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Empirical constraints on alternative gravity theories from gravitational lensing

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ABSTRACT

If it is hypothesized that there is no dark matter, then some alternative gravitational theory must take the place of general relativity (GR) on the largest scales. Dynamical measurements can be used to investigate the nature of such a theory, but only where there is visible matter. Gravitational lensing is potentially a more powerful probe as it can be used to measure deflections far from the lens and, for sufficiently large separations, allow it to be treated as a point-mass. Microlensing within the local group does not yet provide any interesting constraints, as only images formed close to the deflectors are appreciably magnified, but stacking of multiple light-curves and observations of microlensing on cosmological scales may be able to discriminate between GR and non-dark matter theories. Galaxy–galaxy lensing is likely to be a more powerful probe of gravity, with the Sloan Digital Sky Survey (SDSS) commissioning data used here to constrain the deflection law of galaxies to be $A(R) \propto R^{0.1\pm0.1}$ for impact parameters in the range $50\,\mathrm{kpc} \lesssim R \lesssim 1\,\mathrm{Mpc}$. Together with observations of flat rotation curves, these results imply that, in any gravitational theory, photons must experience (close to) twice the deflection of massive particles moving at the speed of light (at least on these physical scales). The full SDSS data set will also be sensitive to asymmetry in the lensing signal and to variation of the deflection law with galaxy type. A detection of either of these effects would represent an independent confirmation that galaxies are dark matter-dominated; conversely, azimuthal symmetry of the shear signal would rule out the typically ellipsoidal haloes predicted by most simulations of structure formation.

Key words: gravitation – gravitational lensing – relativity – dark matter.

1 INTRODUCTION

The hypothesis that the universe is made up primarily of luminous, baryonic matter and that its dynamics are governed by general relativity (GR) is manifestly incorrect. Measurements of galaxy rotation curves provide the clearest invalidation of the above model, but the dynamics of galaxy clusters and the uniformity of the cosmic microwave background radiation both lead to the same conclusion (as reviewed in e.g. Trimble 1987 or Peebles 1993). An obvious possibility is that the universe contains large amounts of (as yet undetected) dark matter, and this hypothesis has become part of the standard cosmological model.

The alternative to dark matter is that GR is incorrect on the large scales or low accelerations not subject to direct investigation within the Solar system, and a number of possible alternative gravity theories have been proposed (e.g. Milgrom 1983; Tohline 1983; Beckenstein & Milgrom 1984; Rood 1984; Mannheim & Kazanas 1989; Beckenstein & Sanders 1994). A generic feature of these theories is that the gravitational force of an isolated point-mass is modified to fall off more slowly than Newton's inverse square law, although the force law must eventually return to the Newtonian form (or steeper), to ensure that the gravitational acceleration caused by an ensemble of masses is finite (e.g. Walker 1994).

In the absence of dark matter, the nature of gravity can be inferred from dynamical observations or gravitational lensing. Aside from their tendency to rely on assumptions of equilibrium, dynamical measurements are subject to the more fundamental limitation that the gravitational field can only be probed in regions where there is visible matter. Conversely, gravitational lensing can be used to measure gravitational effects well beyond the visible extent of the deflector(s). If such measurements can be made sufficiently far from the lens, any internal structure can be ignored, and it can be treated as a point-mass. Such 'simple lensing' scenarios allow the variation of the deflection angle with impact

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parameter to be measured directly. Once this function is known, a number of other tests are possible: it should have the same form for all simple lenses and it should not depend on the orientation of the deflector. Observations of this nature have the power not only to distinguish between various alternative gravity theories, but also to confirm the existence of dark matter independently.

Mortlock & Turner (2001) use some of these ideas to investigate gravitational lensing in the context of modified Newtonian dynamics (MOND; Milgrom 1983), but a more general approach is adopted here. A simple parametrization of the deflection law of a point-mass (Section 2) is constrained using two simple lensing scenarios: galaxy–galaxy lensing (Section 3) and microlensing (Section 4). These results and the future possibilities are summarized in Section 5.

2 GENERALISED DEFLECTION LAWS

In time it may be possible to invert the available lensing data to give the deflection law of a point-mass in a model-independent fashion, but for the moment a parametrization is required. This can specified in terms of the reduced bending angle, $\boldsymbol{\alpha}(\boldsymbol{\theta})$, which relates the angular position of a source, $\boldsymbol{\beta}$, to the angular position of its image(s), $\boldsymbol{\theta}$, via the lens equation, $\boldsymbol{\beta} = \boldsymbol{\theta} + \boldsymbol{\alpha}(\boldsymbol{\theta})$ (e.g. Schneider, Ehlers & Falco 1992). Given that only 'simple' deflectors (i.e. ideally point-masses, but also physically extended lenses, such as galaxies, if the impact parameter is large) are considered here, rotational symmetry can be assumed, and so the vector notation can be omitted without loss of generality.

The deflection law for a point-mass in GR is simply (e.g. Schneider et al. 1992)

$$\alpha(\theta) = -\frac{\theta_{\rm E}^2}{\theta}, \tag{1}$$

where $\theta_{\rm E}$ is the Einstein radius of the lens, given by

$$\theta_{\rm E} = \sqrt{\frac{4GM}{c^2}\frac{d_{\rm ds}}{d_{\rm od}d_{\rm os}}}, \tag{2}$$

where G is Newton's gravitational constant, M is the mass of the deflector, c is the speed of light, and $d_{\rm od}$, $d_{\rm os}$ and $d_{\rm ds}$ are the angular diameter distances from observer to deflector, observer to source and deflector to source, respectively. For a given cosmological model these distance measures are well-defined in GR, but it is unclear how they vary with redshift in an alternative theory. Fortunately the results presented here are not strongly dependent on the distance measure used, and so they are calculated assuming a standard Einstein–de Sitter model (with Hubble's constant taken to be $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$). None the less, a more self-consistent formulation should be adopted when a full analysis of this sort becomes feasible.

If GR is incorrect on large scales (or at small accelerations), the deflection law given in equation (1) will also break down, presumably falling off more slowly with θ. In order to preserve generality no specific alternative gravity theory is adopted here, and instead a more generic point-mass deflection law is adopted. The parametrization used is

$$\alpha(\theta) = -\frac{\theta_{\rm E}^2}{\theta}\left(\frac{\theta_0}{\theta_0+\theta}\right)^{\xi-1}, \tag{3}$$

which matches the Schwarzschild form for $\theta \ll \theta_0$, but falls off as $\alpha(\theta) \propto \theta^{-\xi}$ for $\theta \gg \theta_0$. Note that the deflection angle actually increases with impact parameter if $\xi < 0$. In terms of a physical theory, characterized by a scale r_0 beyond which the physics becomes non-Newtonian, equation (3) suggests the identification $\theta_0 = r_0/d_{\rm od}$. GR (given by $\xi = 1$) is scale-free, and so r_0 can take any value; in a non-dark matter theory dynamical measurements of galaxies imply that $r_0 \simeq 10\,{\rm kpc}$ and $\xi \simeq 0$, although this scale may either vary with mass [e.g. in MOND $r_0 = (GM/a_0)^{1/2}$, where $a_0 \simeq 1.2\times10^{-10}\,{\rm m\,s^{-2}}$; Milgrom 1983] or be a fundamental constant of the theory.

From the deflection law defined in equation (3), the tangential shear of an image is given by (cf. Miralda-Escudé 1991; Fischer et al. 2000)

$$\gamma_{\rm tan}(\theta) \simeq \frac{1}{2}\left[\frac{{\rm d}\alpha}{{\rm d}\theta} - \frac{\alpha(\theta)}{\theta}\right]$$

$$= \frac{\theta_{\rm E}^2}{\theta_0\theta^2}\left(\frac{\xi+1}{2}\theta + \theta_0\right)\left(\frac{\theta_0}{\theta_0+\theta}\right)^{\xi}. \tag{4}$$

Note that this is half the image polarization, $p(\theta)$, as defined by Brainerd, Blandford & Smail (1996). If $\theta \gg \theta_0$ then equation (4) reduces to $\gamma_{\rm tan}(\theta) = (\xi+1)/2(\theta_{\rm E}/\theta_0)^2(\theta_0/\theta)^{\xi+1} \propto \theta^{-(\xi+1)}$.

The magnification of an image is given by (e.g. Schneider et al. 1992)

$$\mu(\theta) = \left|\left[1 + \frac{\alpha(\theta)}{\theta}\right]\left[1 + \frac{{\rm d}\alpha}{{\rm d}\theta}\right]\right|^{-1} \tag{5}$$

$$= \left|\frac{\theta^2\theta_0(\theta_0+\theta)^{\xi}}{\theta_{\rm E}^2\theta_0^{\xi}(\theta_0+\theta) - \theta^2\theta_0(\theta+\theta_0)^{\xi}}\right| \tag{6}$$

$$\times \left|\frac{\theta^2\theta_0(\theta_0+\theta)^{\xi}}{\theta_{\rm E}^2\theta_0^{\xi}(\xi\theta+\theta_0) + \theta^2\theta_0(\theta+\theta_0)^{\xi}}\right|,$$

from equation (3). From this it is also possible to calculate the total magnification of a source, $\mu_{\rm tot}(\beta)$, by solving the lens equation and then summing the magnifications of the resultant images.

Other observables could be calculated, but the shear of an image and the total magnification of a source are chosen as they relate directly to the quantities observed in measurements of galaxy–galaxy lensing (Section 3) and microlensing (Section 4), respectively.

3 GALAXY–GALAXY LENSING

Background galaxies are observed to be tangentially aligned around foreground galaxies because of the latter population's gravitational lensing effect. The angular dependence of the shear signal is consistent with the hypothesis that galaxies are dominated by approximately isothermal haloes (e.g. Brainerd et al. 1996; Fischer et al. 2000), but could also be explained without recourse to dark matter if the (effective) gravitational force decreases as r^{-1} at large distances.

Under the non-dark matter hypothesis, galaxy–galaxy lensing is a very clean probe of the deflection law. Furthermore, as such measurements rely on averaging over many background sources, the signal is only appreciable at large angular separations from the foreground deflectors. Thus, in the absence of dark matter, the foreground galaxies can be regarded as simple lenses and, despite the fact that equation (3) does not match their lensing properties at small θ, the available data can be used to constrain the deflection law directly. The most comprehensive galaxy–galaxy lensing observations made to date are those described by Fischer et al.

(2000), who used Sloan Digital Sky Survey (SDSS; York et al. 2000) commissioning data to measure the mean shear signal out to ~ 600 arcsec around $\sim 3 \times 10^4$ foreground galaxies. A power-law fit of the form

$$\gamma_{\tan}(\theta) = \gamma_{\tan,60} \left(\frac{60\,\text{arcsec}}{\theta} \right)^{\eta} \quad (7)$$

gave one standard deviation limits of $\gamma_{\tan,60} = 0.0027 \pm 0.0005$ and $\eta = 0.9 \pm 0.1$, although the errors are correlated.

Comparing equations (4) and (7), it is clear that θ_0 and ξ are directly constrained by these results, provided that $\theta_{\rm E}$ is known. This is given by integrating over the deflector and source populations (cf. Brainerd et al. 1996), an approach that will be necessary when the full SDSS data set becomes available. However, $\theta_{\rm E}$ can usefully be approximated by a fiducial value here as the shear signal (and hence the deflection law) is a simple power law in the regime probed, and so the angular dependence of equation (3) can be factored out of the integrals, which then only give the normalization. Moreover, the range of angular separations is such that the fitted values of θ_0 and $\theta_{\rm E}$ are degenerate, and so the normalization of the signal cannot place strong limits on the average mass of the foreground galaxies. The local galaxy population is dominated by spirals (e.g. Postman & Geller 1984) and the mean deflector and source redshifts are $\langle z_{\rm d} \rangle = 0.17$ and $\langle z_{\rm s} \rangle = 0.3$, respectively (Fischer et al. 2000), which imply that $\theta_{\rm E} = 1.0 \pm 0.1$ arcsec.[1]

Using the above value for $\theta_{\rm E}$, equation (7) implies the constraints $\theta_0 = 3.65 \pm 0.08$ arcsec (which implies that $r_0 = d_{\rm od}\theta_0 = 10 \pm 2$ kpc, assuming $d_{\rm od} = 600$ Mpc) and $\xi = -0.1 \pm 0.1$, results confirmed by an independent likelihood analysis. Several of these fits, along with the Fischer et al. (2000) data, are shown in Fig. 1. First, it is important to note that the data points cover angles much greater than the inferred value of θ_0; if this were not the case equation (3) could not be used for the deflection law. As expected, GR (i.e. $\xi = 1$) cannot explain the signal without recourse to dark matter, but the data are consistent with the MOND ian lensing formalism investigated by Qin, Wu & Zou (1995) and Mortlock & Turner (2001), which predicts $\xi = 0$. More generally, the concordance between dynamical measurements and these lensing results implies that the relativistic prediction for geodesics (that photons experience twice the deflection of massive particles moving at the speed of light) must also be true in any alternative theory of gravity.

The observations used in the above analysis represent just a few per cent of the eventual SDSS data set, which should allow shear measurements out to several Mpc with uncertainties at about the 5 per cent level. Beyond this scale the signal will be diluted by secondary deflectors along the line of sight, although if the shear signal was observed to drop off faster than $\gamma_{\tan}(\theta) \propto \theta^{-1}$ it could indicate the edge of galaxy haloes (in the dark matter paradigm) or a return Newtonian physics (if there is no dark matter).

The SDSS data will also allow the azimuthal symmetry of the mean shear signal (cf. Natarajan & Refregier 2000) to be measured; this provides a means of distinguishing between the two paradigms. A rotationally invariant signal would imply that dark

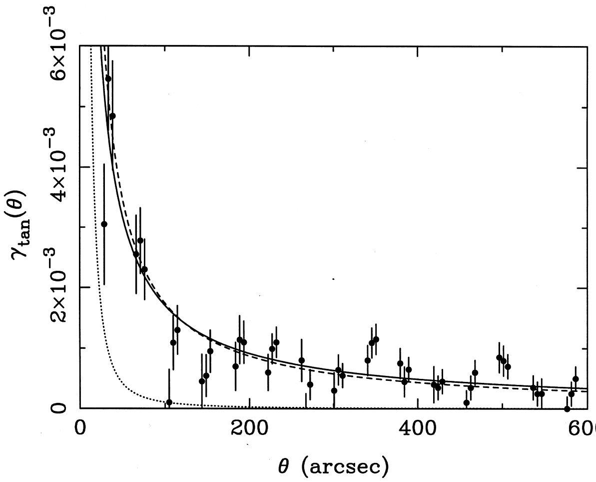

Figure 1. The mean shear around foreground galaxies in the g', r' and i' bands, as measured by Fischer et al. (2000), compared with various theoretical predictions. The data in the three bands are offset for clarity, and the the three models (all of which assume $\theta_{\rm E} = 1$ arcsec) are: $\xi = -0.1$ and $\theta_0 = 3.7$ arcsec (the best-fitting model; solid line); $\xi = 0$ and $\theta_0 = 2.9$ arcsec (MOND or isothermal dark matter haloes; dashed line); and $\xi = 1$ (GR with no dark matter, which is independent of θ_0; dotted line).

matter haloes are typically spherical (or at least circular in projection), in conflict with most collapse models (e.g. Navarro, Frenk & White 1995, 1996), whereas any measured asymmetry much beyond the visible extent of the foreground galaxies would be difficult to reconcile with the hypothesis that they contain no dark matter.

Furthermore, non-dark matter theories, by virtue of their simplicity, are also subject to a number of tests that have no counterpart if there is dark matter. Modulo scaling uncertainties (see Section 2), the point-mass deflection law should always have the same form. In the context of galaxy–galaxy lensing this means that the shear profile should be the same, within statistical uncertainties, around all possible subsets of the foreground galaxy population. For instance, whilst spirals and ellipticals may have slightly different mass-to-light ratios, their deflection laws should have essentially the same form. This principle can also be extended to stellar mass lenses, a possibility explored in Section 4.

4 MICROLENSING

Microlensing of a point-source by a single star provides an unambiguous measurement of the deflection law of a point-mass, but does not necessarily probe regimes in which gravity could be expected to deviate from GR. The family of theories described in Section 2 become non-Newtonian only beyond some scale r_0, which must be several kpc for galaxies but may or may not vary with the mass of the deflector. If it does not, then an isolated star would behave as a Schwarzschild lens out to several kpc, whereas microlensing measurements are only likely to probe subparsec impact parameters. However, if r_0 scales with mass, the deflection law of stars could differ from the standard form as close in as ~ 0.01 pc (e.g. $r_0 \simeq 0.03$ pc for a Solar mass star in MOND). As shown in Mortlock & Turner (2001), microlensing light-curves in such a theory would have narrower peaks and broader wings (relative to GR), although if this was observed, care would have to be taken to exclude more mundane explanations, such as blending and finite source effects.

[1] The value is slightly larger than the actual Einstein radii of the galaxies because of the use of the point-mass lens model, and the large error is a combination of uncertainties in the distance measures and the size of the foreground galaxies.

Observationally, most efforts to detect microlensing have been concentrated in the local group. Several collaborations (e.g. Afonso et al. 1999; Alcock et al. 2000; see Paczynski 1996 for a summary) have monitored millions of stars in the Magellanic Clouds and the Galactic bulge, and over 100 events have been recorded. In several cases the deflector appears to be a binary system (e.g. Alcock et al. 2001), but for the vast majority the approximation that the lens is a single, isolated point-mass is excellent. Unfortunately, the distance scales within the local group are such that single light-curves can only probe Solar system scales, a regime in which Newtonian physics has already been confirmed. Even if magnifications of 0.1 per cent could be measured, microlensing of sources in the Large Magellanic Cloud (LMC; $d_{\rm os} \simeq 60\,{\rm kpc}$) would still only probe the gravitational field of a Solar mass deflector to scales of $\sim 0.001\,{\rm pc}$. One way to escape this limitation might be to 'stack' the light-curves of a large number of lens events, although the effective integration over the deflector population could dilute any non-Newtonian signatures.

A more direct way around this geometrical problem is to search for low optical depth microlensing at cosmological distances. From the definitions in Section 2, and assuming the deflector to be about halfway between observer and source, $\theta_{\rm E} \propto d_{\rm od}^{-1/2}$, whereas $\theta_0 \propto d_{\rm od}^{-1}$, which together imply that $\theta_0/\theta_{\rm E} \propto d_{\rm od}^{-1/2}$. Thus at cosmological distances $\theta_0 \lesssim \theta_{\rm E}$ and the strong lensing regime is subject to non-Newtonian effects, resulting in microlensing light-curves which are markedly more peaked, as described above. The degree of distortion depends upon the value of r_0, but any simple microlensing event with a source redshift close to unity should differ visibly from the GR prediction if $r_0 \lesssim 0.1\,{\rm pc}$ (Mortlock & Turner 2001).

It is possible that microlensing of a cosmologically distant source has already been seen, albeit serendipitously. The redshift 2.04 gamma ray burst (GRB) 000301C (Sagar et al. 2000) was observed to have an achromatic peak in its otherwise smoothly decaying light-curve. Garnavich, Loeb & Stanek (2000) successfully modelled this as microlensing of the expanding fireball by a point-mass, although the fit relied on the assumption that it appeared as a ring on the sky. Unfortunately the photometry of GRB 000301C was not of sufficient quality to facilitate a measurement of the deflection law of the lens, and the uncertain nature of the source only makes such inferences more difficult. More lensed GRBs should be discovered (even if the event rate is low; Koopmans & Wambsganss 2001), but there are also systematic cosmological microlensing searches underway.

Both Walker (1999) and Tadros, Warren & Hewett (2001) describe programs to monitor high-redshift quasars seen through the outskirts of nearby galaxies and clusters. There have not yet been any detection of microlensing, but even a single light-curve should be sufficient to place a lower bound on r_0 (assuming $\xi \simeq 1$; see Section 2). However, the motivation for both these projects was to search for compact dark matter in the foreground objects and so some of the sources were chosen to lie behind their 'outer haloes', regions which might be completely devoid of microlenses if there is no dark matter. This would be unfortunate in the context of light-curve measurements, but also suggests an alternative test of non-dark matter theories.

If there is no dark matter, then the only potential deflectors on these scales are planets, stars and other (known) compact objects. Observations of 'dark' microlensing would tend contradict this hypothesis, but no such detection has been reported. The local group results are consistent with there being no compact dark matter in the Galactic halo, provided that the LMC is extended along the line-of-sight (Afonso et al. 1999; Alcock et al. 2001). The quasar monitoring programs described above have not yet detected any lensing at all, and the GRB microlens could be a star in a galaxy near the line-of-sight (Garnavich et al. 2000). Microlensing has been observed in Q 2237+0305 (Østensen et al. 1996; Woźniak et al. 2000), but this can be attributed to stars in the bulge of the lensing galaxy, and no microlensing has been observed in either image of Q 0957+561 (e.g. Pelt et al. 1998). These results are not only consistent with the non-dark matter hypothesis, but also strongly rule out several popular dark matter candidates.

5 CONCLUSIONS

If there is no dark matter in the universe, then gravitational lensing is an ideal probe of the gravitational field around isolated deflectors. By applying basic symmetries, lensing can be used to distinguish between alternative gravity theories and GR, or used to measure the deflection law in non-dark matter models.

The most suitable data available are the galaxy-galaxy lensing measurements of Fischer et al. (2000). Modelling the deflection law of the foreground galaxies as a power law, the logarithmic slope was constrained to be 0.1 ± 0.1 beyond $\sim 50\,{\rm kpc}$. The full SDSS data set will be 50 times larger, facilitating a measurement of the (as)symmetry of the shear signal. This is a particularly powerful diagnostic, with the power to discriminate unambiguously between the dark matter paradigm and alternative gravity theories.

Non-dark matter models are subject to several other tests, based on the principle that the deflection laws of all isolated deflectors should be the same, modulo possible scaling uncertainties. The galaxy–galaxy lensing results should be independent of galaxy type or luminosity, and the implied physics should be consistent with that inferred from lensing by stars and planets. Observations of microlensing within the local group are fairly insensitive to any putative departures from GR, as non-Newtonian effects are expected only in the wings of lensing events. None the less it may be possible to synthesize sufficiently accurate photometry by stacking multiple light-curves. More promising is the prospect of measuring microlensing events on cosmological scales, either by serendipitous discoveries (Garnavich et al. 2000) or dedicated monitoring programs (Walker 1999; Tadros et al. 2001). With good photometry, even a single light-curve could be used to measure the deflection law of a star out to many parsec, scales on which many theories predict non-Newtonian effects.

All the above ideas pertain to simple lensing scenarios, in which the deflector is not spatially extended; more complex situations permit various observational tests as well, but require additional theoretical development (cf. Mortlock & Turner 2001). Moreover, the tests proposed above are powerful primarily because of their simplicity, and the forthcoming results should place unambiguous constraints on the nature of gravity.

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