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PAPER

Testing the gravitational field generated by a quantum superposition

M Carlesso[1,2], A Bassi[1,2], M Paternostro[3] and H Ulbricht[4]

[1] Department of Physics, University of Trieste, Strada Costiera 11, I-34151 Trieste, Italy
[2] Istituto Nazionale di Fisica Nucleare, Trieste section, Via Valerio 2, I-34127 Trieste, Italy
[3] Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom
[4] School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom

E-mail: matteo.carlesso@ts.infn.it

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Abstract

What gravitational field is generated by a massive quantum system in a spatial superposition? Despite decades of intensive theoretical and experimental research, we still do not know the answer. On the experimental side, the difficulty lies in the fact that gravity is weak and requires large masses to be detectable. However, it becomes increasingly difficult to generate spatial quantum superpositions for increasingly large masses, in light of the stronger environmental effects on such systems. Clearly, a delicate balance between the need for strong gravitational effects and weak decoherence should be found. We show that such a trade off could be achieved in an optomechanics scenario that allows to witness whether the gravitational field generated by a quantum system in a spatial superposition is in a coherent superposition or not. We estimate the magnitude of the effect and show that it offers perspectives for observability.

Quantum field theory is one of the most successful theories ever formulated. All matter fields, together with the electromagnetic and nuclear forces, have been successfully embedded in the quantum framework. They form the standard model of elementary particles, which not only has been confirmed in all advanced accelerator facilities, but has also become an essential ingredient for the description of the Universe and its evolution.

In light of this, it is natural to seek a quantum formulation of gravity as well. Yet, the straightforward procedure for promoting the classical field as described by general relativity, into a quantum field, does not work. Several strategies have been put forward, which turned into very sophisticated theories of gravity, the most advanced being string theory and loop quantum gravity. Yet, none of them has reached the goal of providing a fully consistent quantum theory of gravity.

At this point, one might wonder whether the very idea of quantizing gravity is correct [1–17]. At the end of the day, according to general relativity, gravity is rather different from all other forces. Actually, it is not a force at all, but a manifestation of the curvature of spacetime, and there is no obvious reason why the standard approach to the quantization of fields should work for spacetime as well. A future unified theory of quantum and gravitational phenomena might require a radical revision not only of our notions of space and time, but also of (quantum) matter. This scenario is growing in likeliness [18–20].

From the experimental point of view, it has now been ascertained that quantum matter (i.e. matter in a genuine quantum state, such as a coherent superposition state) couples to the Earth's gravity in the most obvious way. This has been confirmed in neutron [21], atom [22] interferometers and used for velocity selection in molecular interferometry [23]. However, in all cases, the gravitational field is classical, i.e. it is generated by a distribution of matter (the Earth) in a fully classical state. Therefore, the plethora of successful experiments mentioned above does not provide hints, unfortunately, on whether gravity is quantum or not.

The large attention and media coverage about the BICEP2 Collaboration's experiment having shown the quantum origin of primordial gravitational fluctuations [24], subsequently disproved by Planck Collaboration's data analysis [25], testifies the importance and urgency of a pragmatic assessment of the question of whether gravity is quantum or not.

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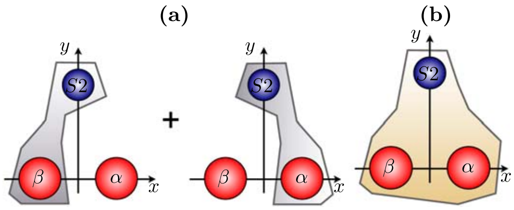

Figure 1. Schematic representation of the two-body setup. S1 is prepared in a spatial superposition along the *x* direction (red balls). S2 is initially prepared in a localized wavepacket (blue ball), and it probes the gravitational field generated by S1. (a) The gravitational field acting on S2 is a linear combination of gravitational fields produced by S1 being in a superposed state. (b) The semi-classical treatment of gravity, where the gravitational field acting on S2 is that produced by a total mass m_1 with density $\frac{1}{2}(|\alpha(\mathbf{r})|^2 + |\beta(\mathbf{r})|^2)$.

In this paper, we discuss an approach where a quantum system is forced in the superposition of two different positions in space, and its gravitational field is explored by a probe (figure 1). Using the exquisite potential for transduction offered by optomechanics, we can in principle witness whether the gravitational field is the superposition of the two gravitational fields associated to the two different states of the system, or not. The first case amounts to a quantum behavior of gravity, the second to a classical-like one.

It is worth noticing at this stage that the scope of this investigation is not testing the existence of gravitons, a goal that would require higher energies. We are instead concerned with the inference of the (potential) quantum nature of the gravitational field, which we address by probing the capability of the latter to be in a superposition of configurations, regardless of whether or not it sustains quanta. An electromagnetic analogy will help to understand our standpoint. We know that the electromagnetic filed generated by a charge located in a spatial superposition is the superposition of the Coulomb fields generated by the charge at the different locations. Yet the field is a (quantum) Coulomb field, with no evidence of photons.

The remainder of this manuscript is organized as follows. In section 1 we define the context considered throughout the manuscript and discuss both the quantum and semi-classical scenarios for gravity. Section 2 presents the theoretical model for the dynamics of the optomechanical platform that we address, while section 3 puts forward our proposals for the inference of the difference between a quantum and classical nature of gravity. Finally, in section 4 we state our conclusions and discuss a few interesting features of our findings.

1. Framework

We consider a setup formed of two systems interacting gravitationally. All non-gravitational interactions are considered, for all practical purposes, negligible. The first system (S1) has a mass m_1, and it is initially prepared in a spatial superposition along the *x* direction. Its wave-function is $\psi(\mathbf{r}_1) = \frac{1}{\sqrt{2}}(\alpha(\mathbf{r}_1) + \beta(\mathbf{r}_1))$, where $\alpha(\mathbf{r}_1)$ and $\beta(\mathbf{r}_1)$ are sufficiently well localized states in position, far from each other in order to prevent any overlap. Thus, we can consider them as distinguishable (in a macroscopic sense), and we approximate $\langle\alpha|\beta\rangle \simeq 0$. The second system (S2) will serve as a point-like probe of the gravitational field generated by S1, it has mass m_2 and state $\phi(\mathbf{r}_2)$. The state $\phi(\mathbf{r}_2)$ is initially assumed to be localized in position and centered along the *y* direction (see figure 1). The question we address is: which is the gravitational field, generated by the quantum superposition of S1, that S2 experiences? We probe the following two different scenarios.

Quantum gravity scenario. Although we do not have a quantum theory of gravity so far, one can safely claim that, regardless of how it is realized, it would manifest in S1 generating a superposition of gravitational fields. As discussed in the introduction, the assessment of this property precedes the quest to ascertain the existence of the graviton and the characterization of its properties, at least as far as the static, low-energy, non-relativistic regime we are considering is concerned. Linearity is the very characteristic trait of quantum theory, and one expects it to be preserved by any quantum theory of gravity.

The reaction of S2 is then to go in a superposition of being attracted towards the region where $|\alpha\rangle$ sits and where $|\beta\rangle$ does. The final two-body state will have the following entangled form

$$\Psi_{\mathrm{QG}}^{\mathrm{final}}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\alpha(\mathbf{r}_1)\phi_\alpha(\mathbf{r}_2) + \beta(\mathbf{r}_1)\phi_\beta(\mathbf{r}_2)}{\sqrt{2}}, \tag{1}$$

where $\phi_\alpha(\mathbf{r}_2)$ ($\phi_\beta(\mathbf{r}_2)$) represents the state of S2 attracted towards the region where $|\alpha\rangle$ ($|\beta\rangle$) rests. The motion in each branch of the superposition is produced by the potential

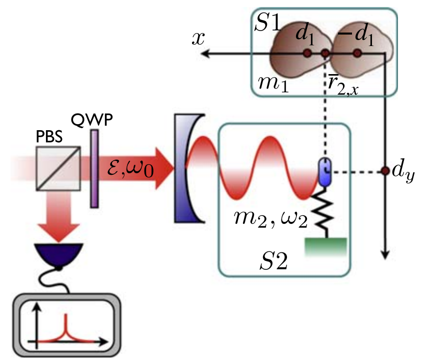

Figure 2. The proposed set-up for the optomechanical falsification of quantum/classical gravity. A system S1 is prepared in a superposition of two localized states at $\pm d_1$ along the x axis. An optomechanical cavity acts as transducer and probe of (potentially quantum) gravity effects S2: the effect of the gravitational coupling between S1 and the mechanical oscillator of an optomechanical cavity induces an effect on the variance of the position fluctuations of the oscillator. The mean position of the latter along the x axis is $\bar{r}_{2,x}$. The cavity is pumped by an external field (frequency ω_0 and coupling rate $\mathcal{E}$).

$$\hat{V}_\gamma(\hat{\mathbf{r}}_2) = -Gm_2\int \mathrm{d}\mathbf{r}_1\frac{\rho_\gamma(\mathbf{r}_1)}{|\mathbf{r}_1-\hat{\mathbf{r}}_2|}, \quad (\gamma=\alpha,\,\beta), \tag{2}$$

where $\rho_\gamma(\mathbf{r}_1)$ is the mass density of S1, centered in $\langle\hat{\mathbf{r}}_1\rangle_\gamma = \langle\gamma|\hat{\mathbf{r}}_1|\gamma\rangle$. We assume that S1 does not move appreciably during the time of the experiment (also quantum fluctuations can be neglected); clearly, such a situation can be assumed only as long as the S1 superposition lives. We further assume that its mass density is essentially spheric, so that the gravitational interaction can be approximated by

$$\hat{V}_\gamma(\hat{\mathbf{r}}_2) \approx -\frac{Gm_1m_2}{|\langle\hat{\mathbf{r}}_1\rangle_\gamma-\hat{\mathbf{r}}_2|}, \quad (\gamma=\alpha,\,\beta). \tag{3}$$

Semiclassical gravity scenario. The second scenario sees gravity as fundamentally classical. In this case, it is not clear which characteristics one should expect from the gravitational field generated by a superposition. However, in analogy with classical mechanics, one can assume that is the mass density $\rho(\mathbf{r}_1) = (\rho_\alpha(\mathbf{r}_1)+\rho_\beta(\mathbf{r}_1))/2$ of the system in superposition that produces the gravitational field. This is also what is predicted by the Schrödinger–Newton equation [5, 26–30]. The final two-body state will be of the form

$$\Psi^{\text{final}}_{\text{CG}}(\mathbf{r}_1,\,\mathbf{r}_2) = \frac{\alpha(\mathbf{r}_1)+\beta(\mathbf{r}_1)}{\sqrt{2}}\phi(\mathbf{r}_2), \tag{4}$$

where the difference with equation (1) is clear. The gravitational potential becomes

$$\hat{V}_{\text{cl}}(\hat{\mathbf{r}}_2) \approx \tfrac{1}{2}\sum_{\gamma=\alpha,\beta}\hat{V}_\gamma(\hat{\mathbf{r}}_2), \tag{5}$$

where $\hat{V}_\gamma(\hat{\mathbf{r}}_2)$ can be eventually approximated as in equation (3).

In the next section, we investigate the difference between the two scenarios by exploiting the sophisticated and powerful machinery provided by optomechanics.

2. Theoretical model

To describe the dynamics that follow the first or second scenario, we take advantage of the quantum Langevin equations, which is the typical description for optomechanical systems. The proposed set-up is schematically presented in figure 2. We assume S2 as trapped harmonically in $\mathbf{r}_{\text{osc}} = (r_{x,\text{osc}},\, r_{y,\text{osc}},\, 0)$ along the x and y directons by means of the cavity fields. The corresponding quantum Langevin equations for the position $\hat{r}_{2,i}$ and momentum $\hat{p}_{2,i}$ operator of S2 read [31]

$$\begin{aligned}\frac{\mathrm{d}\hat{r}_{2,i}(t)}{\mathrm{d}t} &= \frac{\hat{p}_{2,i}(t)}{m_2},\\ \frac{\mathrm{d}\hat{p}_{2,i}(t)}{\mathrm{d}t} &= -m_2\omega_i^2(\hat{r}_{2,i}(t)-r_{i,\text{osc}}) - \gamma_i\hat{p}_{2,i}(t) + \hat{\xi}_i(t)\\ &\quad + \hbar\chi_i\hat{a}_i^\dagger(t)\hat{a}_i(t) + \frac{\mathrm{i}}{\hbar}[\hat{V}_\nu,\,\hat{p}_{2,i}(t)],\end{aligned} \tag{6}$$

where $i = x, y$ (we do not consider the motion along z) and $\nu = \alpha, \beta, \text{cl}$. Here, ω_i is the harmonic frequency of the mechanical oscillator, γ_i is the damping rate for the vibrations, which are characterized by the noise operator

Table 1. Explicit form of the coefficients $C_{n,i}^{(\gamma)}$ entering in equation (9) for the quantum scenario, with $\mathcal{G}_\gamma = Gm_1m_2/h_\gamma^3$ and $h_\gamma = \sqrt{(\bar{r}_{2,x} - s_\gamma d_1)^2 + d_y^2}$. For the classical scenario one obtains that $C_{i,x}^{(\mathrm{cl})} = \frac{1}{2}(C_{i,x}^{(\alpha)} + C_{i,x}^{(\beta)})$.

Quantum scenario		
$C_{n,i}^{(\gamma)}$	$i = x$	$i = y$
$n = 0$	$\mathcal{G}_\gamma(s_\gamma d_1 - \bar{r}_{2,x})$	$\mathcal{G}_\gamma d_y$
$n = 1$	$\frac{\mathcal{G}_\gamma}{h_\gamma^2}[3(\bar{r}_{2,x} - s_\gamma d_1)^2 - h_\gamma^2]$	$\frac{\mathcal{G}_\gamma}{h_\gamma^2}(3d_y^2 - h_\gamma^2)$
$n = 2$	$-\frac{3\mathcal{G}_\gamma}{h_\gamma^2}(s_\gamma d_1 - \bar{r}_{2,x})d_y$	$-\frac{3\mathcal{G}_\gamma}{h_\gamma^2}(s_\gamma d_1 - \bar{r}_{2,x})d_y$

$\hat{\xi}_i$, having the correlation functions defined as $\langle\hat{\xi}_i(t)\rangle = 0$ and

$$\langle\hat{\xi}_i(t)\hat{\xi}_j(s)\rangle = \hbar m\gamma_i\delta_{ij}\int\frac{\mathrm{d}\omega}{2\pi}\mathrm{e}^{-\mathrm{i}\omega(t-s)}\omega\left[1 + \coth\left(\frac{\hbar\omega}{2k_\mathrm{B}T}\right)\right]. \tag{7}$$

The position of S2 is measured by means of the cavity field, whose creation and annihilation operator are $\hat{a}_i^\dagger$ and $\hat{a}_i$. The dynamical equation of the latter is given by

$$\frac{\mathrm{d}\hat{a}_i(t)}{\mathrm{d}t} = -\mathrm{i}[\Delta_{0,i} - \chi_i\hat{r}_{2,i}(t)]\hat{a}_i(t) - \kappa_i\hat{a}_i(t) + \sqrt{2\kappa_i}\hat{a}_{i,\mathrm{in}}(t), \tag{8}$$

where we defined $\Delta_{0,i} = \omega_{c,i} - \omega_{0,i}$, with $\omega_{0,i}$ denoting the frequency of the external laser, $\omega_{c,i}$ the frequency of the cavity mode drived by the laser, $\chi_i = \omega_{c,i}/L_i$ the optomechanical coupling constant between the cavity and the mechanical oscillator with L_i the size of the cavity, and $\mathcal{E}_i = \sqrt{2\kappa_i\mathcal{P}_i/\hbar\omega_{0,i}}$. Here, $\mathcal{P}_i$ is the laser power and κ_i is the cavity photon decay rate. Moreover, we defined $\hat{a}_{i,\mathrm{in}}$ as the annihilation operator of external laser field, whose only non-zero correlation reads $\langle\hat{a}_{i,\mathrm{in}}(t)\hat{a}_{j,\mathrm{in}}^\dagger(s)\rangle = \delta_{ij}\delta(t - s)$. The last term in equation (6) describes the gravitational interaction with S1, whose action is described below.

To be quantitative, we define the mean positions of the two systems in interaction. We consider S1 as holding a steady position that can be approximated to its average value on α or β respectively: $\langle\hat{\mathbf{r}}_1(t)\rangle_\gamma \approx (s_\gamma d_1, 0, 0)$, with $s_\alpha = 1, s_\beta = -1$. Conversely, we consider the position of S2 as an operator, center in $(\bar{r}_{2,x}, d_y, 0)$ (see figure 1). Thus, we have $\hat{\mathbf{r}}_2(t) = (\hat{r}_{2,x}(t), \hat{r}_{2,y}(t), 0) = (\bar{r}_{2,i} + \hat{\delta}_{2,x}(t), d_y + \hat{\delta}_{2,y}(t), 0)$ and $\hat{\mathbf{p}}_2(t) = (\hat{p}_{2,x}(t), \hat{p}_{2,y}(t), 0)$ is its momentum operator.

Assuming that the quantum fluctuations $\delta\hat{\mathbf{r}}_2(t) = (\hat{\delta}_{2,x}(t), \hat{\delta}_{2,y}(t), 0)$ around the initial mean values for S2 are small, we can expand the commutator in the last term of equation (6) up to the first order in the fluctuations. Thus, we have

$$\frac{\mathrm{i}}{\hbar}[\hat{V}_\nu, \hat{p}_{2,i}(t)] = C_{0,i}^{(\nu)} + C_{1,i}^{(\nu)}\hat{\delta}_{2,i}(t) + C_{2,i}^{(\nu)}\hat{\delta}_{2,j}(t), \text{ with } j \neq i. \tag{9}$$

In the quantum scenario, the coefficients $C_{n,i}^{(\nu)}$ entering in equation (9) are defined in table 1, while in the classical scenario one easily obtains: $C_{n,i}^{(\mathrm{cl})} = \frac{1}{2}(C_{n,i}^{(\alpha)} + C_{n,i}^{(\beta)})$.

In the limit of $d_1 \gg \bar{r}_{2,x}$, they become

$$C_{1,x} = \frac{Gm_1m_2}{d^5}(2d_1^2 - d_y^2), \tag{10a}$$

$$C_{1,y} = \frac{Gm_1m_2}{d^5}(2d_y^2 - d_1^2), \tag{10b}$$

$$C_2^{(\gamma)} = -\frac{3Gm_1m_2}{d^5}d_1d_ys_\gamma, \quad \text{and } C_2^{(\mathrm{cl})} = 0, \tag{10c}$$

where $d^2 = (d_1^2 + d_y^2)$. Here only $C_2^{(\nu)}$ depends on the specific scenario (quantum or semi-classical) we are considering. Following conventional approach, one finds:

$$\bar{r}_{2,i}^{(\nu)} = \frac{\hbar\chi_i|\bar{a}_i|^2 + C_{0,i}^{\nu}}{m_2\omega_i^2} + r_{i,\mathrm{osc}}, \quad \text{and} \quad \bar{p}_{2,i}^{(\nu)} = 0. \tag{11}$$

We can remove the radiation pressure contribution by setting the center of the harmonic trap to $r_{i,\mathrm{osc}} = -\hbar\chi_i|\bar{a}_i|^2/m_2\omega_i^2$. Moreover, we assume that $d_1 \gg \bar{r}_{2,x}$, such that one can approximate $h_\gamma \simeq d = (d_1^2 + d_y^2)^{1/2}$ (see table 1), thus finding

$$\bar{r}_{2,x}^{(\gamma)} = \frac{Gm_1d_1}{\omega_x^2d^3}s_\gamma, \qquad \bar{r}_{2,y}^{(\gamma)} = \frac{Gm_1d_y}{\omega_y^2d^3}. \tag{12}$$

These expressions show the first difference between the quantum and the classical scenario. In the quantum scenario S2 is pulled towards positive (or negative) x while in the classical scenario it remains at the center $\bar{r}_{2,x}^{(\mathrm{cl})} = \bar{r}_{2,x}^{(\alpha)} + \bar{r}_{2,x}^{(\beta)} = 0$. However, it also highlights the difficulties one has in discerning the two scenarios. Once the average is taken in the quantum scenario, we have $\langle \hat{r}_{2,x} \rangle_{(\mathrm{qu})} = \frac{1}{2}\sum_\gamma \bar{r}_{2,x}^{(\gamma)} = 0$, which corresponds to the classical result.

Equation (9) shows that the difference between the quantum and the semi-classical scenario is embedded in the coupling between the motions along x and y of S2. Indeed, in the quantum scenario, the gravitation attraction of S1 pulls S2 towards one of the branches of the superposition of S1, leading to correlations between the x and y motions. Conversely, in the semi-classical scenario, for which $C_2^{(\mathrm{cl})} = 0$, the dynamics along the two direction is decoupled, due to the symmetrical attraction of S1 along y. The verification of a coupling of the motion along x with that along y would be sufficient to prefer the quantum scenario over the semi-classical one. Before discussing possible mechanisms that can be exploited for this task, a comment is at order. If S1 in not in the superposition $(|\alpha\rangle + |\beta\rangle)/\sqrt{2}$, but in the statistical mixture $(|\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta|)/2$, one has a coupling between the x and the y motion both in what we called the classical and the quantum scenario; the two situations cannot be discriminated by our proposal. Indeed, the key point of our proposal is that one has to make sure that a quantum superposition of S1 is generated. If necessary, this can be preliminary checked by suitable interferometric techniques. Once quantum coherence in the state of S1 is ascertained, our proposal allows for the discrimination of quantum and classical scenarios. In this regard, our scheme here should be seen as a witness of the potential quantum nature of gravity.

3. Revelation strategies

There are different measurements that one can exploit for witnessing the correlations between the x and y motions, and thus providing a verification of the quantum scenario over the semi-classical one.

(1) *Direct measurement of the density noise spectrum (DNS).* To quantify the difference between the two scenarios, we consider the DNS corresponding to the motion of S2 along the x axis. By working under conditions such that $d_1 \gg \bar{r}_{2,x}$, the Langevin equations for the fluctuations read

$$\begin{aligned}
\frac{\mathrm{d}\hat{\delta}_{2,i}(t)}{\mathrm{d}t} &= \frac{\delta\hat{p}_{2,i}(t)}{m_2},\\
\frac{\mathrm{d}\delta\hat{p}_{2,i}(t)}{\mathrm{d}t} &= -m_2\omega_i^2\hat{\delta}_{2,i}(t) - \gamma_i\delta\hat{p}_{2,i}(t) + \hat{\xi}_i(t) + C_{1,i}\hat{\delta}_{2,i}(t)\\
&\quad + C_{2,i}^{(\nu)}\hat{\delta}_{2,j}(t) + \hbar\chi_i[\bar{a}_i^*\delta\hat{a}_i(t) + \bar{a}_i\delta a_i^\dagger(t)],\\
\frac{\mathrm{d}\delta\hat{a}_i(t)}{\mathrm{d}t} &= -\mathrm{i}\Delta_i^{(\nu)}\delta\hat{a}_i(t) + i\chi_i\bar{a}_i\hat{\delta}_{2,i}(t) - \kappa_i\delta\hat{a}_i(t) + \sqrt{2\kappa_i}\,\hat{a}_{i,\mathrm{in}}(t)
\end{aligned} \tag{13}$$

for $j \neq i$. The coefficients $C_{m,i}^{(\nu)}$ are approximated as in equations (10), $\Delta_i^{(\nu)} = \Delta_{0,i} - \chi_i\bar{a}_i\bar{r}_i^{(\nu)}$, which becomes $\Delta_i \simeq \Delta_{0,i}$ in light of the weakness of the optomechanical coupling.

Equation (13) can be solved in the frequency domain by using the standard approach [31]. By defining $\tilde{r}_{2,i}(\omega)$ as the Fourier transform of $\hat{\delta}_{2,i}(t)$, after lengthly yet straightforward calculations, we find

$$\begin{aligned}
\tilde{r}_{2,i}(\omega) = {}& \frac{1}{m_2[\omega_{i,\mathrm{eff}}^2(\omega) - \omega^2 - \mathrm{i}\gamma_{i,\mathrm{eff}}(\omega)\omega]}\\
&\times\left[\tilde{\xi}_i(\omega) + C_2^{(\nu)}\tilde{r}_{2,j}(\omega) + \hbar\chi_i\sqrt{2\kappa_i}\left(\frac{\bar{a}_i^*\tilde{a}_{i,\mathrm{in}}(\omega)}{\kappa_i + i(\Delta_i - \omega)} + \frac{\bar{a}_i\tilde{a}_{i,\mathrm{in}}^\dagger(\omega)}{\kappa_i - \mathrm{i}(\Delta_i + \omega)}\right)\right],
\end{aligned} \tag{14}$$

where we defined the following effective frequencies and dampings

$$\omega_{i,\mathrm{eff}}^2(\omega) = \omega_i^2 + \frac{2\hbar\chi_i^2|\bar{a}_i|^2\Delta_i(\omega^2 - \kappa_i^2 - \Delta_i^2)}{m_2[(\kappa_i^2 + \Delta_i^2 + \omega^2)^2 - 4\Delta_i^2\omega^2]} - \frac{C_{1,i}}{m_2}, \tag{15a}$$

$$\gamma_{i,\mathrm{eff}}(\omega) = \gamma_i + \frac{4\hbar\chi_i^2|\bar{a}_i|^2\Delta_i\kappa_i}{m_2[(\kappa_i^2 + \Delta_i^2 + \omega^2)^2 - 4\Delta_i^2\omega^2]}. \tag{15b}$$

The effect of such correlation can be seen in the DNS, which can be derived from equation (14) by applying its definition $\mathcal{S}_{ii}(\omega) = \frac{1}{4\pi}\int \mathrm{d}\Omega\,\langle\{\tilde{r}_{2,i}(\omega),\, \tilde{r}_{2,i}(\Omega)\}\rangle$. Then we find

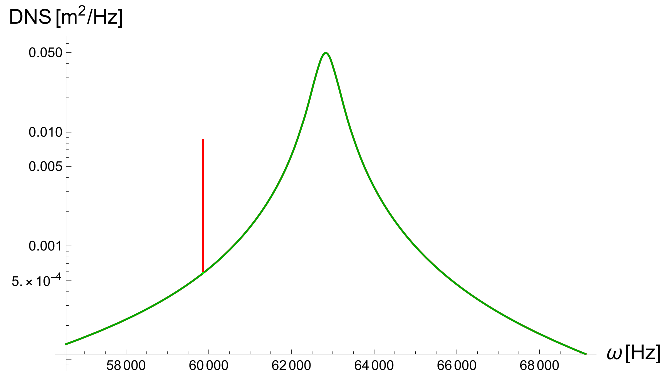

Figure 3. Comparison between the DNS for the classical (in green) and the quantum (in red) scenario. We have taken $m_1 = 5 \times 10^{-14}$ kg, $m_2 = 9.5 \times 10^{-19}$ kg, $d_x = 10^{-9}$ m, $d_y = 2.9 \times 10^{-4}$ m, $\omega_x = 2\pi \times 10^4$ Hz, $\omega_y = 2\pi \times 9.5 \times 10^3$ Hz, $\gamma_x = 2\pi \times 100$ Hz, $\gamma_y = 2\pi \times 3 \times 10^{-3}$ Hz, $T = 4 \times 10^{-3}$ K, $\mathcal{E}_y = 2 \times 10^4\, \mathcal{E}_x = 8 \times 10^{14}$ Hz, $\kappa_x = 10^3 \kappa_y = 9 \times 10^8$ Hz, $\omega_{c,y} = 10^5\, \omega_{c,x} = 2\pi \times 3.7 \times 10^{15}$ Hz.

$$\mathcal{S}_{xx}(\omega) = \frac{m_2 g_y(\omega)\left[\left(\hbar m_2 \gamma_x \omega \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right) + \mathcal{S}_{\mathrm{L}}^x(\omega)\right) + \frac{(C_2^{(\nu)})^2}{m_2^2 g_y^2(\omega)}\left(\hbar m_2 \gamma_y \omega \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right) + \mathcal{S}_{\mathrm{L}}^y(\omega)\right)\right]}{m_2^4 g_x(\omega) g_y(\omega) - 2m_2^2 (C_2^{(\nu)})^2 f(\omega) + (C_2^{(\nu)})^4}, \tag{16}$$

where

$$g_i(\omega) = (\omega_{i,\mathrm{eff}}^2(\omega) - \omega^2)^2 + \gamma_{i,\mathrm{eff}}^2(\omega)\omega^2, \tag{17a}$$

$$\mathcal{S}_{\mathrm{L}}^i(\omega) = \frac{2\hbar^2 \chi_i^2 \kappa_i |\bar{a}_i|^2 (\kappa_i^2 + \Delta_i^2 + \omega^2)}{[(\kappa_i^2 + \Delta_i^2 + \omega^2)^2 - 4\Delta_i^2 \omega^2]}, \tag{17b}$$

and

$$f(\omega) = (\omega_{x,\mathrm{eff}}^2(\omega) - \omega^2)(\omega_{y,\mathrm{eff}}^2(\omega) - \omega^2) - \gamma_{x,\mathrm{eff}}(\omega)\gamma_{y,\mathrm{eff}}(\omega)\omega^2, \tag{17c}$$

with ω_{eff} and γ_{eff} denoting the effective frequency and damping respectively. Equation (16) shows that in the quantum scenario the gravitational interaction leads to an extra contribution in the DNS (last term in squared brackets), which is directly connected to the motion along y. Such a term appears as an extra peak centered in the effective oscillation frequency of the y motion. The amplitude of the peak is related to the coupling between S2 and the cavity field along y. Clearly, the larger the coupling the bigger is the amplitude of the peak. An example of the presence of this second peak is shown in figure 3.

(2) *Indirect measurement of non-classical correlation between cavity fields.* A viable strategy for the inference of the quantum nature of gravity goes through the assessment of possible non-classical correlations between the x and y degrees of freedom induced by the latter. Such a coupling disappears for classical gravity as $C_2^{(\mathrm{cl})} = 0$. The induced all-mechanical correlations could in turn translate into analogous all-optical ones in light of the optomechanical coupling. In an experiment where all other plausible sources of correlations are carefully characterized, the possibility to detect all-optical quantum correlations would pave the way to the inference of the quantum nature of gravity. It is important to stress that such correlations do not need to be as strong as entanglement: any non-zero value of $C_2^{(\gamma)}$ results in non-diagonal elements in the covariance matrix of the overall optomechanical system. The entries of such matrix are $\sigma_{ij} = \langle\{\delta\hat{O}_i, \delta\hat{O}_j\}\rangle$, where the expectation value is taken over the state of the system. Within the validity of the first-order expansion in the fluctuations invoked before, the presence of such non-diagonal elements entails non-classical correlations of the discord form [32]. It is thus sufficient to ascertain the non-nullity of the non-diagonal entries of the covariance matrix of the *all-optical* system embodied by the cavity fields only to infer, indirectly, the non classical nature of their correlations, and thus the quantum nature of the gravitational interaction.

In figure 4(a) we report the total norm $\sigma_{\mathrm{tot}} = \sum_j |\sigma_{jj}^f|$ of the non-diagonal part of the covariance matrix σ^f of the two cavity fields (i.e. we take only the fluctuation operators $\delta\hat{O}_i$ pertaining to the cavity fields) against $C_{1,x}$ for parameters such that $C_{1,x} = C_{1,y}$. We observe a linear growth of the covariances with the strength of the gravity-induced interaction. This gives rise to non-zero values of the discord between such fields, a illustrated in panel (b). Needless to say, the experimental ascertainment of a non-zero value of all-optical discord would pose significant experimental challenges, in light of its weakness. Nevertheless, the link with the strength of the non-diagonal entries of the corresponding covariance matrix offers a potentially viable route towards the goal of this paper: the reconstruction of the entries of an all-optical covariance matrix can indeed be accurately performed via high-efficiency homodyne measurements, as routinely implemented in many laboratories.

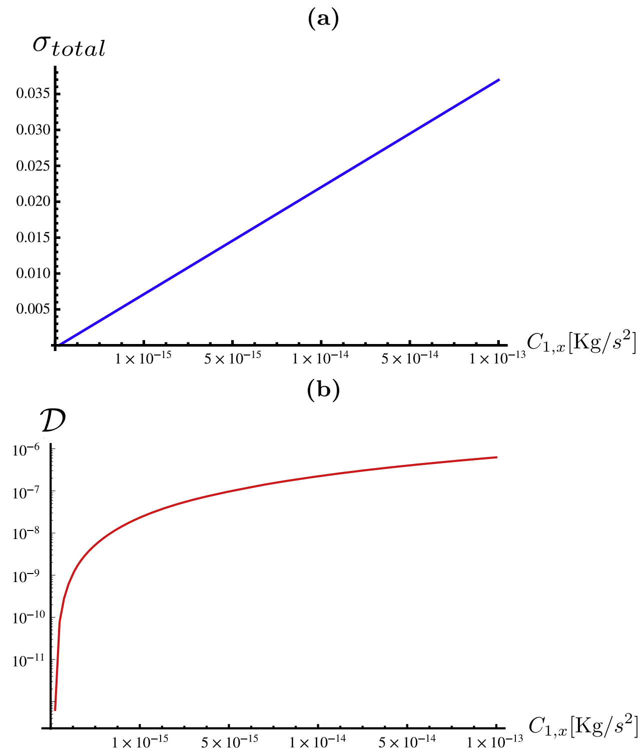

Figure 4. Total norm of the non-diagonal entries of the all-optical covariance matrix (panel (a)) and all-optical discord (panel (b)) plotted against $C_{1,x}$. We have taken $d_{x,y} \sim 10^{-6}$ m, $m_{1,2} = 5 \times 10^{-10}$ Kg, mechanical modes of frequency $2\pi \times 10^7$ Hz, $T = 4$ mK, $\gamma_{x,y} = 2\pi \times 100$ Hz. The cavity has length of 1 mm and finesse of 1.07×10^4.

(3) *Experimental feasibility.* To reduce the decoherence rates from gas collisions and blackbody photons to be smaller than the expected gravity effects, experiments should be done a low temperature and ultra-high vacuum. The calculation of the expected non-classical correlations quantified by discord has been done with typical parameters for optomechanical cantilever or membrane systems [31].

A comment in this regard is useful: while our calculations assume parameters typical of levitated mechanical systems [33–41] and within the grasp or well foreseeable in membrane-based [42] or graphene-based [43] experimental settings, challenges are posed by the arrangement of the geometric configuration specific of our proposal. While the asymmetry of the bidimensional motion addressed in our study does not represent a true difficulty (asymmetric trapping potentials for levitated optomechanical systems are routinely used in current experiments), the small values of the distance between S1 and S2 is the crucial point that requires care. At such distances, in fact, short-range interactions should be considered. van der Waals [44] and Casimir–Polder (CP) [45] forces could be large enough to overtone the gravitational interaction between the two masses, and their influence should be factored in. This problem was already addressed in [46], where the authors showed that the gravitational forces are ten times larger than the CP one at a distance of 200 μm.

It is also worth comparing our scheme to other proposals reported in literature so far, a non-exhaustive list including [47–55, 57]. Among them, let us mention that [47] is based on the possibility to generate Schrödinger cat states of a mechanical oscillator through the use of a superposition of optical field states with exactly 0 and n excitations, which is very challenging to achieve experimentally, on its own. On the other hand, the proposals presented in [52, 53, 55] make use of interferometry between non-classical states of light used to drive the motion of a mechanical systems, which is entirely bypassed by our scheme. Finally, the scheme reported in [54] is based on the trapping and subsequent releasing of a mechanical system, which is not only a different strategy to ours, but also not exempt from technical difficulties due to the need to perform a measurement of position of the decohered particle very accurately during the 'release' time. This comparison helps grasping the differences and potential advantages provided by our own proposal.

4. Conclusions

We have illustrated the dynamics of an optomechanical system probing the gravitational field of a massive quantum system in a spatial superposition. Two different dynamics are found whether gravity is treated quantum mechanically or classically. Here, we propose two distinct methods to infer which of the two dynamics rules the motion of the quantum probe, thus discerning the intrinsic *nature* of the gravitational field. Such methods will be then eventually able to falsify one of the two treatments of gravity.

Recently other interferometric [46] and non-interferometric [56] tests of the nature of gravity were proposed. They are based on the detection of entanglement between two probes, respectively coupled to two different massive systems, which interact through gravity (NV center spins for [46] and cavity fields for [56]). Clearly, to have such entanglement, each of the three couples of interconnected systems (probe 1, system 1, system 2 and probe 2) there considered needs to be entangled on their own. Moreover, the entanglement between the two massive systems is inevitably small due to its gravitational nature. Conversely, our proposal benefits from having only a single massive system involved in the interconnection, which reduces correlation losses. In addition, we provide a second method for discerning the nature of gravity: the individuation of a second peak in the DNS. The latter does not rely on delicate measurements of quantum correlations but can be assessed through standard optomechanical detection schemes.

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ORCID iDs

M Carlesso https://orcid.org/0000-0002-9929-7291
A Bassi https://orcid.org/0000-0001-7500-387X
M Paternostro https://orcid.org/0000-0001-8870-9134
H Ulbricht https://orcid.org/0000-0003-0356-0065

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