Online appendix

A Structural Break Approach to Analysing the Impact of the QE Portfolio Balance Channel on the US Stock Market

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OA. 1. *QE Timeline*- Extending Table 1.

The Fed initiated the first QE programme (QE1, hereafter) on the 25th of November 2008, which was completed in March 2010. QE1 aimed to reduce mortgage discount rates and raise the credit supply for house purchases (Da Costa, 2011; Olsen, 2014). According to Fawley and Neely (2013), the objective of QE1 was to purchase the liabilities of housing association mortgage-backed securities (MBSs) and government-sponsored enterprises (GSE). In addition, the Fed announced plans to purchase $600 billion in treasuries, $100 billion of which were GSEs while the remaining $500 billion were in MBSs. The QE1 programme was expanded further in March 2009 when the Fed announced the purchases of an additional $750 billion in MBSs and $175 billion in GSEs. The QE1 programme ended on the 31st of March 2010, which has resulted in gains in the financial markets convincing the Fed to keep interest rates between 0 and 0.25% (Stroebel and Taylor, 2012).

The Fed launched its QE2 programme on the 3rd of November 2010 and completed it on the 30th of June 2011. The aim of the QE2 programme was to reduce unemployment and increase the rate of inflation to levels consistent with the Fed's target rate. Furthermore, QE2 involved reinvesting payments from the Fed's holdings in long-term bonds, retaining a face value of $2.054 trillion (Fawley and Neely 2013; Krishnamurthy and Vissing-Jørgensen, 2011). Initially, the Fed purchased $600 billion of long-term US Treasuries up until the second quarter of 2011 and continued the programme with $75 billion of purchases monthly, however regularly assessing the pace and the magnitude of the programme. In September 2011, after the completion of QE2, the Fed announced the maturity extension program known as 'Operation Twist' when the US economy experienced a substantial government shutdown. The objective of the maturity extension program was to extend the average maturity of the Fed's holdings of securities by decreasing the long-term interest rates and pushing up the short-term interest rates (Swanson et al., 2011). Moreover, the Fed also reinvested the principal payments from the MBSs and agencies into more MBSs instead of into Treasuries. The focus of this maturity extension program was to push long-term interest rates down and short-term rates up. The program continued until the 20th June 2012 and involved monthly purchases and sales of $45 billion of Treasury securities. Subsequently, the US credit rating was downgraded while the US economy experienced higher unemployment rates and the Eurozone sovereign debt crisis resurfaced (Bowley, 2011; Olsen, 2014).

The Fed then announced the start of the QE3 programme on the 13th of September 2012 along with the maturity extension programme. The program was intended to boost economic growth as well as ensure inflation was within its target. Initially, the program started with monthly purchases of MBSs worth $40 billion along with $45 billion of longer-term US Treasuries. This programme would keep this setting until unemployment rates improved. This made QE3 quite different from QE1 and QE2, because the end of the program would be determined by "goal achievement" rather than by a given date – thereby gaining the nickname "QE-Infinity". Simultaneously with the ongoing maturity extension program, the joint effect was to lower the pressure on long-term interest rates, support the housing market and ensure the financial circumstances were more accommodative. QE3 had no formal end date.

OA. 2: Derivation of mean-variance asset demand function

Equation (4) is derived following Frankel and Engle (1984) and Fraser and Groenewold (2001). We use mean-variance optimisation approach to derive an investor's asset demand since the investor is assumed to maximise a function of the mean and variance of end of period real wealth, W_{t+1} given the information set at the beginning of the period. In this context, the investor chooses the vector of asset shares of the total investment portfolio, λ_t, to maximise wealth. The end of period real wealth is given by:

$$W_{t+1} = W_t \lambda_t^T R_{t+1} + W_t \left(1 - \lambda_t^T \iota\right) R_{t+1}^n \qquad \text{(OA2.1)}$$

This also can be written as:

$$W_{t+1} = W_t \left[\lambda_t^T r_{t+1} + R_{t+1}^n\right] \qquad \text{(OA2.2)}$$

where λ_t^T is the transpose of λ_t, ι a column vector of ones (its dimension equal to the number of assets in the portfolio – i.e. six assets in the case of this paper), R_{t+1} is an a vector of returns to risky assets from t to t+1, and R_{t+1}^r is benchmark numeraire asset (risk free rate of return from t to t+1). The vector of excess returns, r_{t+1}, is defined as $r_{t+1} = R_t - \iota R_{t+1}^r$.

The objective function is defined as follows:

$$\max_{\lambda_t} U(E_t(W_{t+1}), V_t(W_{t+1})) \tag{OA2.3}$$

Subject to the mean and variance constraints: $E_t(W_{t+1})$ and $V_t(W_{t+1})$:

$$E_t(W_{t+1}) = W_t \lambda_t^T r_{t+1} + W_t(1 - \lambda_t^T \iota) E_t(r_{t+1}^n) = W_t\left[\lambda_t^T E_t(r_{t+1}) + E_t(R_t^n)\right] \tag{OA2.4}$$

$$E_t(W_{t+1}) = W_t^2 \lambda_t^T \Omega \lambda_t \tag{OA2.5}$$

where[1]

$$\Omega = E_t\left[(r_{t+1} - E_t(r_{t+1}))(r_{t+1} - E_t(r_{t+1}))'\right] \tag{OA2.6}$$

The first order condition of (OA2.1) gives the following total differential:

$$\frac{dU(E_t(W_{t+1}), V_t(W_{t+1}))}{d\lambda_t} = U_1 \frac{dE_t(Wlt+1)}{d\lambda_t} + U_2 \frac{dV_t(Wlt+1)}{d\lambda_t} = 0 \tag{OA2.7}$$

which is expressed as:

$$U_1 W_t E_t(r_{t+1}) + 2U_2 W_t^2 \Omega \lambda_t = 0 \tag{OA2.8}$$

Define the coefficient of relative risk aversion (i.e. the market price of risk), which is assumed to be constant following Frankel and Engle (1984):

$$z = -2W_t \frac{U_2}{U_1} \tag{OA2.8A}$$

Then, equation (A1.8) can be rewritten as[2]:

$$E_t(r_{t+1}) = \mathrm{z}\Omega\lambda_t \tag{OA2.9}$$

[1] We follow Frankel and Engle (1984) and assume constant variance-covariance matrix, Ω. We relax this assumption and allow the variance-covariance matrix to shift with the conditional mean when applying QP test.

[2] Note that when the coefficient of risk aversion is not assumed constant, we write (OA2.9) as $E_t(r_{t+1}) = z_t \Omega \lambda_t$

Note also we can define excess return as:

$$r_{t+1} = E_t\left(r_{t+1}\right) + \varepsilon_{t+1} \qquad \text{(OA2.10)}$$

where $E_t\left(\varepsilon_{t+1}\right) = 0$. Substituting (OA2.10) into (OA2.9), we obtain:

$$r_{t+1} = \mathrm{z}\Omega\lambda_t + \varepsilon_{t+1} \qquad \text{(OA2.11)}$$

OA. 3 Data Description and Preliminary Statistics

OA. 3.1 Equities and Corporate Bonds

The equity returns and asset shares are derived from the total return index and market capitalisation of the S&P500. This represents a significant and representative part of the equities market in the US. The return index incorporates the aggregate dividend as an increment to the daily change in prices. This is better than using the price index that makes no adjustment for dividend returns.

We use investment grade corporate bonds covering all maturities sourcing the total return and market value from the Barclays Capital Index. As with equities, the index returns include all the cash flows (coupons) and price variations within it. The market value of the bonds and equities is defined as their market capitalization based on the Barclays US corporate bond index and the S&P 500 index respectively, calculated by multiplying the price of an asset by its outstanding securities.

OA. 3.2 Treasury (Sovereign) bonds and money (M2)

We use the Barclays US Treasury index variables and Market Value since Inception (LHUSTRY(MV), LHUSTRY(IN)) to determine the sovereign bond market value and returns. The outstanding value of the total US Treasury securities recorded in this index represents the market value for all US sovereign debt. This provides a comprehensive picture needed for our analysis. When the Fed entered the QE programme, it bought a large proportion of the US Treasuries thus reducing the total available to the private sector. This would be captured in the index as an increase in the total market value.

For money we utilise the US Federal Funds Target Rate (EP) NADJ to calculate the monthly proxy for the returns on M2. This forms the numeraire asset. For market value of broad M2, we use US Money Supply. This measure of money supply includes narrow money, savings deposits, money market mutual funds plus other short-term time deposits. When the Fed purchases large amounts of bonds, this will be reflected in a reduction in the outstanding volumes held by the private sector.

OA. 3.3 Calculating expected excess returns

For equities, corporate bonds and treasuries, we calculate the monthly return and then the year on year return from their respective index:

$$mom_{ix,t} = \frac{Index_{ix,t}}{index_{ix,t-1}} \qquad \text{(OA3.1)}$$

$$yoy_{ix,t} = \frac{mon_{ix,t}}{mon_{ix,t-12}} \qquad \text{(OA3.2)}$$

Where $mom_{ix,t}$ is the monthly gross return on the index where $ix = \{EQ, CB, TR\}$ and t is the time in months. One facet of year-on-year returns is that it avoids any issues with annualising lower frequency outcomes and circumvents the implicit assumption that annualising the change in monthly returns induces a recurrent shock of the same magnitude each month of the year. For money (M2), we use the fed funds rate applicable to that month and calculate the monthly compounded rate, then applies the same year-on-year calculation as above. To calculate the excess returns (that is the returns in excess of the fed funds rate) over money we subtract the year on year return on money from the year-on-year return of the respective asset.

OA. 3.4 Calculating the asset shares

To calculate the share of assets between the four asset classes we use the monthly market valuations thus:

$$AS_{cl,t} = \frac{MV_{cl,t}}{\sum_{c \in cl} MV_{c,t}} \tag{OA3.3}$$

where $AS_{cl,t}$ is the Asset share of the total, $MV_{cl,t}$ is the total market valuation for the asset class, and $cl = \{M2, EQ, CB, TR\}$.

1 OA. 4: Further results and output

Table OA4.1. VAR stability condition and Lag-Length Criteria

	VAR without Dummy
Root (Maximum)	0.9933
SC (Selected Lag)	-39.58 (2)**
HQ (Selected Lag)	-39.84 (2) **
AIC (Selected Lag)	-40.08 (2) **

*Note (1): ** 5% significance level

Table OA4.2 Residual test for autocorrelation LM test for VAR

	LM Test Statistic	P-Value
Lag 1	39.46	0.32
Lag2	30.28	0.73
Lag 3	77.19	0.01

Table OA4.3. Scaling of one standard deviation IRFs to size of QE programmes

	Start of Purchases	Barclays Market Value*	Treasury Securities Outstanding*	Ratio	One Stdev*	Scaling
QE 1 ($300 bn)	Mar-2009	3080	11126	3.61	866	0.10
QE 2 ($600 bn)	Nov-2010	5211	13861	2.66	1168	0.19
QE 3 ($755 bn)	Sep-2012	6067	16066	2.65	1386	0.21

Source: Thomson Reuter's Datastream, Fawley and Neely (2013)
*Note (1): Values in Billions of US Dollar $

Table OA4.4. Regression output from Equation 10

Variable	Coefficient	Std. Error	t-Statistic	P-Value
$\Delta ERP_{t,t-1}$	-4.14	0.12	-34.91	0.00
c	0.07	0.01	18.06	0.00
R^2	0.75			

OA. 5: M-GARCH (1,1) model volatility estimates

The M-GARCH (1,1) model is estimated specifying VAR (2) process in the conditional mean. The estimation process failed to converge using both Stata and EViews. While it was possible to report estimated volatility, inference about the estimated volatility cannot be conducted. Thus, we alternatively estimated a simpler version, in which the conditional mean contains only a constant.

Furthermore, accounting for the shifts in the VAR model leads to different volatility estimates. Fig. OA4.1-5 report the estimated volatility using Multivariate GARCH(1,1) model (M-GARCH(1,1)) based on the full sample and estimated regimes. There are two generally observed patterns. First, the range of volatility when accounting for regime shifts is generally smaller than that estimated for the full sample. Second, Regime 4 volatility estimates are within smaller range for all variables in the system. In addition, we investigate the extent to which Treasuries shares volatility is correlated with the remaining shares and returns series. Fig. A3.6-10 report the pairwise conditional correlation of the volatility between Treasuries share and the remaining series. The full sample conditional correlation, as in Fig. A3.6, indicates, generally, strong correlation – and near perfect in some periods – ranging between negative and positive 1. Incorporating regime shifts produces smaller range of correlation as shown Fig. A3.7-10. Although the range is smaller, relatively strong to near perfect conditional correlation remains the prominent feature across all regimes.

Fig. OA5.1: M-GARCH Estimates of Volatility (Full Sample)

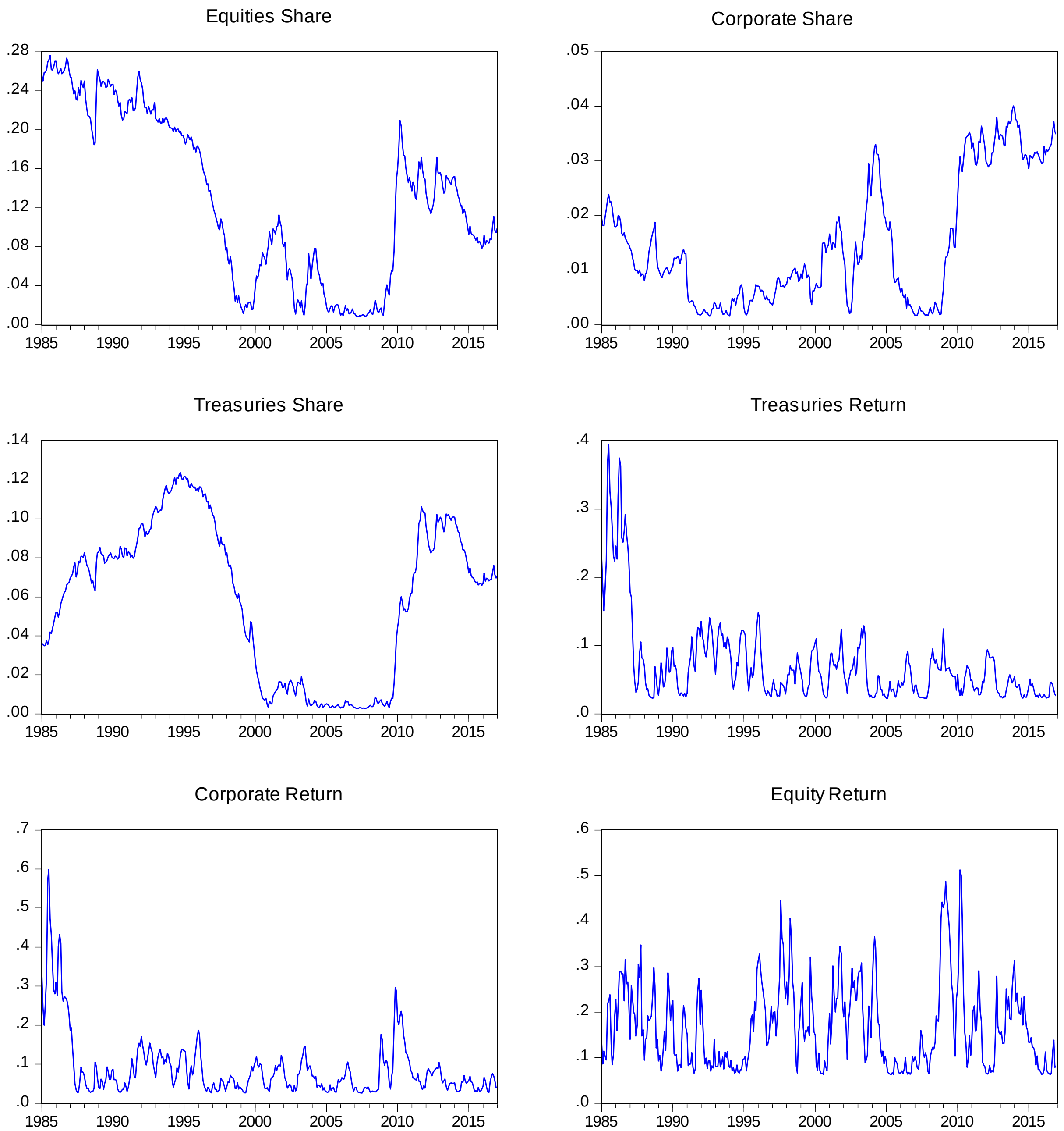


Fig. OA5.2: M-GARCH Estimates of Volatility (Regime 1)

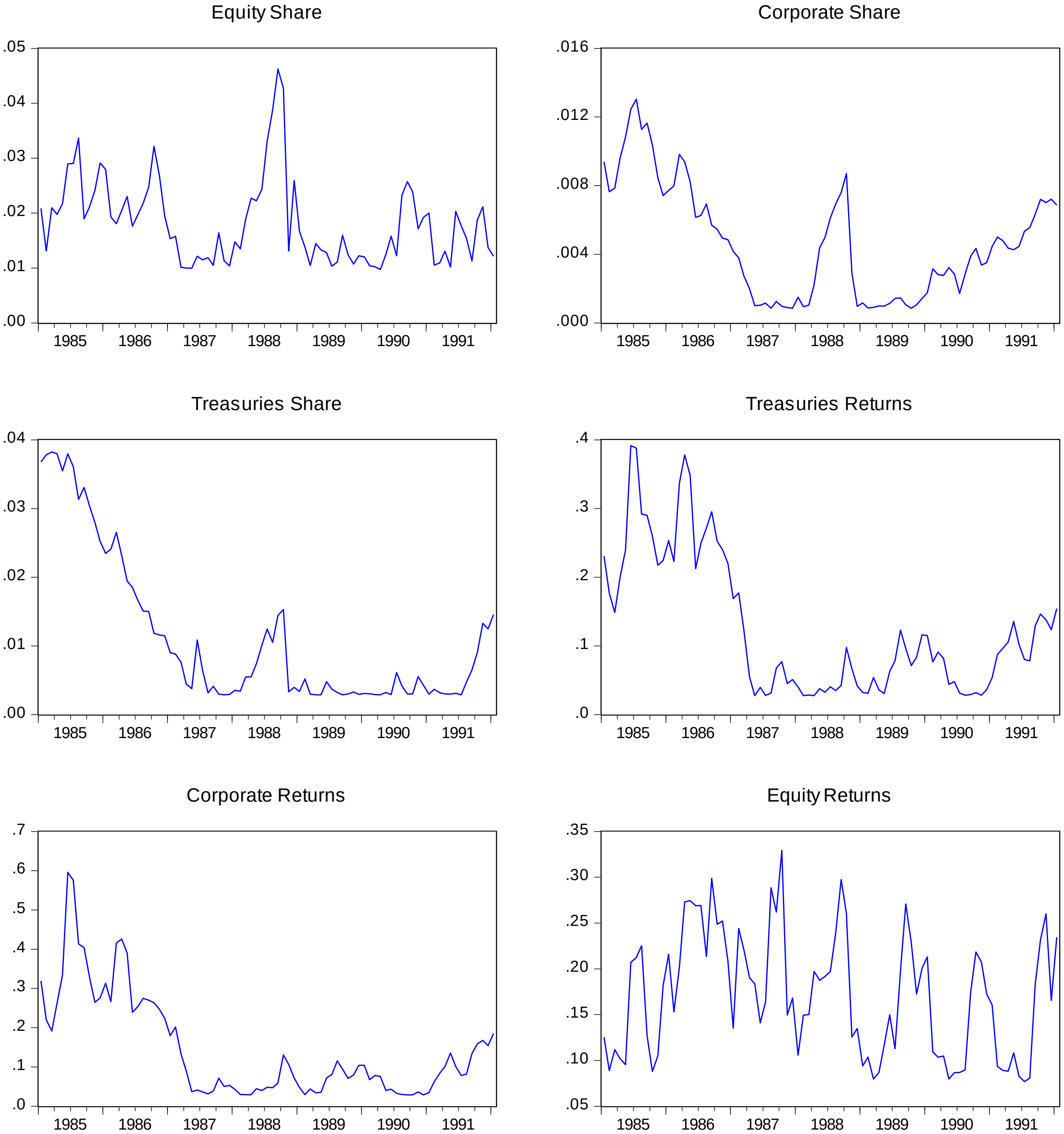


Fig. OA5.3: M-GARCH Estimates of Volatility (Regime 2)

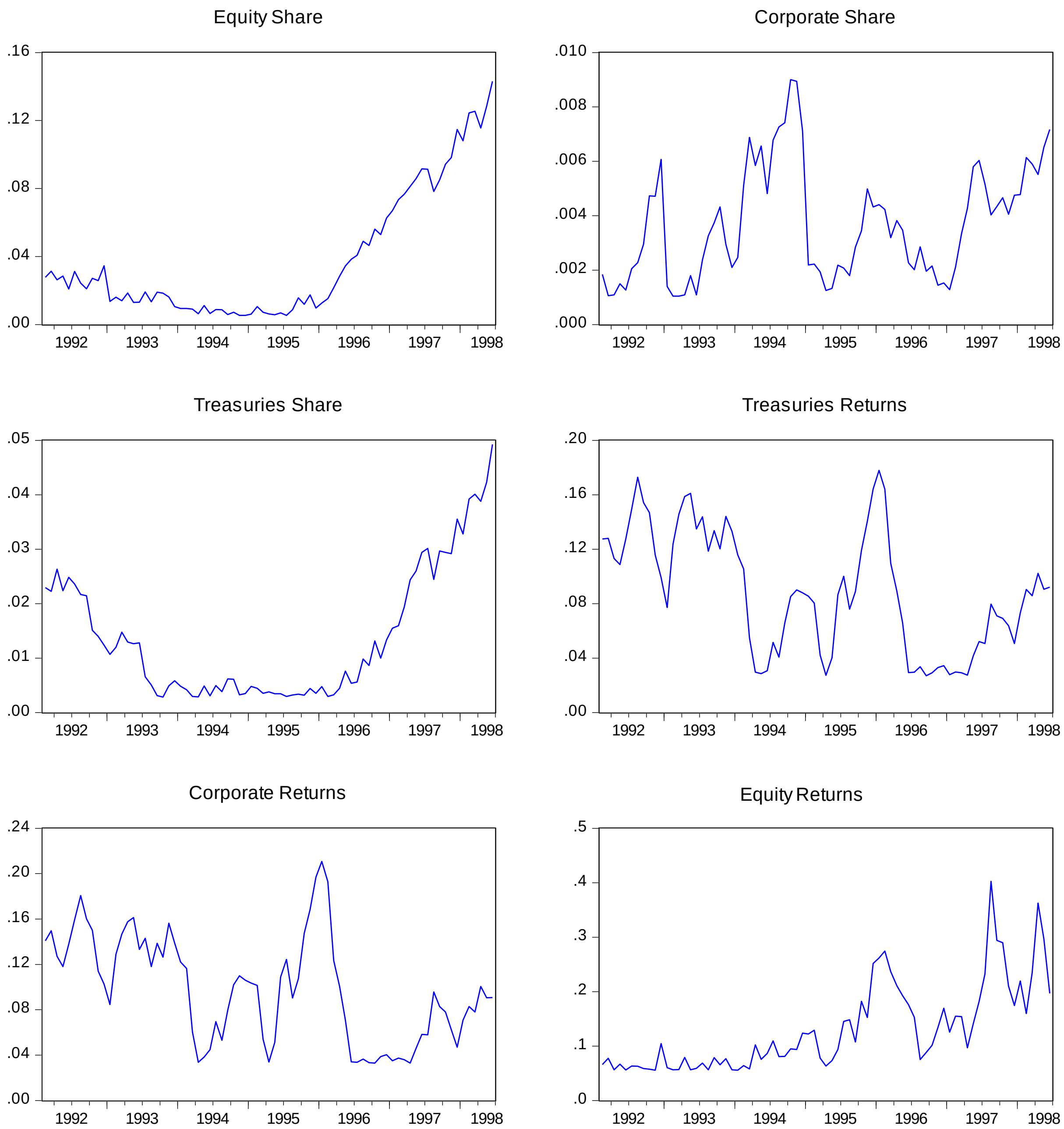


Fig. OA5.4: M-GARCH Estimates of Volatility (Regime 3)

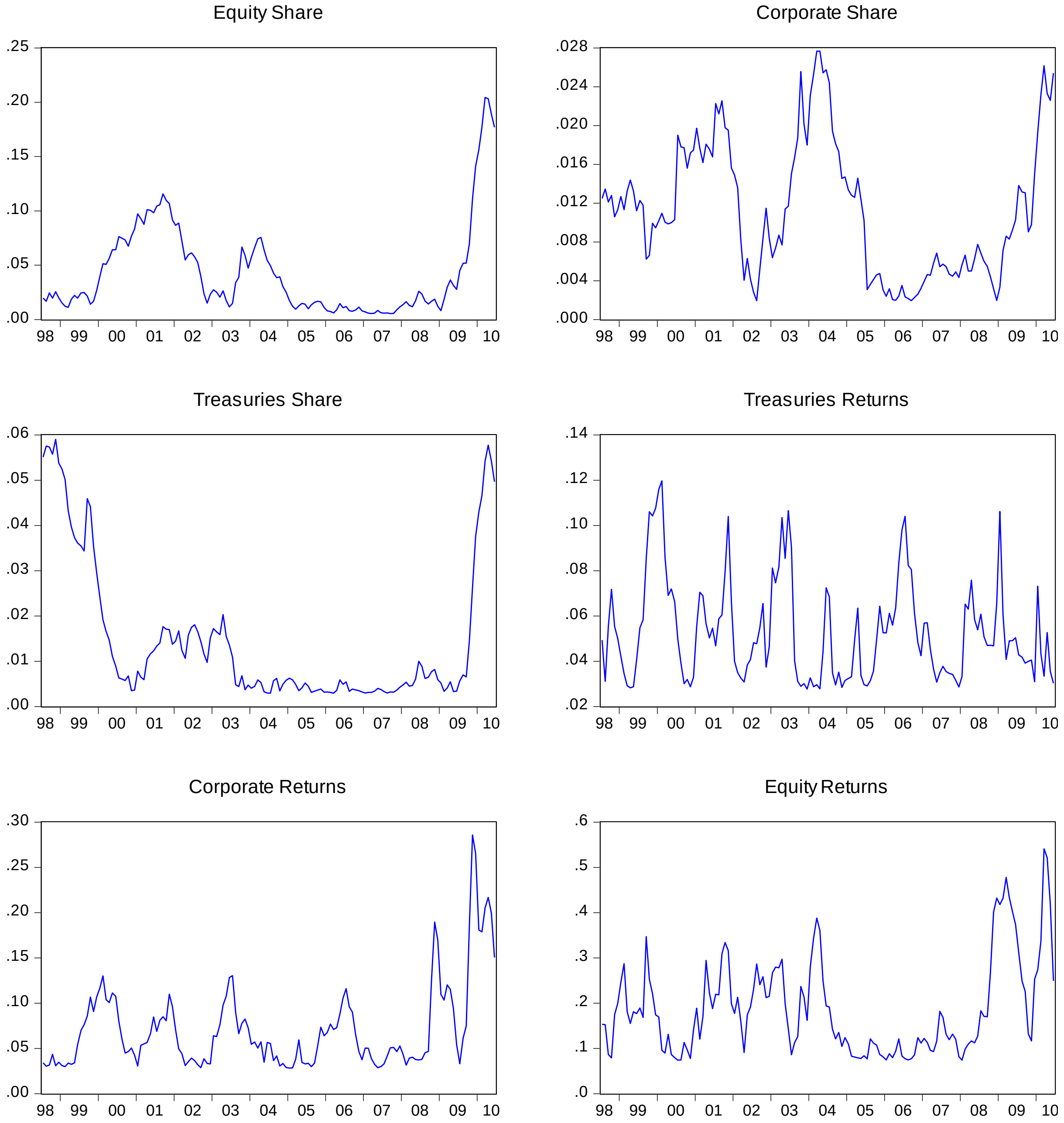


Fig. OA5.5: M-GARCH Estimates of Volatility (Regime 4)

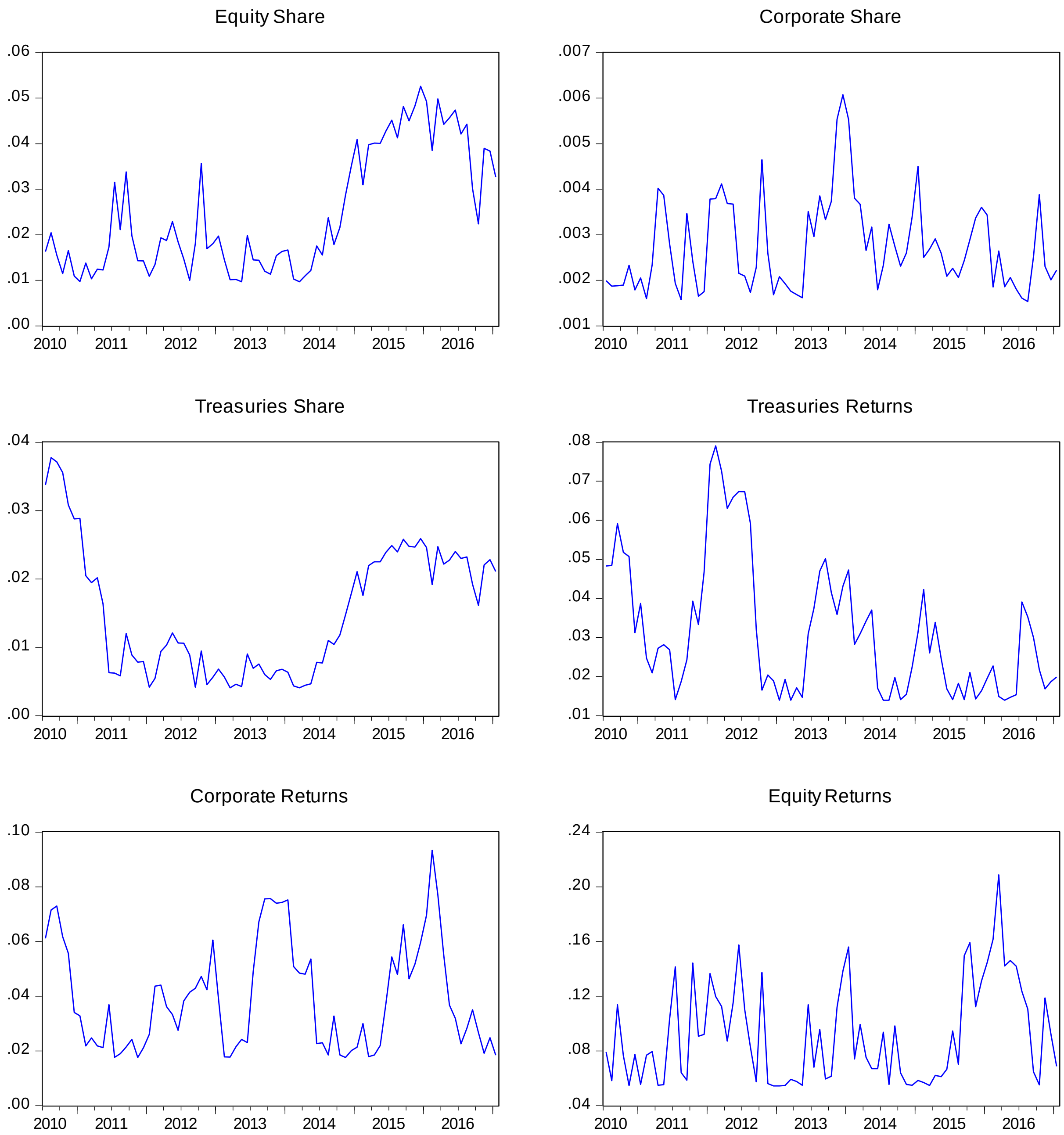


Fig. OA5.6: Full Sample

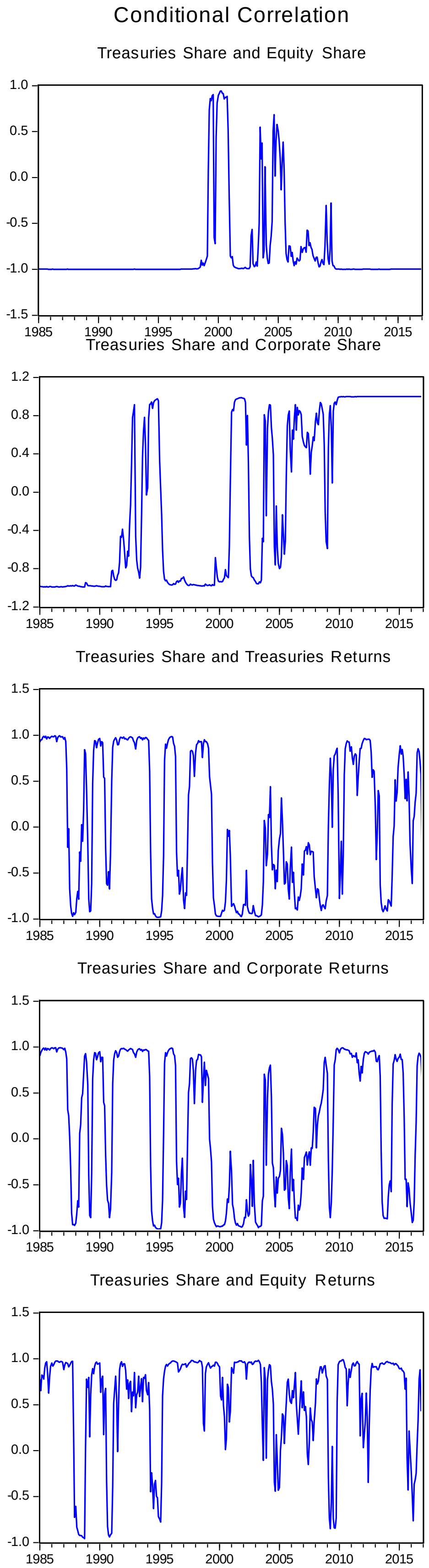


Fig. OA5.7: Regime 1

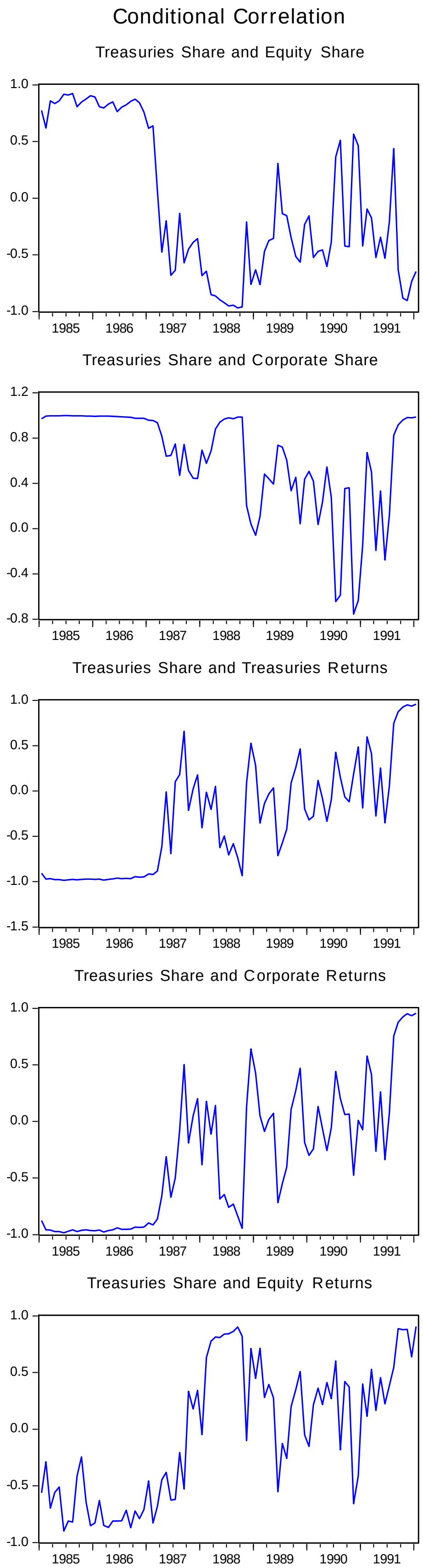


Fig. OA5.8: Regime 2

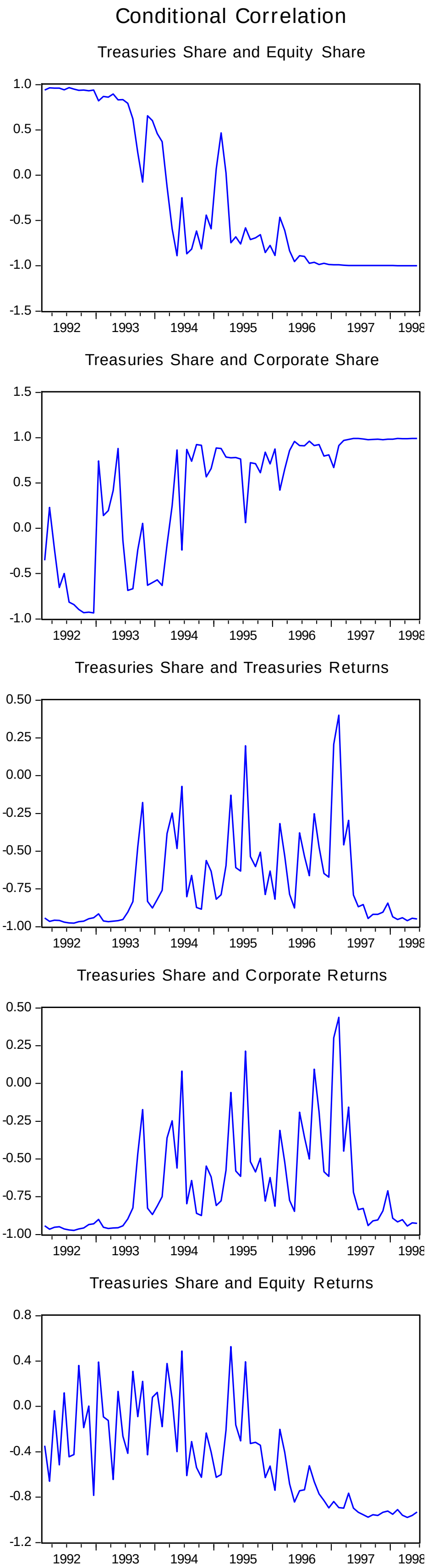


Fig. OA5.9: Regime 3

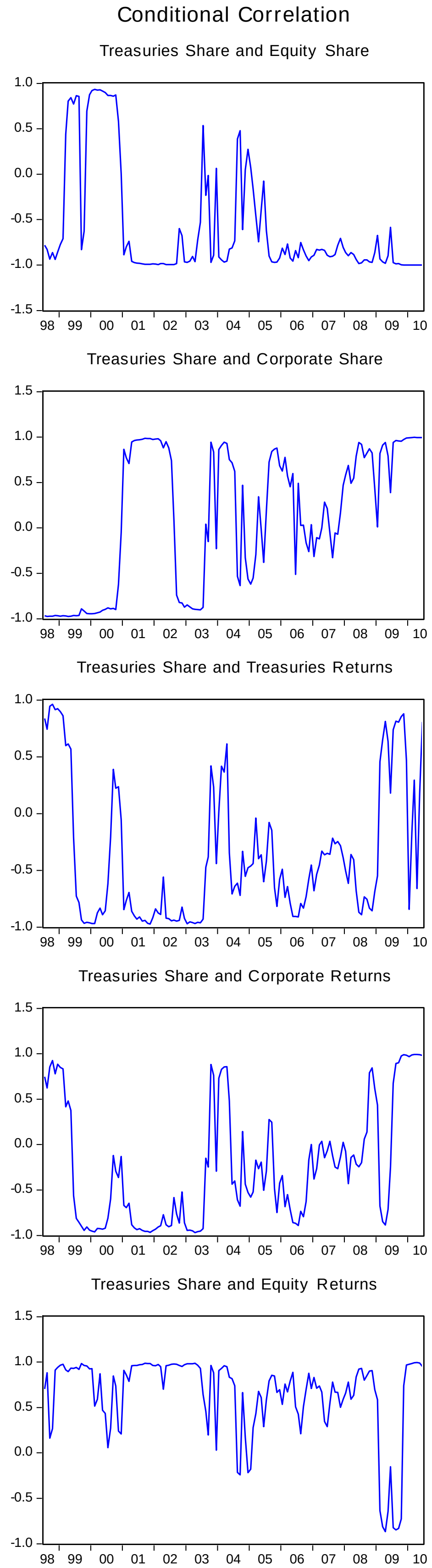
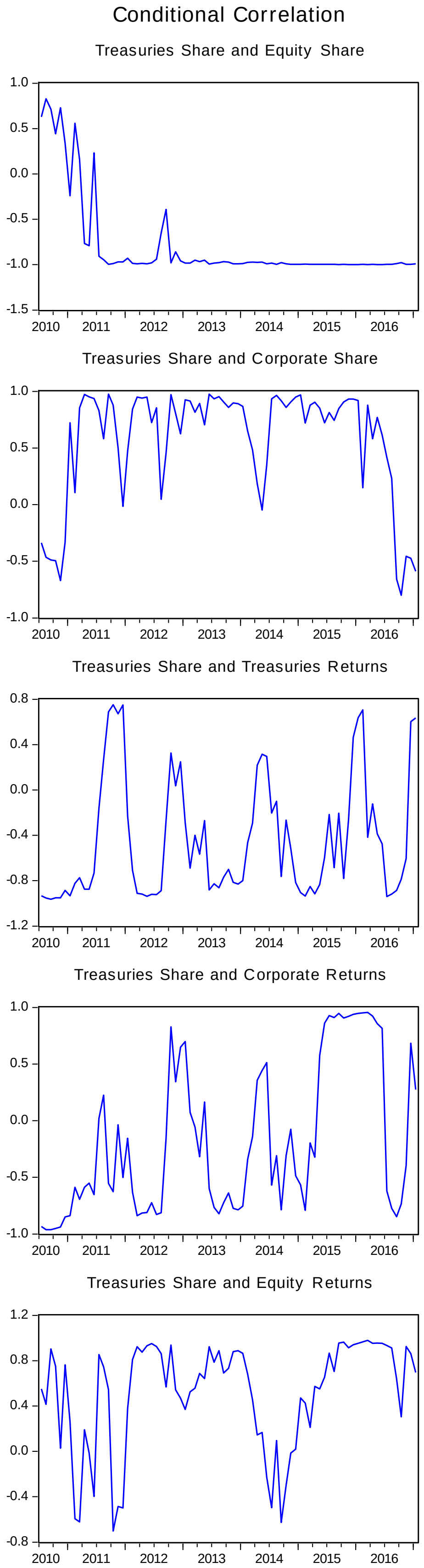


Fig. OA5.10: Regime 4



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