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An overview of surface reconstruction using partial differential equation (PDE)

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Abstract. Surface reconstruction is the main process of reverse engineering where engineering model is reproduced in digital format. Surface reconstruction using PDE can be described as solving the PDE to generate the reconstructed surface of an object of interest. This paper provides a brief introduction to the process of surface reconstruction, partial differential equation and its application in surface reconstruction, as well as summarizing several works that utilize this approach. This paper also outlines the validation method used to assess a reconstruction model.

1. Introduction

Surface reconstruction, as a main part of reverse engineering, is the process of reproducing geometric-based model based on existing physical object [1]. Surface reconstruction allows recovery of engineering models from actual object as digital backup or in order to manipulate the object of interest virtually. Söderkvist [2] divides the process of surface reconstruction in to three separate steps, namely Data Acquisition, Data Organization and Surface Fitting. Further development in the field includes three additional steps in the process, which are Pre-processing, Post-processing and Visualization [3].

PDE is a mathematical tool used to describe a given physical phenomena, where the relation between rates of change with respect to continuous variables can be observed and calculated. Such phenomena that can be described by PDE include heat transfer, acoustics and fluid dynamics. A derivative is the rate of change of a quantity with respect to another variable. When the rate of change includes two or more variables, it is then known as partial derivative. Thus, PDE can be defined as the relations of partial derivatives that make up a given function [4].

Surface reconstruction with PDE can be described as solving the PDE with respect to the boundary curve extracted from an object of interest, to generate the surface that can represent that object. Bae and Weickert [5] describe Partial differential equation (PDE) as a good solution for image interpolation and compression. Ostrov [6] demonstrates the capability of PDE to decrease the time taken to calculate the solution of surface reconstruction. Osechinskiy and Kruggel [7] present an automatic reconstruction method using several PDE modelling stages, and demonstrate the capability of PDE method to handle biomedical image with their algorithm. Previous study shows that to represent a surface, when compared to spline-based techniques, PDE requires smaller amount of parameters, thus is easier to be manipulated. PDE generated surface are also guaranteed a basic degree of smoothness; where the higher the order of the PDE, the higher the smoothness degree of the generated surface would be [8].

The purpose of this paper is to give an overview on previous works on Surface Reconstruction that uses PDE. In this paper, previous works are summarized with the domain and the issues listed in a table, followed by a description on the method proposed and its result. The structure of this paper is as follows, Section 2 provides an overview of several works on PDE-based Surface Reconstruction, Section 3 gives a brief description on validation of reconstruction model, while Section 4 concludes this paper.

2. Overview of Previous Works

This section provides an overview on several works in surface reconstruction that uses Partial Differential Equation.

Table 1. Author, Domain and Issues of previous works on PDE-based Surface Reconstruction

Author	Domain	Issues
Rodrigues et al.	Reconstruction of realistic human faces from 3D facial scans data for facial recognition in airport security.	• Deals with vast amount of data and requires real-time processing. • Requires data encryption for secured transmission over network. • Compression of 3D data acquired from 3D scanners. • Reconstruction of the mesh from the compressed data whilst preserving the quality of the original mesh.
Ugail & Kirmani	Reconstruction of object with complex geometry.	• Reconstruction of complex geometric model using elliptic PDE formulation, where inclusion of only outer edge as boundary condition, may not produce close agreement between original and reconstructed shape.
Eyad & Ugail	Reconstruction of realistic human faces from 3D facial scans data	• Representation of human facial geometry through polygonal based approach require large amount of data to be saved. • Parametric patch approach, in particular spline techniques, requires extra storage, used to define complex geometry.
Linz et al.	Surface Reconstruction from unstructured point cloud data and multiple view 2D images	• Eulerian approach of deformable model requires post-processing to represent the object explicitly and does not guarantee to preserve the topology of the object over time. • Lagrangian approach of deformable model requires the topology information to be defined explicitly when the topology changes thus its implementation is computationally expensive.
Duan et al.	Shape reconstruction from volumetric data, unorganized point cloud and multi-view 2D images.	• PDE-Based surface evolution by applying Eulerian approach is computationally expensive. • Available methods are not flexible enough to handle all three types of data in one algorithm.
Osechinskiy & Kruggel	Reconstruction of cerebral cortex surface from MR Images	• MRI Scans produce images that have limited resolution, noisy, intensity inhomogeneities and partial volume effects. • Automated reconstruction of the cerebral cortex surface from such input can be topologically and geometrically inaccurate.

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Pang et al.	Reconstruction and web visualization of complex geometric shapes	• Large polygon meshes are used to represent complex geometric shapes which require a significant amount of memory to handle. • Level of Detail (LOD) is predefined. • Web visualization using such shapes is impractical due to limited Internet bandwidth.

Rodrigues et al. [9] proposes Fourier-based data compression and PDE-based mesh/surface reconstruction. In preparation step consist of finding the minimum bounding box encompassing the whole mesh, and defining horizontal and vertical oriented cutting planes for vertex sampling. The compression step is comprised of Fourier analysis of data in each plane, saving the coefficients, boundaries and scale into ASCII file according to the prescribed format. The compressed data may be encrypted to ensure secured network transmission. Reconstruction step consists of solving the PDE of each pair of cutting planes with Laplace Equation acquiring patches, which is joined together to reconstruct the original mesh. The result shows data compression of over 96% with minimal loss of accuracy of the reconstructed surface against original mesh.

Ugail and Kirmani [10] use fourth-order elliptic PDE to generate the surface, where four curves acts as the boundary condition. Two curves can be extracted from the outer edges of the shape and is applied as the outer boundary condition, whereas Flexible parameterization is used to determine the position of two inner curves that acts as the inner boundary condition. The surface is reconstructed by solving the PDE with regards of the four previously extracted boundary conditions. Proposed scheme produces close agreement between original shape and reconstructed surface.

In [11], the method requires interaction with the user for initialization. User is prompted to highlight several key features of the facial data, such as tip of the nose, and corner of the eyes and mouth, which is then used to normalize and align the data. A series of plane which are parallel to the symmetry profile of the facial data are defined, where the distance between each two consecutive planes is dependent on the complexity of the topology of that section. The set of points that intersects each planes are identified, and following that, a series of profile curves are extracted from the facial data. These profile curves are used as the boundary curves. Solving the fourth order elliptic PDE with four consecutive boundary curves will generate a surface patch, and by combining these patches, the geometry of a human face may be adequately described. This work also looks into the importance of sampling ratio when extracting the profile curves. Significant saving of data storage whilst retaining the accuracy of the geometry can be observed. Higher sampling ratio guarantees smaller error rate of the generated surface, though depending on the application, such as animation, smaller sampling ratio may reduce the processing time.

Linz et al. [12] proposed a point-based approach to PDE-based surface reconstruction. The generated surface is made up of a set of oriented disks, deformed from initial point surface e.g. a sphere encompassing the object. In sampling, instead of considering the geodesic proximity between each point, spatial proximity, where the structure relies on the notion of k-nearest neighbour, is used for computation. Applying force to a point in a region affects every k-nearest neighbouring point as well, thus displacing the whole neighbourhood. By weighing the force applied to each point in the region with Gaussian kernel, the deformation in that region would still produce a smooth surface locally. The set of oriented disks are then ensured to produce a globally smooth surface by introducing Moving Least-Squares projection. To ensure the points are uniformly distributed, up-sampling scheme is introduced to under-sampled regions, and to control the scheme in order to avoid oversampling, point relaxation method is introduced next. When topology changes are involved, the point-based approach is more flexible and less expensive computationally compared to deformation of triangle meshes for surface reconstruction.

Duan et al. [13] works on the shape reconstruction from 3D and 2D Data using PDE-based deformable surfaces. PDE-based deformable surface model that can evolve from an initial shape to capture the geometric boundaries of the object of interest automatically is proposed. The PDE used is

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the general weighted minimal surface flow. Depending on the data types used for the reconstruction process, the stopping function that is used to stop the deformation of the model when the boundary of the object is reached would differ accordingly. The model only requires for the user to simply use the appropriate data interface function in the initialization phase, as the stopping function for each data type i.e. volumetric data, unstructured point cloud, multi-view 2D images, had already been defined beforehand. The model is capable of automatically processing all three types of data with minimal user interaction. In particular, for reconstruction of multi-view 2D images, the model allows additional images to be incorporated into the mesh representation for progressive refinement when required.

Osechinskiy & Kruggel [7] proposes reconstruction algorithm that is divided into four stages. First stage involves the computation of potential field, where gray matter is treated as dielectric layer that surrounds the conductive white matter, in an electrostatic model. Second stage requires the computation of the distance field from white matter along the streamlines of the potential field that solves the Eulerian framework PDE on a fixed grid. Third stage focuses on extracting the binary skeleton by finding shocks in the distance field, highlighting the areas where gray matter banks are present. Lastly, reconstruction of the cortical surface is performed by advection along the gradient of the potential field or the distance field, using a geometric deformable model which evolution is constrained by the binary skeleton computed previously. Comparison by visual inspection shows that the electric field model used can better describe the sulcal fold position than Laplacian field. The binary skeleton extracted by the method is consistent with the skeleton extracted using ACE algorithm. Overall, the method proposed is capable of producing outer cortical surface that is correct geometrically whilst preserving its initial topology.

Pang et al. [14] applies Patchwise PDE method, where the object is made up of a number of non-overlapping patches with its own parametric coordinate system. The number of patches required to represent an object is dependent on the complexity of said object. The original polygon mesh is first simplified using decimation algorithm to get the base mesh. Each facet of the base mesh is then used to define a surface patch, where the edges of this facet are used as reference to extract the outer boundary curve of each patch from the original object. The inner boundary curve of the patch can then be extracted, by mapping isometric polylines of triangular facet to the initial mesh and sampling accordingly. The boundary curves coordinates are finally stored in a file, which can later be processed by the PDE engine for rendering. Depending on the complexity of the shape and the Level of Details stated by the user, the proposed method requires significantly smaller storage size when compared to the original file.

3. Validation of Reconstruction Model

Validation of a reconstruction model involves assessing the quality of the reconstructed surface produced by the model; how well does the reconstructed surface can represent the original data. These assessments can be performed by either visually or statistically.

The most direct and meaningful assessment is through visual inspection, observation either by user or professional of the field, whether the reconstructed surface is a good fit or in close agreement to the original data. For application such as animation, where faster processing time is of more importance than accuracy, visual assessment is an adequate assessment.

When assessing a model that requires finesse, such as facial recognition, statistical assessment would be a better measurement tool. Through such assessment, the user can quantify the goodness of the fit. Some study compares the difference between each point in the generated surface against its closest point from the original data in Euclidean distance, averaging the calculated values and define it as the error rate; where smaller error rate corresponds to a better fit. Additional measurement approach includes comparing the running time of the model, features of the generated surface i.e. number of vertices, edges and faces, and size of the output file against other methods or models used in a similar domain.

4. Conclusion
Complex geometry shapes are generally represented using polygon meshes or set of points, which in turn requires large data storage. PDE-based approach that makes use of the boundary-value to represent an object is inherently more efficient. Depending on the complexity of the original data, several PDE patches can be blended together to represent a surface. Though proven to be quite expensive to solve and time consuming in computation, PDE surfaces preserve the topological and geometry of original object. It also requires small amount of parameters to define, thus is easier to modify or manipulate, adjust its Level of Detail (LOD), and produce smaller sized output file. These advantages encourages the utilization of PDE surfaces in application that emphasizes on topology and geometric accuracy of generated surface such as geometric modelling and facial recognition, flexible LOD for visualization such as animation, and small file size for transmission and sharing.

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