"New Water in Old Buckets: Hypothetical and Counterfactual Reasoning in Mach's Economy of Science"[1]

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Abstract

Ernst Mach's defense of relativist theories of motion in *Die Mechanik in ihrer Entwickelung* involves a well-known criticism of Newton's theory appealing to absolute space, and of Newton's "bucket" experiment in particular. Sympathetic readers including Norton (1995) and critics including Stein (1967, 1977) agree that there is a tension in Mach's view: he allows for some constructed scientific concepts, but not others, and some kinds of reasoning about unobserved phenomena, but not others. Following Banks (2003), I argue that this tension can be interpreted as a constructive one, as springing from Mach's basic approach to scientific reasoning. Mach's reasoning about the "economy of science" allows for a principled distinction to be made, between natural and artificial hypothetical reasoning. Finally, Mach defends a division of labor between the sciences in a paper for *The Monist* of 1903, "Space and Geometry from the Point of View of Physical Inquiry". That division supports counterfactual reasoning in Mach's system, something that's long been denied is possible for him.

1. Debating Mach's Principle: Empiricism, Counterfactual Reasoning, and Economy

The history of Mach's Principle has been tied closely to the history of the theory of relativity, and thus, more broadly, to the history of relativistic theories of motion. On relativistic theories, a body in motion must be in motion relative to some other body, and not to absolute space. As Norton notes, acceleration often is a "stumbling block" for relativistic theories.[2] For instance, in the "two spheres" thought experiment, two spheres are separated by a string. The spheres rotate around each other. When they are rotating quickly with respect to each other, the string is taut. When the spheres slow down, the string slackens. The string reflects the existence of a force. But on a relativistic theory, since the two spheres are rotating only with respect to each other, there is nothing with respect to which the entire two sphere system is rotating. But if the system is not in motion with respect to any body outside it, then what is the source of the tension on the string?

In Mach's comments on Newton's reasoning, in the famous 'bucket experiment', Mach criticizes Newton's appeal to absolute space and time. Newton argues that the centrifugal force in the bucket experiment, and the force on the string in the two spheres experiment, result from the rotation of the bucket and the spheres with respect to absolute space. Mach asserts three things in response: (1) that "no one is competent to predicate things about absolute space and absolute motion", since these are not facts but creatures of thought,[3] (2) that the water in Newton's bucket, or the two rotating spheres) could be rotating with respect to some more distant inertial frame, such as the fixed stars; and (3) that the physicist's task in evaluating these experiments is to analyze the facts before her, and not "arbitrary fictions of the imagination".[4] Mach concludes that "the phenomena of centrifugal forces compel us not to postulate an absolute reference-system but to recognize the law of inertia as a mere empirical generalization about the motions of bodies relative to the fixed stars".[5]

On Norton's reading, Mach's principle is that "the motion of a body is caused entirely by an interaction with other bodies" (*op. cit.*, p. 10). While Mach himself did not formulate this principle, scientists after him, most famously Einstein, took it as a methodological or physical principle. Moritz Schlick appears to have been among the first to formulate the principle. He "referred to Mach's general proposal for a relativity of all motion, from which, Schlick noted, it follows that 'the cause of inertia must be assumed to be an interaction of masses'".[6] Thus, prima facie, we may read Mach's principle either as a simple statement of a relativist theory of motion, or as a physical statement about the cause of inertia.

Since its formulation, the principle has been controversial. One set of views of its significance appeals to Mach's so-called "phenomenalism". On this reading, which we can attribute to Howard Stein and others, Mach is guilty of an "abusive empiricism" that sets unreasonable criteria for scientific reasoning. *All* conceptual or formal notions used in science must be observable, a requirement Stein finds unreasonable.

Another view, found in Norton (1995), is more sympathetic. On Norton's reading, Mach's criticism of Newton is not intended to lay out a new physical argument or mechanism. Rather, Mach intended only to *redescribe* Newton's experiments without the term "space," which had an indeterminate meaning. He did not intend to give a causal argument or mechanism. As Norton points out, this reading leads to a puzzle. If Mach didn't intend to give a causal argument, why does he accept without comment the many people who said, in his presence or in works he read, that he did?

As Norton points out, though, what Mach meant by a "causal" argument was simply a functional dependence between two observed variables (Norton, *op. cit.*, pp. 27-29). In the case of the bucket experiment, that functional dependence can be captured by pointing out that centrifugal forces are produced by the relative rotation of the water in the bucket with respect to "the masses of the earth and the other celestial bodies".[7]

As Norton notes, even with this heroic save, the original puzzle remains. Einstein wrote to Mach saying that "*inertia* has its origins in a kind of *interaction* of bodies, quite in the sense of your reflections on Newton's bucket experiment" (cited in Norton, *op. cit.,* p. 29). But on Norton's reading, Mach's reflections quite deliberately didn't amount to a general claim about inertia or its sources. Rather, they amount to a re-description of the bucket experiment itself without using the notion of "space" to explain the origins of inertia. As Norton points out, "Einstein's notion of causal interaction extended well beyond the simple functional relations of phenomena. It included relations on hypothetical and counterfactual systems of precisely the type denounced by Mach" (*op. cit.,* p. 29).

Here, there is an unexpected agreement between Stein's critical and Norton's sympathetic reading. As DiSalle summarizes Stein's position,

> Stein characterized Mach's philosophical opposition to Newton as "abusive empiricism"—meaning by this not merely the prejudice against theories involving unobservable entities or far-reaching counterfactual implications, but, more important, the absurd willingness to accept empirically unmotivated hypotheses about cosmic geography, boundary conditions, and so on, just to avoid theories of that sort.[8]

Stein makes a point closely related to Norton's: that Mach's pronouncements on hypothetical reasoning seem inconsistent.

> Mach's opposition to the kinetic-molecular theory is based upon the fact that, as he puts it, atoms are "mental artifices." But what about perfectly ordinary objects? "Ordinary matter," Mach says, is a "highly natural, unconsciously constructed mental symbol for a relatively stable complex of sensational elements"; the only distinction he finds to the disadvantage of atoms is that of the "natural unconscious construction" versus the "artificial hypothetical" one. To conclude, as Mach does, on the basis of this distinction, that atomic theories should eventually he replaced by some "more naturally attained" substitute is is very strange: not only is the argument at right angles to Mach's view of the "economic" objective of science, it actually accords a preference to the instinctive and unconscious over the conceptual and deliberate mental processes.[9]

On the one hand, Mach seems to be arguing that "artificial entities" like "absolute space", "atoms", and the like are to be eliminated from physical reasoning. On the other hand, Mach appears willing to accept "empirically unmotivated" claims about cosmic boundary conditions, the global conditions of matter, and the like, in order to avoid absolute space and the atomic theory.

Moreover, as Norton observes, Mach does not object to Einstein's interpretation of Mach's reasoning about the bucket experiment. But Einstein's interpretation has it that Mach is making a general point about the origins of inertial forces, which implies that Mach is making a strong causal claim beyond functional dependence. That stronger claim involves hypothetical and counterfactual reasoning that is supposed to be anti-Machian.

The accounts of Norton and Stein suggest a criticism of Mach. Mach uses his well-known reasoning about the "naturalness" of concepts and the "economy" of science to oppose concepts like absolute space and atoms. But when it is a matter of his own preferred concepts or results, like ordinary matter, global boundary conditions, or the origins of inertial force, Mach appears willing to violate his own criteria. Those criteria then appear to be ad hoc, not scientific. That is why Stein calls Mach's empiricism "abusive": from a Newtonian perspective, Mach is picking and choosing which "conceptual monsters" he will allow within his system, and violating his own criteria to do so. And, as we've seen, Norton agrees that Mach's position appears internally inconsistent in this way.

2. A Division of Labor within Mach's Economy of Science

If your friends and your foes agree that you have a fault, it may be that you do have that fault. Or, it may be that, despite the heroic efforts even of your sympathetic readers, there is still some misunderstanding.

To a certain extent, Mach has been misunderstood. But the misunderstanding stems, not from his physics, but from his account of the "economy of science", found in Mach's work *The Development of Mechanics, Presented Historico-Critically*.[10] This work is the site of, and crucial context for, Mach's interpretation of Newton's bucket experiment.

A thorough reading of *The Development of Mechanics* leads one to question the usual reading of Mach on the economy of science. According to that reading, Mach argues that the "principles" of mechanics, like the principles of least action or of the straightest path, are "economical" in the sense that they allow for computational efficiency or ease of memory. Many scientific results are encapsulated in the least action principle, for instance, but humans don't have memories that can store and recall all of those results with ease. But the least action principle takes a few minutes to memorize. Moreover, it encapsulates more results. Thus, if the principle is used in an inference, it allows for more computational power: we can more easily determine what is derivable from what.

I don't disagree with this interpretation as a partial reading of Mach on the economy of science. However, Mach's account is much richer than this. Mach presents the economy of science, not just as a static set of principles that allow us to compute more easily, but also as a system that develops over time. That system is dynamic. Certain ways of proceeding will promote the economy of science, while certain others will detract from it.

Typology of the Economy of Science

Methods that Promote Economy	*The Effects of These Methods*
"Instinctive knowledge"	Transparency
The method of differences	Empirical fruitfulness
The law of continuity of experience	The ability to disregard details
Relations of the whole	Computational power
	Minimization
	Completion of experience

One can identify ten elements in *The Development of Mechanics* relevant to the economy of science. The elements on the left hand side of the chart above, the "methods", are ways to promote the economy of science. The elements on the right hand side, the "effects", are results of employing these methods. Computational power, according to this chart, should be seen not (just) as an absolute property of a principle of science, but as the result of employing an economical method.

One example of an economical method is the "method of differences", which Mach discusses in his lecture "On the Conservation of Energy". Experience teaches us that some sensed elements of the world are interdependent - pressure, volume, and heat in a gas, for example. But there are differences as well as dependence.

> Facts may be so nearly related as to contain the same kind of [elements], but the relation be such that the [elements] of the one differ from the [elements] of the other only by the number of equal parts into which they can be divided ... if rules can be given for deducing from one another the numbers which are the measures of these [elements], then we possess in such rules the most general expression of a group of facts ... This is the goal of quantitative investigation… what we have found is that between the [elements] of a group of facts ... a number of equations exists. The simple fact of change brings it about that the number of these equations must be smaller than the number of the [elements]. If the former be smaller by one than the latter, then one portion of the [elements] is uniquely determined by the other portion.[11]

Theories should be analyzed in terms of their observable "elements".[12] The goal of science is to show that some elements of a theory (variables such as pressure and volume) are dependent on other elements of the theory (variables such as motion and density). To show that the independent group is smaller than the dependent group is to promote economy in science. Mach calls this the "method of differences". As Banks notes,

> The tension between the [observed] elements and their ordering into this general manifold of space, time, and matter is a general problem in Mach's philosophy of nature.

> The divide falls between his heraclitean view that the elements are transitory unique events, arising and vanishing and possessing always an individual existence, and his view that space, time, and matter, however unreal they may be on a fundamental level, represent for Mach economical permanencies that must be acknowledged as a task of science... Mach said that he considered the real facts of nature to be the existence of "differences" or inequalities... Mach's elements are the differences of state in the world and, by a careful tracking of their effects on one another, the determinations of the rates and magnitudes of those effects, Mach thought one could deduce the existence of independent potential sources and relations of intensity from this raw data by finding orderings in it.[13]

The tension Banks notes, between "transitory unique events, arising and vanishing and possessing always an individual existence", and "his view that space, time, and matter, however unreal they may be on a fundamental level, represent for Mach economical permanencies", is precisely the tension identified by Norton and Stein as a problem for Mach. But Norton and Stein see the tension as a problem in a negative sense: as an inconsistency, or as an un-scientific element in Mach's view. On Banks's reading, we can see the tension as a "problem" in a positive sense: one aim of science is to resolve the tension between transitory world-elements that are experienced only once, and the "economical permanencies" of space, time, matter, and the like.

Concepts like space, time, and matter can be economical in distinct ways. In *The Development of Mechanics*, Mach provides a thoroughgoing, if disjointed, account of how the economy of science is promoted. And this account can serve as a way to explain why, for him, some concepts can be used to support hypothetical systems and counterfactual reasoning, and some should not. Such an explanation will not make all of Mach's stated positions consistent. But it will clear up the question, asked by Norton, Stein, and others, of why Mach seemed to allow for such reasoning in some cases and not in others.

Stein argues that

> Mach says, in a famous and true remark, that the world is given to us only once, and he concludes that it is "not permitted to us to say how things would be" if the world were to be other than it is… But Mach does not make it a general rule for science that in every statement based upon experience there should appear a list of all the circumstances over which we have no control (the universe being given only once), in order to avoid seeming to claim that we know that the statement would continue to be true even if these things were otherwise. Such a rule would not only grievously violate Mach's "economy of thought", it would make science impossible (*op. cit.* 1977, p. 15).

It's rare, fortunately, to have the dreadful responsibility of contradicting Howard Stein. Even so, Mach does make it a general rule for science that we cannot neglect the rest of the world even when concentrating only on two observed facts or elements.

> even in the simplest case, in which apparently we deal with the mutual action of only two masses, the neglecting of the rest of the world is impossible. Nature does not begin with

> elements, as we are obliged to begin with them. It is certainly fortunate for us, that we can, from time to time, turn aside our eyes from the overpowering unity of the whole, and allow them to rest on individual details. But we should not neglect ultimately to complete and to correct our views by a thorough consideration of the things which for the time being we left out of account… In fact, science can accomplish nothing by the consideration of individual facts: from time to time it must cast its glance at the world as a whole. Galileo's laws of falling bodies, Huygens's principle of vis viva, the principle of virtual velocities, nay, even the concept of mass, could not, as we saw, be obtained, except by the alternate consideration of individual facts and of nature as a totality.[14]

As we learn from Banks, the tension between the elements, the heraclitean world "given only once," and the consideration of "nature as a totality" is a fundamental problem for Mach's account of science. But it is a problem that Mach sees as one we must try to solve, and he thinks that the framework of a solution must be built into science itself.

Mach's "law of continuity of experience" is a crucial element of that framework. This "law" is in fact more of a method or rule. It states that any principle of mechanics must be considered, not as a universal and necessary proposition, but as an assertion that is being checked constantly by experience.

> The most important result of our investigations is that precisely the apparently simplest mechanical principles are of a very complicated character, that these principles are founded on uncompleted experiences, nay on experiences that never can be fully completed, that practically, indeed, they are sufficiently secured, in view of the tolerable stability of our environment, to serve as the foundation of mathematical deduction, but that they can by no means themselves be regarded as mathematically established truths but only as principles that not only admit of constant control by experience but also require it (*op. cit.,* 1883, pp. 237-8).

The law of continuity is the basis of Mach's reasoning about the "completion" of experience through science, one of the products of the economy of science.

> The function of science, as we take it, is to replace experience. Thus, on the one hand, science must remain in the province of experience, but, on the other, must hasten beyond it, constantly expecting confirmation, constantly expecting the reverse. Where neither confirmation nor refutation is possible, science is not concerned. Science acts and acts only in the domain of *uncompleted* experience. Exemplars of such branches of science are the theories of elasticity and of the conduction of heat, both of which ascribe to the smallest particles of matter only such properties as observation supplies in the study of the larger portions. The comparison of theory and experience may be farther and farther extended, as our means of observation increase in refinement (*op. cit.,* 1883, p. 490).

This passage alone contradicts the reading of Mach as a reductive phenomenalist or as an "abusive empiricist" about science. Neither would suggest that science "must hasten beyond" experience. Neither would suggest, either, that science "acts and acts only in the domain of *uncompleted* experience". Certainly, Mach says that our completion of experience is based on observed properties and relations. But, immediately following this passage, Mach explains that those observed properties and relations can be supplemented, and even can be hypothetical:

> When we mentally add to those actions of a human being which we can perceive, sensations and ideas like our own which we cannot perceive, the object of the idea we so form is economical. The idea makes experience intelligible to us; it supplements and supplants experience. This idea is not regarded as a great scientific discovery, only because its formation is so natural that every child conceives it. Now, this is exactly what we do when we imagine a moving body which has just disappeared behind a pillar, or a comet at the moment invisible, as continuing its motion and retaining its previously observed properties. We do this that we may not be surprised by its reappearance. We fill out the gaps in experience by the ideas that experience suggests.[15]

From Mach's statements thus far, we can make a distinction fundamental to his economy of science, between natural and artificial hypothetical reasoning. The distinction is not found in Mach himself, but it is, I believe, a minor and straightforward inference from his own account.

> **Natural hypothetical reasoning** "fills out the gaps in experience by the ideas that experience suggests" with "sensations and ideas like our own".
>
> **Artificial hypothetical reasoning** either:
>
> (a) Does not merely fill in gaps in experience, but rather postulates a speculative kind of experience, or
>
> (b) Uses sensations and ideas that do *not* resemble our own to assemble speculative systems.

The application of this distinction to Newton's bucket, and to the two spheres experiment, is clear. If we appeal to the "fixed stars" or to global boundary conditions that can be cashed out in terms of observations like our own, then we are engaging in natural hypothetical reasoning. We are filling in the gaps of experience, but with experiences that we ourselves could have had. On Mach's reading, if we appeal to Newtonian absolute space and time, then we must construct a speculative hypothetical system based on the sensorium of God. Such an appeal does not fill in gaps in human experience – it goes outside any possible human experience.[16] Newton's account of the bucket experiment is artificial hypothetical reasoning, according to this distinction.

The first conclusion for which I want to argue is that, with a deeper understanding of Mach's economy of science, the vicious tension identified by Stein and Norton can become the more virtuous tension identified by Banks. The question, for Mach, is how to fill out the gaps left by a mere description of observation. To move from the punctiform mosaic of world-elements to a continuous, coherent physical system, we must complete experience with some form of hypothetical reasoning. But that reasoning must be natural, in Mach's terms. It must complete experience using the instruments that experience suggests.

3. Mathematics and Mach's Principle

The account in section 2 above appears to suggest the following way of reading Mach. Mach does not rule out a certain kind of hypothetical reasoning, "natural" hypothetical reasoning. And he does make a principled distinction between natural and artificial hypothetical reasoning, which should clear him of the charge of "abusive" empiricism.

Nonetheless, it seems that Mach still does not allow for *counterfactual* mathematical reasoning, for which we might fault him. After all, Mach was among those engaged in debate over non-Euclidean geometry in the nineteenth century, and he was well aware of the work of Eugenio Beltrami, Nikolai Lobachevsky, Hermann von Helmholtz, Bernhard Riemann, and Carl Friedrich Gauss. Mach's principle is part of the background to relativity theory, but so are the advances in group theory, continuous transformations, non-Euclidean geometry, and allied fields. But these areas all involve robustly counterfactual reasoning – for instance, about the rigid motions possible for a body in certain circumstances, or the paths a body can take through space, or the different ways of determining metric relationships.

In the above section, we saw that Mach's account of scientific reasoning is based on the tension between "transitory unique events, arising and vanishing and possessing always an individual existence", and "his view that space, time, and matter, however unreal they may be on a fundamental level, represent for Mach economical permanencies".[17] Moreover, as Banks remarks, the observed elements for Mach are *real*, while spatiotemporal relationships are not (necessarily): "A full appreciation for Ernst Mach's doctrine of the economy of thought must take account of his direct realism about particulars (elements) and his anti-realism about space-time laws as economical constructions".[18]

Twenty years after the publication of the first edition of *The Development of Mechanics*, we find Mach engaging in an extended reflection on the distinction between physiological, physical, and metric space, in an essay for *The Monist*, "Space and Geometry from the Point of View of Physical Inquiry".[19]

> Our notions of space are rooted in our *physiological* constitution. Geometric concepts are the product of the idealisation of *physical* experiences of space. Systems of geometry, finally, originate in the logical classification of the conceptual materials so gathered. All three factors have left their indubitable traces in modern geometry. Epistemological inquiries regarding space and geometry accordingly concern the physiologist, the psychologist, the physicist, the mathematician, the philosopher, and the logician alike, and they can be gradually carried to their definitive solution only by the consideration of the widely disparate points of view which are here offered.[20]

Here, we find another aspect of Mach's "economy" of science. "Economy" can refer to a way of minimizing or saving, as in effort, computation, or money. But it can refer equally well to a *managed* economy, in which there is a division of labor, for instance, within a society or community. As Schabas details, this latter notion of an economy had a broad currency by the end of the nineteenth century.[21]

Mach refers to the division of labor in the *epistemology* of space and of geometry. The physicist is concerned with what happens once: with the specific setup of an experiment, and with the observed phenomena of the world before us. The physiologist deals with how these Heraclitean sensations arise from an interaction between our physical constitution and the events in question. The geometer is not restricted to the private sphere of "individual intuitive space", but can move freely from physiological to physical to metric space, which gives room for abstract and even counterfactual speculation.[22] Such speculation is not robustly physical − Mach certainly spoke against it in some physical contexts. But in this essay, he argues that the division of labor in the epistemology of space allows the physicist to appreciate geometrical and hypothetical reasoning:

> It little accords with the principles of a physicist to make suppositions regarding the deportment of geometrical constructs in infinity and non-accessible places, then subsequently to compare them with our immediate experience and adapt them to it. He prefers… to regard what is directly given as the source of his ideas, which he considers applicable also to what is inaccessible until obliged to change them. But [the physicist] too may be extremely grateful for the discovery that there exist *several* sufficing geometries, that we can manage also with a *finite space*, etc. ,- grateful, in short, for the abolition of certain *conventional barriers* of thought (Mach, "Space and Geometry", *loc. cit.*, p. 1).

The person who took trouble to type the words above, and then send them to Paul Carus for publication in *The Monist,* is not the Ernst Mach who allegedly opposed all counterfactual or abstract mathematical reasoning. But he is the Ernst Mach who wrote the chapter "On Experiments in Thought" in *Knowledge and Error* (1905). In this chapter, Mach emphasizes that

> the experiment in thought is a necessary *precondition* of the physical experiment. Each experimenter, each inventor must have the arrangement that led there in mind, before he translates it into action. Although [George] Stephenson also knew the train car, the track, and the steam engine from experience, he still had to model in thought the combination of a train car resting on the tracks driven by a steam engine before he could go on to execute it. No less must Galileo see the arrangements of the investigation of falling motion in front of him in imagination before he implements them.[23]

The physicist, the engineer, and the inventor must engage in robustly empirical reasoning. The dramatic narrative of the first chapters of Mach's *Development of Mechanics* follows the failure of the search for a perpetual motion machine. A physicist − or an inventor! − who continues doggedly to build perpetual motion machines will fail. Mach is quite clear that that failure must be accepted, and that recognizing the fact of failure aids in the construction of future experiments and instruments.

But Mach leaves room for truly imaginary counterfactual or hypothetical reasoning that makes no claim to be physical, or to work in practice. Galileo made experiments, and Stephenson made steam engines, that worked. In both cases, a dynamic mental model of how the experimental setup would be, or how the invention would work, was a "necessary precondition" of the experiment itself. But in "Space and Geometry" and in *Knowledge and Error*, Mach emphasizes that mathematicians can help physicists by freeing their imaginations from mental constraints that restrict the construction of new models, abolishing "conventional barriers to thought", and clearing the way for new methods. No harm is done, on Mach's view, as long as the physicist, the physiologist, and the mathematician respect the epistemological division of labor.

It is possible that between Mach's pronouncements on Newton in 1883, and his essay on geometry for *The Monist* twenty years later, Mach had warmed to counterfactual and abstract reasoning. There are reasons why we might think so. As DiSalle (2003, 1993) details, Mach had learned much more about the mathematics of inertial frames and about relativity theory in the meantime, and he may have accepted that some abstract or counterfactual reasoning in this area was productive and even necessary.

But it is also possible that Mach's appreciation of one kind of counterfactual reasoning was in place already in 1883.[24] Most of the classic experiments in thought from Poincaré, Helmholtz, Beltrami, and others that probed the metric structure of the universe (including Poincaré's expanding gas universe, Helmholtz's convex mirror, and Beltrami's pseudosphere) were expressed in the 1860s and 1870s. In the introductory passages of his 1903 essay, Mach cites Riemann's "Über die Hypothesen, welche der Geometrie zu Grunde liegen", which was published in 1867 and given as a lecture in 1854; and letters from Gauss to Bessel from the 1820s and 1830s. Later he refers to Helmholtz's "Ueber die Thatsachen, welche der Geometrie zu Grunde liegen", which he cites as published in the *Göttinger Nachrichten* in 1868.[25]

It is possible, of course, that Mach had been aware of these sources for decades but had recognized their true import only much later. But if we respect Mach's division of labor for the epistemology of space, we might try to argue that his remarks in 1883 are coherent with the views expressed in 1903.

Making the effort to find common ground between the "Mach" of the two time periods, the 1880s and the early 1900s, is instructive.[26] We can try to reconcile the two by recognizing that the Mach of the 1880s was reading the Newton of the bucket and two spheres experiments as a *physicist*, and not as a mathematician. Mach's moralizing about the vocation of the physicist assigns to Newton the task of finding a way to implement his experiments in thought, just as Galileo and Stephenson had done. But Newton's experiments in thought contain artificial elements that cannot be made empirical. Stephenson's tracks, train cars, and engines can be made manifest; Newton's absolute space drives no trains.

What's the difference between Riemann and Newton? Why does the Mach of the 1900s approve of Riemann's investigations in thought, but not of Newton's? Riemann took the object of his study to be the manifold itself. Thus, he was probing a universal geometrical concept. As Mach writes,

> By the recognition of permanency as coincident with spatial displacement, the various constituents of our intuition of space are rendered *comparable* with one another,—at first in the *physiological* sense. By the comparison of different bodies with one another, by the introduction of *physical* measures, this comparability is rendered quantitative and more exact... Thus, in the place of an individual and non-transmittable intuition of space are substituted the universal concepts of geometry, which hold good for all men. Each person

> has his own individual intuitive space; geometric space is common to all. Between the space of intuition and metric space, which contains physical experiences, we must sharply distinguish (Mach, "Space and Geometry", *loc. cit.*, p. 2).

According to Mach, Riemann's metric space is based on comparison via physical measurements.[27] But it is also "universal" and common to all experiences of space, and thus is a valid basis for abstract reasoning. Mach approves of Riemann because he keeps such reasoning within the appropriate realm in the managed economy of science. His arguments are mathematical – Riemann does not argue from the properties of metric space to the existence of a physical force, for instance.

In the 1883 edition of *The Development of Mechanics*, Mach argues that Newton had made two errors. He made assertions about absolute space, a space not derived from comparison of observable phenomena and thus artificial. And he argued from the properties of artificial absolute space to the existence of a physical force.

Later in his life, Mach modified his assessment of Newton himself, though not his assessment of the "monstrous conceptions" of absolute space and time.[28] The Supplement to the third English edition of *The Development of Mechanics* contains Philip Jourdain's transcription and translation of Mach's revisions to the seventh German edition, published in 1912. Those revisions include a number of passages that provide context for Mach's assessment of Newton's bucket and two spheres experiments.

> while Galileo, in his theory of the tides, quite naively chose the sphere of the fixed stars as the basis of a new system of co-ordinates, we see doubts expressed by Newton as to whether a given fixed star is at rest only apparently or really (*Principia*, 1687, p. 11). This appeared to him to cause the difficulty of distinguishing between true (absolute) and apparent (relative) motion. By this he was also impelled to set up the conception of absolute space. By further investigations in this direction – the discussion of the experiment of the rotating spheres which are connected together by a cord and that of the rotating water-bucket (pp. 9, 11) – he believed that he could prove an absolute rotation... "But how we are to collect," says Newton in the Scholium at the end of the Definitions, "the true motions from their causes, effects, and apparent differences, and vice versa; how from the motions, either true or apparent, we may come to the knowledge of their causes and effects, shall be explained more at large in the following Tract." … do not the words quoted in inverted commas give the impression that Newton was glad to be able now to pass over to less precarious questions that could be tested by experience?[29]

In the section following, Mach suggests that Newton was guided by *empirical* considerations in his postulation of absolute space. According to Mach, when Newton read Galileo on the principles of mechanics, Newton rejected Galileo's use of the postulate of an earth at rest in the explanation of the law of inertia. Newton knew that the earth was rotating. Thus, instead, Newton "imagined a momentary terrestrial system of co-ordinates, for which the law of inertia is valid, held fast in space without any rotation relatively to the fixed stars".[30] He did so "in order to have a generally valid system of reference" (*Ibid*.). Newton could assign a system "any initial position and any uniform translation relatively to the above momentary terrestrial system" (*Ibid.*).

Mach emphasizes that the postulate of absolute space was not necessary, for Newton. In fact, as Mach observes, the reference system Newton constructs is relativist: it's just that the target system is moving relative to the hypothetical terrestrial coordinate system. In making this move, Mach urges, "Newton was correctly led by the *tact of the natural investigator*. This is particularly to be noticed, since, in former editions of this book, it was not sufficiently emphasised. How far and how accurately the conjecture will hold good in future is of course undecided".[31]

What we might call "the Mach-Newton conjecture" is the question of whether all target systems can be defined relative to the terrestrial coordinate system defined by Newton, and whether the law of inertia is valid for all motions relative to such a reference system. It is an intriguing question whether this conjecture has been, or can be, empirically or mathematically verified.

Mach concludes in this later edition that Newton was a good physicist, who gave a physically meaningful characterization of the law of inertia in terms of coordinate systems. The postulate of absolute space was not necessary for Newton. Instead, Mach suggests, Newton should have used only the formal apparatus necessary to give a "generally valid" characterization of inertial motion with respect to a system of coordinates. This would satisfy Mach's law of continuity of experience, according to which gaps in the explanation of experience should be filled by the ideas suggested by experience.

Newton's use of a hypothetical terrestrial coordinate system is a nice example of Mach's division of labor between physics and mathematics. When Newton is led to postulate absolute space, Mach accuses him of using an artificial concept and thus of engaging in artificial hypothetical reasoning. But the hypothesis of a terrestrial system at rest with respect to the fixed stars is an empirically possible hypothesis, and is a natural kind of hypothetical reasoning.

Mach's economy of science often is appealed to as a kind of minimization argument, or as an economy of thought or of effort. The above should cast significant doubt on any such reading of Mach on economy. Mach argues that reasoning that allows us to extend what's observable and thus to promote the "continuity" of experience is economical.

Mach allows even for hypothetical and counterfactual reasoning to contribute to the progress of science. It extends the circle of what is observable by appeal to what is accessible via well founded hypothetical reasoning. It is doubtful that Mach's reasoning about the division of labor between mathematics, physics, and physiology can be supported in practice. But understanding Mach's economy of science allows us to understand some of his more notorious arguments, including his polemics against absolute space and time.

Finally, Mach's philosophical reasoning is part of the recognized background to general relativity. The above account pushes against some of the more limited readings of Mach's relativist theories of space, and urges a deeper understanding of Mach's reception of Newton and of Newtonian methods.

[1] I owe a debt to Friedrich Stadler and to the Institute Vienna Circle for inviting me to contribute a talk to the wonderful Ernst Mach centenary conference, held in June 2016, and to contribute a paper for the volume here. At the conference, I received illuminating questions from Erik Banks, Don Howard, Elisabeth Nemeth, Thomas Uebel, and others, which were instrumental in subsequent improvements to the paper. Since then, I gave this paper as the annual Joint Lecture of the Departments of Philosophy and of HPS at Indiana University. Comments from Amit Hagar, Jordi Cat, Jutta Schickore, Kirk Ludwig, Gary Ebbs, Kate Abramson, Mark Kaplan, and Adam Leite gave a new impetus to the project and helped me to find a unifying thread, or so I hope. Sean Morris issued a kind invitation to present the paper at Metropolitan State University, at which Sergio Gallegos, Elizabeth Goodnick, Vijay Mascarenhas, Daniel Krasner, and of course Sean Morris himself pushed the narrative of the paper in novel directions and identified threads of that narrative I hadn't considered. I am grateful to all of those who participated in discussion and analysis of the paper, and regret only that I haven't made better use of their excellent suggestions.

[2] Norton, John. "Mach's Principle Before Einstein", in: Julian Barbour and Herbert Pfister (Eds.). *Mach's Principle: From Newton's Bucket to Quantum Gravity.* Einstein Studies, Vol. 6. Boston: Birkhäuser. 1995, pp. 9-57.

[3] Mach, Ernst. *The Science of Mechanics*. Chicago: Open Court. 1960 / 1883, p. 280.

[4] Mach, *Science of Mechanics* (1960), *loc. cit.*, p. 284.

[5] DiSalle, Robert. "Carl Gottfried Neumann", *Science in Context* 6, 1, 1993, p. 350.

[6] Schlick, Moritz. "Die philosophische Bedeutung des Relativitätsprincips," *Zeitschrift für Philosophie und Philosophische Kritik* 159, 1915, p. 171; Norton, *op. cit.*, p. 47.

[7] Cited Norton, *op. cit.,* p. 29.

[8] DiSalle, Robert. "Reconsidering Ernst Mach on space, time, and motion", in: David B. Malament (Ed.), *Reading Natural Philosophy*. Chicago: Open Court 2002, pp. 169-170.

[9] Stein, Howard. "Some Philosophical Prehistory of General Relativity", in: John Earman, Clark Glymour, and John Stachel (Eds.). *Foundations of Space-Time Theories* (Minnesota Studies in the Philosophy of Science, vol. VIII), Minneapolis: University of Minnesota Press, 1977, pp. 14. See also Stein, Howard. "Newtonian Space-Time", *Texas Quarterly* 10, 1967, pp. 174–200.

[10] The original impetus for this paper came from a discussion of that "economy" on the HOPOS listserv, with references provided from Erik Banks, Don Howard, Alan Richardson, and others, to the works of Banks himself, of Margaret Schabas, and of others. This discussion led me to look more carefully at Mach's work *The Development of Mechanics, Presented Historico-Critically* (*Die Mechanik in ihrer Entwickelung historisch-kritisch Dargestellt*). This work is usually mis-translated, especially the title. As Don Howard has emphasized in personal communication, the second part of the title, *Presented Historico-Critically*, responds to the nineteenth century tradition of Biblical criticism, and to the

reception of hermeneutic, humanistic methods of Biblical criticism by historians including Karl-David Ilgen and Julius Wellhausen. Mach would have expected nineteenth century readers to be aware of this crucial context. His presentation of mechanics in this context implies strongly that the achievements of mechanics, like the doctrines of the Church, are products of human activity. The discussion in Nemeth, Elisabeth. "Freeing up one's point of view – Neurath's Machian Heritage Compared With Schumpeter's", in: Elisabeth Nemeth, Thomas E. Uebel, Stefan W. Schmitz (Eds.) *Otto Neurath's Economics in Context.* Vienna Circle Institute Yearbook 13. Dordrecht: Springer 2007, and in personal discussions that have been very illuminating, emphasize the role of "freeing up one's point of view" for Mach. I explore that notion in section 3, below, from a distinct but complementary standpoint.

[11] Mach, Ernst. "On the Principle of the Conservation of Energy", in: *Popular Scientific Lectures*, Thomas McCormack (Ed.). Chicago: Open Court 1895, pp. 180-181. First published in *The Monist* (1894).

[12] See Banks, Erik. *Ernst Mach's World Elements: A Study in Natural Philosophy*. Dordrecht: Kluwer. 2003, for a wealth of discussion of Mach's "world-elements".

[13] Banks 2003, p. 239.

[14] *Die Mechanik in ihrer Entwickelung historisch-kritisch Dargestellt*. Leipzig: F. A. Brockhaus. 1883, p. 235. All translations cited as from the 1883 edition are by L. Patton for this essay.

[15] Mach, *Science of Mechanics* (1883), *loc. cit.,* pp. 490-1.

[16] Newton did not appeal only to the sensorium of God in giving an account of absolute space and time. For instance, Newton argued that we could model absolute time using the notion of a "fluxion" of natural magnitudes from his calculus, a notion that is robustly observable (Newton, Isaac. "Methodus fluxionum et serriarrum infinitarum cum ejusdem applicatione ad curvarum geometriam", in: J. Castillion (Ed.). *Opuscula mathematica, philosophica et philologica*, Volume 1. Lausanne and Geneva. 1744.).

[17] Banks 2003, p. 239.

[18] Banks, Erik. "The Philosophical Origins of Mach's Economy of Thought", in: *Synthese* 139, 1, 2004, p. 23.

[19] The German version of this essay is reprinted as the twenty-second chapter of *Erkenntnis und Irrtum* (1905). The version in *The Monist* was translated by Thomas McCormack.

[20] Mach, Ernst. "Space and Geometry from the Point of View of Physical Inquiry", *The Monist* 14, 1, 1903, p. 1.

[21] Schabas, Margaret. *The Natural Origins of Economics*. Chicago: University of Chicago Press. 2005.

[22] See Mach 1903, p. 2, as cited below.

[23] Mach, Ernst. *Erkenntnis und Irrtum*. Leipzig: Johann Ambrosius Barth. 1905, p. 184, emphasis in original. Thanks are due to Erik Angner, Erik Banks, Daniel Breazeale, Uljana Feest, Katrin Hohl, Don Howard, Robin O'Keefe, Flavia Padovani, Dirk Schlimm, January Simpson, Thomas Sturm, and Richard Zach for advice on this translation. "Arrangement" is

translated "Anordnung". As several people pointed out, "Anordnung" could be translated "setup" as well, to refer to an experimental setup.

[24] We can make a distinction here similar to the one above between natural and artificial hypothetical reasoning, on the basis of Mach's texts from 1903 and 1905:

Heuristic counterfactual reasoning frees the physicist from conventional barriers of thought, and is the contribution of mathematicians and metric thought to the epistemology of space. It is used to build models in thought and imagination that inform the setup of experiments that work. If successful, this allows us to understand our experience more fully using heuristic methods.

Artificial counterfactual reasoning uses unnatural and non-empirical concepts to expand the range of models and theories beyond observed physical phenomena. It is not used to understand experience more fully. Rather, it is intended to expand the reach of theories and models artificially, using constructed concepts that are never intended to be tested. Thus, it is similar to artificial hypothetical reasoning.

[25] David Hyder has emphasized to me that there is some question about the proper dating of this publication, but it certainly would not have been published after 1883.

[26] Mach's illness makes things much less simple. In 1898, he suffered what was probably a stroke, and was paralyzed on his right side. Gereon Wolters has questioned the authenticity of much of the work that postdates Mach's illness and subsequent paralysis, distinguishing between Mach I and Mach II. However, the work cited here in 1903, 1905, and later 1912 (see below) is consistent with, or at least a natural development of, Mach's earlier views.

[27] Arguably, Mach's reading of Riemann takes Riemann to be much closer to Helmholtz than Riemann really was, because Helmholtz bases metric geometry much more on the observed facts [*Tatsachen*] than Riemann did.

[28] Mach, Ernst. *The Science of Mechanics*. Supplement to the third English edition, trans. and annotated by Philip Jourdain. Chicago: Open Court. 1915, p. xii.

[29] Citations from Newton, Isaac. *Philosophiae Naturalis Principia Mathematica* ("*Mathematical Principles of Natural Philosophy*"), London, 1687. Main text Mach *op. cit.,* 1915, pp. 33-34.

[30] Mach, *Science of Mechanics*(Supplement 1915)*, loc. cit.,* p. 35.

[31] Mach *Science of Mechanics* (Supplement 1915)*, loc. cit.,* p. 36.