Essays on fractional cointegration and spurious long memory

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ABSTRACT

This thesis contains four essays on fractional cointegration and spurious long memory following the introduction and related literature in the first chapter. Chapter 2 provides an analysis of fractional integration and cointegration using the high, the low, and the range stock prices instead of closing prices. We analyze the long memory trends across six Asian stock markets including Korea, Indonesia, Malaysia, Sri Lanka, India, and Pakistan. The empirical analysis provide a univariate analysis which includes the unit root tests and estimation of fractional integration in the highs, the lows, and the ranges. Range, being a linear combination of the nonstationary highs and lows, is a stationary long memory process which specifies the need to model these two extreme values and the range simultaneously in a multivariate fractional cointegration context. The fractional vector error correction model fulfills this specification while considering the short-run and long-run relationships. We also perform a forecast comparison of FVECM with alternative models. The autoregressive fractionally integrated and the Heterogeneous autoregressive models are considered to model the long memory in the ranges. Our results support the use of daily ranges as volatility estimator and FVECM to model the long-run convergence and short-run divergence in the highs and lows at the same time.

In chapter 3, we analyze the true long memory or spurious long memory in the range based volatilities of spot exchange rates across 30 currencies against the USD including the developed, the developing and the emerging exchange rates. The persistence of exchange rates is of much interest for the central bank, for policymakers, and to understand the inflation dynamics in an economy. The frequency domain analysis exhibits the spurious long memory in most of the currencies due to some low-frequency contaminations, level shifts, or structural changes. We proceed with the estimation of structural points with an unknown number of breaks. Our results provide a different number of breaks across currencies which may relate to some shocks, economic crisis, and financial policies.

Chapter 4 contains a detailed analysis of persistent trends in all share index and ten sectoral indices in an emerging stock market of Pakistan. There is a general consideration regarding the inefficiency of emerging markets compared to the developed markets. Our results show the existence of predictable trends across KSE100 and ten sectors. Moreover, we investigate that the existing trends are true or a result of some level shifts with a semiparametric test. According to the adaptive market hypothesis the long-range dependence is not a constant phenomenon and it varies over time corresponding to the market conditions. We analyze this time-varying long memory with a rolling window technique and observe the fluctuating trends such as persistence, antipersistence, efficiency, and inefficiency at different times.

Finally, we analyze the fractional cointegration between the volatilities of the conventional index and Islamic index in chapter 5. Islamic finance attracts the attention of investors and traders regarding its different features such as zero interest rates and profit loss sharing strategies. Some researchers believe that Islamic financial markets can work as a good diversification candidate due to different performance levels during the phases of economic and political shocks. This analysis considers the conventional and Islamic indices across nine Islamic countries: Bahrain, UAE, Oman, Qatar, Malaysia, Indonesia, Egypt, Turkey, and Pakistan. Our results suggest the existence of fractional cointegration and absence of diversification opportunities between the indices in seven out of nine countries in the long-run. This implies that the both type of indices follow same trends while there may exist the diversification alternatives in the other two cases.

Keywords: Spurious Long Memory, Fractional Cointegration, Semiparametric Estimation, Structural breaks, volatility

ZUSAMMENFASSUNG

Diese Arbeit enthält vier Aufsätze über fraktionierte Kointegration und unechte lange Erinnerung nach der Einführung und verwandter Literatur im ersten Kapitel. Kapitel 2 stellt eine Analyse der fraktionierten Integration und Kointegration unter Verwendung der hohen, niedrigen und range Aktienkurse anstelle der Schlusskurse vor. Wir analysieren die Trends des Long Memory an sechs asiatischen Aktienmärkten, darunter Korea, Indonesien, Malaysia, Sri Lanka, Indien und Pakistan, und erstellen eine univariate Analyse, einschließlich Unit-Root-Tests und Schätzung des Long Memory in den Höhen, Tiefen und Bandbreiten. Range, eine lineare Kombination aus Höhen und Tiefen, ist ein stationärer Long Memory Prozess und spezifiziert die Notwendigkeit, diese beiden Extremwerte und den Bereich gleichzeitig im multivariaten fraktionierten Kointegrationskontext zu modellieren. Das fraktionierte Vektor-Fehlerkorrekturmodell erfüllt diese Spezifikation unter Berücksichtigung der kurz- und langfristigen Beziehungen. Wir führen auch einen Prognosevergleich von FVECM mit alternativen Modellen durch und berücksichtigen die autoregressiven fraktionell integrierten und heterogenen autoregressiven Modelle, um den Long Memory in den Bereichen zu modellieren. Unsere Ergebnisse unterstützen die Verwendung von Tagesbereichen als Volatilitätsschätzer und FVECM, um die Konvergenz und Divergenz gleichzeitig in den Höhen und Tiefen zu modellieren.

In Kapitel 3 analysieren wir den wahren Long Memory oder den falschen Long Memory in den bandbasierten Volatilitäten der Spot-Wechselkurse über 30 Währungen gegenüber dem USD, einschließlich der entwickelten, der sich entwickelnden und der aufstrebenden Wechselkurse. Das Fortbestehen der Wechselkurse ist von großem Interesse für die Zentralbank, für die politischen Entscheidungsträger und um die Inflationsdynamik in einer Volkswirtschaft zu verstehen. Die Frequenzbereichsanalyse unterstützt das Vorhandensein von störendem langem Speicher in den meisten Währungen aufgrund von niederfrequenten Verunreinigungen, Pegelschiebungen oder strukturellen Veränderungen. Wir schätzen die strukturellen Punkte mit einer unbekannten Anzahl von Brüchen und die Ergebnisse liefern eine unterschiedliche Anzahl von Brüchen zwischen den Währungen, was sich auf einige Schocks, die Wirtschaftskrise und die Politik beziehen kann.

Kapitel 4 enthält eine detaillierte Analyse der anhaltenden Trends aller Aktienindizes und zehn sektoraler Indizes in einem aufstrebenden Aktienmarkt Pakistans. Es gibt eine allgemeine Überlegung über die Ineffizienz der Schwellenländer im Vergleich zu den entwickelten

Märkten, und unsere Ergebnisse zeigen, dass es in allen KSE100- und zehn Sektoren vorhersehbare Trends gibt. Darüber hinaus untersuchen wir, dass die bestehenden Trends wahr sind oder das Ergebnis einiger Pegelsprünge mit einem semiparametrischen Test. Nach der adaptiven Markthypothese ist die langfristige Abhängigkeit kein konstantes Phänomen und variiert im Laufe der Zeit je nach Marktbedingungen. Wir analysieren dieses zeitvariable Langzeitgedächtnis mit einer Rollfenster-Technik und beobachten die schwankenden Trends wie Persistenz, Anti-Persistenz, Effizienz und Ineffizienz über verschiedene Zeiten hinweg.

Schließlich analysieren wir die fraktionierte Kointegration zwischen den Volatilitäten des konventionellen Index und des islamischen Index in Kapitel 5. Islamische Finanzierungen ziehen die Aufmerksamkeit von Investoren und Händlern auf sich, da sie unterschiedliche Merkmale wie Nullzinsen und Gewinnbeteiligungsstrategien aufweisen. Einige Forscher glauben, dass die islamischen Finanzmärkte als guter Diversifikator in den Phasen wirtschaftlicher und politischer Schocks aufgrund unterschiedlicher Leistungsniveaus während der Krise wirken können. Wir betrachten die konventionellen und islamischen Indizes in neun islamischen Ländern: Bahrain, VAE, Oman, Katar, Malaysia, Indonesien, Ägypten, Türkei und Pakistan. Unsere Ergebnisse zeigen, dass es in sieben von neun Ländern langfristig keine Diversifikationsmöglichkeiten zwischen den Indizes gibt, da sie den gleichen Trends folgen, während es die Diversifikationsalternativen für das Reiben von zwei Fällen ohne fraktionierte Kointegration und unterschiedliche Trends geben kann.

Stichworte: Scheinbar langes Gedächtnis, Semiparametrische Schätzung, Fraktionale Kointegration, Strukturbrüche, Volatilität

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Chapter 1

Introduction

1.1 Introduction

The topics of this thesis are long memory, long-range dependence or fractional integration, cointegration, and spurious long memory. The idea of long memory was first initiated by Hurst (1951) in the field of hydrology while working on the water flows in Nile River. Later on, its existence in economic series was suggested by Mandelbrot (1971) and it was applied to the financial markets data during 1995. In the time domain, the autocorrelation function describes this property with values different than zero at large lags therefore present a hyperbolic decay. A pole near zero frequencies or unbounded spectral density in the vicinity of origin defines the long memory in the frequency domain. This phenomenon has been studied in stock markets (Ding et al., 1993, Greene and Fielitz, 1977, Lo, 1991), in exchange rates (Cheung, 1993, Okimoto and Shimotsu, 2010), in interest rates (Cajueiro and Tabak, 2007, Tabak and Cajueiro, 2005, Tsay, 2000), and in commodity prices (Barkoulas et al., 1997, Dolatabadi et al., 2015) among many others. First estimation method for long-range dependence was the rescaled range (R/S) statistic followed by the Hurst exponent (H), the modified R/S, the variance ratio test and many others. Granger and Joyeux (1980) and Hosking (1981) introduced the first long memory models named fractionally integrated autoregressive models denoted as ARFIMA (p, d, q) to consider the fractional orders of the memory parameters instead of 0 or 1 as an extension of the autoregressive integrated moving average (ARIMA) models. Parametric estimation of these models is based on the maximum likelihood function while the semiparametric methods work with periodogram using the frequencies near origin. We use the semiparametric methods in our analysis just because no prior knowledge of the underlying data generating structure is required by these methods and are based on the user specified bandwidth parameters.

A common extension of fractional integration is fractional cointegration for the multivariate series as introduced by Engle and Granger (1987). It refers the slow adjustment toward equilibrium compared to the standard cointegration. A multivariate time series $X_t \sim I(d)$ is defined cointegrated if $\beta' X_t \sim I(d-b)$ for non-zero vector β and $b > 0$, where $\beta' X_t$ is known as the cointegrating error with cointegrating vector β. Although this definition is very wide and does not impose restrictions on the integer orders of the series and the cointegrating errors, still most of the literature has concentrated on the classical cointegration cases until the mid of the 90's. Subsequently, many methods including (Chen and Hurvich, 2006, Nielsen and Shimotsu, 2007, Robinson, 1994, Robinson and Yajima, 2002, Wang et al., 2015) have been developed to test the existence of fractional cointegration, to estimate the cointegration rank, to test the homogeneity of the memory estimates, and to estimate the cointegration strength.

Johansen (2008) extended the vector error correction models (VECM) to accommodate the fractional cointegration in form of fractional vector error correction models (FVECM). We apply the FVECM technique to model the high and the low stock prices in chapter 2 of this thesis.

Developing interests in the field of long memory has been criticized for the presence of structural breaks in data which may cause the spurious long memory by displaying trends similar to the long-range dependence (Diebold and Inoue, 2001, Granger and Ding, 1996, Granger and Hyung, 2004). Due to the similar finite sample properties of spurious long memory and true long memory, the standard theory fails to distinguish between these two. Some tests including (Ohanissian et al., 2008, Qu, 2011, Shimotsu, 2006) have been developed to differentiate between these two. We estimate the fractional cointegration in chapter 2 and 5 of this thesis while in 3 and 4 test for the spurious long memory with an application of the different semiparametric techniques to the financial time series.

In chapter 2, we analyze the high and the low stock values across six Asian countries including Korea, Indonesia, Malaysia, Sri Lanka, India, and Pakistan. We proceed with the hypothesis of short-run divergence and long-run comovements between the highs and the lows. The existence of comovements is suggested by the unit root analysis of the highs, the lows, and the range series. The ADF unit root test shows the nonstationary dynamics of daily log-highs and log-lows but the range, difference between these two, is stationary. Moreover, the existence of long memory is indicated with the significant autocorrelations at long lags in the range series. The empirical analysis is based on the FVECM of Johansen (2008) since it has the ability to model the short-run and long-run dynamics simultaneously. We estimate long memory with the semiparametric estimators in each series and find nonstationary long memory in the highs and the lows while stationary memory estimates for the range series. The test of homogeneity for the memory estimates and cointegrating rank estimation are performed by using frequency domain methods. Although, the estimated vector is very near to the expectation across all series, the range cannot be considered as a cointegrating vector between the highs and the lows unless it is exactly (1, -1). The estimation of the restricted FVECM model is performed with a restriction on the cointegrating vector. The results of this restriction do not affect the short term estimates and loading factors.

The adjustment coefficients appear with opposite signs and point out the different directions of the highs and the lows toward convergence. Moreover, a fast convergence of the low series is depicted by the greater values compared to the high series. We make an out of sample forecast comparison for FVECM forecasts with the VECM, the naïve model, the MA5, and the MA22 models by using the last 500 data points. The FVECM forecasts show superior performance to the naïve, the MA5, and the MA22 based on the mean square errors (MSE) and the mean absolute errors (MAE). Performance of the VECM is almost equivalent to the FVECM. The range forecasts are obtained by the difference between the forecasted highs and the forecasted lows. Autoregressive fractionally integrated and Heterogeneous autoregressive models are used to model the long-range dependence in daily ranges. The forecast comparison of range series shows the smaller errors in the range forecasts based on FVECM. Overall, our results show the convergence in highs and lows. Therefore, empirical results suggest that the idea to model the long-run and the short-run dynamics the same time may provide useful information. Nonlinear and persistent trends do not exist only in the stock prices but exchange rates also possess such trends. Many studies have found the long memory in spot exchange rates, real effective exchange rates, and nominal exchange rates: (Barros et al., 2011, Cheung, 1993, Marques and Pesavento, 2015, Okimoto and Shimotsu, 2010). Some studies (Holmes, 2002, Taylor, 2006) analyzed the long memory in exchange rates to test the theory of purchasing power parity (PPP) while others (Caporale and Gil-Alana, 2004, De Truchis and Keddad, 2013) find cointegration between the currencies. We extend this literature on the persistence in exchange rates across 30 currencies against the USD in chapter 3. Our analysis is based on the spot exchange rates while we consider a different number of observations. Therefore, it depends on the availability of data in the developed, the developing and the emerging markets. We use the range-based volatilities in contrary to the absolute or squared returns. It has been shown more consistent and good proxy which consider the two extreme values in estimation rather the closing values only (Chou and Liu, 2010). The estimation method is based on the Fourier frequencies and periodogram by using the first m frequencies in the vicinity of origin along with different bandwidth parameters. The graphical analysis of the long memory across a range of bandwidths confirms the declining trends in most of exchange rate volatilities with an increase in the bandwidth. We test the hypothesis of true long memory versus spurious long memory with tests of Shimotsu (2006) and Qu (2011). Our results reject the null hypothesis in 20 out of 30 cases. Presence of the structural breaks or low frequency contaminations results in the inconsistent local Whittle (LW) estimates. Therefore, we estimate the long memory with the modified local Whittle estimator (MLW) of Hou and Perron (2014). It is a robust method of long memory estimation with the consideration of level shifts, low frequency contaminations, and structural breaks. The presence of spurious long memory is also verified with decreased memory estimates in most of the cases while using the MLW estimator.

With the existence of spurious long memory, we use a model with multiple break points in the mean by Bai and Perron (1998b) with an unknown number of breaks. The breakpoints are estimated with OLS techniques in this method. We find a different number of breaks across the volatilities with a minimum number of 2 breaks and a maximum of 7. Moreover, most of the breakpoints are related to some specific periods of crises such as Asian crisis 1998, Brazilian crisis 1999, Global financial crisis 2007-2008, Chinese stock market slowdown, some weather shocks in different regions, oil prices shocks, and different quantitative easing policies.

The analysis of developed, developing and emerging markets suggests the presence of long range dependence in the emerging markets and absence in the developed markets (Auer, 2016, Hull and McGroarty, 2014, Rizvi et al., 2014). Less liquidity, more volatility, and poor governance may be the reasons for this inefficiency in the emerging markets. With this perspective about emerging markets, we analyze the long memory in an emerging market of Pakistan in chapter 4. We use the data of KSE100 index as well as 10 sectoral volatilities in our empirical analysis. Our results suggest the absence of long memory in returns while the existence of persistent or antipersistent trends in volatilities with semiparametric methods including the LW, the exact local Whittle (ELW) and the two-step exact local Whittle (2sELW). Being a volatile market due to economic, political, and international shocks, we test for the hypothesis of true long-memory process. Our results imply the presence of true memory in Pakistan stock exchange (PSX) except four sectoral returns during the sample period. The long memory phenomenon is not constant and varies over time due to the speculative bubbles, financial markets breakdowns and disasters as observed in many studies (Al-Shboul and Alsharari, 2018, Lim et al., 2008). We provide a time-varying analysis by using the nonparametric rescaled range method and semiparametric methods to understand the dynamics of evolving efficiency or inefficiency. Our results of time-varying analysis illustrate the fluctuating behavior in PSX with upward or downward trends of rolling estimates.

Traders and investors always look for some diversification opportunities for the optimization of returns with minimization of risks. Developing interests in the Islamic financial markets are also related to these purposes. Some researchers believe that the Islamic indices behave differently during crisis due to some specific properties such as profit-loss sharing and interest-free trades. Therefore, in chapter 5, we examine the long-run comovements in the Islamic and conventional volatilities across nine Islamic countries including Bahrain, UAE, Oman, Qatar, Malaysia, Indonesia, Egypt, Turkey, and Pakistan for the after global financial crises (GFC) period. The existence of good diversification alternatives depends on the absence of any comovements between these two classes of indices. Equivalently, there exist no diversification advantages with similar trends while return maximization opportunities are available with opposite trends. We get the stationary memory estimates with the ELW method for all indices. Consequently, the presence of fractional integration specifies to study the fractional comovements between these two classes. We investigate the fractional cointegration in the pairs of Islamic and conventional indices across the sampled countries. Furthermore, we test the homogeneity of memory estimates in each pair with a test statistic by Robinson and Yajima (2002) and get no rejections. Moreover, the empirical results for possibility of fractional cointegration with Chen and Hurvich (2006) method exhibit the presence of fractional cointegration in 7 cases out of nine. Additionally, we estimate the cointegration strength accompanying a rolling window technique to find the dynamics of cointegration strength during the phases of crises and recovery in each cointegrated pair. On the basis of our results, we conclude that the level of cointegration strength is high during the periods of crises while it is low during the recovery phases. These results reject our two hypotheses concerning Islamic stocks including the different behavior of Islamic indices during a crisis period as well as the absence of any long-run relationship of Islamic indices with conventional counterparts. So, overall results suggest fractional cointegration between the Islamic and the conventional indices. Therefore, the Islamic indices provide no diversification opportunities in long-run. However, the Islamic indices may provide the diversification chances in short-run for the cointegrated series. The empirical results recommend different trends between these two types of indices regarding Bahrain and Egypt. In this case, the trends in one market cannot be predicted on the basis of other which provide the return optimization chances in these two markets.

Chapter 2

<u>Modeling fractional cointegration between high and low stock prices in Asian countries</u>

Modeling fractional cointegration between high and low stock prices in Asian countries

Co-authored with Philipp Sibbertsen

(The following chapter is under review in *"Empirical Economics")*

2.1 Introduction

The Efficient market hypothesis (EMH) has been a controversial and premeditated area in the literature to comprehend the market excellence. The EMH, proposed by Malkiel and Fama (1970), states that stock returns are random and cannot be predicted on the basis of historical values. In an efficient capital market, present security prices reflect all history and past trends because of a rapid adjustment to incoming information. According to the theory of the EMH, all investors are being effected concurrently by any update or new information in the market. There is no possibility of getting unusual returns because all stocks are retailed at unbiased prices in efficient markets (Al-Shboul and Anwar, 2016). Market efficiency is considered as a critical tool for the growth, investment and efficient resource allocation in an economy (Rizvi et al., 2014). Contrary to the EMH, there may exist some patterns which can be helpful in predicting future stock prices. Modeling, understanding the dynamics of stock returns and predicting future prices based on past values are of interest for policymakers, investors and risk managers. Policies based on the historical price data may provide better forecasts of future stocks behavior and investors may gain higher profits (Caporin et al., 2013).

Long memory or persistent trends in stock prices create a challenge regarding market efficiency. Long memory refers to a situation where present observations are highly correlated to past values. A series has long memory if the autocorrelation function presents a hyperbolic decay. For developed markets, (Breidt et al., 1998, Ding et al., 1993, Garvey and Gallagher, 2012, Kellard et al., 2010) investigated long memory in volatilities of asset markets, exchange rates, inflation rates and interest rates. It is customary to model stock prices as being integrated of order one, $I(1)$ or stationary $I(0)$ stock returns in developed markets, but this may be a limiting approach for developing countries. Fractional integration or long memory techniques can analyze the returns in a wider sense by considering a range of integration orders between zero and one.

It has been conventional to analyze the daily closing or open stock prices although the prices of daily high and low stocks are also available in general. The range (difference between high and low prices) based volatility estimators have been shown to be more efficient than return based estimators in different studies including (Beckers, 1983, Garman and Klass, 1980, Kunitomo, 1992, Parkinson, 1980, Rogers and Satchell, 1991, Yang and Zhang, 2000). The range can provide a good proxy for the volatility but not for the extremes (high and low), so studying these measures simultaneously may be of interest. The long-run relationship and short-run dynamics in high and low prices have been studied in stock prices (Barunik and Dvorakova, 2015, Caporin et al., 2013, Cheung et al., 2010, Cheung et al., 2009), in exchange rates by He and Wan (2009) and in daily oil prices by He et al. (2010).

Long range dependence in stock prices has got attention in both theoretical and empirical terms for developed markets and there is a need to explore such persistence in emerging markets. Some studies in the literature have already analyzed such markets in the context of the EMH. Tan et al. (2011) observed that long memory in developing stock markets of Malaysia, Indonesia, Philippines, and Thailand may not be ruled out, and these markets including India have been reported to be more significant than Latin American markets in the context of long memory (Cajueiro and Tabak, 2004a). Hiremath and Kumari (2015) analyzed a strong evidence of long memory in mean returns of Indian medium and small sized indices. Turkyilmaz and Balibey (2014) found predictable patterns in stock volatilities of the Karachi stock exchange (KSE).

The present study contributes to the literature on Asian stock prices in two ways. First, we use the high, low and range series instead of closing prices. Secondly, by providing a more flexible framework of fractional integration and cointegration rather than a typical $I(0)/I(1)$ vector error correction mechanism. We are using the fractional vector error correction mechanism proposed by (Johansen, 2008) and (Johansen and Nielsen, 2012) to analyze the equilibrium and short-run dynamics. High and low log stock prices of Korea, Indonesia, Malaysia, Sri Lanka, India, and Pakistan are being studied. We examine the stationarity of the highs, lows and ranges by means of the Augmented Dickey Fuller (ADF) and KPSS tests. This study proceeds with the application of semiparametric techniques for long memory estimation in univariate fractionally integrated time series. Bivariate relationships between highs and lows are studied in the fractional cointegration framework. FVECM is the most suitable mechanism to analyze the equilibrium and short-run dynamics between highs and lows simultaneously (Caporin et al., 2013). This work provides forecasting performance of the FVECM over the naïve model, the VECM and the moving average models for highs (lows). Moreover, we are using HAR and ARFIMA models to describe the long-range dependence in range series. A forecasting performance analysis is performed among forecasted range values based on FVECM over the competing models.

This paper precedes by providing some reasoning behind the use of high and low prices instead of closing prices in section 2. Section 3 comprises the methodology for empirical analysis. The data and preliminary analysis are described in Section 4. Section 5 provides the empirical results and discussion. Forecasting performance is evaluated in section 6 and the article ends with concluding remarks in section 7.

2.2 Theoretical framework for high and low prices

The High and low prices refer to utmost and bottommost prices over a fixed sampling interval respectively and these values seem not to float separately from each other over time. Any underlying trend in stock prices affects the highs and lows in the same manner (Cheung, 2007). The analysis of high and low prices may be more helpful by describing the intraday variability in a fixed time span such as one day, week or month. Moreover, other estimators like the range can also be obtained by using highs and lows. It has been common to use absolute returns or squared returns as a volatility proxy, but Parkinson (1980) was the first one to show that a range based estimator is more efficient than the close to close variance estimator. Range, the difference between high and low prices, has been reported as the 5-14 times more efficient estimator of volatility in the literature (Garman and Klass, 1980, Yang and Zhang, 2000). Barrier options (knock-in and knock-out) depend on high and low prices as well as on the underlying volatility. High and low prices correspond to the shifting trends in excess demand and they provide information about the turning points in the series (Cheung, 2007). Sell and buy orders can be placed by using the highs and lows as reference values (Cheung and Chinn, 2001, Xiong et al., 2015). Unexpected shocks and public proclamations affect the highs and lows. Moreover, forecasts for the development of the economy can be based on these reference values (Barunik and Dvorakova, 2015). The high and low prices provide valuable information about price discovery and liquidity provisioning while working as a stop-loss indicator. Highs and lows may be affected by disturbing factors such as transaction costs or market frictions (e.g., price discreteness, stale prices, and tick size) and correspond to ask (bid) quotes (Caporin et al., 2013).

The range series works as bandwidth for the oscillations in extreme values and it varies with the variations in highs and lows. It has proven to be a more efficient estimator for the volatility but highs and lows affect modeling the ranges, so it is necessary to study the ranges using lows and highs. Studying all three series simultaneously can describe the long-run relationship and short-term deviations in extreme bands. Cheung (2007) was the first one to introduce the concept of the long-run relationship between highs and lows and he suggested to model these series like a vector error correction model. This idea proceeded with the presence of common underlying data generating trends in highs and lows. Moreover, these values may move apart from each other but their differences converge over time. Empirical analysis of daily high and low stock prices as a cointegrating relationship was done by Cheung et al. (2009) using the VECM. Similar results with the same methodology were found by He and Wan (2009) for the high and low exchange rates of USD against GBP and JPY

Many studies in the literature investigated the long-term relationships among variables while considering the $I(0)$ or $I(1)$ series but cointegration is not restricted to series integrated of order one or zero. It provides a more flexible environment as two processes X_t and Y_t are fractionally cointegrated if they are $I(d)$ with noninteger d and their linear combination is $I(b)$, where $0 < b < d$. Caporin et al. (2013) considered daily highs and lows as nonstationary series, but range as a stationary long memory process. He modeled highs and lows by using fractional cointegration techniques and applied a fractional vector error correction model (FVECM) to 30 components of the Dow Jones Industrial Average (DJIA). The error correction term in this framework was the linear combination of highs and lows. Moreover, this methodology showed better forecast performance than the MA5, the MA22, and the random walk models. Later, Barunik and Dvorakova (2015) analyzed the four major stock indices, S&P500, NIKKEI, FTSE and DAX with a comparison of the Czech Republic stock index by using daily highs and lows data. They got mixed results for pre-crises, post-crises and overall series from 2003-2012 with an application of restricted and unrestricted FVECM.

2.3 Methodology

This study is based on the theory of fractional integration, long-range dependence or long memory. A time series X_t, with spectral density $f(\lambda)$ and memory parameter d, follows a long memory process in the frequency domain if

$$f(\lambda) \sim \lambda^{-2d}, \lambda \to 0. \tag{2.1}$$

Due to Granger and Joyeux (1980) X_t is called an $I(d)$ process if

$$(1-L)^d X_t = \varepsilon_t \;,\; \varepsilon_t \sim IID(0, \sigma^2). \tag{2.2}$$

The process in equation (2) is short memory with stationary covariance and exponentially decreasing correlations for $d = 0$. X_t is a long memory process with high dependence in successive observations and stationary covariance for $0 < d < 0.5$. It is mean reverting but nonstationary for $0.5 < d < 1$ and represents a unit root process for $d = 1$ (Zhao and He, 2007).

The literature provides us with different methods to estimate the fractional integration parameter d. These methods include parametric, semiparametric and nonparametric estimators. We use semiparametric methods proposed by Robinson (1995), Shimotsu and Phillips (2005) and Geweke and Porter - Hudak (1983) known as Local Whittle (LW), Exact Local Whittle (ELW) and GPH estimator respectively for the estimation of fractional integration in highs, lows and ranges. These methods do not require any short memory estimates and do not depend on the model specification. The LW estimator is obtained by minimizing the Local Whittle objective function in the frequency domain with bandwidth parameter m and total observations N

$$\hat{d} = argmin_d \left(\log \overline{G(d)} - 2d\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j\right), \tag{2.3}$$

where $\overline{G(d)} = \frac{1}{m}\sum_{j=1}^{m} I(\lambda_j)\,\lambda_j^{2d}$, $\lambda_j = \frac{2\pi j}{N}$, $\frac{1}{m} + \frac{m}{N} \to 0$ and $I(\lambda_j) = \frac{1}{2\pi N}\left|\sum_{n=1}^{N} x_n\, e^{i\,\lambda_j\, n}\right|^2$.

The LW estimator was extended to the ELW to provide the estimation of d in the stationary and nonstationary region by Shimotsu and Phillips (2005). The ELW is obtained by minimizing the following objective function

$$Q_m(G,d) = \frac{1}{m}\sum_{j=1}^{m}\left[\log\left(G\lambda_j^{-2d}\right) + \frac{1}{G}I_{(1-L)^d x}(\lambda_j)\right]. \tag{2.4}$$

The estimator is given as

$$\hat{d} = argmin_{d\epsilon(\Delta_1,\,\Delta_2)}R(d), \tag{2.5}$$

where Δ_1 and Δ_2 are lower and upper bounds respectively. Furthermore

$R(d) = \log\hat{G}(d) - 2d\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j$ and $\hat{G}(d) = \frac{1}{m}\sum_{j=1}^{m}\lambda_j^{2d}I_x(\lambda_j)$.

This estimator is applicable whether cointegration is present or absent. The ELW provides a more flexible environment for the estimation of fractional integration in stationary and unit root series. The GPH estimator captures the fractional structure in the lower frequencies and is estimated as slope of the OLS regression of $\ln[I(\lambda_j)]$ on $\ln(\lambda_j)$

$$\ln[I(\lambda_j)] = c - d\ln[4sin^2(\lambda_j/2)] + \varepsilon_j, \tag{2.6}$$

where $I(\lambda_j)$ is the periodogram evaluated at $\lambda_j = \frac{2\pi_j}{N}, (j = 0,1,\ldots,m < N)$, c being the intercept, N are the total observations, and ϵ_j stands for the error term of regression. The condition $\frac{m}{N} \to 0$ as $N \to \infty, m \to \infty$ should be fulfilled while selecting the bandwidth parameter m.

Moving from a univariate framework to a multivariate framework common long-run relationships are often expressed as a common trend in the series. This common trend can be modeled as an equilibrium in a cointegration framework. The equilibrium errors are restricted as $I(0)$ in standard cointegration, but there may exist fractional integration with very persistent deviations from equilibrium. A linear combination $Z_t = Y_t - aX_t$, of two $I(d)$ processes X_t and Y_t, is defined as cointegrated if Z_t is integrated of order $I(d-b)$ and $a > 0,\ 0 < b < d$ (Engle and Granger, 1987). The whole concept of fractional cointegration determines the idea of a common trend in series with an equal order of integration. The departures from the trend are measured by $d - b$, where b is known as integration gap.

The concept of cointegration focuses on integer values of d and b in many previous research works but actually, there is no restriction in the original definition. Fractional cointegration exists for two fractionally integrated series with an equal order of integration if the residual of a linear combination has a memory parameter less than the memory parameter of the series. There are two conditions for two series to be cointegrated

1-Two series have the same order of integration, that is $I(d), d > 0$

2- There exist a linear combination of two series having memory parameter less than d.

Fractional cointegration presents a slow adjustment to the equilibrium and allows errors to possess long memory. The series may diverge from the equilibrium for long terms but ultimately converges back to the equilibrium. We are testing sufficient conditions for equal integration orders in highs and lows by using the semiparametric method of Nielsen and Shimotsu (2007). They provide two different approaches to test the equality of integration orders. One approach uses pairwise comparison and the other is based on an overall comparison. Both of these equations are equal in our bivariate case. The convergence of $\hat{T}_0$ to 0 in presence of cointergation and to $\varkappa^2$ in absence of cointegration is proved by the authors.

$$\hat{T}_0 = m(S\hat{d})' \left(S \frac{1}{4} \widehat{D}^{-1}\left(\hat{G} \circ \hat{G}\right)\widehat{D}^{-1} S' + h(n)^2\right)^{-1} (S\hat{d}), \qquad (2.7)$$

with $\hat{G} = \frac{1}{m}\sum_{j=1}^{m} Re\left\{\widehat{\Lambda}(\lambda_j)^{-1} I_j \widehat{\Lambda}(\lambda_j)^{-1}\right\}$, $S = [1, -1]'$, $h(n) = \log(n)^{-k}$ for $k > 0$, $\hat{d} = (\hat{d}_1, \ldots \hat{d}_p)'$ and $\widehat{D} = diag\{\hat{G}_{11}, \ldots, \hat{G}_{pp}\}$.

As our set-up is bivariate, there exists only one long run relationship between highs and lows. As the next step, we estimate the number of existing cointegrating vectors. Nielsen and Shimotsu (2007) provide a consistent estimate of cointegration rank $\hat{r}$.

$$\hat{r} = \arg \min_{u=1,\dots,p-1} L(u), \tag{2.8}$$

where $L(u) = v(T)(p-u) - \sum_{i=1}^{p-u} \lambda_i$, $v(T) > 0$, with $\lambda_1 > \lambda_2 > \cdots > \lambda_p$ being the eigenvalues of $\hat{G}(d_0) = \frac{1}{m_L} \sum_{j=1}^{m} RE\left[I(\lambda_j)\right]$ and $RE\left[I(\lambda_j)\right]$ stands for the real part of the periodogram $I(\lambda_j)$. Here $\hat{G}(d_0)$ is the spectral density of the cointegrated variables with new bandwidth parameter m_L, such that $m_L/m \to 0$. Moreover, this method does not require the estimation of the cointegrating vector and only depends on the spectral density near the origin with some bandwidth and threshold parameters.

There always exist an error correction mechanism in the presence of cointegration where the first difference of the variables is presented as a function of the cointegrating relation (Engle and Granger, 1987). Fractionally cointegrated vector autoregressive models (FCVAR) or fractionally cointegrated vector error correction models (FVECM) were introduced by Granger (1986) and formalized by Johansen (2008) as an extension of Johansen (1995) vector error correction models (VECM). The FVECM is useful in defining the short-run dynamic and equilibrium relationship simultaneously in the case of fractionally integrated series. These models allow common trends of fractional order d in components of X_t and cofractional of order $d-b$. Johansen and Nielsen (2010) and (Johansen and Nielsen, 2012) have further established the asymptotic theory for inference in these models. Vector error correction models (VAR) are an interesting way to present the long-run relationship and short-term movements from the equilibrium between two variables (Caporin et al., 2013). The cointegrated vector autoregressive (CVAR) model of dimension p, integrated of order one is defined as

$$\Delta Y_t = \alpha \acute{\beta} L Y_t + \sum_{i=1}^{k} \Gamma_i \, \Delta L^i Y_t + \epsilon_t, \tag{2.9}$$

where ϵ_t are the identically and independently distributed error terms having mean zero and covariance term Ω. The operators Δ and L stand for the difference and lag. The operators α, β and Γ are matrices of adjustment factors, cointegrating vectors and the short-run dynamics respectively. Johansen and Nielsen (2012) and Nielsen and Morin (2014) have generalized the CVAR model for fractional cointegration by replacing the lag and difference operators with fractional lag and difference operators as following

$$\Delta^b Y_t = \alpha \acute{\beta} L_b Y_t + \sum_{i=1}^{k} \Gamma_i \, \Delta^b L_b^i Y_t + \epsilon_t, \tag{2.10}$$

which is applied to $Y_t = \Delta^{d-b} X_t$ for the FCVAR model

$$\Delta^d X_t = \alpha\acute{\beta} L_b \Delta^{d-b} X_t + \sum_{i=1}^{k} \Gamma_i \, \Delta^d L_b^i X_t + \epsilon_t. \qquad (2.11)$$

The parameters of the FCVAR model can be interpreted parallel to the CVAR model as α and β are $p \times r$ matrices where $0 \leq r \leq p$ is the number of cointegrating vectors and p is 2 in our case. Moreover, β is a matrix whose columns represent the cointegrating vectors and equilibrium relations are formed by $\beta' X_t$. The matrix α consists of adjustment or loading factors presenting the speed of adjustment for each variable in the system. The short-run dynamics are contained in Γ_i. There are two additional parameters in the FCVAR model, d stands for the integration order of X_t and b as the cointegration gap which defines the reduction of the fractional integration order in comparison of X_t. The CVAR model is nested in the FCVAR model with the restriction $d = b = 1$.

2.4 Data and preliminary analysis

We are studying the daily highs and lows of six Asian countries namely Korea, Indonesia, Malaysia, Sri Lanka, India and Pakistan in our empirical analysis. Specifically, the Indian S&P BSE SENSEX (BSENS), the Korean KOSPI Composite Index (KS11), the Indonesian Jakarta Composite Index (JKSE), the Malaysian FTSE Bursa Malaysia KLCI (KLSE), the Sri Lankan COLOMBO IND ALL SHS (CSE) and the Pakistani KARACHI stock index (KSE100) are examined in this study. We want to formally test the hypothesis that daily highs and lows may diverge in the short-run but in the long-run, they are cointegrated. Data is available on yahoo finance and the considered time span is from January 2007 to December 2016 with a total of 2500 observations. We have omitted missing values in one series for all other series resulting in an equal length of all data series. The present analysis proceeds by taking the logarithm of daily high and low prices. We use log of daily highs and lows denoted by $P_t^H = \log(H_t)$, $P_t^L = \log(L_t)$ in the empirical analysis where P_t^H and P_t^L are the daily highs and lows respectively. The range is calculated as the difference between daily log highs and log lows that is $R_t = P_t^H - P_t^L$. X_t consists of daily highs and lows as $X_t = (P_t^H, P_t^L)$ in the whole empirical analysis. Graphs of daily log high, low prices in Figure 2.1 show a common trend in the highs and lows series. Moreover, we observe the lowest values at the same point in all the series which depicts the financial crises at the beginning of 2007 and was on its peak at the end of 2007. After the crises, there is an increasing trend in all cases and all series reaches to their previous levels with some ups and downs at some points. The overall graphical analysis gives the intuition that all highs and lows are nonstationary.

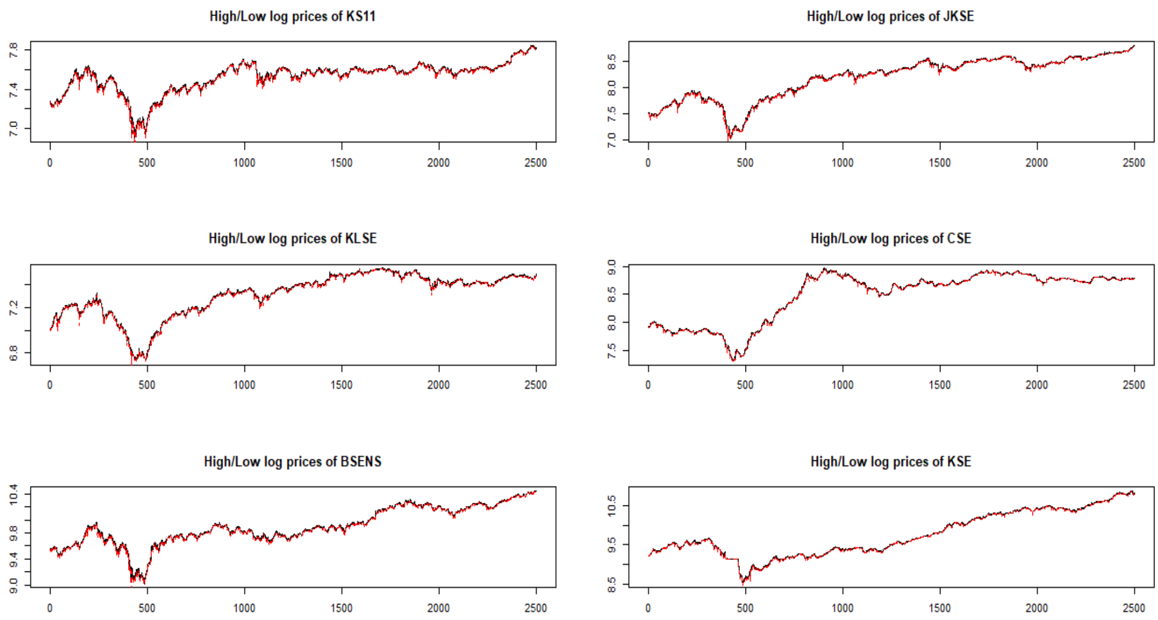


Figure 2.1: Graphs of high/low log prices

	Level	**Δ**	**Level**	**Δ**	**Level**(R_t)
KS11	0.292463	0.01*	0.254118	0.01*	0.01*
	(0.01*)	(0.1**)	(0.01*)	(0.1**)	(0.01*)
JKSE	0.46149	0.01*	0.434495	0.01*	0.01*
	(0.01*)	(0.1**)	(0.01*)	(0.1**)	(0.01*)
KLSE	0.64507	0.01*	0.624038	0.01*	0.01*
	(0.01*)	(0.1**)	(0.01*)	(0.1**)	(0.01*)
CSE	0.876643	0.01*	0.888439	0.01*	0.01*
	(0.01*)	(0.1**)	(0.01*)	(0.1**)	(0.01*)
BSESN	0.176179	0.01*	0.163132	0.01*	0.01*
	(0.01*)	(0.1**)	(0.01*)	(0.1**)	(0.01*)
KSE	0.643596	0.01*	0.652685	0.01*	0.01*
	(0.01*)	(0.1**)	(0.01*)	(0.01**)	(0.01*)

Table 2.1: Critical values of ADF and KPSS (p-values are in parenthesis)

As a first step of the empirical analysis, we apply the traditional tests that only consider stationary and nonstationary series. These tests are helpful in understanding whether a series is affected by one-time shocks permanently or temporarily. Empirical evidence of unit root series is provided by using the ADF of (Dickey and Fuller, 1979). It tests the null hypothesis of unit roots against the alternative of stationarity. The results of the test correspond to the expectation that highs and lows are unit root process at levels and stationary in the first differences. Moreover, the range is not a unit root process at levels in all cases. But the ADF test is not applicable in the case of fractional integration as it is designed to test for a unit root against stationarity.

To confirm our ADF test results, we apply the KPSS test of Kwiatkowski et al. (1992). Here, we test the hypothesis of a stationary series around a possible trend against the alternative of a unit root. Results of both tests are similar and presented in Table 2.1.

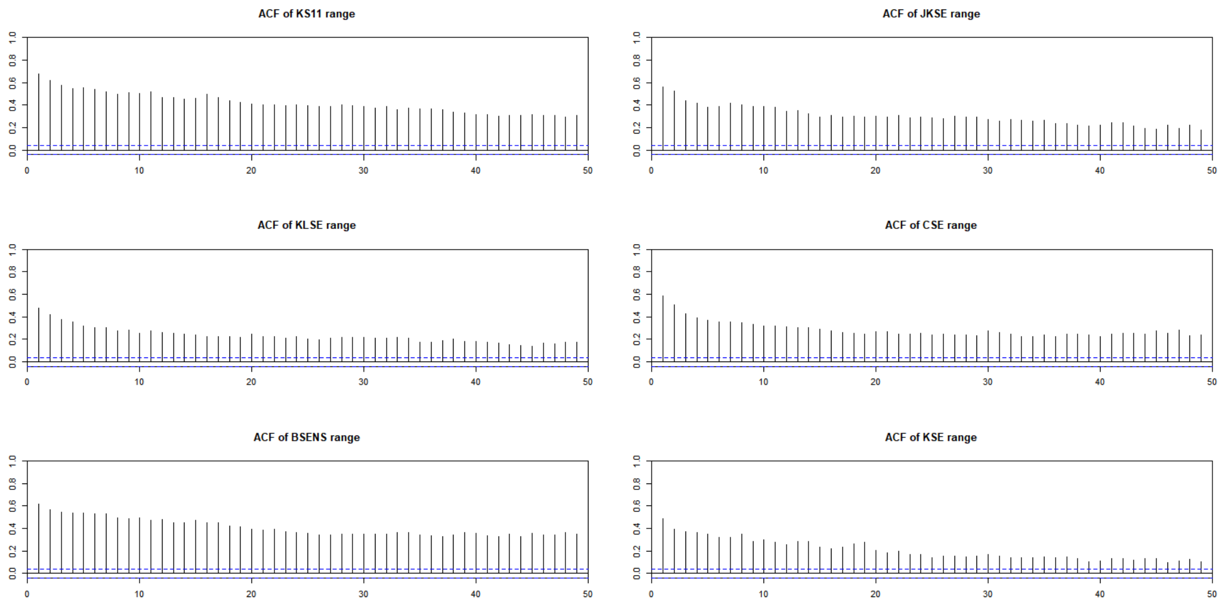


Figure 2.2: Autocorrelations of the range series

Although both tests indicate that the range series are stationary for all stocks, the autocorrelations in Figure 2.2 decay very slowly and do not fit within the $I(0)$ framework. This exponential trend of the range series reflects the long memory or fractional integration of the data. The idea of a long-run relationship between high and low prices have already been modeled in form of a VECM by Cheung (2007). Afterwards, this idea was extended in form of a FVECM by observing a persistent behavior in the range series by Caporin et al. (2013). The graphs of ranges in all cases approve it as volatility estimator by presenting higher range values corresponding to high price variability. We move forward with the idea that the highs and lows in Asian markets share common trends and are thus cointegrated. Moreover, the range as a linear combination of highs and lows is stationary and has no unit roots.

2.5 Empirical results

The idea that daily highs and lows are nonstationary and may have a long-run relationship was suggested by Cheung (2007) who used a VECM to model these prices. This idea was extended to fractional integration by Caporin et al. (2013) for the analysis of the Dow Jones Industrial Average (DJIA) components for the persistence in the range series. Barunik and Dvorakova

(2015) also made use of fractional integration in analyzing the pre-crisis and post-crisis data of five stocks.

With the impression of finding long memory in the range, we proceed with estimating the order of fractional integration in highs, lows and range prices by using semiparametric methods. The LW, the ELW and the GPH estimator are applied to estimate the long memory parameter. The ELW is more general than the LW as it allows for stationary and nonstationary series.

Results of the long memory estimates are provided in Table 2.2. Our estimators are based on two bandwidth parameters $T^{0.5}$ and $T^{0.6}$ similar to Nielsen and Shimotsu (2007) and Barunik and Dvorakova (2015) . The selection of the bandwidth parameter m must be made carefully as inclusion of too few or too many ordinates may produce inaccurate long memory estimates. Regarding to the ELW estimator, we find memory parameters of highs and lows greater than one for both bandwidths except for Korea. We find nonstationary but less than one long memory estimates for P_t^H and L_t^H in all cases with the LW estimator. The GPH estimates are greater than one in the case of Sri Lanka and India. In short, all long memory estimates of highs and lows display long memory and are nonstationary.

		$m = T^{0.5}$			$m = T^{0.6}$		
		P_t^H	P_t^L	R_t	P_t^H	P_t^L	R_t
KS11	**ELW**	0.953592	0.938368	0.541033	0.997425	0.977261	0.57012
	LW	0.922622	0.918284	0.559296	0.98672	0.975618	0.578841
	GPH	0.899524	0.896102	0.624039	0.957781	0.952701	0.575626
JKSE	**ELW**	1.056928	1.061232	0.516087	1.077684	1.070482	0.466365
	LW	0.999955	0.999951	0.535343	0.999922	0.999958	0.472619
	GPH	0.971804	0.969776	0.589597	0.970974	0.962585	0.565594
KLSE	**ELW**	1.044362	1.050971	0.443915	0.996314	0.987519	0.391407
	LW	0.999944	0.999925	0.472498	0.966734	0.960398	0.407928
	GPH	0.997951	1.003244	0.497949	0.938827	0.944008	0.405
CSE	**ELW**	1.125225	1.129161	0.399415	1.113648	1.100404	0.393839
	LW	0.999942	0.999942	0.413086	0.999942	0.999942	0.402645
	GPH	1.027672	1.03074	0.36465	1.0545	1.046833	0.40336
BSESN	**ELW**	1.018676	1.002253	0.455063	1.060746	1.047526	0.623751
	LW	0.999948	0.999927	0.477818	0.999938	0.99994	0.636532
	GPH	1.046711	1.043665	0.470035	0.984128	0.980357	0.66833
KSE	**ELW**	1.00982	1.014954	0.443937	1.062746	1.068951	0.548284
	LW	0.987335	0.992913	0.449937	0.999953	0.99993	0.535261
	GPH	0.946144	0.952309	0.476928	0.995409	1.008221	0.53788

Table 2.2: Long memory estimation in high, low and range prices

Regarding the range series, we observe mixed results of stationary and nonstationary estimates. The range always falls in the stationary region for $T^{0.5}$ except Korea and Indonesia. We observe nonstationary estimates of the range in the case of Korea, India and, Pakistan for $T^{0.6}$.

Overall, the results of all three estimators are in favor of our two primary hypotheses, all series are integrated of an order near one and the range has a persistent decay in autocorrelations.

Our next step is to test whether the estimates of the order of fractional integration are equal for highs and lows in order to find a long-run relationship between them. According to the definition of cointegration, it is compulsory for two series to have an equal order of integration. Test results are obtained by application of the test statistic of Nielsen and Shimotsu (2007). The test statistic converges in probability to 0 for cointegrated series while it converges to a $\varkappa^2$ distribution with no cointegration under the null hypothesis. The Critical value for the $\varkappa^2$ distribution at the 90% significance level is 2.71 with one degree of freedom which is greater than the maximum test statistic value of 0.007. It indicates that highs and lows have an equal order of integration in all cases. The estimation of the cointegration rank is performed with the method of Nielsen and Shimotsu (2007) and our results suggest a common stochastic trend in highs and lows. A suitable linear combination of highs and lows may remove this trend (He and Wan, 2009). We can observe that $L(1) < L(0)$ in all cases which provides strong evidence in favor of one long-run relationship. The results are reported for the bandwidth parameters $m = T^{0.6}$ and $m_L = T^{0.5}$ similar to Caporin et al. (2013) and Barunik and Dvorakova (2015). Results of the cointegration rank estimation and equality of cointegration ranks are reported in Table 2.3.

	$\hat{\mathbf{T}}_0$		$\mathbf{v(T) = m_L^{-0.45}}$			$\mathbf{v(T) = m_L^{-0.05}}$		
	$\mathbf{T^{0.5}}$	$\mathbf{T^{0.6}}$	$\mathbf{L(0)}$	$\mathbf{L(1)}$	$\hat{\mathbf{r}}$	$\mathbf{L(0)}$	$\mathbf{L(1)}$	$\hat{\mathbf{r}}$
KS11	0.0012	0.001	-1.5853	-1.7926	1	-0.3208	-1.1604	1
JKSE	0.0014	0.002	-1.7099	-1.8481	1	-0.3208	-1.1604	1
KLSE	0.007	0.001	-1.7099	-1.8494	1	-0.3208	-1.1604	1
CSE	0.003	0.005	-1.7099	-1.8549	1	-0.3208	-1.1604	1
BSENS	0.0011	0.001	-1.7099	-1.8549	1	-0.3208	-1.1604	1
KSE	0.007	0.002	-1.7099	-1.8549	1	-0.3208	-1.1604	1

Table 2.3: Rank estimation results

Finding equal fractional integration orders, we apply a FVECM to explain the long-run relation between these two series in form of an error correction term. Caporin et al. (2013) stated that the future influences of the adjustment process to settle down the deviations from the equilibrium are nicely interpreted by the FVECM. Table 2.4 reports the results of the unrestricted FVECM. Contrary to the traditional VAR model, only one lag for short deviations is included in the model. As MacKinnon and Nielsen (2014) described the inclusion of one lag is sufficient in this model to remove the serial autocorrelation in the residuals. The parameters $\hat{d}$ and $\hat{b}$ are different from zero and not equal. This also supports our hypothesis that the range is not an $I(0)$ process and this result is consistent with the estimates of the ELW, LW, and GPH estimator. Moreover, the cointegrating vectors look near to our expectation.

The order of integration for daily prices is greater than one in four cases and less than one in two cases. The range is stationary $(d - b < 0.5)$ for Malaysia, Indonesia, Sri Lanka, and Pakistan but nonstationary for India and Korea. Adjustment coefficients α_H and α_L for P_t^H and P_t^L respectively arise with opposite signs and different from zero. Negative values of α_H and positive values of α_L show their opposite movements to settle down the shocks in the system and powers to converge back to equilibrium. It is noticeable that the values of α_H are significantly smaller than the values of α_L in absolute terms which shows the faster convergence to the equilibrium for low stock prices (Barunik and Dvorakova, 2015, Caporin et al., 2013). The range shows a high degree of persistence with the lowest value of 0.418 and highest value of 0.715 in the unrestricted FVECM. These estimated values of the range are close to the values obtained by semiparametric methods. Negative estimates of short-run dynamics show a regressive behavior in most cases for the lagged highs. Moreover, we get spillover effects in form of positive estimates for lagged lows (Cheung, 2007).

	$\hat{d}$	$\hat{b}$	$\hat{\beta}$	α_H	α_L	γ_{11}	γ_{12}	γ_{21}	γ_{22}
KS11	1.012	0.456	(1,-1.003)	-0.158	1.823	-0.443	0.505	-0.552	0.941
	(0.01)	(.456)		(0.199)	(0.397)	(0.187)	(0.188)	(0.303)	(0.318)
KLSE	0.997	0.625	(1,-1.002)	-0.294	0.751	-0.207	0.471	0.007	0.391
	(0.01)	(0.04)		(0.101)	(0.155)	(0.088)	(0.095)	(0.117)	(0.13)
JKSE	0.994	0.588	(1,-1.001)	-0.12	1.252	-0.061	0.261	-0.03	0.378
	(0.01)	(0.051)		(0.118)	(0.239)	(0.107)	(0.108)	(0.152)	(0.168)
CSC	1.001	0.715	(1,-1.001)	-0.171	0.572	0.218	0.244	0.377	0.154
	(0.015)	(0.042)		(0.062)	(0.083)	(0.063)	(0.058)	(0.063)	(0.064)
BSENS	1.004	0.418	(1,-1.003)	-0.268	2.414	-0.701	0.915	-1.365	1.842
	(0.011)	(0.04)		(0.237)	(0.54)	(0.229)	(0.241)	(0.443)	(0.47)
KSE	1.011	0.532	(1,-1.002)	-0.494	1.264	0.33	0.08	0.162	0.453
	(0.016)	(0.045)		(0.118)	(0.225)	(0.098)	(0.092)	(0.139)	(0.162)

Table 2.4: Unrestricted FVECM (standard errors are in parenthesis)

Although the cointegrating vector is close to one, the range cannot be defined as a long-run relationship unless the cointegrating vector is exactly $(1, -1)$. So, in order to interpret the range as long-run relationship, we estimate the model with a restriction on the cointegrating vector. We have also applied a restriction of $d = 1$ to get stationary estimates for the range in the case of $b > 1$ (Caporin et al., 2013). The results of this model are presented in Table 2.5, which are very similar to the results of the unrestricted model. We conclude on the basis of the restricted FVECM results that daily highs and lows share some common trends. Although they may move away from each other for a short period of time as a result of some shocks, some spillover factors are there to bring them back to the equilibrium.

2.6 Out of sample forecasting performance

A model optimizing the in-sample fit does not necessarily provide good out of sample forecasts. We consider an expanding period, the last 500 observations which correspond to the last two years of the data, to investigate the out of sample forecasting performance of the FVECM. We estimate model for the first 2000 observations of the data and make one period ahead forecasts. Let $\widehat{H}_{t+h}$, $\widehat{L}_{t+h}$ denote the forecasted highs and lows for a horizon of $h = 1$. A dynamic recursive method is implemented to construct these forecasts. We estimate the FVECM model to produce one step out of sample forecasts and roll the estimation period to include the next observation.

There always arises the question whether the forecasts from the given model are more accurate compared to competing models. We consider a random walk (naïve) model, MA5, MA22 and a vector error correction model (VECM) as competing models following (Caporin et al., 2013). Here, MA5 and MA22 stand for weekly and monthly moving averages respectively. We estimate forecasts for highs and lows individually in the case of the naïve model, the MA5 and the MA22 model. The FVECM and the VECM provide these forecasts simultaneously. The accuracy of forecasts is evaluated by focusing on the mean squared error (MSE) and mean absolute error (MAE) by using the Diebold- Mariano test statistic. (Diebold and Mariano, 1995) introduced this statistic by using the difference between the squared forecast errors or absolute forecast errors and given as

$$DM = \left[\hat{\gamma}_0 + 2\sum_{j=1}^{h-1}\hat{\gamma}_j\right]^{-1/2}\bar{d}, \tag{2.12}$$

where $\hat{\gamma}_j = \frac{1}{n}\sum_{i=j+1}^{n}(d_i - \bar{d})(d_{i-j} - \bar{d})$, $d_i = e_i^2 - r_i^2 \quad or\ d_i = |e_i| - |r_i|$, e_i and r_i are the forecast errors for the two competing models.

Negative values of the test statistic provide an indication of smaller errors for the first model or alternatively, smaller mean square errors are obtained by better forecasts. These results of the forecasting performance are reported in Table 2.6, where negative values of the test statistic give an indication of forecasting superiority of the FVECM over the competing models. In the case of forecast comparison with the VECM, we observe negative values of the DM statistic in all cases except the P_t^H of Korea and India. Moreover, we have got significant evidence to reject the equality of forecast errors for P_t^H of Pakistan and P_t^L of Indonesia, Malaysia and Sri Lanka. In short, negative values show the preference of the FVECM forecasts over the VECM. Regarding the MA5 and MA22, we get all negative and highly significant results according to expectations. (Caporin et al., 2013) found similar results while comparing with the MA5 and

MA22 forecasts and stated that these averages are helpful in identification of the price trends but cannot identify point estimates. While comparing with the naïve model, we get all negative values, significant in some cases at the 5% level. The overall analysis of the forecasting performance concludes that the FVECM performs better than the naïve, MA5 and MA22 models for highs and lows.

		P_t^H				P_t^L			
		VECM	**Naïve**	**MA5**	**MA22**	**VECM**	**Naïve**	**MA5**	**MA22**
MSE	**KS11**	**0.916563**	-2.68649	-1.47511	-4.52027	**-1.04641**	**-8.42331**	-0.86645	-4.99967
	JKSE	**-1.99315**	**-14.7882**	-2.08565	-13.3255	**-2.82085**	-11.8828	-1.96015	-15.4864
	KLSE	**-4.48611**	1.537687	-2.62033	0.14679	**-7.14755**	**0.268202**	-2.68594	-0.80352
	CSE	**-13.8613**	-1.76647	-11.8707	-1.77071	-12.1663	-0.33087	-15.0632	-2.00631
	BSENS	**0.087945**	**-3.60402**	-0.13368	-4.19233	**-2.94699**	-3.75209	0.911372	-5.92213
	KSE	**-1.63105**	-12.2845	-3.10002	-12.5455	**-3.30423**	-15.738	-1.7919	-15.2497
MAE	**KS11**	**1.251042**	-4.96765	-3.2898	-5.16471	**-3.36701**	-10.675	-1.82451	-7.31197
	JKSE	**-2.83901**	-20.1887	-2.49776	-19.356	-2.64232	-21.0754	-2.34031	-22.8212
	KLSE	**-5.51565**	-0.05462	-3.77872	-2.02842	-9.4374	-0.33923	-4.90546	-0.90158
	CSE	**-20.4411**	-2.75903	-15.9864	-2.28837	-21.0666	-1.29431	-20.0614	-1.46068
	BSENS	**-1.02569**	-5.18936	-1.51167	-5.07838	**-4.86464**	-6.31892	-0.76851	-7.76065
	KSE	-1.4119	-16.4835	-3.02694	-19.4154	**-2.8673**	-21.3372	-1.47875	-22.0507

Table 2.5: Results of forecasts comparison with Diebold Mariano test

So far, we have considered different models such as the naïve, MA5 and MA22 by using the high and low prices as competing models to the FVECM. In the FVECM and the VECM, the range was considered as a long-run relationship between highs and lows, where the error correction term was providing an estimate of the long-run relationship. In this section, we develop the models for the range series as volatility proxy. Volatility in financial markets is characterized by clustering and a very persistent behavior. With the availability of high frequency data, realized variance and realized range estimators of volatility gained importance in analysis and forecasting. Many volatility models such as ARCH, GARCH have been developed.

We observed that the range series is a long memory process with the slowly decaying autocorrelations in all cases. All three estimators of fractional integration in the range give minimum values of 0.36465, 0.391407 and maximum of 0.624039, 0.636532 for the bandwidth parameter $T^{0.5}$ and $T^{0.6}$ respectively. Moreover, the range is nonstationary for Korea and Indonesia while it is stationary for all other stock prices by using $T^{0.5}$ as bandwidth parameter. With $T^{0.6}$ as bandwidth, the range estimators for Korea, India, Pakistan are greater than 0.5 and are in the stationary region in the case of the other three series. Overall, we observed that the range series shows a very persistent behavior as discussed in section 2 and it is a long memory process so that ARCH or GARCH models cannot be used to model this series.

GARCH models have been extended to fractionally integrated GARCH models (FIGARCH) to accommodate fractional integration by (Baillie, 1996). Univariate ARFIMA models were used to model long memory in volatilities by (Ebens, 1999).

Asymmetric interactions of volatilities over different time horizons were empirically analyzed for financial data. Volatility cascade phenomena, followed by a heterogeneous market hypothesis cannot be modeled by the well-known GARCH class of models. The basic idea behind this hypothesis is that participants with different time horizons cause different types of volatilities. Combination of different investment time horizons and volatility views of market contributors were presented in extended GARCH models by (Müller et al., 1997). The same idea was followed by considering the short-term (daily), medium-term (weekly) and long-term (monthly) volatilities in the linear model by (Corsi, 2009) for realized volatilities. We consider HAR models of daily ranges as the first alternative to the FVECM. The HAR specification for daily ranges is presented as follow

$$R_D = \alpha_0 + \alpha_1 R_{D-1} + \alpha_2 R_{D-1}^{(5)} + \alpha_3 R_{D-1}^{(22)} + \varepsilon_t. \qquad (2.13)$$

where R_D are the daily ranges and R_{D-1}, $R_{D-1}^{(5)}$, $R_{D-1}^{(22)}$ stand for daily, weekly and monthly volatilities respectively. We consider univariate ARFIMA models of daily ranges as the second alternative model. Both of these models focus only on the error correction term of the FVECM component which represents the long-run relationship there. Forecasting only the short-run dynamics rather than other components has been motivated in some other studies as well. (Chinn and Meese, 1995, Mark, 1995) obtained the forecasts from the error correction terms. (Cheung et al., 2009) used ARIMA models to forecast the daily ranges of exchange rates. (Chatzikonstanti and Venetis, 2015) modeled daily log ranges in components of the Dow Jones Industrial Average index by using ARFIMA models.

Here we present the forecast comparison of the FVECM range and alternative range models. The FVECM range is calculated as a difference between the forecasted highs and lows. The forecasts based on ARFIMA and HAR models do not provide useful information regarding highs and lows. These forecasts can be used to assess the potential loss/benefit in stripping short-term dynamics from forecasting the range (Cheung et al., 2009). Results of the DM test are reported in Table 2.7. All test statistics with negative values present the preference of the FVECM over competing models. The values are significant for Malaysia and Pakistan in comparison with the HAR forecasts and for Indonesia and Malaysia in comparison with the ARFIMA at the 5% level of significance.

		KS11	JKSE	KLSE	CSE	BSENS	KSE
MSE	**FVECM/HAR**	-0.73503	-1.70932	-3.25784	-0.28971	-0.92967	-2.88622
	FVECM/ARFIMA	-1.3153	-3.66455	-2.9061	-0.99608	-1.97843	-3.30016
MAE	**FVECM/HAR**	-2.53503	-4.40871	-6.1136	-1.069	-3.69332	-6.29465
	FVECM/ARFIMA	-4.0413	-8.74568	-5.5095	-4.45735	-6.4467	-5.58531

Table 2.6: Results of Diebold Mariano test for FVECM/HARR and FVECM/ARFIMA

2.7 Conclusion

This work analyzed the fractional integration and cointegration in six stock markets of Asia. Contrary to a common analysis of close to close prices, this study considers the daily high and low prices. Not only high and low prices but a linear combination the range, based on the difference between highs and lows, has been considered as a volatility proxy. The range has been considered as a more efficient estimator than absolute or squared volatility proxies. Moreover, it also defines the interaction between highs and lows. We considered KS11, JKSE, KLSE, CSE, BSENS, and KSE for our analysis. The data set consists of 2500 observation ranging from January 2007 to December 2016. Graphical analysis of the highs and lows shows an increasing trend and nonstationary series in all cases. Afterward, statistical tests also confirm the presence of unit roots at levels and all series are stationary in the first difference. Tests for unit root series confirm that the range is stationary in all cases, but graphical analysis of the autocorrelations show a very persistent behavior. This persistent behavior is not equivalent to $I(0)$ series and shows that ranges are fractionally integrated series. Different semiparametric estimates of ranges have shown that the range is stationary in three cases out of six.

The FVECM model was employed to take into account the short-run dynamics and long-run relationships simultaneously, where the error correction term presents the long-run relationship and, in our case, this is the range. Results of an unrestricted FVECM models also confirm the semiparametric estimates for ranges. We also estimated restricted FVECM models with a restriction on the cointegrating vector to interpret the range as a linear combination between highs and lows and got similar results to unrestricted models. Overall, our results are similar to Barunik and Dvorakova (2015), who analyzed the S&P500, NIKKEI 225, FTSE 100 and DAX, which come from the developed countries. Caporin et al. (2013) also reported similar results for the components of the Dow Jones Industrial Average (DJIA). The forecasting accuracy of forecasted highs and lows has been compared with alternative models such as the naïve, VECM, MA5 and MA22, and results show that the FVECM model performs better than competitive models. Range forecasts of the FVECM are compared with other range models such as HAR and ARFIMA, and again our results approve the superiority of FVECM over alternative range models.

We analyzed that the high and low stock prices in developing markets are cointegrated in the same way as in developed markets. Moreover, the range was observed to be a good volatility estimator in such small and developing markets. The results of this study contradict the common beliefs that developing markets are more random due to economic instability, lack of information, low trade volumes and having no well- integrated stock markets.

This work supports the hypothesis that an analysis based on highs, lows and ranges may provide better predictions compared to traditional works. More accurate and profitable trading strategies may be developed on the basis of these models. Risk managers and hedgers can get benefits from these forecasts. Moreover, highs and lows in developing markets are cointegrated as in developed markets. The FVECM, involving fractional integration and cointegration is a valuable alternative for estimating range-based volatilities. Other range-based estimators can be used to enhance the forecasting capability. These models can provide guidance not only in risk analysis but in derivative pricing and trading strategies.

Chapter 3

Long memory, spurious memory: Persistence in range-based volatility of exchange rates

Long memory, spurious memory: Persistence in range-based volatility of exchange rates

Co-authored with Philipp Sibbertsen

3.1 Introduction

Persistent level of exchange rates in a country is important to reflect the stability of its currency and it provides helpful information for policy implications in case of sudden shocks (Barros et al., 2011). Less developed countries are more prone to the phases of high exchange rate volatility owing to the depreciation (appreciation) of the currency caused by national or international shocks. Additionally, the exchange rate fluctuations have unlimited impacts on inflation predictability, international trade, and financial asset pricing. The exchange rate volatility was described as an important factor of inflation in Turkey and Mexico (Mendoza V, 2003). Being a risk measure, an increase in the exchange rate volatility reduces trade by raising the cost of the risk-averse investors and traders. Stock markets and foreign exchange rates are characterized by nonlinear and non-stationary behavior in the literature. Taylor (2006) described the transaction costs, the collaboration of various agents in currency markets and official interferences in these markets as three sources of nonlinear trends in the real exchange rates.

Volatility is persistent and predictable while the asset returns are almost random (Choi et al., 2010). High persistence in different volatility measures of a financial time series is a common phenomenon and may be analyzed by the significant correlations at long lags or by using the spectral density analysis. This characteristic is known as long memory and explains the long-lasting effects of shocks to volatility. Moreover, the persistence in exchange rates is important for policymakers to adopt the specific measures according to the persistence level in case of shocks (Barros et al., 2011). Long range dependence is imperative and appealing as it point towards the long-run effects of shocks. Along with the long-run effects, long memory is helpful in future predictions with an indication of some nonlinear dependence among the observations (Charfeddine, 2014). Analysis of the persistent trends has direct policy implications to help the investors in avoiding any losses and to get benefits by following these trends (Ouyang et al., 2002). A unit root series is persistent to the shocks whereas a stationary series is less persistent and mean reverting with temporary effects of the shocks. Therefore, strong policy actions are needed in this case to maintain the higher levels of the positive shocks. In case of negative shocks, some strong measures are needed to converge a series back to its original trend in a unit root series while a stationary series will return to its mean at some future point automatically (Gil-Alana et al., 2014).

Modeling and forecasting the volatility as a risk measurement has an interesting feature in economics and finance literature but it is not directly observable and needs to be measured (Molnár, 2012). Frequently used measures of conventional volatility are based on the squared or absolute returns using the daily closing prices. The idea of range-based volatility estimation by Parkinson (1980) was considered less noisy in comparison to the squared base estimators with 5 times reduced variance. Although it is customary practice to present the candlestick plots based on the range in business newspapers, the range-based volatility is not practiced in general (Li and Hong, 2011). The range-based proxy of volatility is more appropriate and efficient being an unbiased estimator of the standard deviation while using the two quantities (high and low) as compared to the returns based volatility based only on the closing values (Chou and Liu, 2010).

He and Wan (2009) analyzed the highs, the lows, and the ranges of USD with an application of the error correction model. Okimoto and Shimotsu (2010) rejected the null hypothesis of no decline in persistence for the monthly real exchange rates across a sample of 9 out of 17 countries at 10% level of significance. Lima and Tabak (2007) supported the random walk hypothesis in the emerging market exchange rates after adoption of the floating exchange rate regimes. Marques and Pesavento (2015) analyzed an increase in the persistence of real exchange rate for a number of countries after the liberalization period. Gil-Alana and Sauci (2018) observed mean reversion in the real exchange rates of some Latin American countries with an application of the parametric and semiparametric methods. Gil-Alana and Toro (2002) used the ARFIMA technique to model the real effective exchange rates in five industrialized countries.

A hyperbolic decay of the autocorrelation function or an unbounded spectral density may also be a result of short memory model with regime changes in the volatility. Theoretically, a true long memory process takes a long time to eliminate the effects of financial shocks but the autocorrelation function of a short memory process with level shifts should present an exponential decay after a few observations. Apparent persistent trends in a time series were criticized for the possibility of structural breaks or regime switches and this is known as spurious memory. It is important to distinguish between the true long memory and spurious memory. Misspecification followed by the ignored structural breaks can destroy the results by overestimating the fractional integration (Charfeddine, 2014).

Our study focuses on the possibility of long memory or structural breaks in the spot exchange rates. At first step, we study the long range dependence in different exchange rates including the developed, developing and emerging countries in a wider sense of long memory and fractional integration considering the fact that the exchange rate volatility is persistent and can be modeled as a long memory process. Secondly, our analysis is based on the log range volatility contrary to a conventional one approximated by closing values. Thirdly, we provide the empirical evidence of long-range dependence and spurious memory by using the different semiparametric techniques. We use the semiparametric Local Whittle (LW) method to estimate the long memory. To test the presence of true long memory or spurious memory, we use the approaches by Shimotsu (2006) and Qu (2011) with the hypothesis of true long memory in the exchange rate volatilities. Our empirical results show either stationary or nonstationary long memory estimates in all series but later on, the hypothesis of long memory is rejected in favor of the spurious memory. Moreover, graphical analysis of the long memory parameters with different frequencies presents decreasing trends, which might be due to spurious memory. Decreased memory estimates by using the Modified Local Whittle (MLW) estimator, a consistent estimator in case of the random level shifts and low-frequency contaminations, may be caused by the structural breaks.

Rest of the paper is structured as follows. Section 2 discusses the data and methodology of the study. The empirical results are discussed in third section while section 4 concludes the paper with some future recommendations and limitations of this study.

3.2 Data and Methodology

We use the daily log range proxy of volatility in our empirical analysis. Difference between the highest and the lowest value in a fixed sampling interval (1 day for daily data) is defined as the range. We formulate the log range volatility as in equation (3.1)

$$R_t = \ln(\ln(H_t) - \ln(L_t)). \qquad (3.1)$$

Where H_t, L_t, and R_t represent the highest, the lowest, and the range values. We use the data of daily highs and lows across 30 currencies against USD with a different number of observations depending on the availability of data on Eikon Thomson Reuter's database. The starting date is different for each country while the end date is 2/18/2019. A detailed description of the currencies with currency symbols, the starting date, and the number of observations is presented in Table 3.1.

Fractional integration in x_t stationary process with an autocorrelation function ρ_k at lag k, with a finite constant c and a real number H is defined as in equation (3.2).

$$\rho_k = ck^{2(H-1)}\ as\ k \to \infty, \tag{3.2}$$

where H is the Hurst exponent to represent long-range dependence and relate to the fractional integration parameter d as $d = H - 1/2$. The series is a long memory process for $H \in (\frac{1}{2}, 1)$, non-stationary for $H > 1$, and anti-persistent if $H \in \left(0, \frac{1}{2}\right)$. A long memory process has an unbounded spectral density at the low frequencies. An integrated process of order d with a lag

$$(1 - L)^d x_t = \mu_t. \tag{3.3}$$

operator L and μ_t stationary and zero mean process can be represented as

The model in equation (3.3) is a short memory model and belongs to the class of ARIMA models for $d\ \epsilon\ (0, 1)$. Another more general and flexible class of models, autoregressive fractionally integrated moving average (ARFIMA) models for $d\ \epsilon\ (-0.5,1)$ presented by (Granger and Joyeux, 1980, Hosking, 1981). (Cheung and Lai, 1993, Cheung and Lai, 2001, Diebold et al., 1991) used fractional models to find mean reversion and long memory dynamics in the exchange rates.

	Start date	observations		Start date	observations
Japan(JPY)	1/2/1995	6286	**Ukraine(UAH)**	3/15/1996	5658
Brazil(BRL)	1/2/1995	6234	**Kazakhstan(KZT)**	1/17/1996	5839
Indonesia(IDR)	1/2/1995	6076	**Philippines(PHP)**	7/29/1997	5585
Mexico(MXN)	1/2/1995	6279	**New Zealand(NZD)**	3/5/2001	4679
Turkey(TRY)	1/2/1995	6262	**UAE(AED)**	8/3/1998	5327
Hong Kong(HKD)	1/2/1995	6281	**Bahrain(BHD)**	8/3/1998	5319
India(INR)	1/2/1995	6261	**Kuwait(KWD)**	8/3/1998	5353
South Africa(ZAR)	1/2/1995	6280	**Saudi Arabia(SAR)**	8/3/1998	5348
Thailand(THB)	1/2/1995	6276	**Qatar(QAR)**	8/3/1998	5292
Malaysia(MYR)	1/2/1995	5523	**Oman(OMR)**	8/3/1998	5301
Singapore(SGD)	1/2/1995	6282	**Pakistan(PKR)**	10/17/2001	4488
Hungary(HUF)	1/2/1995	6279	**China(CNY)**	9/28/2005	3315
Taiwan(TWD)	1/2/1995	6236	**Sri Lanka(LKR)**	5/24/2002	4129
South Korea(KRW)	1/2/1995	6211	**Vietnam(VND)**	8/4/2008	2677
Russia(RUB)	1/5/1996	5984	**Bangladesh(BDT)**	8/4/2008	2731

Table 3.1: Currency symbols, start date and total number of observations

There are plenty of methods to estimate the long memory including parametric, semiparametric and nonparametric. Lack of the reliable and good estimation methods for long memory may be a possible reason behind such a wide range of methods (Aye et al., 2014). The Local Whittle (LW) estimator by Robinson (1995) with λ_j frequency to compute the periodogram $I(\lambda_j)$ is

$$\hat{d} = argmin_d \left\{ \ln \left(\frac{1}{m} \sum_{j=1}^{m} \frac{I(\lambda_j)}{\lambda_j^{-2d}} - \frac{2d}{m} \sum_{j=1}^{m} \ln \lambda_j \right) \right\}. \tag{3.4}$$

This estimator is based on one researcher specified truncation parameter m. Moreover, the asymptotic distribution of the LW estimator is not effected by the conditional heteroskedasticity in μ_t. A short term bias will be introduced with a too large value of the bandwidth parameter while an increased variance with a too small value (Okimoto and Shimotsu, 2010).

Shimotsu (2006) proposed two tests to explore the true memory or spurious memory. Among them, first is the sample splitting test which is based on the hypothesis that in case of a true long memory process the memory parameter in the subsamples should be equal to the memory of a full sample. This test splits the sample length n into b subsamples of integer length n/b. The null hypothesis is written as

$$H_0: d = d^{(1)} = \cdots = d^{(b)}, \tag{3.5}$$

where d is the true memory of the whole sample and $d^{(i)}$ for $i \in (1,2,\ldots,b)$ are the memory parameters for b subsamples. The test static is given as

$$W_c = 4m\left(\frac{c_m/_b}{m/_b}\right) A\hat{d}_b (A\Omega A')^{-1} (A\hat{d}_b)', \tag{3.6}$$

where $c_m = \sum_{j=1}^{m} v_j^2$, $v_j = \log\lambda_j - \frac{1}{m}\sum_{j=1}^{m}\log\lambda_j$ and

$$\hat{d}_b = \begin{pmatrix} \hat{d} - d \\ \hat{d}^{(1)} - d \\ \cdot \\ \cdot \\ \cdot \\ \hat{d}^{(b)} - d \end{pmatrix}, A = \begin{pmatrix} 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{pmatrix}, \Omega = \begin{pmatrix} 1 & \acute{\tau}_b \\ \tau_b & b_{I_b} \end{pmatrix},$$

for $(b \times b)$ identity matrix I_b and τ_b is $(b \times 1)$ matrix of ones. The limiting distribution of W_c approaches to $\varkappa^2$ with $(b-1)$ degrees of freedom.

The time-domain property of the fractionally integrated process, a stationary process integrated of order zero (i.e. $I(0)$) can be obtained by differencing an $I(d)$ process d times, is the main point in the difference test of Shimotsu (2006). This property may not hold for some spurious memory processes. The $\hat{\mu}$ as purposed by Shimotsu (2006) is used to demean the data as

$$\hat{\mu}(\hat{d}) = \frac{w(d)}{N}\sum_{n=1}^{N} Y_n + (1 - w(d))Y_1, \tag{3.7}$$

where

$$w(d) = \begin{cases} 1 \ if \ d \leq 0.5 \\ 0 \ if \ d \geq 0.75 \end{cases}.$$

The KPSS test for the null hypothesis of stationary series and Z_n test including an intercept term for the null hypothesis of unit root is applied to the d^{th} differenced series. Now the critical values for KPSS and Z_n are different and described in table 1 and 2 of Shimotsu (2006).

We also apply the semiparametric test of Qu (2011) for the null hypothesis of true long memory against the alternative of structural break process. The test statistic is given as follows

$$W_{Qu} = sup_{r\epsilon[\varepsilon.1]}\left(\sum_{j=1}^{m} v_j^2\right)^{-0.5}\left|\sum_{j=1}^{[mr]}\left\{\frac{I_j}{G(\hat{d})\lambda_j^{-2\hat{d}}}-1\right\}\right|, \tag{3.8}$$

with bandwidth parameter m, v_j is described as in splitting test above, the LW estimator $\hat{d}$ and a trimming parameter ε.

With presence of spurious memory in the range-based volatilities, memory estimates with the LW estimator are no more applicable and inconsistent. The long memory estimator by Hou and Perron (2014) named as the Modified Local Whittle estimator (MLW) provides consistent estimates with smallest bias and mean square errors as compared to most of other estimators in case of low-frequency contaminations, random level shifts, and deterministic trends. The asymptotic variance of this estimator is equivalent to the LW with an absence of low-frequency contamination and does not require the underlying process to be Gaussian (Hou and Perron, 2014). The estimator is based on the short or long memory process z_t with a constant term c, a stationary process y_t and u_t low- frequency contaminations as

$$z_t = c + y_t + u_t. \tag{3.9}$$

The estimator is

$$\hat{d} = argmin_{d,\theta} R(d,\theta), \tag{3.10}$$

which is based on the pseudo spectral density obtained by adding one new term $(G_u/T)\lambda_k^{-2}$ in the spectral density of the stationary process to control the low-frequency contaminations.

$$f_z(\lambda_k) = G_0\lambda_k^{-2d} + (G_u/T)\lambda_k^{-2}. \tag{3.11}$$

3.3 Empirical results

As a first step of the empirical work, we perform a graphical analysis of the autocorrelation function and periodogram to detect the persistence in the range series. All series show significant and a hyperbolic decay of the autocorrelations up to lag 100, perhaps an indication of the long-range dependence and fractional integration. Moreover, a pole near zero frequencies in the periodogram specifies the need of differencing to get stationary series. An analysis of the periodogram for differenced series may suggest an over differenced series by displaying values around zero at small frequencies (Gil-Alana and Toro, 2002). The graphs of ACF and periodogram in levels and differenced series for JPY are provided in Figure 3.1. Graphs of the other volatility series also display the similar trends and are presented in Figure 3.3 in appendix. The descriptive statistics are presented in Table 3.2 with the first four moments and the unit root test. We find the positive mean for all the range volatilities, the skewness measure is greater than 0 in all cases presenting the non-symmetrical and right-skewed distributions. High values of the kurtosis show the non-normal (leptokurtic) distribution of the range volatilities in all cases. Moreover, the hypothesis of nonstationary series is rejected in all cases with the ADF test at 5% level of significance.

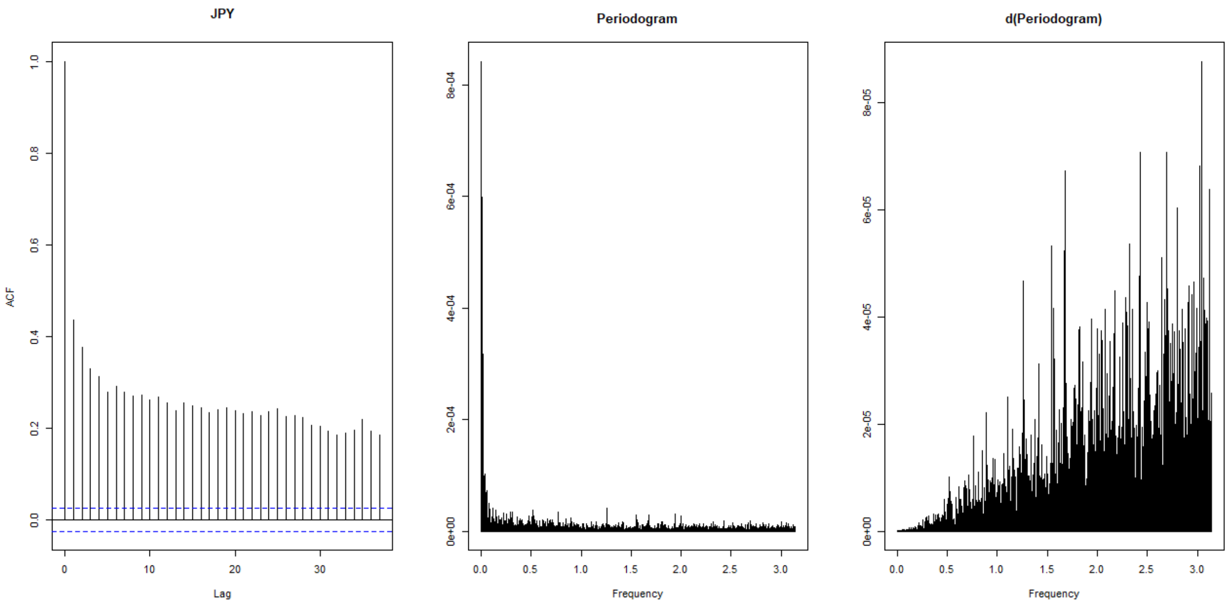


Figure 3.1: ACF, periodogram and differenced periodogram of JPY

Graphical analysis suggests the use of long memory methods to model the trends in the log ranges of spot exchange rates. Long memory estimates by using the LW estimator for different bandwidths are reported in Table 3.3. These estimates change with different bandwidths within stationary and nonstationary regions. The values of d move between 0.2304 and 0.8104 with $m = T^{0.6}$, between 0.2393 to 0.8468 with $m = T^{0.7}$, and between 0.2662 to 0.8862 with $m = T^{0.8}$. Overall results of the long memory estimates in the exchange rate volatilities exhibit stationary or nonstationary long memory depending on the truncation parameter. Generally, results of the semiparametric estimates show mean reversion in the stationary region ($d \leq 0.5$) for nine series while nonstationary ($d \geq 0.5$) in others with the memory estimates greater than 0.8 in 2 cases. We observe that d tend to decrease with an increase in the bandwidth parameter m for JPY, IDR, MXN, TRY, INR, THB, MYR, SGD, HUF, TWD, UAH, PHP, NZD, AED, BHD, KWD, OMR, and CNY. This decrease in the memory estimates may suggests the existence of level shifts or low-frequency contaminations in the exchange rate volatilities (Sibbertsen et al., 2018). One other reason for such a trend may be the failure of semiparametric method to satisfactory deal with the short memory components (Chatzikonstanti and Venetis, 2015). This tendency in the series may be explained by the structural breaks (Perron and Qu, 2010).

	Mean	SD	Skewness	Kurtosis	ADF		Mean	SD	Skewness	Kurtosis	ADF
JPY	0.010	0.006	3.522	31.143	0.01	**UAH**	0.009	0.016	6.854	101.691	0.010
BRL	0.012	0.011	3.195	26.296	0.01	**KZT**	0.002	0.006	20.882	617.244	0.010
IDR	0.012	0.025	6.507	62.943	0.01	**PHP**	0.008	0.009	5.371	55.160	0.010
MXN	0.010	0.010	6.613	75.615	0.01	**NZD**	0.013	0.007	2.625	16.875	0.010
TRY	0.014	0.016	6.956	81.516	0.01	**AED**	0.000	0.000	5.039	52.580	0.010
HKD	0.000	0.000	7.915	137.724	0.01	**BHD**	0.001	0.002	4.765	41.813	0.010
INR	0.006	0.004	2.150	11.442	0.01	**KWD**	0.004	0.004	4.765	51.583	0.010
ZAR	0.017	0.011	2.452	21.278	0.01	**SAR**	0.000	0.001	8.523	111.893	0.010
THB	0.011	0.020	4.047	20.661	0.01	**QAR**	0.001	0.004	8.969	101.914	0.010
MYR	0.005	0.007	5.332	45.744	0.01	**OMR**	0.002	0.003	4.852	36.218	0.013
SGD	0.005	0.004	3.774	32.655	0.01	**PKR**	0.004	0.005	4.041	32.708	0.010
HUF	0.013	0.008	2.202	11.942	0.01	**CNY**	0.002	0.001	2.310	11.458	0.010
TWD	0.005	0.003	2.568	18.925	0.01	**LKR**	0.003	0.003	3.534	27.205	0.010
KRW	0.008	0.010	9.795	197.872	0.01	**VND**	0.003	0.004	6.198	63.141	0.010
RUB	0.010	0.020	12.238	248.952	0.01	**BDT**	0.004	0.005	2.604	14.881	0.010

Table 3.2: Descriptive statistics

	$m^{0.6}$	$m^{0.7}$	$m^{0.8}$		$m^{0.6}$	$m^{0.7}$	$m^{0.8}$
JPY	0.410865	0.388202	0.343197	**UAH**	0.563201	0.353419	0.360383
BRL	0.519366	0.575084	0.541452	**KZT**	0.237609	0.239377	0.266291
IDR	0.606615	0.559214	0.570459	**PHP**	0.578793	0.509767	0.427774
MXN	0.495644	0.459952	0.474757	**NZD**	0.520925	0.457402	0.425035
TRY	0.475162	0.47711	0.442277	**AED**	0.493072	0.462384	0.478125
HKD	0.396736	0.46437	0.478175	**BHD**	0.497132	0.446245	0.419487
INR	0.524059	0.497473	0.46623	**KWD**	0.50063	0.492277	0.435794
ZAR	0.5088	0.513748	0.485871	**SAR**	0.622852	0.551677	0.599708
THB	0.829828	0.757732	0.642622	**QAR**	0.880653	0.846803	0.886298
MYR	0.584721	0.572443	0.555375	**OMR**	0.699366	0.557325	0.504363
SGD	0.49793	0.457032	0.42355	**PKR**	0.507472	0.51974	0.482997
HUF	0.520103	0.495395	0.447246	**CNY**	0.448039	0.430547	0.401974
TWD	0.522827	0.469088	0.414211	**LKR**	0.49506	0.428016	0.422464
KRW	0.661462	0.677352	0.604911	**VND**	0.485797	0.513537	0.517248
RUB	0.519507	0.614267	0.574363	**BDT**	0.604718	0.599206	0.548375

Table 3.3: LW estimates

We also investigate the changing behavior of long memory estimates with the graphs of d against truncation parameter ranging from $m = T^{0.4}$ to $m = T^{0.8}$. The graphs of the long memory estimates against a range of truncation parameters reveal the changing behavior of persistence in the exchange rates and are presented in Figure 3.2. The memory pestimates in case of IDR, THB, MYR, SGD, KRW, KWD, QAR, and PKR are greater than 0.7 at the smaller bandwidths. The memory estimates are greater than one for OMR and PHP. Therefore, they have mean reverting trends in the volatilities. The memory estimates for the remaining exchange rate volatilities are in the vicinity of 0.5 at the lower bandwidths while falls in the stationary region for increasing bandwidths.

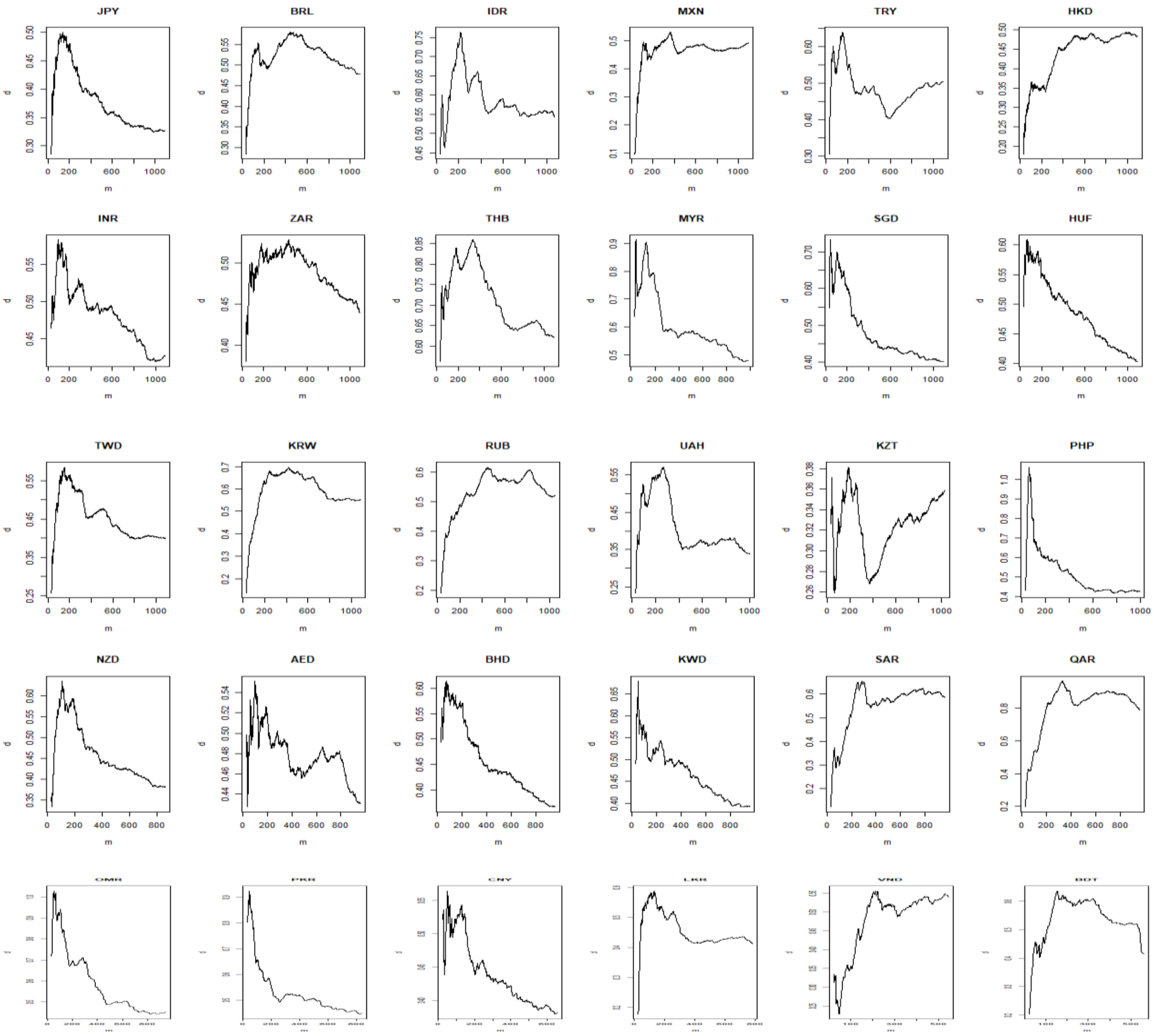


Figure 3.2: Graphs of $\boldsymbol{d}$ against range of frequencies

The graphs of memory estimates for JPY, BRL, IDR, INR, ZAR, THB, MYR, SGD, HUF, TWD, PHP, NZD, AED, BHD, KWD, OMR, PKR, CNY, LKR, and BDT present upward trend at the start of the series for almost first 50 frequencies but afterward a constant decline in the graph is visible with an increase in the bandwidth parameter. Perron and Qu (2010) observed that the memory parameter decreases as the bandwidth parameter increases in case of a short memory series with breaks. Importance of the short memory components increases for higher bandwidth parameter in comparison to the level shift components. Consequently, the memory parameter falls (Chatzikonstanti and Venetis, 2015). The memory estimates in a true long memory process are independent from the values of truncation parameter (McMillan and Ruiz, 2009). Therefore, the visual inspection of these graphs for increasing bandwidth parameter suggests the presence of spurious memory in form of the level shifts or structural breaks in these spot exchange rates volatilities.

	W_{Qu}	W_c				Z_t		KPSS	
		$m = T^{0.6}$		$m = T^{0.7}$		$m = T^{0.6}$	$m = T^{0.7}$	$m = T^{0.6}$	$m = T^{0.7}$
		$b = 2$	$b = 4$	$b = 2$	$b = 4$				
JPY	**1.6782**	0.0975	1.4052	0.0376	1.9237	**-2.0951**	**-1.0785**	0.1117	0.2603
BRL	0.8159	2.2608	3.3561	2.2904	**18.5870**	**-1.7174**	**-2.1347**	0.0886	0.0388
IDR	**2.9118**	**8.1877**	**11.0219**	**5.5820**	**7.8726**	-3.7944	**-1.4631**	0.0406	0.1446
MXN	**1.6908**	0.4580	5.2241	3.2910	**21.9518**	**-2.2251**	**-2.4049**	0.0577	0.0483
TRY	**1.9273**	2.0722	1.1180	2.5599	11.6986	-3.1767	**-2.0321**	0.0337	0.0645
HKD	**1.1570**	2.8079	3.5474	0.3372	5.7323	**-1.6598**	**-2.7018**	0.0957	0.0349
INR	1.0410	2.1290	1.3873	0.3186	1.6349	**-1.3357**	**-1.2804**	0.1495	0.1671
ZAR	0.6877	0.5813	1.3816	0.9503	**23.2297**	**-1.2570**	**-1.2768**	0.1555	0.1488
THB	**2.1958**	**16.5815**	**38.4169**	**39.6156**	**92.8060**	-3.8017	**-2.9417**	0.0276	0.0443
MYR	**3.7320**	**26.3334**	**21.4415**	1.3877	**8.1253**	-5.1514	**-1.8985**	0.0197	0.0883
SGD	**3.1864**	1.1910	**16.8035**	0.2284	3.3556	-3.2017	**-1.6887**	0.0548	0.1043
HUF	**1.8906**	**7.2906**	**11.2519**	**9.4131**	**13.8050**	**-1.6243**	**-1.1098**	0.1116	0.2439
TWD	**1.4433**	0.2405	0.7453	0.3699	1.1089	-3.0092	**-1.8932**	0.0366	0.0983
KRW	**1.4065**	0.0252	1.5188	**13.1288**	**22.9549**	-3.7130	-4.4667	0.0241	0.0211
RUB	**1.6141**	0.5702	0.3030	**14.7397**	**12.3591**	**-2.0960**	-3.6846	0.0657	0.0185
UAH	**3.7738**	**7.8453**	6.2015	**11.4801**	**9.2975**	-3.6079	**-1.6499**	0.0242	0.1021
KZT	0.5132	0.1235	0.9400	2.1930	2.4363	**-1.5541**	**-1.6220**	0.1745	0.1603
PHP	**3.7894**	1.0466	**8.3358**	0.4681	9.2161	**-1.4153**	**-1.0242**	0.0488	0.2150
NZD	**2.0504**	**3.9625**	5.4809	0.4579	4.0304	**-2.4756**	**-1.1951**	0.0696	0.2239
AED	**1.3468**	**4.3634**	4.8763	**19.2294**	**20.3612**	**-1.5556**	**-1.2114**	0.1107	0.1894
BHD	**2.6407**	2.3796	**12.7754**	**30.4419**	**55.1381**	**-2.3108**	**-1.2637**	0.0569	0.1761
KWD	**1.7390**	0.0654	2.7443	**6.2509**	3.1750	**-1.5350**	**-1.3229**	0.1187	0.1630
SAR	**1.4444**	0.0048	1.6367	0.3328	**7.9894**	**-2.0404**	**-2.8858**	0.0684	0.0336
QAR	**1.8642**	**22.3773**	**32.7628**	**135.7163**	**101.8887**	**-2.7511**	-5.1336	0.0359	0.0116
OMR	**5.1073**	**4.0997**	**15.1294**	3.3626	5.6561	**-2.5399**	**-1.2893**	0.0868	0.2098
PKR	**2.6833**	0.4660	**8.3808**	0.0294	6.8956	**-1.3828**	**-0.9284**	0.1842	0.2798
CNY	**1.5396**	0.0131	5.4793	0.2706	3.9666	**-0.2677**	**1.2098**	0.3550	1.0293
LKR	**2.2471**	**7.5240**	7.1882	0.5553	0.7234	**-2.3805**	**-1.3319**	0.0589	0.1819
VND	**1.4579**	1.0517	3.9694	0.0158	2.3638	**-0.6015**	**-1.7493**	0.4152	0.1107
BDT	1.1129	1.1146	2.1572	2.0835	5.6419	**-1.3971**	**-2.1847**	0.1638	0.0642

Table 3.4: True memory versus spurious memory (bold values indicate the rejection of hypothesis)

The graphs for MXN, KRW, SAR, QAR, and VND present some decreases but afterwards, they maintain an almost constant level of persistence with increasing values of m. In case of TRY and KZT, an upward trend is visible after a constant decline while a continuous increasing trend in the persistence of HKD, RUB is obvious. Overall, the graphical analysis suggests that the memory is not constant and decrease with a range of truncation parameter in most of the cases.

As our next step of the empirical analysis, we apply three tests as described in the previous section to distinguish between the true and spurious long memory in the exchange rate volatilities. The results of all these tests are reported in Table 3.4. First column in Table 3.4 reports the results of the Qu (2011) test for the null of true memory against the spurious memory. The results are described for $\varepsilon = 0.05$ with bandwidth parameter 0.7. The critical values at 10%, 5% and 1% levels of significance are 1.022, 1.15, and 1.426. All series reject the hypothesis of true long memory except Brazil, India, South Africa, Kazakhstan, and Bangladesh at 5 % level of significance.

Overall 25 range series out of 30 reject the null hypothesis of true long memory, indicating that the visual memory is not true but spurious. The split Wald test also rejects the null hypothesis of equal memory for the whole sample and subsample for 10 series with $b = 2, 4$ and $m = T^{0.6}$. The number of rejections increases to 14 series with $m = T^{0.7}$. The Z_t test, based on the assumption that an $I(d)$ series will be $I(0)$ after differencing d times, cannot reject the hypothesis of $I(d)$ series in 24 cases at $m = T^{0.6}$.

These results support the hypothesis of true memory in the remaining six series. There is an evidence of true memory only for Russia with $m = T^{0.6}$. The KPSS test for the null hypothesis of $I(0)$ series cannot be rejected on the basis of calculated values using both bandwidths. Hence, these results suggest the existence of true memory in all range series. Al-Shboul and Anwar (2016) obtained similar results with an application of the difference test for sectoral returns of Jordan’s Amman stock exchange. Based on the results of PP test, Split Wald test, and QU test, there is an evidence of spurious memory or level shifts in the exchange rate range volatilities across different countries.

Estimates of the LW are no more consistent with an evidence of spurious memory in the range-based volatilities so we estimate fractional integration by applying the MLW. The estimates are provided for the bandwidth parameter $m = T^{0.65}, T^{0.7}, T^{0.75}$. Hou and Perron (2014) recommends the usage of a bandwidth parameter greater than $m = T^{5/9}$. These results are reported in Table 3.5 and reduction in the memory estimates in most of the cases may be caused by the low-frequency contaminations. The memory estimates are equivalent to the LW results for BRL, MXN, HKD, THB, KRW, RUB, KZT, SAR, QAR, VND, and BDT. Overall, we get mixed results which specify to analyze these series in more detail to better understand the structural breaks.

Lima and Tabak (2007) did not reject the random walk hypothesis in the exchange rates of Indonesia, Malaysia, The Philippines, South Korea, Thailand, Brazil, Mexico, and Russia.

	$m = T^{0.65}$	$m = T0^{.7}$	$m = T0^{.75}$		$m = T^{0.65}$	$m = T0^{.7}$	$m = T^{0.75}$
JPY	0.3040	0.2931	0.2260	**UAH**	0.5728	-0.4999	0.2076
BRL	0.5276	0.5799	0.5434	**KZT**	0.2462	0.2439	0.2687
IDR	0.5869	-0.3907	0.5434	**PHP**	0.1235	0.1394	0.0772
MXN	0.5049	0.4640	0.4767	**NZD**	0.4843	0.1944	0.2403
TRY	0.1380	0.3378	0.3256	**AED**	0.4370	0.3932	0.4479
HKD	0.4057	0.4695	0.4805	**BHD**	0.2772	0.2156	0.2516
INR	0.4886	0.4570	0.4225	**KWD**	0.3855	0.4161	0.3232
ZAR	0.5021	0.4943	0.4605	**SAR**	0.6372	0.5568	0.6023
THB	0.8382	0.7590	-0.3892	**QAR**	0.8965	0.8521	0.8888
MYR	-0.4999	0.0259	0.2799	**OMR**	0.2324	-0.0566	0.0576
SGD	0.0562	0.1507	0.2083	**PKR**	-0.0174	0.2693	0.2940
HUF	0.3869	0.3791	0.3007	**CNY**	0.2603	0.2849	0.2777
TWD	0.5166	0.4003	0.2581	**LKR**	0.3476	0.1234	0.2743
KRW	0.6726	0.6822	0.6059	**VND**	0.5032	0.5217	0.5210
RUB	0.5292	0.6202	0.5751	**BDT**	0.6209	0.6067	0.5315

Table 3.5: Modified local whittle estimator

Li et al. (2017) rejected the hypothesis of true long memory in the daily exchange rates of JPY/USD, CAD/USD, GBP/USD, EUR/USD and applied the random level shift model to specify the short memory plus level shift. Gil-Alana and Sauci (2018) found lack of PPP in the real exchange rates of some Latin American countries by using parametric and semiparametric techniques. Gogas et al. (2013) used the Detrended Fluctuation Analysis (DFA) and rolling window analysis to analyze the persistence in the exchange rates of 23 OECD countries.

The indicated spurious memory in exchange rate volatilities may be caused by some level shifts or structural breaks. As our next step of the analysis, we try to locate the breakpoints by using a method proposed by Bai and Perron (1998a) and Bai and Perron (2003). The multiple mean break model in the range volatilities to test the null hypothesis of constant unconditional mean against the multiple breaks is

$$R_t = c_j + \mu_t \,, t = T_{j-1} + 1, \dots, T_j, j = 1, \dots, m + 1. \tag{3.12}$$

Where T is the sample size, $T_0 = 0$, $T_{m+1} = T$ and c_j is the mean of the range volatility. The number of breaks is considered unknown and the least squares methods are used in estimation. This is a sequential test to estimate the consecutive number of breaks. Procedure of this test divides the sample into two parts after the estimation of the first break point. Further division of the sample dependents on the failure of the constancy parameter hypothesis. Hence this methodology includes l breaks and $l + 1$ regimes. Therefore, it tests the hypothesis of $l + 1$ against l breaks. Choi et al. (2010) found that the persistence in Deutschemark/Dollar, Yen/Dollar, and Yen/Deutschemark can be somewhat explained by the structural breaks in the mean.

Our results present the different number of breaks in each series. List of the total number of breakpoints in all 30 series is provided in Table 3.6. We find a minimum of 2 breaks in Kazakhstan and a maximum of 7 breaks in the volatilities of Brazil, Hong Kong, South Africa, Taiwan, Ukraine, Kuwait, and China. We notice that these structural breaks are related to some specific events such as the financial crisis faced by several Asian countries including Thailand, Indonesia, Malaysia, South Korea, and The Philippines during 1997-98, the Russian crisis in August 1998, Brazilian crisis in 1999, recession crisis during 2007-08, and European crisis 2010.

Considering the Asian crisis 1997-98, started from Thailand, being the most effected countries Thailand, Malaysia, South Korea, Hong Kong, and Indonesia present a structural break at 1997. The less effected countries Singapore and Taiwan also present a break in 1997. This crisis was a result of large external deficits and frequent fluctuations in exchange rates. The depreciation of Japanese yen against USD during 1996-1997 combined with some insignificant effects of the Asian crisis is reflected at the breakpoint in 1997. Moreover, the captured break in 2000 is also a result of financial crisis 1997-2002 (Fukao, 2009). The depreciation in Yen during 2013 was related to some policies known as "Abenomics" to expand Japanese economy and to encourage private investments. First break point in RUB is related to Russian crisis 1998 which badly demolished Russia's trade because of the decreased oil prices accompanying the Asian crisis and instability in balance of payments (Chiodo and Owyang, 2002).

The aftershocks of the Asian crisis caused Brazil in 1998 and the situation became worse along with the Russian crises till August 1998 resulting in 1999 break point. The food inflation related to weather shocks in Brazil may affected the exchange rate volatility which is reflected as 2014 breakpoint. Tequila Crisis, started in 1994-1995 with the devaluation of Mexican currency. It resulted in the worst banking crisis in 1995-1997 and reveals the level shift in Mexican volatility. One other great crisis which caused the structural breaks in the volatilities of almost every country is the great recession of 2008. It started with the sub-prime crisis in USA during 2007 which afterwards scattered worldwide including the emerging markets as well. The breakpoints during 2007, 2008 or in some cases in 2009 are the result of this global financial crisis (GFC). Breakpoints in the Gulf cooperation council (GCC) countries in 2001 are triggered by USD depreciation. All GCC currencies were pegged to USD excluding Kuwaiti dinar and USD lost value due to 11 September 2001 terrorist attacks.

	JPY	**BRL**	**IDR**	**MXN**	**TRY**	**HKD**	**INR**	**ZAR**
	6	**7**	**3**	**5**	**6**	**7**	**6**	**7**
Break dates	12/16/1997	1/12/1999	8/12/1997	5/30/1997	6/16/1997	11/17/1998	8/27/1998	5/21/1998
	6/6/2000	6/15/2001	12/15/1999	2/11/2004	2/21/2001	4/18/2001	3/22/2004	11/28/2001
	2/24/2005	11/13/2003	8/2/2002	9/3/2008	7/22/2003	9/19/2003	3/19/2007	8/27/2004
	7/25/2007	7/23/2007		1/28/2011	5/8/2006	2/16/2006	9/1/2009	1/14/2008
	12/21/2009	6/10/2010		1/5/2016	6/24/2009	10/28/2008	12/20/2013	6/10/2010
	9/19/2013	4/24/2014			9/23/2016	6/4/2012	7/1/2016	7/11/2013
		9/15/2016				1/6/2016		12/8/2015
	THB	**MYR**	**SGD**	**HUF**	**TWD**	**KRW**	**RUB**	**UAH**
	4	**6**	**5**	**6**	**7**	**6**	**5**	**7**
Break dates	6/2/1997	2/14/1997	7/2/1997	7/9/1997	10/16/1997	10/14/1997	8/10/1998	8/14/1998
	10/29/1999	4/5/1999	12/2/1999	12/31/1999	3/14/2000	3/15/2000	12/1/2000	11/24/2000
	12/15/2006	12/2/2005	8/4/2008	8/25/2008	10/6/2003	4/29/2003	8/4/2008	4/16/2004
	5/18/2009	3/5/2008	7/13/2012	1/21/2011	3/7/2006	3/6/2008	7/15/2014	9/20/2007
		8/8/2014	1/1/2015	6/24/2013	8/6/2008	7/30/2010	10/31/2016	12/7/2009
		11/2/2016		6/30/2016	4/14/2011	10/7/2014		1/15/2014
					1/14/2015			5/20/2016
	KZT	**PHP**	**NZD**	**AED**	**BHD**	**KWD**	**SAR**	**QAR**
	2	**4**	**5**	**5**	**5**	**7**	**5**	**3**
Break dates	2/14/2000	9/24/1999	7/25/2007	6/19/2001	9/6/2001	8/22/2000	8/18/2000	11/19/2007
	8/18/2015	11/19/2001	6/25/2009	7/9/2003	1/22/2004	1/21/2004	9/19/2007	3/14/2011
		5/15/2007	12/21/2011	7/26/2005	5/4/2007	11/9/2006	1/28/2010	2/6/2017
		12/17/2010	9/24/2014	11/12/2007	5/19/2009	11/28/2008	1/1/2015	
			9/15/2016	2/16/2010	6/18/2015	12/17/2010	1/31/2017	
						12/12/2013		
						3/1/2016		
	OMR	**PKR**	**CNY**	**LKR**	**VND**	**BDT**		
	5	**6**	**7**	**5**	**4**	**4**		
Break dates	3/7/2001	11/10/2004	7/26/2007	9/3/2009	11/23/2009	2/10/2010		
	3/21/2003	4/14/2008	4/24/2009	2/3/2012	11/30/2010	2/19/2014		
	9/5/2007	1/6/2010	8/11/2010	10/2/2013	6/28/2012	5/9/2016		
	6/30/2011	9/26/2011	4/17/2012	9/3/2015	7/17/2017	9/4/2017		
	7/11/2013	2/26/2015	2/24/2014	6/9/2017				
		5/23/2017	8/10/2015					
			8/7/2017					

Table 3.6: Number of beaks with break date

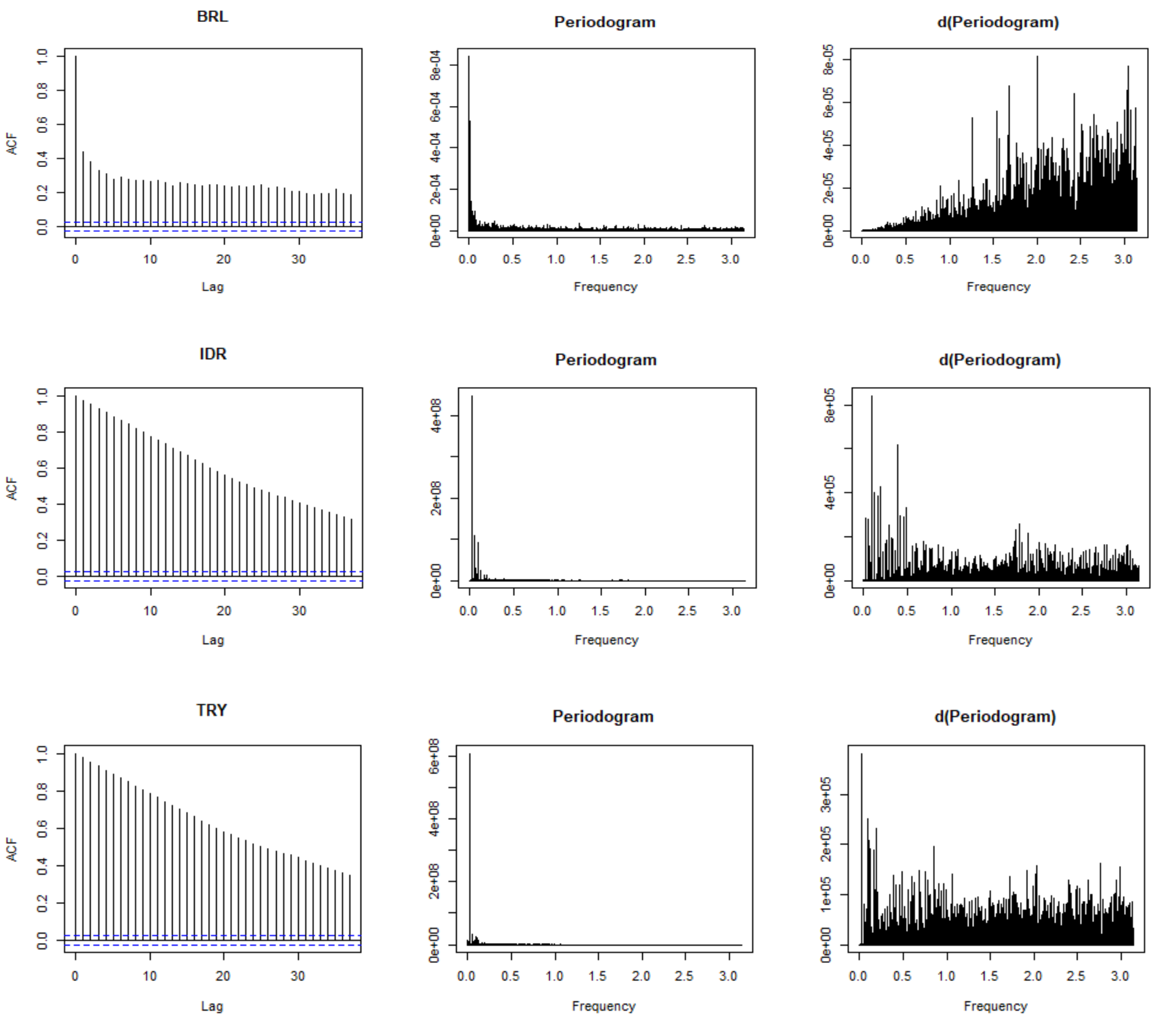
A currency appreciation is believed in oil-exporting countries with the positive oil price shocks and depreciation for oil importing countries (Amin and El-Sakka, 2016). The other breaks may relate to some oil price fluctuations as GCC countries are the major exporters of oil. The financial crisis of 2000-2001 caused by currency collapse has been mirrored by the break point at 2001 in Turkey. The break in volatility series in 2011 regarding Hungary may be caused by the European crisis 2010. China depreciated its currency against USD, EUR, JPY, and KRW such that it gained the lowest value during the last four years in August 2015 which results in a break. Caused by this devaluation of Yuan against USD, many other currencies had to depreciate against the USD. The reported breakpoints during 2014-2015 in some countries may be caused by a decline in oil prices due to surplus supply with weak demands. The breakpoint in 2013 regarding the Indian rupee reflects the currency depreciation of INR against USD in 2013 regarding the US Federal Reserve's policy of Quantitative easing (QE). Not only India but most of Asian currencies went through a devaluation in this regard except China and Bangladesh. The breakpoints in PKR volatility series in 2010 and 2011 are the result of bad economic situation caused by destroyed fertile lands regarding the terrible floods during 2010. The KZT fluctuations in 2015 occurred with an announcement to convert the Tenge as floating currency by the government. The graphs for the break dates in Table 3.6 are presented in Figure 3.4 in appendix with years on the x-axis.

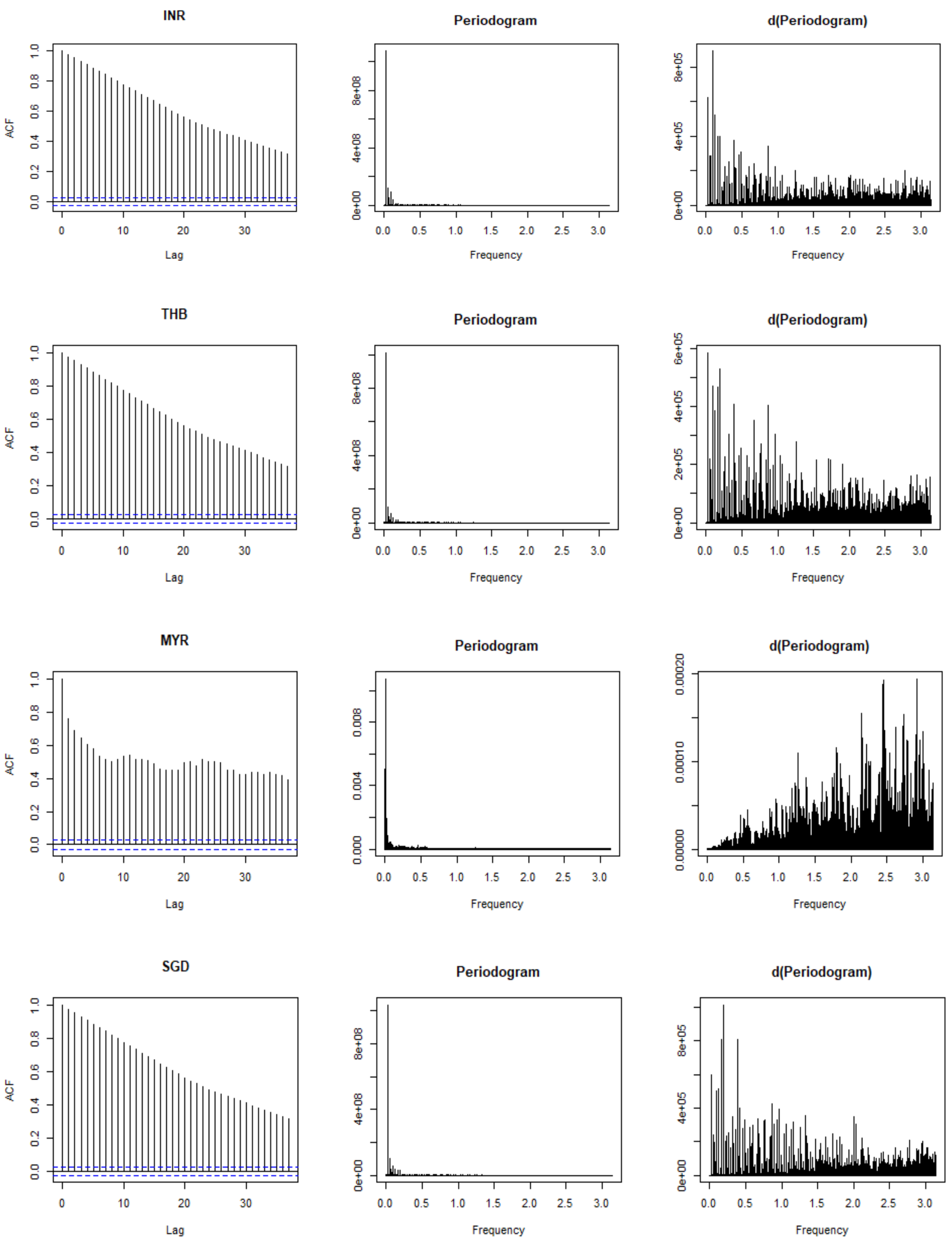
3.4 Conclusion

We analyze the long-range dependence in daily spot exchange rates across 30 different countries. Our results are estimated by using the range-based volatility contrary to the absolute-returns or the squared- returns. We use the semiparametric method to estimate the long-range dependence. The LW estimator is used to estimate the memory for different bandwidths. The results of memory estimates are in stationary as well as in the nonstationary region for the considered bandwidths. We suspected to test for the hypothesis of true long memory versus spurious memory on account of the decrease in the value of memory parameter with an increase in the bandwidth parameter. We tested the true memory hypothesis with an application of two tests by Shimotsu (2006) and a more powerful test by Qu (2011). We obtained the mixed results in favor of the true memory and spurious memory. The graphical analysis of the memory estimates against a range of truncation parameter shows a decrease in the persistence level with an increase in the bandwidth across most of the volatilities. Due to the inconsistency of the LW estimator in case of level shifts or low-frequency contaminations, we estimate the long-range dependence by using the MLW of Hou and Perron (2014). We find the reduced memory estimates in 18 cases compared to the LW estimates. We estimate the number of breaks in the volatility series and find the different number of breaks by using the method of Bai and Perron (2003) with a minimum number of 2 breaks and a maximum of 7. Most of the breakpoints are related to the important disasters and some economic strategies such as the Asian crisis, GFC, Russian crisis, and Brazilian crisis.

Our results seem to suggest that the phenomenon of varying fractional integration in range-based volatilities through different bandwidths may be caused by structural breaks or level shifts. Along with structural breaks, some other economic factors such as the central bank interventions and the speculative effects may cause such fluctuations in exchange rates. Our findings are important for modeling and forecasting the exchange rate volatilities. It is important to distinguish between true and spurious memory to specify the volatility process correctly (Zhou, 2011). Furthermore, exchange rate modeling can be performed by taking into account the structural breaks with an application of random level shifts model. Our results are helpful for the policymakers to develop a better policy considering the persistence and level shifts in exchange rates. In fact, different policy measures can be based on the volatility persistence in exchange rates while studying the true nature of process underlying the data.

3.4.1 Appendix

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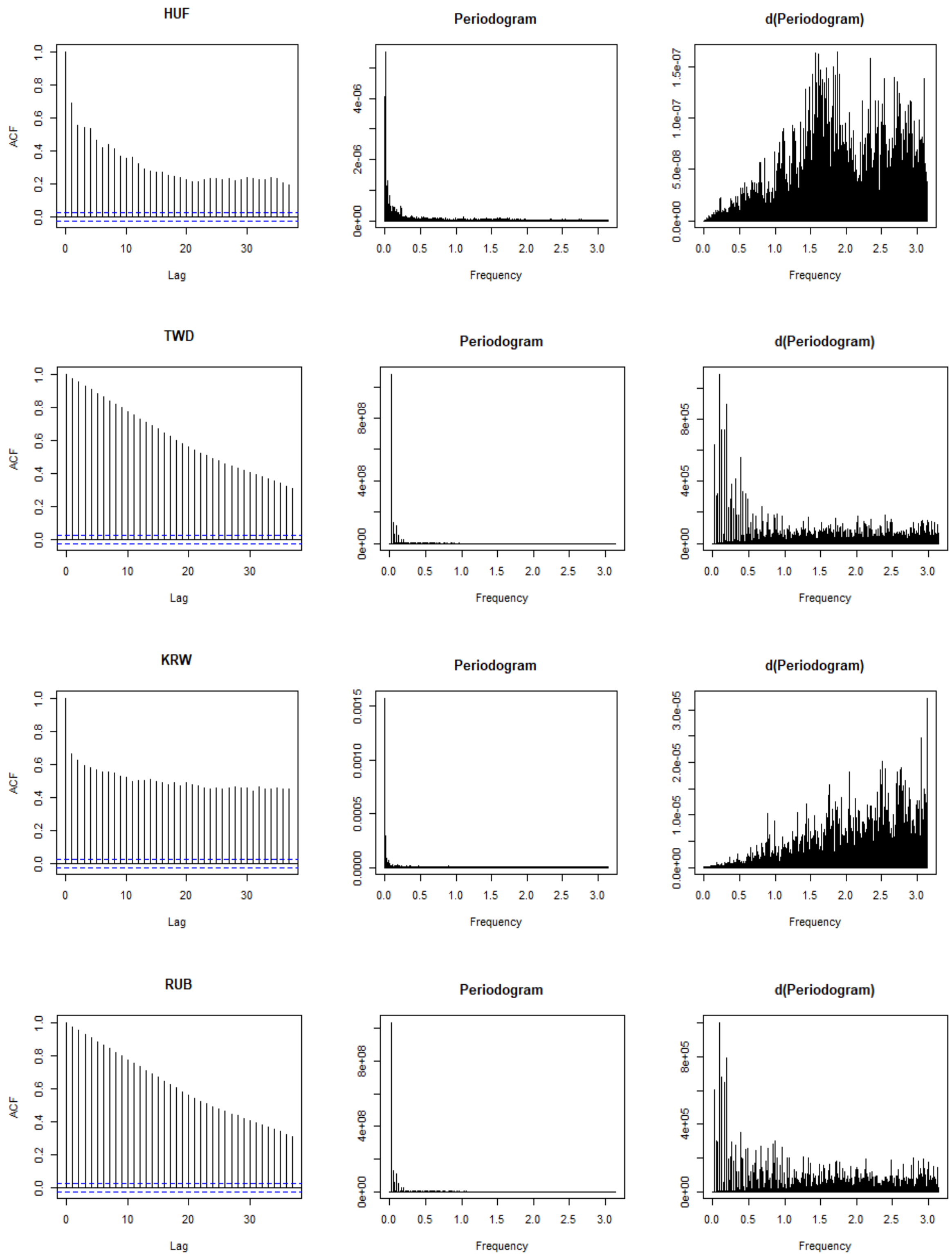
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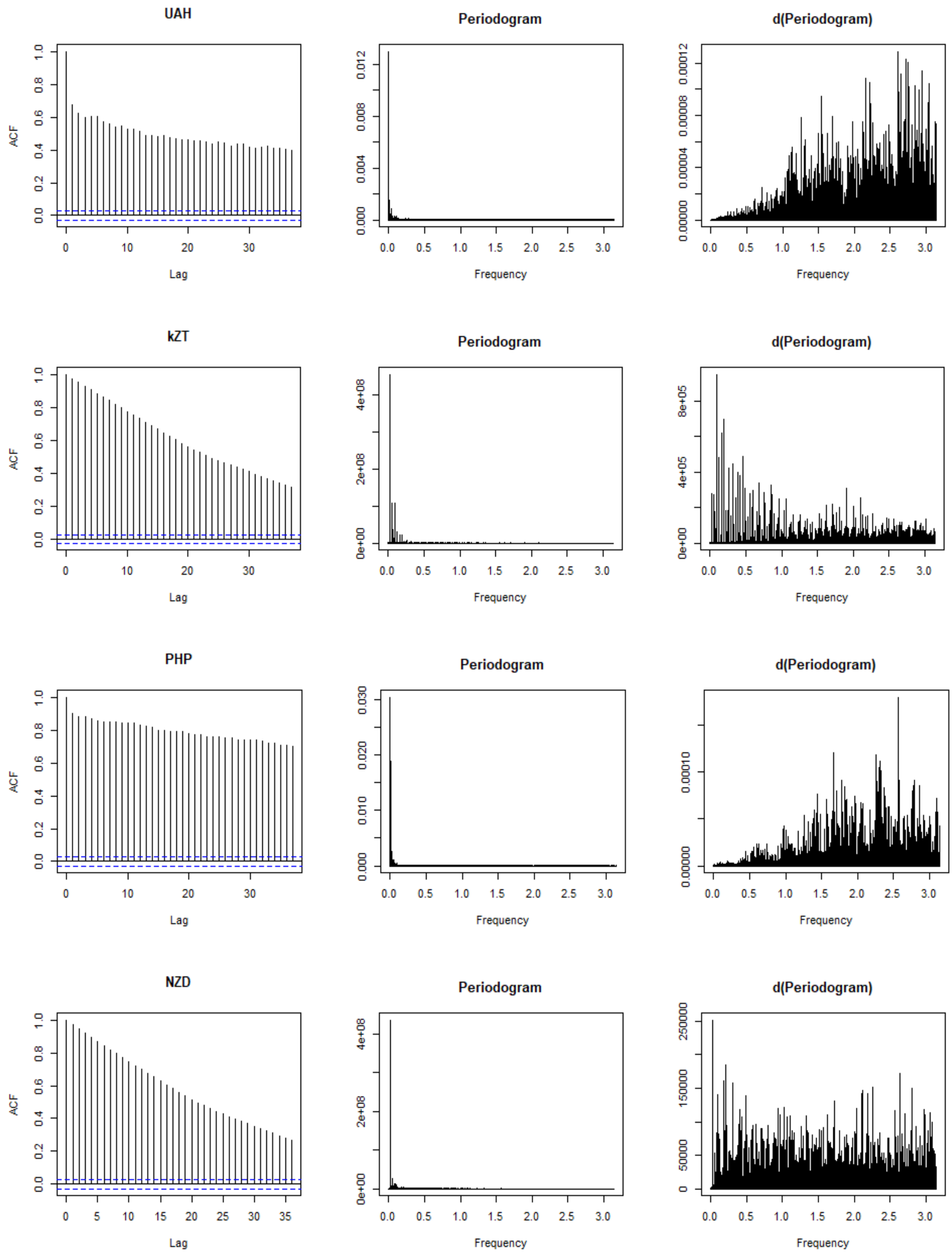
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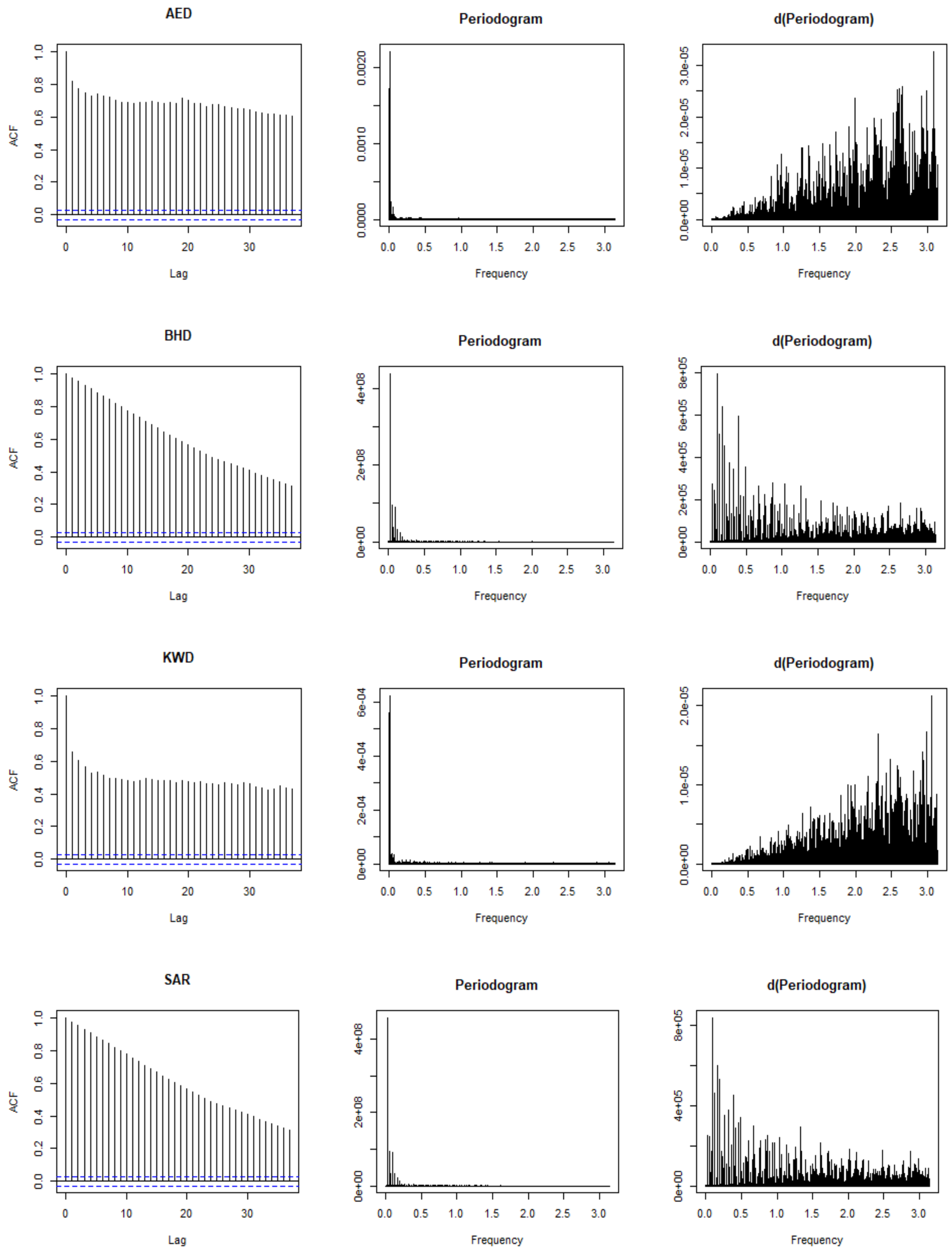
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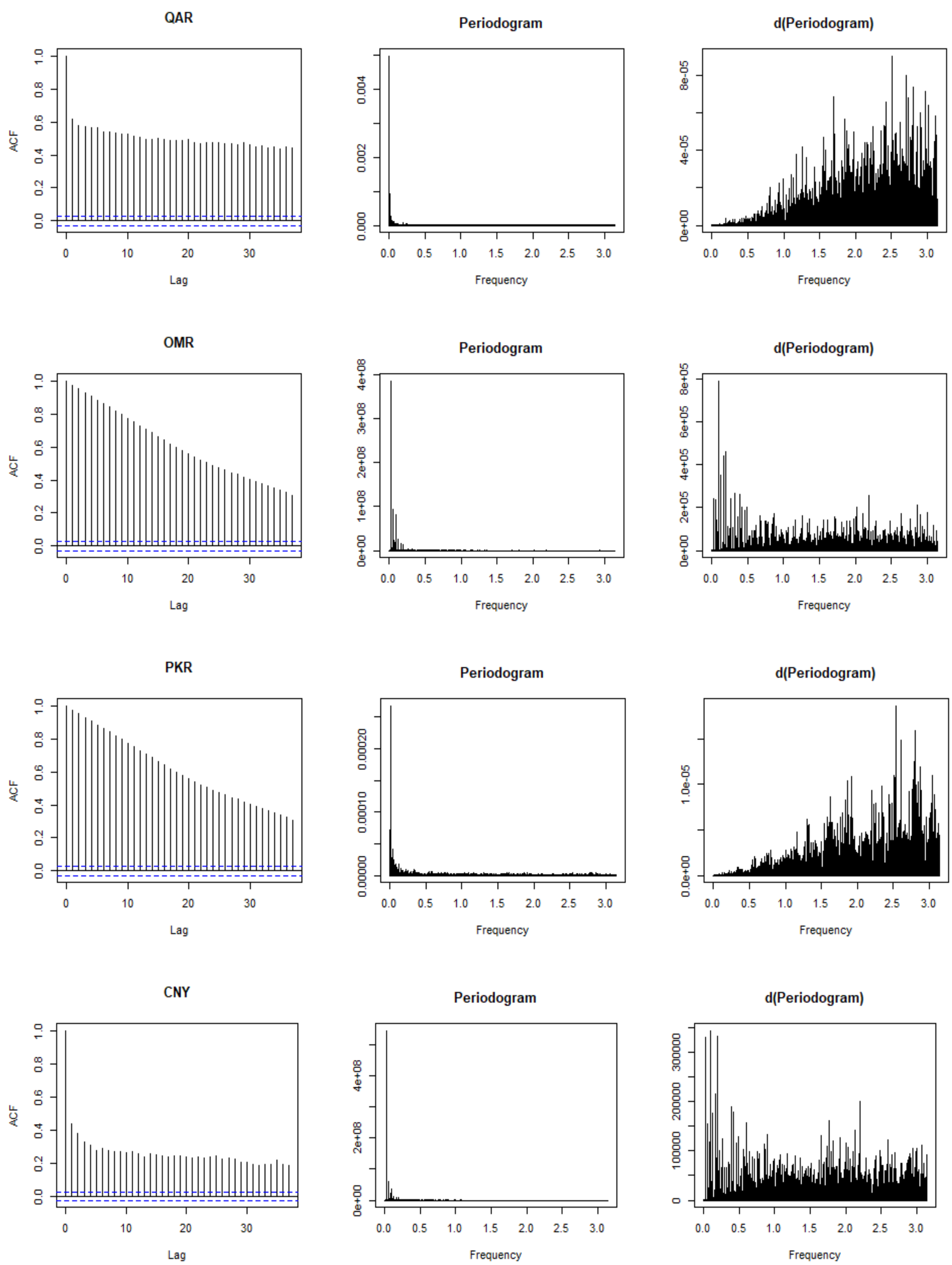
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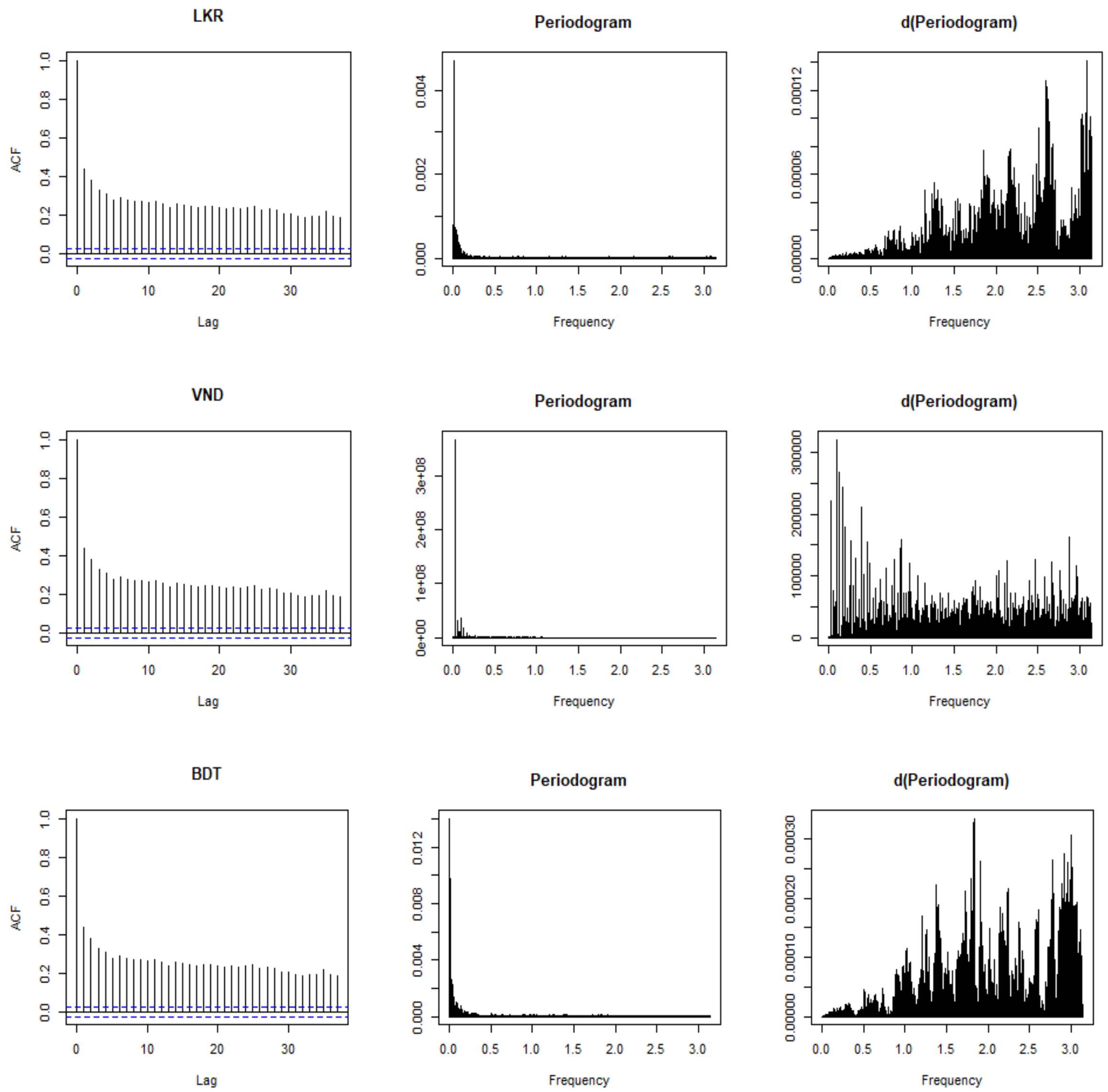
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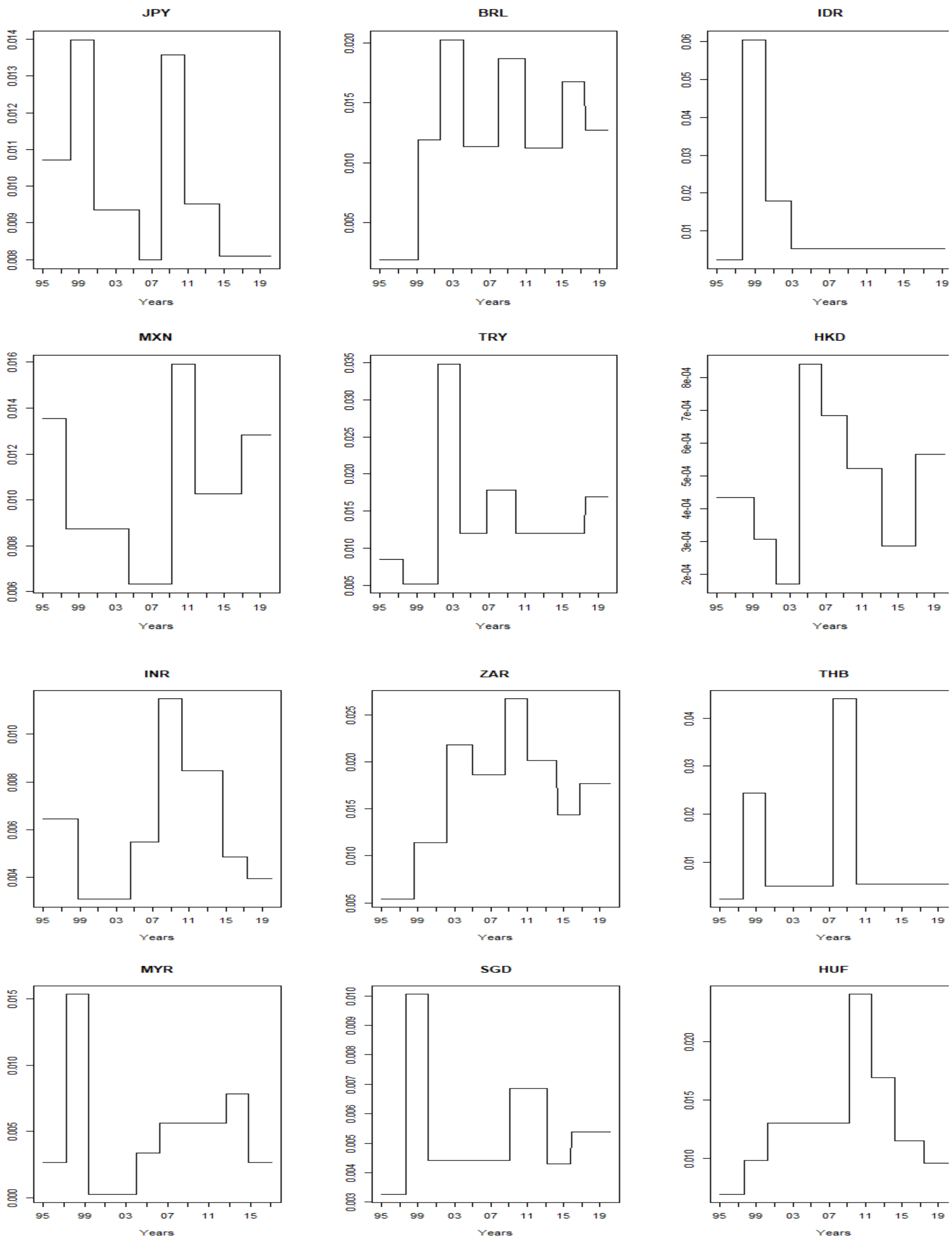
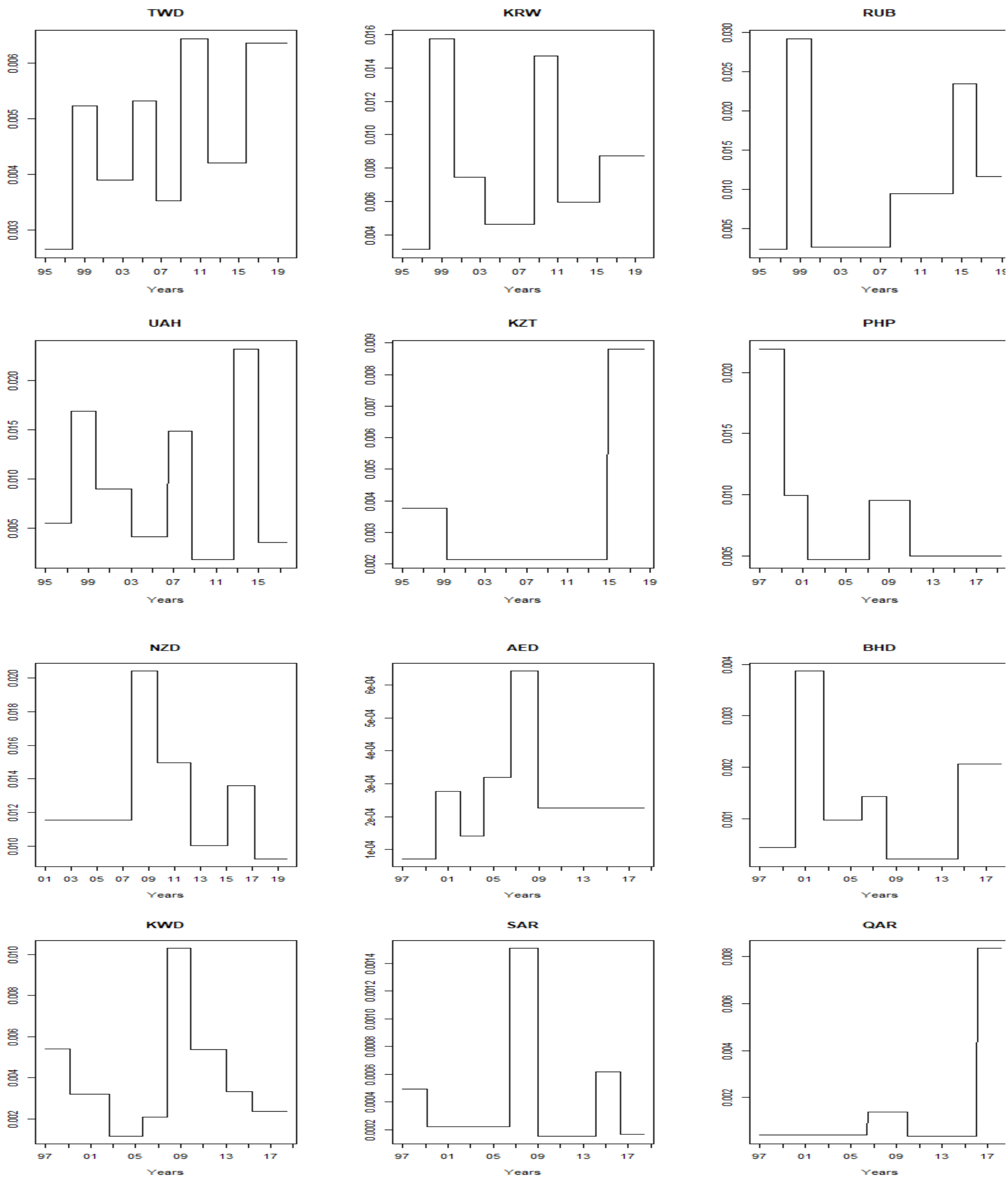
Figure 3.3: ACF, periodogram and differenced periodogram

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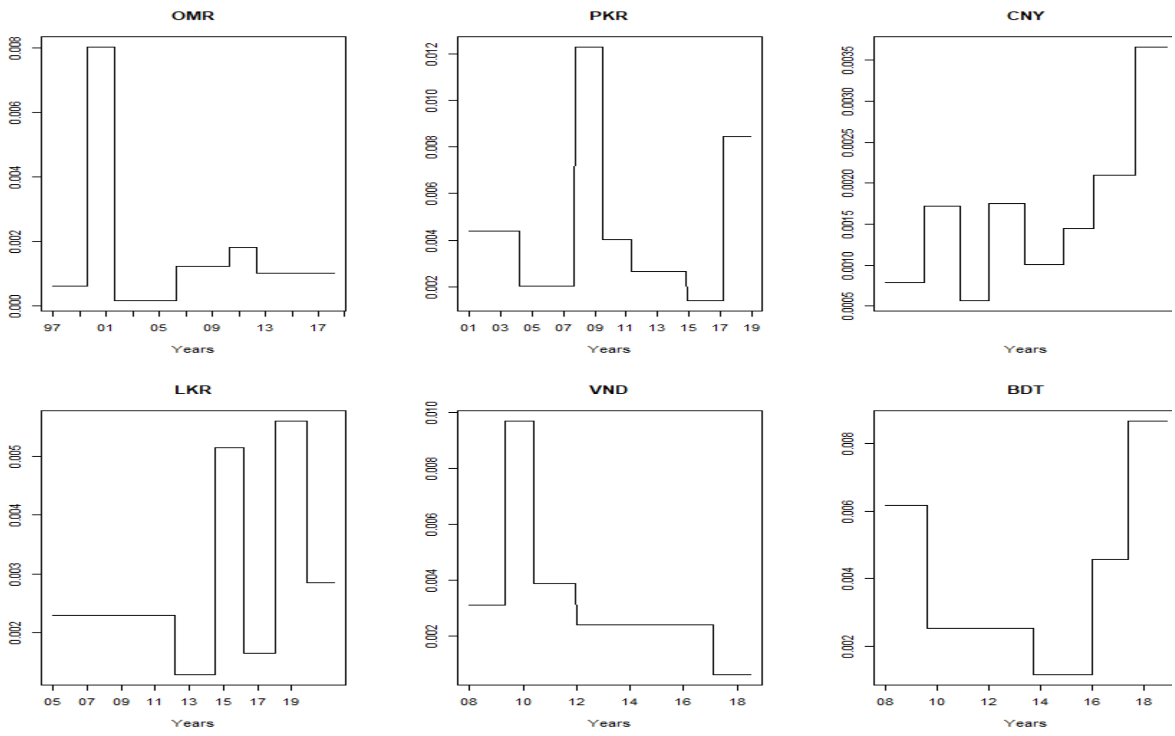


Figure 3.4: Breakpoints graphs

Chapter 4

Long memory, spurious memory or time-varying memory: Evidence from Pakistan stock exchange (PSX)

Long memory, spurious memory or time-varying memory: Evidence from Pakistan stock exchange (PSX)

Co-authored with Philipp Sibbertsen

4.1 Introduction

Importance of a stock market in economic growth and financial development is a worldwide reality. A vital role is played by a stock market in economic growth through boosting the national savings, investments and by efficient resource allocation (Dewandaru et al., 2014). Stability and risk management in these markets are important to fascinate the shareholders and the financiers by providing them risk free and an efficient environment. A market is considered efficient if the returns are not predictable around equilibrium by incorporating all relevant information (Malkiel and Fama, 1970). Market efficiency concerns considerably to the brokers due to exploitable inefficiencies, to risk managers for regulating market asymmetry and to policy makers regarding the technical analysis. An efficient market cannot be outperformed by any updates and it is impossible to predict the future returns by using the past values, resulting in normally and independently distributed asset returns.

A market is regarded as an efficient market if the present prices reflect all past information and the possibility of excessive returns is zero (Al-Shboul and Anwar, 2016). Rapid adjustment of the new relevant information is a requirement of the EMH. However, persistence exists in successive returns in case of gradual evolvement of new information or lack of spontaneous responses by the investors. Such a type of long-range dependence, persistence or antipersistence (mean reversion) in returns contradicts the linear properties of the EMH and makes the future predictions possible. Irrationality of the investors regarding some specific event challenges the random walk hypothesis in stock prices. Long memory is described by a hyperbolic decay of the autocorrelation function in the time domain while with lower frequency contamination in the vicinity of origin in the frequency domain. There exists no perfect arbitrage in presence of persistent stock prices so, models based on asset pricing theory are not suitable. Moreover, long range dependence allows for future predictions by introducing some nonlinear dependencies in the moments of a distribution (Assaf, 2015). Long memory challenges many standard statistical implications and forecasting theories as many models assuming randomness, normality and Brownian motion like CAMP, APT are no more applicable (LeRoy, 1989, Yajima, 1985). Gil-Alana (2006) observed that the trading strategies based on the long memory may provide some unexploited profits by reducing the transaction cost. There is a possibility of earning the abnormal profits in presence of persistence or antipersistence (mean reversion) in stock returns (Hull and McGroarty, 2014).

The presence or absence of an efficient market depends on the financial development as some research works show efficiency in the developed markets while serial dependence in the emerging markets (Auer, 2016, Hull and McGroarty, 2014, Rizvi et al., 2014). Low trading volumes, management issues, and high transaction costs are common features regarding the emerging markets. Auer (2016) explained that not only the existence of exploitable inefficiencies is more likely to hold in the developing or emerging markets versus the developed markets but such markets are also a good candidate for the heterogeneous and adaptive market hypothesis (AMH). Levine (1999) described the lack of adequate information about listing requirements of institutes, deficient number of skilled brokers, use of confidential information and different price manipulation mechanism as prominent features in the emerging markets. These features enhance the speculative activates in such markets. Limam (2003) noticed that informational inefficiency is a result of different institutional dogmatisms in thinly traded markets rather the developed markets. Overreactions or gradual reactions of investors about good or bad news disturb the market equilibrium which result in inefficiency and nonlinearities in stock returns (Bondt and Thaler, 1985). Kristoufek (2012) and Kristoufek (2013) consider higher or lower liquidity levels as a source of volatility and market inefficiency. Lee et al. (2018) stated the bogus hopes concerning a bullish market and utmost worries regarding a bearish market as sources of the market inefficiency. Lim and Brooks (2010) found a more volatile stock market with lower per capita GDP and he concluded that a sufficient and necessary condition for an efficient market is protected private property rights in emerging markets.

Modeling the trends in financial markets is also concerned with identification and estimation of the regime shifts or smooth trends. An extensive amount of work has been done on the structural changes in time series and econometrics literature. Long memory and short memory with shifts share two common aspects: both processes display a hyperbolic decay of the autocorrelation function at long lags and the spectral density of both processes is boundless at origin. It makes difficult to distinguish the long memory from the spurious long memory. (Berkes et al., 2006, Krämer and Sibbertsen, 2002, Ohanissian et al., 2008, Sibbertsen, 2004, Sibbertsen and Kruse, 2009) suggested different tests to distinguish between these two. Shimotsu (2006) proposed a simple splitting test and a difference based test to differentiate the spurious long memory and true long memory. Qu (2011) presented a score type test for the null hypothesis of true long memory. Assaf (2015) analyzed the REITs stocks for spurious long memory and the results supported the presence of true long memory. Al-Shboul and Anwar (2016) analyzed the spurious long memory in insurance and services sectors while Abuzayed et al. (2018) observed long memory in sectoral indices and overall index of Qatar stock exchange. Kristoufek (2010) argues that antipersistence in the important US stock indices is not true but spurious.

It is a well-known fact in literature that the absolute-returns or the squared-returns display long memory with heavy tailed marginal distributions of returns. Present work attempts to analyze the short memory, long or spurious memory in the returns, and volatilities in KSE100 and 10 sectoral indices regarding Pakistan stock exchange (PSX). Long memory in sectoral indices was analyzed by Al-Shboul and Anwar (2016) for five sectors of Jordan Oman stock exchange and Abuzayed et al. (2018) in some sectors of Qatar stock exchange. Cajueiro and Tabak (2005) studied the long-range dependence in individual stocks of the Brazilian market. Khilji and Nabi (1993) observed the long memory in PSX for the sectoral data. Jun and Uppal (1994) recounted the market efficiency by using monthly data of some individual stocks regarding PSX. The sectoral indices and KSE100 index confirmed the inefficiency in PSX over the period 1 January 1989 to 30 December 1993 (Husain and Forbes, 1999). Some other studies rejected the weak form market efficiency by using data of KSE100 index (Hamid et al., 2010, Haque et al., 2011, Nisar and Hanif, 2012). Lee et al. (2018) analyzed an asymmetric market inefficiency in case of Pakistan with an application of the index-based asymmetric –MFDA over different time periods. Kawakatsu and Morey (1999) exposed that the market liberalization did not affect the market efficiency in emerging markets including Pakistan. Cajueiro and Tabak (2008) found the inefficiency in returns and volatilities of the Karachi stock exchange (KSE) considering the time period 1999-2005 with nonparametric techniques.

We contribute to this work in following ways. First, by using the most recent data on daily basis. Second, we observe the predictable trends not only in the KSE100 index but also in some sectors of the PSX. Some studies have reported an evidence of serial dependence but no study has considered such a recent and wide range of data. Thirdly, we contribute by applying the semiparametric and nonparametric techniques to estimate the persistence. Fourthly, we check whether the observed persistent trends are real or spurious. Finally, we observe the evolving (in) efficiency.

In our empirical analysis, we consider the returns and the volatilities with application of different semiparametric and nonparametric techniques. We analyze the KSE100 index and ten sectoral indices: commercial banks (CB), insurance (INS), investment banks (IB), chemical (CHE), cement (CEM), oil & gas (O&G), telecommunication (TC), automobile assembler

(AA), fertilizer (FER), and refinery (REF). Our results indicate that the observed return series are short-memory while a hyperbolic decay of autocorrelation functions and semiparametric memory estimates confirm the existence of long memory in volatilities. Moreover, our results do not provide evidence in favor of spurious memory. An analysis of evolving efficiency shows the fluctuations according to the market conditions which are equivalent to the strong persistence or weak memory over time. We organize this paper by providing some structure of PSX in section 2. Section 3 provides the data and methodology of research work. The graphical analysis, descriptive statistics and other empirical results are presented in section 4. Section 5 comprises the analysis of time-varying market efficiency and section 6 concludes this paper including some recommendations.

4.2 An overview of PSX development

The KSE, Lahore stock exchange (LSE) and Islamabad stock Exchange (ISE) are three main stock markets in Pakistan. KSE was established in 1947, LSE was founded in 1970 as the second oldest market and ISE was established in 1989. Some reforms by the Securities and Exchange Commission of Pakistan (SECP) have been implemented during 2013 to 2017. The corporatization of stock markets in year 2012 was a great reform in history of PSX. All of three markets were not interlinked but the establishment of PSX in 2016 by integrating all three markets was a great step to provide a unique investment market to the overseas financiers. PSX has 573 total listed companies, divided into 35 sectors, and total market capitalization is PKR 7,745.531 billion. KSE100 is the largest index with 100 listed companies and was reported as Asia’s best and fifth best performing index by Bloomberg for year 2016. PSX executed its first planned activity by selling 30 percent shares of PSX to Chinese consortium (china financial futures exchange, Shanghai stock exchange, Shenzhen stock exchange), 5 percent to Pak China Investment Company limited, and 5 percent to HBL. PSX has become the first self-listed stock exchange in 2017. Moreover, it was upgraded to the emerging market in 2017 by Morgan Stanly International Capital (MSIC). Supervision of the PSX by SECP attracted the foreigner investors and investments during 2017.

There are total 35 sectors but we select only 10 and this selection depends on the availability of data on the business recorder website. The Oil & gas index is based on 4 exploration and 8 marketing companies with PKR 118,296 million profit from exploration and RS 37,479 are added by marketing companies. The refinery index contains 4 listed companies and provide returns of RS 16,641 million. The cement sector comprises 20 companies with recorded profit of RS 59,933 in fiscal year 2017-18. The chemical sector with 27 companies adds RS 14,429 million. The contribution of RS 36,497 million by the Technology and communication sector with 10 listing companies. The insurance sector includes 9 life insurers and 41 non-life insurers. The automobile assembler adds RS 36,497.54 and contains 12 companies.

4.3 Data and Methodology

We estimate the long memory in the returns and the volatilities of KSE100 and 10 sectoral indices covering the period from 1-01-2009 to 29-06-2018 with 2351 observations. Time span depends on the availability of data and the data sources are Pakistan stock exchange (PSX) and business recorder (BR) website. A fractionally integrated series has persistent behavior but it looks stationary with the absence of persistent trends in differenced series (Baillie, 1996). Furthermore, we consider the absolute-returns to estimate the volatility as Ding and Granger (1996) and Davidian and Carroll (1987) discussed the domination and robustness of absolute-returns compared to the squared-returns in case of heavy-tailed economic time series.

Suppose p_t denote the closing price at time t, the daily returns r_t are obtained as the logarithmic differences of consecutive daily prices

$$r_t = \log(p_t) - \log(p_{t-1}), \tag{4.1}$$

and we approximate volatility by $|r_t|$.

The stock prices are persistent if they converge back to the equilibrium. A short memory process is represented by an exponential decay while a slow hyperbolic decline in the autocorrelation function specifies a long-memory process. The persistence is measured with long memory parameter d in equation (4.2).

$$(1 - L)^d X_t = u_t, \tag{4.2}$$

with L lag operator and u_t covariance stationary process. Equation (4.2) presents an antipersistent process with negative correlations in interval $-0.5 < d < 0$ while a persistent, fractionally integrated series for $0 < d < 0.5$ with significant positive autocorrelations. The process is nonstationary but mean reverting in interval $0.5 < d < 1$ whereas for $d \geq 1$, the process is nonstationary and non-mean reverting. It is known as explosive process which carry on continually and convergence to equilibrium in this case will be occasional.

We describe different methods to estimate the fractional integration in volatilities of KSE100 and 10 sectoral indices in this section. Additionally, we discuss a method to distinguish between true or spurious long memory. Numerous approaches were suggested to examine the fractional integration including parametric, semiparametric and nonparametric. We estimate the long memory with an application of semiparametric methods as parametric methods require the correct model specification for consistent estimates of fractional integration (Assaf, 2015). Furthermore, the semiparametric methods work under mild regulatory conditions.

Künch (1987) proposed a method to maximize the spectrum of a long memory process by using the Whittle likelihood method of estimation near origin. Robinson (1995) further developed a semiparametric method named local Whittle (LW) to estimate fractional integration parameter d based on this objective function. He also established the asymptomatic properties, consistency, and asymptotic normality of LW estimator in $(-0.5, 0.5)$. The estimator is simply defined as

$$\hat{d} = argmin_d = \left\{\ln\left(\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{\lambda_j^{-2d}} - \frac{2d}{m}\sum_{j=1}^{m}\ln\lambda_j\right)\right\}. \quad (4.3)$$

This method is easy to compute with $I(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t e^{i\lambda_j t}\right|^2$, $\lambda_j = \frac{2\pi j}{T}$, and depends on only one user chosen bandwidth parameter $m, m \to \infty, \frac{m}{T} \to 0$. Phillips and Shimotsu (2004) investigated that the LW estimates are consistent, asymptotically normal, and follow a non-normal distribution in the regions $(0.5,1)$, $(0.5,0.75)$, and $(0.75,1)$ respectively. Robinson (1995) stated that the condition of $m < \frac{T}{2}$ must hold to control the aliasing effects and also proved $\sqrt{m}(\hat{d} - d_0) \to N(0, \frac{1}{4})$ under some mild conditions on the bandwidth parameter.

Due to the irregular asymptotic theory of the LW estimator at $d = 3/4$ and $d = 1$, Shimotsu and Phillips (2005) extended this estimator to satisfy both stationary and nonstationary cases as exact local Whittle (ELW). The estimation of ELW does not require any pre-filters or differencing. The objective function of a series x_t is

$$Qm(j,d) = \sum_{j=1}^{m}\log\left(G\lambda_j^{-2d}\right) + \frac{1}{G}I_{(1-L)^d x}, \quad (4.4)$$

and the estimator is given in equation (4.5)

$$\hat{d}_{ELW} = argmin_{d\epsilon(\Delta_1,\Delta_2)}R(d), \quad (4.5)$$

where Δ_1, Δ_2 present the lower and upper bounds respectively and

$R(d) = \log\hat{G}(d) - 2d\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j$, $\hat{G}(d) = \frac{1}{m}\sum_{j=1}^{m}\lambda_j^{2d}I_x(\lambda_j)$. The estimator $\hat{d}$ has the limit distribution as $\sqrt{m}(\hat{d}_{ELW} - d_0) \to N(0, \frac{1}{4})$ and is consistent for $\Delta_1 - \Delta_2 \leq {}^9/_2$ while $T \to \infty$.

Shimotsu (2010) developed the two-step exact local Whittle (2sELW) to take into account any unknown mean and polynomial trends. At a first stage they used a tapered version of the LW and defined a new objective function. The distribution of 2sELW is $N(0,1/4)$ for $d \ \epsilon \ (-0.5, 2)$ or $d \ \epsilon \ (-0.5, 7/4)$ in the presence of polynomial trends in data. This estimator ensures all the desirable properties of the ELW. An unknown estimator of μ_0 is

$$\tilde{\mu}(d) = \omega(d)\bar{x} + \big(1 - \omega(d)\big)x_1, \tag{4.6}$$

here $\omega(d)$ stands for weight function which is twice continuous differentiable and equals to 1 and 0 for $d \leq 0.5, d \geq 3/4$ respectively. The estimator is

$$\hat{d}_{2sELW} = argmin_{d\epsilon(\Delta_1,\Delta_2)} R(d), \tag{4.7}$$

with lower and upper bounds Δ_1, Δ_2 and $-\infty < \Delta_1 < \Delta_2 < \infty$. Now the ELW objective function is based on $x - \tilde{\mu}(d)$ as

$$R_F(d) = \log \hat{G}_F(d) - 2d\frac{1}{m}\sum_{j=1}^{m} \log \lambda_j, \tag{4.8}$$

where $\hat{G}_F(d) = \frac{1}{m}\sum_{j=1}^{m} \lambda_j^{2d}\, I_{(1-L)^d(x-\tilde{\mu}(d))}(\lambda_j)$ and $I_{(1-L)^d(x-\tilde{\mu}(d))}\big(\lambda_j\big)$ is the periodogram of $(1-L)^d(x - \tilde{\mu}(d))$. He argued that the first value x_1 and empirical mean $\bar{x}$ of the observed series can be used as a suitable estimator of the unknown mean in case of large and small values of d respectively. Moreover, use of x_1 or $\bar{x}$ is appropriate for $d\ \epsilon\ (0.5, 0.75)$.

Qu (2011) proposed a test statistic to test the null hypothesis of true long memory against the alternative of spurious long memory in frequency domain. A special feature of this test is that it does not require the number of breaks and form of the trend so, it is very simple and open. This test is based on the spectral density near origin. Moreover, its null distribution is simple and free from any annoying factors. The test statistic is

$$W_{Qu} = sup_{r\epsilon[\varepsilon.1]}\left(\sum_{j=1}^{m} v_j^2\right)^{-0.5} \left|\sum_{j=1}^{[mr]} \left\{\frac{I_j}{G(\hat{d})\lambda_j^{-2\hat{d}}} - 1\right\}\right|, \tag{4.9}$$

with $v_j = log\lambda_j - \frac{1}{m\sum_{j=1}^{m} log\lambda_j}$, m bandwidth parameter, $\hat{d}$ is the LW estimator of fractional integration and a small trimming parameter ε. He recommends to use the trimming parameter 0.02 and 0.05 for sample size greater than 500 or less than 500 respectively. Consistency and limiting distribution of the test static are also provided for $d\epsilon(0, 0.5)$. Furthermore, he suggested to use the bandwidth parameter $m = T^{0.7}$.

4.4 Empirical results

Before discussing the results of semiparametric long memory estimates, we present a graphical analysis for the ACF graphs of volatilities in Figure 4.1. All series display a slow hyperbolic decay till 30 lags which might indicate the existence of long-range dependence in the absolute-returns.

The descriptive statistics for KSE100 and the sectoral indices are reported in Table 4.1. We observe small mean (m) values while the standard deviations (s) are large. All reported skewness (sk) measures are different from zero and reported negative values regarding 6 indices. These negative values suggest the domination of large negative returns. Large values of the kurtosis (k) measures for 8 returns confirm the non-normality of the observed data. A high degree of kurtosis is also consistent with the literature of financial return series. A Leptokurtic distribution with such skewness and kurtosis properties is common in stock returns (Henry, 2002).

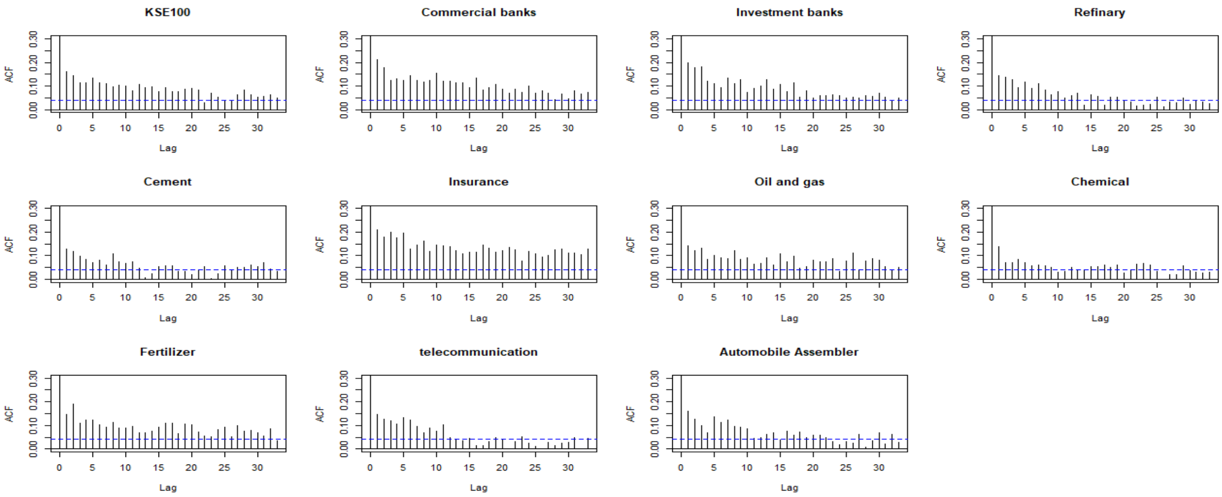


Figure 4.1: ACF of absolute-returns

	KSE100	CB	IB	REF	CEM	INSU	O&G	CHEM	FERT	TELC	AUTO
m	0.001	0.001	0	0.001	0.001	0	0.001	0.001	0.001	0	0.001
s	0.011	0.014	0.026	0.019	0.016	0.013	0.014	0.019	0.023	0.019	0.013
Sk	-0.3	0.03	-6.03	0.18	0.11	-0.05	-2.41	-10.07	24.29	0.06	-0.56
K	7	8	153	4	4	15	46	324	938	5	8
ρ(1)	0.1	0.1	0.16	0.13	0.11	0.18	0.11	0	0.03	0.12	0.2
ρ(2)	0.04	0.08	0.04	-0.01	0.02	0.09	0.02	0.01	0	-0.01	0.06
ρ(10)	0.04	0.03	0.04	0.04	0.03	0.03	0.02	0.02	0.04	0.04	0.02
Q(20)	47	66	96	63	55	142	45	25	11	47	117
ρa(1)	0.159	0.213	0.2	0.144	0.13	0.21	0.143	0.138	0.145	0.148	0.159
ρa(2)	0.143	0.179	0.18	0.138	0.12	0.18	0.12	0.07	0.19	0.126	0.125
ρa(10)	0.1	0.153	0.075	0.076	0.066	0.145	0.093	0.031	0.091	0.074	0.086
Qa(20)	525.5	780	648	333.4	251	1040.6	416.9	187	546.1	345.8	379.5
ADF	-13	-13.5	-11.6	-12.8	-12	-12.7	-13.1	-13	-12.7	-12.8	-12.3
KPSS	0.3	0.1	0.3	0.2	0.3	0.4	0.3	0	0.3	0.1	0.1

Table 4.1: Descriptive statistics

The autocorrelation function at lags 1, 2 and 10 are reported positive in returns (ρ) and volatilities (ρ_a). The existence of linear and non-linear dependence is specified with an application of the Ljung–Box Q-statistic (Q) in daily returns and volatilities (up to lag 20). We start the empirical analysis with test of stationary or unit root series. The Augmented Dickey Fuller (ADF) test of Dickey and Fuller (1979) confirms that the returns and the volatilities are stationary. The ADF may produce inconsistent results for fractionally integrated series so the results of the KPSS test Kwiatkowski et al. (1992) are also reported. KPSS provide better results in a case of fractional integration alternative as Lee and Schmidt (1996) proved its equivalence to R/S statistic of Lo (1991) in presence of long-range dependence. The results of ADF and KPSS are also reported in Table 4.1.

The long memory estimates for the returns with LW, ELW, and 2sELW estimators are reported for different bandwidths in Table 4.2. Different studies used different truncation parameter in previous works such as Charfeddine (2014) used $T^{0.5}$, $T^{0.6}$, Charfeddine and Khediri (2016) used $T^{0.6}$, Abuzayed et al. (2018) used $T^{0.5}$, $T^{0.6}, T^{0.7}, T^{0.8}$, Al-Shboul and Anwar (2016) and Al-Shboul and Alsharari (2018) used $T^{0.4}$, $T^{0.5}, T^{0.55}, T^{0.6}, T^{0.65}, T^{0.7}, T^{0.8}$ and $T^{0.5}$, $T^{0.6}, T^{0.7}$ respectively. The standard errors for the memory estimates are also reported in the last column of Table 4.2. We observe the positive estimates of KSE100 based on all estimators with only exception for $m = T^{0.6}$. The negative estimates of d regarding the commercial banks, the chemical, and the fertilizer sectors are an indication of the negative autocorrelation or antipersistence. The LW and the ELW estimates of the IB sector are significantly different from 0 at 5% level of significance for bandwidth $T^{0.65}$, $T^{0.75}$as well as for all the bandwidths with 2sELW. A significant evidence of long memory is obvious in the cement and the insurance sectors except 2 cases with 2sELW. The long memory estimates are significant for bandwidth $T^{0.75}$ regarding the automobile assembler. The sectoral returns for oil & gas and telecommunication exhibit the insignificant estimates of fractional integration by implementing all three estimators. Moreover, the negative values present antipersistence in the stock returns which implies the more chances for the positive (negative) moves to be followed by a negative (positive) shift. Therefore, there is more frequent reversion in an antipersistent process than the random process (Kristoufek, 2010). The hypothesis of $d = 0$ is rejected at 5% level of significance regarding CB, CEM, and INS sectors which shows the existence of predictable trends in these returns. Overall, our results illustrate a fast exponential decay of autocorrelation function in the stock returns of KSE100 and the sectoral indices.

The estimates of the LW regarding volatilities falls within the stationary region ($0 < d < .5$) and specify the inefficiency and slow reversion to equilibrium (Ijasan et al., 2017). Moreover, all the estimates of long memory are significant at 1% level of significance. The hypothesis of zero persistence ($d = 0$) is rejected in all absolute-returns. The results of the ELW are parallel to the LW which provide the significant and stationary long memory estimates in the interval (0.12, 0.39) except CB. The results with the 2sELW also provide the similar results regarding all volatilities.

		KSE100	CB	IB	REF	CEM	INS	O&G	CHE	FER	TC	AA	s.e
LW	0.6	-0.062	-0.035	0.043	-0.006	***0.100***	0.049	-0.023	-0.025	-0.043	-0.008	0.066	0.049
	0.65	-0.010	-0.045	***0.080***	0.024	***0.086***	***0.090***	-0.006	-0.018	-0.020	0.026	0.061	0.040
	0.7	0.002	-0.044	0.037	0.005	***0.081***	***0.101***	-0.003	-0.033	-0.044	0.017	0.052	0.033
	0.75	0.023	0.027	***0.073***	0.009	***0.069***	***0.102***	0.009	-0.019	-0.027	0.000	***0.076***	0.027
ELW	0.6	-0.026	-0.008	0.067	0.030	***0.121***	0.091	0.005	-0.002	-0.013	0.016	0.084	0.049
	0.65	0.000	-0.031	***0.095***	0.046	***0.097***	***0.117***	0.012	-0.004	-0.002	0.040	0.070	0.040
	0.7	0.011	-0.034	0.046	0.021	***0.088***	***0.118***	0.010	-0.025	-0.032	0.026	0.058	0.033
	0.75	0.030	0.038	***0.083***	0.023	***0.077***	***0.117***	0.020	-0.011	-0.016	0.008	***0.084***	0.027
2SELW	0.6	-0.017	-0.001	***0.104***	0.056	0.077	***0.134***	0.006	-0.002	-0.009	0.017	0.091	0.049
	0.65	0.005	-0.022	***0.110***	0.066	0.073	-0.044	0.024	-0.002	-0.001	0.038	0.075	0.040
	0.7	0.010	-0.030	***0.069***	0.039	***0.071***	***0.130***	0.009	-0.027	-0.022	0.024	0.060	0.033
	0.75	0.032	0.041	***0.098***	0.031	***0.067***	***0.126***	0.026	-0.010	-0.014	0.007	***0.087***	0.027

Table 4.2 Fractional integration in returns (bold Italic values show significance at 5%)

		KSE100	CB	IB	REF	CEM	INS	O&G	CHE	FER	TC	AA	s.e
LW	0.6	0.319	0.364	0.3	0.228	0.225	0.326	0.302	0.227	0.293	0.218	0.27	0.049
	0.65	0.277	0.301	0.276	0.242	0.241	0.32	0.284	0.183	0.271	0.271	0.252	0.040
	0.7	0.26	0.254	0.256	0.26	0.23	0.312	0.243	0.185	0.274	0.269	0.278	0.033
	0.75	0.23	0.226	0.232	0.222	0.181	0.288	0.198	0.159	0.23	0.234	0.199	0.027
ELW	0.6	0.321	0.396	0.307	0.262	0.237	0.321	0.318	0.244	0.327	0.239	0.297	0.049
	0.65	0.281	0.315	0.278	0.262	0.251	0.312	0.294	0.193	0.287	0.287	0.266	0.040
	0.7	0.265	0.263	0.257	0.275	0.238	0.305	0.25	0.192	0.284	0.279	0.288	0.033
	0.75	0.237	0.236	0.236	0.235	0.19	0.286	0.205	0.167	0.24	0.244	0.208	0.027
2sELW	0.6	0.286	0.298	0.252	0.163	0.207	0.258	0.273	0.226	0.355	0.24	0.42	0.049
	0.65	0.264	0.264	0.237	0.337	0.216	0.256	0.258	0.187	0.178	0.274	0.384	0.040
	0.7	0.253	0.235	0.228	0.327	0.212	0.255	0.228	0.189	0.186	0.27	0.384	0.033
	0.75	0.232	0.221	0.217	0.185	0.181	0.247	0.196	0.167	0.18	0.242	0.155	0.027

Table 4.3: Long range dependence in absolute-returns

We conclude on the basis of results provided in Table 4.3 that all the volatilities exhibit stationary and significant long memory. Moreover, these results imply the fractional memory estimates which are not consistent with an ARMA or ARIMA models. The presence of fractional memory ($d > 0$) in PSX volatilities specifies that the effects of shocks take longer time to vanish and have persistent trends which may be useful in future predictions. Furthermore, it implies that the positive (negative) swings are followed by positive (negative) trends in volatilities.

As we discussed in section one that the spurious memory and the long memory share some common characteristics which may confuse these two issues. So our next step is to test for the spurious memory in returns and volatilities. We apply the method of Qu (2011) to test the hypothesis of true long memory against the alternative of spurious memory in sectoral indices and KSE100. The results of this test are described in Table 4.4.

	r	vol		r	vol
KSE100	0.660451	.6524211	**O&G**	0.334898	1.268895
CB	0.471609	***1.675285***	**CHE**	0.631944	0.85524
IB	1.010611	0.922634	**FER**	0.659148	1.048001
REF	0.601967	0.615963	**TC**	0.39597	0.564244
CEM	0.641878	0.758868	**AA**	0.756019	0.512665
INS	0.686491	***1.536351***			

Table 4.4: Long memory against spurious memory (bold and Italic show significance at 5%)

The critical values for this test with $\varepsilon = 0.05$ are reported by the author as 1.022, 1.155, 1.277, and 1.426 at 10%, 5%, 2.5% and 1% respectively. The values in Table 4.4 are reported for $\varepsilon = 0.05, m = T^{0.7}$ for returns and volatilities. Regarding the volatilities, the null hypothesis of true long memory is rejected for commercial banks and insurance indices at 1% level of significance. Overall, we find 2 rejections of true long memory out of eleven indices in support of spurious memory.

4.5 Time-varying long memory

In last section, we assumed a fixed level of market efficiency which implies a constant degree of persistence during the whole time period. This assumption seems unrealistic in an emerging stock market like Pakistan where structural, operational, and environmental changes are frequent. According to (Lo, 2004, 2005), the assumption of permanent equilibrium state in any market is inappropriate. The temporal persistence or evolving market efficiency is examined with time-varying long memory. Lim and Brooks (2011) stated that the level of market efficiency may evolve over time. The reasons behind this evolving efficiency may include the speculative bubbles, the financial disasters, the market breakdowns, and any sudden news announcements (Boubaker, 2017). Dajcman (2012) concluded that the market efficiency is a dynamic feature in eight European stock markets and was coincided with the major financial crises during the study period. Cajueiro and Tabak (2004a, 2004b) applied a rolling window approach to estimate and rank the time-varying market efficiency in emerging markets. Auer (2016) estimated a rolling Hurst exponent (H) by applying different fractal estimation techniques for 21 emerging markets using a 27 year dataset. Hull and McGroarty (2014) examined the returns and the volatilities for evolving efficiency in a sample of 22 countries including the secondary and the advanced markets. Lim and Brooks (2010) observed the more frequent price deviations in emerging stock markets compared to developed markets by employing bicorrelation method. Charfeddine et al. (2018) applied the state-space generalized autoregressive conditional heteroskedasticity in the mean to measure the evolving efficiency in bond markets of USA, UK, South Africa, and India. Al-Shboul and Alsharari (2018) reported an evolving efficiency in the UAE markets. Lim et al. (2008) analyzed the evolving efficiency in 8 Asian markets.

In present section, we estimate a time-varying inefficiency in KSE100 and sectoral volatilities with an application of the rolling window approach. Along with the semiparametric methods discussed in section 3, we use a nonparametric rescaled range R/S method to estimate the Hurst exponent. This statistic introduced by Hurst (1951), hence named Hurst coefficient H, calculates the long memory parameter. The underlying process is white noise for $H = 0.5$, persistent for the values greater than 0.5, and antipersistent for $H < 0.5$. Computation of the R/S statistic for logarithmic returns r_t is performed by dividing the return series into A sub-periods of size n. We use I_a $(a = 1,2,3, \dots A)$ as labels of the sub-series. $N_{k,a}$ $(k = 1,2,3, \dots n)$ defines the elements in I_a. Now, calculate average μ_{I_a} and standard deviation S_{I_a} for all sub-series. The mean adjusted series $Y_{k,a}$ and the cumulative deviations $Z_{k,a}$ are calculated for each sub-series I_a. The range R_{I_a} is computed as the difference between maximum $Y_{k,a}$ and minimum $Y_{k,a}$ in each sub-series I_a. Next step is to normalize the range by dividing with corresponding S_{I_a}. The average of all the rescaled ranges is given as

$$\left({}^{R}/_{S}\right)_n = 1/A \sum_{a=1}^{A} \frac{R_{I_a}}{S_{I_a}}. \tag{4.10}$$

The modified R/S for small sample size by (Annis and Lloyd, 1976) and (Peters, 1996) is

$$\boldsymbol{E}\left({}^{R}/_{S}\right)_n = \begin{cases} \dfrac{n-\frac{1}{2}\Gamma\left(\frac{n-1}{i}\right)}{n\sqrt{\pi}\Gamma\left(\frac{n}{2}\right)} \sum_{i=1}^{n-1} \sqrt{\dfrac{n-i}{n}} & , n > 340 \\ \dfrac{n-\frac{1}{i}}{n\sqrt{\frac{n\pi}{2}}} \sum_{i=1}^{n-1} \sqrt{\dfrac{n-i}{n}} & , n \leq 340 \end{cases}. \tag{4.11}$$

The coefficient H is calculated by a new statistic R/S-AL as

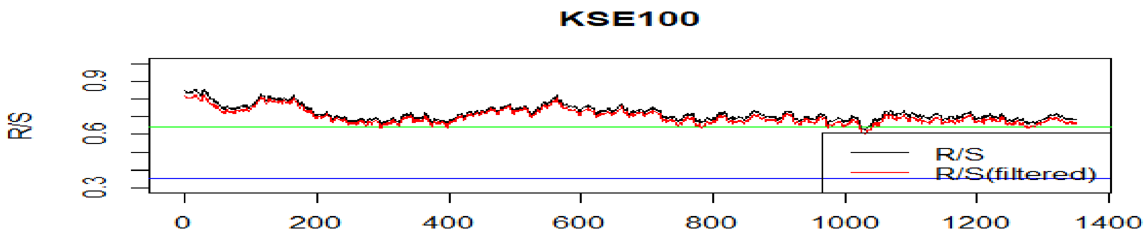
$$0.5 + slope[\left({}^{R}/_{S}\right)_n - \boldsymbol{E}\left({}^{R}/_{S}\right)_n].$$

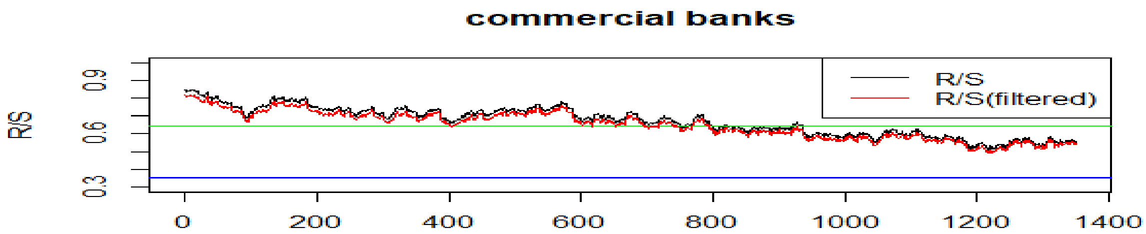
We use a rolling window methodology for the estimation of a time-varying H to consider the possible variations in market (in) efficiency over time. A very long or too short window length may not provide a clear picture of variations in market efficiency over time. Different window lengths were used in different studies: (Charfeddine and Khediri, 2016, Wang and Wu, 2012) used a rolling sample window of 500 values regarding two years of data, Auer (2016) used 240 days window length considering one year, Dajcman (2012) used 300 days as window length. We use a window length of 1000 values regarding four years of data following (Cajueiro and Tabak, 2004b, Jebabli and Roubaud, 2018, Sensoy and Hacihasanoglu, 2014) as they suggest that a four year window can explain the establishment between fluctuations in scaling exponent and analytical stability. Sensoy and Hacihasanoglu (2014) considered this window length sufficient for valid empirical analysis as it coincides to the political cycles in various countries. We estimate the modified R/S for volatilities with window length 1000 corresponding to almost 4 years of the data. We compute the Hurst coefficient for the first 1000 values, roll the window to include a next observation and exclude the first value. In this way, we get a series of total 1349 coefficients for each volatility series. The absence of a natural significance test is the greatest drawback of the nonparametric R/S statistic (Ma et al., 2016, Wang and Wu, 2012). Estimation of the confidence intervals with the Gaussian distribution assumptions for the heavy tailed stock returns is not valid (Jebabli and Roubaud, 2018). However, Qian and Rasheed (2004) considered strong persistence for $H \geq 0.65$, week persistence for $0.5 \leq H \leq 0.65$ and Weron (2002) calculated the 95% empirical confidence interval for RS-AL in equation (4.12) with $N = \log_2 1000$.

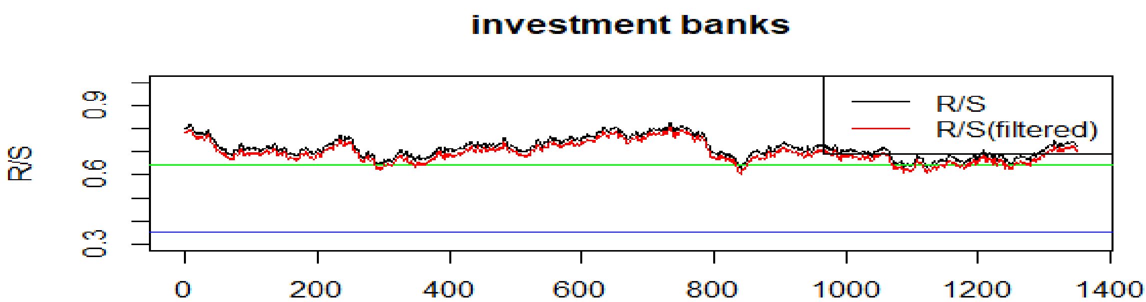
$$\begin{array}{ll} \text{Lower bound} & 0.5 - \exp(-7.33\log(\log N) + 4.21 \\ \text{Upper bound} & \exp(-7.20\log(\log N) + 4.04) + 0.5 \end{array} \tag{4.12}$$

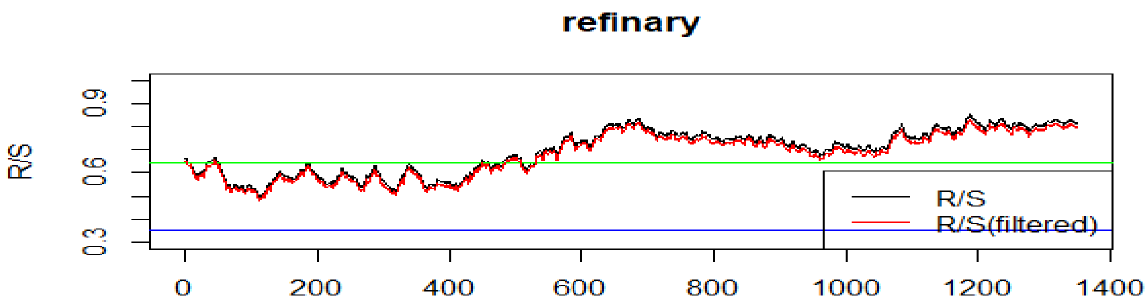
The graphs of time-varying Hurst exponent with 95% confidence interval are presented in Figure 4.2. We observe the random movements in KSE100 graph with a negative trend value. The value of H is greater than 0.65 during the whole period except September 2014 and April 2017, where memory estimates fall within the weak persistence region. The graph of evolving efficiency with nonparametric method for commercial banks shows strong persistent behavior till 2016 but after that, we observe a weak persistence with values greater than 0.5 but less than 0.65. Overall a decreasing trend is dominant in this index. The graph of investment banks index show a mixture of upward and downward trends over time. The evidence of week persistence is observed for March 2014, May 2016, and for some months between May 2017 and January 2018 in case of investment banks. The refinery sector shows an upward trend and strong persistence from March 2015 until June 2018. Before 2015, the values of H moves between (0.5, 0.65) and we observe an antipersistence in June 2013. Weak long memory moving between (0.55, 0.65) can be seen in the cement sector for some months of 2013 and afterwards a smooth persistence moving in interval (0.65, 0.75) prevails.

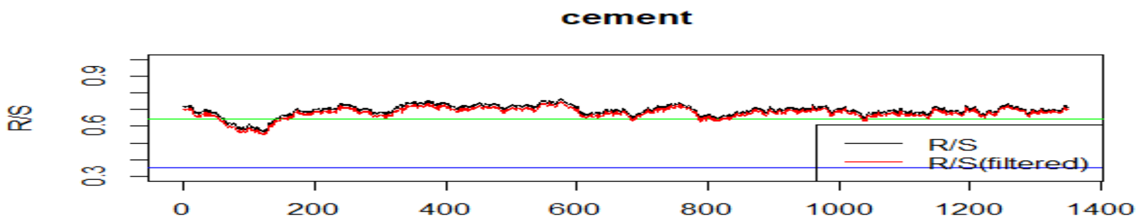
The movements from weak persistence to strong persistence sometimes between the end months of 2014 till starting months of 2017 for insurance sector are apparent. Outside of this time span the persistence lies in the strong region but with a downward trend till 2014 and upward trend afterwards. A graph of the fertilizer sector shows strong persistence (0.67, 0.9) during study period.

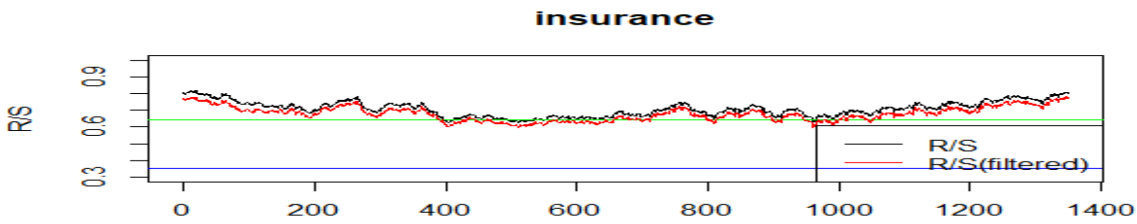


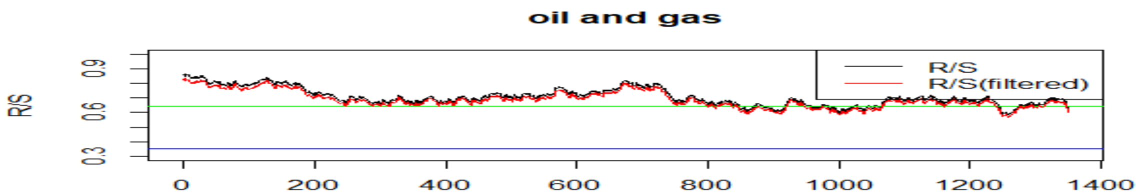


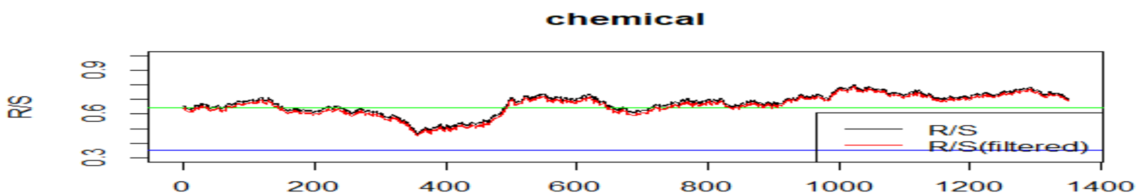





cement
R/S
0.3 0.6 0.9
0 200 400 600 800 1000 1200 1400
R/S
R/S(filtered)


insurance
R/S
0.3 0.6 0.9
0 200 400 600 800 1000 1200 1400
R/S
R/S(filtered)


oil and gas
R/S
0.3 0.6 0.9
0 200 400 600 800 1000 1200 1400
R/S
R/S(filtered)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R/S
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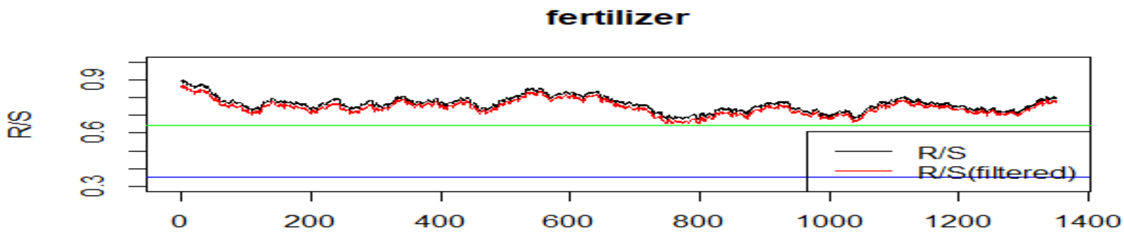
R/S
R/S(filtered)


fertilizer
R/S
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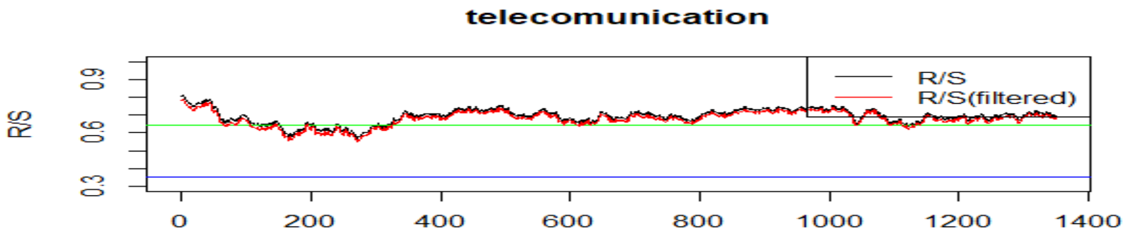
R/S
R/S(filtered)


telecomunication
R/S
0.3 0.6 0.9
0 200 400 600 800 1000 1200 1400
R/S
R/S(filtered)

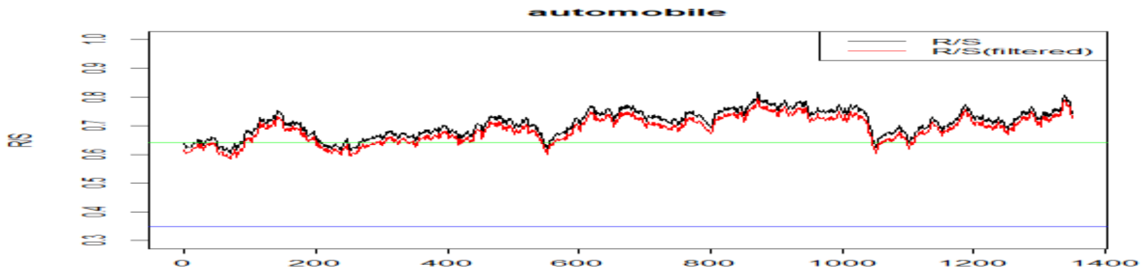


Figure 4.2: Graph of time-varying Hurst exponent for R/S and R/S-(AR-GARCH)

Periods of weak and strong persistence are evident from year 2016 to 2018 while strong before the specified time interval in O&G. An antipersistent trend at mid-2014 was followed by an upward trend of strong persistence in the chemical sector. We observe fragile persistence during 2014 to 2018 in telecommunication. We find an upward persistent trend in automobile assembler during last study period while faint dependence in the years 2013, 2015, and 2017.

	Mean				**SD**			
	R/S	**R/S(A-G)**	**LW**	**ELW**	**R/S**	**R/S(A-G)**	**LW**	**ELW**
KSE100	0.719	0.701	0.148	0.166	0.042	0.042	0.034	0.033
CB	0.67	0.649	0.153	0.17	0.08	0.077	0.029	0.027
IB	0.717	0.696	0.191	0.21	0.043	0.043	0.033	0.032
REF	0.694	0.678	0.191	0.211	0.095	0.093	0.031	0.031
CEM	0.695	0.68	0.173	0.193	0.033	0.034	0.019	0.021
INS	0.709	0.68	0.208	0.227	0.045	0.043	0.029	0.027
O&G	0.706	0.688	0.16	0.177	0.059	0.056	0.031	0.03
CHE	0.673	0.657	0.119	0.137	0.071	0.072	0.054	0.054
FER	0.764	0.744	0.176	0.194	0.041	0.04	0.047	0.046
TC	0.693	0.679	0.213	0.232	0.041	0.042	0.023	0.025
AA	0.708	0.687	0.229	0.246	0.045	0.044	0.031	0.029
	Skewness				**Kurtosis**			
	R/S	**R/S(A-G)**	**LW**	**ELW**	**R/S**	**R/S(A-G)**	**LW**	**ELW**
KSE100	0.963	0.808	1.477	1.16	0.619	0.148	3.388	2.337
CB	-0.052	-0.073	1.645	1.429	-0.971	-0.99	3.225	2.523
IB	0.403	0.399	0.06	0.061	-0.431	-0.46	-1.038	-1.065
REF	-0.313	-0.305	-0.408	-0.458	-1.232	-1.231	-0.936	-0.873
CEM	-1.247	-1.367	-0.294	-0.268	2.491	2.763	-0.697	-0.91
INS	0.283	0.266	0.948	0.837	-0.735	-0.765	0.726	0.533
O&G	0.556	0.502	0.107	-0.03	-0.338	-0.397	-0.277	-0.571
CHE	-0.788	-0.734	0.24	0.27	0.196	0.059	-1.251	-1.256
FER	0.421	0.268	0.48	0.412	0.228	0.003	-0.461	-0.65
TC	-0.475	-0.573	-0.131	-0.118	0.204	0.146	-1.124	-1.202
AA	-0.192	-0.175	0.99	0.875	-0.882	-0.852	0.781	0.72

Table 4.5: Descriptive statistics for time-varying series

The graphical analysis of time-varying memory with the nonparametric technique suggests that the (in) efficiency evolves over time in sectoral indices and KSE100. Along with R/S, we estimate R/S-(AR-GARCH) for the filtered series with application of AR(1)-GARCH(1,1) pursuing Cajueiro and Tabak (2004b) to eliminate any short term volatility effects. The graphs of R/S-(AR-GARCH) are also presented in Figure 4.2. We estimate a time-varying long memory estimator d with application of LW and ELW. Following Wang and Wu (2012), we use bandwidth as $1/10$ of series length. As our window length is 1000, so we use $m = 100$. It is difficult to report the long memory results for each window so we present the results graphically in Figure 4.3 in appendix.

Although, the LW rolling window results are not directly comparable to the parametric ones, the trends are equivalent to the latter method. The graph of KSE100 displays a slow decline in autocorrelation function with average value of 0.165. We observe the lowest values over the period mid-2014 till the first quarter of 2015. The high values at start of the graph may relate to the decline of foreign investments (Arshad et al., 2016). Graphs of commercial banks and investment banks show persistence equivalent to previous trends in 2016. The refinery, the insurance, the automobile, and the chemical sectors present an upward trend while the inefficiency levels in cement and refinery sectors are parallel to the parametric estimates. Figure 4.3 displays the graphs for time-varying windows by applying LW and ELW. Overall, the long memory estimates in all volatilities are not constant and vary over time. As we get several estimates of H and d based on the rolling sampling techniques, we report the descriptive statistics for the dynamic of long memory estimates in Table 4.5. We test the existence of some consistent trend by employing a nonparametric Mann-Kendal (MK) trend test to the rolling windows H series. MK investigates the null hypothesis of no monotonic trend against the alternative of trend. The test statistic is written as

$$Z = \begin{cases} (S-1)/\sqrt{var(S)}, & S > 1 \\ 0, & S = 0 \\ (S+1)/\sqrt{var(S)}, & S < 1 \end{cases}. \tag{4.13}$$

Where $S = \sum_{i=1}^{n} \sum_{j=j+1}^{n} sgn(R_i - R_j)$ with the relative ranks $R_1, R_2, R_3, \ldots R_n$ for n time series values $X_1, X_1, X_1, \ldots, X_n$. Moreover R_i is an indicator function with values $(-1, 0, 1)$. An increasing (decreasing) trend is indicated with positive (negative) value of the test statistic. The Z statistic is normally distributed with critical values: 1.645, 1.96 and 2.57 at 10%, 5% and 1% significance levels respectively. Results in Table 4.6 are for window length 1000 and for three series: R/S, R/S-(AR-GARCH), and LW. Negative value of Z indicates a downward trend but only this information is not sufficient to conclude that sectoral indices are becoming efficient. The decreasing trend may be due to the evolving efficiency from strong to weak efficiency (Auer, 2016). Our results suggest that degree of inefficiency changes over time due to market circumstances and institutional elements. With confirmation of trend, we estimate the slope β of trend by using a nonparametric method of Sen (1968).

$$\beta = \text{Median}\left(\frac{(x_i - x_j)}{j-i}\right), \tag{4.14}$$

where $1 < i < j < n$. The estimator β is the median of slope for all possible pairs of data. The increasing or decreasing trend can be determined with positive or negative value of β.

A negative slope coefficient regarding KSE100 indicates an evolving inefficiency in PSX. The slopes of the commercial bank, investment banks, oil & gas, and fertilizers are also negative with parametric and nonparametric methods. We observe a time-varying inefficiency in refinery, chemical, telecommunication, and automobile assembler sectors with positive slope coefficients. All this inspection suggests that efficiency or long-range dependence is not a static phenomenon but it varies over time due to financial, political, and business events.

	R/S		**R/S(AR-GARCH)**		**LW**	
	Z	**β**	**Z**	**β**	**Z**	**β**
KSE100	-0.3456	-0.5590	-0.38064	-0.6110	-0.01161	-0.0150
CB	-0.77549	-1.9450	-0.77012	-1.8610	-0.32241	-0.3290
IB	-0.16675	-3.3320	-0.17982	-3.1120	-0.19819	-0.6420
REF	0.59651	-4.7180	0.589902	-4.3630	0.321374	-0.9560
CEM	0.002367	-6.1050	0.045491	-5.6140	-0.1442	-1.2700
INS	0.05486	-7.4910	0.040892	-6.8650	-0.2846	-1.5830
O&G	-0.42044	-8.8770	-0.4243	-8.1150	0.293054	-1.8970
CHE	0.486451	-10.2640	0.49049	-9.3660	0.316925	-2.2110
FER	-0.24309	-11.6500	-0.21786	-10.6170	-0.58582	-2.5240
TC	0.1465	-13.0370	0.160704	-11.8680	0.420487	-2.8380
AA	0.419893	-14.4230	0.428747	-13.1190	-0.34827	-3.1510

Table 4.6: Nonparametric trend estimation

Being an emerging market, the existence of time-varying long memory in PSX is not surprising. The level of market efficiency is linked to the information inflow which cause the efficiency to evolve over time. Some authors cited the herding behavior of traders as the main cause of evolving efficiency because the investors follow each other blindly and do not pay attention to the current market information (Al-Shboul and Alsharari, 2018, Hull and McGroarty, 2014). Irrational and herding behavior of the investors due to lack of information may cause such long memory dynamics in our study. The absence of transparency in PSX may be a contributing factor behind our results since Charfeddine and Khediri (2016) considered the failure to match the international standards of transparency and corporate disclosures as a reason of evolving efficiency. The market shocks regarding to the political instabilities and the economic crises may be responsible for inefficient PSX market. Political and economic disasters are interlinked to the market efficiency rather the market booms (Kim et al., 2011). A low number of market-makers and fewer speculation activities may also be a factor of this evolving (in) efficiency since these features develop a thin trading phenomenon by decreasing the trade volume as well as increasing the ask-bid spread (Al-Shboul and Alsharari, 2018). The less development level of PSX can be a potential factor. According to Rizvi et al. (2014), the level of market development is positively correlated with the market efficiency. Therefore, there is a need of development in the Islamic countries markets for an efficient resource allocation. Time-varying efficiency was characterized by factors such as the variations in market circumstances, the number and configuration of market participants, and the profit opportunities (Khuntia and Pattanayak, 2018). A positive correlation between liberalization and market efficiency was examined by Cajueiro et al. (2009) for the Athens stock market. So, less foreign investment may be a possible cause of fractional integration in PSX. We can consider less liquidity and thin trading as the factors parallel to Thai markets, where evolving long memory was characterized by the market size and liquidity (Bariviera, 2011). Exchange rate fluctuations due to the budget deficits along the borrowed money, and the lack of appropriate price mechanism may also be contributing factors.

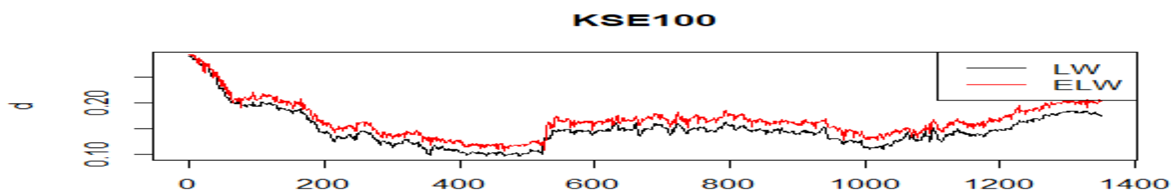
4.6 Conclusion

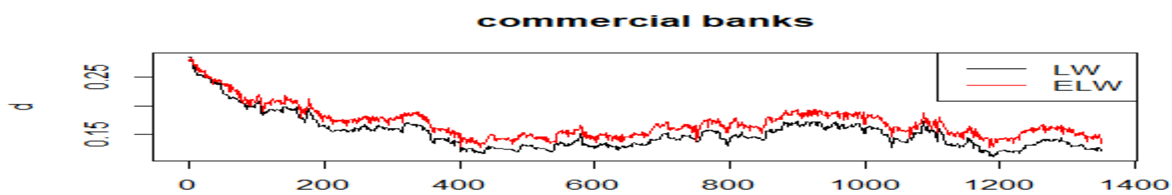
This study analyzes the presence of long memory in returns and volatilities of the KSE100. Furthermore, we test for the true or spurious long memory. We use the absolute-returns as volatility estimator in this study. The study focus on 10 sectors and KSE100 with 2351 observations over the period 13-05-2008 to 01-03-2018. The descriptive statistics show that all the series are nonlinear with negative skewness measure and high kurtosis values. The test of normality and independent autocorrelations are also rejected. The results of the ADF test show that all series are unit root process at levels while stationary in first differences. We apply three semiparametric estimators including the LW, the ELW, and the 2sELW, in order to estimate the persistence or antipersistence in the returns and the volatilities regarding the PSX. The long memory estimates are based on the bandwidths $m = T^{.6}, T^{.65}, T^{.7}, T^{.75}$ and suggest the predictable trends in returns and volatilities except four return series. All memory estimates lie in the stationary region $0 < d < 0.5$. Moreover, our results reject the hypothesis of trend stationary series regarding the volatilities.

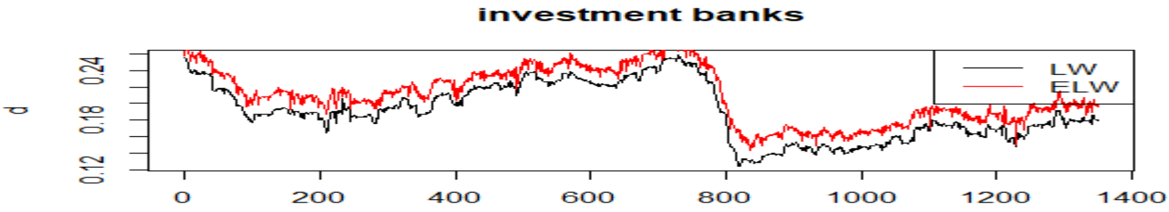
We test for the true or spurious long memory by using a LM type test of Qu (2011) with $m = T^{.7}$ and $\varepsilon = 0.05$. Results reject the hypothesis of true memory for 2 indices at 1% level of significance. In order to check whether the market (in) efficiency is static or evolve over time, we apply a rolling window technique. The estimated results of the time-varying analysis show that the (in) efficiency is not a static phenomenon. Therefore, it changes according to the market conditions and other important events. We find a decreasing trend in KSE100, commercial banks, oil & gas and fertilizer sectors with a negative slope coefficient in favor of evolving inefficiency. The presence of long range dependence in KSE100 and sectoral indices indicates the predictable trends. Moreover the results suggest the arbitrage opportunities and informational asymmetry in PSX. Inefficiency in the PSX market is also related to the corruption, political and economic instability, poor security, thin trading, speculative and irrational attitude of the investors, and bad market infrastructure.

The predictable trends in PSX are interesting for researchers, investors, and policymakers for risk management and development of efficiency in PSX. The existence of long-range dependence in stock prices show an imperfect market. Moreover, the behavioral rationality does not exist in this market. The investors should react rationally and get the proper information before investing in PSX. It is essential to develop some efficient information sources in order to avoid the speculation which develops an inefficient market. The market efficiency and trade liberalization may be promoted by providing reliable private property rights to attract the foreign investors. Some policy implications are required regarding homogenous market structure to remove the arbitrage opportunities. A good governess and financial liberalization are obligatory to promote better resource allocation and market efficiency. Further research can be planned by using other volatility estimators or some intraday data rather closing prices and by considering other features like volatility clustering, asymmetry and fat tails.

4.6.1 Appendix



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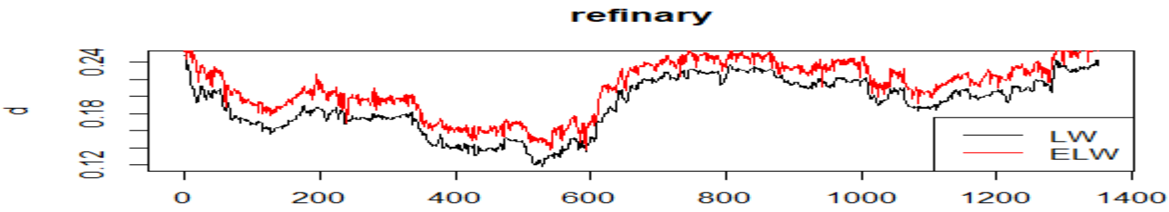
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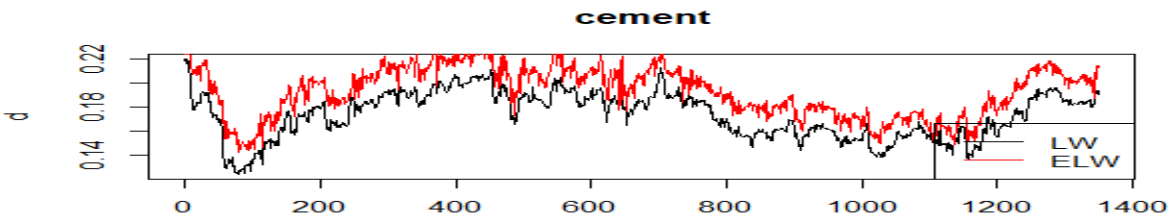
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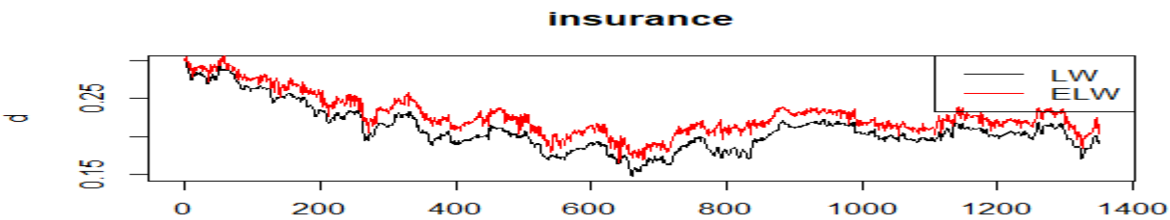
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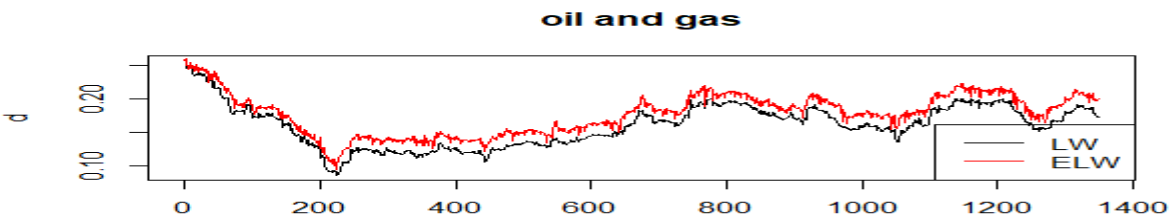
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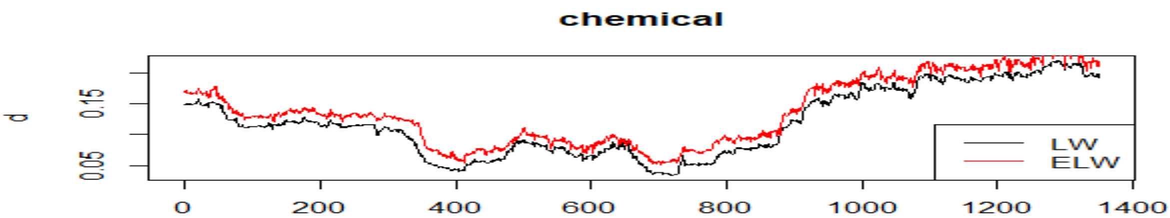
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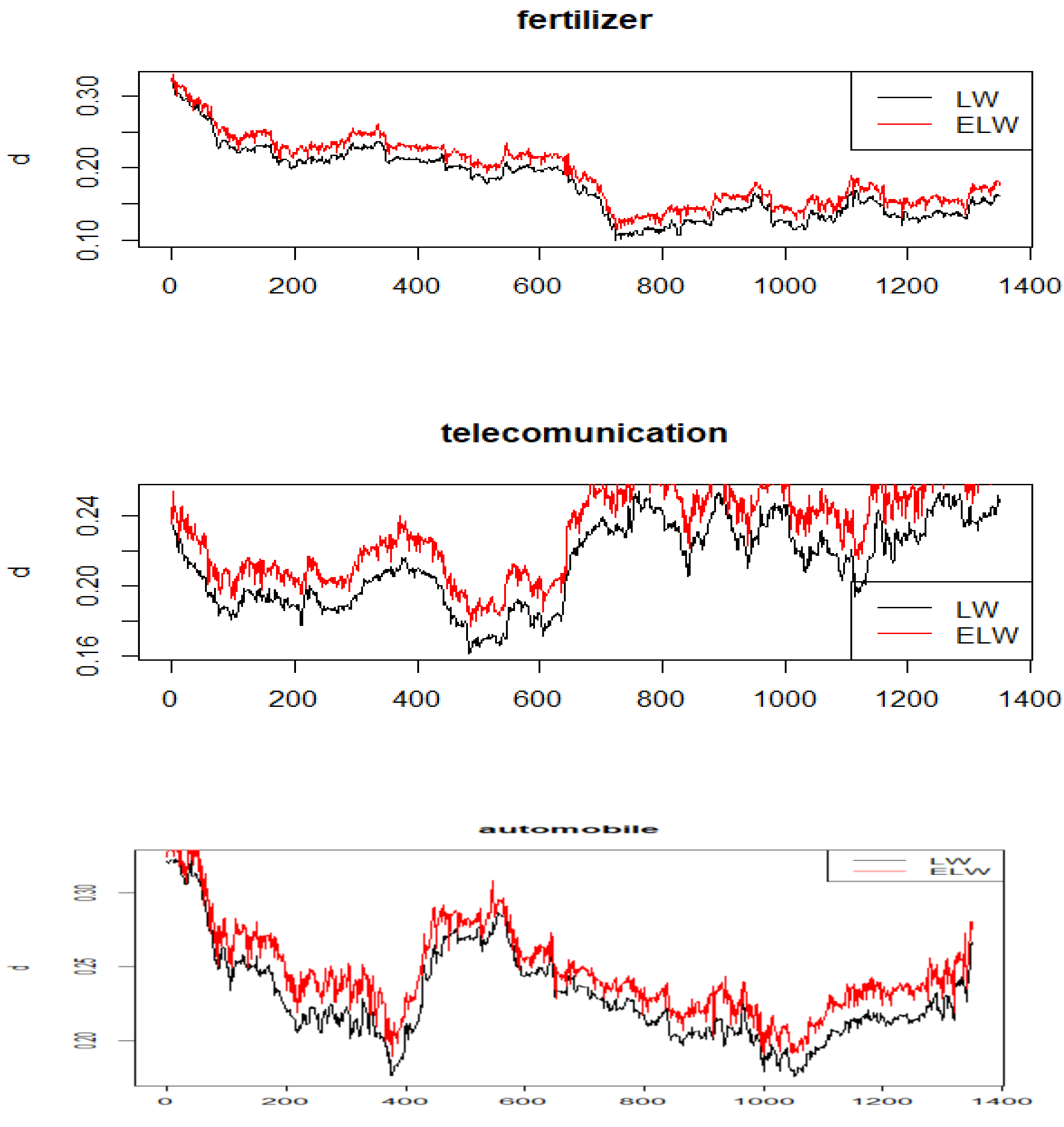


Figure 4.3 Graph of time-varying long memory for LW and ELW

Chapter 5

Fractional cointegration between Islamic stock index and conventional stock index

Fractional cointegration between Islamic stock index and conventional stock index

Co-authored with Philipp Sibbertsen

5.1 Introduction

With the growing interest in Islamic finance (IF), the question of (in) efficiency in Islamic financial markets is also of interest for researchers and investors. Many studies including (Al-Khazali et al., 2016, Rizvi et al., 2014, Uddin et al., 2018) have already pointed out the inefficiency of emerging and Islamic markets compared to the developed markets. All relevant information is reflected instantly in the current prices of an efficient market and there are no chances to predict the future trends by exploiting the historical prices. Inefficiency in the emerging markets is a result of small trade weights, less liquidity, more volatility, poor governance, absence of efficient environment, trade barriers, and higher trading costs (Rizvi et al., 2014). The awareness of market efficiency is important for the traders to get the advantage of inefficiencies while optimizing the returns and for the regulators to regulate the market conditions (Al-Khazali et al., 2016). The development level of a market accompanying its performance and risk level is considered as a main determinant of investment choices in any market by modern finance theory. The risk and the return are defined as the main differences between the conventional and the Islamic equity markets. Moreover, the Islamic stocks can be used as diversification alternatives in case of positive but low correlation with the conventional ones. Therefore, the investment in these indices can also provide insurance against losses (Uddin et al., 2018).

Cointegration refers to the similar trends in financial markets and it defines the long-run relationships or comovements in indices (Gulzar et al., 2019). Risk management and return optimization with an investment portfolio are the driving factors of interest and intensive research to investigate the comovements in financial markets. Medhioub and Chaffai (2016) observed a high degree of synchronization between the Sharia Islamic indices and the conventional ones regarding Malaysia, the Dow Jones, the FTSE, and the MSCI but not in case of Indonesia by using the data over 2000 to 2004. Al-Khazali et al. (2016) studied the market efficiency through the world, the developed and the emerging Islamic markets. They concluded that the Islamic markets are less efficient compared to the conventional markets as well as there exist cointegration between these two during the crisis. Rizvi et al. (2014) analyzed the stock market efficiency in developed versus Islamic markets including Bahrain, Egypt, Indonesia, Kuwait, Malaysia, Oman, Pakistan, and Turkey. They concluded inefficiency in the markets of

Islamic countries. Moreover, they suggested to improve the market development for efficient resource allocation in Islamic countries.

It is important to study the persistent behavior of stock markets as the predictability and risk management are related to the persistence levels. Moreover, the comovements between these two types of indices provide good portfolio diversification opportunities. In case of the cointegrated series, the trends in one market can be predicted on the basis of other market which consequently reduce the diversification opportunities. On the other side, the absence of long run relationship provides more opportunities and insurance against risks in case of shocks. There will exist diversification prospects in the absence of cointegration while diversification will no more be beneficial in the presence of cointegration between these two indices (El Khamlichi et al., 2014). Furthermore, it is also a topic of debate that Islamic markets perform in a different way compared to the conventional benchmarks in the case of crisis. The diversification advantages for the portfolio managers diminish in the presence of equity market integration. Additionally, the investors cannot make profits in cointegrated markets.

Some studies measure the efficiency of Islamic indices while others make a comparison of efficiency between Islamic and conventional indices. We contribute to this literature on (in) efficiency by analyzing the market efficiency in Islamic and conventional indices within each country. Therefore, we consider the stock indices of Bahrain, UAE, Oman, Qatar, Indonesia, Malaysia, Egypt, Turkey, and Pakistan. We consider the market efficiency and diversification in fractional integration and cointegration framework which is equivalent to a univariate and multivariate context. We analyze the comovements between these two indices for each country. The analysis is based on the semiparametric techniques, which work under mild regulatory conditions with no restrictions on the short-run dynamics of the process. The results show fractional integration in the absolute-returns of all Islamic and conventional counterparts. The semiparametric estimates of fractional cointegration show comovements across 7 out of 9 markets. Despite Sharia rules applied to the Islamic indices, both types of indices have fractional long-run relationships regarding 7 countries according to the results for the considered time period. Our results support an existence of the identical trends in Islamic and conventional indices during the crisis.

The rest of this paper is structured as follows. We provide the definition, overview and some factors behind the interest in Islamic finance (IF) or Islamic indices in section 2. Section 3 comprises the data and methodology of the empirical analysis. The results of empirical analysis are reported in section 4 and we conclude our work in section 5 with some suggestions and future recommendations.

5.2 An overview of Islamic index

The Sharia index or Islamic finance (IF) works as an alternative to the conventional index considering the religious concerns of investors. The working rules are the central difference between these two classes of indices. IF was developed to fulfill the demand of a large number of Muslim community in accordance with the Sharia complaint regulations (El Mehdi and Mghaieth, 2017). It follows the laws of Islam and the Sharia board which include the profit-loss sharing (PLS) and physical asset-based financial transactions while excluding speculation, interest rates which includes derivatives with predefined interest rates and gambling (Cevik and Bugan, 2018). The main Islamic principles are "Halal" and "interest free investments"(Medhioub and Chaffai, 2016). Moral, ethical, religious, and social sentiments of investors and traders are considered in the IF. Therefore, they are provided with a platform that does not conflict with moral, social or religious aspects (Al-Khazali et al., 2016). The Islamic markets are apparently different and have dissimilar features than the conventional markets. Moreover, the speculative transactions which do not include any real transactions such as financial derivatives are not permissible in these markets. The investment in financial sectors is prohibited while in real sectors is permitted in Islamic equities.

IF includes the stock indices, the hedge funds, and the bonds (sukuks). There exists a Sharia board or Sharia Supervisory Committee in every Islamic financial institution to regulate it according to the different Islamic financial thoughts (El Mehdi and Mghaieth, 2017). The ethical investment is criticized for extra costs of screening process and inefficiency. The objective of IF is to offer such investment opportunities which are parallel to the conventional ones considering the Sharia laws as well as equivalent to the ethical funds considering the social and moral values (Guyot, 2011). It is not permissible to pay or receive any interest in Islamic finance. The IF investments include real state, telecommunication, engineering, transportation, and utilities. Two Islamic indices named as DMI 150 (Dar al Mal al-Islami) and SAMI (Socially Aware Muslim Index) were introduced in 1998 with Dow Jones Islamic Market Index (DJIMI) and FTSE Global Islamic Index Series (GIIS) in 1999, S&P Global Benchmark Sharia indices in 2006, and MSCI Islamic indices in 2007 (El Khamlichi et al., 2014).

Al-Khazali et al. (2016) described some factors of rapid diversification and remarkable growth in IF such as the PLS strategy of Islamic indices, security of the growing wealth in Middle East due to sharp increase in oil prices, the stability of Islamic banks during global financial crises 2008-2009, and less risk in adverse economic conditions in Islamic stocks based on PLS. Uddin et al. (2018) discussed three main concepts of association between Islamic and conventional indices such as the integration, the diversification, and the efficiency. A relationship between the markets is implied in case of inefficiency which results in market integration. Equivalently, this market integration is used to predict the movements in interlinked markets which provide reasons for diversification.

5.3 Data and methodology

We use the data of Islamic and conventional indices across nine countries including Bahrain, UAE, Oman, Qatar, Indonesia, Malaysia, Egypt, Turkey, and Pakistan. The data of Islamic indices is obtained from the Thomson Reuters IdealRatings Islamic indices master list over the period 1-04-2009 to 9-7-2018 with a total number of observations 2419. The data of conventional indices also obtained from the Thomson Reuters DataStream except Pakistan. Data of both indices regarding Pakistan is obtained from the website www.Psx.com.pk as Thomson Reuters IdealRatings master list contains no data regarding Islamic index of Pakistan.

	Islamic index	**Conventional index**
Bahrain	Thomson Reuters IdealRatings Islamic Bahrain index	BAHRAIN ALL SHARE
UAE	Thomson Reuters IdealRatings Islamic UAE index	ADX GENERAL
Oman	Thomson Reuters IdealRatings Islamic Oman index	MSM30
Qatar	Thomson Reuters IdealRatings Islamic Qatar index	QE ALL SHARE INDEX
Indonesia	Thomson Reuters IdealRatings Islamic Indonesia index	IDX composite
Malaysia	Thomson Reuters IdealRatings Islamic Malysia index	FTSE Bursa Malaysia KLCI
Egypt	Thomson Reuters IdealRatings Islamic Egypt index	EGYPT EGX 30
Turkey	Thomson Reuters IdealRatings Islamic Turkey index	Istanbul SE National 100
Pakistan	KMI 30	Karachi SE 100

Table 5.1: Data description

The start date for Pakistan data is 9-06-2009 and end date is 9-7-2018 with 2255 observations. In order to analyze the market efficiency in the context of fractional integration and cointegration, we convert the daily prices in absolute-returns which represent the volatilities. We present a detail description of Islamic and conventional indices in Table 5.1.

The presence of high degree autocorrelations at long lags in the volatility measures is a well-known and extensively observed feature in the literature. This feature is commonly known as long memory, fractional integration or long-range dependence. The long memory or fractionally integrated models are used to represent this phenomenon in the stock's volatility. A model with lag operator $Lx_t = x_{t-1}$, an order of integration d, and μ_t stationary short memory process for a series x_t is defined as

$$(1-L)^d x_t = \mu_t. \quad (5.1)$$

The equation (5.1) presents a fractionally integrated stationary model within the interval $d \leq 1/2$ and nonstationary for $d \geq 1/2$. We use the exact local Whittle estimator (ELW) of Shimotsu and Phillips (2005) to estimate the fractional integration in Islamic and conventional indices. This estimator has been proved more efficient than the periodogram-based regression estimators as well as the LW estimator. The ELW estimator works in the frequency domain using only first m frequencies, hence does not consider the short-run dynamics. Moreover, it provides the consistent and asymptotically normal estimates for all possible values of d and the estimator is

$$\hat{d} = argmin_{d\epsilon(\Delta_1,\Delta_2)} R(d), \quad (5.2)$$

where $(d) = \log \hat{G}(d) - 2d\frac{1}{m}\sum_{j=1}^{m} \log \lambda_j$, $\hat{G}(d) = \frac{1}{m}\sum_{j=1}^{m} \lambda_j^{2d} I_x(\lambda_j)$, and Δ_1, Δ_2 represent lower and upper bounds respectively. Moving from univariate to multivariate always involve the problem of cointegration. Engle and Granger (1987) introduced the idea of cointegrated series and defined it as the stationary linear combination of two nonstationary series. More formally, two or more nonstationary series are related in the long-run although they may have no relationship in the short- run. The basic concept was introduced for $I(1)$ and $I(0)$ series but later on extended for the ARFIMA models, which are defined as

$$\phi(L)(1-L)^d Yt = \theta(L)\varepsilon_t, \quad (5.3)$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$, and

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)}.$$

Where Γ denote the gamma function and parameter d is allowed to assume any real value. Fractional cointegration exits for fractionally integrated variables in a vector process X_t with two variables, each with memory parameter $d1 = d2 = d$ (non-integer) and there exist a non-zero linear combination of the variables whose residuals are integrated of order $I(d-b), 0 < b \leq d$. We estimate the individual memory parameters of Islamic and conventional indices for each country. A test for the homogeneity of estimated memory estimates in each pair is performed with an application of the test statistic by Robinson and Yajima (2002)

$$\hat{T}_0 = (S\hat{d})' \left(S\frac{1}{4}\hat{D}^{-1}(\hat{G}\circ\hat{G})\hat{D}^{-1}S' + h(n)^2 I_{n1-1}\right)^{-1} (S\hat{d}), \quad (5.4)$$

where $\hat{G} = \frac{1}{m}\sum_{j=1}^{m} Re\left\{\hat{\Lambda}(\lambda_j)^{-1} I_j \hat{\Lambda}(\lambda_j)^{-1}\right\}$, $\hat{d} = (\hat{d}_1, \ldots \hat{d}_p)'$, and $\hat{D} = diag\{\hat{G}_{11}, \ldots, \hat{G}_{pp}\}$. It tests the null hypothesis of equal memory parameters and converges to zero and $\varkappa^2$ in absence or presence of fractional cointegration respectively.

We test for the presence of fractional cointegration with an application of Chen and Hurvich (2006) method. This method is based on the memory estimates of cointegrating residuals. The eigenvectors of the tapered averaged periodogram are used to construct the cointegrating residuals. The memory of residuals is estimated with the LW estimator but using the shifted frequencies to consider the tapering effects. The test statistic for the null hypothesis of no fractional cointegration is based on the comparison of residual memory estimates and given as

$$T_n = m_n^{\frac{1}{2}}(\hat{d}_{11} - \hat{d}_{qq}). \tag{5.5}$$

The test statistic is normally distributed with mean 0 and upper bound $\Phi_p/2$. Here m_n is the bandwidth and $\hat{d}_{11}$, $\hat{d}_{qq}$ are the residual memory estimates corresponding the smallest and the largest eigenvalues. The hypothesis of no fractional cointegration is rejected if

$$T_n > \left(\Phi_p/2\right)^{1/2} z_{\alpha/2}. \tag{5.6}$$

Estimation of the long-run relationships in the frequency domain is the most suitable mechanism by representing the cointegration relation near zero frequencies (Berger et al., 2009). We estimate the cointegration strength b with the method of Chen and Hurvich (2006) which is based on the eigenvalues and the eigenvectors.

5.4 Empirical results

The graphs of autocorrelations function for the Islamic and the conventional absolute-returns in Figure 5.1 show significant autocorrelations at long lags. Such high values of autocorrelation function indicate the presence of long-range dependence in Islamic and conventional volatilities. We describe the statistical features for the absolute-returns including average, standard deviation, skewness, kurtosis, normality test, and unit root test in Table 5.2. The average value is near zero across all indices whereas the identical level of volatility, measured as the standard deviation, in case of both indices is presented by Oman, Qatar, Malaysia, Indonesia, Egypt, and Turkey. The conventional indices of Bahrain and UAE present higher volatilities with greater values of the standard deviation.

The symmetry of distribution is depicted by the third moment of data and measured as zero skewness while a negative value of skewness is an indication of the larger left tail and large right tail results in the positive value of skewness. We get all negative values of the skewness referring to the non-symmetric distribution except the conventional index of Bahrain and thus lead to higher volatility and risk. The normality of the distribution is measured by the fourth moment named as kurtosis and describe how the data is centered on average.

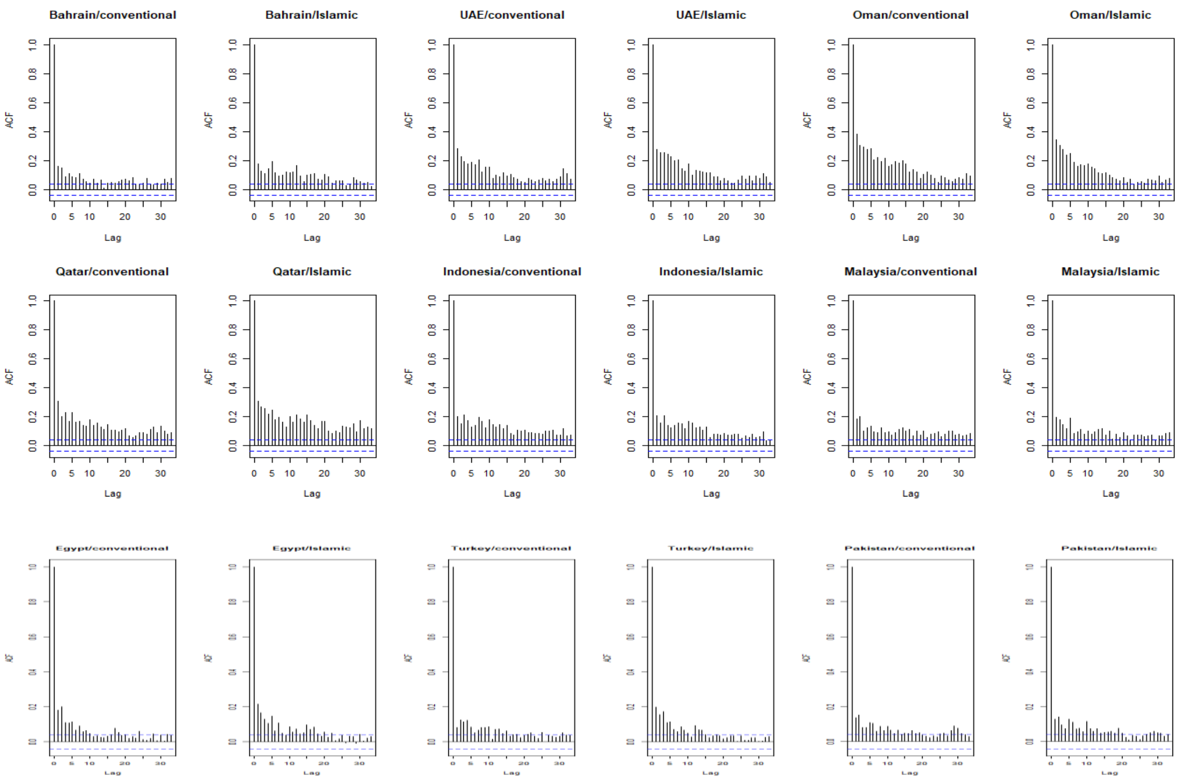


Figure 5.1: Graphs of ACF

	Mean	**St.Dev**	**Skewness**	Kurtosis	J-B	ADF	KPSS
			Conventional indices				
Bahrain	-0.00007	0.004937	-0.62009	10.47431	5785.787	-12.0806	0.327014
UAE	0.000256	0.009359	-0.26116	14.05273	12340.5	-13.3101	0.102737
Oman	-0.00001	0.006877	-0.55469	16.83951	19428.88	-13.3029	0.457368
Qatar	0.000444	0.010162	-0.41514	13.64252	11485.46	-13.2642	0.493145
Indonesia	0.000578	0.010882	-0.384	8.751943	3394.127	-13.5196	0.760454
Malaysia	0.000269	0.005728	-0.20578	6.09992	985.6311	-13.2806	0.98556
Egypt	0.000548	0.014809	-0.51834	8.434732	3085.334	-12.4638	0.102248
Turkey	0.000558	0.014076	-0.40099	6.762788	1491.893	-12.8271	0.34074
Pakistan	0.00692	0.006569	1.895177	7.929407	3631.366	-8.99468	0.55254
			Islamic Indices				
Bahrain	-0.00011	0.011879	0.712809	11.52225	7525.225	-12.2966	0.160266
UAE	0.000156	0.01235	-0.11887	14.97033	14448.01	-12.4828	0.153893
Oman	-0.00004	0.006817	-0.47151	13.63478	11489.03	-12.8414	0.813432
Qatar	0.000304	0.011637	-0.28444	14.81016	14091.03	-12.9361	0.325919
Indonesia	0.000375	0.012448	-0.48779	9.001896	3726.724	-14.6417	0.607187
Malaysia	0.000313	0.005551	-0.36812	6.919086	1602.718	-12.9512	0.605368
Egypt	0.000315	0.013451	-0.05606	8.403245	2943.885	-12.9272	0.122881
Turkey	0.000436	0.011263	-0.86686	10.02702	5279.948	-12.5956	0.582257
Pakistan	0.007492	0.007197	1.890699	8.023335	3712.796	-9.35625	0.396042

Table 5.2: Descriptive statistics

The kurtosis value is 3 for normal distribution while greater (smaller) value indicates a leptokurtic (platykurtic) distribution. We get kurtosis measure greater than 3 in all cases indicating peaked distribution which result in more volatile and risky prices. Moreover, the results of ADF unit root test reject the null hypothesis of unit root and the null of stationary series using KPSS test is not rejected across all indices.

The results of fractional integration estimation with the ELW using bandwidth $T^{0.7}$ are reported in Table 5.3. We get a minimum value of 0.23 and a maximum value of 0.366 within the conventional indices while the smallest memory estimate 0.219 and the greatest is 0.368 among the Islamic indices.

All memory estimates fall within the stationary region and reject the hypothesis of zero memory parameter. The presence of significant fractional integration reflects the inefficiency of conventional and Islamic stock indices in sampled emerging markets. Moreover, the existence of long memory indicates the predictable trends in these markets which may cause integrated conventional and Islamic markets.

At a next step of the empirical analysis, we test for the homogeneity of memory estimates with test statistic $\hat{T}_0$ as discussed in section 3. Homogenous memory estimates are a necessary condition for the existence of a long-run relationship. The results are presented in Table 5.4 and confirm the homogeneity of memory estimates across all combinations of the conventional and Islamic indices. We also tabulate the p-values for the test in column two of Table 5.4. The maximum value of test statistic is 1.048 which is less than the $\varkappa^2$ critical value 2.71 with one degree of freedom at 10% level of significance. All results in Table 5.4 are presented for the truncation parameter $T^{0.7}$.

	Bahrain	**UAE**	**Oman**	**Qatar**	**Indonesia**
Conventional	0.235180	0.335076	0.366188	0.323227	0.315240
Islamic	0.303509	0.368762	0.348059	0.329640	0.295400
	Malaysia	**Egypt**	**Turkey**	**Pakistan**	
Conventional	0.255131	0.230028	0.238913	0.245779	
Islamic	0.261171	0.231166	0.219269	0.253882	

Table 5.3: Long memory estimates with ELW

Results for the presence of fractional cointegration are also reported in column 3 of Table 5.4 with critical value 1.3859038. The empirical results suggest the existence of long-run comovements between these two classes of indices regarding UAE, Oman, Qatar, Indonesia, Malaysia, Turkey, and Pakistan. On the basis of these results, the long-run relationships do not exist between conventional and Islamic indices for Bahrain and Egypt. Our results propose the fractional cointegration between Islamic and conventional indices for 7 pairs out of nine. There exist a long-run relationship between the two types of indices in spite of short term divergence with respect to UAE, Oman, Qatar, Indonesia, Malaysia, Turkey, and Pakistan. The absence of long term relationship in stocks of Bahrain and Egypt provides the chances of diversification and risk management during the phases of crisis. The existence of fractional cointegration in the absolute-returns shows the longer time periods regarding the convergence to equilibrium. Moreover, these results suggest slow level of mean convergence as one can expect in the standard cointegration case. On the basis of empirical results, the conventional and the Islamic indices are interdependent, integrated, and each index can be predicted on the basis of other. Overall, our results find the fractional long-run relationship between conventional and Islamic stock indices. Therefore, the diversification benefits will be reduced in the long-run as the volatilities of the studied stocks tend to move in the same direction.

	T_0	p-values	T_n	decision	b
Bahrain	1.048049	0.147308	1.021331	FALSE	0
UAE	0.834309	0.202054	3.059948	TRUE	0.173625
Oman	0.466086	0.320577	4.031286	TRUE	0.236863
Qatar	0.224478	0.411193	2.798316	TRUE	0.169554
Indonesia	0.580961	0.280633	2.926533	TRUE	0.185908
Malaysia	0.148523	0.440965	1.796293	TRUE	0.112579
Egypt	0.016343	0.49348	-0.10036	FALSE	0
Turkey	0.367397	0.356661	2.115944	TRUE	0.124631
Pakistan	0.194518	0.422885	1.913518	TRUE	0.118228

Table 5.4: Empirical results

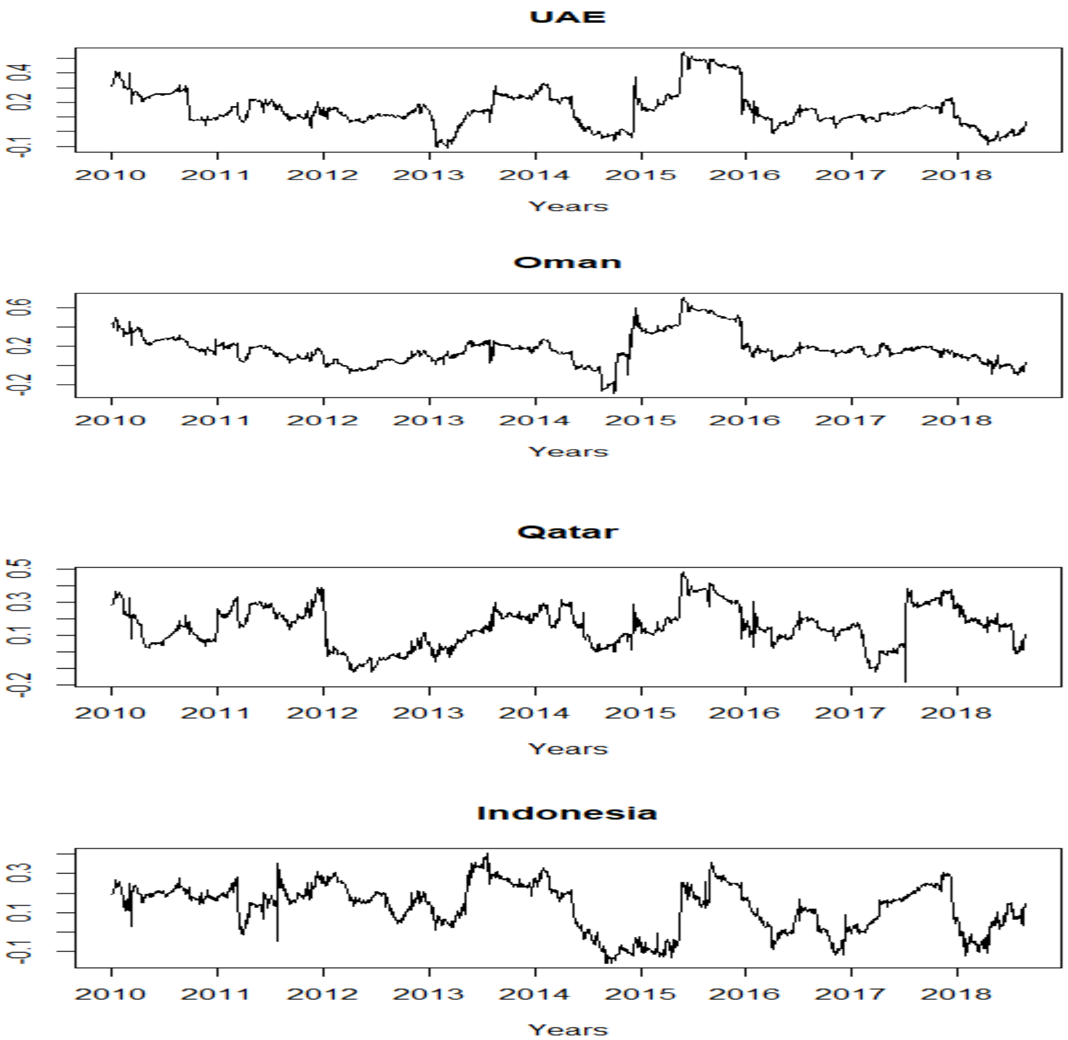
Jebran et al. (2017) observed a long-run relationship between the Islamic and the conventional index of Pakistan by using the error correction models (ECM). Pranata and Nurzanah (2016) detected less volatility in the Jakarta Islamic index (JII) in comparison to conventional index except 2010 and concluded that there is no significant performance difference between these two in Indonesia. Kuala Lumpur Sariah (KLSI) and Kuala Lumpur Composite Index are cointegrated and fluctuations in one can be used to forecast the other (Albaity and Ahmad, 2008). Ata and Buğan (2015) observed a causal relationship between the Dow Jones and the Morgan Stanley indices during the positive or negative shocks for Turkey. El Khamlichi et al. (2014) tested the random walk hypothesis in the daily Islamic and conventional indices including the Standard & poor, the Morgan Stanley, the Dow Jones and the Financial Times. They estimated the same level of (in) efficiency in both classes of indices while the diversification opportunities corresponding to the Dow Jones and the S&P indices due to absence of cointegration.

We estimate the cointegration strength b by using the eigenvectors of the differenced tapered averaged periodogram with the method of Chen and Hurvich (2006) and results are reported in column 5 of Table 5.4. The degree of potential diversification depends on the degree of relationship between assets. There exists an inverse link as the high potential for diversifications is related to the low levels of cointegration and vice versa (Guyot, 2011). The opportunities of abnormal gains for the portfolio managers are limited in the long term amongst the integrated markets. Main factors to explain the cointegration among financial markets are economic integration and market features. The presence of similar trends and market integration are described by the comovements in financial markets where, the comovements depict the existence of cointegrating trends in Islamic and conventional markets (Gulzar et al., 2019). Different price configurations in asset prices also result in diversification possibilities (Guyot, 2011).

Barberis et al. (2005) described three different models for the integrated markets such as fundamental theory states that the comovements in fundamental values are reflected by the cointegrated prices, the sentiments of the investors who are trading a subset of securities affect the prices of coordinated demands by transferring funds from one category to the other according to the trading-induced model, and the category based comovements according to which the investors consider the categories rather than individual securities while making the portfolio choices. There always exist some common factors for the value settlement which may cause the integrated markets as assumed by the standard theory of the portfolio maximization. Some important information may be exchanged through cross trading of the conventional (Islamic) securities to the Islamic (conventional) markets (Hoque et al., 2016). The use of developed technology with the computerized systems has increased the trader's awareness about market circumstances which results in increased integration and interdependence with decreased diversification opportunities (Hoque et al., 2016).

It is of great interest for the investors in Islamic or conventional markets to figure out the time-varying cointegration between the volatilities of Islamic and conventional markets to manage the risks along the diversification profits. Thus we make a time-varying analysis of the cointegration strength to understand the dynamic fractional cointegration between these two types of indices by using a rolling window technique. This technique enables us to observe the changing association between the volatilities by observing a gradual convergence (Mylonidis and Kollias, 2010). It reflects the strength of relationship at every step by moving one step ahead while considering the same sample size and a fixed window length.

We choose a window length of one year which consists 256 days. We move forward by including a next value and excluding the first one. Graphs of the rolling window analysis for cointegration strength are presented in Figure 5.2. A rolling window analysis offers an opportunity to understand the long-run relationships before and after the crisis periods as the current world is more integrated due to interlinked trade and electronic trading systems. A rolling window analysis is superior to the recursive analysis because with the latter one it cannot be differentiated either the change in the result is due to a new added observation or due to the increasing power of the test with more observations (Mylonidis and Kollias, 2010). Concerning UAE, the variations in cointegration strength during 2010 are caused by the 2009 Dubai debt standstill as a result of Global financial crises 2007-2008. Dubai World (DW), assisting the business portfolio and projects to Dubai government, was not able to fulfill the debt deadline and the government of Dubai announced on 25 November 2009 to standstill all the financers. This announcement affected the global stock markets. The next movements are related to the Dow Jones financial crash of 2010 and S&P bearish trend over 2011 which affects the cointegration strength to decrease. The 2015-2016 Chinese stock market crisis hit the worldwide economies as reflected by an increased level of cointegration between the Islamic and the conventional index in graphs of each country.



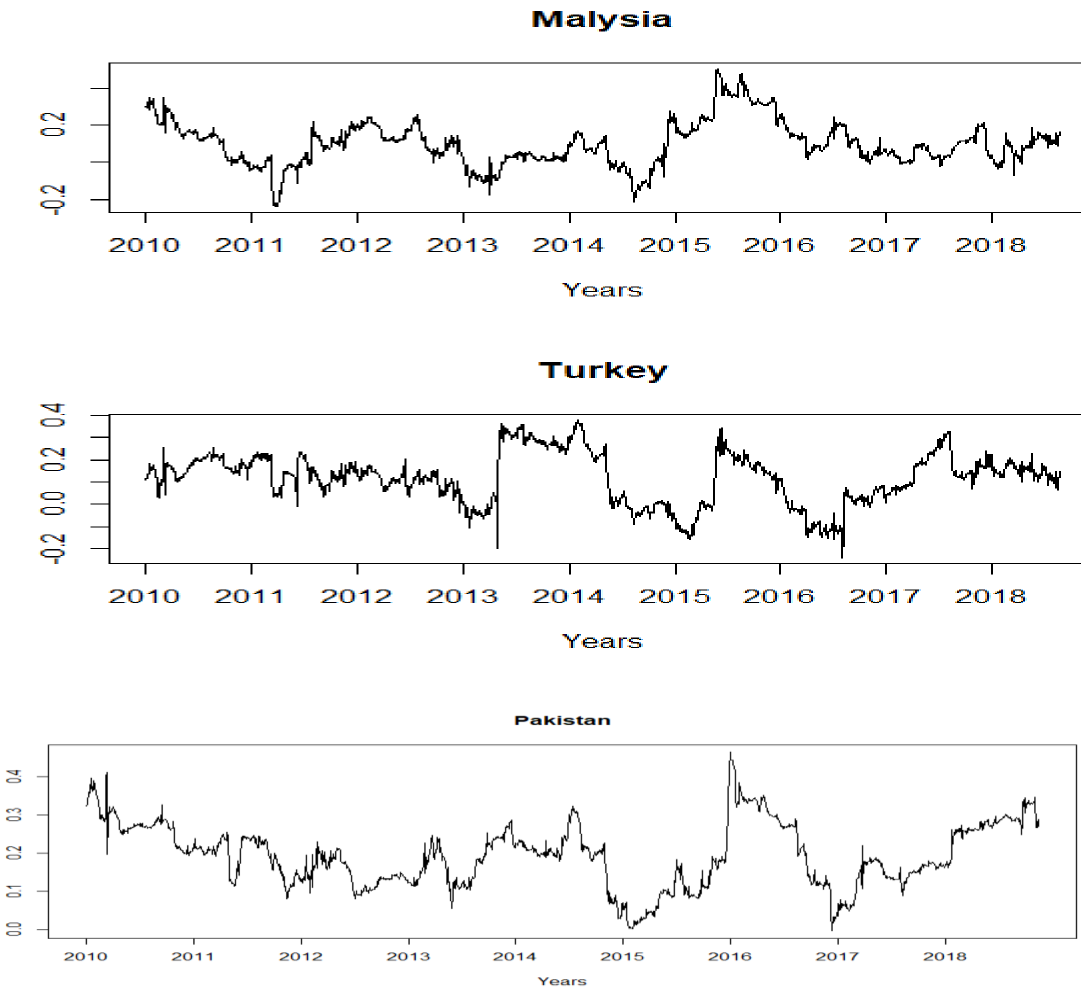


Figure 5.2: Time-varying strength of cointegration

This increased level of the cointegration shows the similar behavior in both types of indices during the crisis. Results of the graphical analysis negate the concept of a different performance level regarding the Islamic indices during shocks. Additionally, a price decrease in different commodities including a drop in oil prices during the 2015-2016 stock market plunge may also affect the cointegration strength. We notice a higher level of relationship in 2013 for Turkish indices caused by the European debit crisis and the US announcements of declined bound buy-outs (Ari and Cergibozan, 2014). An increased level of cointegration during 2014 may be a result of strong USD, which caused the fluctuations in exchange rates of many currencies and created volatility in most of the financial markets. The higher integration levels between Islamic and conventional indices of Qatar, Indonesia, and Turkey with moderately increased levels in case of UAE, Oman, Malaysia, and Pakistan may be triggered by this market volatility. An increased cointegration level during 2018 may be a result of the US stock market downturn.

Overall, our results of rolling window analysis, to understand the dynamics of fractional cointegration, show the higher levels of cointegration during the crisis times while the lower levels in recovery times. Moreover, on the basis of empirical results, the used methodology and considered time period, we find no diversification opportunities to manage the risks during the crisis period between Islamic and conventional indices. These results confirm the presence of predictable trends between the pairs and hence confirm the inefficiency of the emerging and developing conventional and Islamic indices. Moreover, our results show equal performance levels for both indices in case of shocks. There may exist few chances to get abnormal returns in case of long-run relationships between the stocks while the possibility of short term gains from the arbitrage actions remains there (Hoque et al., 2016). Álvarez-Díaz et al. (2014) investigated the forecast ability for the Dow Jones Industrial Average (DJIA) and the Dow Jones Islamic Market (DJIM). They observed a parallel performance levels between these two types of indices and concluded that the Islamic index did not perform significantly better during the crisis period. Therefore, Islamic indices might not be a good risk diversification candidate in asset allocation and hedges against risk exposures. El Mehdi and Mghaieth (2017) argue that holding Islamic stocks in a portfolio is more beneficial in a time of crisis by providing enhanced performance as these are not vulnerable to the interest rate risk. Uddin et al. (2018) found that the Islamic indices provide diversification, hedging, and risk minimizing opportunities to the investors over different time horizons.

5.5 Conclusion

Islamic finance has gained attention in the recent years and this study investigates the market inefficiency of Islamic and conventional stocks across 9 countries by using the Thomson Reuters IdealRatings Islamic indices and the conventional indices over the time period 1-4-2009 to 9-07-2018. Most of the investors attract to these markets in search of diversification alternatives and interest free environment. We estimate the univariate long memory parameters with the semiparametric ELW estimator using truncation parameter $T^{0.7}$ due to its flexibility through stationary and nonstationary cases. All the memory estimates are stationary and show persistent trends. Moreover, the condition of equal memory estimates for cointegration existence is tested with a semiparametric test statistic of Robinson and Yajima (2002) at 5% level of significance. The hypothesis of equal memory estimates is accepted across all pairs of memory estimates. Chen and Hurvich (2006) provided a test statistic for the hypothesis of no fractional cointegration. The statistic is based on the memory estimates of cointegrating residuals of the differenced tapered averaged periodogram. The empirical results support the existence of fractional cointegration between these two classes of estimates across seven out of nine pairs. The strength of the relationship is also measured in case of the cointegrated indices.

Empirical results of this study suggest an evidence of fractional cointegration between the Islamic and the conventional indices. Therefore, the trends in one market can be forecasted on the basis of other. Moreover, they may diverge in the short-run but will converge in long-run. Hence, there are no potential benefits of investments in the Islamic markets compared to the conventional counterparts. Moreover, these markets do not provide any potential diversification opportunities to cope the risk measures. A rolling window analysis of the cointegration strength is also performed to observe the dynamics of the fractional relationship between these conventional and Islamic indices. Graphical analysis indicate the absence of any diversification alternatives during the financial or the political shocks. Moreover, an increased level of cointegration during the crisis periods and decreased level of comovements during the recovery periods is obvious.

The development of stock markets is related to the efficiency and our research can be helpful for the policymakers to regulate the markets toward efficiency in the emerging economies. These results can help the investor to understand the trends and inefficiency in the conventional and Islamic markets. Furthermore, the integration and vulnerability of the Islamic markets to their conventional parts can help the investors and portfolio managers to make a decision regarding portfolio diversification (Gulzar et al., 2019). Additionally, this study suggests the policymakers to develop strategies in such a way that markets tend toward efficiency in these emerging Islamic stock markets.

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