been 1.85 kr./kg. However, with the entrant's bid the price ended up being 3.10 kr./kg. The entrant did not buy the remaining 20 tonnes as this was short of the 437 tonnes demanded. The striking feature about this auction is that the individual bid schedules for all three incumbents drop off sharply right before the intersection with the grandfathered quota share of the supply.

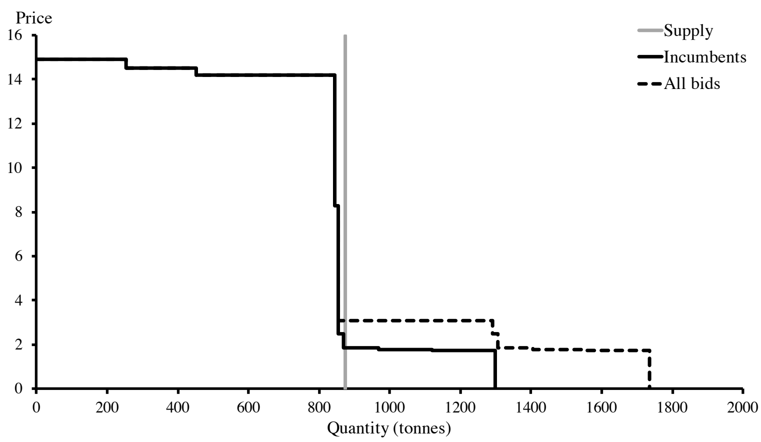
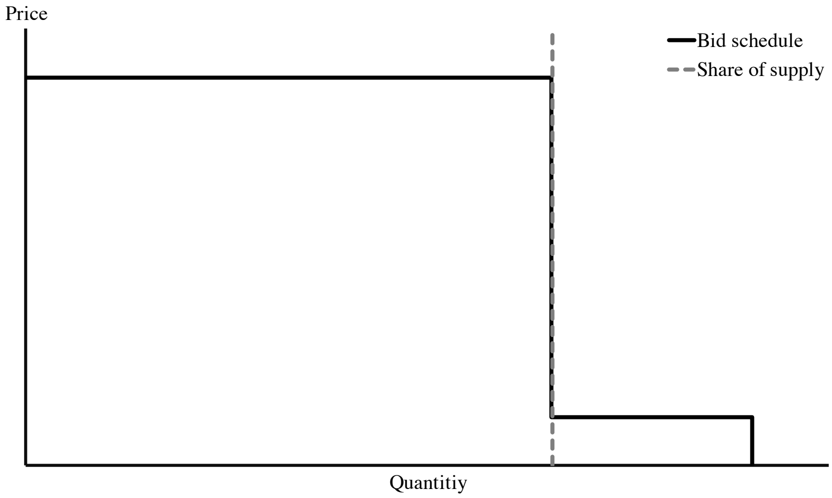
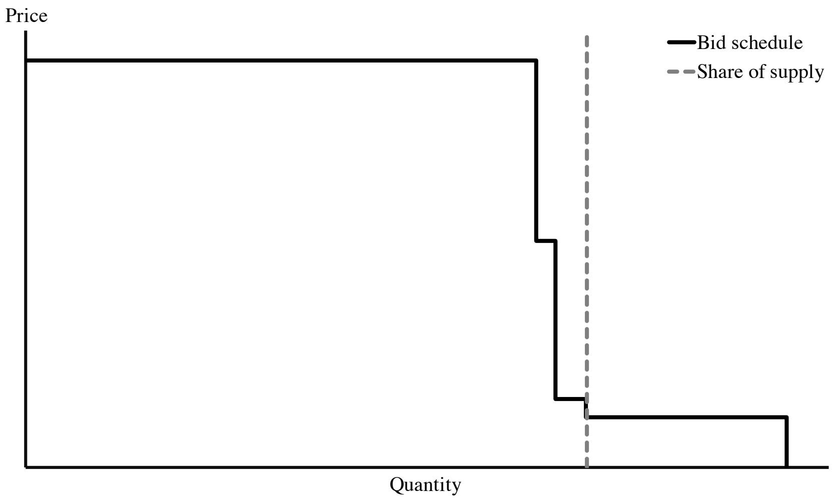


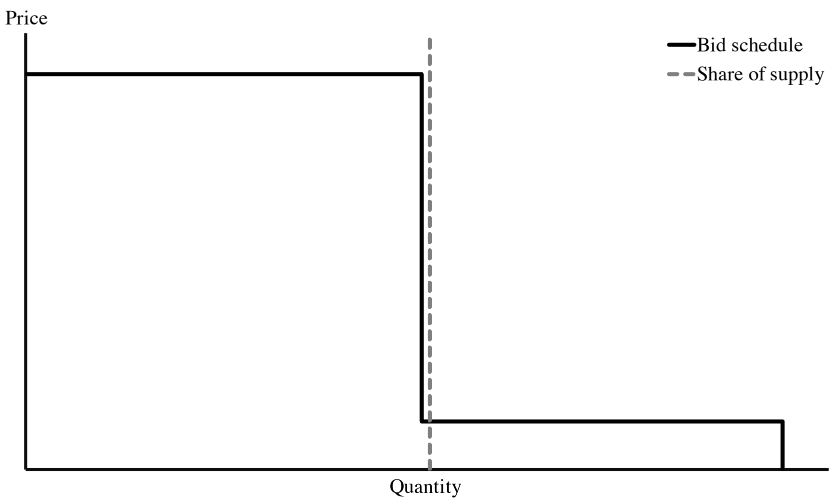
Figure 3.15: Aggregate bid schedule in the 2018 auction for one-year quota for demersal fish in the Norwegian part of the Barents Sea



(a) Bidder A



(b) Bidder B



(c) Bidder C

Figure 3.16: Individual bid schedules in the 2018 auction for one-year quota for demersal fish in the Norwegian part of the Barents Sea and grandfathered quota share of supply (prior to the auction)

3.13 Appendix F: Auctions for Demersal Fish Quota in the Russian Part of the Barents Sea in 2016 and 2017

F.1 Quota for Demersal Fish in the Russian Part of the Barents Sea 2016

The first evidence of crank-handle bidding was in the auction for one-year quota for demersal fish in the Russian part of the Barents Sea. The uniform-price sealed bid auction took place on 21 July 2016 and bidders were allowed to submit one bid per vessel. Five bids were submitted by four companies — three incumbents and one entrant. If looking at the bids of the incumbents only, the sharply declining portion of the aggregate bid schedule takes place at quantities between 550 to 575 tonnes, with an aggregate supply of 600 tonnes. The final step occurs less than 4.2 percent from where aggregate demand and supply would have intersected with only the incumbents' bids. The price in this case would have been 2.03 kr./kg. However, with the entrant's bid the price was pushed up to 3.06 kr./kg. The entrant did not end up buying the remaining 50 tonnes as this was short of the 385 tonnes demanded.

Table 3.12: All bids in the 2016 auction for one-year quota for demersal fish in Russian waters

Vessel	Company	Price kr./kg	Kg	Agg. demand	Leftover
Enniberg	Enniberg	8.31	300,000	300,000	
Gadus	JFK Trol	8.12	250,000	550,000	
Sjagaklettur	Jókin	3.25	385,000	935,000	50,000
Sjúrðarberg	JFK Trol	3.06	25,000		
Akraberg	Framherji	2.03	300,000		

Table 3.13: Incumbents' bids in the 2016 auction for one-year quota for demersal fish in Russian waters

Vessel	Company	Price kr./kg	Kg	Agg. demand	Leftover
Enniberg	Enniberg	8.31	300,000	300,000	
Gadus	JFK Trol	8.12	250,000	550,000	
Sjúrðarberg	JFK Trol	3.06	25,000	575,000	
Akraberg	Framherji	2.03	300,000	875,000	275,000

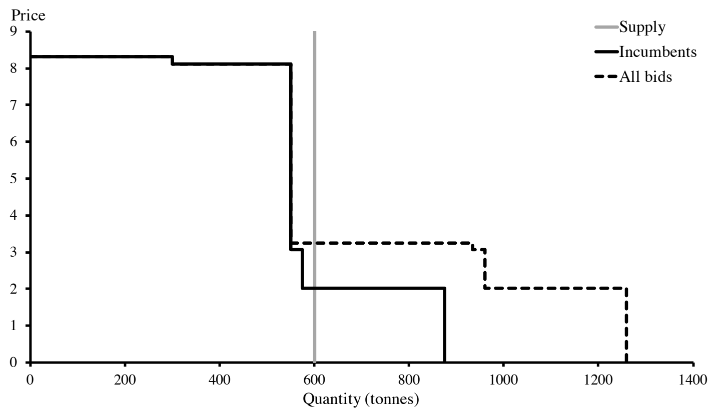


Figure 3.17: Aggregate bid schedule in the 2016 auction for one-year quota for demersal fish in Russian waters

F.2 Quota for Demersal Fish in the Russian Part of the Barents Sea 2017

Crank-handle bidding was also observed in the auction of one-year quota for demersal fish in the Russian part of the Barents Sea on 24 August 2017. Companies were allowed to submit three bids per vessel in 2017 following a change in the executive order concerning the auctions of fishing quota. A total of eight bids were submitted by four companies — the same three incumbents and one entrant. The sharply declining portion of the incumbents' aggregate bid schedule takes place at quantities between 1,055 and 1,095 tonnes, with the aggregate supply being 1,107 tonnes. The final step occurs at less than 1.1 percent from the intersection of aggregate demand (of incumbents) and supply. In the absence of the entrant the price would have been 1.75 kr./kg which was the reserve price set by the government. However, with the entrants bid the price was pushed up to 3.01 kr./kg. The entrant did not buy the 52 tonnes offered as this was significantly less than the demanded quantity of 450 tonnes.

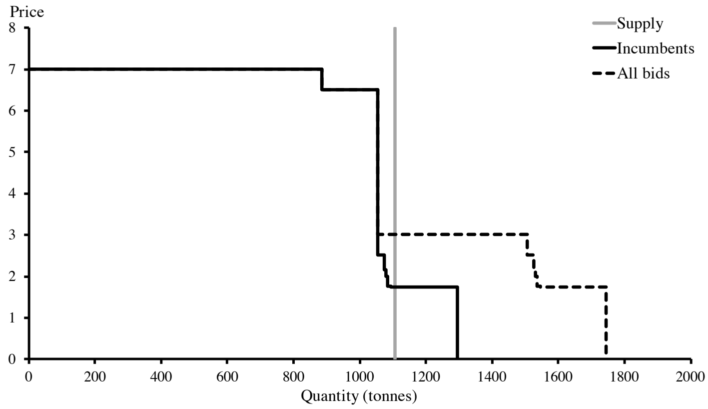


Figure 3.18: Aggregate bid schedule in the 2017 auction for one-year quota for demersal fish in Russian waters

Chapter 4

Declining Price Trends: Evidence from a Faroese Fish Market

4.1 Introduction

The declining price anomaly (or the afternoon-effect) has imposed an intriguing puzzle in the literature of sequential auctions. The term was popularised by Ashenfelter (1989) who showed that the price of identical bottles of wine sold on auction was more than twice as likely to decrease than to increase throughout the day. Since the discovery of this phenomena many economists have attempted to test for this feature in different markets and auction settings.[1,2] Others have made attempts to explain the phenomena theoretically (Engelbrecht-Wiggans and Kahn, 1999; Black and de Meza, 1992; McAfee and Vincent, 1993; Milgrom and

[1]Ashenfelter (1989); McAfee and Vincent (1993); Ginsburgh (1998) found the declining price anomaly in wine auctions, Deltas and Kosmopoulou (2004) and Kells (2001) in rare book auctions. Ashenfelter and Genesove (1992) and Lusht (1994) identified it in real estate auctions and Buccola (1982) and Engelbrecht-Wiggans and Kahn (1999) in livestock auctions.

[2]The declining price anomaly has also been supported by a number of controlled experiments. For some examples, see Burns (1985), Pitchik and Schotter (1988), Estenson (1995) and Keser and Olson (1996).

Weber, 1982a; Jeitschko, 1999, 1998; Bernhardt and Scoones, 1994).

The declining price anomaly has also been tested in a handful of fish markets (e.g. Gallegati et al., 2011; Pezanis-Christou, 2000; Salladarré et al., 2017; Fluvià et al., 2012) with contradictory results. The aim of this paper is to test for this anomaly in fish market auctions in the Faroe Islands. The Faroese fish market has several particular features which make it an interesting case for analysis. Firstly, the Faroese fish market has significantly fewer bidders compared to sellers than any other fish market studies on the topic. Secondly, it is a small market where sellers and bidders are quite well informed about each other. Thirdly, in contrast to most other fish markets studied, these auctions are run as pooled auctions (or right-to-choose auctions) where winning bidders can choose the desired units at every round.

This paper tests for the declining price anomaly in the Faroese fish markets using several methods. It applies the original ratio method conducted by Ashenfelter (1989). To add to the analysis, reduced form analysis is conducted by running various regressions on the whole dataset and on subsets corresponding to certain species. This way we can determine whether the findings also hold on the species level where goods are homogenous. The paper finds that declining prices are a strong feature of the Faroese fish market both overall and for major species.

The paper starts by reviewing the relevant literature related to the declining price anomaly, before introducing the fish market, the design of the auctions and the dataset. The following sections contain a description of the methodology, followed by a presentation of the results. Finally, the findings of the paper will be discussed in relation to existing theories on this anomaly before the final section concludes and proposes some avenues for further research.

4.1.1 Price Trends in Sequential Auctions

Sequential auctions of multiple items have been studied substantially since the theoretical work of Weber (1983). He showed that in the indepent private value (IPV) model, with risk-neutral bidders having unit demands, the price sequence in a first (or second) price auction of identical objects is a martingale — i.e. on average prices should not drift upwards nor downwards over time, but remain constant. Milgrom and Weber (1982b) extended the model of multiple identical units sold sequentially to include the case where bidders' valuations are affiliated. Milgrom and Weber (1982a) showed that in the first-price sequential auction with affiliation and price announcements the price sequence is a submartingale. Without price announcements they proved that the price sequence in first (and second) price auctions at equilibrium is upward-drifting, i.e. given the price in any stage, the expected price in the next stage would be at least as great. The upward-drifting prices are driven by the affiliation assumption that results in a lower risk of the winner's curse, as information about the bidders' valuations is revealed in earlier rounds. These famous results have been challenged empirically, originally by Ashenfelter (1989), who showed that prices in sequential auctions of identical bottles of wine sold in the mid 1980s were more than twice as likely to decrease as to increase. This result is often referred to as the *declining price anomaly* (or the *afternoon-effect* as better known among wine traders). McAfee and Vincent (1993) confirmed the existence of the declining price anomaly by using data from sequential wine auctions at Christie's auction house in Chicago in 1987.

Declining price trends are not limited to wine auctions and have been observed in many other markets.[3] Early evidence of such price trends was detected by Buccola (1982) who showed downward trending prices in livestock (fall yearling steer) auctions taking place at Winchester market, Virginia, from 1958–1979. Milgrom and Weber (1982a) reported strictly declining prices in transponder lease auctions at Sotheby's in 1981 contradicting the theoretical results of price sequences in the same paper. Kells (2001) studied Australian rare book sales in three different decades (1970–1990s) and found that prices in sequential auctions of identical books decreased more often than they increased or remained constant in all three decades.

Declining prices have been identified in studies focusing on auctions of real-estate (Ashenfelter and Genesove, 1992; Lusht, 1994),[4] Dutch flowers (van den Berg et al., 2001), jewellery (Chanel et al., 1996), cattle (Engelbrecht-Wiggans and Kahn, 1999; Buccola, 1982) and art (Beggs and Graddy, 1997; Pesando and Shum, 1996).[5]

Although the majority of empirical studies on price trends in sequential auctions point toward declining prices, there is some evidence of markets exhibiting increasing price trends.[6]

[3]Most of these markets do not sell completely identical objects like in the case of wine bottles. Exceptions include some book auctions. See Deltas and Kosmopoulou (2004) and Kells (2001).

[4]Ashenfelter and Genesove (1992) studied prices in sequential auctions for almost identical condominium units and Lusht (1994) analysed commercial real-estate auctions with unidentical units.

[5]Beggs and Graddy (1997) looked at auctions of impressionist and modern paintings and Pesando and Shum (1996) studied sales of modern Picasso prints.

[6]Examples include Chanel et al. (1996) for watches, Raviv (2006) for used cars, Gandal (1997) for television licenses, Jones et al. (2004) for wool.

The declining price anomaly has been attributed to many different factors. Many auction theorists have attempted to account for the phenomenon by considering e.g. risk aversion, supply uncertainty, heterogeneity of objects and participation costs. Ashenfelter (1989) and McAfee and Vincent (1993) suggested risk averse buyers were the cause of the anomaly, although McAfee and Vincent (1993) highlighted that this is only a sufficient explanation for bidders with *non-decrasing absolute risk aversion* — which is unlikely to be satisfied in practice — they (McAfee and Vincent, 1993) therefore argue that risk-aversion alone is not a sufficient explanation for the anomaly. Mezzetti (2011) attributed declining prices more specifically to bidders being averse to price risk, although he found a countervailing effect when there were informational externalities.

Pezanis-Christou's (2000) study of sequential descending fish market auctions with buyers with asymmetric preferences found that prices were not significantly affected by the asymmetries. He also used risk aversion to explain the reasons behind the declining prices, but concluded — like Neugebauer and Pezanis-Christou (2007) — that uncertain supply is another important factor in determining price trends, as price trends were positive when supply was high, whilst the opposite was true in low supply conditions. Similarly, Jeitschko (1999) considers settings in which supply was uncertain and found that the price formation depends on the information regarding supply and that an uncertain supply will lead to declining prices.

Aspects relating to competition have also been connected to the declining price anomaly. Von der Fehr (1994) argued that predatory bidding in earlier rounds reduces competition due to participation costs, whilst Engelbrecht-Wiggans and

Kahn (1999) highlighted a limited capacity for purchases by buyers, which leads to decreasing competition in later rounds and thereby, lower prices.

The work on the declining price anomaly has often assumed that goods are identical but there is often some heterogeneity in objects despite their shared characteristics. As an example, consider two lots of cod of the same size, fished at the same location, on the same day, using the same fishing gear, but by different vessels. These lots would be 'identical', although the experienced and well-informed buyer would know that the lot fished by one vessel would be of superior quality and therefore this heterogeneity would influence his behaviour at the auction. Several studies (Engelbrecht-Wiggans, 1994; Bernhardt and Scoones, 1994; Gale and Hausch, 1994) have related 'heterogeneity' to the declining price anomaly. Engelbrecht-Wiggans (1994) showed that as long as there is a large enough number of stochastically equivalent objects with bounded values, this will on average lead to a downward trend in prices, whereas Gale and Hausch (1994) found that right-to-choose auctions, where the winner can choose the preferred items first, guarantee declining prices and efficiency.

Finally, Black and de Meza (1992) have questioned the existence of the declining price anomaly, highlighting that the explanation for the observed price decreases by Ashenfelter (1989) was the buyer's option, where buyers could purchase additional units at the same price. They highlighted that this feature of wine auctions would automatically lead to declining prices. Having reviewed the key literature related to the declining price anomaly, the context and relevant auction market on which this chapter focuses will be described, before we go fishing for the anomaly at the Faroese fish market.

4.2 Faroe Fish Market

Fish prices in the Faroe Islands were originally set individually by fish factories across the islands. This led to huge dissatisfaction among fishermen, due to the different prices fishermen received depending on where they landed their fish. Several attempts were made to deal with this problem, first in 1965, when representatives from fish factories and the fishermen in common would agree on the fish prices. This system was further developed in 1975 — following a collapse of the fish prices in connection with the global oil crisis in 1973 — with the establishment of 'Ráfiskagrunnurin' (wet fish fund). The aim of the fund was to ensure that fishermen received more stable prices for their fish. This eventually led to a plan economy that did not serve the Faroese fishing industry very well (Gørtz et al., 1994; Astrup Hansen, 2007). After the 'failed' plan economy, calls were made for a more market oriented approach, and this represented the start of the Faroese fish auctions. The first auctions of wholesale fish in the Faroe Islands were run in 1991 where public auctions were organised through telephones. These auctions were well received, and soon proved to be a better solution than the price setting mechanisms used up until then. However, one big disadvantage with these auctions was that buyers could only buy fish in bulk rather than choose exactly what was needed. This sparked the idea of a 'floor' auction, where buyers could see and choose their preferred lots. In September 1992, the Faroe Fish Market was founded and in March 1993, it held the first auction of this kind. The Faroe Fish Market, which has been running auctions continuously since its establishment, remains the only fish market in the Faroe Islands.

4.2.1 Design of the Auctions

The auctions of wholesale fish in the Faroe Islands are organised as clock auctions. Faroe Fish Market runs both Dutch (descending) and Japanese (ascending) auctions and sometimes a mix of the two. The price is set by the highest or second highest bidder depending on the type of auction. The seller can set a reserve price and the auctioneer chooses the starting price. If any bidders enter the auction arena at the starting price, the price clock will increase until only one bidder is left in the auction arena – and the price is set by the second highest bidder.

If no bidders enter the auction at the chosen starting price, the price decreases until a bidder enters the auction. If there is only one bidder he will win at the price at which he entered and pay the first price. If more than one bidder enters as the price clock decreases, the clock will reverse, and the price will increase again, until only one bidder is left in the auction arena. The winner will pay the price equal to the second highest bid.

Participants can see which other bidders are active in the auction arena, and the winners of each lot are announced following each auction round. Buyers have detailed information about the fish on supply prior to the auction. This includes:

- Estimated quantity and size of each fish species
- Where and when the fish was caught
- Fishing method — trawl or line
- Method of storage
- Date, time and place of landing

There are two types of auctions: 'floor' auctions and 'telephone' auctions. In the floor auctions the fish is present at the fish market and buyers can see the fish before buying. The telephone auctions may only be based on approximate quantities and size of fish. In some cases, the fish has not yet been landed.[7]

All fish of the same species and size is sold in one pool. Buyers have the right to choose from that pool and the remaining fish will be sold in the following round. This process goes on until all the fish is sold — then the auctioneer moves on to the next size of the same fish (if there is more than one size) or the next species.

Fish factories located around the islands constitute the majority of buyers. Private (smaller) buyers along with foreign buyers occasionally participate. All Faroese vessels can sell their fish on the fish market. Since 2018 Faroe fisheries legislation (Føroya Løgting, 2017) has required that 25 percent of landed fish is sold through the fish market.[8]

4.2.2 Presentation of Data

The dataset used in this paper contains every transaction of fish sold at the fish market in the Faroe Islands from 2011 to 2017. The transactions are registered daily in the fish market's databases. The analysis in this paper will focus on data from January 2017 to December 2017. This represents a sufficiently big sample (5398 transactions) to test for the declining price anomaly. The database consists of data taken from two separate databases of the Faroe Fish Market — one containing all the auction data and the other one a record of all purchases.

[7]For further information on regulations of the auctioneer please see Faroe Fish Market (2007)

[8]Fish caught using purchased quota on auction is exempt from this requirement.

To obtain the final dataset, the two databases had to be merged in order to obtain quantities sold at each auction round. In a few instances, where there were inconsistencies between the two databases, auction rounds have been deleted from the dataset.[9]

The auctions analysed in this paper took place at the Faroe Fish Market, in Toftir, Faroe Islands. Auctions at the fish market are organised through an online system and buyers can decide to be present at the fish market or to participate remotely. Auctions take place every morning at 10 from Monday to Friday. On some days auctions are also run in the afternoon if there is additional fish to sell.

It has been highlighted before that one of the weaknesses of many of the studies on the declining price anomaly is that they focus on unidentical goods. Kells (2001, p. 12) argues that "differences between the assets in the sequences expose auction outcomes to the influence of sellers' quality ordering strategies and bidders' idiosyncratic preferences over asset characteristics". Due to this, we try to ensure the largest degree of homogeneity as possible by excluding rounds where lots contain mixed sizes of a specific fish. In other words, we only analyse transactions where lots contain fish of the same species and size.

Once inconsistencies between the databases and heterogeneous lots have been removed from the sample, 5398 transactions of a total of 25 fish species remain in the dataset. These species are further split into different sizes so there are a total of 63 different products sold on these auctions. The dataset included 2243 transactions from round 2 or higher. See Table 4.1 for total transactions in each

[9] A total of 4 percent of the transactions have been removed from the dataset for various reasons. Mismatches in price and buyers were the most common reason for inconsistencies. In order to keep the analysis as accurate as possible, inconsistent data was excluded from the analysis.

round. The auctions took place on 226 days in 2017. The fish sold in the sample were landed by 314 vessels at 19 different ports and the total number of bidders participating in these auctions was 25. See Appendix A for a summary of the data and the fish sold in 2017.

Table 4.1: Number of transactions in each round

Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Trans.	3155	1051	538	287	151	86	55	34	17	10	7	3	2	1	1

4.3 The Declining Price Anomaly

The aim of this paper is to look at the price dynamics in these auctions and to see whether the declining price anomaly is a feature of Faroese fish auctions. Several methods will be employed to test for declining prices in these auctions. In addition to applying these methods to all the data simultaneously, we also restrict our attention to certain species to identify price trends on the species level. Firstly, we start by employing the method used by Ashenfelter (1989) in his influential paper on the declining price anomaly, where he calculated price ratios to identify price patterns in sequential auctions of wine.

Secondly, we run regressions to establish how the auction round affects absolute and relative price changes — both with respect to the first price of the day and to the price in the previous round. We also explore the impacts of the number of bids, quantity sold and the price of the first round of each species on price trends.

Finally, we employ the method used by Gallegati et al. (2011) by looking at average prices of a particular fish species in the whole period and measure the strength of auction round as a predictor for average prices.

4.3.1 Ratios of Prices

We start by applying the method of Ashenfelter (1989) where we calculate price ratios to establish whether prices increase, decrease or remain constant most frequently. First we look at the ratio of the prices in round 2 or higher to the price in the previous round. We calculate the ratio $\frac{(k+1)^{th}\ price}{k^{th}\ price}$ for all $k \geq 1$ where k represents the round in the sequence of auctions of a particular species and size in a day.

By calculating these ratios we get a price comparison of all consecutive auctions of the same type (species and size) sold sequentially each day. These ratios are then set equal to unity to establish whether prices are increasing, decreasing or constant. Once these ratios have been calculated the proportion of decreasing (increasing and constant) prices can be found to establish whether there is a tendency of prices decreasing more often and hence possible evidence of the declining price anomaly.

Since comparisons are only made of the same type of fish in a day, there are some days where only one auction is of a specific fish type, because the first winner purchases all the fish on supply in the first round. When these auctions have been subtracted, there are 2243 auction rounds left to analyse. Table 4.2 shows the number of auction rounds where prices are increasing, decreasing and constant. The table also contains the mean, standard error and standard deviation of the price ratios. The table shows very strong evidence of declining prices as decreases in prices occur in approximately 81 percent of the cases. In 12 percent of the cases, prices increase and in 7 percent of the cases prices remain unchanged. The mean of ratios is 0.94 (standard error = 0.0022), meaning that prices decrease by an average of 6 percent per round. By looking at the price ratios on the aggregate level, we have found a strong tendency for prices to decrease. However, it is worthwhile examining whether this tendency also applies on the species level. When restricting attention to the species most frequently sold on auction (and with the highest quantities overall), the results are similar. Cod is the most common species sold on auction followed by haddock, and a total of 90 percent of comparisons are from auctions of cod or haddock. The percentages of transactions with decreasing prices are 78 and 87 percent, respectively. The mean of ratios are 0.95 and 0.94 for the two species and the standard errors remain low (0.0025 and 0.0033). The results found here are significantly stronger than those of Ashenfelter (1989) who showed that prices were more than twice as likely to decrease as to increase, and in approximately half of the cases prices remained constant.

Table 4.2: Distribution of price patterns on the aggregate and species level

	All auctions		Cod		Haddock	
Later price higher	267	11.9%	205	14.0%	40	7.7%
Later price lower	1813	80.8%	1148	78.1%	452	87.3%
Later price identical	163	7.3%	116	7.9%	26	5.0%
Mean of ratios	0.9422		0.9534		0.9396	
Standard deviation	0.1034		0.0963		0.0743	
Standard error of mean	0.0022		0.0025		0.0033	
Number of comparisons	2243		1469		518	

Note: Prices compared to the price in the previous round.

The number of cases where the ratios are less than or greater than unity does not necessarily constitute evidence of the declining price anomaly as prices may increase by more than they decrease even though decreases occur more often. Therefore it is worth examining the magnitude of the price changes in more detail. Table 4.3 shows the average ratios of increasing and decreasing prices seperately. The table shows that price decreases are more substantial than price increases, with an average decrease of 8 percent and an average increase of 4 percent. For cod and haddock the respective figures are nearly 5 and 2 percent for increases and around 7 percent for decreases for both species.[10]

Table 4.3: Average increases and decreases in price

		All auctions	Cod	Haddock
Increasing	Average ratio	1.0437	1.0467	1.0202
	Standard deviation	0.1090	0.1144	0.0225
	Standard error of mean	0.0067	0.0080	0.0036
Decreasing	Average ratio	0.9221	0.9321	0.9290
	Standard deviation	0.0962	0.0854	0.0733
	Standard error of mean	0.0023	0.0025	0.0034

Note: Prices compared to the price in the previous round.

4.3.2 Regression Analysis

Thus far, we have established that prices from one round to another are substantially more likely to decrease than increase, and that the prices decrease by a greater magnitude than they increase. To explore price trends in the data and their significance, we do some regression analysis on the data. First we regress prices on the rounds in each day and take account for the different species by absorbing the fish type variable. To ensure that species and prices do not confuse the overall picture, we also control for the starting price of each fish type everyday. It is important to control for the first price of the day due to two reasons: Firstly,

[10] We have conducted the same analysis comparing prices to the price in the first round. For an overview, see Appendix B.

the prices can vary depending on the day of the week,[11] and secondly, buyers' valuations are affiliated meaning that the signals sent through the price have an impact on later prices. In the first regression, we have price as the dependent variable, round as the independent variable, whilst our control variable is the first price. Figure 4.1 below shows a binned scatterplot nicely demonstrating a clear decreasing price trend throughout the day with a fairly precise and highly significant coeffient of -0.5529 (standard error $= 0.0173$, $p = 0$). For the plot, the data has been grouped into 20 equally sized bins, where each dot represents the mean of the bin. As can be seen from the figure below, the data points are a tight fit to the regression line and this indicates a significant downwards trend. Table 4.4 shows the estimation coefficients and their significance from running this regression.

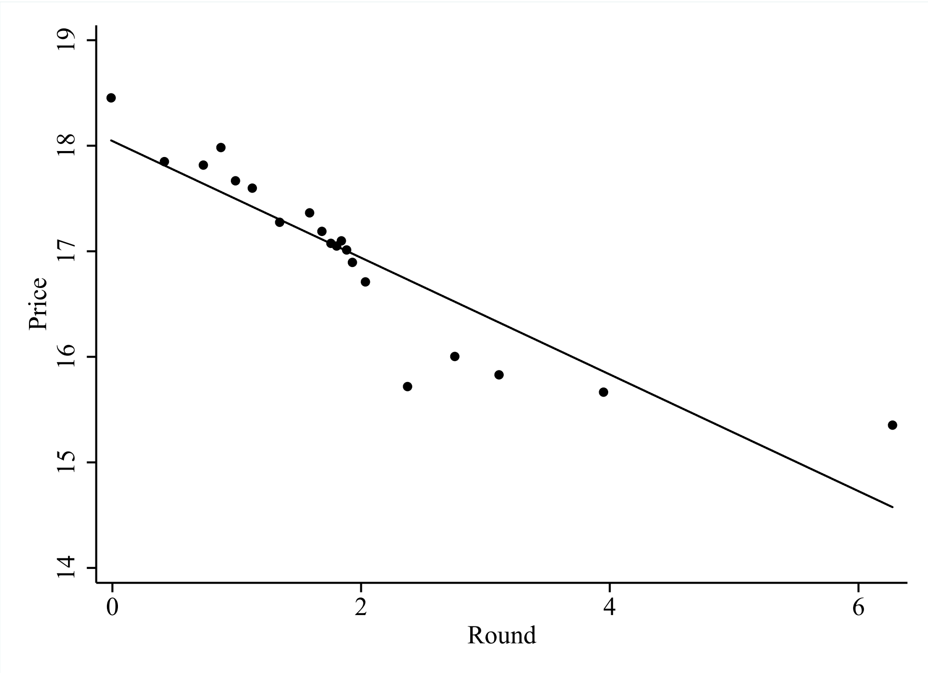


Figure 4.1: Binned scatterplot of price on round

[11]We do not analyse prices on different weekdays, but Salladarré et al. (2017) demonstrated that weekday is a predictor of price in a French fish market.

Table 4.4: Estimation result of price on round

Round	-0.5529	***	(0.0173)
First price	0.8906	***	(0.0052)
Constant	2.2351	***	(0.0976)
Number of observations: 5398			
$F(2, 5333) = 14521.73$ ***, $R^2 = 0.9765$			
Note: Standard errors in brackets. ***$p < 0.001$.			

Given that our data consists of many different fish types it is worthwhile checking that the negative trend observed in the data is not specific to some species only and that the trend holds across species. To model the variability of the trend across clusters, we run a linear mixed regression model with a random intercept and slope. By doing so we are able to test whether the effect of round on prices varies depending on the type of fish. In our regression the data is grouped into 63 fish types; we use price as the dependent variable and round as our independent variable. We measure both random and fixed effects with respect to our round variable. We also control for the first price of the day and the number of bids, since this usually has an impact on prices in auctions. The results clearly demonstrate a negative relationship between round and price, as the round coefficient of -1.3209 is highly significant. As expected, the first price and the number of bids both have positive and highly significant coefficients. The random effect estimates show that there is considerable variation across fish types, but the overall effect of round on prices is negative. Checking the individual fixed effect coefficients for the round, we find that 62 out of 63 have a negative coefficient, the minor exception being halibut of size 1 with only six transactions.

Table 4.5: Mixed regression model for estimating prices

Fixed effects			
Round	-1.3209	(0.2299)	***
First price	0.8825	(0.0049)	***
Bids	0.4231	(0.0203)	***
Constant	2.6610	(0.4105)	***
Random effects			
SD(Round)	1.3186	(0.1870)	
SD(Constant)	2.8754	(0.3106)	
SD(Residual)	1.6296	(0.0159)	
Number of observations: 5398			
Number of groups: 63			
Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.			

The previous regressions assume that the relationship between the price and the round is linear. However, this may not be the case and therefore it is worthwhile running similar regressions where each round is treated as a separate variable. Our dependent and control variables remain the same whilst the independent variables now become the individual rounds. Due the small samples in later rounds, we have merged the data from the last eight rounds (see Table 4.1 for the number of transations in each round). Figure 4.2 shows the coefficients of running a regression of prices on the rounds and the first price of each type every day (the constant and first price coefficients have been dropped from this plot). Table 4.6 shows the estimated coefficients and significance levels and the plot shows the estimated absolute decrease in prices at each round relative to the price in the first round of the day. It also shows the 95% confidence interval of all the round coefficients. Although all the coefficients are significant, the standard errors are lowest for round 2, and then gradually increasing for each round — this is to be expected with the number of observations in each round. As well as showing that prices decrease throughout the day, the coefficients also demonstrate that the price decreases are largest early in the day.[12]

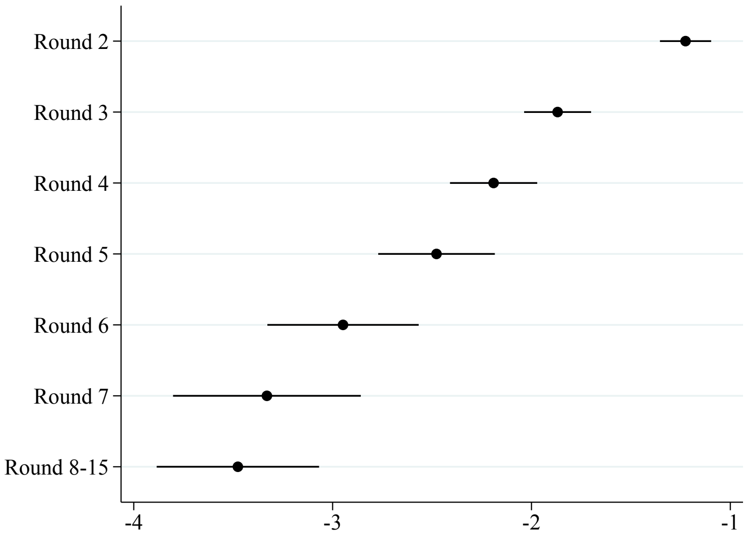


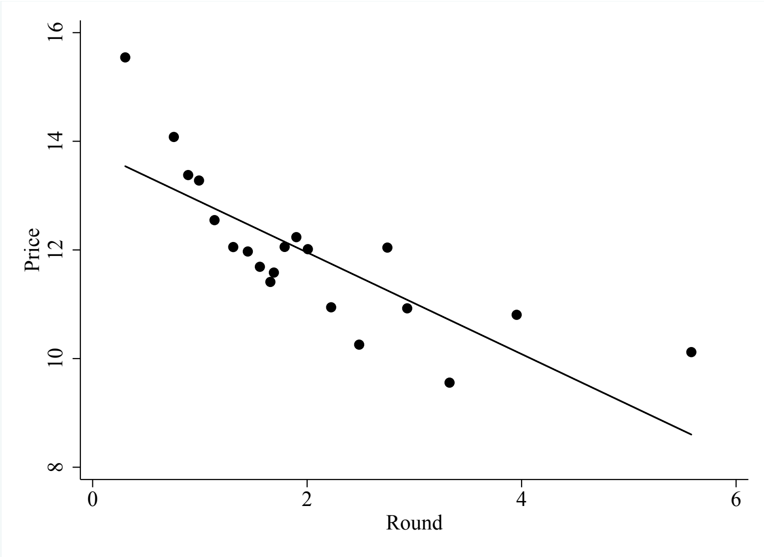
Figure 4.2: Regression coefficients for estimating price

Table 4.6: Estimation results of prices on rounds and the first price

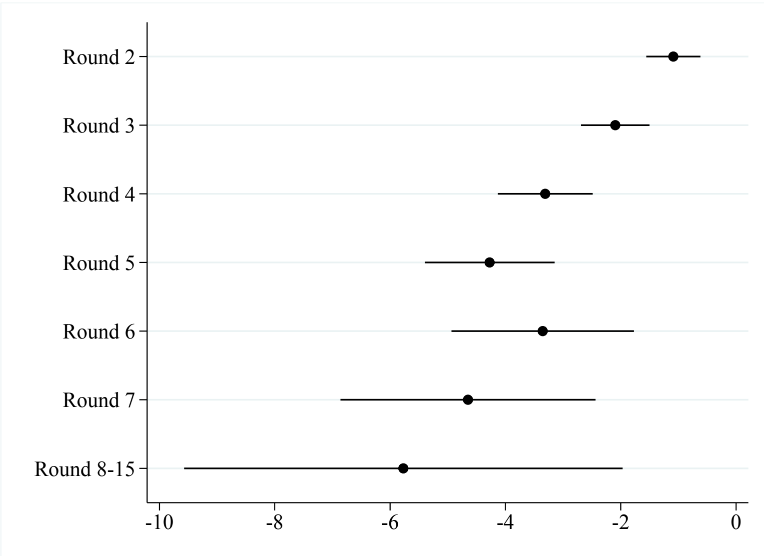
Round 2	-1.2252	***	(0.0657)
Round 3	-1.8686	***	(0.0859)
Round 4	-2.1905	***	(0.1119)
Round 5	-2.4775	***	(0.1496)
Round 6	-2.9477	***	(0.1941)
Round 7	-3.3304	***	(0.2408)
Round 8-15	-3.4765	***	(0.2083)
First price	0.8979	***	(0.0052)
Constant	1.7822	***	(0.0941)
Number of observations: 5398			
$F(8, 5327) = 3789.71^{***}$, $R^2 = 0.9774$			
Note: Standard errors in brackets. $^{***}p < 0.001$.			

By running the same two regressions as above, on separate fish types, we observe a similar trend. In Figure 4.3a we see a binned scatterplot showing the relationship between the price of cod 5 on the round whilst controlling for

[12]For a similar regression, where all individual rounds are included separately in the regression model (instead of merging the last eight rounds) see Table 4.15 in Appendix C.



(a) Binned scatterplot



(b) Coefficient plot

Figure 4.3: Binned scatterplot and coefficient plot for price of cod 5

the first price of the day. The figure nicely demonstrates that the previously observed negative relationship between price and round is even greater for cod 5 than overall. The round coeffient is -0.9457 and significant at the 1 percent level ($p < 0.001$). Figure 4.3b shows a plot of the round coefficients and their confidence intervals, from running the regression of prices on indivual rounds whilst controlling for the first price. The plot demonstrates a strong statistical significance at each round. See Table 4.17 in Appendix D, for the coefficients and their individual significance. We find similar results for all sizes of cod and haddock, see Appendix D, for binned scatterplots and estimation results.

To explore further when the greatest price changes occur, we look at relative price changes from round to round. To do this, we calculate the difference between current and previous price and divide it by the previous price. Here we regress the price change on the rounds without controlling for the first price but still absorbing the fish types. As demonstrated in Figure 4.4, prices decrease at every round, but they decrease by the greatest magnitude in the earlier rounds, with the largest absolute change of 7.25 percentage points occuring between the first and the second round. All of the price changes are significantly different from zero (see Table 4.7). For a similar regression comparing prices to the price in the first round, see Table 4.20 and Figure 4.8 in Appendix E.[13]

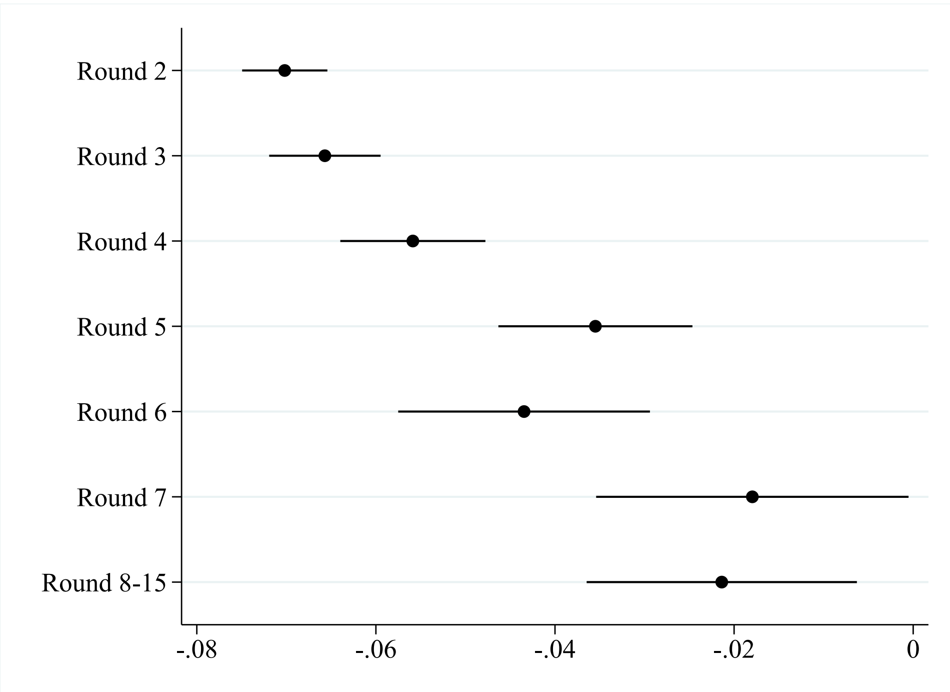


Figure 4.4: Regression coefficients for estimating price changes relative to the previous price

[13]For estimation results of relative price change compared to the first and previous round using all fifteen rounds separately, see Table 4.16 in Appendix C.

Table 4.7: Estimation results of price change relative to previous price

Round 2	-0.0725	***	(0.0024)
Round 3	-0.0687	***	(0.0032)
Round 4	-0.0591	***	(0.0042)
Round 5	-0.0397	***	(0.0056)
Round 6	-0.0482	***	(0.0072)
Round 7	-0.0226	*	(0.0090)
Round 8-15	-0.0248	**	(0.0077)
Number of observations: 5398			
$\mathrm{F}(7, 5328) = 162.19$***, $R^2 = 0.2086$			
Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.			

4.3.3 Average Price Trends Between Rounds

To see how our results compare to that of other fish markets, we apply the method used by Gallegati et al. (2011) in their study of price trends in the Ancona fish market. This method calculates the average price of each round of all the auctions in the sample, comparing the average prices between rounds. This is an alternative method to see if there is a tendency for prices to decrease throughout the day.

Here, we have applied the method to the auctions of cod (size 4), which is the most frequently sold fish. Like Gallegati et al. (2011), we find that the prices are decreasing throughout the day, although they found that prices increased for the last transactions of the day due to limited supply. As can be seen in Figure 4.5, the average price is higher for the first round of the day and lowest for the last auction round of the day. The price varies from 18.1 kr./kg to 16 kr./kg, meaning there is a reduction of nearly 12 percent in price from the first to the last round. When regressing prices on the rounds, it is clear that there is a negative

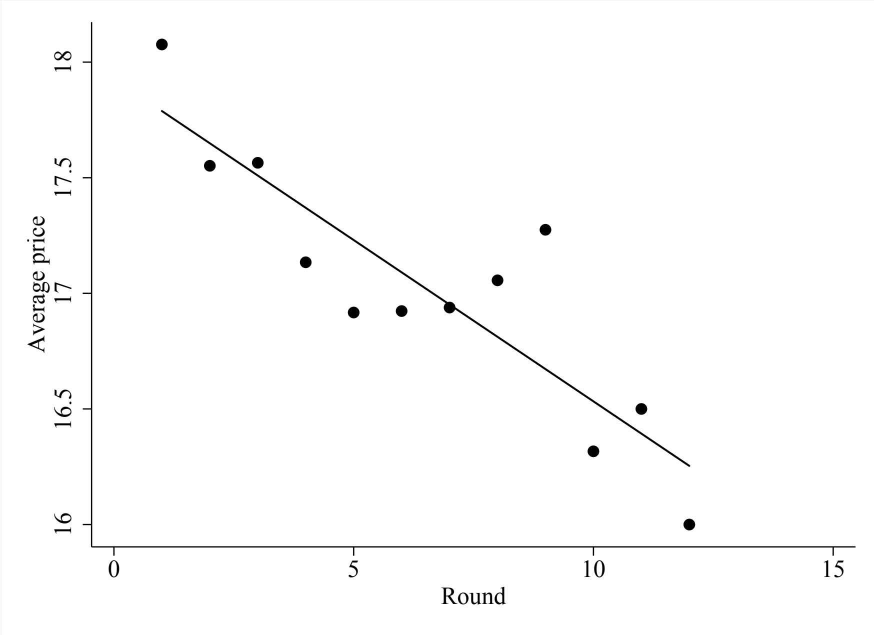


Figure 4.5: Average price of cod 4 in each round and regression line

relationship between the two variables meaning that later in the sequence a lot is sold, the lower the price.

To see the correlation between average prices and the rounds of the auctions, a linear regression was run to predict the price from the auction round. The results show that the negative relationship (coefficient is -0.1395) between round and average prices of cod is statistically significant at the 1 percent level ($p < 0.001$), implying that our independent variable, round, is a significant predictor of average prices for cod 4. See Table 4.8 for estimation results.

Table 4.8: Estimation results for average price of cod 4

Round	-0.1395	***	(0.0240)
Constant	17.928	***	(0.1765)
Number of observations: 12			
$F(1, 10) = 33.83$***, $R^2 = 0.7719$			
Note: Standard errors in brackets. ***$p < 0.001$.			

4.4 Discussion

After employing several different methods to test for the declining price anomaly in the Faroe Fish Market, we can conclude that declining prices are a strong feature of this particular market. We have established that the round is a significant predictor of price in later auctions, even when controlling for the first price and number of bids. Using Ashenfelter's (1989) measure of the declining price anomaly, we can conclude that the declining price anomaly is also present at the fish market studied here. However, there are two important differences between Ashenfelter's paper and the present study. Firstly, the goods considered here are not as homogeneous as the bottles of wine studied by Ashenfelter (1989), and secondly, the auctions at the Faroe Fish Market are right-to-choose auctions, allowing bidders to pick their preferred goods. Decreasing prices could be expected in right-to-choose auctions as this would imply that the 'best' goods would be picked first. This has also been highlighted by Gale and Hausch (1994) who argued that the format of right-to-choose auctions itself guarantees declining prices and efficiency. This could be one potential explanation for the decreasing prices.

In some cases, a potential explanation for declining prices could be reducing quality throughout the day, i.e. fish sold in the afternoon is not as fresh as the first lot sold in the morning, provided it was caught at the same time. However, in this particular market setting, auctions start at 10 in the morning, and all the fish is typically sold within 30 minutes. Since all lots of one fish type are normally sold in the space of 1 to 3 minutes, we can rule out decreasing quality due to time as a potential explanation for the declining price trends in our context.

Several papers have suggested that uncertainty in supply leads to decreasing prices (Jeitschko, 1999; Neugebauer and Pezanis-Christou, 2007; Pezanis-Christou, 2000). The context in which the Faroese fish auctions take place does involve uncertain supply, especially due to the buyer's option, where winning bidders can choose to purchase the whole lot immediately. This means that potential buyers are not guaranteed the chance to purchase the relevant items in later rounds. This is also likely to be a contributing factor to the declining prices observed here. There are several reasons why it is not possible to analyse the exact impact of uncertain supply within this context. Firstly, the auction format means that the overall demand is unknown. Secondly, there is no available data on buyer expectations, which would need to be collected in a qualitative manner. Thirdly, all the data in our current dataset is from the same auction format. Data from another auction format in a similar setting would allow for a deeper exploration of the exact contribution of uncertain supply to declining price trends.

Our findings support those of Salladarré et al. (2017) who found that prices decrease more dramatically in periods with low supply, suggesting that the supply at the auction is a significant predictor of price trends. We briefly examined whether the supply of cod (which is the most frequently sold species) was correlated with the changes in price. We found that low supply is negatively correlated with relative price change to previous round, at a significant level. Therefore, in line with Salladarré et al. (2017), we can conclude that if supply of cod is low, the decreases in price from round to round are greater than under other supply conditions. When using the round and low supply variables to predict price change, low supply has a much greater negative impact on prices than the round, both negative and significant at the 1 percent level (see Model 1 in Table 4.10 for this estimation result). We also estimated relative price changes using quantity purchased in each round as one of the independent variables. The results showed that the effect of quantity purchased in each round is negligible. Furthermore, we found that including quantity as a control variable in the regression model has a minute effect on the round coefficient (see Table 4.9 and Table 4.21 in Appendix E for the coefficients and their significance).

Table 4.9: Predicting relative price changes (first round)

	Model 1		Model 2		Model 3	
Round	-0.027802	***	-0.026483	***	-0.028356	***
	(0.0008)		(0.0007)		(0.0008)	
Bids			0.015089	***	0.013852	***
			(0.0009)		(0.0009)	
Quantity					0.000006	***
					(0.0000)	
Constant	0.015702	***	-0.021694	***	-0.020047	***
	(0.0018)		(0.0030)		(0.0029)	
Observations	5398		5398		5398	
F-test	1283.03	***	799.25	***	615.56	***
R^2	0.2497		0.2839		0.3087	

Note: Standard errors in brackets. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$. Here quantity refers to the amount sold in each round.

As discussed in Section 4.1.1 many economists attribute the declining price anomaly to risk aversion in buyers. Risk aversion also seems to be one plausible explanation for the price dynamics at the fish market studied here. Most buyers are fish factories located in different parts of the islands. These factories only have one means of getting fresh fish for their production, and therefore it is crucial that they manage to secure their resources at the fish market for their production. The combination of uncertain supply — exacerbated by the right-to-choose and buyer's option format of the auctions — and risk averse buyers would likely be a contributor to decreasing prices. Testing such an hypothesis is out of the scope of this paper, as this would require collection of qualitative data in relation to risk aversion, and additional data from several auction formats to allow for comparisons.

In this context, it is worth considering another relevant explanation for the decreasing price anomaly. Engelbrecht-Wiggans and Kahn (1999) attribute declining prices to decreased competition in later rounds because buyers have limited capacity for purchases and therefore may purchase the necessary quantities in earlier rounds. This leads to an overall decrease in buyers and hence decreased competition in later rounds.

We tested this by running a regression of the number of bids on the round. The results show that the number of bidders is decreasing by almost 10 percent per round. The round coefficient of -0.0874 was highly significant ($p < 0.001$) with a standard error of 0.01. Since this is the case, it is interesting to further explore whether the number of bids can predict price patterns. We find that

Table 4.10: Estimating price change relative to previous price of cod

	Model 1		Model 2		Model 3	
Round	-0.00322	***	-0.00281	**	0.00004	
	(0.00087)		(0.00088)		(0.00086)	
Low supply	-0.01365	**	-0.01124	**	-0.00501	
	(0.00403)		(0.00421)		(0.00404)	
Bids			0.01015	***	0.01431	***
			(0.00108)		(0.00106)	
First price					-0.00645	***
					(0.00041)	
Constant	-0.00936	*	-0.04164	***	0.05959	***
	(0.00465)		(0.00644)		(0.00894)	
Observations	2439		2439		2439	
F-test	9.45	**	42.3	***	95.73	***
R^2	0.0077		0.0794		0.1632	

Note: Standard errors in brackets. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

the number of bids is positively correlated with price. When trying to estimate relative price change to the first round, and using the number of bids and round as our independent variables, the bid coefficient of 0.0151 is significant at the 1 percent level. The round coefficient is still negative, -0.0265 ($p < 0.001$), implying that the number of bids alone, is not sufficient to explain the declining prices. See Table 4.9 for these results. We obtain similar results by regressing the relative price change to the previous round on the same two variables.[14]

Therefore we can support Engelbrecht-Wiggans and Kahn (1999) by saying decreasing competition seems like a plausible contributing factor to the declining prices. In this context, it is also relevant to highlight that the ratio between buyers and sellers at the Faroe Fish Market differs significantly from that of other fish markets studied thus far (Gallegati et al., 2011; Fluvià et al., 2012; Salladarré et al., 2017; Pezanis-Christou, 2000), with very few buyers compared to sellers. It is likely that this feature, with few buyers, will mean that they get what they need early on, causing reduced competition later on, resulting in a price decrease, an observation already made by Engelbrecht-Wiggans and Kahn (1999).

To get an indication of whether the above reasons can adequately explain the anomaly, we can run a regression for cod controlling for all the measureable contributors (first price, bids, low supply). Here we find that the effect of round on price change compared to the previous round is minute (0.00004) and insignificant. Table 4.10 demonstrates these results and also shows that including more of these control variables makes the round coefficient less negative (and finally positive and insignificant). This suggests that the reasons above are sufficient in

[14]See Table 4.21 in Appendix E for these estimation results. The round coefficient is -0.0071 and the bid coefficient 0.0120, both significant at the 1 percent level.

explaining the anomaly for cod. Future work should test whether the contributors also can account for the declining prices for the entire dataset.

4.5 Concluding Remarks

This paper has tested for the declining price anomaly, which was first identified by Ashenfelter (1989). It has since been the subject of many studies. We have employed several different methods to test for the declining price anomaly in the Faroe Fish Market. First, the original ratio method conducted by Ashenfelter (1989) was employed. To add to the analysis, various regressions on the whole dataset and on subsets corresponding to major species were run in order to determine whether the findings also were relevant on the species level where goods are more homogenous. Our conclusions from employing all of the methods demonstrate that declining prices are a strong feature of the Faroese fish market both overall and for major species. We found that the round was a significant predictor of price trends.

From considering the context studied here and comparing to conclusions in the literature, declining prices should perhaps not come as a surprise as indicated in the discussion above. One could expect declining prices in connection with the right-to-choose (Gale and Hausch, 1994) and buyers option (Black and de Meza, 1992) format of this market.

In line with (Salladarré et al., 2017; Engelbrecht-Wiggans and Kahn, 1999; Jeitschko, 1999), our analysis also found that both competition and supply conditions had an impact on price patterns. Like Salladarré et al. (2017), we found that low supply exacerbated the declining prices with steeper price drops in such contexts.

We also found that the number of bidders was positively correlated with price, and as the number of bidders reduced throughout the day, this likely contributed to the declining prices. As the number of bidders tends to be quite low at the Faroe Fish Market, moments with low competition can happen quite frequently.

However, there are many avenues that should be explored further. We analysed data from one year. As the quantities sold through the auction vary significantly from year to year, it would be interesting to extend the analysis to the whole dataset to explore whether declining prices are a feature throughout the period, and examine how different supply conditions and seasonal variation affect price trends.

Faroe Fish Market is a small market, where sellers and bidders are quite well informed about each other. It would be worthwhile to explore the behavioural patterns of bidders, such as which buyers purchase from which sellers, when particular buyers tend to enter the auctions, and the prices they pay relative to others. This would allow us to test whether the declining prices can be attributed to this particular pattern. As the dataset already contains this information, this should be relatively easily achieved. Finally, efforts should focus on extending the analysis to other fish markets to gauge which of the findings are specific to the Faroese context, and which can be generalised to a wider setting. This would enable us to make more general conclusions with regards to which particular factors are most influential in determining prices.

4.6 Appendix A: Summary Statistics

Auctions Analysed in the Paper

- 5398 auctions
- 63 different fish types (grouped in species and sizes)
- Up to 15 lines for every fish type
- 2243 auctions in round 2 or later
- 226 days from January to December 2017
- Fish landed by 314 vessels at 19 different ports
- 25 bidders in total (all winning at least 1 auction)

Table 4.11: Fish types, number of transactions, quantities and average prices in 2017

ID	Fish	Trans.	Quantity (kg)	Average price (kr./kg)
1	Cod 1	369	258,254	22.29
2	Cod 2	457	435,105	20.87
3	Cod 3	565	1,044,138	18.93
4	Cod 4	649	802,592	17.54
5	Cod 5	399	57,550	11.93
6	Cod	29	7,702	13.48
7	Cod 1-2	2	75	19.55
8	Haddock 1	299	52,781	18.00
9	Haddock 2	366	325,695	17.47
10	Haddock 3	337	237,454	14.89
11	Haddock 4	82	8,960	2.87
12	Haddock	79	10,578	11.19
13	Haddock 1-2	18	957	15.91
14	Whiting	105	20,204	7.55
15	Whiting 1	2	239	9.25
16	Coalfish 1	39	6,469	7.78
17	Coalfish 2	10	1,947	8.41
18	Coalfish 3	12	741	7.53

ID	Fish	Trans.	Quantity (kg)	Average price (kr./kg)
19	Coalfish 4	6	213	5.52
20	Coalfish 5	4	791	4.18
21	Coalfish	33	7,189	7.16
22	Coalfish 1-2	8	968	7.73
23	Cusk 1	49	17,303	7.36
24	Cusk 2	37	32,139	7.31
25	Cusk 3	25	25,604	7.18
26	Cusk 4	16	6,330	5.68
27	Cusk	43	9,168	4.32
28	Cusk 1-2	5	3,295	7.80
29	Cusk 2-3	3	229	6.13
30	Ling 1	94	75,012	14.95
31	Ling 2	62	156,005	15.60
32	Ling 3	49	51,508	14.71
33	Ling 4	36	13,512	11.75
34	Ling 5	13	417	3.92
35	Ling	35	10,768	10.50
36	Ling 1-2	3	408	14.30
37	Ling roe	1	120	12.00
38	Greenland halibut	9	24,770	30.58
39	Norwegian haddock 1	1	128	8.00
40	Norwegian haddock 2	2	44	7.10
41	Norwegian haddock 3	-	-	-
42	Norwegian haddock	94	14,833	8.00
43	Blue ling	85	78,557	12.78
44	Wolf fish	129	14,993	6.77
45	Spotted wolf fish	64	29,029	10.88
46	Monkfish 1	1	18	24.00
47	Monkfish	194	36,683	23.20
48	Halibut 1	36	5,251	54.49
49	Halibut 2	42	2,358	81.80
50	Halibut 3	29	685	94.72
51	Halibut 4	6	28	104.00
52	Halibut	1	391	55.00
53	Plaice	46	3,630	5.99
54	Pollack	95	5,377	7.45
55	Smear dab	29	4,438	24.21
56	Ray (Skate)	53	17,569	7.67
57	Ray sides	42	9,681	20.44
58	Dab	14	428	3.99
59	Cod roe	4	67	9.58
60	Turbot	3	10	41.00
61	Porbeagle	1	25	7.80
62	Opak	1	40	11.50
63	Mackerel	1	7	4.80
64	Greenland halibut (sek)	75	131,401	19.05

Table 4.12: Summary statistics of price and quantity per fish type

ID	Fish	Price		Quantity	
		Mean	S.D	Mean	S.D
1	Cod 1	22.29	3.69	699.88	2,397.34
2	Cod 2	20.87	3.18	952.09	2,137.45
3	Cod 3	18.93	2.60	1,848.03	3,354.01
4	Cod 4	17.54	3.62	1,236.66	5,262.24
5	Cod 5	11.93	3.17	144.24	228.18
6	Cod	13.48	3.98	265.59	482.36
7	Cod 1-2	19.55	0.78	37.50	17.68
8	Haddock 1	18.00	3.55	176.53	252.99
9	Haddock 2	17.47	3.50	889.88	1,568.21
10	Haddock 3	14.89	2.97	704.61	1,227.53
11	Haddock 4	2.87	0.75	109.27	118.35
12	Haddock	11.19	3.56	133.90	183.44
13	Haddock 1-2	15.91	3.87	53.17	47.29
14	Whiting	7.55	2.98	192.42	200.14
15	Whiting 1	9.25	1.63	119.50	92.63
16	Coalfish 1	7.78	0.98	165.87	284.33
17	Coalfish 2	8.41	0.54	194.70	271.90
18	Coalfish 3	7.53	0.62	61.75	96.38
19	Coalfish 4	5.52	1.36	35.50	23.05
20	Coalfish 5	4.18	0.62	197.75	118.53
21	Coalfish	7.16	1.32	217.85	155.31
22	Coalfish 1-2	7.73	0.78	121.00	189.94
23	Cusk 1	7.36	0.82	353.12	489.31
24	Cusk 2	7.31	0.94	868.62	1,015.85
25	Cusk 3	7.18	0.82	1,024.16	901.12
26	Cusk 4	5.68	0.83	395.63	226.92
27	Cusk	4.32	0.98	213.21	353.25
28	Cusk 1-2	7.80	0.45	659.00	598.58
29	Cusk 2-3	6.13	0.23	76.33	96.10
30	Ling 1	14.95	1.92	798.00	1,084.12
31	Ling 2	15.60	1.63	2,516.21	3,465.52
32	Ling 3	14.71	2.25	1,051.18	1,278.77
33	Ling 4	11.75	2.41	375.33	637.14
34	Ling 5	3.92	0.29	32.08	29.03
35	Ling	10.50	2.62	307.66	317.89
36	Ling 1-2	14.30	1.95	136.00	140.39

ID	Fish	Price		Quantity	
		Mean	S.D	Mean	S.D
37	Ling roe	12.00	.	120.00	.
38	Greenland halibut	30.58	0.71	2,752.22	4,056.12
39	Norwegian haddock 1	8.00	.	128.00	.
40	Norwegian haddock 2	7.10	0.14	22.00	21.21
41	Norwegian haddock 3	-	-	-	-
42	Norwegian haddock	8.00	3.02	157.80	150.58
43	Blue ling	12.78	4.80	924.20	646.72
44	Wolf fish	6.77	1.87	116.22	200.96
45	Spotted wolf fish	10.88	5.98	453.58	458.45
46	Monkfish 1	24.00	.	18.00	.
47	Monkfish	23.20	3.20	189.09	220.54
48	Halibut 1	54.49	8.54	145.86	134.19
49	Halibut 2	81.80	24.99	56.14	69.03
50	Halibut 3	94.72	25.10	23.62	49.92
51	Halibut 4	104.00	36.44	4.67	2.58
52	Halibut	55.00	.	391.00	.
53	Plaice	5.99	2.89	78.91	105.58
54	Pollack	7.45	1.74	56.60	82.80
55	Smear dab	24.21	12.65	153.03	125.30
56	Ray (Skate)	7.67	2.00	331.49	519.12
57	Ray sides	20.44	3.67	230.50	277.68
58	Dab	3.99	1.73	30.57	24.86
59	Cod roe	9.58	2.19	16.75	6.18
60	Turbot	41.00	18.62	3.33	2.31
61	Porbeagle	7.80	.	25.00	.
62	Opak	11.50	.	40.00	.
63	Mackerel	4.80	.	7.00	.
64	Greenland halibut (sek)	19.05	2.22	1,752.01	1,058.78

4.7 Appendix B: Price Ratios

Here we analyse price changes in comparison to the first round of each species. The price ratio is now given by $\frac{k^{th}\ price}{1^{st}\ price}$ for all $k \geq 2$. As can be seen from Table 4.13 below, over 84 percent of all auctions in round 2 or later have a lower price than that of the first round. These figures are 83 percent for cod and 89 percent for haddock. When looking at the average magnitude of the increases and decreases in price (see Table 4.14) we can conclude that when prices are higher than the first price, they are an average of 5 percent above, whilst lower prices are an average of nearly 12 percent below the first price.

Table 4.13: Distribution of price patterns (first price)

	All auctions		Cod		Haddock	
Later price higher	242	10.8%	184	12.5%	37	7.1%
Later price lower	1897	84.6%	1219	83.0%	460	88.8%
Later price identical	104	4.6%	66	4.5%	21	4.1%
Mean of ratios	0.9093		0.9108		0.9244	
Standard deviation	0.1251		0.1271		0.0837	
Standard error of mean	0.0026		0.0033		0.0037	
Number of comparisons	2243		1469		518	

Note: Prices compared to the price in the first round each day.

Table 4.14: Average increases and decreases in price (first price)

		All auctions	Cod	Haddock
Increasing	Average ratio	1.0484	1.0517	1.0247
	Standard deviation	0.0766	0.0711	0.0247
	Standard error	0.0049	0.0052	0.0041
Decreasing	Average ratio	0.8866	0.8847	0.9128
	Standard deviation	0.1196	0.1209	0.0814
	Standard error	0.0027	0.0035	0.0038

Note: Prices compared to the price in the first round each day.

4.8 Appendix C: All Rounds

Table 4.15: Estimation results of price on all rounds

Round 2	-1.2247	***	(0.0656)
Round 3	-1.8680	***	(0.0857)
Round 4	-2.1891	***	(0.1116)
Round 5	-2.4757	***	(0.1493)
Round 6	-2.9454	***	(0.1936)
Round 7	-3.3267	***	(0.2403)
Round 8	-3.4757	***	(0.3031)
Round 9	-2.7102	***	(0.4251)
Round 10	-3.4791	***	(0.5526)
Round 11	-3.3270	***	(0.6594)
Round 12	-5.2606	***	(1.0048)
Round 13	-7.3004	***	(1.2305)
Round 14	-8.5211	***	(1.7403)
Round 15	0.6789		(1.7403)
First price	0.8978	***	(0.0052)
Constant	1.7848	***	(0.0939)
Number of observations: 5398			
$F(15, 5320) = 2032.04$***, $R^2 = 0.9775$			
Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.			

Table 4.16: Estimation results of price changes on all rounds

	Relative to first price			Relative to previous price		
Round 2	-0.0725	***	(0.0029)	-0.0725	***	(0.0024)
Round 3	-0.1032	***	(0.0038)	-0.0688	***	(0.0031)
Round 4	-0.1162	***	(0.0049)	-0.0592	***	(0.0041)
Round 5	-0.1259	***	(0.0066)	-0.0398	***	(0.0055)
Round 6	-0.1473	***	(0.0086)	-0.0484	***	(0.0071)
Round 7	-0.1613	***	(0.0106)	-0.0228	*	(0.0089)
Round 8	-0.1583	***	(0.0134)	-0.0478	***	(0.0112)
Round 9	-0.1195	***	(0.0189)	-0.0419	**	(0.0157)
Round 10	-0.1719	***	(0.0245)	-0.0290		(0.0204)
Round 11	-0.1679	***	(0.0293)	0.0471		(0.0243)
Round 12	-0.2596	***	(0.0446)	-0.0515		(0.0371)
Round 13	-0.3517	***	(0.0546)	-0.0214		(0.0454)
Round 14	-0.3858	***	(0.0772)	-0.0187		(0.0643)
Round 15	0.0195		(0.0772)	0.6408	***	(0.0643)
Observations	5398			5398		
F-test	132.53	***		91.58	***	
R^2	0.3098			0.2264		

Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.

4.9 Appendix D: Cod and Haddock

Table 4.17: Estimation results for price of cod 5

Round 2	-1.0882	***	(0.2387)
Round 3	-2.0947	***	(0.3015)
Round 4	-3.3104	***	(0.4182)
Round 5	-4.2737	***	(0.5726)
Round 6	-3.3533	***	(0.8050)
Round 7	-4.6492	***	(1.1246)
Round 8-15	-5.7705	**	(1.9335)
First price	0.7402	***	(0.0285)
Constant	3.1123	***	(0.3697)
Number of observations: 399			
$F(8, 390) = 85.88$***, R^2= 0.6379			
Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.			

Table 4.18: Estimation results for price of cod

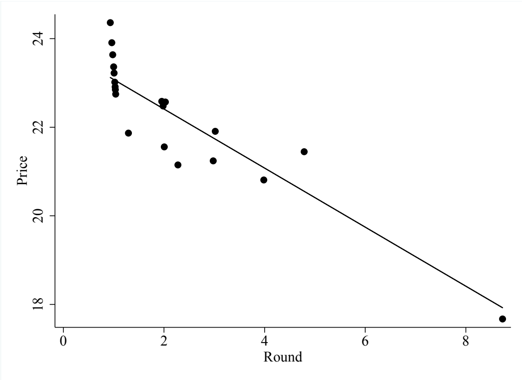
	Number of observations: 369, $R^2 = 0.7614$			
Cod 1	Round	-0.6665	***	(0.0484)
	First price	0.8556	***	(0.0271)
	Constant	3.7634	***	(0.6477)
	Number of observations: 457, $R^2 = 0.6616$			
Cod 2	Round	-0.4247	***	(0.0519)
	First price	0.7434	***	(0.0252)
	Constant	5.6200	***	(0.5535)
	Number of observations: 565, $R^2 = 0.6966$			
Cod 3	Round	-0.2846	***	(0.0312)
	First price	0.7579	***	(0.0211)
	Constant	4.8422	***	(0.4000)
	Number of observations: 649, $R^2 = 0.5151$			
Cod 4	Round	-0.3607	***	(0.0450)
	First price	0.5866	***	(0.0227)
	Constant	7.3959	***	(0.4453)
	Number of observations: 399, $R^2 = 0.6316$			
Cod 5	Round	-0.9357	***	(0.0700)
	First price	0.7286	***	(0.0281)
	Constant	4.1075	***	(0.3628)

Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.

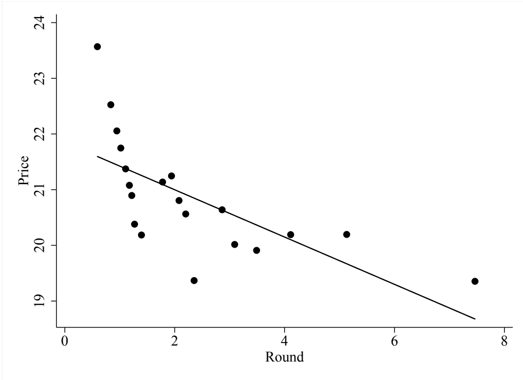
Table 4.19: Estimation results for price of haddock

Haddock 1	Number of observations: 299, $R^2 = 0.8790$			
	Round	-0.8610	***	0.0793
	First price	0.9551	***	0.0206
	Constant	1.5837	***	0.3782
Haddock 2	Number of observations: 366, $R^2 = 0.8805$			
	Round	-0.5206	***	0.0518
	First price	0.8966	***	0.0173
	Constant	2.2352	***	0.3218
Haddock 3	Number of observations: 337, $R^2 = 0.8765$			
	Round	-0.5269	***	0.0455
	First price	0.8740	***	0.0180
	Constant	2.3192	***	0.2859
Haddock 4	Number of observations: 82, $R^2 = 0.9213$			
	Round	-0.7387	***	0.0744
	First price	0.9529	***	0.0314
	Constant	0.8746	***	0.1088

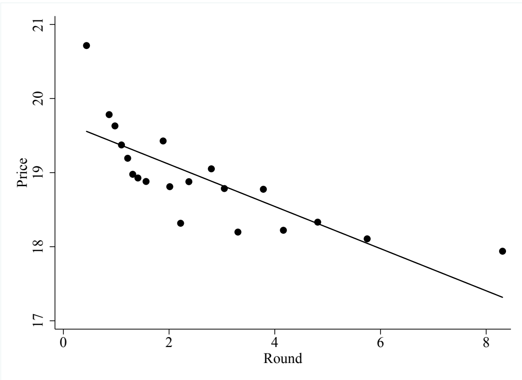
Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.



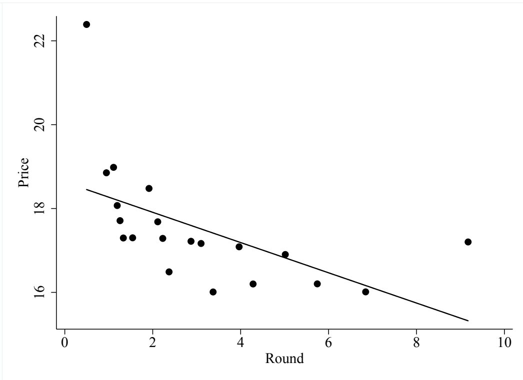
(a) Cod size 1



(b) Cod size 2



(c) Cod size 3



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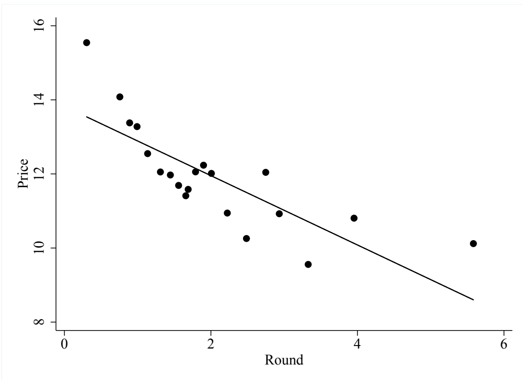
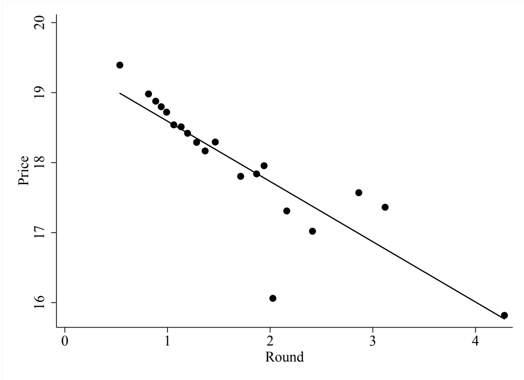
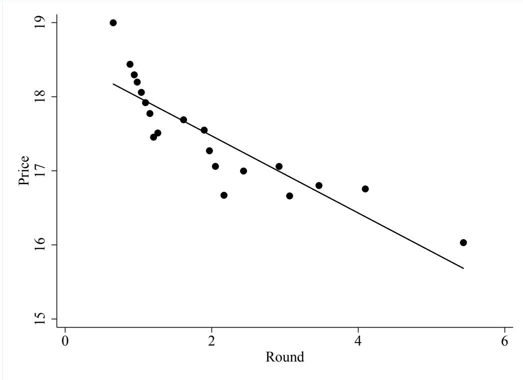


(e) Cod size 5

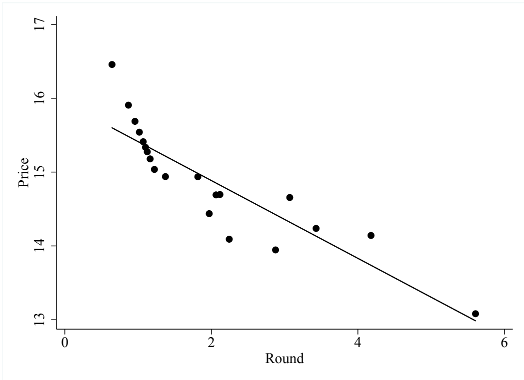
Figure 4.6: Binned scatterplots for price of cod



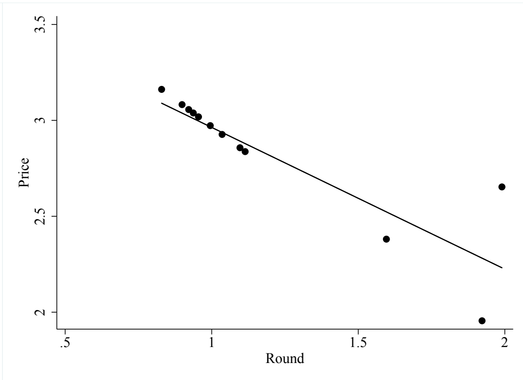
(a) Haddock size 1



(b) Haddock size 2



(c) Haddock size 3



(d) Haddock size 4

Figure 4.7: Binned scatterplots for price of haddock

4.10 Appendix E: Estimation Results

Table 4.20: Estimation results of price change relative to first price

Round 2	-0.0725	***	(0.0029)
Round 3	-0.1032	***	(0.0038)
Round 4	-0.1163	***	(0.0050)
Round 5	-0.1259	***	(0.0066)
Round 6	-0.1474	***	(0.0086)
Round 7	-0.1615	***	(0.0107)
Round 8-15	-0.1622	***	(0.0092)
Number of observations: 5398			
$\mathrm{F}(7, 5328) = 258.47$***, $R^2 = 0.3051$			
Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.			

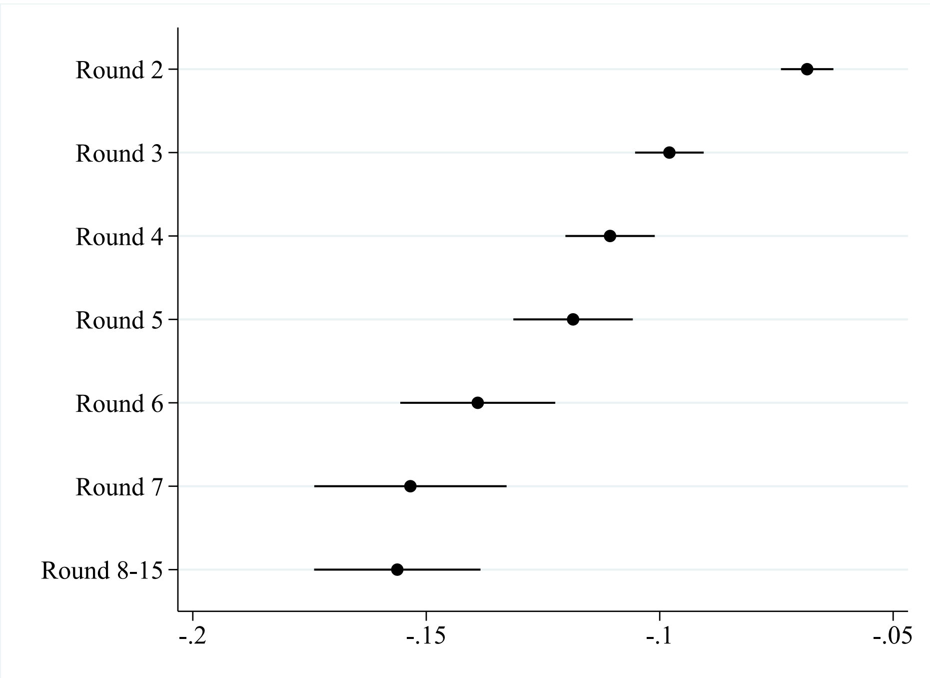


Figure 4.8: Regression coefficients for estimating price change relative to first price

Table 4.21: Predicting relative price changes (previous round)

	Model 1		Model 2		Model 3	
Round	-0.008141	***	-0.007088	***	-0.008687	***
	(0.0007)		(0.0007)		(0.0007)	
Bids			0.012043	***	0.010928	***
			(0.0008)		(0.0008)	
Quantity					0.000006	***
					(0.000)	
Constant	-0.008361	***	-0.038209	***	-0.036725	***
	(0.0016)		(0.0026)		(0.0026)	
Observations	5398		5398		5398	
F-test	142.92	***	178.72	***	190.49	***
R^2	0.0651		0.1004		0.133	

Note: Standard errors in brackets. ***$p < 0.001$, **$p < 0.01$, *$p < 0.05$. Here quantity refers to the amount sold in each round.

Conclusion

This thesis has addressed the topic of resource allocation within the broad field of game theory, covering both cooperative and non-cooperative game theoretical approaches. In the four chapters, largely focusing on allocation methods within fisheries, both theoretical and empirical methods have been employed. Three different allocation mechanisms have been considered within the thesis: auctions, grandfathering and the Shapley value, which is discussed in a more general setting. Efforts have been made to take into account the contextual setting to the extent possible and real life examples have been analysed to add to our understanding of the behaviour of players and their interactions.

In the first chapter on the Shapley value, we introduced a cooperative game in which some of the information may be missing from the characteristic function. A universal assumption of game theory is that the money worths of the coalitions are known. In reality, this is often not the case, since it might be in the interest of players to withhold information or coalitions might not have formed. In the chapter, two different methods for extending the Shapley value to the missing information domain were proposed. First, intuitive generalizations of the classical axioms of Shapley (1953) were suggested. Following this, the paper looked for domain restrictions on the set of known coalitions upon which the Shapley axioms characterize a unique solution. The chapter found that when the set of known coalitions is permutational, and there are as many known coalitions as players, or the set of known coalitions is singleton-defined, there is a value which satisfies the Shapley axioms. Somewhat surprisingly, the axioms characterize the value. The value proposed (φ^*) is an obvious extension of the Shapley value on the missing information domain. The value gives each player the average of their marginal contributions to their predecessors in each of the known marginal vectors. However, when the set of known coalitions is not permutational, then the value (φ^*) cannot be applied to the game. Even if the set of known coalitions is permutational, when assigning payoffs to the players, the value (φ^*) may fail to use all the information contained in the characteristic function.

The second solution, which draws on ideas in social choice and economic justice, largely succeeds in remedying these shortcomings. The solution consists of two stages in which (i) an impartial observer could use the information contained in the characteristic function to fill in the missing information, using an agreed upon normative principle, in our case to "maximize the minimum payoff" (ii) the Shapley value would then be applied to the 'filled in' game. Therefore, the paper concluded that there are ways to extend the Shapley value to the missing information domain and thereby find a 'fair and efficient solution' in potential situations where coalitions have not formed or information is withheld. Finally, we highlighted that both of the proposals put forward rest on the assumption that the worths of coalitions are independent of the behaviour of players outside the coalition and that there are many situations where this might not be the case. Therefore developing ways of extending the Shapley value to partition function games — where worths are dependent on behaviour of players outside the coalition — with missing information would make our methods relevant in a wider range of settings.

In the second chapter, the first steps were taken towards developing a theoretical model for exploring how different combinations of two allocation mechanisms — grandfathering which allocates quota based on historical catches and auctions, which any firm is allowed to participe in — perform in relation to key parameters. Since an important component of fisheries management is to limit the negative externalities of the activity, the developed model allows the government to incentivize investment into greener and more environmentally friendly fishing technologies. In our model, the government did this by applying an environmental tax, which could be reduced through investment, to achieve these environmental objectives. Therefore, the firms' level of investment directly affects its valuation of quota sold at auction. This reduces the firms' externalities and decreases their marginal costs, leading to a higher valuation of the fishing quota sold at the auction. This meant that bidder valuations were endogenous within the model, which we argued, better captures the complexity related to agents' decisions. By applying the model in a theoretical setting, several important results were obtained, which are very relevant in the fisheries context. This included how changing different government controlled variables (how much is auctioned or the resource fee of grandfathered quota), affects the government revenue, total welfare, the firms' level of investment as well as the equilibrium price at auction. We concluded that the model needs to be further developed to improve its applicability in real-life settings. In particular, we highlighted that the model must include decreasing marginal costs as well as potential anti-trust rules to improve its relevance in the fisheries context. However, as currently developed, the model still provides policy-makers with a tool to explore trade-offs between government revenue and investment into environmentally friendly technologies and thereby understand how to incentivise investments in the industry. The model also captures another important policy concern by making evident how particular policy decisions and changing the amount or price of grandfathered quota — and thereby also the amount that is auctioned — would impact on the industry and the revenue of the government. We found that the majority of the results seemed to fit to what one might expect indicating that the model could be applied in the field of fisheries, although as indicated above, the model and its underlying assumptions need to be refined and developed further to improve its applicability and relevance in the fisheries context

The third chapter focused on the auctioning of fishing rights that took place in the Faroe Islands from 2016 to 2018. In terms of fraction of GDP, these auctions were some of the largest ever held. Two different types of auctions took place: the ascending (or second-price) and uniform-price multiunit auctions. The performance of the auctions with respect to collusion, entrants, and price equilibria was evaluated. The chapter found that signalling, bidding rings and low-price equilibria all featured at the auctions, indicating that there were severe problems with the auction design. Several examples of crank handle bidding, leading to low-price equilibria as well as evidence of a bidding ring and signalling by a dominant player were all observed. After having identified what we believe to be the first case of all three of the above-mentioned phenomena happening in the same bidding environment, we argued that the underperformance of ascending and uniform-price auctions are not just theoretical curiosities, but rather a pervasive phenomenon in practical auction design.

Finally, straightforward improvements to the auction design, which could have mitigated the problems identified in the Faroese auctions, were suggested. More specifically, we argued that the first-price sealed-bid package auction would allow bidders to express preferences over operational scale which would make it harder to lock out entrants. Furthermore as sealed bid auctions transmit little information, the formation of bidding rings would be less likely. Moreover, pay-as-bid auctions reduce the value advantage of incumbent bidders compared to ascending or second price auctions, as well as any incentives for crank handle bidding in uniform price auctions. In view of this, such a format would probably be more suitable in the Faroese context. The chapter demonstrated that the underperformance of ascending and uniform price auctions, which is predicted by various models of auctions with asymmetric bidders and collusive behaviour, was borne out in practice, even among bidders who had little experience of bidding in such auctions. As a result, we ended by cautioning against the use of (sequential) ascending or uniform-price auctions without flexible supply in cases where there is serious bidder asymmetry and opportunities for industry coordination.

The fourth chapter analysed the auctions at the Faroe Fish Market. Using auction data from 2017, the paper focused on identifying price trends at the market. Since Ashenfelter's (1989) discovery of the declining price anomaly in sequential wine auctions, there has been a steadily growing body of literature on the topic with contradictory findings, although the majority of empirical studies have found declining prices. Several methods were employed to test for declining prices at the Faroe Fish Market. First, the method used by Ashenfelter (1989) in his influential paper on the declining price anomaly was employed. Second, regressions to establish how the auction round affected absolute and relative price changes — both with respect to the first price of the day and to the price in the previous round — were run. Furthermore, we explored the impact that the number of bids, quantity sold and the price of the first round of each species had on price trends. Finally, we applied the method used by Gallegati et al. (2011) by analysing the average price of cod in each round across the whole period. Here we also measured the strength of auction round as a predictor for average prices. The paper found that declining prices were a strong feature of the auctions at the Faroe Fish Market, both overall and for major species. This result was independent of the method employed. In 81 percent of all cases, the prices were declining from one lot to another, with the highest percentage for the second largest species in terms of quantity and transactions — haddock — where the corresponding figure was 87 percent. We also established that the auction round was a significant predictor of price trends, even when controlling for number of bids, quantities and the first price. Once declining prices had been identified, the paper went on to discuss potential reasons for the price patterns. It highlighted that there were two important differences with regards to the auctions studied by Ashenfelter (1989). Firstly, the goods considered in our chapter were not as homogenous as the bottles of wine studied by Ashenfelter and secondly, the auction format differed as the auctions at the Faroe Fish Market were right-to-choose auctions, allowing bidders to pick their preferred goods.

After considering the context of the auctions and comparing to findings from the literature, our paper argued that the declining prices were perhaps not so surprising. We highlighted that one could expect declining prices in connection with the format of the market. In addition to the right-to-choose, the auctions also involve a buyers option — where winning bidders can choose to purchase the whole lot immediately. We argued that these two features would expectably lead to lower prices. Our analysis demonstrated that both competition and supply conditions had an impact on price patterns. We found that low supply exacerbated the declining prices with steeper price drops in such settings. The paper also found that the number of bidders was positively correlated with price, and that it reduced throughout the day, which likely also contributed to the declining prices. Therefore, we concluded that there were many contributing factors to the declining price anomaly identified, which perhaps is not such an anomaly after all.

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