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15 June 2018

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Abstract

Classical models of decision-making do not incorporate for the role of influence and honesty that affects the process. This paper develops on the theory of influence in social network analysis. We study the role of influence and honesty of individual experts on collective outcomes. It is assumed that experts have the tendency to improve their initial predilection for an alternative, over the rest, if they interact with one another. It is suggested that this revised predilection may not be proposed with complete honesty by the expert. Degree of honesty is computed from the preference relation provided by the experts. This measure is dependent on average fuzziness in the relation and its disparity from an additive reciprocal relation. Moreover, an algorithm is introduced to cater for incompleteness in the adjacency matrix of interpersonal influences. This is done by analysing the information on how the expert has influenced others and how others have influenced the expert.

Keywords: Honesty; group decision making; social network analysis; confined influence; predilection

1. Introduction

Group decision making (GDM) does not include the role of influence and honesty in the process of achieving final decisions. However, in real life, these factors may alter the process significantly and hence they need to be incorporated. In social networks, significant personalities have impact on choices of the masses. However, it is up to an individual to admit the intensity of influence and propose his revised opinion with complete honesty.

GDM is a process in which collective choices are exhibited by a panel of experts. The aim of this process can be articulated as finding a satisfactory solution for a given problem. In a complex social environment, opinions are rarely in agreement. It is important for the experts of the group to interact and reason with each other about the choices that they have made. A more realistic model must encompass the changes that may take place in opinions of the judges as they interact and influence one another.

At the beginning of the process of decision making, experts present their predilections for a particular alternative over others. But these initial opinions may undergo modifications after discussions due to social influence (Capuano, Chiclana, Fujita, Viedma, & Loia, 2018). In (Qian, Liao, & Liu, 2017) a social influence is defined in terms of changes incurred by an individual after interaction with another individual or a group. In this paper we model the possible changes in experts' opinions after they express their predilections for one alternative over others based on how honest they are in the decision making process.

We introduce the notion of honesty and its role in an influence based decision making problem. Researchers who explore a dispositional notion of honesty focus heavily on the identification of character traits that allow for the consistent prediction of behaviour. In this paper, we propose that intensity of predilection for one alternative over other alternatives may alter as experts interact and influence one another. However, this does not ensure that they will present their revised predilections exactly as they are. Experts may misrepresent or withhold their predilections to manipulate the outcome or to preserve a certain outcome. There can be various other reasons to withhold the true information. In order to study this, we measure honesty of an expert based on the preference relation provided by them. We propose two indicators, average fuzziness of the relation and how adrift the relation is from additive reciprocity, to measure the degree of honesty of an expert. If a preference relation has less average fuzziness and is closer to an additive reciprocal relation then the expert has a higher degree of honesty on a scale of 0 to 1. Since honesty is a diagonal matrix, the iterative model remains convergent.

Social influence network (SIN) builds on the idea that there exists interdependence among actors and their actions (John & Carrington, 2011; Stanley & Faust, 1994). The network structural environment may facilitate the individual actions or put constraints on them. Relationship among social bodies is studied in SIN accompanied with the patterns and implications of these relationships (Pérez, Mata, Chiclana, Kou, & Herrera-Viedma, 2016).A fuzzy adjacency matrix $W = \left(\omega_{ij}\right)_{m\times m}$ summarizes the social influence network involving a set of experts $E = \{e_1, e_2, \ldots, e_m\}$. In (DeGroot, 1974) it is suggested that the weights $\omega_{i1}, \omega_{i2}, \ldots, \omega_{im}$ are directly chosen by the expert e_i before she has information about preferences expressed by other experts. The expert assigns relative importance to the opinion of other experts, including herself. It should be noted that normalization property is to be satisfied by these weights $\sum_{j=1}^{m} \omega_{ij} = 1$ for all $i \in \{1,2,\ldots,m\}$. This means that the influence of peers on each expert must add up to 1. In this paper, we study the problem when W is not complete and some information has been confined by the expert. An expert may be somewhere between influenced or not influenced by others, which is why degree is assigned from the unit interval. But an expert may not be able to state the exact degree by which another expert has influenced his decision (Khalid & Beg, 2019). This incompleteness may arise due to many reasons. However, the available information states how other experts have influenced this expert and how this expert has influenced others on average. Since, the adjacency matrix of interpersonal influences has a row sum of 1, we derive an interval to which the missing influence degrees may belong to. We propose an algorithm that utilizes the information provided by this particular expert to estimate the confined information in W. This algorithm ensures that the normalization property is not disturbed.

In influence based decision making, opinion of an expert is revised after each session of interaction with other experts. We have not used preference relations provided by experts in the model. Preference relations are first converted into matrices of predilections before using them in the influence and honesty based group decision model. So the focus is more on predilections than it is on the original preference relations. As discussed before, our model is affected by honesty of each expert. If degree of honesty of all experts is 1 then it means that all experts are completely honest in expressing their revised predilections for an alternative over the set. This is a specific case where the diagonal matrix of honesty becomes the identity matrix. If degree of honesty is closer to 0 then the expert has withheld the revised information and expresses a dishonest predilection for every alternative. We use the honesty induced ordered weighted averaging operator to aggregate the final opinions. The final opinions are then ranked to find the best possible alternative.

The paper is arranged in the following manner: Section 2 presents preliminaries for the sequel. Section 3 proposes treatment for incomplete interpersonal influences. Section 4 describes the role of honesty in modelling influence based decision making. This is followed by ranking of alternatives from the aggregated matrix of predilections. Finally, section 5 concludes the paper and proposes some future directions.

2. Preliminaries

In this section, we summarize a few concepts that are required to understand the proposed work. In group decision modelling, a panel of experts decides which alternative is the best to solve a problem. Each expert provides a preference intensity for every possible pair of alternatives in a non-empty and finite set $X = \{x_1, x_2, \dots x_n\}$.

Definition 1:

A fuzzy preference relation R on X is defined by the membership function $\mu_R: X \times\ X \rightarrow\ [0,1]$. The membership function $\mu_R(x_i, x_j) = r_{ij}$ is interpreted as follows:

If x_i is absolutely preferred over the alternative x_j when $r_{ij} = 1$. x_j is absolutely preferred over the alternative x_i when $r_{ij} = 1$. x_i is preferred over x_j when $r_{ij} \in (0.5, 1]$. x_j is preferred over x_i when $r_{ji} \in (0.5, 1]$. There exists indifference between the alternatives x_i and x_j when $r_{ij} = 0.5$.

A fuzzy preference relation R is additive reciprocal if it satisfies the property $r_{ij} + r_{ji} = 1$ for all $i, j \in \{1,2, \dots, n\}$ (Bezdek, , Bonnie , & Spillman, 1978; Hannu, 1981). Moreover, for all $i, j, k \in \{1,2, \dots, n\}, i \neq\ j$ the fuzzy preference relation R satisfies additive consistency or additive transitivity if it satisfies $r_{ij} = r_{ik} + r_{kj} - 0.5$ (Herrera-Viedma, Herrera, Francisco, & Luque, 2004; Tanino, 1984)

We define ordered weighted averaging operator OWA (Yager, 1988) quantifier guided OWA (Yager, Quantifiers in the formulation of multiple objective decision functions, 1983) and Induced ordered weighted averaging operators (IOWA) operators in the following definition. The main difference between an OWA and an IOWA operator is the re-ordering step of the argument variable. In OWA operators, it is the magnitude of the values to be aggregated that determine the reordering step whereas in the case of IOWA operator an order-inducing vector is used to induce the reordering. These aggregation operators will be used in the aggregation phase.

Definition 2:

An OWA operator of dimension n is a mapping

$$\phi: R^n \rightarrow\ R$$

with associated weights $W_{OWA} = (w_1, w_2, \dots, w_n)^T$ such that $w_i \in\ [0,1],\ \sum_{i=1}^{n} w_i = 1$,

$$\phi(a) = \phi(p_1, p_2, .., p_n) = \sum_{i=1}^{n} w_i\, p_{\sigma_i}$$

where $\sigma_i: \{1,2, .., n\} \rightarrow \{1,2, .., n\}$ is a permutation function such that $p_{\sigma_i} \geq p_{\sigma_{i+1}}$ for all $i = 1,2, \dots, n-1$. These weighting vectors can be obtained using the soft majority concept fuzzy majority by using quantifier guided aggregation given by Yager (Yager, Quantifiers in the formulation of multiple objective decision functions, 1983). We use the linguistic quantifier (Zadeh, 1983) "necessary for good solution" in the aggregation phase. We use regular increasing monotone (RIM) quantifier Q to compute the weights using the following expression:

$$w_i = Q\left(\frac{1}{n}\right) - Q\left(\frac{i-1}{n}\right)$$

A generalization of OWA operator is the IOWA operator (Mitchell & D, 1997).An IOWA operator of dimension n is a mapping $\phi_I: R\ \times R \rightarrow R$ with associated set of vectors $W = (w_1, w_2, \dots, w_n)^T$ such that $w_i \in\ \ [0,1], \sum_{i=1}^{n} w_i = 1$ such that

$$\phi_I(< u_1, p_1 >, \dots, < u_n, p_n >) = \sum_{i=1}^{n} w_i p_{\sigma_i}$$

where σ is a permutation function $p_{\sigma_i} \geq p_{\sigma_{i+1}}$ for all $i \in \{1,2,..,n-1\}$. The vector of values $(u_1, u_2, \dots, u_n)$, is the order inducing vector whereas $\{p_1, p_2, \dots, p_n\}$ are the values of the argument variable. Here, the reordering of $\{p_1, p_2, \dots, p_n\}$ is induced by the reordering of $(u_1, u_2, \dots, u_n)$, which is based on their magnitude (Yager, Induced aggregation operators, 2003; Yager & Dimitar, Induced ordered weighted averaging operators, 1999).

Now we define importance IOWA (I-IOWA) operator (Yager, Quantifiers in the formulation of multiple objective decision functions, 1983). This method associates an importance degree to each expert (Chiclana, Herrera-Viedma, Francisco , & Alonso, 2007). The importance degree of an expert i is $u_i \in [0, 1]$. An expert with no importance is assigned a weight of zero and for some expert, $u_i = 1$ if the importance is the highest. For quantifier guided aggregation, a procedure to evaluate overall satisfaction of Q important criteria by alternative x is presented in (Yager, Quantifiers in the formulation of multiple objective decision functions, 1983), where Q is a linguistic quantifier. Once satisfaction values have been ordered, the associated OWA weighting vector is calculated using Q as defined the following definition.

Definition 3:

Let $E = \{e_1, e_2, \dots, e_m\}$ be a non-empty set of m experts and $U = \{u_1, u_2, \dots, u_m\}$ denote the vector associated to their importance where $u_i \in [0,1]$. Then, an I-IOWA operator ϕ_{II} of dimension n has U as the order inducing vector and the associated set of weights $W = (w_1, \dots, w_n)$ defined as follows:

$$\phi_{II}(< u_1, p_1 >, \dots, < u_n, p_n >) = \sum_{i=1}^{n} w_i \, p_{\sigma(i)}$$

with $w_i = Q\left(\frac{S(i)}{S(n)}\right) - Q\left(\frac{S(i-1)}{S(n)}\right)$ where $S(i) = \sum_{s=1}^{i} u_{\sigma(s)}$ where σ is a permutation as defined above.

Definition 4:

(Giles, 1976; Siegfried, 1983) Let $\nabla\colon [0, 1] \times [0, 1] \rightarrow [0, 1]$ denote a t-conorm, then for $all\ x, y, z \in [0, 1]$

i. $\nabla(x, y) = \nabla(y, x)$ (commutativity)
ii. $\nabla(x, y) \leq \nabla(x, z)$ if and only if $y \leq z$ (monotonicity)
iii. $\nabla(x, \nabla(y, z)) = \nabla(\nabla(x, y), z)$ (associativity)
iv. $\nabla(x, 0) = x$

An example of a tconorm that is used in later sections is probabilistic tconorm defined as $\nabla(x, y) = x + y - xy$.

3. Estimation of confined interpersonal influences

Human beings are embedded in social networks that affect their lives in different ways. Social networks are elaborate, complex and in fact, ubiquitous. It is important to understand how they are formed, what they are and how they affect us. In real life problems, decision makers form their opinions in a complex interpersonal environment in which preferences change due to social influence.

Liang et al (Qian, Liao, & Liu, 2017) define social influence as changes in individuals' thoughts, feelings, attitudes or behaviours resulting from interaction with another individual or a group. In (DeGroot, 1974; Friedkin & Johnsen, 1999) it was suggested that in social influence network (SIN), influence can be modelled with the help of a directed graph among the experts in set E. According to this, each arc between e_i and e_j has a weight $w_{ij} \in [0,1]$ representing the intensity of influence of the $j-th$ expert on the $i-th$ expert. This is represented by the adjacency matrix $W = (\omega_{ij})_{m\times m}$. The weights $\omega_{i1}, \dots, \omega_{im}$ are chosen by the expert e_i before knowing the preferences of other experts. These weights satisfy the normalization property so that influence of peers on each expert sums up to 1. In this section, we will consider the situation when W is incomplete.

To start with, the fuzzy preference relations provided by the experts are converted into matrices of predilections. The initial opinion of the experts on a given alternative is collected in an $m \times 1$ vector denoted as $y^{(1)}$. After interacting with the group, due to interpersonal influence, the opinion may change to $y^{(2)} = W\, y^{(1)}$. Similarly, when each experts knows the opinions of other experts in the group, it is probable that their own opinion will change as well. The expert's opinion after time t is given as follows:

$$y^{(t)} = Wy^{(t-1)}$$

Consider the following matrix of interpersonal influence among two experts as

$$W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Suppose that expert 1 has initial predilection for one solution over others with a degree of 0.6 whereas the second expert states it as 0.8. Accordingly,

$$y^{(2)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$$

Whereas,

$$y^{(3)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$$

This results in a never ending cycle and because of such a choice of W the iterative scheme will not converge. This problem has been tackled in (DeGroot, 1974). It is suggested that choice of W is such that there exists a positive integer t for which $W^{(t)}$ is positive. Because of this condition, the opinions will converge to a value.

Friedkin et al (Friedkin & Johnsen, 1999) suggested to include susceptibility of each expert to the interpersonal influence as $a_{ii} = 1 - \omega_{ii} \in [0,1]$. The closer the degree of a_{ii} is to 1, the more susceptible the expert is to interpersonal influence. This helps in forming the matrix of susceptibility $A = diag(a_{11}, \dots, a_{mm})$. The improved opinions are obtained iteratively as

$$y^{(t)} = AWy^{(t-1)} + (I - A)y^{(1)} \qquad (1)$$

where I is an $m \times m$ identity matrix. Here, opinion of an expert at time t is stated as a linear combination of his initial opinion and the influenced opinion at the time $t-1$.

For this iteration to reach an equilibrium, $I - AW$ must be non-singular. That is, if $y^{(\infty)} = \lim_{t\to\infty} y^{(t)}$ exists, then,

$$y^{(\infty)} = (I - AW)^{-1}(I - A)y^{(1)} \qquad (2)$$

Note that equation (1) can be re-written as

$$y^{(t)} = (AWy^{(t-1)} + (I - A)\sum_{i=0}^{t-2}(AW)^{i})\, y^{(1)}$$

$$= (AW)^{(t-1)}y^{(1)} + (I - A)(AW)^{(t-2)}y^{(1)} + \cdots + (I - A)(AW)y^{(1)} + (I - A)y^{(1)}$$

Note that from the second term onwards, it is a geometric series that is bounded and monotonic hence convergent. In this section, we study the problem where an expert withholds the intensity with which he is influenced by other experts. Instead, he displays an interval in which this value lies. Using the property of the matrix W, we complete the missing degrees of interpersonal influences.

Example 1:

Consider a panel of four experts $E = \{e_1 = Faisal\,, e_2 = Amy\,, e_3 = \ Maaz\,, e_4 = Junain\}$ in a household. After an interactive session, Faisal knows the extent to which he is influenced by his wife Amy, but he has withheld information about the degree of influence that his children Maaz and Junain has made on him. The situation is summarized in the adjacency matrix of interpersonal influences among this family as follows:

$$W = \begin{pmatrix} 0.3 & 0.2 & 0.3 & 0.2 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0.3 & - & 0.1 & - \\ 0.2 & 0.3 & 0.4 & 0.1 \end{pmatrix}$$

Faisal may be indecisive or he may be withholding this information because of some reason. This is represented graphically in diagram 1. The dashed line represents withheld information.

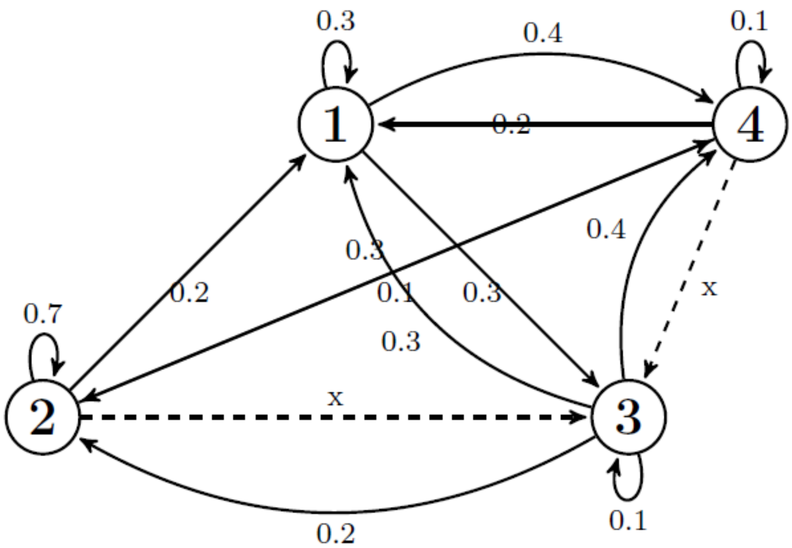

Diagram 1: Graphical representation of confined interpersonal influence matrix W

We propose an algorithm with the help of which withheld interpersonal influence can be estimated by using the influence that the third expert has had on other experts, and the average degree of influence that other experts have on this expert. In the following, an algorithm is designed to estimate the withheld or missing degrees of interpersonal influences.

If degrees of interpersonal influences $\omega_{ik'}, k' \neq i$ and $\omega_{il'}, l' \neq i$ exhibiting influence of experts k' and l' on expert i are withheld, then we can estimate them using the following algorithm.

Algorithm:

Case 1: Suppose that $\left(\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2 \geq 4\left(\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1} \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}\right)$, then if,

Case (1a): $\max\left\{\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}, \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}\right\} = \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}$ find,

$$\omega_{il'} = \max\left\{\begin{array}{c}\frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}, \\ \frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)-\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}\end{array}\right\}$$

And,

$$\omega_{ik'} = \min\left\{\begin{array}{c}\frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}, \\ \frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)-\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}\end{array}\right\}$$

Case (1b): Otherwise if, $\max\left\{\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}, \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}\right\} = \frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}$ then find,

$$\omega_{ik'} = \min\left\{\begin{array}{c}\frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}, \\ \frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)-\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}\end{array}\right\} \text{ and}$$

$$\omega_{il'} = \min\left\{\begin{array}{c}\frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}, \\ \frac{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)-\sqrt{\left(1-\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2-4\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}}}{2}\end{array}\right\}$$

Case 2: On the other hand, if $\left(\sum_{t=1,t\neq l',k'} \omega_{it}\right)^2 < 4\left(\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1} \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}\right)$ then there are two options that need to be considered.

Case (2a): Firstly if, $\max\left\{\frac{\sum_{i=1,k=k'}^{k} \omega_{ik}}{m-1}, \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}\right\} = \frac{\sum_{i=1,l=l'}^{k} \omega_{il}}{m-1}$, then $\omega_{il'} = 1 - \sum_{t=1,t\neq l',k'}^{m} \omega_{it}$ and $\omega_{ik'} = 0$

Case (2b): Whereas if, $\max\left\{\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1},\frac{\sum_{i=1,l=l'}^{k}\omega_{il}}{m-1}\right\}=\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}$ then $\omega_{ik'}=1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}$ and $\omega_{il'}=0$

Note that if expert i does not display, or is indecisive of the degree to which he is influenced by just one other expert k' then this problem is easier to handle. What we are suggesting is that suppose $\omega_{ik'},k'\neq i$ is unknown, then it can easily be determined as $1-\sum_{j=1,j\neq k'}^{m}\omega_{ij}$. However, if expert i has has more than one unknowns in the row of an adjacency matrix, then following theorem proves that the unknowns can be determined using the proposed algorithm.

Theorem 1:

If degrees of interpersonal influences $\omega_{ik'},k'\neq i$ and $\omega_{il'},l'\neq i$ exhibiting influence of experts k' and l' on expert i are hidden but all $\omega_{ik},k\neq k'$ and $\omega_{il},l\neq l'$ are known then $\omega_{ik'}\in[0,1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}]$ and $\omega_{il'}\in[0,1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}]$ can be estimated using algorithm 1. Moreover, the estimated values do not void the normalization property.

Proof:

If expert i is unable to express the degree of influence of just one expert say k' on him, then it can easily be estimated as $\omega_{ik'}=\sum_{i=1,k\neq k',k'}^{k}\omega_{ik}$. Also, the $i-th$ row will satisfy $\sum_{i=1}^{k}\omega_{ik}=1$. We assume here that at most one influence value is withheld in one column.

To prove the theorem, we have two unknown values in a row that satisfy the property

$$\omega_{ik'}+\omega_{il'}=1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it} \tag{3}$$

Suppose that $\sum_{t=1,t\neq l',k'}^{m}\omega_{it}<1$ because otherwise if $\sum_{t=1,t\neq l',k'}^{m}\omega_{it}=1$ then the missing influence values $\omega_{il'}$ and $\omega_{ik'}=0$. Also, consider the average of the column sum that has missing influence values, $\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}$ and $\frac{\sum_{i=1,l\neq l',k'}^{k}\omega_{il}}{m-1}$. This depicts that on average, the influence of the $k-th$ expert on all the other experts is $\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}$. Similarly, expert $l-th$ expert has influenced other experts on average by an intensity of $\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}$. The following formula is based on the fact that since $\sum_{j=1}^{n}\omega_{ij}=1$, therefore, the unknowns $\omega_{ik'}$ and $\omega_{il'}$ are inversely proportional.

$$\omega_{ik'}\omega_{il'}=\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1} \tag{4}$$

Suppose, $\omega_{ik'}\omega_{il'}\neq 0$ as we prove Case 1.

There are two cases to be discussed. According to case (1a) of Algorithm 1, if

$$\max\left\{\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1},\frac{\sum_{i=1,l=l'}^{k}\omega_{il}}{m-1}\right\}=\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}$$

then find $\omega_{il'}$ using equations 2 and 3,

$$\omega_{il'}+\frac{1}{\omega_{il'}}\left(\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}\right)=1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}$$

which implies that

$$\omega_{il'}^2 + \frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1} = \omega_{il'}\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)$$

That is,

$$\omega_{il'}^2 - \omega_{il'}\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right) + \frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1} = 0$$

which implies that

$$\omega_{il'} = \max\left\{\frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}, \frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}\right\}$$

and the smaller value belongs to $\omega_{ik'}$. That is,

$$\omega_{ik'} = \min\left\{\frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}, \frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}\right\}$$

Similarly, according to case (1b) of Algorithm 1, if

$$\max\left\{\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}, \frac{\sum_{i=1,l=l'}^{k}\omega_{il}}{m-1}\right\} = \frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}$$

then find $\omega_{ik'}$ using the above two equations as

$$\omega_{ik'} + \frac{1}{\omega_{ik'}}\left(\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k}\omega_{il}}{m-1}\right) = 1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}$$

which implies that

$$\omega_{ik'}^2 - \left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)\omega_{ik'} + \frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l=l'}^{k}\omega_{il}}{m-1} = 0$$

This means that,

$$\omega_{ik'} = \max\left\{\frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}, \frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}\right\}$$

And

$$\omega_{il'} = \min\left\{\frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}, \frac{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)+\sqrt{\left(1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it}\right)^2-4\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\frac{\sum_{i=1,l\neq l'}^{k}\omega_{il}}{m-1}}}{2}\right\}$$

It is easy to see that the row sum will be 1 in case 1a and 1b. Also, if case 2a or 2b is followed, then again, the missing influence values will be estimated and the row sum will be equal to 1.

In figure 1, we note that the third expert, Faisal, has withheld information on degree to which he is influenced by his children, expert 2 and expert 4. In this case, we already know that that both the unknown degrees belong to the interval $(0, 0.6]$. According to the algorithm, we see that

$$1-\sum_{t=1,t\neq l',k'}^{m}\omega_{it} \geq 4\left(\frac{\sum_{i=1,k\neq k'}^{k}\omega_{ik}}{m-1}\,\frac{\sum_{i=1,l=l'}^{k}\omega_{il}}{m-1}\right)$$

Therefore, the example falls under case 1 of the algorithm. Here, note that $\max\left\{\frac{\sum_{i=1,i\neq 3}^{4}\omega_{i2}}{3}, \frac{\sum_{i=1,i\neq 3}^{4}\omega_{i4}}{3}\right\} = \frac{\sum_{i=1,i\neq 3}^{4}\omega_{i2}}{3}$

Therefore, $\omega_{32} = 0.492$ and $\omega_{34} = 0.108$. The reason for allocating both values to the unknowns in this manner is because on average, the second expert has influenced the experts more as compared to the fourth expert. Because of this information, we claim that the greater value should be ω_{32} inferring that the second expert has influenced the third expert with a greater intensity.

4. Role of honesty in modelling influence

Once the confined interpersonal influences are estimated, we work our way towards the influence model. In this section, we modify the recursive definition of the influence process by introducing the degree of honesty of the experts. Consider $\wp$ to be the set of all preference relations. Then $p_j, j \in \{1,2,\ldots,m\} \in \wp$ is the fuzzy preference relation provided by the $j-th$ expert. An important aspect of this iterative model is the diagonal matrix of honesty denoted as $\mathcal{H}$. Honesty of an expert is calculated from the preference relation provided by the expert. It is measured as a function of average fuzziness in the matrix and how adrift this matrix is from an additive reciprocal preference relation. That is, $\mathcal{H} = \nabla(\overline{\mathbb{F}}, \overline{AR})$ where $\overline{\mathbb{F}}$ is an $m \times m$ diagonal matrix defined as $\overline{\mathbb{F}} = diag(\mathbb{F}(P_1), \ldots, \mathbb{F}(P_m))$ where average fuzziness $\mathbb{F}: \wp \to [0, 1]$ is defined as follows

$$\mathbb{F}(P_k) = 1 - \sum_{i \neq j, i,j=1}^{n} \frac{F(a_{ij}^k)}{n(n-1)}$$

Where $F: [0,1] \rightarrow [0,1]$ is defined in the following equation.

$$F(a_{ij}^k) = 1 - \begin{cases} \frac{a_{ij}^k}{0.5-\epsilon} & if \quad a_{ij}^k \in [0, 0.5-\epsilon) \\ 1 \quad if & a_{ij}^k \in [0.5-\epsilon, 0.5+\epsilon) \\ \frac{a_{ij}^k - 1}{\epsilon - 0.5} & if \quad a_{ij}^k \in (\epsilon + 0.5, 1] \end{cases} \quad (5)$$

The idea is that if the preference values are closer to 0.5, then the matrix has more fuzziness. If all values provided by an expert in his preference relation are 0.5 then it means that there is maximum fuzziness in his preference relation which in turn reflects in the degree of honesty with which he has proposed his preferences. But, this must also depend on the decision problem under consideration. For instance, if all experts in the panel express preferences closer to 0.5 then there may be ambiguity in the decision problem. This is the reason why $\epsilon \in [0,1]$ is introduced in the function. It varies according to how the panel collectively behaves. If there is ambiguity in the decision problem, the preference values provided by all experts may be closer to 0.5 and hence the function proposed above is dependent on it and will vary from problem to problem. We define ϵ as the average dispersion of all preference values from 0.5.

$$\epsilon = 0.5 - \sum_{k=1}^{m} \sum_{i \neq j, i,j=1}^{n} \frac{|a_{ij} - 0.5|}{mn(n-1)}$$

It needs to be noted that ϵ will vary depending on how all experts have responded to a decision problem. Suppose we have a 3 by 3 preference relation comprising of 0s and 1s as the non-diagonal entries, then $\epsilon = 0.5 - \frac{0.5(6)}{3(2)} = 0$. Accordingly, $F(a_{ij})$ will be 1, indicating that there is no fuzziness in the choices of the expert.

Whereas, if all non-diagonal entries in the supposed matrix are 0.5 then $\epsilon = 0.5 - \frac{0.5(6)}{3(2)} = 0$. Accordingly, $F(a_{ij})$ will be 0 indicating that there is maximum fuzziness in the choices of the expert.

Now we define the second function $\overline{AR}$. It is dependant on how different the preference relation is from an additive transitive preference relation

$$\overline{AR} = diag(\widetilde{AR}(P_1), \ldots, \widetilde{AR}(P_m))$$

where,

$$\widetilde{AR}(P_k) = 1 - 2 \sum_{i \neq j, i,j=1}^{n} \frac{AR(a_{ij}^k, a_{jI}^k)}{n(n-1)}$$

Here $AR: [0,1] \times [0,1] \rightarrow [0,1]$ is defined as $AR(a_{ij}^k, a_{ji}^k) = |a_{ij}^k - a_{ji}^k + 1|$.

For the $k-th$ expert, this measures the distance between the sum of a_{ij} and a_{ji} from 1.

Again, we emphasize that preference relation has more average fuzziness $\mathbb{F} \in [0,1]$ if the value is closer to 0. Similarly, fuzziness in the preference relation is less if it is closer to 1. Moreover, a preference relation is farther from being an additive reciprocal relation if $AR \in [0,1]$ is closer to 1 and it is closer to an additive reciprocal relation if the value is closer to 0.

Now we define the honesty function $\mathcal{H} = diag(h_1, \ldots, h_m) = \nabla(\overline{\mathbb{F}}, \overline{AR})$ by using the probabilistic t-conorm defined in section 2. Here, $h_i = \nabla\left(\mathbb{F}(P_i), \widetilde{AR}(P_i)\right)$. We elaborate the calculation of honesty of an expert by finding average fuzziness and distance to additive reciprocity of his preference relation in the following example.

Example 2: Suppose that there are four options $X = \{x_1, x_2, x_3, x_4\}$ for investing a fixed amount of money, where, $x_1 =$ buying a house, $x_2 =$ investing in Silicon valley, $x_3 =$ invest in sports car rental business, $x_4 =$ invest in airplane ownership. A panel of experts $E = \{e_1, e_2, e_3, e_4\}$ have proposed their preference relations P_1, P_2, P_3, P_4 over X as follows. These preference relations will be converted into matrices of predilections later.

$$P_1 = P_3 = \begin{pmatrix} 0.5 & 0.4 & 0.6 & 0.9 \\ 0.7 & 0.5 & 0.5 & 0.3 \\ 0.2 & 0.6 & 0.5 & 0.1 \\ 0.3 & 0.2 & 0.3 & 0.5 \end{pmatrix}, \; P_2 = \begin{pmatrix} 0.5 & 0.3 & 0 & 0.2 \\ 0.1 & 0.5 & 1 & 0.3 \\ 0.2 & 0 & 0.5 & 0.6 \\ 0.8 & 0.4 & 0.2 & 0.5 \end{pmatrix} \text{ and}$$

$$P_4 = \begin{pmatrix} 0.5 & 0.9 & 0.8 & 0.4 \\ 0.1 & 0.5 & 0.7 & 0.9 \\ 0.2 & 0.3 & 0.5 & 0.6 \\ 0.6 & 0.1 & 0.4 & 0.5 \end{pmatrix}$$

In this example, we calculate $\epsilon = 0.25$. Accordingly,

$$F\left(a_{ij}^k\right) = 1 - \begin{cases} 4a_{ij}^k \quad if \quad a_{ij}^k \in [0, 0.25) \\ 1 \quad if \quad a_{ij}^k \in [0.25, 0.75) \\ -4(a_{ij}^k - 1) \quad if \quad a_{ij}^k \in (0.75, 1] \end{cases}$$

We calculate, $\mathbb{F}(P_1) = \mathbb{F}(P_3) = 0.08$, $\mathbb{F}(P_2) = 0.28$ and $\mathbb{F}(P_4) = 0.233$

Accordingly, $\overline{\mathbb{F}} = diag\left(\mathbb{F}(P_1), \mathbb{F}(P_2), \mathbb{F}(P_3), \mathbb{F}(P_4)\right)$. That is,

$$\overline{\mathbb{F}} = \begin{pmatrix} 0.08 & 0 & 0 & 0 \\ 0 & 0.28 & 0 & 0 \\ 0 & 0 & 0.08 & 0 \\ 0 & 0 & 0 & 0.233 \end{pmatrix}$$

Similarly, $\widetilde{AR}(P_1) = 0.7167, \widetilde{AR}(P_2) = 0.6834, \widetilde{AR}(P_3) = 0.7167, \widetilde{AR}(P_4) = 1$. Consequently,

$$\overline{AR} = \begin{pmatrix} 0.7167 & 0 & 0 & 0 \\ 0 & 0.6834 & 0 & 0 \\ 0 & 0 & 0.7167 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With the help of these matrices, we use the t-conorm to state the diagonal matrix of honesty as follows.

$$\mathcal{H} = \begin{pmatrix} 0.739 & 0 & 0 & 0 \\ 0 & 0.772 & 0 & 0 \\ 0 & 0 & 0.739 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If all experts are honest, then the diagonal entries in $\mathcal{H}$ will be 1. This is a specific case, and it will keep the model as it originally is because $\mathcal{H}$ will be an identity matrix. After interactions, the interpersonal influences revise the initial opinions and since the expert is honest, he states the revised opinion as is. However, in a more general case, if experts are not completely honest, then the revised opinions are not exhibited as they are but they are misinformed. This affects the revised opinions in each iteration. The expert wants the group to believe that their opinion is different from what it really is.

We now reform the iterative method $y^{(t)} = AWy^{(t-1)} + (I - A)y^{(1)}$, (for $t = 2,3,\ldots$), that is used to model the process of influence among experts belonging to the social network, to also incorporate for honesty. The idea is that when an expert undergoes the process of discussion and reforms his opinion because of interpersonal influence, he may not express it exactly as it has been reformed. This information may be changed due to various reasons.

$$x^{(t)} = AWy^{(t-1)} + (I - A)y^{(1)} \qquad (7)$$

$$y^{(t)} = \mathcal{H}x^{(t)} \qquad (8)$$

for $t = 2,3,\ldots$ where $y^{(1)}$is an $n \times 1$column vector of experts' initial opinion in a decision model. W is an $n \times n$ matrix of interpersonal influences such as $\omega_{ij} \in [0,1]$ and $\sum_{j=1}^{n} \omega_{ij} = 1$. Also, $A = diag(a_{11}, \ldots a_{nn})_{n\times n}$ where $a_{ii} \in [0,1], a_{ii} = 1 - \omega_{ii}$, is the diagonal matrix of experts' susceptibility to interpersonal influences on the problem. Note that, $x^{(t)}$ is the $n \times 1$ column vector of the experts' revised opinion at time t whereas, $y^{(t)}$ is the $n \times 1$ column vector of experts' exhibition of their reviewed opinion at time t.

It is easy to see that this recursive definition of honesty based influence process is convergent. Assume that the process reaches equilibrium eventually and that $I - \mathcal{H}AW$ is non-singular. We have, $\lim_{t\to\infty} y^{(t)} = y^{(\infty)} = k < \infty$, so the equation is rewritten as,

$$y^{(t)} = \mathcal{H}x^{(t)}$$

$$y^{(t)} = \mathcal{H}(AWy^{(t-1)} + (I - A)y^{(1)})$$

$$\lim_{t\to\infty} y^{(t)} = \lim_{t\to\infty} \left(\mathcal{H}\left(AWy^{(t-1)} + (I - A)y^{(1)}\right)\right)$$

$$y^{(\infty)} = \mathcal{H}(AWy^{(\infty)} + (I - A)y^{(1)})$$

$$(I - \mathcal{H}AW)y^{(\infty)} = \mathcal{H}(I - A)y^{(1)}$$

Where $V_{\mathcal{H}} = (I - \mathcal{H}AW)^{-1}\mathcal{H}(I - A)$ represents the total interpersonal effects that transform the initial opinion along with the role of honesty on transforming the actual revisions into final ones.

We now portray how to model influence by incorporating the role of honesty in a decision model. We base this model basically on two steps. The first step is to transform each preference relation into matrix exhibiting predilection of each alternative over others. We do not alter or improve the given preference relation because we do not want to void originality of the data provided by the decision makers. This provides us with m matrices, each representing predilection of expert i on each alternative over the other $n - 1$ alternatives. This is represented as

$$\rho_i = \begin{pmatrix} p_{i1} \\ \cdot \\ p_{in} \end{pmatrix}, i = 1,2,..,m$$

This represents the predilection of expert i over all the alternatives. We now model how these choices evolve as they interact with one another. Moreover, we assert that honesty plays a vital role when it comes to deliverance of opinions. Before the expert expresses his predilections for a certain alternative over all the other alternatives, he may choose to alter the information. Experts may differ with one another, but with the exchange of dialogues, they mend and improve their wants. However, experts who are honest, portray their predilections as they are but those that are not, make the amendments that they deem necessary before expressing their revisions publicly. Since, experts are providing preference relations on n alternatives, the matrices of predilections are column vectors of order $n \times 1$ and hence the model is applied to each alternative. So, for $i = \{1,2,..,n\}$,

$$y_{x_i}^{(\infty)} = V_{\mathcal{H}}(y^{(1)})$$

We transform the information provided by the decision maker so that it can be used in the influence model. Therefore, $(y_{x_i}^{(1)})$ represents the initial opinion of the experts over alternative x_i. Hence, relevant information is gathered in the following column vector.

$$y_{x_i}^{(1)} = \begin{pmatrix} p_{1i} \\ \cdot \\ p_{mi} \end{pmatrix}, i = 1,2,..,n$$

We compute $V_{\mathcal{H}} = (I - \mathcal{H}AW)^{-1}\mathcal{H}(I - A)$ and apply the honesty based influence model, the final opinion are obtained as follows.

$$y_{x_i}^{(\infty)} = \begin{pmatrix} y_{1i}^{(\infty)} \\ \cdot \\ y_{mi}^{(\infty)} \end{pmatrix}, \qquad i = 1,2,..,n$$

This information is used to obtain the final vectors for each expert

$$\rho_i^{(\infty)} = \begin{pmatrix} y_{i1}^{(\infty)} \\ \cdot \\ y_{in}^{(\infty)} \end{pmatrix}, \qquad i = 1,2,..,m$$

For illustration, we consider example 2. The first step is to transform the preference relations into matrices of predilections. For this purpose, we measure the dominance degree by using the weighting vector $W_{OWA} = (0.5,0.21,0.16,0.13)$, calculated with the linguistic quantifier "most of" defined as $(r) = r^{\frac{1}{2}}$.

$$\rho_1 = \rho_3 = \begin{pmatrix} 0.708 \\ 0.574 \\ 0.45 \\ 0.387 \end{pmatrix}, \rho_2 = \begin{pmatrix} 0.345 \\ 0.66 \\ 0.437 \\ 0.595 \end{pmatrix}, \rho_4 = \begin{pmatrix} 0.747 \\ 0.69 \\ 0.479 \\ 0.482 \end{pmatrix}$$

Adjacency matrix of interpersonal influence among the experts is defined in example 1 as:

$$W = \begin{pmatrix} 0.3 & 0.2 & 0.3 & 0.2 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0.3 & 0.492 & 0.1 & 0.108 \\ 0.2 & 0.3 & 0.4 & 0.1 \end{pmatrix}$$

The diagonal elements represent susceptibility of each expert to interpersonal influence. The closer the value w_{ii} is to 1, the more susceptible the expert is to interpersonal influence. Similarly, $w_{12} = 0.2$ indicates the degree of direct relative influence of the second expert on the first expert. We have already calculated degree of honesty of each expert based on the preference values that they provided. Now, we calculate

$$V_{\mathcal{H}} = \begin{pmatrix} 0.2866 & 0.1809 & 0.0222 & 0.01792 \\ 0.00626 & 0.6714 & 0.00608 & 0.00413 \\ 0.0701 & 0.3013 & 0.0892 & 0.01454 \\ 0.0863 & 0.3542 & 0.0415 & 0.1204 \end{pmatrix}$$

to apply in the process. Here,

$$A = \begin{pmatrix} 0.7 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0.9 \end{pmatrix}$$

It is obvious that all elements in $(I - \mathcal{H}AW)^{-1}\mathcal{H}(I - A)$ are well defined. The final opinion on alternative x_1 is

$$y_{x_1}^{(\infty)} = \begin{pmatrix} 0.2866 & 0.1809 & 0.0222 & 0.01792 \\ 0.00626 & 0.6714 & 0.00608 & 0.00413 \\ 0.0701 & 0.3013 & 0.0892 & 0.01454 \\ 0.0863 & 0.3542 & 0.0415 & 0.1204 \end{pmatrix} \cdot \begin{pmatrix} 0.708 \\ 0.345 \\ 0.708 \\ 0.747 \end{pmatrix} = \begin{pmatrix} 0.2945 \\ 0.2435 \\ 0.2276 \\ 0.3062 \end{pmatrix}$$

It can be studied that the first expert, for instance, has an initial predilection for alternative x_1 over other alternatives with a degree of 0.708. Due to interpersonal influences, his final opinion changed which was presented to the panel affected by his degree of honesty as 0.437. Similarly,

$$y_{x_2}^{(\infty)} = \begin{pmatrix} 0.2866 & 0.1809 & 0.0222 & 0.01792 \\ 0.00626 & 0.6714 & 0.00608 & 0.00413 \\ 0.0701 & 0.3013 & 0.0892 & 0.01454 \\ 0.0863 & 0.3542 & 0.0415 & 0.1204 \end{pmatrix} \cdot \begin{pmatrix} 0.574 \\ 0.66 \\ 0.574 \\ 0.69 \end{pmatrix} = \begin{pmatrix} 0.309 \\ 0.453 \\ 0.3003 \\ 0.3902 \end{pmatrix}$$

$$y_{x_3}^{(\infty)} = \begin{pmatrix} 0.2866 & 0.1809 & 0.0222 & 0.01792 \\ 0.00626 & 0.6714 & 0.00608 & 0.00413 \\ 0.0701 & 0.3013 & 0.0892 & 0.01454 \\ 0.0863 & 0.3542 & 0.0415 & 0.1204 \end{pmatrix} \cdot \begin{pmatrix} 0.45 \\ 0.437 \\ 0.45 \\ 0.479 \end{pmatrix} = \begin{pmatrix} 0.2266 \\ 0.301 \\ 0.2103 \\ 0.27 \end{pmatrix}$$

$$y_{x_4}^{(\infty)} = \begin{pmatrix} 0.2866 & 0.1809 & 0.0222 & 0.01792 \\ 0.00626 & 0.6714 & 0.00608 & 0.00413 \\ 0.0701 & 0.3013 & 0.0892 & 0.01454 \\ 0.0863 & 0.3542 & 0.0415 & 0.1204 \end{pmatrix} \cdot \begin{pmatrix} 0.387 \\ 0.595 \\ 0.387 \\ 0.482 \end{pmatrix} = \begin{pmatrix} 0.2403 \\ 0.3266 \\ 0.2374 \\ 0.2583 \end{pmatrix}$$

Accordingly,

$$\rho_1^{(\infty)} = \begin{pmatrix} 0.2945 \\ 0.309 \\ 0.2266 \\ 0.2403 \end{pmatrix}, \rho_2^{(\infty)} = \begin{pmatrix} 0.2435 \\ 0.453 \\ 0.301 \\ 0.3266 \end{pmatrix}, \rho_3^{(\infty)} = \begin{pmatrix} 0.2276 \\ 0.3003 \\ 0.2103 \\ 0.2374 \end{pmatrix}, \rho_4^{(\infty)} = \begin{pmatrix} 0.3026 \\ 0.3902 \\ 0.27 \\ 0.2583 \end{pmatrix},$$

In the next section, we find the collective matrix of predilection. This matrix is ranked to find the alternative that is most liked by the group of experts.

5. Ranking of collective matrix of predilection

We have calculated the final matrices of predilections that incorporate for the changes in the initial opinions due to interpersonal communications and also the experts' honesty to present their actual predilections for each alternatives over others. Now we need to draw meaning and identify the best possible alternative according to the experts' revised opinions. This is done by aggregating the matrices of predilections using honesty based induced ordered weighted averaging operator defined below.

Definition 5:

Let E be the set of experts and $U = \{h_1, \dots h_m\} \in [0,1]^n$ be the vector of their associated degree of honesty. An n dimensional $\mathcal{H} - IOWA$ operator $\phi_W^{\mathcal{H}}$ is an operator in which the vector of degree of honesty is the order inducing vector and the associated set of weights are $W = (w_1, \dots, w_n)$. So,

$$\phi_W^{\mathcal{H}}(< h_1, p_1 >, < h_2, p_2 >, \dots, < h_n, p_n >) = \sum_{i=1}^{n} w_i p_{\rho(i)}$$

Here, $w_k = Q\left(\frac{S(k)}{S(n)} - Q\left(\frac{S(k-1)}{S(n)}\right)\right)$ where $S(k) = \sum_{l=1}^{k} u_{\sigma(l)}$ and σ is a permutation such that $h_{\sigma(i)} \geq h_{\sigma(i+1)}$ for all $i = 1,2,..,n-1$.

In this paper, we use honesty based IOWA operator in the selection phase. In literature, the use of quantifier guided dominance degree for this purpose. With the help of this, the best alternative for majority (Q) is computed.

Honesty based Influence model

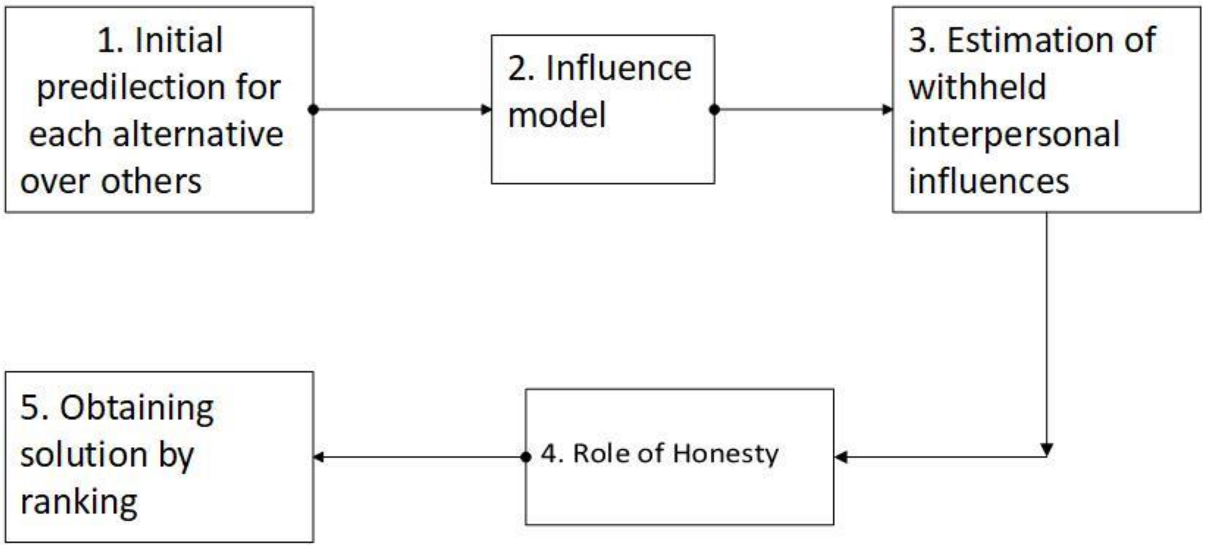

Figure 1: Honesty based influence model

Definition 6:

(Chiclana, Herrera-Viedma, Francisco , & Alonso, 2007) To calculate the dominance of one alternative over all others in a fuzzy majority sense, the quantifier-guided dominance degree $(QGDD_i)$ is defined as

$x_S = \{x_i \ : \ x_i \in X, QDDD_i = sup_j \ \ QGDD_j\}$ where $QGDD_j = \phi_Q(\widehat{p_{\iota 1}}, \dots, \widehat{p_{\iota n}})$ and $(\widehat{p_{\iota 1}}, \dots, \widehat{p_{\iota n}})$ represents a tuple of collective outcome.

We use the $\mathcal{H} - IOWA$ operator to accomplish the selection phase. Here,

$$\phi_W^{\mathcal{H}}(< 1,0.3026 >, < 0.772,0.2435 >, < 0.739,0.2957 >, < 0.739,0.2276 >)$$

$$= 0.5(0.3026) + 0.21(0.2435) + 0.16(0.2957) + 0.13(0.2276) = 0.279335$$

Similarly, for all alternatives, we have

$$\rho_s = \begin{pmatrix} 0.279335 \\ 0.3781 \\ 0.26811 \\ 0.2670 \end{pmatrix}$$

This implies that all experts exhibit predilection for the second alternative over other alternatives by the highest degree. Hence, order of alternatives from best to worst is (x_2, x_1, x_3, x_4). Some of the important steps undertaken in the process are stated in figure 1.

6. Conclusion

Classical models of decision making do not incorporate for the role of influence and honesty that affects the process. However, in real life, these factors may alter the process significantly and hence they need to be incorporated. In social networks, significant personalities have impact on choices of the masses. However, it is up to an individual to admit the influence and propose his revised opinion with complete honesty.

This paper further develops the theory of social network analysis by introducing the notion of honesty of experts. Honesty plays a vital role in the decision making process and hence needs to be incorporated in modelling of decision problems. Preference relations provided by the experts are used to measure degree of honesty of the experts. These relations are transformed into matrices of predilections and used in influence based model inspired by degree of honesty of the experts.

We calculate the degree of honesty from the preference relation provided by the expert. Honesty is estimated with the help of two measures; average fuzziness in the preference relation and how adrift the relation is from being additive reciprocal. Another contribution of this paper is that it deals with incompleteness in matrix of interpersonal influence. At times, an expert may deliberately withhold information about the degree to which other experts have influenced him or he may just not be sure to put down an exact number from the unit interval. We propose an algorithm to deal with this problem using the information available in the matrix of interpersonal influences.

Social networks are very important in the modern day. Decisions are based and affected by interpersonal interactions. It is important to understand how these models vary according to how honest they are in representing their revised opinions. In future, more traits from human psychology can be studied to improve the honesty based influence model. Moreover, improved ranking techniques (Benferhat, et al., 2016; Beg & Rahid, 2017) can be included to rank the alternatives for final outcome and applications of crowd detection (Benferhat, et al., 2016; Bezdek, , Bonnie , & Spillman, 1978; Chaudhry, Karim, Abdul Rahim, & BenFerhat, 2017) can be used in future to develop the theory of influence further.

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