Quantifying relevant uncertainties on the solution model of reflection tomography

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Summary

Reflection tomography determines the velocity model that best fits the travel time data associated with reflections of seismic waves in the subsurface. This solution model is only one model among many possible models. Indeed, the uncertainties on the observed travel times (resulting from an interpretative event picking on seismic sections) and more generally the underdetermination of the inverse problem lead to uncertainties on the solution model. An a posteriori uncertainty analysis is then crucial to delimit the range of possible solution models. The analysis of the a posteriori covariance matrix (inverse of the Hessian matrix) gives the uncertainties on the solution model but its computation is generally expensive (the matrix is huge for 3D problems) and the physical interpretation of the results is difficult. A formalism based on linear combinations of model parameters (macro-parameters) allows to compute uncertainties on relevant geological quantities for a reduced computational time (the matrices to be manipulated are reduced to the macro-parameter space). A first application on a synthetic example with basic macro-parameters shows their potentialities. The generality of the formalism allows a wide flexibility for the construction of the macro-parameters.

Introduction

Reflection tomography aims to determine the subsurface velocity model that best fits the travel time data associated with the main reflectors in the earth. The picking of the observed travel times on seismic sections leads to data corrupted by noise due to the difficulties in following a continuous event. Modeling errors (approximations in the forward problem, parameterization errors) come in addition to the errors on the input data. These uncertainties on the data and, more generally, the underdetermination of the inverse problem lead to uncertainties on the solution model. Classically, the analysis of the solution of the inverse problem consists in checking the misfits between observed travel times and calculated travel times by computing characteristic values of the misfit distribution (the RMS (Root Mean Square) value and the maximum misfit value) and by studying the spatial distribution of the misfits (detection of local zones with high misfits and trends). However, this analysis is not sufficient even if these quality control criteria are verified, the determined model is only one of many possible models that match the data. An uncertainty analysis should be performed to quantify the range of admissible models we can obtain from these data.

Let us consider a simple synthetic example (Figure 1) to illustrate this. This 3D model is composed of one layer with lateral velocity variations ($v(x,y)$, there is no vertical variations). We model synthetic data by ray tracing in the exact model. We add to these exact travel times a white noise (described by a center Gaussian probability law) of $5ms$ standard deviation. The inversion of these noisy data gives a solution model close to the exact model (model 1 of Figure 2) with a RMS misfit of $5ms$ and a maximum misfit of $20.47ms$, result which is consistent with the introduced noise on the data. But three other solution models could be obtained with an equivalent RMS misfit (Figure 2). Without knowing the exact model or introducing discriminant a priori information on the model, we could not choose between the four solution models: they all match the data within the expected accuracy. This example illustrates the necessity to be able to estimate the range of possible models. If the resulting uncertainties on the solution model is considered too high, we should introduce other data or a priori information on the model to reduce them.

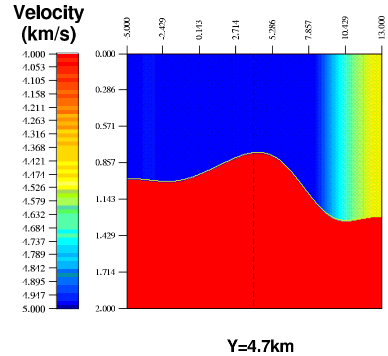

Fig. 1: Vertical slices in the 3D exact velocity model (Left: along x axis, right: along y axis).

In this paper, after the description of the chosen formulation of reflection tomography, we apply different methods to access the uncertainties on the solution model of the example of Figure 1. Following Tarantola (1987) and Gouveia and Scales (1997), we study the a posteriori covariance matrix and we discuss the limitations of the different methods to analyze this matrix. We then propose a new method to quantify uncertainties on geological macro-parameters (combinations of parameters).

Formulation of reflection tomography

In our approach (Jurado et al., 1996), the subsurface model m is composed of 2D or 3D B-spline functions describing velocity variations in a layer ($v(x,y) + k.z$ or $v(x,y,z)$) and 2D B-spline functions describing the interfaces ($Z(x,y)$, $Y(z,x)$ or $X(y,z)$).

The forward problem is a two-point ray tracing problem (sources and receivers are fixed) solved by a bending method (Jurado et al., 1998). We denote by $T^{cal}(m)$ the travel times computed by ray tracing in the model m.

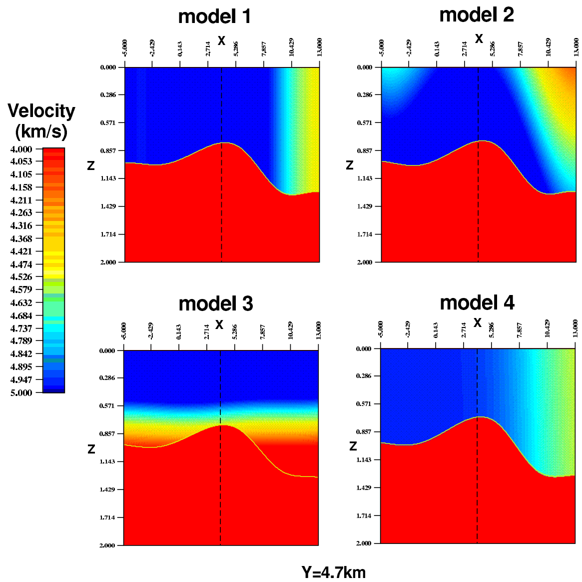

Fig. 2: Vertical slices along x axis in 4 different 3D solution velocity models obtained by tomography. Model 1: solution model with no vertical velocity variations ($k = 0/s$) - RMS of the travel time misfits $= 5ms$ - maximum value of the travel time misfits $= 20.47ms$. Model 2: solution model with a fixed vertical gradient $k = 0.5/s$ - RMS $= 5ms$, Max. $= 20.46ms$. Model 3: solution model with an erroneous parameterization of the velocity: $v(x, y, z)$ instead of $v(x, y)$ - RMS $= 5.5ms$ - Max. $= 22ms$. Model 4: solution model with a velocity parameterization by $v(x, y, z)$ and a constraint that keeps the vertical velocity gradient positive - RMS $= 5.5ms$ - Max. $= 22ms$.

The inverse problem consists in minimizing the objective function (Delprat-Jannaud and Lailly, 1993)

$$C(m) = ||T^{cal}(m) - T^{obs}||^2_{C_d^{-1}} + \epsilon^2 ||D^2(m - m^{prior})||^2_{\mathcal{M}} \quad (1)$$

with $||D^2(m)||^2$ a regularization term which is necessary to well pose the inverse problem (this term is composed of the sum of the second derivatives of the velocity variations and the interface depths) where m is a subsurface model defined by N_I interfaces and N_V layer velocities. $||.||_{C_d^{-1}}$ is the norm in the data space, C_d being the a priori covariance operator on the data. Its diagonal elements specify the uncertainties on the data (errors on the data themselves and modeling errors), and its off-diagonal elements specify the correlations between these uncertainties. $||.||_{\mathcal{M}}$ is the norm in the model space (here the L^2 norm). ϵ^2 is a regularization weight that allows to tune the regularization effect.

The objective function (1) is not quadratic since the forward modeling operator $T^{cal}(m)$ is non linear. We solve this non linear minimization problem with a Gauss-Newton algorithm, which consists in successive linearizations of the forward operator. The resulting quadratic objective function

$$\tilde{C}_n(\delta m) = \frac{1}{2} ||J_n \delta m - \delta T_n^{obs}||^2_{C_d^{-1}} \quad (2)$$

$$+ \frac{\epsilon^2}{2} ||D^2(m_n + \delta m - m^{prior})||^2_{\mathcal{M}} \quad (3)$$

is minimized at each Gauss-Newton iteration n where m_n is the current model, δm is the model perturbation, $J_n = \frac{\partial T^{cal}}{\partial m}(m_n)$ is the Jacobian matrix evaluated at m_n, and $\delta T_n^{obs} = T^{obs} - T^{cal}(m_n)$. Solving this minimization problem is equivalent to solving the linear system

$$(J_n^T C_d^{-1} J_n + \epsilon^2 \mathcal{Q}_{reg}) \delta m = J_n^T C_d^{-1} \delta T_n^{obs} \quad (4)$$

where $\mathcal{Q}_{reg}$ is the matrix made up of the regularization terms.

Quantifying uncertainties on solution model

A classical approach consists in the analysis of the Hessian matrix (or its inverse: the a posteriori covariance matrix) associated with the linearized problem (2) around the solution model m_∞. This approach is valid in the vicinity of the solution model, the size of the vicinity depending on the non linearity of the forward map.

The Hessian matrix measures the influence of a model perturbation δm on the quadratic cost function defined around m_∞

$$\tilde{C}_\infty(\delta m) - \tilde{C}_\infty(0) = \delta m^T (J_\infty^T C_d^{-1} J_\infty + \epsilon^2 \mathcal{Q}_{reg}) \delta m \quad (5)$$

with $\tilde{C}_\infty(0) = C(m_\infty)$. The a posteriori covariance matrix is defined by

$$C'_m = (J_\infty^T C_d^{-1} J_\infty + \epsilon^2 \mathcal{Q}_{reg})^{-1}. \quad (6)$$

The space of admissible models could be characterized by the ellipsoids of center m_∞, contour lines of

$$(m - m_\infty)^T C_m'^{-1} (m - m_\infty). \quad (7)$$

The axes of the ellipsoids (7) are defined by the eigenvectors of C'_m, the square root of the eigenvalues giving the uncertainties on the associated eigenvector.

The square root of diagonal terms of C'_m are the uncertainties on the B-spline parameters describing the model and the off-diagonal terms are the correlations between these uncertainties. In Figure 4, we notice the very high values for the boundary B-spline coefficients. It is not surprising: the sensitivity of the B-spline function to these coefficients is small. We observe in Figure 3 also high correlations between the uncertainties on these coefficients: this is the effect of the regularization, these coefficients being mostly determined by this information. The uncertainties on the B-spline parameters are not very interesting in practice: the very high uncertainties on the boundary coefficients are intrinsic to the B-spline definition. We would rather compute uncertainties on physical quantities, for instance, the evaluation of the B-spline functions in the physical domain.

As Delprat-Jannaud and Lailly (1992), we have studied the eigenvector decomposition of the a posteriori covariance matrix to have access to the worst/best determined model directions and the associated uncertainties.

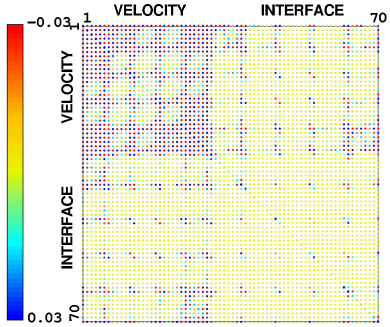

Fig. 3: A posteriori covariance matrix of the solution model 1 of Figure 2.

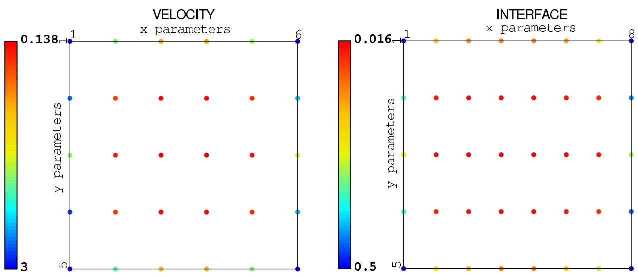

Fig. 4: Uncertainties associated to model parameters (square root of the diagonal terms of the a posteriori covariance matrix) associated with the model 1 of Figure 2. The horizontal (resp. vertical) axis represents the index of the B-spline parameter in x (resp. y) direction.

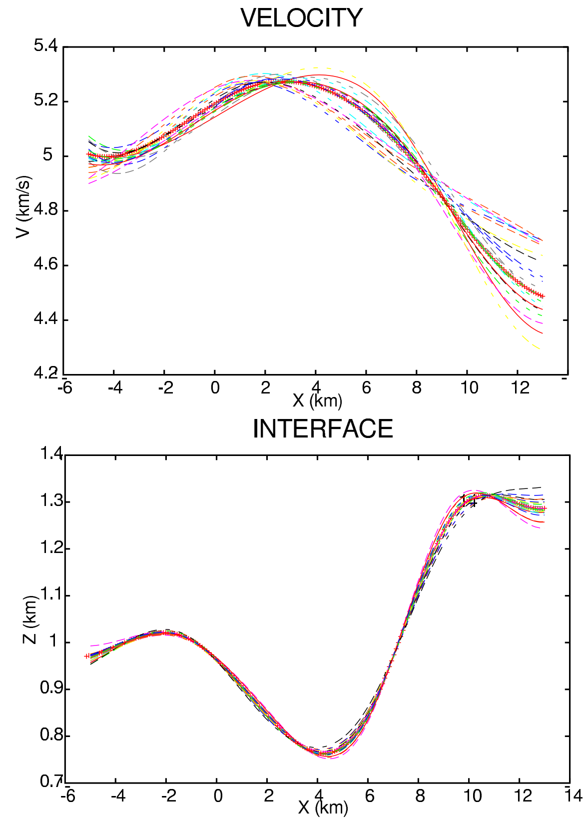

Fig. 5: Slices along x axis in the 3D simulated velocity models. Left: velocity variations. Right: interface depth variations.

The best (resp. worst) determined model direction corresponds to the smallest (resp. highest) eigenvalue of the a posteriori covariance matrix , i.e. highest (resp. smallest) perturbation of the quadratic cost function. The uncertainties associated with these eigenvectors, namely the square root of the eigenvalues, are meaningless, the eigenvectors being composed of mixed velocity and interface parameters. A careful choice of the norm in the model space to be used for the decomposition of the a posteriori covariance matrix has to be done (Tarantola (1987), Delprat-Jannaud and Lailly (1992)). We should check that the method furnishes intrinsic results which do not depend on the physical units or on the dicretization for instance. We do not recommend this approach in applications mainly because of the difficult physical interpretation of the eigenvectors. The eigenvectors are a combination of interface and velocity parameters, what we compute is thus the uncertainties on these combinations which are usually difficult to link to physical quantities. Moreover, this approach is expensive: the diagonalisation of the huge a posteriori covariance matrix has to be performed. Another approach is the simulation of admissible models from the a posteriori probability density function that allows a more complete analysis than the sole analysis of the diagonal terms of the a posteriori covariance matrix. Indeed, the correlations between the uncertainties are taken into account. Moreover, the simulations furnish directly interpretable results, i.e. physical models. The method (see, for instance, Parker (1994)) consists in random simulations of model pertubations from the probability density function

$$\exp\{-\frac{1}{2}\delta m^T C_m' \delta m\}.$$

Figure 5 shows 20 simulated models obtained by this method: slice of the interfaces and lateral variations of the velocities along x direction. The highest uncertainties (≈100m) on the interface depth are located at the boundaries of the model, areas that are not well illuminated by the rays. In other areas, the order of magnitude of the uncertainties of the depth is a few meters. Concerning the lateral velocity variations, we observe uncertainties of $100 - 200m/s$ distributed on the whole domain. This method is quite attractive for the straightforward interpretation of the results despite its cost.

Uncertainties on geological macro-parameters

The methods described in the previous section present some interesting features but remain expensive for 3D applications. Here, we propose a method that allows to deal with large models at a reasonable cost. Moreover, we want to give to the geophysicist some uncertainties on physical quantities, and not on some numerical parameters that we invert for in the tomography process. The proposed approach consists in building macro-parameters with a geophysical interest. These macro-parameters are linear combinations of the inverted parameters: for instance, the mean of the vertical velocity variations in a zone, the slope of an interface, the average thickness of a layer, etc. Grenié (2001) has introduced this notion of macro-parameter (his main motivation being to avoid numerical problems in the inversion of the complete Hessian). We propose here a generalization of his work (general definition of macro-parameters) which allows the computation of uncertainties for huge 3D problems.

We apply this method on our example. We choose simple macro-parameters: the mean of the velocity variation in the layer, the mean of its first derivatives, the mean of the interface depth and of its first derivatives. The uncertainties on those macro-parameters are calculated on the solution model 1 and on the solution model 4 of Figure 2, the latter model being obtained with a parameterization of the velocity by a 3D B-spline $v(x, y, z)$ instead of a 2D B-spline $v(x, y)$. The results are listed in the Table 1. For the inversion with a $v(x, y, z)$ parameterization, we notice the bad determination of the velocity and especially of its vertical variations (uncertainty of $0.17/s$, twice the standard deviation) (as already shown with the different solution models presented in Figure 2). This result is not surprising: the rays propagate close to the vertical direction, the travel times are thus not very sensitive to vertical velocity variations. This method with its general formalism allows to compute uncertainties on relevant geological quantities with a reasonable computation cost. Gaussian simulations of the macro-parameters (using the reduced a posteriori covariance matrix) can also be performed: it furnishes interesting information on the correlations of macro-parameter uncertainties and has a lower computational cost than simulations of the model parameters with the complete a posteriori covariance matrix.

Conclusions

Reflection tomography furnishes the velocity model that best fits the travel time data: however, this solution model is only one out of many admissible models. An a posteriori uncertainty analysis is crucial to delimit the range of possible solution models that would fit the data and the a priori information with the expected accuracy. In this paper, we describe different methods to perform a linearized a posteriori analysis, approach valid only in the vicinity of the solution model. The methods based on the analysis of the a posteriori covariance matrix (huge matrix for 3D models) are generally too expensive and the physical interpretation of the results is difficult. We propose a general formalism to reduce the a posteriori analysis to geological quantities of the model: we evaluate the uncertainties on macro-parameters (linear combinations of model parameters) that have a geological interest. This method allows the manipulation of reduced matrices and thus becomes feasible in 3D. A first application on a synthetic example with basic macro-parameters shows the potentialities of the method. The generality of the formalism allows a wide flexibility for the construction of the macro-parameters. Nevertheless, we should keep in mind that this approach is only valid in the vicinity of the solution model (linearized framework) and complex cases may require a non linear approach.

Acknowledgements

The authors would like to thank also Pr. Michel Cuer, from Montpellier II University, and Andreas Ehinger, from IFP, for many fruitful discussions.

References

Delprat-Jannaud, F., and Lailly, P., 1992, What information on the earth model do reflection travel times hold ?: Journal of Geophysical Research, **97**, 19827–19844.

Delprat-Jannaud, F., and Lailly, P., 1993, Ill-posed and well-posed formulations of the reflection travel time tomography problem: Journal of Geophysical Research, **98**, 6589–6605.

Gouveia, W., and Scales, J., 1997, Resolution of seismic waveform inversion: Bayes versus occam: Inverse problems, pages 323–349.

Grenié, D., 2001, Tomographie de temps de trajet adaptée au problème de l'imagerie 3D en exploration pétrolière: théorie, mise en oeuvre logicielle et applications: Ph.D. thesis, Université Paris VII.

Jurado, F., Sinoquet, D., and Ehinger, A., 1996, 3D reflection tomography designed for complex structures: 66^{th} Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 711–714.

Jurado, F., Lailly, P., and Ehinger, A., 1998, Fast 3D two-point raytracing for traveltime tomography: Proceedings of SPIE, Mathematical Methods in Geophysical Imaging V, **3453**, 70–81.

Parker, R., 1994, Geophysical inverse theory: Springer, Berlin.

Tarantola, A., 1987, Inverse problem theory: Elsevier.

MODELS		**Macro-parameters**	
		Function	**DZ**
Model 1 ($v(x, y)$)	velocity	3m/s	–
Model 4 ($v(x, y, z)$)		126m/s	0.085/s
Model 1 ($v(x, y)$)	interface	0.76m	–
Model 4 ($v(x, y, z)$)		0.76m	–

Table 1: Standard deviations (square roots of the diagonal terms of $\tilde{C}''_m$) associated with different macro-parameters: mean of the B-spline function, of its first derivative in z on the definition domain. This uncertainty analysis is performed for the solution model 1 of Figure 2 obtained with the exact parameterization and also for the solution model 4 of Figure 2 obtained with a velocity parameterization $v(x, y, z)$.