The Rise of Services and Balanced Growth in Theory and Data*

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August, 2019

Abstract

We investigate the effect of structural transformation on the process of economic growth. Using a two-sector growth model we show that, in addition to Baumol's cost disease, structural transformation from goods to services generates other predictions that are in line with cross-country growth facts: an increase in the real investment rate, a decline in the real interest rate and the marginal product of capital and an acceleration of investment-specific technological change as the share of services increases. The model calibrated to U.S. data can account for the elasticity of real investment rates to the share of services measured in cross-country data.
JEL Classification: E22; E24; E31; O41.
Keywords: Structural transformation, NIPA measurement, two-sector model.

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1 Introduction

Since the seminal contribution of Baumol (1967), several papers have discussed the effect of structural transformation (ST hereafter) on aggregate productivity, often labeling this phenomenon as Baumol's cost disease. As ST occurs, the economy experiences a transition from a high TFP growth sector (goods) to one with low TFP growth (services). This transition implies that aggregate productivity slows down, and so does GDP growth. Baumol's cost disease has been extensively discussed and measured both for the U.S. and in a cross-country dimension.[1] However, while the effect on aggregate productivity is well understood, the implications of ST on other aspects of the growth process have received little attention. For instance, a robust observation of the growth process is an increasing real investment rate as income grows, which is typically attributed to a declining price of investment (relative to consumption). ST contributes to this process by moving resources from goods to services. As long as investment is more intensive in goods than in services relative to consumption, the relative price of investment declines. In this light, ST endogenously generates investment-specific technological change, and also the pace at which it grows over time. As a result, the evolution of the real investment rate is affected by the pattern of ST.

Here we investigate, theoretically and quantitatively, the effect of ST on elements of the growth process that have received less attention in previous work, namely the real interest rate (RIR), the marginal product of capital (MPK), the nominal and real capital/output ratios, and the nominal and real investment/output ratios. In the first part of the paper, we assess the the role of ST on growth in the U.S. by using an off the shelf two-sector growth model of ST, first proposed in Boppart (2014). The model displays balanced growth when aggregate output is measured in units of an appropriately chosen numeraire (i.e. the capital good). However, we show that, when aggregate output is measured using standard NIPA methodology to construct GDP from the model's equilibrium, growth becomes "unbalanced" and it is possible to measure the effect of ST on the variables shaping the growth process. Since the post-war period, the U.S. economy has experienced an increase in the share of services in consumption, a decline in the relative price of goods to services, an increase in the real investment to output ratio, and an increase in the ratio of real capital services to GDP.[2] These facts suggest that, while ST occurs, the U.S. experiences "unbalanced" growth. We thus

[1]See Echevarria (1997), Nordhaus (2008), Moro (2015) and Duernecker, Herrendorf, and Valentinyi (2017) for instance.

[2]Most of the literature focuses on the constancy of the real capital/GDP ratio as measured by NIPA. However, when using BLS estimates of the capital stock (i.e. capital *services*) as in Fernald (2012) and Gourio and Klier (2015), the real capital/GDP ratio displays a positive trend. The capital services measured by the BLS are a more appropriate measure of an input in a production function, while the NIPA estimate is more appropriate as a measure of real wealth in the economy.

calibrate the model to replicate certain features of the U.S. economy in the past 65 years: the *average* rate of growth of GDP, the observed change in the share of services in consumption, the increase in the real investment/GDP ratio, and the relative price goods/services. The calibrated model replicates the data targets well and predicts the following patterns over the period: i) a fall in the marginal product of capital (and increase in the real capital to output ratio) of 34% in units of GDP and of 41% in units of aggregate consumption; ii) a decline in the real interest rates of 0.35 percentage points (from 7.39% to 7.04%) in terms of GDP units and 0.45 percentage points (from 7.26% to 6.81%) in terms of consumption units; and iii) a decline of the per capita GDP growth rate of 0.35 percentage points (from 2.31% to 1.96%) from the beginning to the end of the sample period. Our quantitative results suggest that ST has a non-negligible effect on the growth process in the U.S., a country that is typically considered to follow a well defined balanced growth path.

In the second part of the paper we turn to assessing the role of ST using cross-country data. Specifically, we focus on the well known observation that real investment rates increase with economic development (Barro (1991), Hsieh and Klenow (2007)). To do this, we compare the performance of the ST model with that of a standard investment-specific technical change (ISTC) theory in which the relative price of investment declines exogenously over time at a constant pace.

We first show that the ST model displays a set of additional predictions with respect to the ISTC model which are qualitatively consistent with cross-country data: 1) the rise of the services share in GDP as income grows (Herrendorf, Rogerson, and Valentinyi (2014)); 2) the decline in the growth rate of GDP as the share of services grows (Echevarria (1997), Moro (2015)); 3) a declining real interest rate as income grows (Barro and Sala-i-Martín (2004, p. 13)); and, importantly, 4) an acceleration of ISTC as income grows (Samaniego and Sun (2016)). The ST model produces an acceleration of ISTC because the share of services in consumption increases over time, thus making the relative price of investment decline faster at higher income levels. Thus, in the same vein as for the growth rate of GDP, the marginal product of capital, and the real interest rate, the key variable in determining the pace of ISTC in the model is the share of services. Motivated by this prediction of the model and the empirical observation of an acceleration of ISTC as income grows, we investigate the relationship between the pace of ISTC and the share of services, finding a positive and significant correlation between the two variables in cross-country data.

Second, we use the model to assess to what extent the process of ST can account for the elasticity of real investment rates to the share of services. By using data from the International Comparisons Program (PWT) we compute this elasticity for the benchmark years 1980, 1985, 1996, 2005 and 2011, finding an average value across years of 0.61. We tie our hands by using the same parametrization arising from the U.S. calibration to calculate the elasticity of the real investment rate with respect to the share of services that arises along the growth path of the model, and compare it with that estimated in the data. The model provides an elasticity of 0.59, virtually the same as in the data. When we use the ISTC model calibrated to the U.S. to compute the elasticity of real investment rates to the income level, we find a substantial difference: 0.35 in the model versus 0.19 in the data. Thus, the ST model performs substantially better than the ISTC in predicting real investment rates. The main reason behind this finding is the acceleration of ISTC as the share of services grows. A model of exogenous ISTC cannot capture this acceleration by construction. Instead, the model of ST endogenously generates it due to the changing composition of final consumption expenditure.

The mechanics of the effect of ST on the growth process can be explained as follows. In the model, sectoral TFP grows at a constant (but different) rate in the two sectors, which implies a constant decline in the relative price of goods/services, as observed in the U.S. in the post-war period. This minimal assumption, when paired with non-homothetic preferences, leads to a change in the composition of consumption (i.e. structural change) given by the rise of the services sector.[3] As ST occurs, growth in the model is balanced in the sense that the growth rate of aggregate output, the real and the nominal capital/output ratios, the real and the nominal investment rates and the real interest rate in units of output are all constant over time. This is what happens in the models in Ngai and Pissarides (2007) and Boppart (2014).[4] The typical definition of balanced growth in these *models*, however, relies on expressing all variables, including aggregate output, in terms of a numeraire. This is usually the price of capital. We show here that the constancy of the growth rate of aggregate output, and of the real investment/output and capital/output ratios strictly depend on the units variables are expressed in.[5] This is relevant when bringing the *model* to the *data*, because GDP in the data differs from nominal aggregate output divided by the price of one good. Instead, real GDP in the data is constructed using a chain-weighted Fisher index. Roughly speaking, the

[3]Buera and Kaboski (2012a) and Buera and Kaboski (2012b) investigate the theoretical mechanism that lead the services sector to grow along the growth path, linking this process to the rise of the skill premium, economies of scale, and home production.

[4]The model in Kongsamut, Rebelo, and Xie (2001) also displays balanced growth. However, in contrast with Ngai and Pissarides (2007) and Boppart (2014), it does not model heterogeneous productivity growth across sectors, which is key here to generate the effects of structural transformation on the growth process.

[5]We note here that Boppart (2014) already points out that in an economy with structural change, when GDP is deflated by the true cost of living price index, the growth rate is not constant and a Baumol's cost disease can emerge. Boppart (2014) also computes the quantitative importance of this effect if the postwar trends are extrapolated into the future. Also, Ngai and Pissarides (2007) point out that the real rate of interest (in consumption units) declines along the balanced growth path of their model. Here we use the model to quantify these effects for the U.S. economy during the period 1950-2015 together with the other variables of interest.

Fisher index weights the growth rate of individual components of GDP by their shares in GDP. This implies that, even if variables grow at a constant rate, if these rates are different and there is ST, the growth rate of GDP is non-constant over time. This non-constancy of the growth rate is then associated with trends in the marginal product of capital, the real interest rate, the real capital-output ratio, the real investment-output ratio and the pace of ISTC.

Our work is related to several streams of the literature. Here we discuss those most closely related, in addition to the ones mentioned above. First, this paper belongs to a broad ongoing research project pointing out that the measurement of the multi-sector model with NIPA methodology is key for the model to generate aggregate dynamics that are comparable with the data. Within this line of research, Duernecker, Herrendorf, and Valentinyi (2017) study the effect of ST on the slowdown of aggregate productivity in the U.S. and make predictions on the future path of this variable. They consider a sequence of static economies in a three sector model and focus mainly on the different evolution of TFP within services sectors.[6] Our focus here is to analyze the effect of ST on several variables which, in addition to GDP growth, are important to describe the growth process within and across countries. For this purpose, we study a two sector model and focus on the distinction between balanced growth in theory and unbalanced growth in the data using a model with capital, which allows us to measure the evolution of the marginal product of capital and the real investment rate along the growth path. Duernecker, Herrendorf, and Valentinyi (2018) provide analytical expressions for the differences in GDP growth obtained using NIPA-consistent Fisher index and various numeraires in different versions of the multisector growth model. Their focus is on the time-series evolution of GDP growth in the U.S. economy. Here, instead, we mainly focus on the effect of ST for the evolution of great ratios and also study its consequences in a cross-country perspective, with special focus on the endogenous acceleration of ISTC generated by ST.

This paper also relates ST and growth in a cross-country perspective. While ST appears as a robust regularity across countries, few papers study its role in shaping the various aspects of the growth process in a cross-country dimension, with two exceptions being Echevarria (1997) and Moro (2015). Echevarria (1997) is the first paper explicitly investigating the effect of ST on growth in a multi-sector growth model, finding a hump-shaped pattern of growth rates depending on the stage of ST. Moro (2015) studies the effect of ST on growth using Fisher chain-weighted index to measure GDP but abstracts from capital accumulation, and focuses on the effect of structural transformation on GDP growth and volatility. Modeling capital accumulation is key here, as it allows to measure the effect of ST on the real interest

[6] IMF (2018) also emphasizes the heterogeneity of the service sector in productivity trends.

rate, the marginal product of capital, the real investment-output and capital-output ratios, and ISTC, something not explored in either Echevarria (1997) or Moro (2015), who focus mainly on GDP growth. By doing this, we relate ST to the well documented cross-country increase in real investment rates as income grows discussed, for instance, in Barro (1991), Hsieh and Klenow (2007) and, more recently, in García-Santana, Pijoan-Mas, and Villacorta (2016), and find that a model of ST without distortions can account well for this pattern.[7]

The remainder of the paper is organized as follows. Section 2 presents some data facts for the U.S. economy; in section 3 we present the model and in section 4 we show how to measure the model's equilibrium outcomes in a data-consistent way. In section 5 we calibrate the model to U.S. data and use it as a measurement tool to assess the implications for the marginal product of capital, the real interest rate, and the growth rate of GDP. In section 6 we discuss the international evidence and use the model to assess how much ST can explain of cross-country differences in investment rates and we compare the predictions of our model with those of a model with investment-specific technical change. In section 7 we conclude.

2 Stylized facts for the U.S.

We present a set of facts that describe the U.S. growth process and that we use to calibrate the model in section 5. We pay special attention to the measurement of variables in a way that is consistent with the two-sector model presented below. The key variables are the relative price of goods over services, the investment to GDP ratio measured in real terms, the capital-GDP ratio measured in real terms, and the nominal share of services consumption in total personal consumption expenditure. In the two-sector model below we assume that the goods sector produces an output that can be used both for investment and for consumption. Thus, in the data we construct a *price of goods* which is a Fisher chain-weighted price index of consumption goods (including durable and non-durable consumption) and gross (private and government) domestic investment (GDI).[8]

The relative price goods/services is obtained from NIPA tables as the *price of goods* (constructed as described above) relative to the price of services.[9] The real GDI to GDP ratio is calculated as the ratio of real investment to real GDP. We deflate nominal GDI[10] using

[7]García-Santana, Pijoan-Mas, and Villacorta (2016) also find that nominal investment rates display a hump shaped pattern with development. Our focus here is mainly on real investment rates, which increase with economic development.

[8]In Appendix A we present a three-sector model and data for the relative prices goods/services and investment/services. As we show there, the main message in the data and in the model is confirmed.

[9]NIPA Table 1.1.4 for private expenditures and Table 3.9.4 for public at http://www.bea.gov/iTable/index_nipa.cfm

[10]NIPA Table 1.1.5. for private investment and Table 3.1 for government investment.

the same *price of goods* used to construct the goods/services price ratio. Note that when using the investment deflator from NIPA tables to deflate investment, the trend observed in the investment/GDP ratio is similar and statistically significant, but less pronounced.[11] This is discussed further below because replicating a measure of the investment-output ratio deflated by the investment price requires a three-sector model.[12] Finally, real GDP is given by nominal GDP deflated by the GDP deflator.

Additionally, we present evidence on the evolution of the real capital-GDP ratio. We are interested in the ratio between the real capital stock and real GDP, i.e. where each nominal measure is deflated by its own price. Note that this differs from the ratio of the two nominal measures as long as the relative price deflators for capital and GDP are different.

The measurement of capital is more controversial than that of investment. For this reason the estimates should be taken with more caution. We use the measure coming from the BLS Multi Factor Productivity (MFP) project which calculates total capital services for the private business sector.[13] The measure uses a Jorgensonian perpetual-inventory method aggregating different types of capital according to their real user costs. An extensive explanation of how capital services are calculated is available in the online Appendix section A. As pointed out in Gourio and Klier (2015), BLS estimates are a more appropriate measure of factor inputs than BEA fixed assets accounts, as they use weights based on real user costs rather than asset prices to aggregate different types of capital assets. For comparison, we also show the measure of Fernald (2012), which accounts for the total business sector and is adjusted for capital utilization. In practice, the trends displayed by these two measures are very similar, as they mostly differ only in terms of business cycle volatility.

Finally, the share of services in total consumption expenditure is calculated as the nominal share of personal consumption expenditure on services over total personal consumption expenditures (i.e. on services and goods). The data also come from NIPA (Table 1.1.5).

Figure 1 presents the data in logs (except for the consumption share of services) and a fitted trend line. The figure also contains the investment (GDI) to output (GDP) ratio in nominal terms from NIPA accounts for comparison. The price of consumption goods relative to services displays a very well defined negative trend implying a yearly growth of -1.49%. This is accompanied by an increase in the share of services in total private consumption

[11] See Appendix A.

[12] The price of total goods including investment and consumption relative to services displays a very similar trend to that of the price of consumption goods relative to services. The former falls at a rate of 1.49% per year between 1950 and 2015, and the latter at a rate of 1.61% per year.

[13] We used the historical multifactor productivity dataset from Bureau of Labor Statistics (2016) available at `https://www.bls.gov/mfp/special_requests/mfptablehis.xlsx` March 2016 release for the private sector. To calculate the real capital-output ratios we used the real value added measure provided in the same database and transformed the resulting series into an index number with base year 1950.

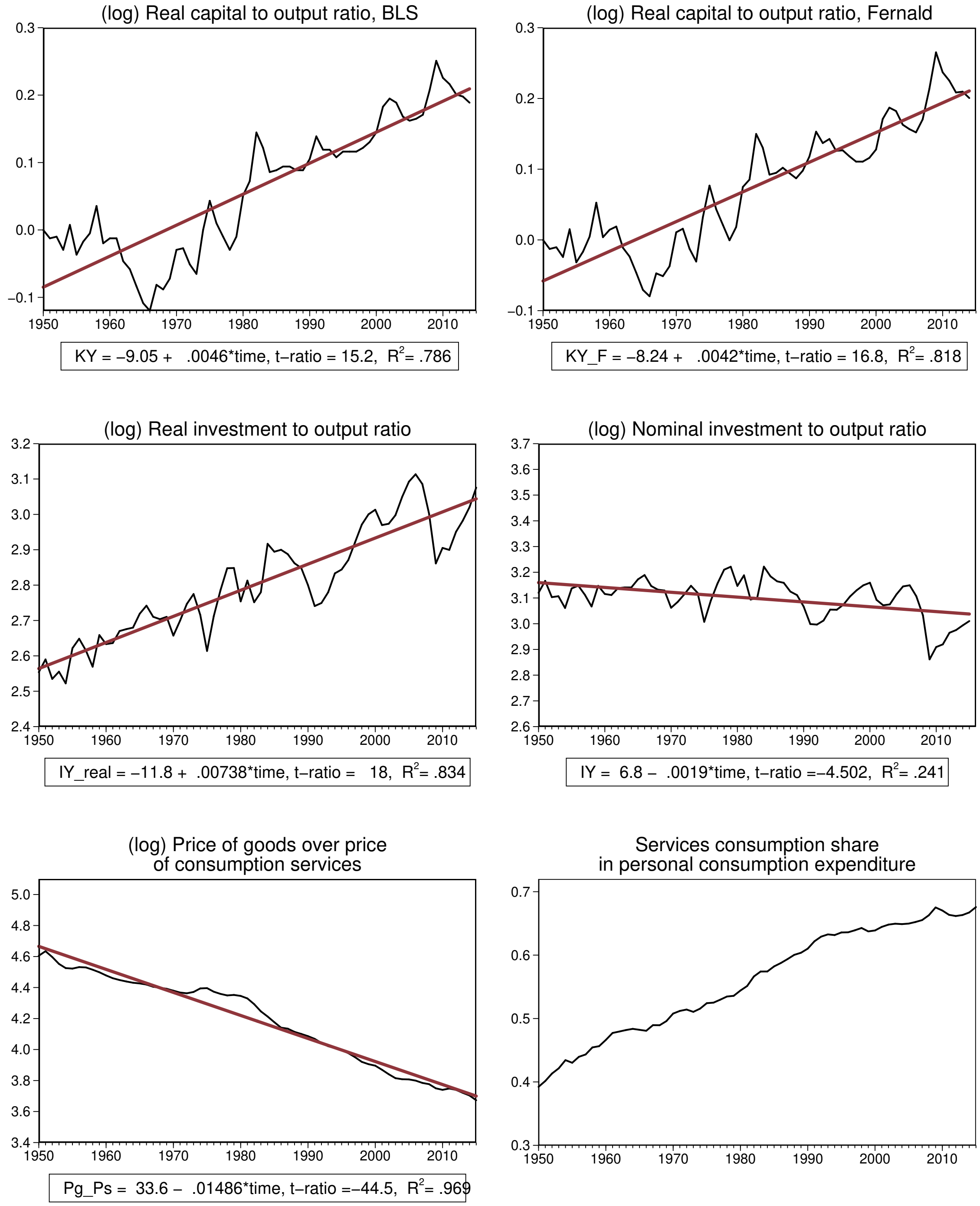

Figure 1: BLS real capital-output ratio, Fernald real capital-output ratio, real investment-output ratio, nominal investment-output ratio, price of goods relative to services consumption, and share of services consumption in total consumption expenditure. All variables in logs and with a fitted linear trend except for the services consumption share.

expenditure from 39% in 1950 to 68.5% in 2015, which appears to be leveling off slightly during the last 15-20 years. The increase in the share of services in consumption and GDP is a well known fact in literature on the process of structural transformation (see Herrendorf, Rogerson, and Valentinyi (2014)). The data in figure 1 suggest that this process has been accompanied by a steady increase in the real measures of the investment to GDP ratio and the capital to GDP ratio. The former increases at a rate of 0.74% per year and the latter at a rate of 0.46% per year (0.42% if using the measure by Fernald (2012) adjusted for capacity utilization). In contrast, the nominal investment-output ratio displays a mild (and significant) negative trend during the sample period. However, the negative slope is entirely driven by the post 2008 recession years. When these are dropped, the trend becomes insignificant.[14]

While none of the facts presented in this section is new, the relationship between the growth facts discussed here and structural transformation has received little attention in the literature. We aim at investigating this relationship by using a model of structural transformation that displays a theoretical balanced growth path. When appropriately compared to the data, this model can account *jointly* for all the facts presented in this section.[15]

3 Model

This section presents a two-sector model of structural change with balanced growth. The model is a simplified version of Boppart (2014), where we abstract from household heterogeneity and focus on features related to structural transformation between goods producing and services producing sectors, since we then use the model as a measurement tool.

3.1 Households

Time t is discrete. There are two consumption goods (*goods* and *services*) and one investment good (that we we simply label *investment*). The representative household in this economy has preferences given by

$$U = \sum_{t=0}^{\infty} \beta^t V(p_{st}, p_{gt}, E_t), \tag{1}$$

[14]Specifically, it takes a value of -0.0005 with a t-value of -1.28. This is not the case for the real investment-output ratio. In that case, the trend up to year 2007 is 0.082 and highly significant. Note also that the capital services data are for the private business sector only, as BLS does not produce historical statistics for capital services for the overall economy. However, our investment data include the whole economy.

[15]It is well known that models of investment-specific technical change (ISTC) can also generate different trends for real and nominal investment rates. In section 6 below we discuss the differences between ISTC models relative to models of structural transformation in terms of their consequences for the growth process.

where β is the subjective discount factor, $V(p_{st}, p_{gt,} E_t)$ is an instantaneous indirect utility function of the household, p_{st} is the price of services, p_{gt} the price of goods, and E_t is total nominal consumption expenditure. The explicit functional form for V is

$$V(p_s, p_{g,} E) = \frac{1}{\epsilon}\left[\frac{E}{p_s}\right]^{\epsilon} - \frac{\nu}{\xi}\left(\frac{p_s}{p_g}\right)^{-\xi} - \frac{1}{\epsilon} + \frac{\nu}{\xi}, \tag{2}$$

where $0 \leq \epsilon \leq \xi \leq 1$ and $\nu > 0$. These non-homothetic and non-Gorman type of preferences are the key to obtaining balanced growth in the original model by Boppart (2014). Within the indirect utility function $1 - \epsilon$ governs the exponential evolution of expenditure shares, both ϵ and ξ govern the elasticity of substitution between the two goods, and ν is a shift parameter.

The household owns the capital stock of the economy and rents it out to firms in the market. It also inelastically supplies a unit of labor to firms each period in exchange for a wage. The budget constraint in nominal terms (i.e. dollars) is

$$E_t + p_{gt}K_{t+1} = w_t + p_{gt}K_t(1 + r_t - \delta), \tag{3}$$

where w_t is the wage rate, K_t is the amount of capital owned by the household, r_t is the rental rate of capital denominated in units of goods and before depreciation, and δ is the depreciation rate. Thus, the problem of the household is to maximize (1) subject to (2) and (3).

The indirect utility function $V(p_{st}, p_{gt,} E_t)$ encompasses the static problem in which the household decides, given the level of consumption expenditure E_t, how much to spend in goods and services such that instantaneous utility is maximized and

$$E_t = p_{st}C_{st} + p_{gt}C_{gt},$$

holds, where C_{st} and C_{gt} are the optimal consumption levels of services and goods.

The Euler equation of the household is

$$\frac{E_{t+1}}{E_t} = \{\beta[1 + r_{t+1} - \delta]\}^{\frac{1}{1-\epsilon}} \left(\frac{p_{st}}{p_{st+1}}\right)^{\frac{\epsilon}{1-\epsilon}}. \tag{4}$$

In the online Appendix section B we show that, from the Euler equation, it is possible to derive the *intertemporal elasticity of substitution of expenditure* which is equal to $1/(1-\epsilon)$.

3.2 Firms and Market Clearing

There are two representative firms in the economy operating under perfect competition. The first firm produces goods with technology

$$y_{gt} = k_{gt}^{\alpha}(n_{gt}A_{gt})^{1-\alpha}, \tag{5}$$

where k_{gt}, n_{gt} and $A_{gt}^{1-\alpha}$ are capital, labor, and total factor productivity (TFP) of the goods producing firm. This output can be used to build the capital stock or as consumption of goods.[16] The second firm produces services with technology

$$y_{st} = k_{st}^{\alpha}(n_{st}A_{st})^{1-\alpha}, \tag{6}$$

with k_{st}, n_{st} and $A_{st}^{1-\alpha}$ being capital, labor, and TFP of the service producing firm. The output of this firm is used as services consumption.

The efficiency terms in the two sectors evolve according to

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s, \tag{7}$$

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g, \tag{8}$$

where γ_s and γ_g are exogenous constant growth rates, and we assume that $\gamma_s < \gamma_g$.

In equilibrium, all markets clear and the following must hold:

$$y_{gt} = C_{gt} + K_{t+1} - (1-\delta)K_t,$$

$$y_{st} = C_{st},$$

$$k_{gt} + k_{st} = K_t,$$

and

$$n_{gt} + n_{st} = 1.$$

3.3 A model displaying both balanced and unbalanced growth

In this subsection we show how growth in the model can be balanced or unbalanced depending on the units it is measured in. To do this, we first consider the properties of the economy at

[16] In Appendix A we consider the case in which consumption and investment goods are produced in different sectors.

time t in two extreme cases, one in which only goods are produced, and the other in which only services are produced. Next, we use the production possibility frontier of the economy to study the case in which aggregate output is a composite of goods and services.

At a point in time t, there are two possible aggregate production functions that can be defined. One mapping total capital and labor in the economy into the maximum amount of goods that can be produced, and another mapping the same inputs into the maximum amount of services that can be produced. Boppart (2014) shows that the aggregate production function in goods units is

$$Y_{gt} = K_t^{\alpha}(A_{gt})^{1-\alpha}. \tag{9}$$

Boppart (2014) also shows that there is a dynamic equilibrium which represents a balanced growth path (BGP). Along this BGP, aggregate capital, wages, consumption expenditure and output in terms of the numeraire (goods) grow at the same rate of A_{gt}, γ_g.[17]

The Cobb-Douglas technologies in the two sectors determine a linear production possibility for this economy. This implies that, at any t, giving up production of a unit of goods allows to produce p_{gt}/p_{st} units of services. It follows that the maximum amount of services that can be produced at t is:

$$Y_{st} = \frac{p_{gt}}{p_{st}} Y_{gt}, \tag{10}$$

where

$$\frac{p_{st}}{p_{gt}} = \frac{A_{gt}^{1-\alpha}}{A_{st}^{1-\alpha}}. \tag{11}$$

To derive the aggregate production function in services units we can either plug (9) and (11) into (10), or simply consider the production function of services when all capital and labor in the economy are employed in the services sector, to obtain:

$$Y_{st} = K_t^{\alpha}(A_{st})^{1-\alpha}. \tag{12}$$

Along the BGP in goods units, K grows at rate γ_g and A_{st} grows at rate γ_s so Y_{st} grows at the constant rate $\gamma_{ys} = \alpha\gamma_g + (1-\alpha)\gamma_s$. Note, however, that if output is measured in services units, the real capital output-ratio is non-constant over time, as the numerator grows at γ_g while the denominator at $\gamma_{ys} < \gamma_g$.

Consider now the aggregate marginal product of capital in the economy (MPK) in the two extreme production cases, which is obtained by deriving (9) and (12) with respect to K_t:

$$MPK_t^g = \alpha K_t^{\alpha-1}(n_t A_{gt})^{1-\alpha} = \alpha \frac{Y_{gt}}{K_t} = \alpha \frac{p_{gt} Y_{gt}}{p_{gt} K_t} = r_t = constant, \tag{13}$$

[17]See online Appendix section C for details.

and

$$MPK_t^s = \alpha K_t^{\alpha-1}(n_t A_{st})^{1-\alpha} = \alpha \frac{Y_{st}}{K_t} = \alpha \frac{p_{st}Y_{st}}{p_{st}K_t} = \alpha \frac{Y_{gt}}{K_t}\frac{p_{gt}}{p_{st}} = \frac{p_{gt}}{p_{st}} MPK_t^g. \tag{14}$$

While MPK_t^g is constant along the BGP, as long as p_{gt}/p_{st} is non-constant over time, the marginal product of capital in services units is also non-constant in this model. A time varying MKP means that an additional unit of capital provides a different amount of output at two points in time, implying that the real capital/output ratio is also time varying.

We now turn to the real interest rate in the two extreme production cases. To derive the real interest rate in the model we introduce a one period bond denominated in nominal terms (i.e. dollars) and then we create derivatives on that bond denominated in goods and services units. The budget constraint of the consumer becomes:

$$E_t + p_{gt}K_{t+1} + B_{t+1} = w_t + p_{gt}K_t(1 + r_t - \delta) + (1 + r_t^b)B_t$$

where in addition to variables appearing in (3), B_{t+1} is the cost in dollars at t of a bond which gives $(1 + r_{t+1}^b)B_{t+1}$ dollars at $t + 1$. Consider now an arbitrageur who creates two derivatives from this bond denominated in dollars. One derivative is sold at a price in goods units and provides a gross return in goods units. The other derivative is sold at a price in services and provides a gross return in service units. By the no-arbitrage condition, the first derivative costs B_{t+1}/p_{gt} goods units today and gives $(1 + r_{t+1}^b)B_{t+1}/p_{gt+1}$ goods units tomorrow. For the same reason, the second derivative costs B_{t+1}/p_{st} services units today and gives $(1 + r_{t+1}^b)B_{t+1}/p_{st+1}$ services units tomorrow. Using a standard asset pricing formula (see Cochrane (2009)) to compute the gross return of the three assets we obtain the return in dollars

$$R_{t+1}^b = \frac{(1 + r_{t+1}^b)B_{t+1}}{B_{t+1}} = (1 + r_{t+1}^b),$$

that in goods units

$$R_{t+1}^g = \frac{(1 + r_{t+1}^b)p_{gt}B_{t+1}}{p_{gt+1}B_{t+1}} = \frac{(1 + r_{t+1}^b)p_{gt}}{p_{gt+1}},$$

and that in services units

$$R_{t+1}^s = \frac{(1 + r_{t+1}^b)p_{st}B_{t+1}}{p_{st+1}B_{t+1}} = \frac{(1 + r_{t+1}^b)p_{st}}{p_{st+1}}.$$

From utility maximization we have that

$$[1 + r_{t+1} - \delta] = [1 + r_{t+1}^b]\frac{p_{gt}}{p_{gt+1}} = R_{t+1}^g = constant_a, \tag{15}$$

where the last equality follows from the fact that real return on capital in units of goods

r_{t+1} is constant in equilibrium. Now consider the return in services units. Note first that $R^s_{t+1} = R^g_{t+1}\left[\left(p_{gt+1}/p_{gt}\right)/\left(p_{st+1}/p_{st}\right)\right]$. The ratio $\left(p_{gt+1}/p_{gt}\right)/\left(p_{st+1}/p_{st}\right)$ is given by

$$\frac{p_{gt+1}/p_{gt}}{p_{st+1}/p_{st}} = \left(\frac{A_{st+1}/A_{st}}{A_{gt+1}/A_{gt}}\right)^{1-\alpha} = \left(\frac{1+\gamma_s}{1+\gamma_g}\right)^{1-\alpha} = constant_b,$$

so the return in services units is also constant and given by

$$R^s_{t+1} = R^g_{t+1}\left(\frac{1+\gamma_s}{1+\gamma_g}\right)^{1-\alpha} = constant_c < R^g_{t+1} = constant_a. \tag{16}$$

Thus, in both cases the real interest rate is constant, but it is larger when the economy produces only goods. Thus, as in the one-sector growth model, the larger the growth rate of aggregate TFP, the larger the real interest rate in the economy.

Note that in units of services the economy displays a *non-constant* MPK_s together with a *constant* R^s_t. To see why this is the case, it is useful to see MPK_s as the net payoff in services units of holding a unit of capital at $t+1$. This is given by

$$MPK^s_{t+1} = \frac{p_{gt+1}}{p_{st+1}} r_{t+1}, \tag{17}$$

which, as described above, is declining at the same rate as p_{gt+1}/p_{st+1}. How does the cost of obtaining a unit of capital evolve over time? To obtain one unit of capital at t, one has to give up p_{gt}/p_{st} units of services. This implies that both the price and the payoff of the investment are declining at the same rate, thus making the real return, which is the ratio of the two, constant over time.

Finally, we conclude this section by discussing the theoretical concept of GDP in a multisector model. Figure 2 shows the evolution of the production possibility frontier over time, under the assumption that $\gamma_g > \gamma_s$.[18] When the economy produces only one product, the definition of aggregate output (i.e. GDP) is straightforward and maximum growth at any time t is attained if the economy produces only goods, while minimum growth is given if the economy produces only services. To measure growth in all other intermediate cases, one has to take a stand on what aggregate output (i.e. GDP) is when the economy produces both goods. This choice is typically avoided in multisector models as GDP is represented as output in units of one of the goods in the economy. To define GDP growth in the multisector case, note that in the two extreme cases in Figure 2 this is given by the Euclidean distance

[18] Note that the production possibility frontier is linear at each point in time because the weight of capital in the production function is the same in both sectors. That is, the model does not feature an effect of structural change on relative prices.

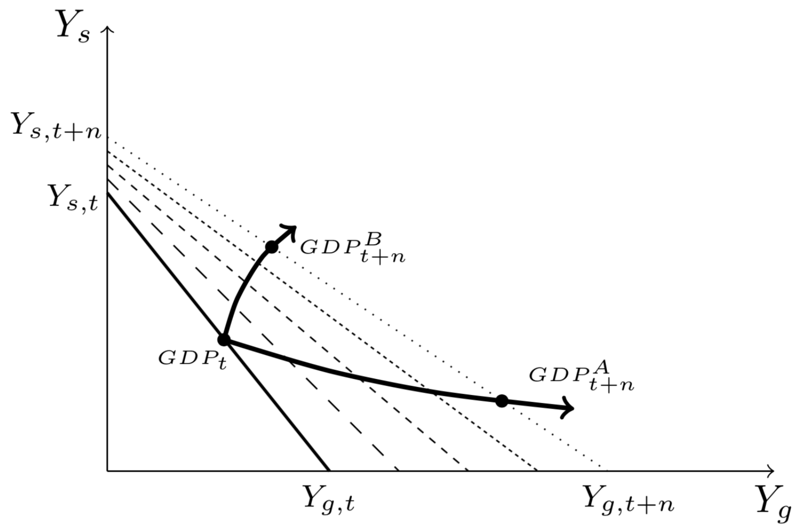

Figure 2: Evolution of the production possibility frontier and two possible GDP trajectories.

between the corresponding two points on the frontiers (for instance between $Y_{s,t}$ and $Y_{s,t+n}$). We can extend this measure to all intermediate cases and define GDP growth between two periods as the Euclidean norm between two points on two different frontiers. Note that this measure depends on both the quantities of the two products and the relative prices (which determine the change in the slope of the frontier) in the two periods considered. Figure 2 reports two possible paths for GDP between t and $t+n$, suggesting that the more the economy is intensive in services, the smaller GDP growth will be.[19]

4 Rates of Return in units of GDP

In this section we describe how we use NIPA methodology to measure real GDP and the GDP deflator from model outcomes and the implications for the MPK and the real interest rate. In Appendix B we report detailed formulas from NIPA that we use to construct real GDP and the GDP deflator. There, we show that GDP as measured by the chain-weighted Fisher index also depends on the quantities of the two products and the relative price of the two products in the two periods, in the same way as the Euclidean norm discussed in the previous section. Here, we focus on the predictions of the model for the marginal product of capital (MPK) and the real interest rate.

[19]Note that the solid lines in the figure are stylized GDP trajectories, and not the Euclidean norm measuring GDP growth, which is linear by definition.

4.1 The MPK

Equations (13) and (14) suggest that the MPK can be expressed in different units by simply dividing the MPK in goods units by a relative price. The question that naturally arises then is which is the appropriate deflator in multi-sector models when confronting them with the data. In one-sector models, this issue does not arise as all goods are produced with the same technology and output, investment, and consumption share the same price, commonly assumed to be the numeraire. In multi-sector models, instead, the common practice is to express aggregate variables such as total output (GDP) and aggregate consumption in terms of the numeraire of the economy, usually the investment good. However, this is in contrast with standard aggregate measures in national accounts, that are used to contrast the model with the data.

In the U.S., the NIPA construct real GDP using a chain-weighted Fisher index of sectoral value added. This is similar to a Divisia index, in which the growth of the various components of GDP is weighted by their shares in nominal GDP. As the shares change over time, the weights of the various components also change. Thus, if GDP is constructed in the model as it is in the data, even if all its individual components (consumption of goods and services and investment in the context of our model) grow at constant but different rates over time, structural transformation implies a non-constant growth of GDP over time.[20] Equally, to construct measures of the economy-level MPK one needs to decide in terms of which units this is expressed. In fact, the aggregate MPK is given by the ratio between the new aggregate output produced by some additional capital, and the amount of that additional capital. As the measure of aggregate output in NIPA accounts is GDP, we measure the MPK in the data as the additional amount of GDP that is generated by an additional unit of capital.[21] This requires to construct GDP from the model's equilibrium path as it is constructed in the data. Hence, we take the following steps:

1. We find the solution of the model;

2. We use the solution of the model to construct real GDP through a Fisher index :

$$GDPreal_t = \sqrt{Q_t^L Q_t^P},$$

where Q_t^L and Q_t^P are the Laspeyres and Paasche chain-weighted indices.[22]

[20] Moro (2015) for instance, shows that in a model calibrated to the U.S., structural transformation from goods to services implies a decline in the growth rate of GDP as measured with a Fisher index.

[21] Or, equivalently, the MPK in terms of aggregate consumption is the extra units of aggregate consumption obtained from an extra unit of capital in production.

[22] See Appendix B for the formulas defining these indices.

3. By using this measure of real GDP and GDP in terms of the numeraire in the model (Y_{gt}) we construct a measure of the GDP deflator ($P_{GDP,t}$):

$$P_{GDP,t} = \frac{Y_{gt}}{GDPreal_t}.$$

4. Since the marginal revenue product of capital is equalized across sectors, we can write $p_g MPK_g = P_{GDP,t} MPK_{GDP,t}$, where MPK_g is the physical marginal product of capital in the goods sector, and $MPK_{GDP,t}$ the marginal product of capital in GDP units. We thus find $MPK_{GDP,t}$ as

$$MPK_{GDP,t} = \frac{p_g MPK_g}{P_{GDP,t}}. \tag{18}$$

   This equation shows that the relevant variable affecting $MPK_{GDP,t}$ over time is the relative price $P_{GDP,t}/p_g$.

5. We repeat steps 2, 3 and 4 by substituting GDP with aggregate consumption to obtain a measure of the MPK in consumption units.

4.2 The real interest rate

The discussion in section 3.3 suggests that in an economy experiencing structural transformation the real interest rate is bounded from above by $R_t^g - 1$ and converges asymptotically to the lower bound $R_t^s - 1$ as the economy transfers resources from the goods to the services sector. Thus, in a context of structural transformation, the real interest rate in GDP or consumption units is non-constant. To show this, we derive the real interest rate as the return of an investment opportunity of an investor holding a unit of GDP at t. This investment opportunity then measures the units of GDP that the investor can buy at $t+1$ if she gives up a unit of GDP at t and invest it in capital. A similar reasoning holds for aggregate consumption.

At time t, the investor uses an amount of GDP, say $\bar{y} = 1$ units, whose price is $P_{GDP,t}$, to purchase some capital such that $P_{GDP,t}\bar{y} = p_{gt} K_t$ holds. As the price of capital is the numeraire in each period, then the gross return in GDP units is given by

$$R_{t+1} = \frac{1 + \bar{r}_{t+1}}{1 + \pi_{t+1}^y}, \tag{19}$$

where $\bar{r}_{t+1} = r_{t+1} - \delta$ and π_t^y is the inflation rate of the GDP deflator, while the net return, i.e. the real interest rate $\tilde{r}_t$, is given by

$$\tilde{r}_{t+1} = \frac{1+\bar{r}_{t+1}}{1+\pi^y_{t+1}} - 1, \tag{20}$$

which is the gross return in GDP units minus the initial unit of GDP invested. The last equations show that the relevant variable affecting the real interest rate over time is the relative price change over time in the inflation rate of the GDP deflator.[23] The real interest rate reflects the fact that a unit of GDP tomorrow costs $P_{GDP,t+1}$ while a unit of GDP today costs $P_{GDP,t}$, so the real return has to be adjusted for the change in the relative price $1+\pi^y_t = P_{GDP,t+1}/P_{GDP,t}$.[24] This change in the price of GDP, however, is not constant when we measure the model's outcome as in the data. This is because structural change modifies the weight of different consumption components. Since the share of services in consumption increases along the growth path, and since the price of services grows faster than the price of goods, π^y_t also increases along the balanced growth path, and hence the real interest rate falls. The real interest rate in GDP units in the data is then computed using the inflation rate of the GDP deflator as measured in section 4.1 and formula (19). A similar methodology is used to compute the real interest rate in consumption units.

5 Quantitative analysis

In this section we calibrate the model to aggregate targets of the U.S. economy to measure the effect of structural transformation on the process of growth. We set some parameters to standard values in the literature. Thus we have β= 0.95, α= 0.34, and δ= 0.06 as in Caselli and Feyrer (2007).

By normalizing TFP levels in the two sectors in the first period to 1, we then need to calibrate three preference parameters ϵ, ξ and ν, and two growth rates of TFP, γ_g and γ_s. To calibrate these we choose the following targets in the data over the period 1950-2015: 1)

[23] To derive (19), note that the real return to capital at $t+1$ is $\bar{r}_{t+1}$, so the investor obtains $p_{gt+1}K_t(1+\bar{r}_{t+1})$. This, by using $P_{GDP,t}\bar{y} = p_{gt}K_t$, is equivalent to $p_{gt+1}\frac{P_{GDP,t}\bar{y}}{p_{gt}}(1+\bar{r}_{t+1})$. Such return on investment can be used to purchase GDP at $t+1$ at price $P_{GDP,t+1}$, so the real return on investment is $\frac{p_{gt+1}}{p_{gt}}\frac{P_{GDP,t}}{P_{GDP,t+1}}\bar{y}(1+\bar{r}_{t+1})$, which is equation in the text as $\bar{y}=1$.

Note that we could have derived the real interest rate in GDP and consumption units using the bond introduced in section 3.3 and the standard asset prices formula. In that case we would have a gross return at t+1

$$R_{t+1} = \frac{(1+r^b_{t+1})P_{GDP,t}B_{t+1}}{P_{GDP,t+1}B_{t+1}} = \frac{1+\bar{r}_{t+1}}{1+\pi^y_{t+1}}.$$

A similar formula applies to aggregate consumption.

[24] An equivalent reasoning is made when measuring the real return in units of consumption. In that case, we would use the relative inflation rate for the consumption price index as constructed in the previous section.

Table 1: Parameter Values

β	α	δ	ϵ	ξ	ν	A_{g1}, A_{s1}	γ_g	γ_s
0.95	0.34	0.06	0.22	0.51	0.64	1	2.77%	0.51%

Table 2: Model's fit

	Targeted Statistics					Untargeted Statistics	
	Goods GDPpc Growth	Initial share of services	Final share of services	Real I/Y growth	Growth of p_s/p_g	GDPpc Growth	Inv/Out Ratio (nom)
Data	2.77%	0.393	0.685	0.74%	1.49%	2.12%	0.222
Model	2.77%	0.390	0.685	0.64%	1.48%	2.12%	0.215

the average growth rate of GDP per capita in units of goods (i.e. nominal GDP per capita deflated by the price of goods); 2) the share of services in the initial period (1950); 3) the share of services in the final period (2015); 4) the average growth rate of the real investment to output ratio; and 5) the average growth in the relative price services/goods. In the model, we assume that the goods sector produces both investment and the consumption good. Thus, as explained in section 2, to construct our target 5 we compute a Fisher index from the price of investment and the price of goods in the data, and take the ratio of this index and the price of services.

Formally, we proceed as follows. From the model's equilibrium we know that the growth rate of GDP in units of goods is equal to γ_g. So this parameter is uniquely pinned down by the average growth rate of GDP in units of goods during the 1950-2015 period (2.77%). Using (11) we then obtain that the growth in the relative price of services is equal to $(1-\alpha)(\gamma_g-\gamma_s)$ and so we use this equation together with the average growth of p_s/p_g in the data (1.49%) to find γ_s. Finally, given γ_g and γ_s we search for the values of ϵ, ξ and ν that minimize a quadratic loss function. The latter is given by the sum of squares of the differences of the following data targets from the model counterparts: the share of services in the first period (0.393), the share of services in the last period (0.685) and the average growth rate of I/Y (0.74%). Table 1 reports all parameter values while table 2 shows the fit of the calibrated model.[25]

[25]Note that Boppart (2014) provides estimates of two of the preference parameters from cross-sectional micro U.S. data. He reports a value of of ϵ of 0.22, and a value of ξ of 0.405. Our estimate of ϵ (0.22) coincides with the micro estimate in Boppart (2014) while that of ξ is slightly larger (0.51). By using $\xi = 0.405$ together with the other parameters of our baseline calibration reported in Table 1, we obtain a growth rate of GDP per capita of 2.14%, an initial share of services of 0.39, a final share of services of 0.65 and a growth of I/Y of 0.62%. Thus, even using micro estimates of the parameters, the quantitative properties of the model are

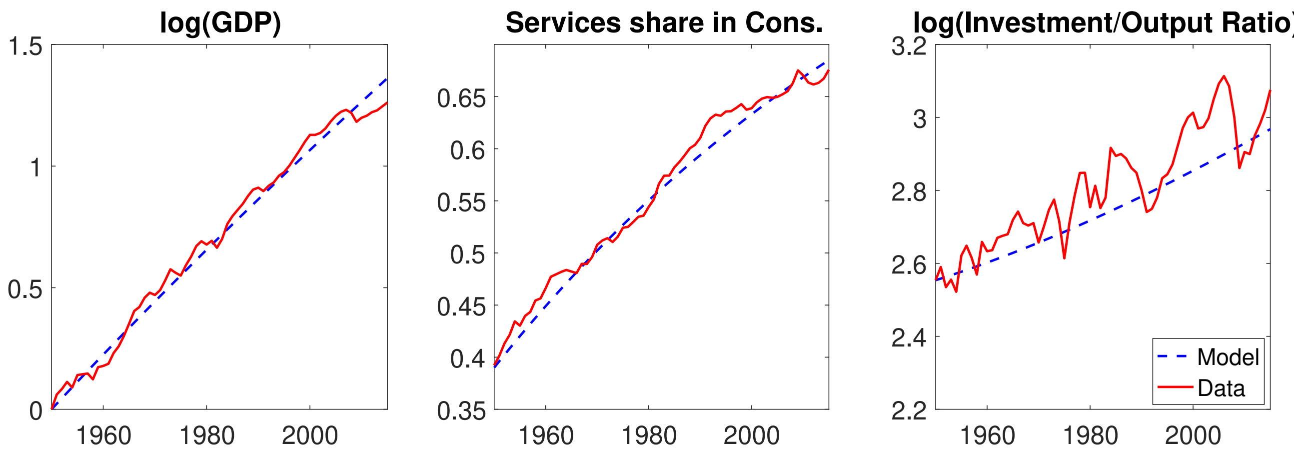

Figure 3: Model versus Data.

Figure 3 reports the visual fit of the model for GDP, the share of services and the investment-output ratio. The model does a good job at replicating the long run evolution of the services share. The evolution of the investment-output ratio is also reproduced fairly well, although this series in the data displays high volatility. The model produces a 0.64% average growth compared to a 0.74% in the data. Note also that, without being targets for the calibration, the model matches remarkably well both the average growth rate of GDP measured with NIPA methodology (2.12% in model and data) and the average nominal share of investment in output, which is constant in the model (0.215 versus 0.222). The first panel of Figure 4 compares the behavior of the model versus that of a linear trend in predicting the evolution of log-GDP. The model predicts a declining growth rate of GDP, due to structural transformation between goods and services. The growth rate of GDP in the model goes from 2.31% in the first period to 1.96% in the last period of the simulation. Such concavity in the evolution of GDP in the model helps to fit better the data. We find that the sum of squared residuals of the log deviations of the model from actual GDP is 36% lower than the corresponding measure of the log deviations from a linear trend. The second panel of Figure 4 plots the percentage difference between the model and a linear trend.

Thus, even if GDP appears to grow at a constant rate in the data, the model suggests that the rate of growth declines over time. Given the size of the U.S. business cycle, which displays a standard deviation of GDP growth of 2.3% over the period considered, it is difficult to detect such trend decline in the data. Using state space models allowing for a change in the long-run growth rate of GDP, however, Antolín-Díaz, Drechsel, and Petrella (2017) find that there is a slow moving fall in the growth rate of real GDP in the U.S.[26] They report a fall

in line with those of our benchmark calibration obtained using macro data.

[26]Previous evidence in Bai, Lumsdaine, and Stock (1998) and Eo and Morley (2015), also suggests that

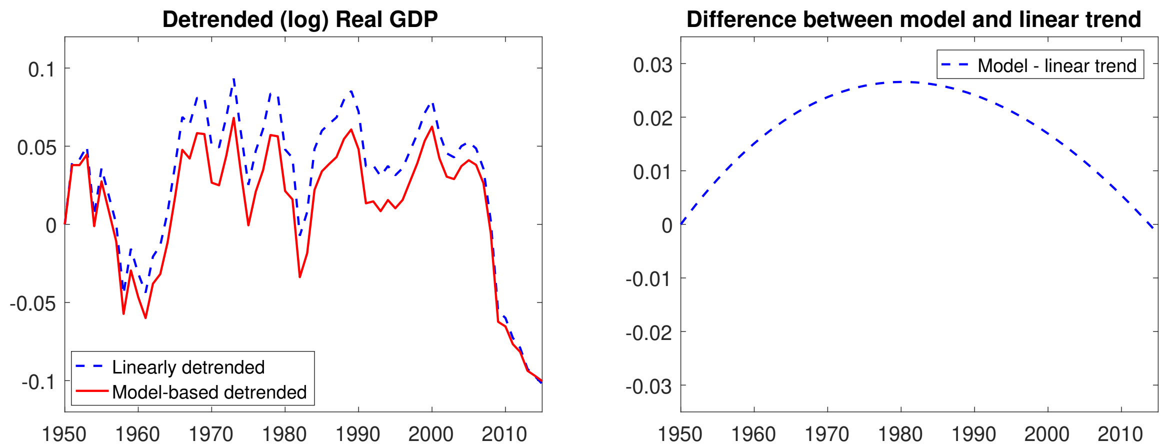

Figure 4: GDP: Model versus linear trend.

from an estimated long-run growth from 3.5% in the 1950s to 2% in recent years (a decline of almost 43%). Their estimates correspond to real GDP growth and are not in per-capita terms. Given the decline in the rate of population growth of about 1 percentage point (1.7% in the 1950s to 0.7% in the current decade) this implies a decline in the rate of growth of per capita GDP of around 28%.

Figure 5 reports the effect of structural transformation on the growth facts we focus on, i.e. the MPK, the real interest rate and GDP growth. The left panel of Figure 5 shows that the MPK declines by 34% over the period considered (0.66 in 2015) in units of GDP and by 41% in units of aggregate consumption (0.59 in 2015). Thus, if an additional unit of capital in 1950 provides an additional unit of GDP, in 2015 this additional unit of capital provides only 0.66 units of GDP. The difference between 1950 and 2015 in terms of units of consumption is even more striking. Note that this is consistent with the findings in Caselli and Feyrer (2007) using a cross-country comparison of MPKs. Their results suggest that the MPK is equalized across countries, regardless of the income level. However, if measured in units of GDP, the MPK would display a different value across countries, depending on the level of income (i.e. depending on the share of services in GDP). This, as pointed out in Caselli and Feyrer (2007), is due to the different relative price of capital across countries.[27] A related argument for cross-country comparisons is made in the next section.

there is a fall in the growth rate of real GDP in the U.S. In these papers, the fall takes the form of abrupt structural breaks. Antolín-Díaz, Drechsel, and Petrella (2017), instead, allow for the growth rate to drift gradually over time. Consistent with our model, their evidence points to a gradual decline in the growth rate.

[27]Note that this trend in the MPK and the real capital-output ratio generated by structural change could potentially relate to the recent decline in the labor share if the elasticity of capital-labor substitution were larger than one, as argued by Karabarbounis and Neiman (2014).

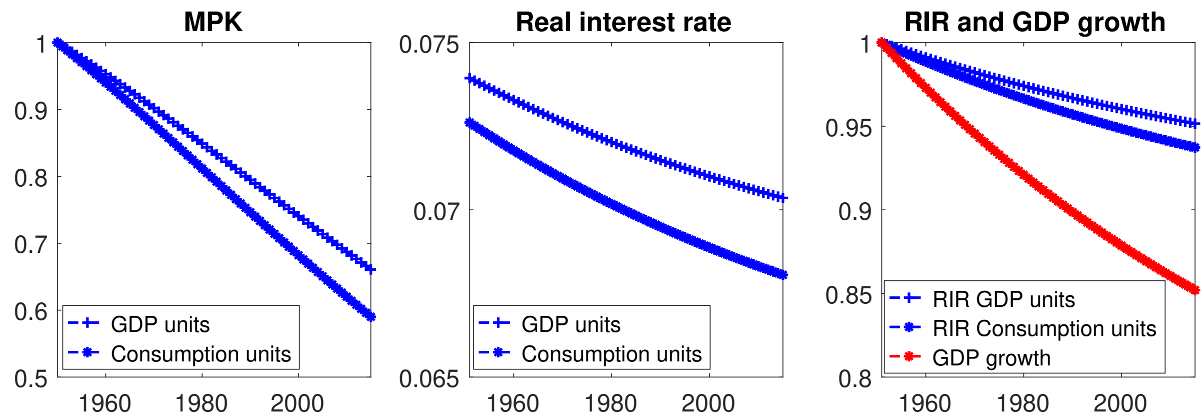

Figure 5: Evolution of model-derived variables.

The middle panel of Figure 5 shows that while the effect on the MPK is large in magnitude, the corresponding effect on the real interest rate is more contained. It goes from 7.39% to 7.04% in GDP units and from 7.26% to 6.81% in consumption units. The difference between the decline in the MPK and the real interest rate lies in the fact that, while the units of GDP obtained from an additional unit of capital decline strongly, the cost of buying that unit of capital in GDP terms also falls substantially. Finally, the third panel of Figure 5 shows the comparison between the decline in the real interest rate and the growth rate of GDP, when both are normalized to one in 1950. The growth rate of GDP declines faster than the real interest rate, regardless of the units the latter is measured in (i.e. GDP or consumption).[28]

Finally, we note here that what matters for calculation Baumol's cost disease is the evolution of the price of GDP (and aggregate consumption), both in level and in percentage change. This is evident by analyzing formulas (18) and (19). For instance, even without the model, we can calculate that, since the weight of services in consumption increased by 0.29 between 1950 and 2015, and the relative price of services to goods increases on average by 1.49% per year, the price deflator for aggregate consumption suggests that the real interest rate in consumption units was 0.29*1.49%=0.43 percentage points higher in 1950 compared to 2015. Also, since consumption accounts for roughly 80% of GDP there will be a difference between 1950 and 2015 of about 0.43*0.8=0.34 percentage points when the GDP deflator is used which is, up to approximations, the number we obtain above (2.31%-1.96% = 0.35 percentage points). However, the model allows us to infer the evolution of structural change outside the sample period considered, and so to make out of sample predictions, which is what we do in the next section.

[28]Note that this is due to the normalization of both rates to one in 1950. In percentage points both GDP growth and the interest rate decline by 0.35.

6 Cross country evidence

The previous section shows that the model of structural transformation measured with NIPA methodology fits well the growth experience of the U.S. both qualitatively and quantitatively. However, ST is a phenomenon that can also be observed across countries and, thus, we want to know whether its consequences for the growth process are also consistent with cross-country data. In addition to this, as in the structural change model, a standard theory of investment specific-technological change (ISTC) also predicts the following cross-country growth facts: i) the positive relationship of real investment rates with income levels (Barro (1991)); ii) the absence of correlation between nominal investment rates and income levels (Hsieh and Klenow (2007)); and iii) the absence of correlation between the MPK in units of capital and income levels (Caselli and Feyrer (2007)).

For the above reasons, we devote this section to evaluating the performance of the structural transformation model in predicting cross-country facts. We focus on the positive relationship between real investment rates and income levels (and the absence of a relationship between nominal investment rates and income). We do this in three steps. We first show that modeling structural transformation provides additional predictions on the growth process with respect to a standard theory of ISTC, and that these predictions are *qualitatively* in line with the cross-country evidence.

Second, we test the ability of the model in providing out of sample *quantitative* predictions. We show that the model calibrated for the U.S. can account well for the relationship between real investment rates and the share of services in consumption that we document in the data. This result has at least two important implications. On the one hand, it suggests that the processes structural transformation by itself has the potential to determine the evolution of real investment rates along the growth path, as no inefficiencies are present in the model. On the other hand, it suggests that the growth and structural transformation experience observed in cross-country data is remarkably consistent with that of the U.S.

Finally, we ask whether the predictions of the ST model calibrated for the U.S. regarding the evolution of real investment rates are comparable to those displayed by a standard model of ISTC, also calibrated to the U.S. By contrasting the two models with the cross-country data, we find that the ST model substantially outperforms the ISTC model along this metric. This result suggests that explicitly modeling structural transformation not only provides additional predictions on the growth process with respect a model of ISCT, but also provides a quantitatively better performance in terms of the predictions shared by the two models.

6.1 A standard model of ISTC

We first modify our benchmark model to be a standard theory of ISTC. To do this, we reduce the model to one consumption sector and one investment sector by setting $\nu = 0$.[29] In this way, preferences depend only on services consumption. Thus, the services sector produces the only consumption good (i.e. the only one that enters utility) and the goods sector produces the investment good (i.e. the only one used to build capital). We then drop the terminology services and goods in this model to refer to *consumption* and *investment* sectors.

The utility function becomes

$$V(p, E) = \frac{1}{\epsilon}\left[\frac{E}{p}\right]^{\epsilon} - \frac{1}{\epsilon}, \tag{21}$$

where p is the price of consumption relative to investment (i.e. the investment good is the numeraire) and E is again nominal consumption expenditure.

The consumption good is produced with a Cobb-Douglas technology

$$y_{ct} = k_{ct}^{\alpha}(n_{ct}A_{ct})^{1-\alpha}, \tag{22}$$

and the investment firm produces with technology

$$y_{It} = k_{It}^{\alpha}(n_{It}A_{It})^{1-\alpha}. \tag{23}$$

This two-sector representation implies that the relative price of consumption is

$$p_t = \frac{(A_{It})^{(1-\alpha)}}{(A_{ct})^{(1-\alpha)}}, \tag{24}$$

and assuming that $\gamma_I > \gamma_c$ the relative price consumption/investment increases over time, so there is ISTC, which is given by $1/p_t$. Thus, as in a typical ISTC model, the growth rate of ISTC is constant.

Note that this technology specification is isomorphic to explicitly assuming that the investment-good producer uses a linear technology that turns x units of consumption good into qx units of investment good with technology

$$I_t = q_t x_t. \tag{25}$$

[29] See online Appendix section D for more details.

In this case the current state of ISTC is q_t which, as above, is equal to $1/p_t$ in equilibrium.

As the structural transformation model (ST hereafter) this ISTC model predicts a positive relationship between the real investment rate and the income level, a constant nominal investment rate, the absence of correlation between the MPK in units of capital and the level of income and a declining trend in the MPK in GDP or consumption units.

6.2 ISCT or Structural Transformation?

As a starting point to compare the two models, we note that the model of structural change endogenously produces ISTC, as the price of investment relative to aggregate consumption (and to GDP) declines as income grows. This decline is due to the changing composition of consumption, which becomes more intensive in services relative to goods.[30] Thus, the ST model can also be interpreted as an endogenous theory of ISTC. A clear advantage of this modeling choice is that the ST model provides additional predictions with respect to a standard theory of exogenous ISTC, which can be contrasted with cross-country growth facts. These are: 1) the rise of the services share in GDP as income grows; 2) the decline in the growth rate of GDP as the share of services grows; 3) a declining real interest rate as income grows; 4) an acceleration of ISTC as income grows.[31] The first fact is well established (Herrendorf, Rogerson, and Valentinyi (2014)). The second relates to Baumol's disease and has been documented in cross-country data in Echevarria (1997) and Moro (2015) who also shows that structural transformation can account for the bulk of differences in per-capita GDP growth between middle income and high income economies. The third fact is more controversial due to difficulties in constructing real interest rates that are comparable across countries, but the evidence discussed in Barro and Sala-i-Martín (2004, p. 13) leads the authors to claim that "*it seems likely that Kaldor's hypothesis of a roughly stable real rate of return should be replaced by a tendency for returns to fall over some range as an economy develops*".

The fourth fact has been recently documented by Samaniego and Sun (2016), and represents a key dimension to evaluate whether the endogenous theory of ISTC performs better than the exogenous theory. We reproduce this evidence in Figure 6, in which we use the Penn World Table 7.1 to regress the average yearly growth rate of the inverse of the rela-

[30] Structural transformation would generate ISTC even if the investment good were made up of both goods and services as long as the proportion of goods in investment is larger than that in consumption, which is the empirically relevant case (see García-Santana, Pijoan-Mas, and Villacorta (2016) on this point). However, in this case structural change within investment would make the mechanism generating ISTC from structural change less potent.

[31] As stated at the end of section 6.1, the MPK in GDP or in consumption units shows a declining trend both in the ISTC model and in the ST model. This is in contrast with the effect on the real interest rate and real GDP growth which both fall because of Baumol's cost disease, a feature only present in the ST model.

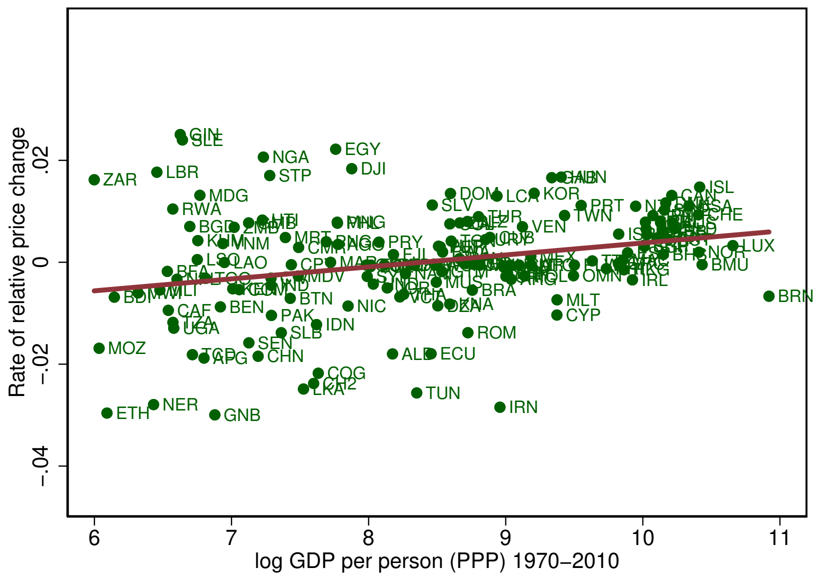

Figure 6: Investment-specific technological change by income level. Own calculations following the methodology in Samaniego and Sun (2016).

tive price investment/consumption (i.e., the average growth rate of ISTC) over the period 1970-2010, against the log of average GDP per capita in the same period. We use version 7.1 because, as discussed in Samaniego and Sun (2016), in later versions of the PWT the sampling method focuses on goods that are comparable across countries instead of being representative of goods purchased in any given country as was the case in PWT versions 7.1 and earlier. Since the focus here on the growth rate of the relative price of investment goods, using a single benchmark as in the PWT 7.1. is appropriate for our purposes.[32] We find a positive coefficient of 0.0023 (s.e. 0.00073) which is statistically significant at the 1% level.[33] The ST model produces an acceleration of ISTC as income grows because the share of services in consumption increases over time, thus making the relative price of investment decline faster at higher income levels.[34] Thus, as for the growth rate of GDP, the marginal product of capital, and the real interest rate, the key variable in determining the pace of ISTC in the model is the share of services. To test whether this prediction of the model is also confirmed in the data, we regress the growth of the rate of ISTC between 1970 and

[32]Note that, if we combine PWT 7.1 for the relative price of investment with the higher quality data from PWT 9.0 for GDP per person, the relationship in Figure 6 holds and the slope of the fitted line is essentially the same.

[33]Samaniego and Sun (2016) show that a similar relationship holds when considering the 1950-2010 period. In that case, however, the number of countries for which data are available covering that period is substantially reduced. For this reason we report here the regression for the 1970-2010 period.

[34]Comparisons between models are made by measuring both with NIPA methodology, so the differences that we highlight do not depend on measurement issues.

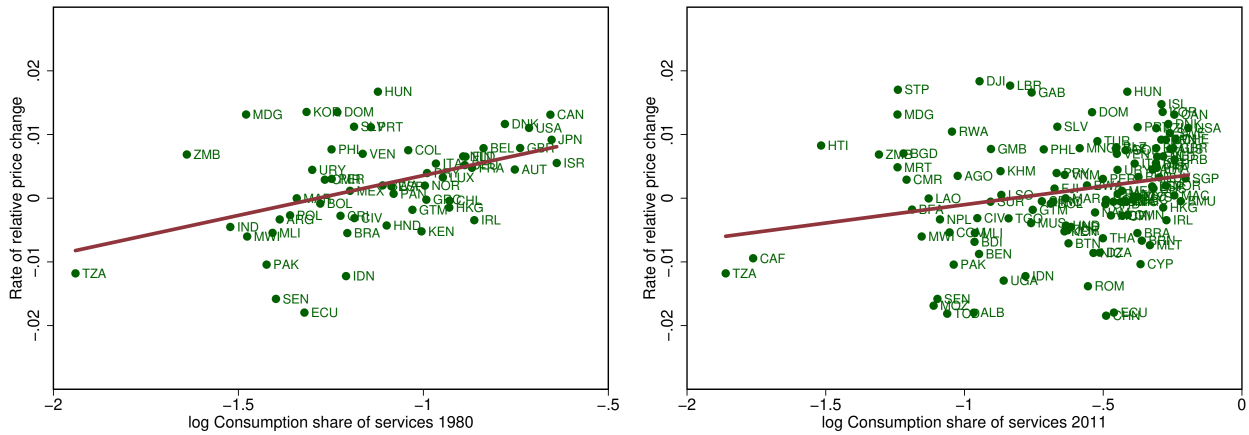

Figure 7: Investment-specific technological change against services share. Left panel: 1980. Right panel: 2011.

2010 on the log consumption share of services using using the Penn World Table 7.1. As discussed in the next section and in Appendix C, the consumption share is only available for benchmark years, so we use the first and the last of these (1980 and 2011). For 1980 there are 55 countries after matching services share and ISTC data while for 2011 there are 119.[35] Figure 7 reports the results. The coefficient for 1980 is 0.013, while that in 2011 is 0.006. In both cases, the coefficients are significant at the 5% statistical level. We note that using the 1980 data, the coefficient is twice as large as for the 2011 data while the dispersion is smaller. The difference between the two cases is due to the fact that the 1980 sample of 55 observations mainly contains developed economies while the 2011 sample includes more countries at lower income level and smaller share of services. As ISTC appears to accelerate with income level, the correlation coefficient between ISTC and share of services is larger when only higher income countries are considered.[36]

The evidence discussed in this section suggests that structural transformation affects the growth process along several dimensions which hold in cross-country data, and that are absent in a standard theory of ISTC. In addition, the ST model represents a theory of endogenous ISTC that is consistent with the cross-country evidence on this type of technological change.

[35] We trimmed the sample for countries with implausibly large or low rates of ISTC above 3% or below -3%. As mentioned below, we also used robust regressions to control for the effect of outliers.

[36] Lower income countries also display higher variability, which suggests that the significance of the regression might be due to outliers. To exclude this possibility, we run robust regressions to control for the effect of outliers in both years. The results remain significant, and the effect is stronger than in the original regressions. For the 1980 regression the coefficient increases to 0.014 significant at the 1% level, and for 2011 it increases to 0.007 also significant at the 1% level. The robust regression results remain significant when we do not trim the sample.

6.3 Out of sample predictions of the ST model

Given the qualitative predictions of the model discussed in the previous subsection, we now ask how well the model can account *quantitatively* for the cross-country evidence on real investment rates. We choose real investment rates since the positive trend with the level of income appears to be one of the most robust cross-country observations regarding the growth process. To test the model's predictive power we focus on the share of services in consumption. We choose this variable because, in the model, given the constant growth of TFP in the two sectors, the evolution of the share of services is the key variable which determines the magnitude of the change in the relative price of investment. A faster pace of structural transformation implies that the relative price of investment declines faster, implying a higher rate of ISTC and a faster increase in the real investment rate. Thus, if the model is a good predictor of the cross-country growth process, it should provide an elasticity of the real investment rate to the share of services comparable to that observed in the data.

To report the empirical evidence on the relationship between the share of services in consumption and real investment rates we use data from the International Comparisons Program (ICP) used to construct the Penn World Tables for the years 1980, 1985, 1996, 2005 and 2011. We focus on these years as they contain the benchmark data with details on expenditure components measured in local currency (nominal) and in purchasing power parity (PPP) dollars (real). Appendix C describes in detail data sources and methodology. We construct data for the cross-section of countries for the real and nominal shares of investment in GDP, and the share of services in private consumption expenditures. In table 3 and figure 8 we report, for each year, the estimated elasticity of the real and nominal investment rates with respect to the share of services in private consumption.[37] Similar to the results in Hsieh and Klenow (2007), who use the income level as a proxy of development, we find a positive and significant relationship between the real investment rates and the share of services in consumption, with an average elasticity across years of 0.61.[38] Also, consistent with Hsieh and Klenow (2007), nominal investment rates do not correlate or correlate very mildly with development indicators. As discussed above, the two-sector model employed in this paper is qualitatively consistent with both observations.

To contrast the model with cross-country data, we need to calibrate the model independently from such data. To do this, we tie our hands by using the calibration of the previous section for the U.S. growth path. This exercise amounts to assuming that the U.S. path

[37] We also provide estimates for all countries and years pooled. To account for different intercepts, in that particular regression we take the dependent and independent variables relative to the value for the U.S. for the corresponding year, hence normalizing all values to make them comparable.

[38] We also estimate robust regressions to account for the potential impact of outliers. The results do not change significantly any of the elasticities.

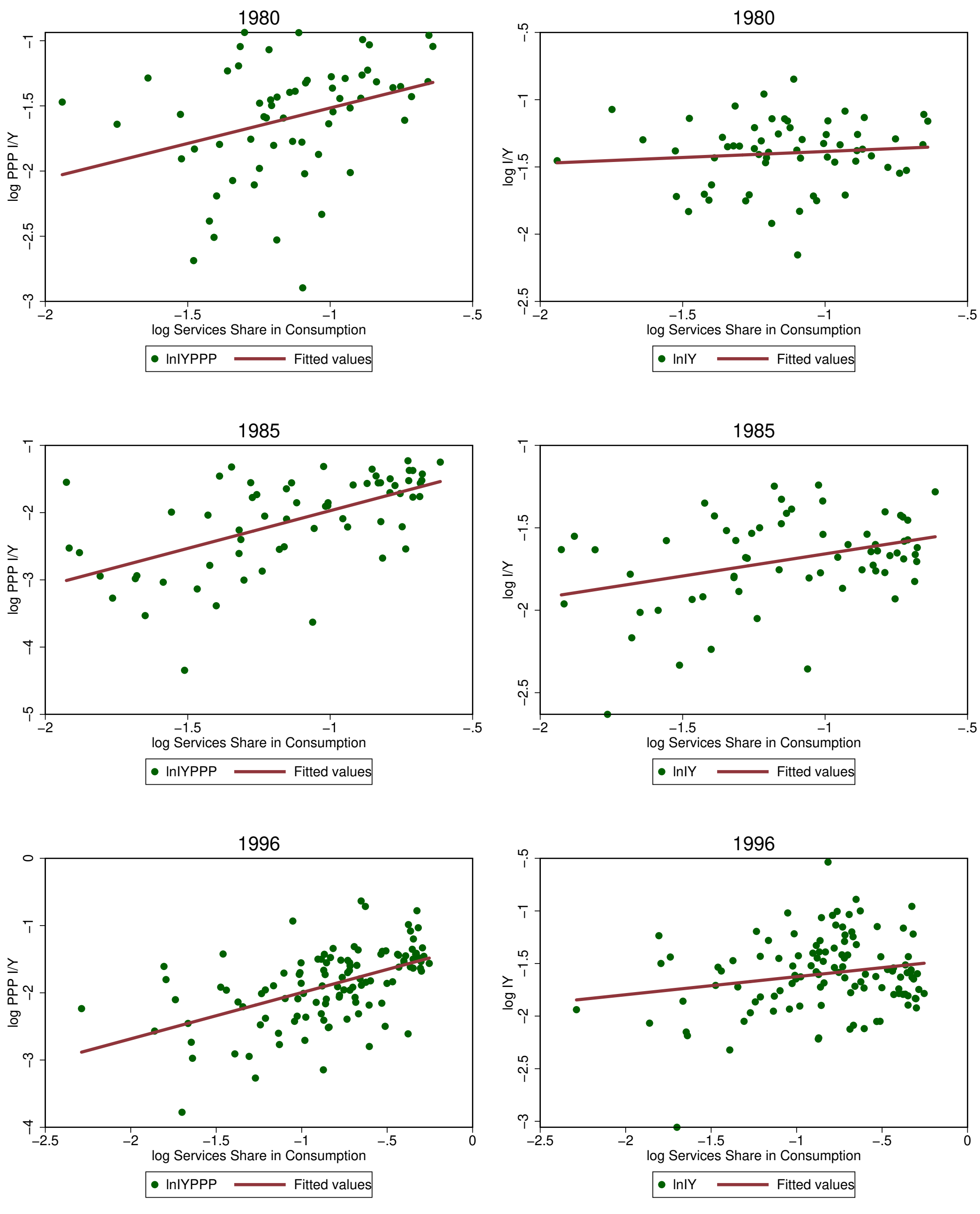

Figure 8: Investment to GDP ratio measured in PPP dollars (left column) and in nominal terms (right column) versus consumption share of services. Data from the International Comparisons Program, 1980, 1985, 1996, 2005, 2011. See Appendix C for construction.

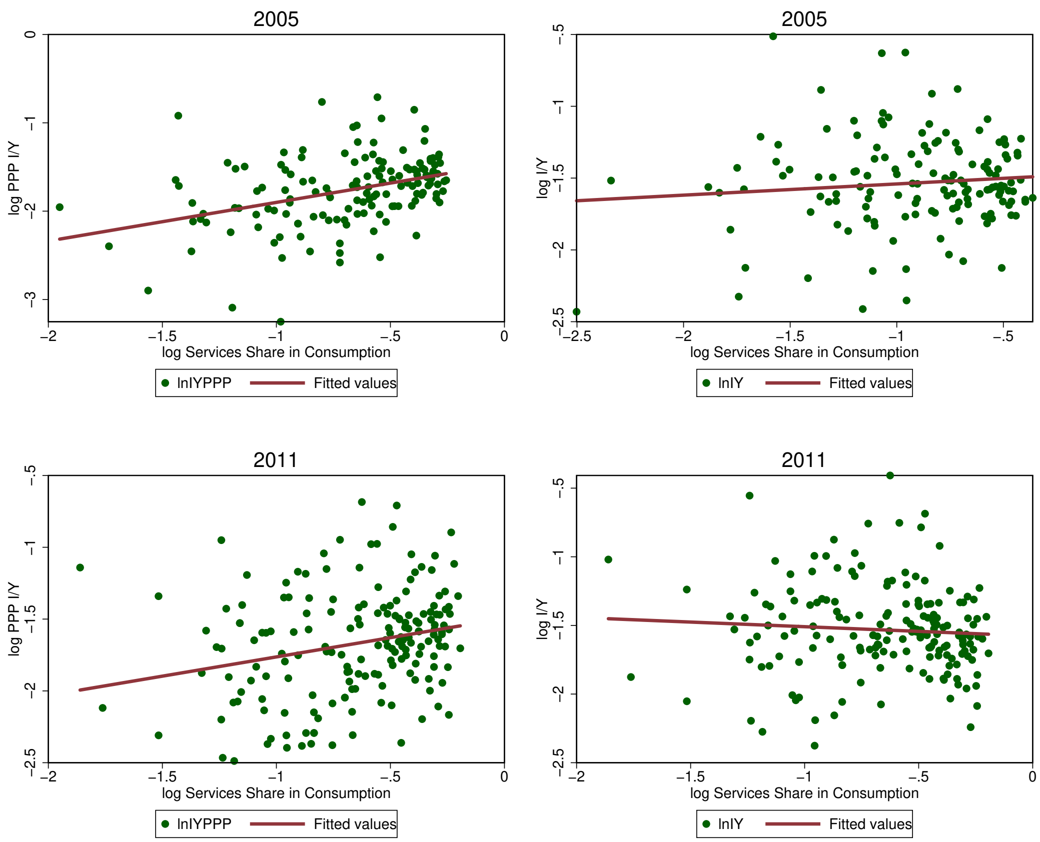

Figure 8: Continued

Table 3: Coefficient of PPP investment rates and domestic prices investment rates regressed on consumption share of services. All variables in logs.

year	PPP I/Y	Nominal I/Y	No. Observations
1980	**0.544**	0.089	N=61
	(0.193)	(0.100)	
	R^2=0.11	R^2=0.01	
1985	**1.122**	**0.268**	N=64
	(0.218)	(0.097)	
	R^2=0.34	R^2=0.13	
1996	**0.688**	0.171	N=115
	(0.123)	(0.087)	
	R^2=0.28	R^2=0.04	
2005	**0.437**	0.092	N=145
	(0.098)	(0.096)	
	R^2=0.14	R^2=0.01	
2011	**0.269**	-0.068	N=180
	(0.091)	(0.078)	
	R^2=0.06	R^2=0.01	
All years	**0.560**	0.075	N=565
	(0.064)	(0.043)	
	R^2=0.15	R^2=0.01	

Notes: robust standard errors in parentheses. Bold indicates significant at the 5% level.

of structural transformation is a representative one, and to asking whether such experience produces an evolution of the real investment to GDP ratio that resembles the cross-country evidence. Technically, we proceed as follows. Starting from period 1 of the simulation in the previous section, we discount TFP levels in each sector using the constant growth rates of TFP in the two sectors reported in table 1 for a number of periods. This way we are able to reconstruct, along the theoretical balanced growth path, the equilibrium of the model at earlier stages of development in which the share of services is smaller. The number of periods backwards is pinned down by the minimum level of the share of services we want to achieve, which we choose to be the minimum value across countries and years in the data (0.10 for Tanzania in 1996). This implies that, given the growth rates of TFP in table 1 and starting from period 1 of the U.S. simulation, we need to project the model back by 38 periods. This exercise leaves us with 104 years of data for the artificial economy with the same parameter values as the U.S. economy between 1950 and 2015. We then calculate the real investment to GDP ratio of this artificial economy for the 104 periods and the corresponding average elasticity with respect to the consumption share of services.[39] This yields a model elasticity of 0.59. The average elasticity in table 3 for the five benchmark years considered is 0.61. The elasticity obtained by pooling the data for all years is 0.56 (row "all years" in table 3). Figure 9 shows the scatter plot for all country-years and the log-linear fit together with the model-implied log-linear fit.[40] The two lines are virtually indistinguishable, showing a striking resemblance between the model and the cross-country elasticity of the real investment to GDP ratio with respect to the services share in consumption. Thus, even without resorting to transitional dynamics, the behavior of the structural transformation model, measured with NIPA conventions, can account well for the international evidence on real investment rates.[41] This suggests that most countries experience a growth process that resembles the one of the U.S.

6.4 ISCT vs ST out of sample predictions

In this subsection we compare the quantitative performance of the ISTC and the ST model in the out of sample predictions. We address the following question: if both the ST and the ISTC models are calibrated to U.S. data, which model provides a more accurate prediction

[39] The elasticity of the real investment rate to the share of services in consumption is given, period by period, by the percentage change in the first variable divided by the percentage change in the second variable.

[40] The intercept of the model implied log-linear fit in Figure 9 is chosen such that it crosses the data fit line at the average value of the services shares.

[41] Note that the empirical relationship between real investment rates and proxies for development is unlikely to be driven by transitional dynamics. If it were, countries that are far away from steady state and hence grow faster, would display higher investment-output ratios. In our data, we could not find a systematic relationship between growth and real investment rates. Results available on request.

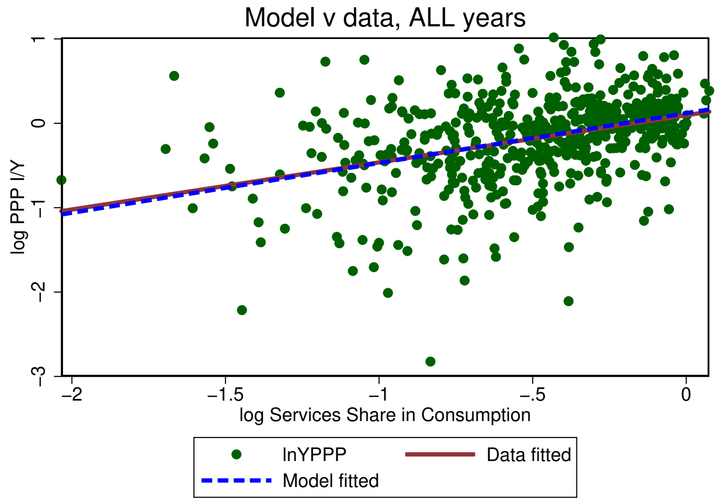

Figure 9: Cross country investment to GDP ratio in PPP dollars for all years v consumption share of services. The red line is the linear data fit, and the blue dashed line the fit arising from the model calibration.

of the cross-country variability in real investment rates? To answer this, we first note that, in the ST model, the key variable affecting the growth process is the share of services. In the ISTC model, absent this variable, we focus on the growth of per-capita GDP to compare the model calibrated to the U.S. with cross-country data.

In contrast with the ST model, in the ISTC model there are only three parameters that we need to calibrate, ϵ, γ_g and γ_s. We choose the following targets in the data: 1) the average growth rate of GDP per capita; 2) the average growth rate of the real investment to output ratio and 3) the average nominal investment rate. The three targets are computed for the 1950-2015 period.[42]

With an average GDP growth of 2.12%, real investment needs to grow at 2.86% to obtain a growth in I/Y of 2.86%-2.12%= 0.74% per year. The calibration then gives $\gamma_I = 2.86\%$, $\gamma_c = 1.24$ and ϵ=0.3454, with the three targets perfectly matched. The ISTC model thus accounts perfectly for the income elasticity of the real investment rate in the U.S., which is $0.74/2.12 = 0.35$. The income elasticity of the real investment rate computed using cross-country data for our 5 benchmark years and using the same specification as in Hsieh and Klenow (2007) is 0.19. In contrast with the ST model, the ISTC model predicts an elasticity that is almost double the one obtained using cross-country data.[43]

[42]Online Appendix E presents more details about the calibration of the ISTC model.

[43]It is worth noting that a possible reason why the ST model performs better than the ISTC model is that the former cannot as accurately account for the growth in the real investment rate in the U.S. The ST model predicts a value of 0.64% compared to 0.74% in the data. We can then ask what would the performance of

Table 4: Models Projection to 2065

	ST model				ISTC model			
	1950	2015	2050	2065	1950	2015	2050	2065
Services Share	0.390	0.685	0.779	0.811	-	-	-	-
GDP growth	2.31%	1.96%	1.85%	1.82%	2.12%	2.12%	2.12%	2.12%
Investment Growth	2.77%	2.77%	2.77%	2.77%	2.86%	2.86%	2.86%	2.86%
Investment/GDP	1	1.51	2.03	2.33	1	1.61	2.09	2.33

This result emerges due to the fact that, as discussed in subsection 6.2, ISTC accelerates with the level of income. The standard ISTC model delivers, by construction, a constant elasticity of the real investment rate with respect to income. Also, the ISTC model allows to perfectly match such elasticity for the U.S., 0.35. However, calibrating the model to a high-income country like the U.S., delivers an elasticity which is too high when compared to cross-country data because this country is at high income levels, at which ISTC grows fast. Instead, in the ST model, the acceleration of ISTC occurs due to structural transformation, and the model provides an elasticity that is closer to the data.

We conclude by reporting the predicted GDP growth, the investment growth and the real investment/output ratio (normalized to one in 1950) in the two models in 2050 and 2065. This is shown in Table 4. It is worth noting that, while the predictions for GDP growth are sensibly different between the two models over time, the difference in the real investment rate is less striking. The ST model displays a lower level of the real investment/output ratio in 2015, with a value of 1.51 compared to 1.61 of the ISTC model. However, in the ST model GDP growth slows down over time, implying a continuous acceleration in the growth of the real investment/output ratio, such that in 2050 this model has almost catched up with the ISTC one, with a value of 2.03 compared to one of 2.09. In 2065 the two models predict the same real investment/output ratio.

7 Conclusions

We study the consequences of structural transformation for the growth process. While the extant literature has mainly focused on productivity growth effects (i.e. Baumol's "cost disease") here we study the effects on growth, real interest rates, the marginal product of capital, and the real investment to output ratio. We argue that, because of its effects on growth, structural transformation has the potential for generating "unbalanced" growth when

the ISTC model be if it reproduced a growth of the real investment rate of 0.64%. In this case, we would have 0.64%/2.12% = 0.30, which is still 58% larger than in the data.

variables are measured using NIPA conventions. We analyze the consequences of structural transformation for both the U.S. time-series and a cross-section of countries.

In the post-war U.S. economy we observe that the *real* investment-output and capital-output ratios display significant upward trends, whereas the rate of growth of per capita GDP displays a mild decline. We show how structural transformation can affect these variables using a two sector model of structural change from goods to services displaying "balanced" growth. In this model, balanced growth occurs when variables are measured in terms of a numeraire (the price of goods). When taken to the data, however, we need to measure the aggregate variables in the model using the same NIPA conventions that are used to construct national accounts. Our quantitative results suggest that structural transformation has a non-negligible effect on the growth process in the U.S. during the past 65 years.

From an international perspective, the model implies that countries at a more advanced stage of structural transformation should display higher real investment to output ratios. Using the parameter calibration arising from the model for the U.S. economy, we then ask how much of the cross-country variability in investment rates can be accounted for by structural transformation alone. The elasticity of the real investment-output ratio with respect to the share of services in consumption is 0.61 in the data. The elasticity arising from the model is 0.59. That is, we can interpret the well known fact that real investment-output ratios increase as economies develop as a consequence of economies being at different stages of structural transformation along the same growth path. It follows that the two-sector model of structural transformation represents a simple and very tractable tool that can be used to study the process of economic growth. In particular, to explain the long run evolution of real investment-output and capital-output ratios it is not necessary to assume that different countries are on transitional dynamics converging asymptotically to a balanced growth path. The model does not even require assuming differences in preferences, taxation, or other deep parameters to explain cross-country differences in investment rates. The key assumption to generate these differences is a constant differential TFP growth between the goods and the services sector along the growth path, something that is motivated by the well established constant decline of the relative price goods/services in U.S. data. We also compare the performance of the structural transformation model with that of a standard investment-specific technical change model and conclude that the former outperforms the latter in terms of its predictions for cross-country real investment rates. The main reason is that, unlike a standard ISTC model, a model of structural transformation endogenously generates an acceleration of investment-specific technical change as income grows.

Thus, our results suggest that a growth model of structural transformation, when appropriately taken to the data, can account well for the time series evidence for the U.S. and for the international evidence on investment-output ratios. The measurement of the model with NIPA conventions is a key aspect of our approach, which is overlooked in most applications comparing multi-sector models to the data.

Appendix

A A three sector model

In this appendix we extend the model to three sectors: a consumption good sector, a services sector and an investment sector. There are now three representative firms in the economy operating in perfect competition. The first firm produces the consumption good with technology

$$y_{gt} = k_{gt}^{\alpha}(n_{gt}A_{gt})^{1-\alpha}, \tag{26}$$

where k_{gt}, n_{gt} and $A_{gt}^{1-\alpha}$ are capital, labor and total factor productivity (TFP) of the firm. The second firm produces services with technology

$$y_{st} = k_{st}^{\alpha}(n_{st}A_{st})^{1-\alpha}, \tag{27}$$

with k_{st}, n_{st} and $A_{st}^{1-\alpha}$ being capital, labor and TFP. The output of this firm is used as services consumption. Finally, the third firm produces the investment good with technology

$$y_{It} = k_{It}^{\alpha}(n_{It}A_{It})^{1-\alpha}, \tag{28}$$

with k_{It}, n_{It} and $A_{It}^{1-\alpha}$ being capital, labor and TFP.

TFP in the three sectors evolves according to

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s, \tag{29}$$

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g, \tag{30}$$

$$\frac{A_{It+1}}{A_{It}} = 1 + \gamma_I, \tag{31}$$

where γ_s , γ_g and γ_I are exogenous constant growth rates.

In equilibrium all markets clear and the following must hold

$$y_{gt} = C_{gt},$$

$$y_{st} = C_{st},$$

$$y_{It} = K_{t+1} - (1-\delta)K_t$$

$$k_{gt} + k_{st} + k_{It} = K_t,$$

Table 5: Parameter Values

ϵ	ξ	ν	A_{g1},A_{s1},A_{I1}	γ_g	γ_s	γ_I
0.17	0.50	0.64	1	3.19%	0.75%	2.42%

Table 6: Data targets

Target	(1)	(2)	(3)	(4)	(5)	(6)
Data	2.42%	0.393	0.685	0.30%	1.61%	1.10%
Model	2.42%	0.390	0.685	0.30%	1.59%	1.09%

and

$$n_{gt} + n_{st} + n_{It} = 1.$$

By normalizing TFP levels in the three sectors in the first period to 1, we then need to calibrate three preference parameters ϵ, ξ and ν, and three growth rates of TFP, γ_s, γ_g and γ_I. Thus we need an additional target with respect to the two-sector model. Also, in the two-sector model, as a first target we use a Fisher index of the price of consumption goods and investment, because we assume that goods and investment are produced in the same sector. Thus, given that goods and investment are separate in this version of the model, here we use the growth of GDP in units of investment (2.42%) as target (1). Targets (2) and (3) are given by the initial and final share of services, while target (4) is the average growth of the real investment/output ratio (0.30%). Also, instead of the growth in the relative price of services where goods were a composite of investment and consumption goods in the data, here we target (5) the average growth in the relative price services/goods (1.61%, where now we use the price of consumption goods as the numerator) and (6) the average growth in the relative price services/investment (1.10%). Table 5 reports the relevant parameter values (we keep $\beta = 0.95$, $\alpha = 0.34$, $\delta = 0.06$). Table 6 and figure 10 show the fit of the calibrated model.[44]

In this case, the real investment-GDP ratio increases by 0.30% per year, compared to the 0.74% figure in section 3. The three sector model performs well in fitting GDP growth, which is 2.12% (not reported in table 10) as in the data and in the two sector model, and the nominal investment rate (0.211 versus 0.222 in the data). It also reproduces a growth of the real investment-GDP ratio as in the data. The decline in the MPK in this case is 18% in terms of GDP and 22% in terms of aggregate consumption.

[44]Note that the small difference in relative prices growth between the model and the data is given by the approximation used in computing growth rates.

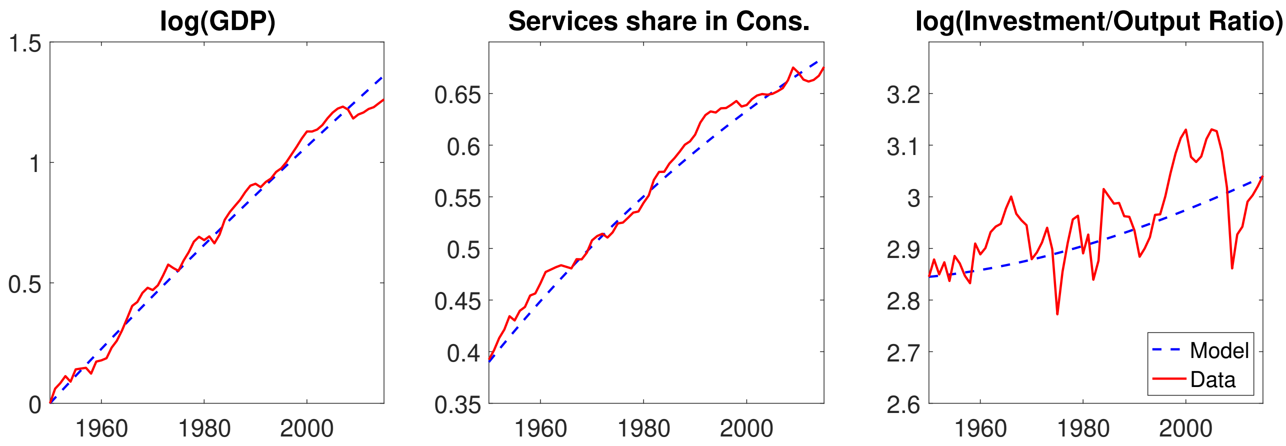

Figure 10: Three-sector model versus Data.

B Fisher Index

The Laspeyres and Paasche quantity indices as computed by NIPA are given by

$$Q_t^L = \frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}},$$
$$Q_t^P = \frac{\sum p_t q_t}{\sum p_t q_{t-1}},$$

where the sum is over all the goods and services included in the bundle, p represents prices, and q quantities. The Fisher quantity index is then given by a weighted average of Laspeyres and Paasche

$$Q_t^F = \sqrt{Q_t^L Q_t^P}.$$

Consider the case of two goods. The Laspeyres is

$$Q_t^L = \frac{p_{1,t-1}q_{1,t} + p_{2,t-1}q_{2,t}}{p_{1,t-1}q_{1,t-1} + p_{2,t-1}q_{2,t-1}}.$$

Note that the Laspeyres quantity index is independent of the numeraire chosen. This is because it is a function of *relative prices*. To see this, divide numerator and denominator by the same price at $t-1$:

$$Q_t^L = \frac{q_{1,t} + \frac{p_{2,t-1}}{p_{1,t-1}}q_{2,t}}{q_{1,t-1} + \frac{p_{2,t-1}}{p_{1,t-1}}q_{2,t-1}},$$

thus implicitly choosing good 1 as the numeraire, or

$$Q_t^L = \frac{\frac{p_{1,t-1}}{p_{2,t-1}} q_{1,t} + q_{2,t}}{\frac{p_{1,t-1}}{p_{2,t-1}} q_{1,t-1} + q_{2,t-1}},$$

implicitly choosing good 2 as the numeraire. The same argument can be made for the Paasche index. This implies that the same argument extends to the Fisher index, which is a weighted average of the two. The bottom line is that the Fisher quantity index is *independent of the numeraire*.

The Fisher price index instead, is *not independent of the numeraire*. To see this we can proceed in two different ways, a direct one and an indirect one. The direct one requires constructing the Fisher price index using the NIPA formula. This is a weighted average of a Laspeyres and a Paasche price indices:

$$P_t^L = \frac{\sum p_t q_{t-1}}{\sum p_{t-1} q_{t-1}}$$

$$P_t^P = \frac{\sum p_t q_t}{\sum p_{t-1} q_t},$$

where again the sum is over the goods and services included in the bundle. The Fisher index is then given by

$$P_t^F = \sqrt{P_t^L P_t^P}.$$

Consider the case of two goods. The Laspeyres is:

$$P_t^L = \frac{p_{1,t} q_{1,t-1} + p_{2,t} q_{2,t-1}}{p_{1,t-1} q_{1,t-1} + p_{2,t-1} q_{2,t-1}}. \tag{32}$$

It should be clear that this formula is not independent of the numeraire. To see this, consider that in (32) the numeraire each period is current dollars, as prices are expressed in dollar units. If instead, the numeraire each period is the price of good 1, equation (32) becomes

$$\widetilde{P}_t^L = \frac{q_{1,t-1} + \frac{p_{2,t}}{p_{1,t}} q_{2,t-1}}{q_{1,t-1} + \frac{p_{2,t-1}}{p_{1,t-1}} q_{2,t-1}}. \tag{33}$$

Clearly

$$P_t^L \neq \widetilde{P}_t^L.$$

The same argument can be made for the Paasche price index.

The other way to see this is to use the indirect method to construct the Fisher price index, that is dividing nominal GDP (i.e. in current dollars) by the Fisher index of real

GDP computed above. Then

$$P_t^F = \frac{GDP_t}{Q_t^F}.$$

While real GDP Q_t^F is independent of the numeraire, nominal GDP, given by GDP_t in the formula, is not. For instance, if we express nominal GDP in units of apples instead of dollars, the Fisher price index that we obtain is different. The result should not be surprising, as a price is always an *exchange rate* of some units of one good for a unit of another good.

C Cross country data sources

The data used to construct cross-country series for investment to output ratios and the share of services in final household consumption come from four waves of the benchmark years of the International Comparisons Program used to construct the PWT dataset. We obtained data for benchmark years 1980, 1985, 1996, 2005, and 2011.[45] We collected data in purchasing power parity (PPP) dollars and in local currency. The series for real (PPP) investment to GDP ratios are ratios of investment to GDP in PPP dollars. The series for nominal investment to GDP ratios are ratios of investment to GDP in local currency. Investment consists of gross investment in fixed assets (excluding inventories). For the services consumption share, we summed the (nominal) expenditures on services and divided them by (nominal) household consumption. Because different benchmark years contain different detail of information on expenditure items, we list below the items considered as services consumption. We follow Herrendorf, Rogerson, and Valentinyi (2014) whenever possible as they suggest a sector assignment for years 1985 and 1996. The following are considered services consumption:

- 1980: services correspond to items 55, 59–62, 67, 79-80, 84–90, 94, 98–102, 109–111, 114–118, 123–125.
- 1985: services correspond to items 48, 52, 53-55, 62, 69, 73, 74, 78-81, 85, 88-93, 98-100, 102-104, 108-111.
- 1996: services correspond to gross rent and water charges, medical and health services, operation of transportation equipment, purchased transport services, communication, recreation and culture, education, restaurants, cafes and hotels, other goods and services.

[45] Benchmark data for 1980, 1985 and 1996 was obtained from Groningen Growth and Development Centre (1992-2019), and for 2005 and 2011 from World Bank (2005-2011). Links provided in references.

- 2005: services correspond to miscellaneous goods and services, restaurants and hotels, education, recreation and culture, communication, transport, health. Because the 2005 data contains an item called housing, water, electricity, gas and other fuels, it does not distinguish between rents and the consumption of housing goods such as fuel. To separate rents out, we imputed rents according to the proportion of rents in total housing costs in 1996. The results without this imputation remain very similar and are available on request.

- 2011: services include health, transport, communication, recreation and culture, education, restaurants and hotels, miscellaneous goods and services. Housing expenditure is obtained as the difference between "individual consumption expenditure by households" and "individual consumption expenditure by households without housing".

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