

Developing a coupled turbine thrust methodology for floating tidal stream concepts: Verification under prescribed motion

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Abstract

Floating systems offer an opportunity to expand tidal energy resource through an increase in viable sites and greater flow speeds near the free surface. However, the close proximity of the free surface provides uncertainty regarding power delivery and survivability due to the presence of waves, which could be addressed through a numerical model that is capable of considering all components of a floating tidal system simultaneously. This paper presents the first step in the development of such a tool: using the open-source CFD libraries of OpenFOAM as a basis, a computationally efficient HATT model has been developed for generalised incident flow conditions using actuator theory. A thorough evaluation of the model's sensitivity to key considerations in the simulation of entire floating tidal systems, such as flow speed and mesh alignment, showed that the model is robust, ensuring that it is suitable for future extension to wave-driven environments and integration into a framework for such systems.

Keywords:

HATT model, CFD, OpenFOAM, Actuator theory, Marine renewable energy, Velocity deficit

1. Introduction

2 Development of the Offshore Renewable Energy (ORE) sector is of high
3 national importance for the UK and tidal stream represents a renewable energy
4 source with a number of desirable characteristics: it is more predictable than
5 other sources (such as wind and wave energy) providing simplified power grid
6 management; the resource tends to be concentrated by topography resulting
7 in desirable sites with high energy densities close to land masses (and to end
8 users), reducing costs in terms of installation and maintenance as well as cabling,
9 and; the majority of present device concepts, particularly Horizontal Axis Tidal

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10 Turbines (HATTs), benefit from technology that has been developed through
11 existing industries, such as hydro and wind power, accelerating the maturity of
12 the tidal stream industry.

13 However, the majority of the proposed tidal stream devices (particularly
14 those with the highest level of technology readiness) tend to be seabed-mounted
15 or gravity-based devices. Use of these concepts limits the number of viable sites
16 as the water depth has to fall within a narrow range, due to constraints on the in-
17 stallation and the required clearance above the turbine blades. The bathymetry
18 also has to be favourable, i.e. relatively flat and horizontal. Furthermore, due to
19 boundary layer effects, the flow speed at depth tends to be lower and so seabed
20 mounted devices do not exploit the full tidal stream resource at deeper sites,
21 providing a further constraint on water depth. Finally, seabed-mounted devices
22 tend to suffer from time-consuming and difficult installation, maintenance and
23 recovery procedures, greatly increasing the overall cost of the projects.

24 Consequently, a number of floating tidal stream concepts have been pro-
25 posed. These concepts have a number of distinct advantages over seabed-
26 mounted devices. Floating devices are not limited by water depth, bathymetry
27 or the presence of mobile sediments resulting in a greater number of potential
28 sites and hence a higher potential extractable resource. In addition to this,
29 floating tidal stream concepts tend to be easier and quicker to install, maintain
30 and recover since the majority of them can be towed to site using basic tug
31 boats, reducing the need for expensive specialist vessels. Floating devices also
32 benefit from positioning the turbine towards the top of the water column where
33 the flow velocity is maximum, again increasing the available resource compared
34 with seabed-mounted devices at certain sites.

35 However, despite the advantages of floating tidal stream concepts, it should
36 be noted that sites ideally suited for bottom mounted turbines tend to be shall-
37 lower, and hence generally experience faster flow speeds than deeper sites. Fur-
38 thermore, the flow observed by bottom-mounted and floating designs would be
39 similar at these shallower sites since the location of the turbine will be closer to
40 mid-water in both cases, limiting the advantage of a floating approach. Float-
41 ing devices also require additional considerations to be made, regarding their
42 location at the free-surface: firstly, these devices have an additional impact on
43 other stakeholders at the site, e.g. obstruction of navigation and visibility is-
44 sues, and; secondly, these devices are exposed to free-surface effects and waves
45 leading to concerns over both the power delivery and the survivability of the de-
46 vices. With so few deployments and limited operating hours to date, the effect
47 of proximity to the free-surface and wave-induced motion/loading on these de-
48 vices is not presently understood and crucial, under-pinning research is required
49 before these devices will become commercially viable.

50 As with other emerging ORE industries, modelling (both physical and nu-
51 merical) has now become an essential part of the development process. Numerical
52 modelling, in particular, is being relied upon more and more to overcome the
53 costs and scaling issues associated with physical modelling as well as to provide
54 the high resolution measurements and the quantitative descriptions required for
55 engineering design.

56 The modelling of floating tidal stream concepts, however, is incredibly com-
57 plex, combining: complicated hydrodynamics, such as the interaction between
58 waves and currents; floating structure; mooring system, and; (possibly multiple)
59 submerged turbines. Existing numerical codes are rarely capable of including
60 all of these elements and, for those that are, the behaviour tends to be linearised
61 and each of the elements treated separately, i.e. a ‘decoupled’ model. This cre-
62 ates considerable uncertainty in the power delivery and survivability predicted
63 by these models. A model which fully resolves the contribution of all elements
64 simultaneously as well as the fully nonlinear hydrodynamics and floating-body
65 motion is therefore desirable when assessing the behaviour of these devices, the
66 loads (in the mooring lines for example) and the power output from the tur-
67 bine. Unfortunately, such a model, if available, would likely suffer from excessive
68 CPU requirements making the use of such a tool prohibitive in routine design
69 processes.

70 Therefore, this paper details the first step in an incremental development
71 of an efficient numerical tool that is capable of predicting the fully nonlinear,
72 coupled behaviour of floating tidal stream systems. The article concentrates
73 on the methodology used to generate a computationally efficient HATT model
74 that predicts accurately the coupled forces on the turbine, and the fluid, while
75 remaining numerically stable under arbitrary motion. Using the open-source
76 Computational Fluid Dynamics (CFD) libraries of OpenFOAM as a basis, the
77 HATT model has been developed for generalised incident flow conditions using
78 actuator theory. It should be noted that actuator approaches have been im-
79 plemented in OpenFOAM in previous work, but they have largely focused on
80 validation of fixed wind turbine wake predictions [25, 26, 33, 44], and the impli-
81 cations for wind farm layout [32, 49]. Although these previous methods for fixed
82 turbines provide a basis for floating applications, they are not directly applicable
83 since they often require a very specific mesh layout to maximise alignment with
84 the turbine, which could not be achieved if the turbine position is constantly
85 updating. Therefore, the approach presented here incorporates the effects of the
86 turbine model on the fluid dynamics in the fully nonlinear Reynolds-Averaged
87 Navier-Stokes (RANS) solver via a ‘body-force’, momentum-sink-type method-
88 ology which allows the turbine position to move independently of the mesh.
89 This results in a strongly coupled model that is rigorously characterised, using
90 steady-state simulations, and demonstrated to be robust in a series of test cases
91 in which the turbine has prescribed motion.

92 **2. Background**

93 High-fidelity numerical methods, such as CFD, have been used extensively in
94 mature industries, like wind energy, to assess the behaviour and performance of
95 horizontal axis turbines. The development of tidal stream turbines has benefited
96 greatly from the knowledge gained in the wind industry, however; it is important
97 to recognise that tidal turbines can be subject to free-surface effects (such as
98 ventilation), possible cavitation and bi-directional flow and that the established
99 methods must be adapted to provide an accurate prediction of tidal stream

100 turbine behaviour [35]. Moreover, existing methods rarely take into account the
101 arbitrary motion of a freely moving turbine which, in the case of a floating tidal
102 stream device, could result in a turbine velocity comparable to the free-stream
103 velocity of the fluid. The following is a brief review of the most commonly used
104 methods for turbine modelling in CFD.

105 Arguably, the most realistic methods are ‘bladed-resolved’ techniques, in
106 which the turbine is directly meshed into the computational domain, allow-
107 ing the flow to be resolved as it passes the turbine blades [15, 19, 21, 43, 50].
108 Computationally, these methods are extremely expensive; the spatial resolution
109 must be fine enough in the vicinity of the turbine to accurately model both the
110 complex geometry of the turbine and the small-scale flow structures around the
111 blades; and the time step must be shorter than the temporal scales of these
112 small-scale structure, in order to resolve them accurately. Furthermore, the
113 mesh needs to be updated at every time step to accommodate the rotation of
114 the turbine, which often involves complex remeshing techniques (e.g. arbitrary
115 mesh interface (AMI) [46]), or additional interpolation overheads (e.g. overset
116 grid [21, 50]), further reducing the computational efficiency. Thus, although
117 potentially very accurate, the computational cost of blade-resolved methods is
118 often considered to be prohibitive in routine design processes and so cheaper al-
119 ternatives have been developed, aiming to represent the required characteristics
120 of the turbine without the need to resolve the flow around the turbine blades.

121 Actuator methods are a common approach when representing a horizontal
122 axis turbine in a fluid flow. The simplest cases are actuator disc models which,
123 based on momentum theory, apply a ‘resistance’ to the incoming flow over the
124 swept area of the turbine, similar to that of a porous disc (which is often used in
125 physical laboratory experiments as a simple representation of a turbine [3, 22,
126 31]). The applied resistance typically takes the form of a momentum sink, the
127 magnitude of which is based on the relationship between the free-stream velocity
128 and the thrust on (and power generated by) the turbine [6]. In these methods
129 a coarser, static mesh can be used, greatly reducing the computational costs
130 relative to blade-resolved methods. A number of authors have utilised actuator
131 disc models, in a wide range of numerical models including CFD simulations of
132 both wind [2, 7, 36, 45] and marine current [1, 4, 5, 13] turbine applications.
133 However, it has often been found that an actuator disc approach suffers from
134 the absence of rotational effects (particularly when the focus is on an accurate
135 prediction of the turbine wake [13, 12, 38]). Since the area of the actuator
136 disc is fixed, vorticity is shed into the wake as a continuous sheet from the
137 edges of the disc instead of from the tips of the blades [38]. To increase the
138 accuracy in unsteady flows and capture rotational effects, extensions to the
139 actuator disc methodology have been developed. These include: actuator line
140 [8, 16, 29, 42, 41] and actuator surface [20, 40, 47] methods in which the applied
141 momentum sink is distributed into finite lines or surfaces to represent the blades
142 of the turbine. Furthermore, in these methods the momentum sink is considered
143 to be transient with the position of the blades being updated based on the flow
144 speed and the characteristics of the turbine. In these methods, the torque on
145 the generator can then be calculated from the angular velocity of the rotation.

146 In addition, by discretising the blades in this way, distinct tip vortices can be
147 calculated (rather than the continuous vorticity sheet arising from an actuator
148 disc model) giving an improved representation of the turbine wake [38].

149 Blade element momentum theory (BEMT) [4, 12, 24, 27] is another extension
150 of the actuator disc model, combining blade-element and actuator methodologies
151 to calculate the lift and drag forces on each section of the discretised turbine
152 blades [6]. With the inclusion of a ‘tip loss correction factor’ [27, 30, 39], which
153 accounts for vortex effects at the blade tips, BEMT has been shown to have
154 good agreement with physical measurements and blade-resolved CFD models
155 [23, 27, 24]. Masters et al. [27] also suggests that further improvements can be
156 made by including a ‘hub loss correction factor’ [30] which, in a similar way to
157 the tip loss correction, accounts for vortex effects caused by the presence of the
158 rotor hub.

159 3. Methodology

160 A new library, allowing for the representation of tidal turbines, has been
161 designed and implemented in OpenFOAM (v. 4.1 [48]), an open source tool-
162 box aimed at solving continuum mechanics problems (including CFD). The
163 software is written in C++ and is based around the Object Orientated Pro-
164 gramming (OOP) paradigm, offering a large collection of solvers and shared
165 libraries. Consequently, the new turbine library is easily coupled with many
166 of the existing solvers. However, in this study the focus is on three solvers
167 of increasing complexity, that solve the incompressible RANS equations using
168 the Finite Volume Method (FVM): `simpleFOAM`, for steady-state simulations;
169 `pisoFOAM`, for transient single fluid cases; and `interFOAM` for simulating free
170 surface flows using a two-phase Volume Of Fluid (VOF) approach [37].

171 Since it is computationally expensive to resolve the flow structure around
172 the turbine, a simpler, more efficient approach for modelling the turbine is
173 adopted (compared with a blade-resolved method). Furthermore, since the over-
174 arching aim of this work is to develop a tool for modelling complete floating tidal
175 stream devices, the focus here is on facilitating the key aspects required for a
176 coupled system, i.e. accurate prediction of the coupled forces and numerical
177 stability with arbitrary motion of the turbine, rather than on developing a new
178 turbine model. Therefore, in this study, an actuator disc model has been used
179 to demonstrate the methodology (it is, however, worth noting that, due to the
180 object oriented nature of the code developed here, it is relatively straightforward
181 to include more sophisticated turbine models, such as a BEMT approach, in the
182 future).

183 To allow for arbitrary movement of the turbine through the computational
184 domain, the turbine model here is based upon a ‘weighted body force imple-
185 mentation’ which, at each time step, identifies and applies weights to a finite
186 ‘region’ of the computational domain (representing the turbine). This requires
187 no constraints on the local mesh structure (a requirement for the complete,
188 coupled device), contrary to common methods used in static cases which often
189 require the mesh to be highly contrived in the disc region [7, 12, 24]. These

190 weights are then used to determine the local velocity at the turbine position as
 191 well as to add an additional, equal and opposite force (based on the thrust on
 192 the turbine) to the momentum equations, ensuring that the model is two-way
 193 coupled.

194 3.1. Actuator Disc Theory

195 Actuator disc theory states that, in a steady current, the mass flow rate must
 196 be conserved. Hence, the stream-wise velocity at the disc, u_t , can be determined
 197 through the relationship

$$u_t = (1 - a)u_\infty, \quad (1)$$

198 where u_∞ is the free stream velocity, and a is the axial induction factor [6]. Using
 199 momentum theory it is then possible to formulate expressions for the thrust, T ,
 200 on and power, P , generated by the disc as functions of the free stream velocity

$$T = \frac{1}{2}\rho C_t A u_\infty^2, \quad (2)$$

201

$$P = \frac{1}{2}\rho C_p A u_\infty^3, \quad (3)$$

202 where A is the area of the disc and C_t and C_p are the thrust and power coeffi-
 203 cients respectively, where

$$C_t = 4a(1 - a), \quad (4)$$

204

$$C_p = 4a(1 - a)^2. \quad (5)$$

205 Actuator disc methods are common in numerical models due to their sim-
 206 plicity, requiring only knowledge of the thrust coefficient and the free-stream
 207 velocity. In this study, however, the turbine methodology is required to work
 208 in transient flows such as those experienced in wave-driven environments, and
 209 hence the free stream velocity is not known *a priori*. Therefore, the actuator
 210 disc methodology is reverse engineered based on the known velocity at the tur-
 211 bine in order to estimate the instantaneous free stream velocity and the thrust
 212 on the disc (explained further in Section 3.4).

213 3.2. Weighting Function

214 The first stage in the turbine model is the calculation of the weighting func-
 215 tion (or field), W , which determines the contribution of each cell in the compu-
 216 tational domain to the local flow velocity at the turbine (W also determines the
 217 distribution of the thrust force on the turbine and the corresponding distributed
 218 momentum sink (see Section 3.4)).

219 In this study, actuator disc theory is used to represent a HATT and so
 220 a cylindrical region is selected to represent the turbine. The cylinder has: a
 221 radius, R , equal to the radius of the swept area of the turbine blades, and;
 222 an axis coincident with that of the turbine. At each time step, all cells in the
 223 computational domain are evaluated to find the distance between their centre

224 and the central line of this ‘turbine region’, i.e. the turbine axis. For a turbine
 225 axis parallel to the global x -axis,

$$\mathbf{dx} = \mathbf{x}_{cell} - \mathbf{x}_{hub} = (dx, dy, dz), \quad (6)$$

226 where \mathbf{x}_{cell} and \mathbf{x}_{hub} are the coordinates of the cell centre and the hub position of
 227 the turbine respectively. The x -component, dx , corresponds to the axial distance
 228 from the turbine plane and the axial width of the turbine is $2N\sigma$ (Figure 1).
 229 The radial components of the cylindrical region, dy and dz , are used to define
 230 another vector

$$\mathbf{r} = (0, dy, dz), \quad (7)$$

231 whose length, $|\mathbf{r}|$, determines the radial distance from the turbine axis. The
 232 edge of the turbine will then be located on the line $|\mathbf{r}| = R$. Consequently, the
 233 turbine region is made up of cells that have r values that fall within the range
 234 $R_{hub} \leq |\mathbf{r}| \leq R$, where R_{hub} is the hub radius (Figure 1a).

235 In order to ensure mathematically smooth values for the calculated local
 236 flow speed (and thrust forces) through time, for the general case in which the
 237 motion of the turbine is not concurrent with the motion of the mesh cells, the
 238 turbine region is given a finite width. The width is defined as $2N\sigma$ ($N\sigma$ either
 239 side of the centreline in the axial direction), where σ is the Gaussian root mean
 240 square width as shown in Figure 1b and N is a user-defined coefficient to limit
 241 the width of the turbine region (set by default to 2 according to the sensitivity
 242 analysis in Section 4.2). A Gaussian weighting, is then determined for each cell
 243 in the computational domain

$$W = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{dx^2}{2\sigma^2}\right) & \text{if } |dx| \leq N\sigma \text{ and } R_{hub} \leq |\mathbf{r}| \leq R, \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

244 with cells closer to the central plane of the turbine region having the largest
 245 weights and, therefore, contributions in the proceeding calculations. Using this
 246 method, any number of turbine regions can be represented simultaneously with-
 247 out a significant increase in computational effort. The sensitivity of the model
 248 to various parameters is discussed in Section 4. Note that, for simplicity, the
 249 presented model considers the weighting inside the turbine region to be uni-
 250 form in the radial direction and for all cells outside of the turbine region the
 251 contribution to the local velocity is zero.

252 3.3. Orientation

253 For generality, the turbine model has been developed to allow the turbine
 254 to be placed in any orientation relative to the coordinate system of the com-
 255 putational domain. This is achieved via the orientation matrix, \mathbf{Q}_0 , defined as

$$\mathbf{Q}_0(\alpha, \beta, \gamma) = \mathbf{R}_x\left(\frac{\alpha}{2}\right)\mathbf{R}_y\left(\frac{\beta}{2}\right)\mathbf{R}_z(\gamma)\mathbf{R}_y\left(\frac{\beta}{2}\right)\mathbf{R}_x\left(\frac{\alpha}{2}\right), \quad (9)$$

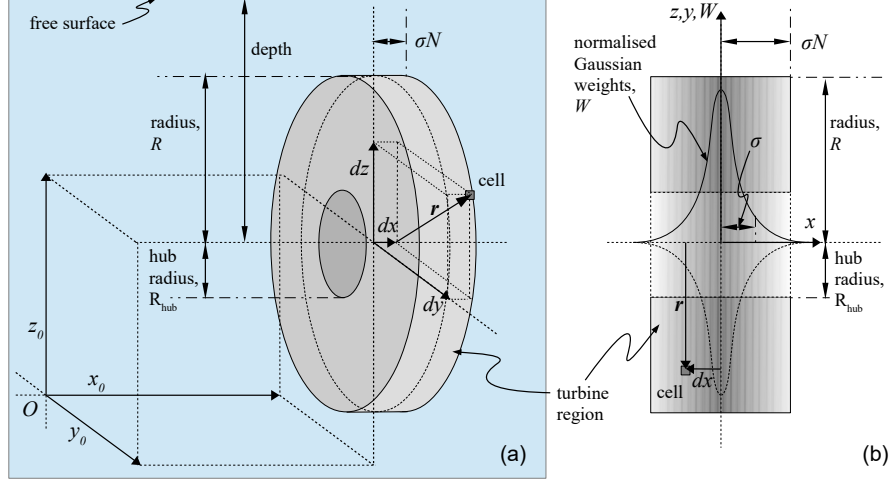


Figure 1: Schematic representation of a) the ‘turbine region’ and b) the Gaussian weighting function used in the turbine model.

257 where \mathbf{R}_x , \mathbf{R}_y , \mathbf{R}_z are matrices defining a rotation about the global x , y and z
 258 axes respectively. These are defined as

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix}, \quad (10)$$

259

$$\mathbf{R}_y = \begin{pmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{pmatrix}, \quad (11)$$

260

$$\mathbf{R}_z = \begin{pmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (12)$$

261 where α , β and γ are the angles of roll, pitch and yaw respectively.

262 A new coordinate system, based on the orientation matrix, is then generated
 263 and each cell in the domain is assigned a new set of coordinates in the rotated
 264 system according to

$$\mathbf{dx}' = (dx', dy', dz') = \mathbf{Q}_0^T \cdot \mathbf{dx} \quad (13)$$

265 The turbine weighting is then calculated (as described in Section 3.2) but with
 266 \mathbf{dx} replaced by \mathbf{dx}' .

267 3.4. Free Stream Velocity Calculation

268 As described in Section 3.1, the free stream velocity, u_∞ , is required to
 269 calculate the thrust on, and power generated by, the turbine. However, in

270 general and particularly in wave and current cases, u_∞ is time-varying and is
 271 not known in advance. Therefore, a method to determine the instantaneous free
 272 stream velocity from the known velocity field, local to the turbine region, has
 273 been developed. Using the weighting function, W , as described in Section 3.2,
 274 the weighted average relative velocity, \mathbf{u}_{av} in the turbine region can be calculated
 275 using

$$\mathbf{u}_{av} = \left(\frac{1}{V} \sum_{i=1}^N \mathbf{u}_i V_i W_i \right) - \mathbf{v}_{hub}, \quad (14)$$

276 where N is the total number of cells in the turbine region, \mathbf{v}_{hub} is the velocity
 277 of the turbine and V is the total weighted volume of the turbine region based
 278 on the volume of each cell, V_i ,

$$V = \sum_{i=1}^N V_i W_i. \quad (15)$$

279 The local speed in the axial direction, u_t , is then determined using

$$u_t = |\mathbf{u}_{av} \cdot \mathbf{x}_{axis}|, \quad (16)$$

280 where

$$\mathbf{x}_{axis} = \mathbf{Q}_0^T \cdot \hat{\mathbf{x}}, \quad (17)$$

281 is a unit vector parallel to the axis of the turbine and $\hat{\mathbf{x}}$ is a unit vector in the
 282 global x-direction. For a turbine with known axial induction factor, a , or thrust
 283 coefficient, C_t , the instantaneous free stream velocity is then calculated using a
 284 rearrangement of equation (1).

285 The instantaneous thrust on the turbine, T , and the instantaneous power
 286 generated, P , can then be calculated using equations (2) and (3) respectively.

287 3.5. Update Momentum Equation

288 Assuming laminar flow and neglecting surface tension, the incompressible
 289 (unsteady) RANS equations take the following form

$$\frac{\partial(\mathbf{u})}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla \frac{p}{\rho} + \nabla^2(\nu\mathbf{u}) + \mathbf{g} + \mathbf{T}, \quad (18)$$

290

$$\nabla \cdot \mathbf{u} = 0, \quad (19)$$

291 where p is the pressure, ρ is the fluid density, $\mathbf{u} = (u, v, w)$ is the fluid velocity,
 292 ν is the kinematic fluid viscosity, \mathbf{g} is acceleration due to gravity and \mathbf{T} is the
 293 momentum sink due to the presence of the turbine.

294 To achieve coupling between the calculated thrust on the turbine, T , and
 295 the associated momentum sink in the fluid, \mathbf{T} , the thrust force per unit mass
 296 is distributed across the turbine region using the weighting function, W , (from
 297 Section 3.2) such that

$$\mathbf{T}_i = \pm \frac{TV_i W_i}{\rho V} \mathbf{x}_{axis}, \quad (20)$$

298 where \pm is positive for flow in the direction of the turbine axis or negative for
299 reversed flow.

300 The governing equations for the fluid (equation 18) can then be solved for the
301 next time step (transient) or iteration (steady-state) and the processes described
302 in Sections 3.2-3.5 repeated until the maximum time (transient) or convergence
303 (steady-state) is reached.

304 4. Steady-State Analysis

305 In this section, the sensitivity of the turbine model, to the various imple-
306 mentation parameters discussed above, is demonstrated using the single phase,
307 steady-state solver `simpleFoam`. For this analysis, a single static turbine (i.e.
308 $|\mathbf{v}_{hub}| = 0$) is considered, in isolation. The objective here is to demonstrate the
309 robustness of the model as well as verify that the model returns the expected
310 results for idealised cases.

311 The modelling technique described in Section 3, relies fundamentally on pre-
312 dicting the free stream velocity, u_∞ , from the local velocity in the turbine region,
313 u_t . Hence, to quantify the accuracy of the model and assess the sensitivity of the
314 approach to the key parameters, comparisons are made between the predicted
315 value of u_∞ and the user defined value at the inlet. The convergence criteria are
316 kept constant throughout this analysis and are set to have maximum residual
317 values of 10^{-4} and 10^{-5} for the pressure and velocity fields respectively.

318 4.1. Initial Setup

319 Unless stated otherwise, the sensitivity test cases use a turbine model with
320 a radius of 2 m (the hub radius is set to zero), a $C_t = 0.9$, $N = 2$, and $\sigma = 0.15$.
321 The prescribed free stream velocity is 1 m s^{-1} .

322 The required cell size around the turbine was evaluated using a mesh con-
323 vergence study. The initial numerical domain was a $200 \times 20 \times 20 \text{ m}$ cuboid
324 consisting of cubic cells (side length = 0.5 m) and a 5 m cubic region around the
325 turbine, refined by one level using the octree refinement strategy [9]. The re-
326 finement in the turbine region was then incrementally increased until the value
327 of the predicted free stream velocity changed by less than 0.1% between meshes.
328 This was found to occur for cells 3.125 cm in length (4 levels of refinement). All
329 remaining simulations, in the sensitivity analysis, use cells of this size in the
330 turbine region unless otherwise stated.

331 To optimise the dimensions of the computational domain, with respect to
332 a reduction in both the blockage effects arising from the boundaries and the
333 required computational effort, a series of tests focusing on the inlet, outlet and
334 side wall distances were performed. By incrementally increasing one of these
335 distances (whilst fixing the other two), the minimum distance from the turbine
336 was determined based on convergence of the predicted free stream velocity. By
337 applying this method to all boundaries, a $100 \times 100 \times 100 \text{ m}$ numerical domain
338 was selected for the sensitivity analysis, running from -50 m to 50 m in each
339 dimension, with the centre of the turbine located at the origin (Figure 2).

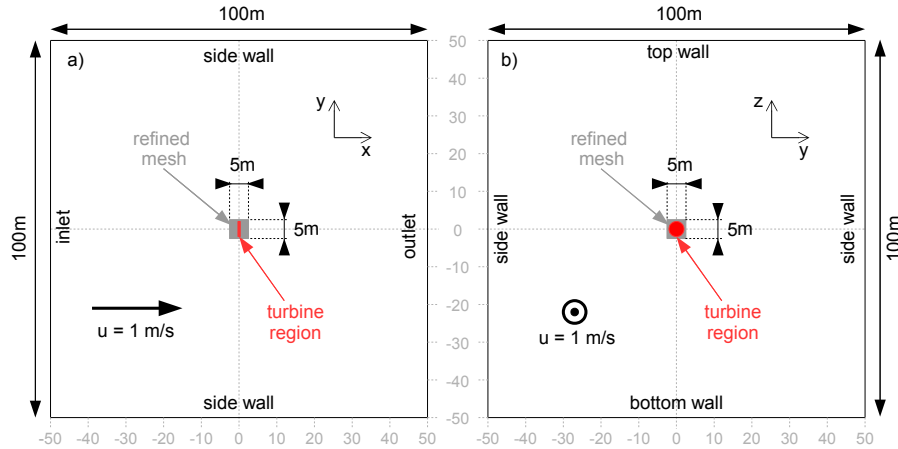


Figure 2: Sketch of the numerical domain used for the steady-state sensitivity analysis in a) the $x - y$ plane at $z = 0$, and b) the $y - z$ plane at $x = 0$, with the turbine region indicated in red.

340 The side walls of the domain are considered to be solid and have no-slip
 341 boundary conditions applied to them, the inlet boundary has the free stream
 342 velocity prescribed as a boundary condition and the outlet has zero gradient
 343 conditions to represent flow leaving the domain. The initial condition is the
 344 prescribed free stream velocity used at the inlet, and turbulence modelling has
 345 not been considered in this analysis.

346 4.2. Width Parameters

347 Actuator theory is based on an infinitesimally thin disc [11, 18] and therefore,
 348 in the present model, a delta function to describe the turbine width would
 349 likely give the most accurate solution. This would be possible in finite volume
 350 methods, if the turbine was fixed and aligned perfectly with the cell centres (or
 351 faces). However, in the present study, the turbine model is coupled to a floating
 352 structure which (as well as having arbitrary mesh motion and structure), in
 353 general, has arbitrary alignment with the mesh. It is, therefore, necessary to
 354 assign a finite region in which to ascertain the local flow velocity and apply
 355 the corresponding momentum sink, as an infinitesimally thin region would not
 356 perform well in cases (or time steps) in which there is a misalignment with the
 357 computational mesh. In this section, the sensitivity of the model to the width
 358 of this finite turbine region, $2N\sigma$, is considered.

359 A series of simulations were performed using different combinations of N
 360 and σ and, using the converged solution for u_∞ , the error [%] was calculated
 361 with respect to the prescribed inlet flow speed (1 m s^{-1}).

362 Figure 3a shows the error, as a function of $\sigma\sqrt{N}$, for the case in which
 363 the central plane of the turbine is coincident with cell faces. For higher values

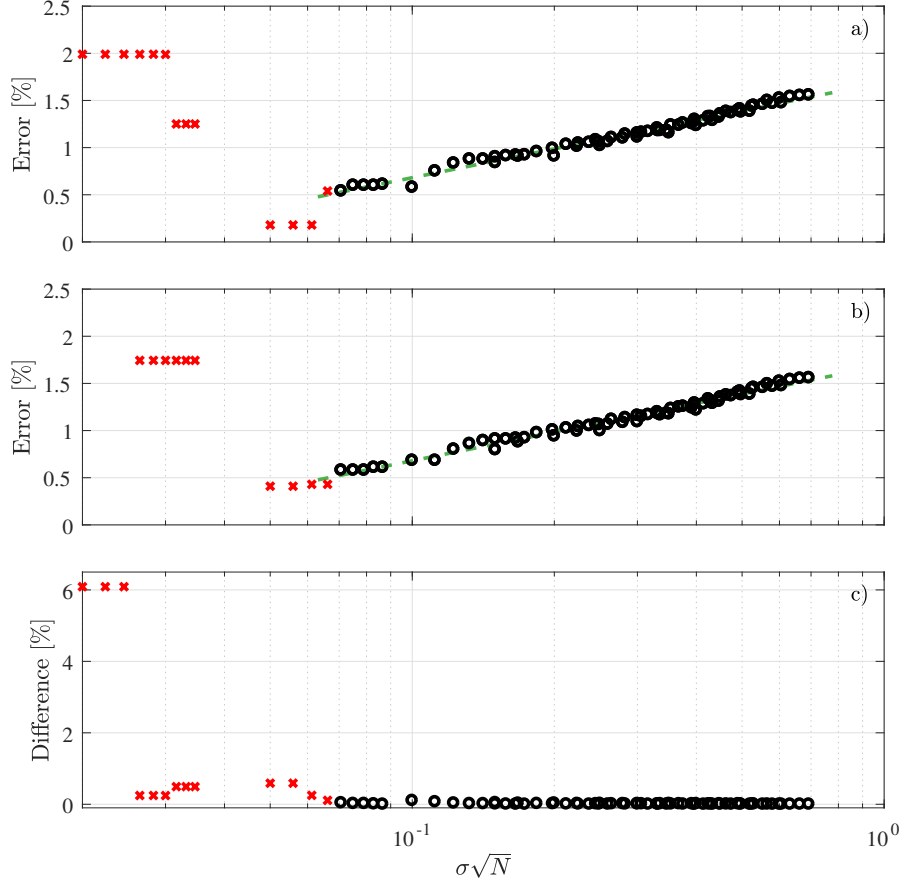


Figure 3: Error [%] in u_∞ as a function of $\sigma\sqrt{N}$, when the turbine is aligned with a) cell faces, b) cell centres. Values which follow a log law (---) are indicated as circles (o) otherwise they are represented by crosses (x). Also shown (c) is the difference [%] between the solutions obtained with the turbine aligned with cell faces and with cell centres.

364 of $\sigma\sqrt{N}$ (marked with o), as the width moves towards the ideal case of an
 365 infinitesimally thin disc, the error decreases with a logarithmic trend (indicated
 366 by the dashed line (---)). However, below a certain ‘cut-off’ the logarithmic
 367 trend breaks down (x); the error initially decreases, before rapidly increasing.
 368 The rapid increase in error at low $\sigma\sqrt{N}$ values is a consequence of the total
 369 width of the turbine region approaching the width of a single cell. Based on
 370 this, it seems that the optimal turbine width would be the value at which this
 371 cut-off occurs, i.e. small enough to minimise the error, but large enough that
 372 the results lie in the well-behaved, logarithmic region. For the combination
 373 of mesh, turbine and flow speed used here, the cut-off value was found to be
 374 $\sigma\sqrt{N} = 0.07$, with an error of 1% or less for $0.07 \leq \sigma\sqrt{N} \leq 0.11$.

375 In general, the turbine’s central plane will be positioned arbitrarily relative

376 to each mesh cell and hence the solution must be independent of this parameter.
 377 Figure 3b shows the error for the case in which the central plane of the turbine
 378 is coincident with the mesh cell centres. With respect to the staggering of
 379 the mesh cells relative to the turbine central plane, this represents the most
 380 extreme alternative to the case used above which was coincident with the mesh
 381 faces. From Figure 3b, it can be seen that the error for a turbine central plane
 382 coincident with the cell centres has the same trend as that for that coincident
 383 with the cell faces (Figure 3a).

384 The difference between the two solutions is presented in Figure 3c. For low
 385 values of $\sigma\sqrt{N}$ there are unacceptable differences between the solutions; it
 386 is anticipated that, in the general case of a turbine moving arbitrarily through
 387 the mesh, these would generate unphysical fluctuations in critical values (e.g.
 388 the thrust on the turbine). Although small differences can be observed around
 389 $\sigma\sqrt{N} = 0.1$, the differences between the two solutions are much smaller in the
 390 logarithmic region. This is due to the increased turbine width distributing the
 391 weighting over more cells and reducing the sensitivity to single values (including
 392 the difference between coincidence with a cell centre or a cell face).

393 The results presented in Figure 3 imply that the model is not overly sensitive
 394 to the coincidence of the mesh cells and the turbine central plane, provided $\sigma\sqrt{N}$
 395 is reasonably large. This is essential for the model to be successful in a moving
 396 mesh simulation. However, the results also demonstrate that for high levels
 397 of accuracy the turbine region, i.e. $\sigma\sqrt{N}$, should be kept relatively small. As
 398 a compromise, in this particular case, N and σ have been chosen to achieve
 399 $\sigma\sqrt{N} = 0.11$ giving an error of around 1%, whilst maintaining a solution that
 400 is suitably independent of the coincidence of the mesh cells and turbine central
 401 plane.

402 4.3. Mesh Dependency

403 As mentioned above, the error in the predicted value of u_∞ appears to have
 404 some mesh dependence at low $\sigma\sqrt{N}$ values. Consequently, further simulations
 405 were performed, with varying σ and N , with the mesh in the region of the turbine
 406 one octree level finer (1.5625 cm) or one level coarser (6.25 cm) compared to the
 407 mesh used in Section 4.2 (see Table 1 for details).

408 Figure 4 shows the error [%], as a function of $\sigma\sqrt{N}$, for each of the three
 409 mesh resolutions (original (\circ), finer (\triangle) and coarser (\diamond)). The dashed lines
 410 represent the logarithmic trends of the mesh in the corresponding colour. The

Table 1: Mesh resolution, aspect ratio, octree level and total size used in each of the steady-state, static cases.

| Mesh | Background | | Refined Region | | Total Cells |
|--------|----------------|----|----------------|----------------|-------------|
| | Δx [m] | AR | Oct. lvl. | Δx [m] | |
| Coarse | 0.5 | 1 | 3 | 0.0625 | 8.5M |
| Medium | 0.5 | 1 | 4 | 0.0313 | 12.2M |
| Fine | 0.5 | 1 | 5 | 0.0156 | 40.7M |

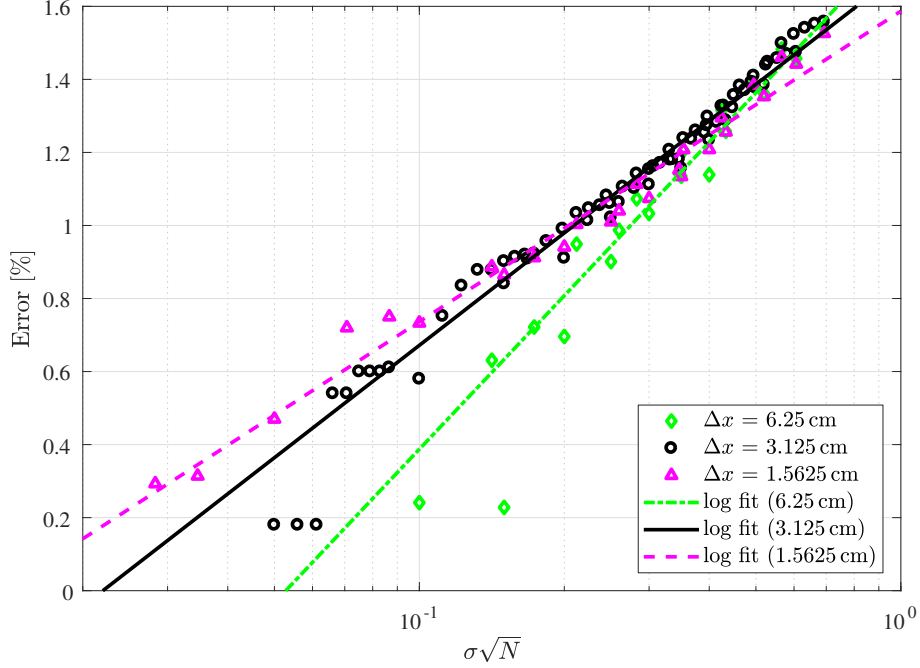


Figure 4: Comparison of the error [%] as a function of $\sigma\sqrt{N}$ for three different mesh resolutions: $\Delta x = 6.25$ cm (\diamond), 3.125 cm (\circ) and 1.5625 cm (\triangle). The lines represent the logarithmic error trend for each case dash-dotted, solid, and dashed, respectively.

411 gradient of the trend lines decreases with increasing resolution, as does the cut-
 412 off width, \mathcal{C} , defining the end of the logarithmic region, i.e. for finer meshes, the
 413 logarithmic region holds for much lower $\sigma\sqrt{N}$ and so higher accuracies can be
 414 achieved before the model becomes too sensitive to motion through the mesh.
 415 For higher $\sigma\sqrt{N}$ the solutions from the three meshes are very similar, indicating
 416 mesh independence. It is unclear whether the logarithmic regions continue for
 417 much higher values of $\sigma\sqrt{N}$, but it is unlikely that a width greater than those
 418 considered here would be beneficial due to the increased error.

419 Based on the logarithmic trends observed in Figure 4, the error in the predicted
 420 value of u_∞ takes the form

$$\mathcal{E}(\%) = \mathcal{A} \ln(\sigma\sqrt{N}) + \mathcal{B}, \quad \text{if } \sigma\sqrt{N} \geq \mathcal{C} \quad (21)$$

421 where \mathcal{A} , \mathcal{B} and \mathcal{C} are all functions of mesh resolution Δx .

422 Figure 5 shows that \mathcal{A} and \mathcal{C} (and to a reasonable degree \mathcal{B}) are linear
 423 functions of mesh resolution Δx , and for this case

$$\mathcal{A} = 5.0807\Delta x + 0.288, \quad \mathcal{B} = 4.0155\Delta x + 1.5418, \quad \mathcal{C} = 2.56\Delta x - 0.01. \quad (22)$$

424 The coefficients in equation (22) are likely to be functions of the turbine diameter
 425 (4 m) and the incident flow speed (1 m s^{-1}), however, by combining equations

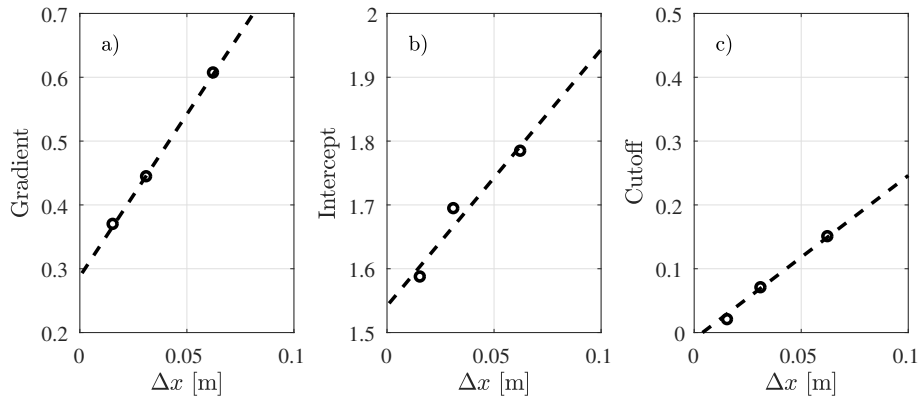


Figure 5: Properties of the logarithmic error trend as a function of mesh resolution: a) gradient, b) intercept, and c) the minimum $\sigma\sqrt{N}$ cut-off value for which the trend holds.

426 (21) and (22) the error in this case could be estimated for any given mesh
 427 discretisation, allowing a suitable value of $\sigma\sqrt{N}$ to be chosen.

428 4.4. Flow Speed

429 Over a tidal cycle, a turbine will experience a wide range of flow speeds;
 430 furthermore, for floating tidal energy applications the turbine will be subject
 431 to a combination of both tidal currents and waves, i.e. oscillatory flow. Con-
 432 sequently, in order to model a full floating tidal energy concept, in realistic
 433 conditions, it is vital that the performance of the turbine model is not overly
 434 sensitive to the flow speed.

435 To assess the performance of the present model as a function of free stream
 436 velocity, u_∞ , a series of simulations were run with different prescribed flow
 437 speeds (in the range $0.25 - 4 \text{ m s}^{-1}$). Figure 6a presents the predicted u_∞ values
 438 against the prescribed inlet velocities, with the red, dotted line representing
 439 perfect prediction. The predicted and prescribed values generally agree very
 440 well, although as the flow speed increases the deviation does appear to increase.
 441 Considering Figure 6b (which shows the error as a function of prescribed in-
 442 let speed), it is clear, however, that the relative error remains very similar
 443 throughout, i.e. $\approx 0.95\%$ for all of the 16 flow speeds tested. It can therefore
 444 be concluded that, the model developed here performs equally well over the
 445 required range of incident flow speeds.

446 4.5. Turbine Characteristics

447 So far in this section, the turbine characteristics have been fixed to repre-
 448 sent a generic turbine with a radius of 2 m and a thrust coefficient of 0.9. In
 449 realistic applications, these parameters will be determined by the turbine man-
 450 ufacturer and, although it may be constant during operational conditions, the
 451 thrust coefficient could potentially change in order to produce favourable out-
 452 put characteristics or reduce the chance of damage to the generator at high flow

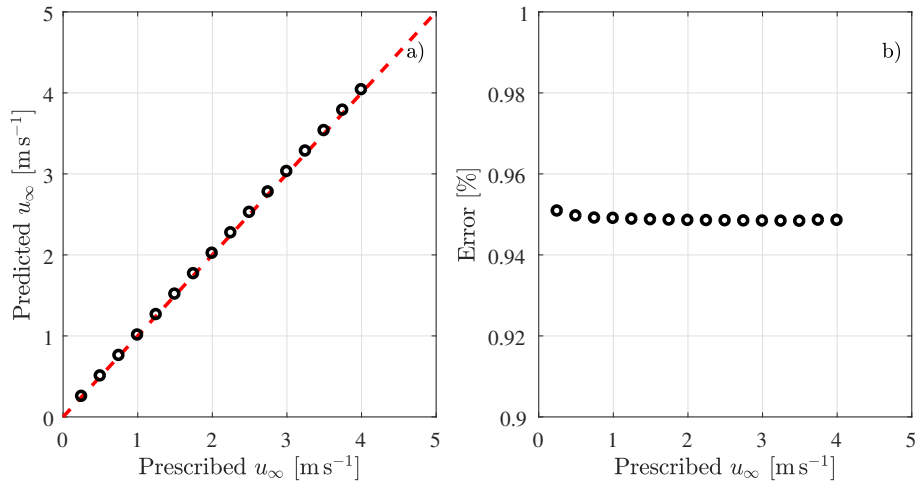


Figure 6: Sensitivity of the model to flow speed: a) predicted free stream value as a function of the prescribed value at the inlet, and b) the relative error as a function of prescribed flow speed.

453 speeds. Hence, for completeness, and generality, the performance of the model
 454 is assessed for turbines of different thrust coefficients, utilising the same mesh
 455 and simulation setup as presented earlier.

456 Figure 7 shows the error [%] as a function of thrust coefficient, C_t . In this
 457 case: For low thrust coefficients ($C_t < 0.65$) the free stream velocity is slightly
 458 under-predicted; for high thrust coefficients $C_t \geq 0.65$ the free stream velocity
 459 is over-predicted. Further work is required to understand this behaviour for
 460 different turbine characteristics, flow speeds and domain sizes but it appears
 461 that, for all except the very highest C_t values, the predicted free-stream velocity
 462 is well within 1% of the true value.

463 5. Prescribed Motion Cases

464 The aim of this work is to develop a turbine methodology that can be used
 465 in the simulation of entire floating tidal stream systems. In Section 4, the
 466 methodology is shown to predict with good accuracy the free stream velocity,
 467 and hence the thrust, in the case of a static turbine. However, when simulat-
 468 ing the complete coupled system, the movement of the device (in any of six
 469 degrees of freedom) leads to a time-varying turbine position with arbitrary lo-
 470 cation and alignment with the numerical grid. This prevents the use of highly
 471 contrived meshes designed solely to capture the turbine well [7, 12, 24] and
 472 requires a methodology capable of seamlessly transitioning through the mesh
 473 without causing numerical instabilities. In this section the ability of the present
 474 method, to meet this requirement, is demonstrated via a series of test cases in
 475 which the turbine is given prescribed motion through the computational domain.

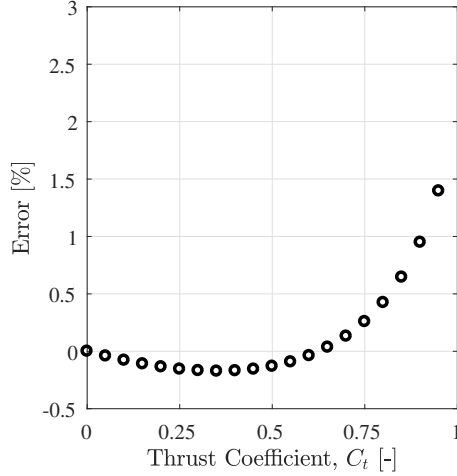


Figure 7: Sensitivity of the model accuracy to turbine thrust coefficient, C_t .

476 In each case, a turbine with $R = 2$ m, $C_t = 0.9$, $N = 2$ and $\sigma = 0.15$ is moved
477 with prescribed velocity, \mathbf{v} , through initially still water and the predicted free
478 stream velocity is again compared to the true u_∞ . The tests were run for
479 60 s using the modified transient solver `pisoFoam` (see Section 3) and the same
480 $100 \times 100 \times 100$ m domain as that described in Section 4 (see Figure 2). Only
481 the refined region in the path of the turbine, its initial location/orientation and
482 prescribed velocity vary in each case. The boundary conditions are the same as
483 described in Section 4 (with inflow speed of 0 ms^{-1}), the initial conditions are
484 zero flow conditions, and turbulence modelling has not been considered in this
485 section.

486 In all cases, the speed of the turbine is ramped up to avoid effects arising
487 from instantaneous movement of the turbine. The ramp up is described by the
488 sinusoidal function

$$\mathbf{v}_{hub} = \begin{cases} \frac{1}{2}\mathbf{v} \left[1 - \cos\left(\frac{\pi}{t_{ramp}}t\right) \right] & \text{if } t < t_{ramp} \\ \mathbf{v} & \text{if } t \geq t_{ramp}, \end{cases} \quad (23)$$

489 where t_{ramp} is the ramp up time (set to 20 s in this work) and the position of
490 the turbine is updated based on the integral of this function.

491 5.1. Constant Linear Velocity

492 In the first two test cases, the turbine is given a constant velocity through
493 the mesh, $|\mathbf{v}_{hub}| = 1 \text{ ms}^{-1}$. These cases are considered to be equivalent to the
494 idea of a physical towing tank and it is anticipated that the turbine behaves the
495 same as if it were fixed in uniform flow with velocity equal to the prescribed
496 motion, i.e. the relative flow over the turbine is the same.

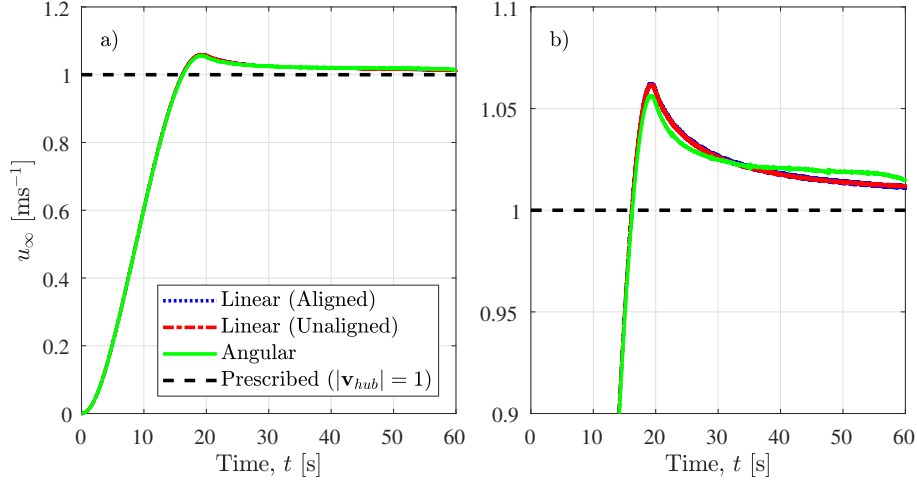


Figure 8: Time series of the prediction of u_∞ in the aligned (\cdots), misaligned ($-\cdot-$) and angular ($—$) prescribed velocity cases, along with the prescribed velocity ($---$). Both the full time series (a) and a magnified view (b) are presented.

Table 2: Initial turbine position, orientation and refined region for each of the prescribed motion cases. The mesh resolution is given by the ‘coarse’ mesh in Table 1, and the refined region is $[-2.5, 2.5]$ in the z direction in all cases.

| Case | Init. Hub Pos. | | | Refined Region | | | # Cells | Timestep Δt [s] |
|------------|----------------|-----|------------|----------------|---------------|------------|---------|-------------------------|
| | x | y | γ | x | y | γ | | |
| Aligned | 30 | 0 | 0° | $[-31, 31]$ | $[-2.5, 2.5]$ | 0° | 14.6M | 0.01 |
| Misaligned | 18 | -18 | 45° | $[-22, 40]$ | $[-20, 20]$ | 45° | 14.6M | 0.01 |
| Angular | 0 | -8 | 0° | $[-13, 13]$ | $[-13, 13]$ | 0° | 19.1M | 0.01 |

497 The first test demonstrates the case of the turbine moving parallel to the
498 x -axis, i.e. aligned with the mesh. At time $t = 0$ s the centre of the turbine
499 is located at $\mathbf{x}_{hub} = (30, 0, 0)$ m and, after the period of ramp up, the turbine
500 moves with the constant prescribed velocity, $\mathbf{v}_{hub} = (-1, 0, 0)$ ms^{-1} . The mesh
501 in the region along the path of the turbine (Table 2) is refined by three octree
502 levels ($\Delta x = 0.0625$ m), which (based on the information in Section 4) gives an
503 anticipated error in the predicted free stream velocity of approximately 1%.

504 Figure 8a presents a time series of the predicted free stream velocity, u_∞ ,
505 for the aligned case (\cdots) and the final, prescribed speed, i.e. the true solution
506 ($---$). The initial ramp up of the turbine velocity can be observed, along with
507 an over-shoot ($\sim 6\%$) as the ramp up period ends. After this, the prediction
508 converges towards the anticipated solution with an error of approximately 1%
509 (observed at time $t = 60$ s). Crucially, the prediction of u_∞ is relatively smooth,
510 indicating that the present methodology works well for turbines moving parallel
511 to the axes of the mesh. There are some very small fluctuations (~ 0.001 ms^{-1})
512 in the prediction that can be observed when considered more closely (Figure 8b),
513 which are likely due to the instantaneous position of the turbine (see Section 4.2)

514 but these are considered to be negligible and are not expected adversely to affect
515 the stability of the simulation.

516 The second case considers turbine motion that is not aligned with the axes of
517 the mesh. This is achieved by rotating the turbine axis by 45° and prescribing a
518 constant velocity $\mathbf{v}_{hub} = (-0.707, 0.707, 0) \text{ ms}^{-1}$. The turbine is initially located
519 at $\mathbf{x} = (18, -18, 0)$ and the mesh along the path of the turbine (Table 2) is again
520 refined by three octree levels (again with an anticipated error of 1%). The
521 predicted value of u_∞ , in this case, is presented in Figure 8a ($-\cdot-$). The time
522 series is very similar to that observed for the aligned case; after the initial ramp
523 up period, the prediction overshoots before converging to 1% of the prescribed
524 speed by $t = 60 \text{ s}$. The error is marginally larger than that observed in the
525 aligned case but, interestingly, the fluctuations in the prediction are smaller
526 (Figure 8b). This is thought to be caused by the ‘mis-alignment’ of the turbine:
527 In the aligned case, the edge of the turbine region crosses cell faces at all points
528 simultaneously; hence, any slight differences between the cells also contribute to
529 the solution simultaneously resulting in a more noticeable change. In the non-
530 aligned case, the edge of the turbine region crosses the cell faces arbitrarily and
531 hence the differences contribute asynchronously resulting in lower fluctuations.

532 5.2. Constant Angular Velocity

533 In Section 5.1 the present turbine methodology is shown to be robust and
534 accurate, when moving at a constant linear velocity (either aligned and mis-
535 aligned with the computational mesh). Floating tidal stream devices, however,
536 are capable of moving in all six degrees of freedom and so, it is crucial that the
537 methodology can also accommodate rotational motion through the mesh.

538 The third prescribed motion test case considers the turbine rotating about
539 the z -axis with a constant angular velocity, $\omega = 0.125 \text{ rad s}^{-1}$. The turbine’s
540 velocity is given by the instantaneous tangential velocity (at the turbine hub),
541 $\mathbf{v}_{hub} = \omega \mathcal{R}$, where \mathcal{R} is the orbital radius. At each time step the value of γ
542 has been updated (relative to the centre of the orbit), and \mathbf{R}_z (equation 12) is
543 applied an additional time in equations (13) and (17) to capture the rotation of
544 the turbine. In this case $\mathcal{R} = 8 \text{ m}$ and so $|\mathbf{v}_{hub}| = 1 \text{ ms}^{-1}$. One complete orbit
545 takes approximately 60 s, ensuring the turbine region does not interact with the
546 wake from the previous orbit. The mesh along the path of the turbine (Table 2)
547 is refined to the same discretisation as in the linear velocity cases.

548 The predicted free stream velocity in the rotating case is presented in Fig-
549 ure 8a ($-$). The results show the same trend as in the linear velocity cases: An
550 initial over-shoot in the prediction after the ramp up period, before converging
551 to within 2% of the expected solution. This error is slightly larger than in the
552 linear velocity cases, however, this might be anticipated as the underlying the-
553 ory behind the expected solution is based on uniform flow across the turbine
554 (and this is not true in this case). Crucially, again, there are only negligible
555 fluctuations in the predicted solution (Figure 8b) indicating that the present
556 methodology performs well even with arbitrary mesh alignment and rotational
557 motion through the computational mesh.

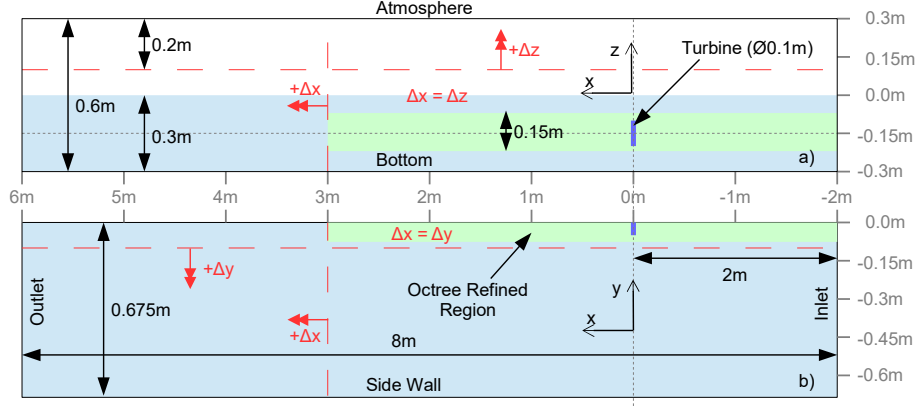


Figure 9: Numerical domain used for the two-phase simulations, in the $x - z$ (a), and $x - y$ (b) planes. Information regarding the mesh resolution is indicated in red, with double headed arrows representing mesh grading. The green shading indicates the refined region.

558 6. Velocity Deficit Validation

559 In Sections 4 and 5 the numerical model is shown to be robust and capable of
 560 capturing turbine loads when moving through a mesh, which is the primary moti-
 561 vation for the model. However, a secondary objective is to determine whether
 562 the turbine models influence on the fluid is captured accurately. Therefore, in
 563 this section the model is validated against existing experimental data for the
 564 velocity deficit behind a porous disc [31, 14]. These experiments were conducted
 565 in the Chilworth research laboratory flume at the University of Southampton,
 566 which is 21 m in length, 1.35 m wide and used a nominal water depth of 0.3 m
 567 [31]. Small scale discs ($\text{\O}0.1$ m, 0.001 m width) of varying porosity ($Ct = 0.61$,
 568 0.86 and 0.94) were evaluated, with wake profile measurements taken at a point
 569 location (varied between runs) using an Acoustic Doppler Velocimeter (ADV).

570 The `interFoam` solver (see Section 3) coupled with the developed turbine
 571 model is used to simulate the problem. Current speeds of 0.2487 ms^{-1} are
 572 generated using the expression based boundary condition and relaxation zone
 573 technique provided as part of the `waves2Foam` toolbox [17]. The $k - \omega$ SST
 574 turbulence closure scheme [28] is used to model the turbulent effects. For com-
 575 putational efficiency, the numerical model simulates one half of the flume (and
 576 disc), assuming that the flow is symmetric at the $y = 0$ plane. The water depth
 577 is set to 0.3 m ($-0.3 \leq z \leq 0.3$) and the tank width is 0.675 m ($-0.675 \leq y \leq 0$),
 578 consistent with the experiments [31]. The simulated length of the tank is set to
 579 8 m ($-2 \leq x \leq 6$), to accommodate an inlet region ($-2 \leq x \leq 0$), a working
 580 region $30D$ in length ($0 \leq x \leq 3$) and a relaxation zone ($3 \leq x \leq 6$). The initial
 581 mesh is designed such that the Aspect Ratio (AR) is set to 1 in the working
 582 region, with a mesh resolution of $\Delta x = 0.01$ m (Figure 9). Mesh grading is used
 583 to reduce computational cost in all directions: $x \geq 3$, $y \leq -0.1$ and $z \geq 0.1$.

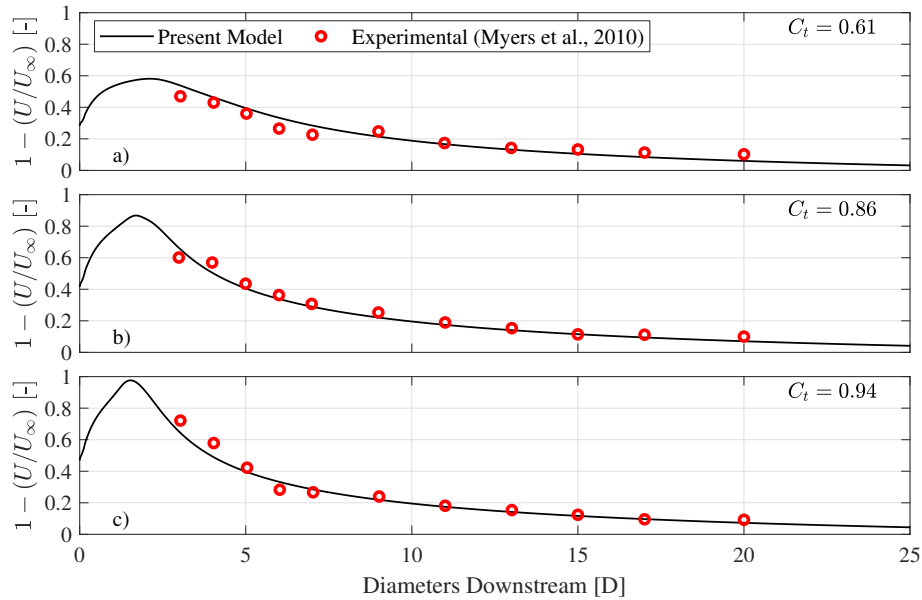


Figure 10: Comparison of experimental [31] (\circ) and numerical predictions (—) of centreline, horizontal velocity deficit profiles for $C_t = 0.61$ (a), 0.86 (b) and 0.94 (c).

584 Two levels of additional octree refinement [10] are used in the region of the wake
 585 of the turbine ($-2 \leq x \leq 3$, radius 0.1 m, $\Delta x = 0.0025$ m).

586 The disc is centred at $x = 0$, $z = -0.15$, with $R = 0.05$ m, $\sigma = 0.005$
 587 and $N = 2$, which gives an expected error of approximately 1% based on the
 588 results in Section 4, and each case is run for 120 s of simulation time. The inlet
 589 and outlet boundaries for velocity are both set to the prescribed free stream
 590 velocity (0.2487 ms^{-1}) in the water phase (0 ms^{-1} in the air phase). The top
 591 boundary is modelled as an atmosphere condition with a total pressure condition
 592 applied. The bottom and side boundaries are considered to be walls and hence
 593 are modelled with no-slip conditions. Wall functions are used for the turbulent
 594 parameters at these boundaries and hence mesh refinement is applied adjacent to
 595 these boundaries to achieve a suitable y^+ value ($y^+ \approx 40$). The inlet turbulent
 596 conditions are determined based on an inlet turbulent intensity of 5%, with
 597 zero gradient conditions applied at the outlet and atmosphere boundaries. The
 598 initial conditions for velocity and turbulence parameters is set to the values
 599 specified at the inlet.

600 Figure 10 presents a comparison of experimental (\circ) and numerical prediction
 601 (—) for the disc's centreline ($y = 0$ m, $z = -0.15$ m) velocity deficit
 602 profile as a function of diameters downstream, for $C_t = 0.61$ (a), 0.86 (b)
 603 and 0.94 (c). In all cases, the numerical predictions agree well with the experimen-
 604 tal data. The near wake region ($x \leq 5D$) was observed to increase with thrust
 605 coefficient in the experimental data. The numerical model captures this effect
 606 due to thrust coefficient well, with progressively increasing velocity deficit: the

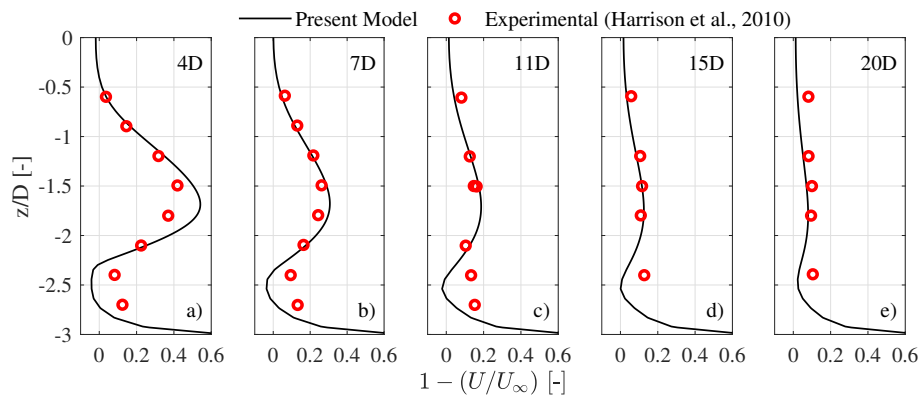


Figure 11: Comparison of experimental [12] (\circ) and numerical predictions (—) of vertical velocity deficit profiles for a $C_t = 0.86$ disc, at $x = 4D$ (a), $7D$ (b), $11D$ (c), $15D$ (d) and $20D$ (e).

607 predictions of maximum velocity deficits are 0.58, 0.87 and 0.98 for $C_t = 0.6$,
 608 0.86 and 0.94, respectively, although it should be noted that these can not be
 609 validated since the experimental campaign only considered positions for $x \geq 3D$.

610

611 A comparison of the experimental (\circ) and numerical predictions (—) of
 612 vertical profiles is presented in Figure 11 for the $C_t = 0.86$ at a number of
 613 horizontal locations: $x = 4D$ (a), $7D$ (b), $11D$ (c), $15D$ (d) and $20D$ (e). At
 614 $4D$ in the experimental data, there is a region of high velocity deficit, which
 615 extends from $z/D \approx -2$ to $z/D \approx -1$, i.e. the position of the disc. This is
 616 also observed in the numerical predictions, and the maximum occurs slightly
 617 below the centreline of the disc at this location, which has also been observed in
 618 previous CFD studies of the wake structure behind an actuator disc in a marine
 619 environment [4, 34]. Moving further away from the disc, the experimental data
 620 shows that this region reduces in magnitude and increases in height, which is
 621 also captured by the numerical model. However, the maximum value gets lower
 622 with increasing x in the numerical predictions, which although not obvious in the
 623 point measurements presented in Figure 11, could be observed in spatial plots
 624 presented by Myers and Bahaj [31]. In this work, the behaviour is more clearly
 625 observed in spatial plots of the numerical data (Figure 12), and is consistent
 626 for each of the discs considered. The spatial plots also show that the wake
 627 distribution for $x \geq 8D = 0.8$ m is very similar for the three discs. This indicates
 628 that the far wake structure is independent of the properties of the disc, and is
 629 in-line with the observations of Myers and Bahaj [31].

630 Overall, the numerical model captures velocity deficit to a similar standard
 631 as other numerical models [12, 4, 34], and distributions are comparable with
 632 experimental data [31]. Therefore, it is concluded that the model would be
 633 suitable for investigating both the effect of the turbine on a structures motion,
 634 and the implications for the fluid flow, in future work.

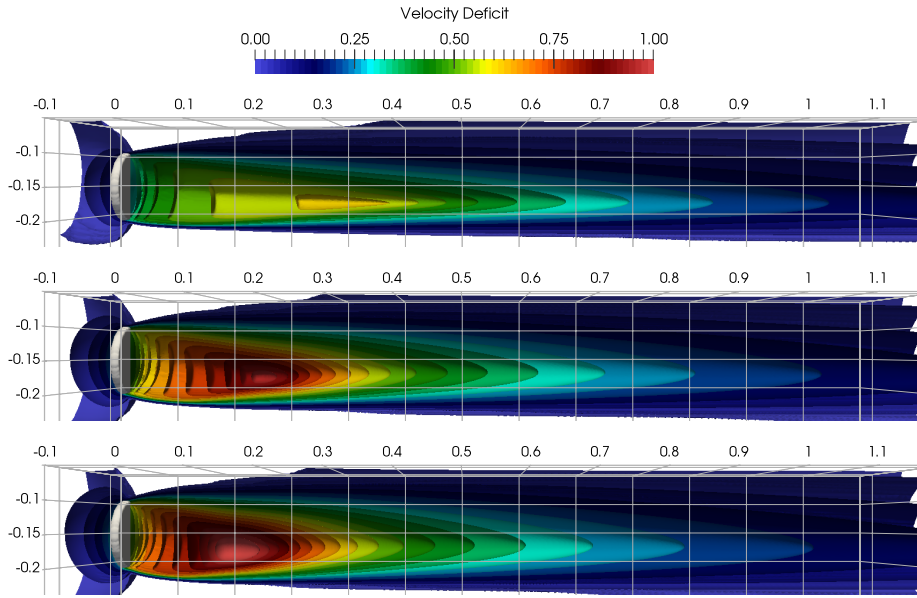


Figure 12: Velocity deficit contours for different turbine thrust coefficients: $C_t = 0.61$ (a), $C_t = 0.86$ (b) and $C_t = 0.94$ (c).

635 7. Conclusions

636 A new turbine model which can be used as a component of a framework
 637 for simulating entire floating tidal systems has been presented. Analysis of
 638 the new model in steady-state conditions showed that the prediction of the
 639 free stream velocity could be replicated to within a 2% accuracy relative to
 640 theoretical solutions, and this could be further reduced by tuning the width
 641 parameter and mesh resolution. However, the key aspects of the model were
 642 defined by the requirement to use the model for simulation of entire floating tidal
 643 systems: the model has been shown to be insensitive to flow velocity, performs
 644 well in any alignment with the mesh, and is capable of predicting the free stream
 645 velocity while moving through the mesh under both linear and angular velocity.
 646 These properties are crucial when simulating floating tidal systems, since such
 647 systems will be required to survive in complex, non-linear environments driven
 648 by strong wave-current interactions, requiring the turbine methodology to be
 649 robust during changing flow velocity. Furthermore, the system will be capable
 650 of moving in 6 degrees of freedom, and hence, the turbine will generally be
 651 arbitrarily aligned with a mesh, and must be able to accurately predict the free
 652 stream velocity under both linear and angular movement.

653 Following the success of the turbine model presented in this work, future re-
 654 search will focus on the development of a new coupled framework for simulating
 655 entire floating tidal systems, including integration of the present model.

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660 Wave Structure Interaction (CCP-WSI) [EP/M022382/1], which aims to bring
661 together the community of researchers, data, code and expertise within the area
662 of wave structure interaction (WSI) and has the objective of building an open-
663 source, community-driven numerical wave tank (NWT) facility and capacity for
664 high quality research in an area of high importance ([https://www.ccp-wsi.
665 ac.uk](https://www.ccp-wsi.ac.uk)).

666 **References**

- 667 [1] Abolghasemi, M., Piggott, M., Spinneken, J., Viré, A., C.J., C., Cram-
668 mond, S., 2016. Simulating tidal turbines with multi-scale mesh optimisa-
669 tion techniques. *Journal of Fluids and Structures* 66, 69–90.
- 670 [2] Ammara, I., Leclerc, C., Masson, C., 2002. A viscous three-dimensional
671 method for aerodynamic analysis of wind farms. *J. Sol. Energ. Eng.* 124,
672 345–356.
- 673 [3] Aubrun, S., Loyer, S., Hancock, P., Hayden, P., 2013. Wind turbine wake
674 properties: Comparison between a non-rotating simplified wind turbine
675 model and a rotating model. *J. Wind Eng. Ind. Aerodyn.* 120, 1–8.
- 676 [4] Batten, W., Harrison, M., Bahaj, A., 2013. Accuracy of the actuator disc-
677 RANS approach for predicting the performance and wake of tidal turbines.
678 *Philosophical Transactions of the Royal Society A* 371.
- 679 [5] Blackmore, T., Batten, W., Bahaj, A., 2014. Influence of turbulence on
680 the wake of a marine current turbine simulator. *Proc. R. Soc. A* 470, 17.
- 681 [6] Burton, T., Jenkins, N., Sharpe, D., Bossanyi, E., 2001. *Wind Energy*
682 *Handbook*. 2 ed., John Wiley & Sons, Ltd.
- 683 [7] Castellani, F., Vignaroli, A., 2013. An application of the actuator disc
684 model for wind turbine wakes calculations. *Appl. Energ.* 101, 432–440.
- 685 [8] Churchfield, M., Li, Y., Moriarty, P., 2013. A large-eddy simulation study
686 of wake propagation and power production in an array of tidal-current
687 turbines. *Phil. Trans. R. Soc A* 371.
- 688 [9] Greaves, D., Borthwick, A., 1999. Hierarchical tree-based finite element
689 mesh generation. *Int. J. Numer. Meth. Eng.* 45, 447–471.
- 690 [10] Greaves, D.M., 2004. A quadtree adaptive method for simulating fluid flows
691 with moving interfaces. *Journal of Computational Physics* 194, 35–56.

- 692 [11] Hansen, M., 2000. Aerodynamics of wind turbines. 1 ed., James & James
693 (Science Publishers) Ltd.
- 694 [12] Harrison, M., Batten, W., Bahaj, A., 2010a. A blade element actuator disc
695 approach applied to tidal stream turbines, in: Proceedings of OCEANS,
696 IEEE, pp. 1–8.
- 697 [13] Harrison, M., Batten, W., Bahaj, A., Myers, L., 2009. A comparison be-
698 tween CFD simulations and experiments for predicting the far wake of
699 horizontal axis tidal turbines, in: Proceedings of the 8th European Wave
700 and Tidal Energy Conference, Uppsala, Sweden. pp. 566–575.
- 701 [14] Harrison, M., Batten, W., Myers, L., Bahaj, A., 2010b. Comparison be-
702 tween CFD simulations and experiments for predicting the far wake of
703 horizontal axis tidal turbines. IET Renewable Power Generation 4, 613–
704 627.
- 705 [15] Holst, M., Dahlhaug, O., Faudot, C., 2014. CFD analysis of wave-induced
706 loads on tidal turbine blades. IEEE J. Ocean. Eng. 40, 506–521.
- 707 [16] Howard, R., Pereira, J., 2006. A study of wind turbine power generation
708 and turbine/tower interaction using large eddy simulation. Wind Struct.
709 9, 95–108.
- 710 [17] Jacobsen, N.G., Fuhrman, D.R., Fredsøe, J., 2012. A wave generation
711 toolbox for the open-source CFD library: OpenFOAM[®]. International
712 Journal for Numerical Methods in Fluids 70, 1073–1088.
- 713 [18] Johnson, B., Francis, J., Howe, J., Whitty, J., 2014. Computational actu-
714 ator disc models for wind and tidal applications. J. Renew. Energ. 2014,
715 10.
- 716 [19] Lawson, M., Li, Y., Sale, D., 2011. Development and verification of a com-
717 putational fluid dynamics model of a horizontal-axis tidal current turbine,
718 in: Proceedings of the 30th International Conference on Ocean, Offshore,
719 and Arctic Engineering, Rotterdam, The Netherlands. pp. 1–10.
- 720 [20] Leclerc, C., Masson, C., 2004. Toward blade-tip vortex simulation with an
721 actuator-lifting surface model. AIAA Paper 2004-0667, 9.
- 722 [21] Li, Y., Paik, K.J., Xing, T., Carrica, P., 2012. Dynamic overset CFD
723 simulations of wind turbine aerodynamics. Renew. Energ. 37, 285–298.
- 724 [22] Lignarolo, L., Ragni, D., Ferreira, C., van Bessel, G., 2016. Experimental
725 comparison of a wind-turbine and of an actuator-disc near wake. J. Renew.
726 Sustain. Energ. 8, 26.
- 727 [23] Madsen, H., Bak, C., Døssing, M., Mikkelsen, R., Øye, S., 2010. Valid-
728 ation and modification of the Blade Element Momentum theory based on
729 comparisons with actuator disc simulations. Wind Energ. 13, 373–389.

- 730 [24] Malki, R., Williams, A., Croft, T., Togneri, M., Masters, I., 2013. A
731 coupled blade element momentum - computational fluid dynamics model
732 for evaluating tidal stream turbine performance. *Appl. Math. Model.* 37,
733 3006–3020.
- 734 [25] Martínez, L., Leonardi, S., Churchfield, M., Moriarty, P., 2012. A com-
735 parison of actuator disk and actuator line wind turbine models and best
736 practices for their use, in: 50th AIAA Aerospace Sciences Meeting, p. 13.
- 737 [26] Martínez-Tossas, L., Churchfield, M., Leonardi, S., 2015. Large eddy sim-
738 ulations of the flow past wind turbines: actuator line and disk modeling.
739 *Wind Energy* 18, 1047–1060.
- 740 [27] Masters, I., Chapman, J., Willis, M., Orme, J., 2011. A robust blade
741 element momentum theory model for tidal stream turbines including tip
742 and hub loss corrections. *J. Mar. Eng. Technol.* 10, 25–35.
- 743 [28] Menter, F.R., 1994. Two-equation eddy-viscosity turbulence models for
744 engineering applications. *AIAA Journal* 32, 1598–1605.
- 745 [29] Mikkelsen, R., 2003. Actuator disc methods applied to wind turbines. Ph.D.
746 thesis. Technical University of Denmark.
- 747 [30] Moriarty, P., Hansen, A., 2005. AeroDyn Theory Manual. Technical Report
748 NREL/TP-500-36881. NREL.
- 749 [31] Myers, L., Bahaj, A., 2010. Experimental analysis of the flow field around
750 horizontal axis tidal turbines by use of scale mesh disk rotor simulators.
751 *Ocean Engineering* 37, 218–227.
- 752 [32] Naderi, S., Parvanehmasiha, S., Torabi, F., 2018. Modeling of horizontal
753 axis wind turbine wakes in Horns Rev offshore wind farm using an improved
754 actuator disc model coupled with computational fluid dynamic. *Energy*
755 *Conversion and Management* 171, 953–968.
- 756 [33] Naderi, S., Torabi, F., 2017. Numerical investigation of wake behind a
757 HAWT using modified actuator disc method. *Energy Conversion and Man-*
758 *agement* 148, 1346–1357.
- 759 [34] Nguyen, V., Guillou, S., Thiébot, J., Cruz, A., 2016. Modelling turbulence
760 with an Actuator Disk representing a tidal turbine. *Renewable Energy* 97,
761 625–635.
- 762 [35] Pintar, M., Kolios, A., 2013. Design of a novel experimental facility for
763 testing of tidal arrays. *Energies* 6, 4117–4133.
- 764 [36] Rajagopalan, R., Rickerl, T., 1990. Aerodynamic interference of vertical
765 axis wind turbines. *J. Propul. Power* 6, 645–653.

- 766 [37] Rusche, H., 2002. Computational fluid dynamics of dispersed two-phase
767 flows at high phase fractions. Ph.D. thesis. Imperial College of Science,
768 Technology & Medicine.
- 769 [38] Sanderse, B., van der Pijl, S., Korne, B., 2011. Review of computational
770 fluid dynamics for wind turbine wake aerodynamics. *Wind Energy* 14,
771 799–819.
- 772 [39] Shen, W., Zhang, J., Sørensen, J., 2005. Tip loss correction for
773 actuator/Navier-Stokes computations. *J. Sol. Energ.* 127, 5.
- 774 [40] Shen, W., Zhang, J., Sørensen, J., 2009. The actuator surface model: A new
775 Navier-Stokes based model for rotor computations. *J. Sol. Energ.* 131, 9.
- 776 [41] Sørensen, J., Mikkelsen, R., Henningson, D., Ivanell, S., Sarmast, S.
777 and Andersen, S., 2015. Simulation of wind turbine wakes using the actu-
778 ator line technique. *Phil. Trans. R. Soc. A* 373, 16.
- 779 [42] Sørensen, J., Shen, W., 2002. Numerical modeling of wind turbine wakes.
780 *J. Fluid. Eng.* 124, 393–399.
- 781 [43] Sørensen, N., Michelsen, J., Schreck, S., 2002. Navier-Stokes predictions of
782 the NREL phase VI rotor in the NASA Ames 80ft \times 120ft wind tunnel.
783 *Wind Energ.* 5, 151–169.
- 784 [44] Svenning, E., 2010. Implementation of an actuator disk in OpenFOAM.
785 Technical Report. Chalmers University of Technology.
- 786 [45] Viré, A., Xiang, J., Piggott, M., Cotter, C., Pain, C., 2013. Towards
787 the fully-coupled numerical modelling of floating wind turbines. *Energy*
788 *Procedia* 35, 43–51.
- 789 [46] Wang, Q., Zhou, H., Wan, D., 2012. Numerical simulation of wind turbine
790 blade-tower interaction. *J. Mar. Sci. Appl.* 11, 321–327.
- 791 [47] Watters, C., Masson, C., 2010. Modeling of lifting-device aerodynamics
792 using the actuator surface concept. *Int. J. Numer. Meth. Fluids* 62, 1264–
793 1298.
- 794 [48] Weller, H., Greenshields, C., Bainbridge, W., Janssens, M., Santos, B.,
795 2017. OpenFOAM 4.1. URL: <https://openfoam.org/version/4-1/>.
- 796 [49] Wilson, J., Davis, C., Venayagamoorthy, S., Heyliger, P., 2015. Compar-
797 isons of horizontal-axis wind turbine wake interaction models. *Journal of*
798 *Solar Energy Engineering* 137, 8.
- 799 [50] Zahle, F., Sørensen, N., Johansen, J., 2009. Wind turbine rotor-tower in-
800 teraction using an incompressible overset grid method. *Wind Energ.* 12,
801 594–619.