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A Historical Account and Technical Reassessment of the Broyden-based Input Space Mapping Optimization Algorithm

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***Abstract* — The Broyden-based input space mapping (SM) algorithm, better known as the aggressive space mapping (ASM) algorithm, is revisited in this article. The most fundamental SM-based optimization methods developed until now, in which ASM is framed, are overviewed. More than two decades of ASM evolution are briefly accounted, evidencing its popularity in both academia and industry. The two main characteristics that explain its popularity are emphasized: 1) simplicity, and 2) efficiency (when it works, it works extremely well). The fundamentals behind the Broyden-based input SM algorithm are illustrated, accentuating key steps for its successful implementation, as well as typical scenarios where it may fail. Finally, some future directions regarding ASM are ventured.**

***Index Terms* — aggressive space mapping, Broyden, EM-based design, non-linear systems, optimization.**

I. Introduction

Space Mapping (SM) optimization methods belong to the general class of surrogate-based optimization algorithms [1], which aim at efficiently optimizing computationally expensive objective functions. Prof. John Bandler invented the space mapping (SM) technique in 1994 [2]. This paper, based on [3], is focused on the Broyden-based input space mapping (SM) algorithm, better known as the aggressive space mapping (ASM) algorithm, which is the simplest and by far the most widely used SM design optimization approach [3].

To place ASM into proper context, the most fundamental SM-based optimization methods developed until now are briefly mentioned. ASM evolution over more than two decades is accounted, in terms of both theoretical developments and applications, confirming its popularity in academia and industry. The fundamentals behind the Broyden-based input SM algorithm are briefly described, emphasizing key steps for its successful implementation, as well as typical cases where it may fail. Finally, some future directions regarding ASM are ventured.

II. Space Mapping Optimization Evolution

ASM emerged in 1995 [4]. Since then, many other SM-based design optimization algorithms have been developed, as illustrated in Fig. 1. These more advanced algorithms aim at making SM optimization more general, more robust, and more efficient. Excepting perhaps implicit space mapping [8], most of them have a significantly higher complexity than ASM, making them less accessible to practicing engineers [3].

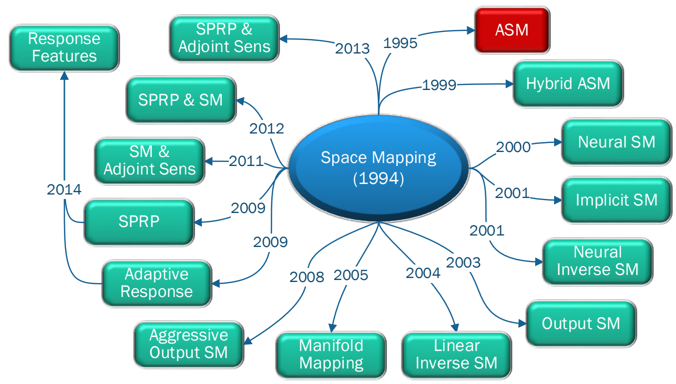

Fig. 1 Fundamental design optimization methods emerged from the SM concept: aggressive SM [4]; hybrid ASM [5]; neural SM [6,7]; implicit SM [8]; neural inverse SM [9]; output SM [10]; linear inverse SM [11]; manifold mapping [12]; aggressive output SM [13]; adaptive response correction (ARC) [14]; shape-preserving response prediction (SPRP) [15]; SM with adjoint sensitivities [16]; SPRP exploiting SM [17]; SPRP using adjoint sensitivities [18]; response features [19,20] (emerged from ARC and SPRP). From [3].

III. Two Decades of ASM Applications

The most important theoretical contributions to the Broyden-based input SM algorithm, as well as its main publicly documented applications are graphically summarized in Figs. 2 and 3, in which the fine and coarse models utilized on each application case are also illustrated. Several observations can be inferred from those two figures:

a. Diverse engineering disciplines. ASM has been applied not only to electromagnetics-based design optimization of RF and microwave circuits, as originally intended, but also to several other areas, including magnetic circuits, mechanical engineering, materials design, medical instrumentation, environmental sciences, etc.

b. Diverse of CAD tools. Models of the optimized structures have been implemented using a variety of numerical simulators, including commercially available CAD tools and internal tools. Physical data obtained from direct measurements have also been used as “fine models”.

c. Diverse contributors. A very significant number of theoretical contributions and applications have been made from research groups outside the originator group at McMaster University, especially for the second decade of evolution.

d. Stable production of applications. A quite steady

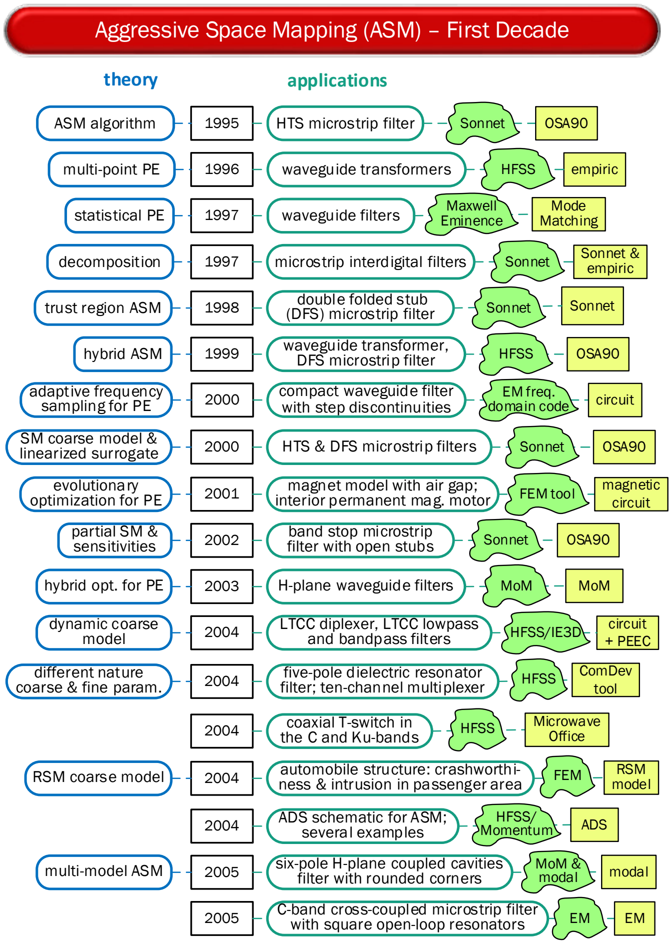

Fig. 2. First decade of evolution of ASM: key theoretical elements contributed to ASM and main applications to design optimization using ASM (indicating fine and coarse models employed). From [3].

generation of engineering applications of ASM over time, spanning over two decades, with no signs of an imminent end to ASM applications.

IV. The Essence of ASM

ASM starts by finding the optimal coarse model design $\boldsymbol{x}_c^*$ that yields the target coarse model response, $\boldsymbol{R}_c(\boldsymbol{x}_c^*) = \boldsymbol{R}_c^*$, and serves as the starting point for the fine model design parameters $\boldsymbol{x}_f$. The central part of the Broyden-based input SM algorithm is the parameter extraction process [3], which can be interpreted as a multidimensional vector function $\boldsymbol{P}$ representing the mapping between both design parameter spaces, $\boldsymbol{x}_c^{(i)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i)})$. If the current extracted parameters $\boldsymbol{x}_c^{(i)}$ correspond approximately to $\boldsymbol{x}_c^*$, then the current fine model response approximates the desired response, $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}) \approx \boldsymbol{R}_c^*$. It is seen that the ASM algorithm iteratively finds a solution to the following system of nonlinear equations

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^* \tag{1}$$

since any root $\boldsymbol{x}_f^{SM}$ of the above system of equations $\boldsymbol{f}(\boldsymbol{x}_f)$ implies that $\boldsymbol{R}_f(\boldsymbol{x}_f^{SM}) \cong \boldsymbol{R}_c^*$.

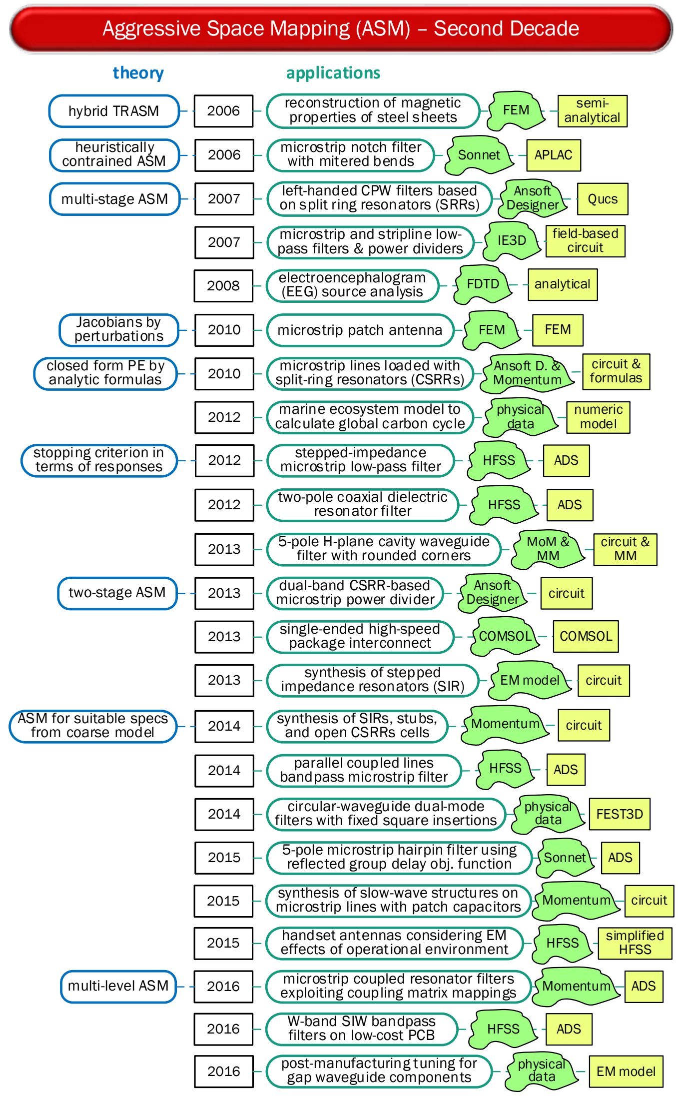

Fig. 3. Second decade of evolution of ASM: key theoretical contributions and main applications to design optimization. See [3].

A typical evolution of ASM from the perspective of the system of nonlinear equations associated to the mapping function is illustrated in Fig. 4, where the Broyden matrix $\boldsymbol{B}$ is initialized with the identity (a one-dimension design optimization problem is considered for simplicity). In this illustration, it is assumed that the initial design is very bad (or a very deviated coarse model), implying a very large value of $\|\boldsymbol{f}(\boldsymbol{x}_f^{(0)})\|$. In spite of that, ASM converges very quickly to a space mapped solution $\boldsymbol{x}_f^{SM}$.

Plots in Fig. 4 also provide some insight regarding the famous efficiency of ASM, by which many highly complex problems are frequently solved in just a few fine model evaluations, regardless of the number of optimization variables, even in cases were the initial fine model response $\boldsymbol{R}_f(\boldsymbol{x}_c^*)$ is very much deviated from the target response $\boldsymbol{R}_c^*$. As seen in Fig. 4, the efficiency of ASM depends on the degree of nonlinearity of $\boldsymbol{f}(\boldsymbol{x}_f)$, which in turns depends on the degree of nonlinearity of the mapping $\boldsymbol{P}$ between both model parameter

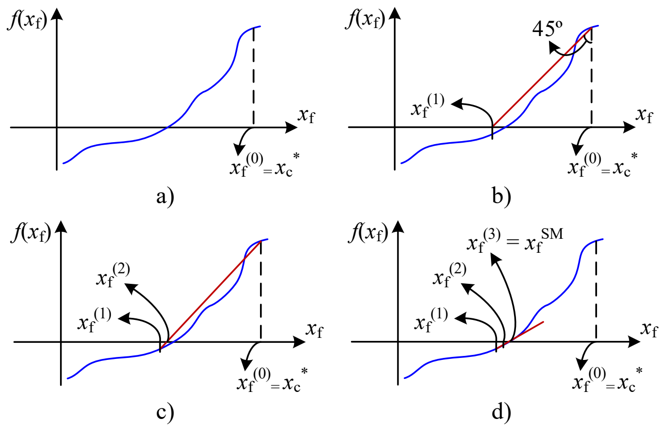

Fig. 4. Typical evolution of the aggressive space mapping (assuming a one-dimension design optimization problem and a very bad initial design): a) initial fine model response calculated, first extracted parameters are very different to $\boldsymbol{x}_c^*$; b) Broyden matrix is initialized with the identity and first iterate is predicted; c) Broyden matrix is updated with formula and next iterate calculated; d) Broyden matrix is updated and next iterate is practically a root (extracted parameters are practically equal to $\boldsymbol{x}_c^*$). From [3].

spaces. If the mapping is relatively linear (even with a large offset), ASM solves the design problem in a few iterations, regardless of the problem dimensionality, even when the initial fine model response is significantly deviated from the target.

The parameter extraction process is the weakest part of ASM, since this optimization sub-problem may present multiple local minima, some of them yielding a good match (several coarse model designs able to approximate with acceptable accuracy the current fine model response). The non-uniqueness of the parameter extraction solution may lead to oscillations or even divergence in the ASM algorithm [21]. Several successful strategies are known to overcome this difficulty [22].

V. Future Direction and Conclusion

Based on the most recent applications of ASM, it is seen a trend towards the development of fully automated CAD tools based on ASM, for efficient and accurate synthesis and design optimization algorithms dedicated to particular structures in specific technologies. This trend might lead to the future incorporation of ASM into industrial CAD tools as built-in design functions for specific structures.

This paper recognizes Prof. Bandler as one of the founders and most influential figures of the MTT-S CAD arena.

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