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TORQUE VECTORING CONTROL FOR FULLY ELECTRIC SAE CARS

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Abstract. *Fully electric vehicles with individually controlled powertrains can achieve significantly enhanced vehicle response, in particular by means of Torque Vectoring Control (TVC). This paper presents a TVC strategy for a Formula SAE (FSAE) fully electric vehicle, the “T-ONE” car designed by “UninaCorse E-team” of the University of Naples Federico II, featuring four in-wheel motors. A Matlab-Simulink double-track vehicle model is implemented, featuring non-linear (Pacejka) tyres. The TVC strategy consists of: i) a reference generator that calculates the target yaw rate in real time based on the current values of steering wheel angle and vehicle velocity, in order to follow a desired optimal understeer characteristic; ii) a high-level controller which generates the overall traction/braking force and yaw moment demands based on the accelerator/brake pedal and on the error between the target yaw rate and the actual yaw rate; iii) a control allocator which outputs the reference torques for the individual wheels. A driver model was implemented to work out the brake/accelerator pedal inputs and steering wheel angle input needed to follow a generic trajectory. In a first implementation of the model, a circular trajectory was adopted, consistently with the "skid-pad" test of the FSAE competition. Results are promising as the vehicle with TVC achieves up to ≈ 9% laptime savings with respect to the vehicle without TVC, which is deemed significant and potentially crucial in the context of the FSAE competition.*

1 INTRODUCTION

Formula SAE is an international student design competition which challenges worldwide students to conceive, design, fabricate and compete with small formula-style racing cars [1]. While the competition was historically based on internal combustion engines (since 1981), recently there has been an increasing interest towards electric-powered Formula SAE vehicles, with the first Formula SAE Electric competition taking place in 2013 [2]. Most of the solutions adopted so far include two or four in-wheel electric motors, without differential. That allows Torque Vectoring Control (TVC), i.e., the individual control of each drivetrain [3-7]. By imposing an uneven distribution of torque demand between the left and right side of the vehicle, a direct yaw moment can be generated and appropriately exploited to improve vehicle performance and, ultimately, reduce laptime.

This paper deals with the development and assessment of a torque vectoring strategy for the Formula SAE vehicle T-ONE of the UninaCorse E-team from the University of Naples Federico II (Fig. 1), featuring four in-wheel motors, and with main parameters shown in Table 1. The vehicle model and simulations were implemented into Matlab-Simulink.

Section 2 describes the vehicle model. Details regarding the torque vectoring algorithm are given in Section 3. Section 4 deals with the reference trajectory and the driver model. Preliminary results are presented in Section 5, and conclusions are in Section 6.

Figure 1: The Formula SAE vehicle "T-ONE".

Quantity	*Symbol*	*Value and unit*
Wheel radius	R_r	0.26 m
Front semi-wheelbase	a_1	0.990 m
Rear semi-wheelbase	a_2	0.660 m
Vehicle mass	m_v	350 kg
Moment of inertia along the vertical axis	I_z	400 kg m^2
Wheelbase	l	1.650 m
Track	t	1.200 m
Height of the centre of mass	h	0.32 m

Table 1: Main parameters of the Formula SAE vehicle T-ONE.

2 VEHICLE MODEL

A double-track vehicle model was implemented. The Adapted ISO sign convention [8] and the vehicle reference frame and schematic in [9] were adopted in this study. Hence, the x-axis represents the forward direction, the y-axis indicates the lateral direction (positive to the left), the z-axis is vertical (positive upwards). The longitudinal and lateral components of the velocity of the centre of mass of the vehicle are respectively u and v, while r is the vehicle yaw rate. F_{xij} and F_{yij} are, respectively, the longitudinal and lateral forces at the corner ij, where $i = 1,2$ for front and rear axles, and $j = 1,2$ for left and right sides. The wheel steering angle, δ, is assumed to be the same for both front wheels.

The longitudinal equilibrium equation is

$$m_v\dot{u} = F_{x11}cos\delta + F_{x12}cos\delta - F_{y11}sin\delta - F_{y12}sin\delta + F_{x21} + F_{x22} - \frac{1}{2}\rho C_x S\, u^2 + m_v(vr) \tag{1}$$

which includes the aerodynamic drag, where ρ is the air density, C_x the drag coefficient, and S the frontal area of the vehicle.

The lateral equilibrium equation is

$$m_v\dot{v} = F_{x11}sin\delta + F_{x12}sin\delta + F_{y11}cos\delta + F_{y12}cos\delta + F_{y21} + F_{y22} - \frac{1}{2}\rho C_y S\, v^2 - m_v(ur) \tag{2}$$

The moment balance equation in the z direction leads to:

$$J_z\dot{r} = \frac{t_1}{2}(-F_{x11}cos\delta + F_{x12}cos\delta + F_{y11}sin\delta - F_{y12}sin\delta) + \frac{t_2}{2}(-F_{x21} + F_{x22}) + a_1(F_{x11}sin\delta + F_{x12}sin\delta + F_{y11}cos\delta + F_{y12}cos\delta) - a_2\,(F_{y21} + F_{y22}) + M_z \tag{3}$$

where M_z is the yaw moment generated via the TVC (see Section 3).

The congruence equations, under the assumption of small sideslip angles, read

$$\alpha_{11} = \delta - \left(\frac{v + ra_1}{u - r\,\frac{t_1}{2}} \right) \quad (4)$$

$$\alpha_{12} = \delta - \left(\frac{v + ra_1}{u + r\,\frac{t_1}{2}} \right) \quad (5)$$

$$\alpha_{21} = -\left(\frac{v - ra_2}{u - r\,\frac{t_2}{2}} \right) \quad (6)$$

$$\alpha_{22} = -\left(\frac{v - ra_2}{u + r\,\frac{t_2}{2}} \right) \quad (7)$$

where α_{ij} is the tyre slip angle at the corner ij.

The constitutive equations were implemented using a PAC2002 Pacejka formulation, starting from the .tir file of the used tyre, i.e. Hoosier 13". The adopted formulation provides the lateral forces F_{yij} as functions of camber angle, γ, vertical load, F_{zij}, slip angle, α_{ij}, and wheel radius, R_r, in pure lateral conditions. F_{xij}, instead, were obtained with an even distribution among the four wheels of the overall desired longitudinal force, F, provided by the driver model (see Section 3). The vertical loads are

$$F_{z11} = F_{z10} + F_{z1long} - F_{z1lat} \quad (8)$$

$$F_{z12} = F_{z10} + F_{z1long} + F_{z1lat} \quad (9)$$

$$F_{z21} = F_{z20} + F_{z2long} - F_{z2lat} \quad (10)$$

$$F_{z22} = F_{z20} + F_{z2long} + F_{z2lat} \quad (11)$$

where the downforce and longitudinal load transfer contributions are

$$F_{z1long} = \frac{C_{zf} S \rho}{2} u^2 - \frac{m_v a_x h}{l} \quad (12)$$

$$F_{z2long} = -\frac{C_{zr} S \rho}{2} u^2 + \frac{m_v a_x h}{l} \quad (13)$$

and the lateral load transfers are

$$F_{z1lat} = \frac{1}{t_1} \left[\frac{m_v a_y d_1 a_2}{l} + k_{\varphi rel1} (h - d) \left(m_v a_y \right) \right] \quad (14)$$

$$F_{z2lat} = \frac{1}{t_2} \left[\frac{m_v a_y d_2 a_1}{l} + k_{\varphi rel2} (h - d) \left(m_v a_y \right) \right] \quad (15)$$

where h is the height of the centre of mass, d the height of the roll centre below the centre of mass, C_{zf} and C_{zr} the front and rear aerodynamic lift coefficients, $k_{\varphi rel1}$ e $k_{\varphi rel2}$ the front and rear relative roll stiffness values, $F_{z10} = \frac{m_v g a_2}{l}$ and $F_{z20} = \frac{m_v g a_1}{l}$ the static front and rear vertical loads are, g the gravity acceleration, a_x and a_y the vehicle longitudinal and lateral acceleration.

3. TORQUE VECTORING CONTROL

The developed TVC strategy is based on the scheme proposed in [10]. A reference yaw rate value, r_{ref}, is generated through a look-up table which takes as input the wheel steering angle, δ, and the vehicle velocity, V. The look-up table was built considering steady-state conditions and a desired vehicle cornering response (a.k.a. understeer characteristic), shaped as in Equation 26. With respect to the baseline vehicle, i.e. the vehicle without TVC, the cornering response is designed so as to: i) decrease the understeering gradient; ii) extend the region of linear relationship between dynamic steering angle, δ_{dyn}, and lateral acceleration, up to a_y^*; iii) increase the maximum lateral acceleration achievable, $a_{y,max}$, which is very important in the interest of laptime minimisation. Specifically, the look-up table was built by defining vectors of a_y and V, then using the following relationships:

$$r_{ref} = \frac{a_y}{V} \tag{25}$$

$$\delta_{dyn} = \begin{cases} K a_y & if a_y < a_y^* \\ K a_y^* - (a_{y,max} - a_y^*) K \ln\left(\frac{(a_y - a_{y,max})}{(a_y^* - a_{y,max})}\right) & if\ a_y > a_y^* \end{cases} \tag{26}$$

Then, to relate the dynamic steering angle to the overall steering angle, the kinematic steering angle (Ackermann angle), δ_{kin}, was obtained as

$$\delta_{kin} = \frac{a_y l}{V^2} \tag{27}$$

and added to the dynamic contribution to obtain the total wheel steering angle:

$$\delta = \delta_{dyn} + \delta_{kin} \tag{28}$$

Finally, the table was inverted in order to have wheel steering angle and vehicle velocity as input, and reference yaw rate as output.

A PID controller was implemented to track the yaw rate, specifically taking as input the error between the reference yaw rate and the current yaw rate, r, and giving as output the value of yaw moment, M_z, to be generated.

Once the value of desired overall force and yaw moment are known, a "control allocator" block [4] calculates the four wheel torque demands, τ_{ij}, as:

$$\tau_{11} = \tau_{21} = \left(F - \frac{2M_z}{t}\right)\frac{R_r}{4} \tag{29}$$

$$\tau_{12} = \tau_{22} = \left(F + \frac{2M_z}{t}\right)\frac{R_r}{4} \tag{30}$$

4. REFERENCE TRAJECTORY AND DRIVER MODEL

Among the Formula SAE dynamic tests, this study selected the Skid-pad test [11], in which the car goes through a figure-of-eight shaped track including two circles with diameter 15.25 m. The car performs two laps in one of the circles, then it moves to the other circle and it performs other two laps. The best laptime is selected between the second attempt at each circle. Hence, in a first implementation of this work, it is sufficient to design a circular trajectory with radius R, to be negotiated twice. Specifically, the vehicle starts in (0,0) so the circle has centre (0, R). The equations for the reference position are thus:

$$x_{ref}(s) = -R + R\cos(s/R) \tag{16}$$

$$y_{ref}(s) = R\sin(s/R) \tag{17}$$

where s is the curvilinear abscissa, which can be calculated as:

$$s = \int \sqrt{u^2 + v^2}\, dt \tag{18}$$

The driver model used in this study is inspired to [12]. It calculates: i) the wheel steering angle, δ, through a Proportional controller based on errors on position and orientation of the vehicle; ii) the acceleration/brake inputs, i.e. the overall longitudinal force demand, F, to achieve the maximum possible vehicle speed.

The reference trajectory is obtained via Equations 16, 17 and 18. The reference orientation of the vehicle, σ_{ref_visual}, is taken after a speed-dependent "visual" distance, l_{steer}, defined as

$$l_{steer} = Vt_{resp} + \frac{a_x t_{resp}^2}{2} \tag{19}$$

where t_{resp} depends on the driver's behaviour (herein assumed as 0.3 s) and V is the vehicle speed, i.e. $V = \sqrt{u^2 + v^2}$. Denoting the current position with (x, y), the position error is

$$e_p = \left(x - x_{ref}\right)\cos\left(\sigma_{ref}\right) + \left(y - y_{ref}\right)\sin\left(\sigma_{ref}\right) \tag{20}$$

and the orientation error is

$$e_o = \sigma_{ref\_visual} - \int r\, \mathrm{d}t + \frac{\pi}{2} \tag{21}$$

where the constant $\frac{\pi}{2}$ is needed to guarantee the use of consistent reference frames. Finally,

$$\delta = K_p e_p + K_o e_o \tag{22}$$

where K_p and K_o are calculated according to [12].

The maximum, i.e. target, vehicle speed, V_{max}, depends on the maximum allowable lateral acceleration,$a_{y,max}$:

$$V_{max} = \sqrt{a_{y,max} R} \tag{23}$$

The target longitudinal acceleration, $a_{x,ref}$, can be worked out as a function of the maximum allowable longitudinal acceleration, $a_{x,max}$:

$$a_{x,ref} = a_{x,max} \sqrt{1 - \frac{|a_y|}{a_{y,max}}} \tag{24}$$

The overall longitudinal force demand, F, is composed of a feedforward contribution, $m_v a_{x,ref} \pm \frac{1}{2} \rho C_x S V^2$, to improve the driver promptness (the sign in front of the aerodynamic drag is positive in acceleration and negative during braking), and a feedback contribution which is a Proportional Integral controller based on the error $(V_{max} - V)$. Due to the specific electric motors used in this project, the individual motor torques are saturated to 21 Nm.

5 PRELIMINARY RESULTS

Based on the vehicle model described in Section 2 integrated with the TVC algorithm described in Section 3, and on the driver model presented in Section 4, simulations were performed in Matlab-Simulink to assess the performance of the proposed control strategy. The circumference radius to be followed by the centre of mass of the car was set to 8.3 m, as it takes into account the size of the vehicle.

Figure 2 shows the reference trajectory and the actual trajectory during the second lap. The reference trajectory is perfectly followed, demonstrating the effectiveness of the driver model. Figure 3 shows the curvilinear abscissa and the yaw rate (negative, as the vehicle is negotiating a right turn, based on the adopted sign conventions) as a function of time for the baseline vehicle and the TVC vehicle. With the baseline vehicle, the time taken to complete the trajectory is 4.26 s. By activating the TVC, the laptime decreases to 3.84 s. So, there is a laptime improvement of ≈9% by using TVC with respect to the baseline vehicle.

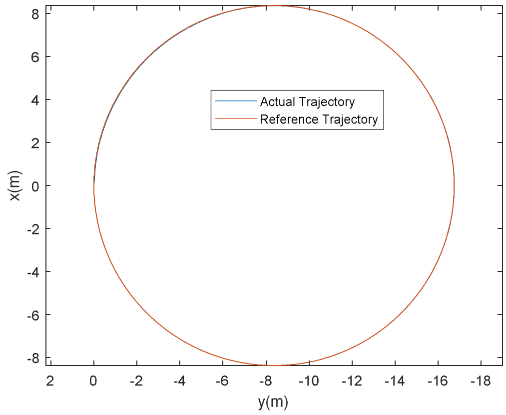

Figure 2: Actual and reference trajectory, coinciding thanks to the driver model.

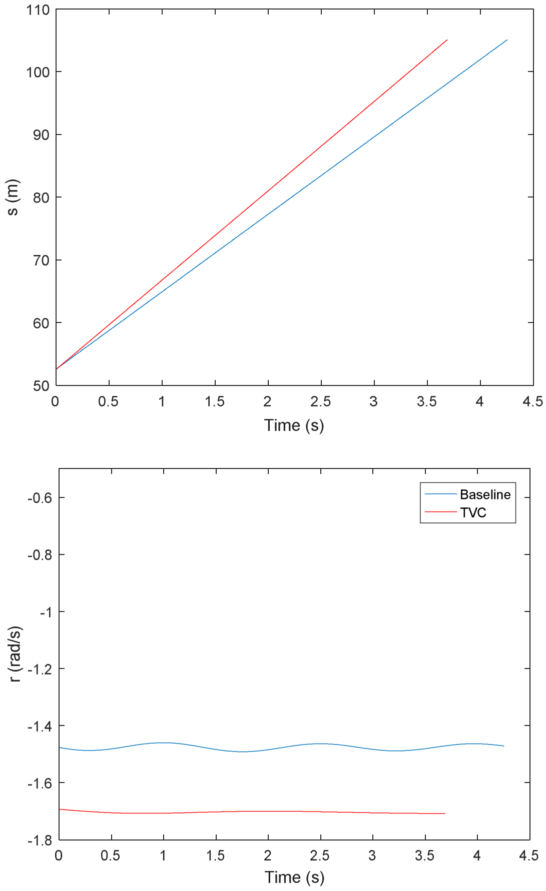

Figure 3: Comparison between baseline vehicle and TVC vehicle: (top) curvilinear abscissa as a function of time; (bottom) yaw rate as a function of time.

6 CONCLUSIONS

In this paper, a Torque Vectoring Control strategy was presented for a Formula SAE electric vehicle. Matlab-Simulink was used to implement a double track vehicle model featuring Pacejka tyres, and a driver model providing the steering angle and the acceleration/braking input. The implemented Torque Vectoring Control strategy allowed a time saving of around 9% during a skidpad test. Further developments will include the improvement of the simulation model adopted (e.g. by including tyre combined interaction), the assessment of the benefits of the proposed technique along a simulated lap, and the experimental validation on the T-ONE vehicle.

REFERENCES

[1] https://www.fsaeonline.com/, last accessed 18 May 2019.

[2] https://www.sae.org/attend/student-events/formula-sae-electric/about, last accessed 18 May 2019.

[3] Esmailzadeh, E., Goodarzi, A. and Vossoughi, G.R., *Optimal yaw moment control law for improved vehicle handling*, Mechatronics, 13(7), pp.659-675, 2003.

[4] Tota, A., Lenzo, B., Lu, Q., Sorniotti, A., Gruber, P., Fallah, S., Velardocchia, M., Galvagno, E. and De Smet, J., *On the experimental analysis of integral sliding modes for yaw rate and sideslip control of an electric vehicle with multiple motors*, International Journal of Automotive Technology, 19(5), pp.811-823, 2018.

[5] Lenzo, B., Sorniotti, A., De Filippis, G., Gruber, P., Sannen, K., *Understeer characteristics for energy-efficient fully electric vehicles with multiple motors*, EVS29 Proceedings, Montreal, Quebec, 2016.

[6] Lenzo, B., Bucchi, F., Sorniotti, A., Frendo, F., *On the handling performance of a vehicle with different front-to-rear wheel torque distributions*, Vehicle System Dynamics (2018, in press).

[7] Zhang, X., Göhlich, D., *Integrated traction control strategy for distributed drive electric vehicles with improvement of economy and longitudinal driving stability*, Energies, 10(1), p.126, 2017.

[8] Pacejka, H.B., *Tyre and vehicle dynamics*, Butterworth-Heinemann, 2006.

[9] Guiggiani, M., *The science of vehicle dynamics*, Springer, 2018.

[10] De Novellis, L., Sorniotti, A., Gruber, P., *Driving modes for designing the cornering response of fully electric vehicles with multiple motors*, Mechanical System and Signal Processing, 64- 65, 2005.

[11] Formula Sae rules, last accessed 21 February 2019:
https://www.fsaeonline.com/content/2017-18%20FSAE%20Rules%209.2.16a.pdf

[12] Braghin, F., Cheli, F., Melzi, S. Sabbioni, E., *Race driver model,* Computers and Structures, 86, 1503-1516, 2008.