RedFame

Applied Economics and Finance
Vol. 1, No. 1; May 2014
ISSN 2332-7294 E-ISSN 2332-7308
Published by Redfame Publishing
URL: http://aef.redfame.com

# Martingales in Daily Foreign Exchange Rates: Evidence from Six Currencies against the Lebanese Pound

Samih Antoine Azar

Correspondence: Samih Antoine Azar, Professor, Faculty of Business Administration & Economics, Haigazian University, Mexique Street, Kantari, Beirut, Lebanon.

Received: March 1, 2014 Accepted: March 20, 2014 Available online: March 24, 2014

doi:10.11114/aef.v1i1.362 URL: http://dx.doi.org/10.11114/aef.v1i1.362

## Abstract

The purpose of this paper is to test whether the Lebanese foreign exchange rate market is weak form efficient by studying the stochastic behavior of six foreign currencies against the Lebanese pound on a daily basis. Efficiency requires that the data meet more than one condition. The first condition is the presence of a unit root process. The second one is that increments are random and uncorrelated. The third is the long term persistence of shocks. The fourth is the absence of breaks in the samples. The last one is the insignificance of pair-wise Granger causality tests. These five conditions describe a statistical behavior known as a martingale. All five conditions are found to apply to the six data series. Non-normality, conditional heteroscedasticity, other non-linear dependencies, and contemporaneous cross-correlations of the log returns of the exchange rates are features that are present in the data but that do not invalidate the general designation of a martingale. Finally, the descriptive statistics of the six series under consideration are quite similar to those of other major currencies, even when compared for different time periods, implying that daily foreign exchange rates share quasi the same characteristics globally.

**Keywords:** Lebanese pound, six foreign currencies, daily frequency, martingale, weak form efficiency, unit roots, autocorrelation, runs tests, variance ratio tests, calendar breaks, Granger-causality, normality, conditional heteroscedasticity, non-linear dependence, cross-correlation, descriptive statistics.

**JEL Classification codes:** F31, G14, G15, C22.

## 1. Introduction

The purpose of this paper is to determine whether the Lebanese foreign exchange market is weak form efficient by studying the statistical behavior of six daily foreign currencies against the Lebanese pound. Notably the paper seeks to examine whether these six currencies follow each a martingale. The requirements for such a process are many. Among them is verifying that these currencies incorporate one unit root, that increments are random, that shocks are very persistent, that the samples do not suffer from any calendar breaks, and finally, that the currency rates are not predictable from the past history of bilateral relations. The issue of financial efficiency is crucial especially because research has unveiled inefficiencies in distant samples, inefficiencies that tend to disappear later on in most recent samples, and this is probably due to a learning process (Van de Gucht et al., 1996; Chang, 2004; Qi and Wu, 2006). Some research has consisted in testing for unit roots (Ibrahim et al., 2011), while other research has applied variance-ratio tests (Liu and He, 1991; Van de Gucht et al., 1996; Chang, 2004; Rufino, 2014). No research has tested as comprehensively the stochastic process of foreign exchange rates as is done in this paper. The presence of a unit root and the finding of persistence of shocks by variance-ratio tests are only parts of the requirements for informational efficiency (Grossman and Stiglitz, 1980). Orthogonality to past information, randomness, periodic homogeneity of the underlying process, and unpredictability by cross rates are other requirements. The paper also tests for supplementary characteristics such as non-normality, non-linear dependence, conditional heteroscedasticity, and contemporaneous cross correlations, although these "anomalies" do not affect the appropriateness of the hypothesis of a martingale.

The paper is organized as follows. In the second section the empirical results are presented and discussed. Subsections include the source of the data, unit root tests, tests on uncorrelated increments, variance ratio tests, non-linear dependencies, normality tests, tests for breaks, Granger causality tests, tests for GARCH effects, cross correlation tests, and end with some global descriptive statistics. One last subsection carries out some hypothesis tests that are of interest. The paper concludes that the Lebanese foreign exchange market is efficient by the standards that are initially set up, and that there is evidence for a global market of foreign exchange rates.

## 2. The Empirical Results

### 2.1 Source of the Data

The data is taken from the web site of the Bank of Lebanon, the central bank of Lebanon, and span the daily period from January 4, 2010 to January 31, 2014, i.e. 990 observations per variable. Six currencies are selected and these are the Australian dollar (AUD), the Canadian dollar (CAD), the Swiss franc (CHF), the Euro (EURO), the British pound (GBP), and the Japanese yen (JPY). The six currencies are quoted indirectly in terms of the number of units of the Lebanese pound per one unit of the foreign currency. All series are logged, and log returns are calculated by taking the first difference of the natural logs.

### 2.2 Unit Root Tests

The first condition for a martingale that needs to be verified is whether there is a unit root in the logged data series. A unit root is a requirement of weak form efficiency as discussed by Fama (1965, 1970, and 1991). A martingale is defined as a process whereby the expected future value of the variable, based on current information, is equal to its current value. Another term for such a process is a random walk. If $Z$ is the foreign exchange rate, $\alpha$ is a constant, $\varepsilon$ is a well-behaved residual, and $E$ is the expectation operator for information at time $t$, then a (sub)martingale adheres to the following relation in period $t+1$:

$$\log(Z_{t+1}) = \alpha + \log(Z_t) + \varepsilon_{t+1} \quad \Rightarrow \quad \mathrm{E}(\log(Z_{t+1})) = \alpha + \log(Z_t) \tag{1}$$

Since the data is logged, and is of a daily frequency, the first difference of the logs is a very close approximation to a proportionate change, and a stationary process of the latter implies that expected returns are stable, which is also another requirement of weak form efficiency:

$$E(\log(Z_{t+1})) - \log(Z_t) \approx E\left(\frac{Z_{t+1} - Z_t}{Z_t}\right) \approx \alpha \tag{2}$$

Although there are more recent tests for unit roots than the Phillips-Perron test, (Phillips and Perron, 1988), this test is selected because it is robust to the presence of heteroscedasticity, which is a salient feature of the data, as will be shown later on. The test includes a constant and a trend. Table 1 presents the results of the individual unit root tests.

Table 1. Unit root tests with an individual constant and trend

| Z | Test on log Z | Test on Δ(log Z) |
|---|---|---|
| **Individual Phillips-Perron tests** | | |
| AUD | -1.365952 (0.8703) | -32.79569 (0.0000) |
| CAD | -2.114074 (0.5368) | -34.95555 (0.0000) |
| CHF | -1.845859 (0.6816) | -31.48207 (0.0000) |
| EURO | -2.524542 (0.3161) | -31.91371 (0.0000) |
| GBP | -3.055582 (0.1178) | -33.48230 (0.0000) |
| JPY | -1.337255 (0.8779) | -34.12272 (0.0000) |
| **Panel tests** | | |
| (1) Null: Unit root (assumes common unit root process) | | |
| Levin, Lin & Chu t* | 0.09701 (0.5386) | -41.3383 (0.0000) |
| Breitung t-statistic | 0.55488 (0.7105) | -12.6431 (0.0000) |
| (2) Null: Unit root (assumes individual unit root process) | | |
| Im, Pesaran and Shin W-statistic | 0.40889 (0.6587) | -36.2428 (0.0000) |
| ADF-Fisher Chi-square | 8.85128 (0.7156) | 902.535 (0.0000) |
| PP-Fisher Chi-square | 9.13112 (0.6917) | 1580.34 (0.0000) |
| (3) Hadri tests: | | |
| Hadri Z-statistic | 37.7979 (0.0000) | -0.95924 (0.8313) |
| Heteroscedastic Consistent Z-statistic | 28.5201(0.0000) | -1.01623 (0.8452) |

Notes: Log is the natural logarithm and Δ is the first-difference operator. AUD stands for the Australian dollar, CAD for the Canadian dollar, CHF for the Swiss franc, EURO for the euro, GBP for the British pound, and JPY for the Japanese yen. Each Z variable consists of 990 daily observations. All tests have the null of a unit root except the Hadri tests which have the null of a stationary process.

The presence of a unit root, and of only one unit root, is strongly supported for the logs of all six foreign currencies. Since panel unit root tests may be more powerful, seven panel unit root tests are implemented (Maddala and Wu, 1999;

Breitung, 2000; Hadri, 2000; Choi, 2001; Levin, Lin, and Chu, 2002; Im, Pesaran, and Shin, 2003). All tests have the null hypothesis of a unit root except the two Hadri tests which have the null of stationarity. The results are presented in the same table, Table 1. All these panel unit root tests confirm the presence of a unit root, and of only one unit root, in the stacked series. Therefore it is ascertained that the logged series are integrated of order 1, or I(1), but that the log returns are I(0), or definitely stationary in process. The first requirement for a martingale is therefore met with success.

### *2.3 Ljung-Box Q-statistics and Runs Tests*

The Ljung-Box Q-statistics are applied on the demeaned log returns (Ljung and Box, 1979). Since the data is daily four values for the lag length are used: 1, 5, 10, and 15. The results are depicted in Table 2. Whatever the lag length, the null hypotheses of no serial correlation fail to be rejected for all 6 currencies at a marginal significance level of 1%, although serial correlation for the Canadian dollar is marginally significant when a 5% marginal significance level is chosen (Table 2). Therefore the evidence in support of serial correlation in log returns is quite weak. This supports weak form efficiency and a martingale process, because, when there is no serial correlation, increments are random. It must be mentioned that the Ljung-Box Q-statistic is a weighted-average of the squares of the autocorrelation coefficients, which implies that these autocorrelation coefficients must be relatively small in absolute magnitude. Two other tests for serial correlation are implemented: the Breusch-Godfrey LM test with 5 lags (Table 2), and the runs tests (Table 3). The results of the Breusch-Godfrey tests are identical to the results of the Ljung-Box Q-statistic for the same lag length (i.e. 5). This is not surprising. The runs tests check for randomness and can have two test values, the mean and the median. Again the null of randomness fails to be rejected for both test values and for all 6 currencies. The conclusion is strong that these six series of log returns are random and not auto-correlated. This means that these returns are orthogonal to any information available previously and this supports weak form efficiency and a martingale process.

Table 2. Serial correlation tests on the residuals of $\Delta \log(Z)$ regressed on a constant

| | Ljung-Box Q-statistic | | | | Breusch-Godfrey LM test with k = 5 lags | |
|---|---|---|---|---|---|---|
| $Z$ | K=1 | K=5 | K=10 | K=15 | F-statistic (5, 983) | Chi-Square (5) |
| AUD | 0.199 | 0.849 | 0.545 | 0.117 | 0.8502 | 0.8493 |
| CAD | 0.004 | 0.065 | 0.127 | 0.035 | 0.0511 | 0.0513 |
| CHF | 0.926 | 0.043 | 0.096 | 0.319 | 0.0407 | 0.0409 |
| EURO | 0.608 | 0.652 | 0.539 | 0.532 | 0.6333 | 0.6319 |
| GBP | 0.053 | 0.374 | 0.676 | 0.750 | 0.3726 | 0.3715 |
| JPY | 0.014 | 0.213 | 0.304 | 0.385 | 0.2048 | 0.2043 |

Notes: See notes under Table 1. Actual p-values are reported.

Table 3. Runs tests for randomness of $\Delta \log(Z)$

| $Z$ | Test value is the mean | Test value is the median |
|---|---|---|
| AUD | 0.874 | 0.831 |
| CAD | 0.679 | 0.889 |
| CHF | 0.045 | 0.086 |
| EURO | 0.975 | 0.842 |
| GBP | 0.464 | 0.459 |
| JPY | 0.924 | 0.924 |

Notes: See notes under Table 1. Actual two-tailed asymptotic p-values are reported.

### *2.4 Variance Ratio Tests*

The variance ratio test is for the null hypothesis of a long run martingale, or for a stochastic process that is highly persistent. The reference to this test is Lo and MacKinlay (1988) which allows for heteroscedastic robust standard errors. The test is defined as:

$$\frac{variance(\log(Z_{t+k+1})-\log(Z_t))}{variance(\log(Z_{t+1})-\log(Z_t))}\frac{1}{k+1} \tag{3}$$

The test can be joint or individual. The joint test on the maximum absolute value of the z-statistic under the normal distribution fails to reject the null of a martingale at marginal significance levels as high as 10% (column 2, Table 4). The stacked panel and the joint Fisher combined tests on the stacked logs of the six foreign currencies also fail to reject a martingale at the same high marginal significance levels.

This variance ratio is exactly equal to +1 for a martingale. In Table 4 the values of these variance ratios are reported for individual $k$ that takes the values 2, 4, 8, and 16 months. All variance ratios are insignificantly different from +1, giving support to the hypothesis that there is no significant serial correlation, except for the Canadian dollar for which the evidence is mixed. A given variance ratio is a weighted-average of the autocorrelation coefficients. When the variance ratio is close to +1 this means that these autocorrelation coefficients may change in sign and may be sizeable in magnitude.

Table 4. Variance ratio test statistics on $\log(Z)$. The null hypothesis is a martingale

| Variable $Z$ | Joint tests for maximum $\lvert z-statistic \rvert$ | Individual tests for the variance ratio for period: | | | |
|---|---|---|---|---|---|
| | | 2 | 4 | 8 | 16 |
| AUD | 1.2218 (0.9767) | 0.9603 (0.2218) | 0.9542 (0.4705) | 0.9413 (0.5694) | 0.9541 (0.7709) |
| CAD | 2.4074 (0.2157) | 0.9108 (0.0161) | 0.8528 (0.0371) | 0.7751 (0.0417) | 0.6781 (0.0494) |
| CHF | 0.7158 (0.9999) | 0.9988 (0.9840) | 1.0062 (0.9584) | 1.0584 (0.7368) | 1.1615 (0.4741) |
| EURO | 0.5735 (1.0000) | 0.9854 (0.6867) | 0.9762 (0.7187) | 1.0193 (0.8535) | 1.0851 (0.5874) |
| GBP | 1.5394 (0.8620) | 0.9373 (0.1479) | 0.8900 (0.1516) | 0.9187 (0.4819) | 0.9738 (0.8740) |
| JPY | 1.6747 (0.7725) | 0.9233 (0.0940) | 0.9119 (0.2734) | 0.9227 (0.5124) | 0.9272 (0.6604) |
| Stacked panel | 2.3206 (0.2649) | 0.9570 (0.0220) | 0.9399 (0.0918) | 0.9468 (0.3238) | 0.9707 (0.6975) |
| Fisher combined | 3.9279 (0.9847) | | | | |

Notes: See notes under Table 1. Standard errors are heteroscedasticity robust. Actual p-values are in parenthesis.

### *2.5 Non-linear Dependence*

The absence of serial correlation is evidence against linear dependence. But a martingale can have non-linear dependence. Two tests of non-linear dependence are carried out: the BDS test (Brock et al., 1996), and the Ljung-Box Q-statistic test on the squares of the demeaned log returns. The results of the first test are presented in Table 5. In general non-linear dependence fails to be rejected at all dimensions selected 2, 3, 4, 5 and 6. Another symptom of non-linear dependence is conditional heteroscedasticity (Table 6). Again this kind of non-linear dependence fails to be rejected at very low marginal significance levels. ARCH tests with 5 lags also show the presence of conditional heteroscedasticity (Table 6). It will be shown later that GARCH and EGARCH statistical models can remove all of the conditional heteroscedasticity that is present in the data (Table 10). Although non-linear dependence does not invalidate a martingale process, it is still an unwanted stochastic behavior.

Table 5. BDS independence tests on $\Delta\log(Z)$

| $Z$ | Dimension | | | | |
|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 |
| AUD | 0.0256 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| CAD | 0.0020 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| CHF | 0.0034 | 0.0085 | 0.0058 | 0.0021 | 0.0009 |
| EURO | 0.0046 | 0.0017 | 0.0004 | 0.0000 | 0.0000 |
| GBP | 0.0419 | 0.0236 | 0.0024 | 0.0002 | 0.0001 |
| JPY | 0.0449 | 0.0019 | 0.0002 | 0.0000 | 0.0000 |

Notes: See notes under Table 1. Actual p-values are reported.

Table 6. Conditional heteroscedasticity tests on the residuals of $\Delta\log(Z)$ regressed on a constant

| $Z$ | Ljung-Box Q-statistics on squared residuals | | | | ARCH test with k = 5 lags | |
|---|---|---|---|---|---|---|
| | K=1 | K=5 | K=10 | K=15 | F-statistic (5, 978) | Chi-Square (5) |
| AUD | 0.733 | 0.002 | 0.000 | 0.000 | 0.0055 | 0.0056 |
| CAD | 0.012 | 0.000 | 0.000 | 0.000 | 0.0000 | 0.0000 |
| CHF | 0.000 | 0.000 | 0.000 | 0.000 | 0.0000 | 0.0000 |
| EURO | 0.001 | 0.000 | 0.000 | 0.000 | 0.0000 | 0.0000 |
| GBP | 0.000 | 0.000 | 0.000 | 0.000 | 0.0000 | 0.0000 |
| JPY | 0.000 | 0.000 | 0.000 | 0.000 | 0.0000 | 0.0000 |

Notes: See notes under Table 1. Actual p-values are reported.

### *2.6 Non-normality*

In Table 7 six normality tests are carried out on the six series of log returns: Kolmogorov-Smirnov, Jarque-Bera, Lilliefors (D), Cramer-von Mises, Watson (U2), and Anderson-Darling (A2). All computations use the EViews 8 (2013) statistical software. The six series show significant departure from normality. A random walk designation necessitates the normality of residuals. However, normality is not a requirement for a martingale. Therefore the evidence of non-normality is to be added to the above evidence of non-linear dependence as an unwanted statistical feature that does not invalidate a martingale process. Moreover since the sample sizes are large t-tests and F-tests can still be applicable and will be applied later by invoking the Central Limit Theorem and asymptotic normality.

Table 7. Normality tests of $\Delta \log(Z)$

| $Z$ | Kolmogorov-Smirnov | Jarque-Bera | Lilliefors (D) | Cramer-von Mises | Watson (U2) | Anderson-Darling (A2) |
|---|---|---|---|---|---|---|
| AUD | 0.044 | 0.00000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| CAD | 0.000 | 0.00000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| CHF | 0.001 | 0.00000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| EURO | 0.002 | 0.00000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| GBP | 0.033 | 0.00000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| JPY | 0.000 | 0.00000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Notes: See notes under Table 1. Actual p-values are reported.

### *2.7 Structural Breaks*

The data may suffer from structural breaks although the time span is not that long (between early 2010 and early 2014). Structural breaks may force a series to look like a unit root process while in fact there is no unit root. Therefore suspicion of breaks is detrimental to the martingale hypothesis. The Bai-Perron test (Bai, 1997; Bai and Perron, 1998) is applied on the six demeaned log returns. The results are in Table 8. In total 5 breaks are allowed for but none is found. To be more certain about the inexistence of breaks Quandt-Andrews unknown break point tests are additionally implemented (Andrews, 1993; Andrews and Ploberger, 1994). Six different test statistics are computed (Table 8), out of which 3 are reported. In total, and for each currency, 692 break points are compared. The actual p-values are reported in Table 8.

Table 8. Tests for breakpoints of the residuals of $\Delta \log(Z)$ regressed on a constant

| $Z$ | Bai-Perron tests (Maximum breaks = 5) | | | Quandt-Andrews unknown breakpoint tests (Number of breaks compared: 692) | | |
|---|---|---|---|---|---|---|
| | F | Scaled F | Critical value | Maximum F | Exponential F | Average F |
| AUD | 3.4049 | 3.4049 | 8.58 | 0.4745 | 0.2479 | 0.1768 |
| CAD | 2.0544 | 2.0544 | 8.58 | 0.7722 | 0.4563 | 0.3893 |
| CHF | 7.0997 | 7.0997 | 8.58 | 0.0989 | 0.5062 | 0.6733 |
| EURO | 2.8504 | 2.8504 | 8.58 | 0.5857 | 0.6788 | 0.6746 |
| GBP | 1.9336 | 1.9336 | 8.58 | 0.8022 | 0.8602 | 0.8440 |
| JPY | 7.0621 | 7.0621 | 8.58 | 0.1006 | 0.0404 | 0.0248 |

Notes: See notes under Table 1. Actual F-statistics and critical F-statistics for the Bai-Perron tests are reported. Actual p-values for the Quandt-Andrews tests are reported.

Since the null hypothesis is the absence of breaks, and since all p-values are larger than 10%, the conclusion is strong that there are no breaks, that the samples are therefore homogeneous, and that the martingale process is an inherent feature of the data and is not artificially reproduced.

### *2.8 Granger Causality Tests*

A martingale implies that increments are random, non-auto-correlated, orthogonal to information known in advance, and literally unpredictable. One way to test for orthogonality and unpredictability is to carry out Granger causality tests. This consists of regressing the log returns of a given currency on its own lagged values and the lagged values of the log returns of another currency. Of course it is already known from section 2.3 above that all the six series are not auto-correlated, but own lagged values will nonetheless be included in the tests. The results are reported in Table 9, where pair-wise Granger causality tests are conducted with a specified lag length of 5. The smallest actual p-value is 0.0543, the next highest is 0.0975, and the rest are all above 10%. The evidence is therefore strong that the series do not suffer from predictability. Weak form efficiency and a martingale behavior hold well.

Table 9. Pair-wise Granger causality tests between $\Delta \log(Z)$ . The number of lags is set to 5

| Null hypothesis | Observations | F-statistic | Probability |
|---|---|---|---|
| Δ(log CAD) does not Granger cause Δ(log AUD) | 984 | 0.51572 | 0.7645 |
| Δ(log AUD) does not Granger cause Δ(log CAD) | 984 | 1.36753 | 0.2340 |
| Δ(log CHF) does not Granger cause Δ(log AUD) | 984 | 0.97944 | 0.4292 |
| Δ(log AUD) does not Granger cause Δ(log CHF) | 984 | 1.08198 | 0.3687 |
| Δ(log EURO) does not Granger cause Δ(log AUD) | 984 | 1.62649 | 0.1502 |
| Δ(log AUD) does not Granger cause Δ(log EURO) | 984 | 2.18018 | 0.0543 |
| Δ(log GBP) does not Granger cause Δ(log AUD) | 984 | 1.15080 | 0.3317 |
| Δ(log AUD) does not Granger cause Δ(log GBP) | 984 | 1.02831 | 0.3995 |
| Δ(log JPY) does not Granger cause Δ(log AUD) | 984 | 0.69068 | 0.6306 |
| Δ(log AUD) does not Granger cause Δ(log JPY) | 984 | 1.65946 | 0.1417 |
| Δ(log CHF) does not Granger cause Δ(log CAD) | 984 | 0.68867 | 0.6321 |
| Δ(log CAD) does not Granger cause Δ(log CHF) | 984 | 1.27046 | 0.2744 |
| Δ(log EURO) does not Granger cause Δ(log CAD) | 984 | 0.97136 | 0.4342 |
| Δ(log CAD) does not Granger cause Δ(log EURO) | 984 | 1.86709 | 0.0975 |
| Δ(log GBP) does not Granger cause Δ(log CAD) | 984 | 0.95652 | 0.4435 |
| Δ(log CAD) does not Granger cause Δ(log GBP) | 984 | 1.11230 | 0.3520 |
| Δ(log JPY) does not Granger cause Δ(log CAD) | 984 | 1.03998 | 0.3927 |
| Δ(log CAD) does not Granger cause Δ(log JPY) | 984 | 1.55526 | 0.1701 |
| Δ(log EURO) does not Granger cause Δ(log CHF) | 984 | 0.72073 | 0.6079 |
| Δ(log CHF) does not Granger cause Δ(log EURO) | 984 | 0.73291 | 0.5988 |
| Δ(log GBP) does not Granger cause Δ(log CHF) | 984 | 1.51867 | 0.1812 |
| Δ(log CHF) does not Granger cause Δ(log GBP) | 984 | 1.78838 | 0.1125 |
| Δ(log JPY) does not Granger cause Δ(log CHF) | 984 | 1.37973 | 0.2293 |
| Δ(log CHF) does not Granger cause Δ(log JPY) | 984 | 1.31924 | 0.2535 |
| Δ(log GBP) does not Granger cause Δ(log EURO) | 984 | 0.67057 | 0.6459 |
| Δ(log EURO) does not Granger cause Δ(log GBP) | 984 | 0.88355 | 0.4914 |
| Δ(log JPY) does not Granger cause Δ(log EURO) | 984 | 1.41805 | 0.2151 |
| Δ(log EURO) does not Granger cause Δ(log JPY) | 984 | 1.15918 | 0.3274 |
| Δ(log JPY) does not Granger cause Δ(log GBP) | 984 | 0.66295 | 0.6517 |
| Δ(log GBP) does not Granger cause Δ(log JPY) | 984 | 1.62266 | 0.1512 |

Notes: See notes under Table 1.

### *2.9 GARCH and EGARCH Models*

Since there is evidence for conditional heteroscedasticity it is natural to model it and see whether such modeling can remove this non-linear dependence. Baillie and Bollerlslev (1989) and Hsieh (1989) are the first to adopt such a modeling for daily foreign exchange rates. Baillie and Bollerslev (1989) fit a GARCH (Bollerslev, 1986) model, while Hsieh considers ARCH (Engle, 1982), GARCH (Bollerslev, 1986), and EGARCH (Nelson, 1991) models. By carrying goodness-of-fit tests Hsieh's preference goes to the last model, i.e. EGARCH, also known as exponential GARCH because the specification of the conditional volatility is logarithmic, while Baillie and Bollerslev stress on a GARCH(1,1) model with different distributional assumptions. Abdalla (2012) studies daily data of foreign exchange rates for Arab countries, including for Lebanon. He favors also EGARCH and finds evidence for a leverage effect: a negative shock in the conditional mean equation increases the conditional volatility. The two specifications, GARCH(1,1) and EGARCH, are both tried on the six data series in this paper. By minimizing the Akaike information criterion a GARCH(1,1) functional form is preferred for the Swiss franc and the Euro, while an EGARCH model is preferred for the other four currencies. While there is a leverage effect for the Australian dollar, the Canadian dollar, and the British pound, there is no such effect for the Japanese yen. The latter has however an asymmetric effect in its conditional variance equation. What is interesting and noteworthy is that all such models for the conditional volatility remove entirely heteroscedasticity in the standardized residuals, which is the original purpose of the exercise. The standardized residuals are computed as the ratio of the residuals over the conditional standard deviations. The Ljung-Box Q-statistic is applied on the squares of the standardized residuals and this for 3 lag lengths: 5, 10, and 15. The smallest actual p-value of the Ljung-Box Q-statistic on the squares of the standardized residuals is 0.053, and the next in line is 0.064. The remaining sixteen p-values are all higher than 10%.

Table 10. Regressions of $\Delta \log(Z)$ with a conditional variance equation

| Variable $Z$ | AUD | CAD | CHF | EURO | GBP | JPY |
|---|---|---|---|---|---|---|
| **Conditional mean equation:** | | | | | | |
| Constant | -0.000196 | -0.000189 | 0.000210 | 2.96E-06 | -0.000153 | -0.000105 |
| | (0.864062) | (1.148078) | (1.093991) | (0.016796) | (0.954955) | (0.594900) |
| **GARCH(1,1) model of the conditional variance:** | | | | | | |
| Constant | | | 6.64E-07 | 4.17E-08 | | |
| | | | (2.661444) | (0.421069) | | |
| ARCH(1) | | | 0.063846 | 0.026731 | | |
| | | | (9.169786) | (4.086406) | | |
| GARCH(1) | | | 0.924457 | 0.971694 | | |
| | | | (109.0385) | (125.0206) | | |
| **EGARCH model of the conditional variance:** | | | | | | |
| Constant | -0.151224 | -0.165921 | | | -0.054814 | -0.828633 |
| | (3.181799) | (3.048632) | | | (3.647116) | (5.396224) |
| ABS(RESID(-1)/SQRT(GARCH(-1))) | 0.093103 | 0.097067 | | | 0.011154 | 0.209019 |
| | (4.816267) | (4.641838) | | | (1.423939) | (7.414821) |
| RESID(-1)/SQRT(GARCH(-1)) | -0.050045 | -0.025758 | | | -0.037237 | 0.076274 |
| | (4.673094) | (1.877535) | | | (4.786598) | (4.620167) |
| LOG(GARCH(-1)) | 0.991887 | 0.991098 | | | 0.995688 | 0.933914 |
| | (237.0618) | (234.5258) | | | (787.8607) | (67.38600) |
| Ljung-Box Q-statistics on the squares of the standardized residuals: | | | | | | |
| K=5 | 0.134 | 0.516 | 0.064 | 0.163 | 0.808 | 0.862 |
| K=10 | 0.053 | 0.860 | 0.210 | 0.370 | 0.761 | 0.796 |
| K=15 | 0.162 | 0.929 | 0.499 | 0.187 | 0.648 | 0.934 |

Notes: See notes under Table 1. Absolute t-statistics are in parenthesis. Actual p-values for the Ljung-Box Q-statistic are reported. The lag length is K.

### *2.10 Cross Dependencies*

Table 11 presents the cross correlations between the six currencies. At the 1% two-tailed marginal significance level all pairwise correlations are significantly different from zero and significantly positive except the correlations that involve the Japanese yen. The latter has a marginally significant cross correlation with the Euro and with the British pound but statistically insignificant correlations with the Australian and Canadian dollars. Cross correlation between currencies does not invalidate the martingale hypothesis. As a matter of fact significant cross correlations are expected because the Lebanese currency is the reference point for all six foreign currencies, which means that these six currencies are expected to have a similar statistical behavior (see, nonetheless, Azar, 2013). There are also implications for portfolio diversification. Whenever correlation coefficients are significantly less than +1 diversification of risk is possible. It seems that the Japanese yen is the currency that should be held in the portfolio because of its low cross correlations.

Table 11. Pair-wise cross-correlation coefficients between $\Delta \log(Z)$

| | AUD | CAD | CHF | EURO | GBP | JPY |
|---|---|---|---|---|---|---|
| AUD | 1 | | | | | |
| CAD | 0.722829** | 1 | | | | |
| CHF | 0.354725** | 0.255762** | 1 | | | |
| EURO | 0.583899** | 0.485707** | 0.588560** | 1 | | |
| GBP | 0.513091** | 0.452947** | 0.410786** | 0.643986** | 1 | |
| JPY | 0.051396 | -0.054116 | 0.225297** | 0.069392* | 0.072992* | 1 |

Notes: See notes under Table 1. ** denotes significance at the 1% 2-tailed significance level. * denotes significance at the 5% 2-tailed significance level. No star denotes statistical insignificance.

### *2.11 Descriptive Statistics*

Table 12 reproduces descriptive statistics on the log returns of the six foreign currencies. Hsieh (1989) reports summary statistics on 4 currencies, common with this paper, but against the US dollar. The standard deviations range between 0.002234, for the Canadian dollar, to 0.007889 for the Swiss franc, with a value of 0.005921 for the British pound, and a value of 0.006260 for the Japanese yen. This compares with a range in Table 12 between 0.005412 for the British pound and 0.007894 for the Australian dollar, with a value of 0.005744 for the Canadian dollar, a value of 0.007377 for the Swiss franc, and a value of 0.006315 for the Japanese yen. The maximum return in Hsieh (1989) is for the Swiss franc with a value of 0.04466. This compares with a maximum of 0.047542, again for the Swiss franc, in Table 12. The minimum in Hsieh (1989) is for the Canadian dollar with a value of -0.018677, while it is -0.024694 for the Euro in Table 12. In addition the currencies in Hsieh (1989) all suffer from excess kurtosis like the results in Table 12. It is remarkable how close these statistics are between currencies against the US dollar and the same currencies against the Lebanese pound. One notable exception is the Canadian dollar.

Table 12. Descriptive statistics on $\Delta \log(Z)$

| Variable $Z$ | AUD | CAD | CHF | EURO | GBP | JPY |
|---|---|---|---|---|---|---|
| Mean | -3.37E-05 | -7.53E-05 | 0.000136 | -6.26E-05 | 1.39E-05 | -0.000101 |
| | (0.134312) | (0.412379) | (0.580507) | (0.311897) | (0.080932) | (0.501705) |
| Median | 0.000195 | 0.000101 | 0.000325 | 0.000150 | 0.000187 | 0.00000 |
| Maximum | 0.030186 | 0.027628 | 0.047542 | 0.020842 | 0.024694 | 0.027992 |
| Minimum | -0.042255 | -0.032814 | -0.075868 | -0.024123 | -0.029017 | -0.037140 |
| Standard deviation | 0.007894 | 0.005744 | 0.007377 | 0.006311 | 0.005412 | 0.006315 |
| | (44.45222) | (44.4522) | (44.4522) | (44.4522) | (44.4522) | (44.4522) |
| Skewness | -0.380897 | -0.303098 | -1.161151 | -0.357981 | -0.401377 | -0.573966 |
| | (4.8902) | (3.8914) | (14.908) | (4.5960) | (5.1532) | (7.3690) |
| Kurtosis | 5.151929 | 5.449254 | 19.24701 | 3.812348 | 5.030923 | 7.244424 |
| | (13.814) | (15.723) | (104.296) | (5.2148) | (13.037) | (27.247) |

Notes: See notes under Table 1. Absolute t-statistics are in parenthesis.

The same comparisons are made with Van de Gucht et al. (1996). In the latter the standard deviations for the whole sample vary between 0.002464 for the Canadian dollar to 0.007665 for the Swiss franc. The remaining currencies have standard deviations between 0.006140 and 0.006594. This compares with a range between 0.005412 and 0.007894 in Table 12. For example the standard deviation for the Japanese yen is 0.006140 in Van de Gucht et al. (1996) while it is 0.006315 in Table 12. Although all currencies suffer from skewness and excess kurtosis in Van de Guicht et al. (1996), the results in Table 12 show that all currencies have indeed excess kurtosis but suffer from negative skewness, a feature different from Van de Guicht et al. (1996).

Similar comparisons are made with Chang (2004). In Chang (2004) the standard deviations range from 0.0058, for the Canadian dollar, to 0.0145 for the Japanese yen. In Table 12, the standard deviation of the Canadian dollar is 0.005744, very close to the figure of 0.0058 in Chang (2004). In this case the Canadian dollar is not an exception.

In Qi and Wu (2006) the standard deviations range between 0.0026 for the Canadian dollar to 0.0076 for the Swiss franc, while the remaining five currencies have standard deviations between 0.0062 and 0.0065. This compares with a range between 0.005412 and 0.007377 in Table 12. Again the highest standard deviation is for the Swiss franc in both places. The maximum log return is 0.0563 in Qi and Wu (2006) and it is 0.047542 in Table 12. The minimum log return is -0.0626 in Qi and Wu (2006) while it is -0.075868 in Table 12. All currencies suffer from negative skewness in Qi and Wu (2006) except for the Japanese yen, while in Table 12 all currencies suffer from negative skewness without exception.

Abdalla (2012) has summary statistics for daily data of the US dollar against the Lebanese pound for a different time period. He finds a standard deviation of 0.008745, a maximum log return of 0.04951, and a minimum log return of -0.05001. This compares with a standard deviation of 0.007894 for the Australian dollar, a maximum log return of 0.047542 for the Swiss franc, and a minimum log return of -0.075868 for the Swiss franc in Table 12.

The magnitudes of the standard deviations, the maximum log returns, and the minimum log returns, are therefore of the same order of magnitude in this paper compared to the literature, although most of the currencies in the literature are against the US dollar while they are against the Lebanese pound in Table 12. It seems that daily foreign exchange rates are more similar in statistical behavior than dissimilar, whatever the reference points of the currencies.

### *2.12 Hypothesis Tests*

A battery of hypothesis tests is carried out. The first is an F-test on the means. The hypothesis that the mean log returns of all six currencies are equal to each other fails to be rejected with a p-value of 0.9736 for the usual F-test, that assumes equal variances, and a p-value of 0.9785 for the Welch test that assumes unequal variances. Individual t-tests on the mean log returns show that all means are insignificantly different from zero. See the t-tests in Table 12. This means that all currencies are martingales and not sub-martingales, or, in other terms, that the estimates of $\alpha$ in equations (1) and (2) are all zero. In addition the hypothesis of equality of medians is not rejected with actual p-values ranging between 0.6790 and 0.9017 depending on the test adopted. However the three tests on the joint equality of variances, Bartlett, Levene, and Brown-Forsythe, reject the null of the equality of variances at marginal significance levels much lower than 0.0001.

Table 13 presents pair-wise comparisons of variances. When a one-tailed Type I error of 5% is selected, only the two variances of the Euro and the Japanese yen are equal to each other statistically, while all the remaining currencies have unequal variances. Some of the pair-wise variances have marginal significance, like, for example, the variances between the Canadian dollar and the British pound.

Table 13. F-tests of pair-wise equality of variances

| | AUD | CAD | CHF | EURO | GBP |
|---|---|---|---|---|---|
| CAD | 0.0000 | | | | |
| CHF | 0.0167 | 0.0000 | | | |
| EURO | 0.0000 | 0.0016 | 0.0000 | | |
| GBP | 0.0000 | 0.0307 | 0.0000 | 0.0000 | |
| JPY | 0.0000 | 0.0015 | 0.0000 | 0.4920 | 0.0000 |

Notes: See notes under Table 1. Actual upper-tailed p-values are reported.

## 4. Conclusion

The purpose of this paper is to test for weak form efficiency of the Lebanese pound. Six foreign currencies are selected and daily data, that became available lately, are used. Efficiency requires that the data meet at least six conditions. The first one is for the presence of one unit root process. The second one is to have increments that are random and uncorrelated. The third one is that shocks should persist for the long run. The fourth one is that the samples should be homogeneous, with the absence of calendar breaks. The fifth one is the statistical insignificance of pair-wise Granger causality tests. The last one is that the means of the log returns should be statistically insignificantly different from zero. These six conditions describe a statistical behavior known as a martingale. All six conditions are found to apply to the six data series. Non-normality, conditional heteroscedasticity, other non-linear dependencies, and contemporaneous cross-correlations of the log returns of the exchange rates are features that are present in the data but that do not invalidate the general designation of a martingale. Finally, the summary statistics of the six series under consideration are quite similar to those of other major currencies, even when compared for different time periods. This is especially true for standard deviations, maxima, minima, and kurtosis. Hence, daily foreign exchange rates seem to share quasi the same characteristics globally.

## References

Abdalla, S. Z. S. (2012). Modeling exchange rate volatility using GARCH models: Empirical evidence from Arab countries. *International Journal of Economics and Finance, 4*, 216-229. http://dx.doi.org/10.5539/ijef.v4n3p216

Andrews, D. W. K. (1993). Tests for parameter instability and structural change with unknown change point. *Econometrica, 61*, 821-856. http://dx.doi.org/10.2307/2951764

Andrews, D. W. K., & Ploberger, W. (1994). Optimal tests when a nuisance parameter is present only under the alternative. *Econometrica, 62*, 1383-1414. http://dx.doi.org/10.2307/2951753

Azar, S. (2013). The post-war behavior of foreign exchange rates in Lebanon. *The Empirical Economics Letters, 12*(9), 1005-1010

Bai, J. (1997). Estimating multiple breaks one at a time. *Econometric Theory, 13*, 315-352. http://dx.doi.org/10.1017/S0266466600005831

Bai, J., & Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica, 66*, 47-78. http://dx.doi.org/10.2307/2998540.

Baillie, R. T., & Bollerslev, T. (1989). The message in daily exchange rates: A conditional-variance tale. *Journal of Business & Economic Statistics, 7*, 297-305. http://dx.doi.org/10.1080/07350015.1989.10509739

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics, 31*, 307-327. http://dx.doi.org/10.1016/0304-4076(86)90063-1

Breitung, J. (2000). The local power of some unit root tests for panel data. In B. Baltagi (Ed.) *Advances in Econometrics*, Vol. 15: *Nonstationary Panels, Panel Cointegration, and Dynamic Panels*, (pp. 161-178). Amsterdam: JAI Press.

Brock, W., Dechert, D., Sheinkman, J., & LeBaron, B. (1996). A test for independence based on the correlation dimension. *Econometric Reviews, 15*, 197-235. http://dx.doi.org/10.1080/07474939608800353

Chang, Y. (2004). A re-examination of variance-ratio test of random walks in foreign exchange rates. *Applied Financial Economics, 14*, 671-679. http://dx.doi.org/10.1080/0960310042000233449

Choi, I. (2001). Unit root tests for panel data. *Journal of International Money and Finance*, *20*, 249-272. http://dx.doi.org/10.1016/S0261-5606(00)00048-6

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, *50*, 987-1007. http://dx.doi.org/10.2307/1912773

EViews 8. (2013). IHS Global Inc., Irvine, California.

Fama, E. (1965). The behavior of stock market prices. *Journal of Business*, *38*, 34-105. http://dx.doi.org/10.1086/294743

Fama, E. (1970). Efficient capital markets: a review of theory and empirical work. *Journal of Finance*, *25*, 383-417. http://dx.doi.org/10.2307/2325486

Fama, E. (1991). Efficient markets II. *Journal of Finance*, *46*, 1575-1618. http://dx.doi.org/10.1111/j.1540-6261.1991.tb04636.x

Grossman, S. J., & Stiglitz, J. E. (1980) On the impossibility of informationally efficient markets. *American Economic Review*, *70*(3), 393-408.

Hadri, K. (2000). Testing for stationarity in heterogenous panel data. *Econometric Journal*, *3*, 148-161. http://dx.doi.org/10.1111/1368-423X.00043

Hsieh, D. A. (1989). Modeling heteroscedasticity in daily foreign-exchange rates. *Journal of Business & Economic Statistics*, *7*, 307-317. http://dx.doi.org/10.1080/07350015.1989.10509740

Ibrahim, J., Long, Y., Ghani, H. A., & Salleh, S. I. M. (2011). Weak-form efficiency of foreign exchange market in the Organization for Economic Cooperation and Development countries: Unit root test. *International Journal of Business and Management*, *6*, 55-65. http://dx.doi.org/10.5539/ijbm.v6n6p55

Im, K. S., Pesaran, M. H., & Shin, Y. (2003). Testing for unit roots in heterogenous panels. *Journal of Econometrics*, *115*, 53-74. http://dx.doi.org/10.1016/S0304-4076(03)00092-7

Levin, A., Lin, C. F., & Chu, C. (2002). Unit roots in panel data: asymptotic and finite-sample properties. *Journal of Econometrics, 108*, 1-24. http://dx.doi.org/10.1016/S0304-4076(01)00098-7

Liu, C. Y., & He, J. (1991). A variance-ratio test of random walks in foreign exchange rates. *Journal of Finance, 46*, 773-785. http://dx.doi.org/10.1111/j.1540-6261.1991.tb02686.x

Ljung, G., & Box, G. (1979). On a measure of lack of fit in time series models. *Biometrika*, *66*, 265-270. http://dx.doi.org/10.1093/biomet/66.2.265

Lo, A. W., & MacKinlay, A C. (1988). Stock market prices do not follow random walks: Evidence from a simple specification test. *The Review of Financial Studies*, *1*, 41-66. http://dx.doi.org/10.1093/rfs/1.1.41

Maddala, G. S., & Wu, S. (1999). A comparative study of unit root tests with panel data and a new simple test. *Oxford Bulletin of Economics and Statistics, 61*, 631-652. http://dx.doi.org/10.1111/1468-0084.61.s1.13

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, *59*, 347-370. http://dx.doi.org/10.2307/2938260

Phillips, P. C. B., & Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, *75*, 335-346. http://dx.doi.org/10.1093/biomet/75.2.335

Qi, M., & Wu, Y. (2006). Technical trading-rule profitability, data snooping, and reality check: Evidence from the foreign exchange market. *Journal of Money, Credit, and Banking*, *38*(8), 2135-2158. http://dx.doi.org/10.1353/mcb.2007.0006

Rufino, C. C. (2014). Martingales in floating ASEAN+3 currencies. *De La Salle University Business & Economic Review*, *23*(2), 65-79.

Van De Gucht, L., Dekimpe, M. G. & Kwok, C. C. Y. (1996). Persistence in foreign exchange rates. *Journal of International Money and Finance, 15*, 191-220. http://dx.doi.org/10.1016/0261-5606(96)00001-0

Verbeek, M. (2012). *A guide to modern econometrics* (4th ed.). Chichester: John Wiley & Sons Ltd.