THE EFFECTS OF GRAVITATIONAL ACCELERATION ON MICROPOLAR FLUID MODEL OF BLOOD FLOW IN A BIFURCATED STENOSED ARTERY

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To my beloved family, supervisors and friends -
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ABSTRACT

Gravity is a fundamental force regulating the cardiovascular system in our body. However not many previous studies on bio-fluids take into consideration of the variation of gravitational acceleration. Besides, the geometry of the bifurcated artery is chosen to be investigated since it is significant in human cardiovascular networking, where stenoses tend to form around branching junctions. Blood flow in the segment of artery is assumed to be axisymmetric, unsteady, laminar, fully developed, and two-dimensional. This research investigates the effects of gravity on micropolar fluid model of blood flow along a bifurcated artery segment which consists of a single stenosis at the parent branch. Meanwhile, to proceed with this study, blood is initially modelled as Newtonian fluid and micropolar fluid respectively in a straight stenosed artery segment. Then, the effects of gravity on Newtonian blood flow in bifurcated artery are explored. Here, a non-dimensional parameter G is introduced to describe the condition of gravity, where G is directly proportional to gravitational acceleration. The governing equations are solved numerically using the explicit finite difference method with prescribed condition of pressure and the computational algorithms are developed in Matlab software. Generally, with consideration of gravity variation, increment of gravitational acceleration causes decrement of axial velocity and increment of wall shear stress. Thus the consideration of gravity term in fluid model is necessary so that results obtained are closer to realistic conditions. Further, flow abnormalities are noticed at the branching junction from graphs of wall shear stress. This can be a crucial cause of stenosis overlapping and restenosis, which means that the structures of artery is significant in influencing blood flow patterns.

ABSTRAK

Graviti ialah salah satu daripada daya asas yang mengawal atur sistem kardiovaskular dalam badan kita. Tetapi tidak banyak penyelidikan bendalir-bio terdahulu yang mempertimbangkan perubahan pecutan graviti. Selain itu, struktur dwicabang pada arteri telah dipilih dalam penyelidikan ini sebab struktur ini adalah sangat penting dalam rangkaian kardiovaskular badan manusia, di mana stenosis sering wujud di bahagian cabang salur darah. Aliran darah dalam arteri diambil kira sebagai berpaksi simetri, tak mantap, berlamina, terbentuk sepenuhnya dan dua dimensi. Penyelidikan ini menyiasat tentang kesan-kesan daya tarikan graviti ke atas pengaliran darah mikropolar melalui arteri dwicabang yang berstenosis di cabang utama. Untuk meneruskan kajian tersebut, darah dimodelkan sebagai bendalir Newtonan dan bendalir mikropolar yang mengalir melalui arteri lurus berstenosis. Selepas itu, kesan daya tarikan graviti terhadap darah yang dimodelkan sebagai bendalir Newtonan melalui arteri dwicabang juga diterokai. Di sini, satu parameter tak berdimensi, G, telah diperkenalkan untuk menggambarkan keadaan graviti, di mana G adalah berkadar langsung dengan pecutan graviti. Persamaan yang mengawal diselesaikan dengan kaedah berangka beza terhingga berserta tekanan sebagai syarat yang telah ditetapkan; algoritma pengiraan dibangunkan dengan menggunakan perisian Matlab. Secara umumnya, dengan mempertimbangkan perubahan graviti, penambahan pada pecutan graviti akan menyebabkan penurunan pada halaju dan peningkatan pada tegaran ricih dinding. Oleh itu, dalam penyelidikan bendalir-bio, graviti perlu diambil kira supaya keputusan yang didapati adalah dekat dengan situasi sebenar. Seterusnya, merujuk kepada graf tegaran ricih dinding, aliran tidak normal didapati di bahagian cabang. Keadaan ini boleh menjadi penyebab utama kepada penindihan mendapan dan stenosis berulangan.

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LIST OF SYMBOLS / NOTATIONS

a	-	Constant radius of the non-stenotic region at main vessel
A_0	-	Constant amplitude of pressure gradient
A_1	-	Amplitude of pulsatile component
BC	-	Boundary Conditions
C	-	Couple stress tensor
d'	-	Segment length from origin to stenosis
D_{normal}	-	Diameter of vessel without constrictoin
D_{ste}	-	Minimum lumen diameter at stenotic region
f_p	-	Pulse frequency
Fr	-	Froude number
g	-	Gravity acceleration
G	-	Dimensionless expression of gravity acceleration
I_0, I_1	-	Bessel's function of zeroth and first order
IC	-	Initial Conditions
j	-	micro-inertia coefficient
k	-	Time step
l_0	-	Stenosis length
p	-	Pressure
r_0	-	Radius of curvature at lateral junction of bifurcation
r_0'	-	Radius of the curvature of flow divider at bifurcation
r_1	-	Constant radius of the non-stenotic region at daughter branch
$R_1(z,t)$	-	Radius function for outer wall of bifurcation
$R_2(z,t)$	-	Radius function for inner wall of bifurcation

$R(z,t)$	-	Radius function of the arterial segment
		For bifurcation model, $R(z,t)=R_1(z,t)-R_2(z,t)$
Re	-	Reynolds number
U_0	-	Cross sectional average velocity
$u(r,z,t)$	-	Radial velocity component
$w(r,z,t)$	-	Axial velocity component
α	-	(Section 3.1.2) Angle of bifurcation of daughter branch
α	-	(Section 3.2.2) Womersley number
β	-	Combination factor
δ_{ij}	-	Kronecker delta function
Δt	-	Time increment
Δx	-	Increment in radial direction
Δz	-	Increment in axial direction
ε_{ijk}	-	Alternating unit tensor of Levi-Civita
λ	-	Second viscosity coefficient
κ	-	Dynamic micro-rotation viscosity
ρ	-	Fluid (blood) density
μ	-	Dynamic Newtonian viscosity
υ	-	Kinematic viscosity
θ	-	Vertical angle between vessel and gravity direction
ϕ	-	Momentum flux
$\boldsymbol{\tau}$	-	Stress tensor
τ_m	-	Maximum height of stenosis
ω	-	Micro-rotation
ω_0	-	Over relaxation parameter
ω_p	-	Angular frequency
ω_u	-	Under relaxation parameter
γ	-	Coefficient of angular viscosities
$\frac{\partial p}{\partial z}$	-	Pressure gradient

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CHAPTER 1

INTRODUCTION

1.1 Background of Problem

Healthcare problems are apparently concerned by people all over the world. For over centuries, cardiovascular diseases have been noticed as one of the major illnesses where numerous people suffer from them. The diseases such as stroke and atherosclerosis etc. are closely related to abnormality, disorder and malfunction of blood flow characteristics in human body. Among the blood vessels, arteries play the role to carry oxygenated blood away from the heart to other parts of the body. Healthy arteries have smooth inner lining which promote unobstructed streaming blood. However, constriction in artery has been noticed to be one of the most common conditions which bring several serious illnesses. Arterial plaque, which is made up of deposits of fatty substance, cholesterol, calcium, fibrin or other cellular waste products, develop in inner lining of arteries and cause segmental narrowing. This will lead to abnormal blood flow and forming of blood clots. Also, this kind of unhealthy deposition or stenosis in a vessel tends to raise blood pressure and reduce vessel elasticity. Due to inadequate blood supplement, infected part of the body (especially the limbs) will get numbness. More seriously, a heart attack may occur if oxygenated blood supply to the heart is reduced. Also, a stroke may occur if the blood supply is cut off to the brain. Furthermore, this abnormality of blood flow would probably cause the present stenosis to further get worse by forming additional constrictions and coupling effects.

Hence, numerous medical studies have been carried out since decades to obtain more understandings about cardiovascular blood flow and stenosis in blood vessels. Furthermore, mathematicians started to model blood flow using knowledge such as fluid mechanics. Blood used to be modelled as Newtonian and non-Newtonian fluid by adopting several justifiable assumptions. Chakravarty *et al.* (1996) pointed out that blood behaves as Newtonian fluid when it flows through wider arteries such as the aorta; and oppositely, non-Newtonian behaviours are observed in tinier arteries. Some researchers such as Sankar and Md. Ismail (2009) even proposed two-fluid model, whereby blood possesses Newtonian behaviour near the vessel wall (namely the peripheral layer) and behaves non-Newtonian at the vessel core region. Eringen (1965) introduced a theory of a specialised non-Newtonian fluid, namely the micropolar fluid, which considers the involvement of micro-structure, to describe some kind of fluid suspensions. These mathematical bio-mechanical studies provided remarkable and extensive advancement in the field of cardiology. Despite, in current clinical field, constrictions and abnormal behaviours of blood in vessel is still commonly observed using various invasive methods. Thus, persistency in mathematical modelling of blood flow and stenosis is important to give deeper understanding on blood flow rheology, and provide more ideas on the hemodynamic. One can even able to speculate the cause of some common phenomena such as stenosis overlapping and restenosis by investigating the blood flow characteristics.

On the other hand, gravitational force is one of the fundamental forces regulating biological and physiological systems. From previous researches, it is interesting to find out that gravitational acceleration does not only differ on earth and in space. On the earth itself, different altitude and latitude will give different values of gravity acceleration. Also, during postural changes, blood assembles at certain part of body because of gravity attraction, and this will lead to increment of blood pressure at that body part. However, not many of the previous studies included gravity as a body force in modelling blood flow in arteries, thus this becomes the main motivation to this research. Throughout this research, variation of gravitational acceleration is taken into consideration to provide deeper insight about its effects on blood flow in constricted arteries.

Apart from quantitative factors such as gravity force, physical factors such as vessel structures are also found to be one of the major factors affecting blood flow behaviour in human cardiovascular system. Blood vessels are a series of branches expanding throughout the body, therefore bifurcation structure is very common among the branching system. This kind of structure has been clinically proven to be significant on atherogenesis, which means the augmentation of arterial wall deposition. This phenomenon is also clearly remarked where clinical investigations point out that arterial stenosis often occurs at bifurcated vessel regions. Due to this, branching structures have become one of the interested and important aspects in modelling blood flow to find out properties of flow rheology passing through vessel bifurcations. In this current research, blood flow through a mildly stenosed artery bifurcation is included. Main interest in this study is to model blood as micropolar fluid and study its effects when passing through an artery bifurcation under variation of gravity acceleration.

Despite researches about blood flow in living body have been carried out since decades, characteristics of blood flow in cardiovascular system has not yet been developed and interpreted thoroughly. This is because apart from quantitative parameters, too many subjective factors are affecting behaviours of human anatomy, including environment and even the internal individual emotions. However from the angle of mathematical modelling, even though a lot of assumptions need to be made, such analyses progressively give larger picture to successfully relate to clinical circumstances. Currently, this thesis formulates problems regarding the effects of gravitational acceleration, structure of artery segment (straight and bifurcation), involvement of single stenosis, and modelling of blood flow in Newtonian and micropolar fluids. Besides, problems discussed in this thesis are solved by numerical methodologies, which include the Marker and Cell (MAC) method and the finite difference method. For the proposed problems, streaming blood is assumed to be two-dimensional, laminar, time-dependent, and fully-developed. Further, some preliminaries for this research will be introduced in the next subsection. Also, problem statements, research objectives, scopes, research significance, and the thesis outline are discussed in next few sections.

1.1.1 Preliminaries of the Research

This research is about investigating blood flow patterns in stenosed artery. So, choice of artery geometry is the first concern before proceeding to problem formulation. In this research, blood vessels as well as streaming blood are assumed to be two-dimensional and axisymmetric. There are two kinds of artery geometry involved in this research, which are a finite segment of straight artery (Problems 1 and 3) and a finite segment of bifurcated artery (Problems 2 and 4). Both kinds of artery geometry are presumed to consist of a single constriction (stenosis) occurring at the main vessel. The stenosis is said to be 'single' since it has only one peak, where the tiniest diameter of vessel lumen takes place. Severity of the stenosis can be calculated by its percentage of areal occlusion, which will later be shown in Chapter 3 - Equation (3.1).

From previous researches, various shapes of stenosis were modelled by different researchers, in order to mimic the real circumstances in constricted blood vessel. Two main presumed models made to describe conditions of stenosis are smooth surface and rough surface (also named as 'irregular'). Commonly, stenosis with smooth surface is modelled by a continuous cosine function; while stenosis with rough surface needs to be modelled by a series of discrete data which is likely to be obtained from clinical data. In this research, straight artery is assumed to have a single rough stenosis with 48% areal occlusion, where the irregular surface data is taken from Back *et al.* (1984); whilst, bifurcated artery is assumed to have a single smooth stenosis at the parent branch, also with 48% areal occlusion, where the stenosis is modelled by a continuous function established by Chakravarty and Mandal (1997).

In this research, for both kinds of artery geometry, the presumed stenosis is chosen to be 48% occluded, which is considered as mild. However, any of these stenosis geometry profiles can be modified to possess higher percentage of areal occlusion, i.e. to become a severe stenosis.

1.2 Problem Statement

With any choice of blood vessel and stenosis geometry, blood flow problems are modelled in various ways in previous researches, by treating blood to have different behaviours. Each model of fluid has its own pros and cons, because blood is hard to be modelled as it is one of the most complicated basic bio-fluids in living bodies. Newtonian models have been widely proposed as it is the simplest and most direct way to investigate elementary blood flow patterns. Nonetheless, various non-Newtonian models describe blood flow considerably through different points of view. Micropolar fluid is one of the non-Newtonian models describing fluid as a suspension with rigid microstructures affecting the fluid motion.

There are inadequate studies pointing out influences of gravity force or gravitational acceleration on blood flow, despite gravity is the fundamental force regulating our body systems. Gravity does not only vary in term of location (for example, on earth versus outer space, difference of altitude and latitude on earth, and so on), it also varies even when a person changes his posture, for example, when a head-up tilt is carried out. Hence, further investigation on the effects of gravity should be related to each fluid model.

Another interest in the current study is the structure of the artery vessel. Many researchers choose a straight finite segment of vessel with stenosis to study the fluid flow patterns. Yet, the branching structure of artery has been noticed to be one of the reasons to stenosis forming and restenosis phenomena. Hence from this concern, blood flow in the structure of bifurcation of an artery shall gain further understandings.

Main problem in this research is to investigate the effects of gravitational acceleration on micropolar fluid model of blood flow through a stenosed bifurcated artery. However, before proceeding to this study, three related problems are studied as anticipations. For all the studies, effects of gravitational acceleration and artery structures are main concerns. Firstly, blood is modelled as Newtonian fluid in a straight stenosed artery. Then, micropolar fluid model is considered where other parameters remain unchanged. After that, the geometry of artery is assumed to be a bifurcated vessel with single stenosis located at parent branch, where blood flow through the bifurcated artery is modelled as Newtonian fluid. And finally, extended from the third problem, micropolar fluid model is considered where other physical parameters remain the same.

1.3 Research Objectives

This research aims to investigate and analyse blood flow patterns under certain circumstances as stated in the problem statements. The problems are solved numerically in Matlab programming software. Solution procedures comprise of mathematical models development, choices of vessel and stenosis geometry profiles, formulation of the problem, and numerical methods by respective computational approaches.

Main objective of this research is to investigate the influence of gravity on micropolar fluid model of blood flow in a bifurcated artery that consists of a stenosis at the parent branch. This study is carried out computationally using finite difference method and the programming code is developed using Matlab software.

Meanwhile, in order to achieve this main objective, the prior objectives are considered, which are to study the gravitational effects on Newtonian and micropolar fluid models of blood flow through a straight stenosed artery, and also to study the effects of gravitational acceleration on micropolar fluid model of blood flow in a bifurcated artery that consists of a stenosis at the parent branch. These problems are carried out computationally using Matlab programming based on finite difference scheme.

1.4 Research Scopes and Methodology

Scope of this research comprises of four main aspects. Firstly, involvement of gravitational acceleration as external force acting on blood flow in stenosed artery is studied. For the first three problems, gravity condition is represented by a dimensionless parameter namely the Froude number; while for the main problem, a dimensionless parameter G is used.

Next, two different structures of constricted artery segment are taken into account. The straight stenosed artery profile consists of a single irregular stenosis referred from proofed clinical data developed by Back *et al.* (1984). Then, the stenosed bifurcation geometry is introduced by Chakravarty and Mandal (1997), which consists of a single stenosis at parent branch. Both geometries of stenosed artery are assumed to be two-dimensional, cylindrical and axisymmetric.

The third aspect focused in the research is the fluid model. Streaming blood is assumed to be unsteady, two-dimensional, incompressible, fully developed and laminar. For the first two problems, blood is modelled as Newtonian fluid; whilst for the third and last problems, blood is modelled as micropolar fluid as introduced by Eringen (1965).

Last but not least, in this research, the methods of solution comprise of two: the Marker and Cell (MAC) method for the first three problems, and the explicit finite difference method for the last problem. MAC method grants the obtainability of pressure-velocity fields, and it is convincingly converging in similar problems, but it involves a comparatively complex solution algorithm. Main difference between finite difference method and MAC method is the definition of grids, where the explicit finite difference is discretized based on non-staggered grids while MAC method is based on staggered grids. Also, a specific pressure condition has to be prescribed to perform the finite difference method while MAC method does not need to have pressure boundary conditions, because for MAC method, discretization of pressure is not taken at boundaries. The algorithm of MAC method involves multiple parts for every time loop, includes calculating of time step and combination factor, solving the velocity fields, obtaining the pressure field by Successive-Over-Relaxation method, and also performing the pressure-velocity corrections (see Appendix A). Whilst for the main objective, a complicated problem involving micropolar fluid modelling, structure of bifurcation artery with mild stenosis, and the additional of gravity force is performed. Hence, here the MAC method algorithm is overlooked; while the explicit finite difference method is chosen to access this problem, with prescribed pressure gradient and radial velocity component, which yields a more direct computation. Both of the numerical calculations are done by computational programming in Matlab software.

1.5 Significance of Research

Gravity force differs when approaching to or away from the atmosphere around earth, where microgravity or hyper-gravity is experienced. Yet, gravity can also be influenced by altitude and latitude. On the other hand, due to the gravity attraction, distribution of blood to the body can be influenced by postures. Hence, the study provides deeper understandings on effects of gravity on blood flow in stenosed artery. Particularly, blood flow patterns and the physical impacts can be clearly determined when a person travels to different locations, such as from equator tropical to northern pole, from flatlands to mountains; or, during ascending and descending of flights. Also, when a patient changes posture such as encountering a head-up tilt (which is a test widely used to diagnose dizziness or syncope), the physiology of blood flow can be evaluated. Studies which include the gravity term are comparably more accurate in describing real-life cases. Gravity is one of the fundamental physical forces; hence with deeper exploration on effects of gravity, additional precautions and medications can be improved.

Secondly, from the physiological point of view, structure and geometry of a blood vessel is important in affecting hemodynamic. Circulatory system in body comprises branching network of blood vessels, where bifurcation is a common and significant structure. Investigation on blood flow along a bifurcated vessel provides further understandings to relate to the phenomena of stenosis overlapping and restenosis which often occur at branching junctions.

Last but not least, in medical field nowadays, most diagnoses of blood flow problems are invasive. Thus, researches which are done from mathematical points of view provide more comprehensive ideas on behaviour of blood flow and stenosis in vessel. With appropriate mathematical modelling, diagnoses can be done non-invasively. Non-invasive diagnoses and medications are safer, promote faster recovery time, and at the same time medical costs can be reduced. Further, nowadays, decisions of medications to stenosed arterial diseases are mostly considered based on a doctor's previous experiences. Mathematical modelling and computational analyses enable predictions of illness, so that treatments and precautions can be done at earlier stage. These effectively reduce the risks for diseases to get more severe.

1.6 Thesis Outline

This thesis consists of six chapters investigating hemodynamics of blood flow through stenosed artery under the effects of gravity. The first chapter presents background of the current research and introduces some preliminaries about stenosis in artery. Also, problem statements, research objectives, scope, methodology, and significance of the research are clarified.

Then, second chapter presents literature reviews on previous researches related to the current study and highlights the research gap, which include blood flow in stenosed arteries, stenosis in artery branches, relativity of gravity and blood flow, behaviours of blood together with introduction of different fluid models, and finally, the numerical or computational methods used to solve blood flow problems.

Next, Chapters 3 presents formulation of the problems, which include construction of vessel geometry and derivation of governing equations. In the first section, geometry of straight artery with a single irregular stenosis is shown, together with the clinical data established by Back *et al.* (1984) which constructs the stenosis surface roughness. Then, geometry of the bifurcated artery with a single stenosis at parent branch is also discussed. After that, derivation of the governing equations for micropolar fluid model is shown, which include the derivation of continuity equation, momentum equations and angular momentum equations for both straight artery and bifurcated artery models. Initial and boundary conditions of micropolar fluid models for both artery geometries are also discussed.

Chapter 4 shows solution procedures of the main problem in this research - micropolar fluid model of blood flow in a bifurcated stenosed artery. Here, radial coordinate transformation and derivation of radial velocity component are shown. Then, before performing computational algorithm, the involved governing equations and boundary conditions are discretized according to finite difference scheme. Finally, finite difference algorithm is explained.

Chapter 5 presents results and discussions of the four problems. The results are categorised into two main sections by vessel geometry, where the first section discusses about blood flow in straight artery with a single stenosis; while another section discusses about blood flow in an artery bifurcation with also a single stenosis at its parent branch. For all the investigations, a gravity vector is included in the momentum equations as an external body force. Results are presented in graphical form and discussed accordingly. In the first section, streaming blood is treated as Newtonian fluid and micropolar fluid respectively. In this section, both studies are done under same physical geometry - the straight artery, which is assumed to be two-dimensional, axisymmetric and each tube is cylindrical. Also, the stenosis profile is chosen to be a single irregular stenosis.

Second section in this chapter discusses about blood flow in an arterial bifurcation with a single smooth stenosis at parent branch under the influences of gravity. Governing equations for this problem are similar to the previous section; while the boundary conditions are slightly modified to suit the situation of bifurcation geometry. A different radial coordinate transformation is imposed due to difference in artery geometry. Similar to the previous section, this section is also divided into two part, where in the first part, streaming blood is treated as Newtonian

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