STEADY AND UNSTEADY ALIGNED MAGNETOHYDRODYNAMICS FREE CONVECTION FLOWS OF MAGNETIC AND NON MAGNETIC NANOFLUIDS ALONG A WEDGE, VERTICAL AND INCLINED PLATES

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To my family

Without whose loving support
this thesis could not have been written

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ABSTRACT

Nanofluids are a new type of heat transfer fluid engineered by uniform and stable suspension of nanometer sized particles into liquids. The heat transfer in nanofluids is important especially in the context of chemical engineering, aerospace engineering and industrial manufacturing processes. The reason is that, nanofluids were found to transfer heat more efficiently than the conventional fluids. Therefore, nanofluids research could lead to a major breakthrough in developing next generation coolants for numerous engineering applications. Due to this reason, several flow problems related to heat transfer over vertical flat plate, inclined plate and wedge were studied in this thesis. The main purpose of this study was to investigate the characteristics of two dimensional flow and surface heat transfer for two cases which are steady and unsteady convection flows. Nanofluids with two different base fluids (water and kerosene) containing magnetic and non magnetic nanoparticles were considered. The effect of magnetohydrodynamics (MHD) on the flow and heat transfer was also studied. The study starts with the formulation of the mathematical models that governed the fluid flow and heat transfer. Next, the governing nonlinear equations in the form of partial differential equations were reduced into ordinary differential equations using appropriate similarity transformation. The resulting systems of ordinary differential equations were then solved numerically using Keller box method. The numerical values of the skin friction coefficient, the local Nusselt number which represents the heat transfer rate at the surface as well as the velocity and temperature profiles were obtained for various values of the magnetic field inclination angle, magnetic interaction, plate inclination angle, nanoparticles volume fraction, wedge angle, moving wedge, unsteadiness, Grashof number and thermal buoyancy. All results obtained, were displayed graphically in addition to tabular form. The comparisons of results with previous studies were made to validate the results. For both steady and unsteady problems, it is found that magnetic field inclination angle can be used as controlling factor for certain situation because it enhances the skin friction and heat transfer rate. The plate inclination angle parameter and nanoparticles volume fraction parameter have tendency to increase momentum and thermal boundary layers thickness. For unsteady problems, it is observed that the unsteadiness parameter has significant effect on the nanofluids motion and heat transfer characteristic.

ABSTRAK

Nanobendalir adalah sejenis bendalir pemindahan haba baharu yang direka bentuk oleh zarah berukuran nanometer yang seragam dan ampaian stabil yang di masukkan ke dalam cecair. Pemindahan haba dalam nanobendalir adalah penting terutamanya dalam konteks kejuruteraan kimia, kejuruteraan aeroangkasa dan proses pembuatan industri. Ini adalah kerana, nanobendalir didapati memindahkan haba dengan lebih cekap berbanding dengan bendalir konvensional. Oleh itu, penyelidikan berkenaan nanobendalir boleh membawa kepada penemuan baharu dalam proses penyejukan untuk pelbagai aplikasi kejuruteraan. Disebabkan oleh faktor ini, beberapa masalah aliran berkaitan dengan permindahan haba ke atas plat rata menegak, plat cenderung dan baji dikaji dalam tesis ini. Tujuan utama kajian ini adalah untuk menyiasat ciri aliran dua matra dan pemindahan haba permukaan bagi dua kes aliran iaitu aliran olakan mantap dan tidak mantap. Nanobendalir yang terdiri daripada dua bendalir asas yang berlainan (air dan minyak tanah) yang mengandungi zarah nano bermagnet dan tidak bermagnet dipertimbangkan. Kesan hidrodinamik magnet (MHD) ke atas aliran dan pemindahan haba juga dikaji. Kajian ini dimulakan dengan memfomulasi model-model matematik yang mengawal aliran dan pemindahan haba. Seterusnya, persamaan menakluk tak linear dalam bentuk persamaan pembezaan separa diturunkan kepada persamaan pembezaan biasa menggunakan penjelmaan keserupaan yang bersesuaian. Sistem persamaan pembezaan biasa yang terhasil kemudiannya diselesaikan secara berangka menggunakan kaedah kotak Keller. Nilai berangka bagi pekali geseran kulit, nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan, serta profil halaju dan suhu diperoleh untuk pelbagai nilai sudut kecenderungan medan magnet, interaksi medan magnetik, sudut kecenderungan plat, pecahan isipadu zarah nano, sudut baji, pergerakan baji, ketidakstabilan, nombor Grashof dan keapungan haba. Semua keputusan yang diperoleh dipersembahkan secara graf dan dalam bentuk jadual. Perbandingan keputusan dengan kajian terdahulu dibuat untuk mengesahkan keputusan yang diperoleh. Bagi kedua-dua masalah aliran mantap dan tidak mantap, didapati bahawa sudut kecenderungan medan magnet boleh digunakan sebagai faktor pengawal pada keadaan tertentu kerana ia meningkatkan geseran kulit dan kadar pemindahan haba. Parameter sudut kecenderungan plat dan parameter pecahan isipadu zarah nano mempunyai kecenderungan untuk meningkatkan ketebalan lapisan sempadan momentum dan terma. Untuk masalah aliran tidak mantap, diperhatikan bahawa parameter ketidakstabilan mempunyai kesan yang signifikan terhadap gerakan bendalir nano dan ciri pemindahan haba.

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LIST OF MATTERS

Al_2O_3 - Alumina Oxide
Ag - Silver
Fe_3O_4 - Magnetite
TiO_2 - Titatium dioxide
Cu - Copper
Co - Cobalt
Ni - Nickel
Mn-Zn - Manganese-zinc ferrite

LIST OF SYMBOLS

Roman Letters

A_b - Rate of change of energy inside fluid element
$\mathbf{a}$ - Acceleration of the element
B_b - Total heat flux into the fluid element
$\mathbf{B}_0$ - Applied magnetic field
B_0 - Magnitude of applied magnetic field
$\mathbf{B}$ - Total magnetic field
C_b - The rate work done on element by the force
C_f - The skin friction coefficient at the surface
c_p - Heat capacity of base fluids
$\left(c_p\right)_s$ - Heat capacity of solid nanoparticles
$\left(c_p\right)_{nf}$ - Heat capacity of nanofluids
$\mathbf{D}$ - Rate of strain tensor
$\frac{d}{dt}$ - Material time derivative
$\mathbf{E}$ - Total electric field
$\mathbf{F}$ - Net force on the fluid element
$\mathbf{F}_b$ - Volume forces
F_x - Volume forces in the $x-$coordinate
F_y - Volume forces in the $y-$coordinate
F_z - Volume forces in the $z-$coordinate
Gr - Grashof Number
Gr_x - Local Grashof Number

g - Gravitational acceleration

$\mathbf{I}$ - Identity tensor

$\mathbf{i}$ - Cartesian unit vector in the $x-$direction

$\mathbf{J}$ - Current density

$\mathbf{J}\times\mathbf{B}$ - Lorentz force

$\mathbf{j}$ - Cartesian unit vector in the $y-$direction

K - Unsteadiness parameter

L - Characteristic Length

$\mathbf{k}$ - Cartesian unit vector in the $z-$direction

k - Thermal conductivity

k_p - Thermal conductivity of solid nanoparticles

k_f - Thermal conductivity of base fluids

k_{nf} - Thermal conductivity of nanofluids

M - Magnetic interaction parameter

m_0 - Mass of the element

Nu - Nusselt number

$\Pr$ - Prandtl number

p - Pressure

q_w - Surface heat flux

Re - Reynold's number

$\mathbf{T}$ - Cauchy stress tensor

T - Temperature

T_w - Temperature at the surface

T_∞ - Temperature at the free stream

t - Time

U, U_∞, u_e - Velocity in free stream

u_w - Velocity at wedge surface

u - Velocity in $x-$direction

$\mathbf{V}$ - Velocity vector field

V - Magnitude of velocity

V_0 - Volume

v - Velocity in $y-$direction

w - Velocity in $z-$direction

y - Dimensionless coordinate axis normal to the plate

Greek Letters

α - Magnetic field inclination angle

δ - Boundary layer thickness

δ_t - Thermal boundary layer thickness

δ_1 - Time dependent length scale

γ - Plate inclination angle

Ω - Total wedge angle

ρ - Density

ρ_p - Density of solid nanoparticles

ρ_f - Density of base fluids

ρ_{nf} - Density of nanofluids

ρ_∞ - Density of fluids at free stream

β_p - Volumetric coefficient of thermal expansion of solid nanoparticles

β, β_f - Volumetric coefficient of thermal expansion of base fluids

β_{nf} - Volumetric coefficient of thermal expansion of nanofluids

β_1 - Hartree pressure gradient

μ - Dynamic viscosity of fluids

μ_f - Dynamic viscosity of base fluids

μ_{nf} - Dynamic viscosity of nanofluids

λ - Moving wedge parameter

λ_T - Thermal buoyancy parameter

ϕ - Nanoparticles volume fraction

ν_f - Kinematic viscosity of fluids

ψ - Stream function

ω - Unsteady parameter for leading edge accretion

τ - Anisotropic viscous stress

τ_1 - Dimensionless time variable

τ_w - Wall shear stress

τ_{ij} - Shear stress

σ - Electrical conductivity

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

In this chapter, the strength of this thesis is explained concisely. The background of the research is presented in Section 1.2. The problem statements as well as objective and scope of the research are given in Section 1.3, 1.4 and 1.5 respectively. Consequently, the significance of the research is presented in Section 1.6. Lastly, Section 1.7 gives the outline of the whole thesis.

1.2 Research Background

Fluid dynamics refers to the science involving the movement of fluids and gases, forces causing such movements and the interactions between the solids and fluids. This field of study is an important component of engineering science and technology and is used in our daily lives. Fluid dynamics, in some way, affect different areas like transportation, energy, defence, environment, manufacturing, medicine, etc (Anderson and Wendt, 1995). Furthermore, an advanced knowledge of this field is very useful for predicting weather, aerodynamic behaviour of the moving vehicles, movement of the biological fluids within the body, cooling pattern of the electronic components, and performance of the micro fluidic devices (Anderson and Wendt, 1995). Because of the massive complexity of this subject and its various applications, it is viewed as very challenging and exciting. Need for a better understanding of this subject has led to its development and has also brought about the development of the related fields like numerical computing, applied mathematics and the experimental techniques. The field of fluid dynamics is based on the Navier-Stokes equations, which could be used along with the other transport equations for concentration, energy or magnetic fields. Owing to their nonlinear nature, exact solutions are rare and many scenarios have to depend on the approximate computational or analytical solutions. This has led to the development of novel diagnostic techniques for the experimental and computational processes, which helped the scientists in their exploration of the complexities in this field (Kays *et al.*, 2005).

Since the humans discovered fire, they have been very fascinated with energy and heat. Also, after the industrial revolution, the mathematicians and scientists became very interested in predicting and modelling heat transfer. Many studies, involved in converting the energy and heat into power (led to the inventions like combustion and steam engines), highlighted the manner in which heat transfer occurred between the mediums and stated that this technique was important for energy conservation and for developing effective processes and devices (Kays *et al.*, 2005). Heat transfer takes place in three ways such as conduction, radiation and convection. Conduction refers to the direct heat transfer between the adjacent particles and occurs at the microscopic level. Radiation refers to the energy or heat transfer directly through the electromagnetic waves. On the other hand, convection is defined as the heat flow through the gases or liquids along with the mass flow. Hence, this heat transfer process occurs on a larger scale. Out of the three heat transfer processes, convection is very difficult to model as it requires a thorough understanding of the flow process of the fluids.

Convection is further divided into three different types such as forced, free and mixed convections. When the motion of the fluids is induced by external resources like blowers, fluid machinery pumps or vehicle motion, it is known as the forced convection and this process is called as the forced convection flow. When the fluid motion is induced by the body forces like centrifugal or gravitational forces, it is called as the natural or free convection. Furthermore, a mixed convection flow takes place when the forced and free convection mechanisms occur simultaneously and bring about a significant heat transfer (Kays *et al.*, 2005).

The free convection process has garnered a lot of interest from the researchers as it occurs in nature and can be successfully used in several engineering applications. In nature, the convection heat transfer takes place when air rises above the hot land or water surfaces and is an important feature of the weather systems. Convection helps in the formation of the sea winds, oceanic currents and the rise of the plume of hot air from the fires. In the case of engineering applications, convection occurs in the microstructure configuration when the molten metals are cooled and also takes place when the fluid flows around the solar ponds and covered heat-dissipation fins. One of the most common industrial applications of the convection process is the cooling of the free air without using fans. This occurs at a smaller scale (computer chips) or in the larger scale process equipment (Bejan, 2013).

Several of the industries have expressed a need for better fluids which allow efficient heat transfer. However, the inherent poor thermal conductivity of a majority of the convection fluids has also fundamentally limited their heat transfer. Hence, a lot of research is being conducted for improving the fundamental limit (Bejan, 2013). One of the conventional methods for improving heat transfer in the thermal systems involves increasing the heat transfer surface areas of the cooling devices along with their flow velocity or dispersing the solid particles in the heat transfer fluids. Maxwell (1873) proposed the idea of suspending solid particles in the conventional heat transfer fluids for enhancing their thermal conductivity. Maxwell dispersed micrometre or millimetre-sized particles in the base liquids and improved their thermo-physical properties. However, this led to many problems like abrasion, sedimentation, clogged microchannels and a high pressure drop which prevented the use of the microparticle slurries as the heat transfer fluids. Thus, the use of the millimetre or micrometre-size particle suspension in the various heat transfer application was rejected. This idea was re-investigated after more than a century by Masuda *et al.* (1993) and Arnold Grimm *et al.*(2012). However, they also faced the problem of the sedimentation of solid particles.

In one study, Choi and Eastman (1995) used the nanoparticles suspension in the conventional base fluids. They observed a stable solution with no dispersed particles and were able to overcome a majority of the above-mentioned problems. This solution was called as the ‘Nanofluids’. Their experiments showed that the addition of the nanoparticles in the conventional base fluids improved its thermal conductivity. Additionally, this method was preferable than the micrometre-size particle addition, due to valid scientific observations like (i) Long suspension times (better stability), (ii) Larger surface area or volume ratio (1000 times larger), (iii) Low clogging or erosion, (iv) Low requirement for the pumping power, (v) Decrease in the inventory of the heat transfer fluid and (vi) Better energy saving. The novel properties displayed by the nanofluids help in their heat transfer-related applications. They help in more effective heat transfer than the conventional base fluids. Hence, when they are used for improving the performance and the design of the thermal management systems, they offer many benefits, like better reliability, a decrease in the cooling system sizes, a lesser requirement of the pumping power, high fuel and energy efficiency and low pollution (Sheikholeslami and Ganji, 2014). Also, the nanofluids show a significant effect in cooling the high-heat-flux systems and devices used in many industrial, consumer or defence industries.

The field of magnetohydrodynamics (MHD) involving fluid mechanics includes the phenomena resulting from applying a magnetic field to an electrically conducting fluid. In recent years, the research around the field of MHD has developed quickly (Davidson, 2001). In an electrically conducting fluid, many applications have been derived through the study of flow and heat transfer when a magnetic field is applied past a heated surface. Some of these applications include manufacturing processes such as nuclear reactor, cooling down metallic plate and extrusion of polymers (Davidson, 2001). In electrically conducting fluid, the MHD flow can also control the heat transfer rate at the surface, which results in achieving the desired cooling effect. Although advanced nanofluids like Carbon Nanotubes(CNT)–water diamond–water are available, they are too expensive for practical uses. Thus, economically cheaper nanofluids were considered, which also offered better heat transfer improvements. Ferrofluids are basically magnetic nanofluids that are normally stable liquids comprising dispersed colloidal magnetic nanoparticles like cobalt (Co), Magnetite (Fe_3O_4) and nickel (Ni). Unlike conventional non magnetic nanofluids, ferrofluids provide various advantages when employed as heat transfer media for example, (i) the solute nanoparticles' properties such as thermal conductivity and viscosity can be modified by using external magnetic field to achieve a specific design requirements, (ii) the thermomagnetic convection in a ferrofluids can be controlled and enhanced by employing the external magnetic field, and (iii) the size and cost of components can be decreased by applying ferrofluids in heat transfer devices. In this context, few researchers (Sheikholeslami and Ganji (2014), Sheikholeslami and Gorji (2014)) have confirmed that ferrofluids gives excellent heat transfer improvements.

In this day and age, different geometries also improve the thermal conductivity process. Many studies have investigated and proposed novel techniques for improving the heat transfer (Bejan, 2013). Also, researchers have investigated the role played by the variation in the device geometry on the heat transfer rate. Several numerical and experimental studies were published which determined the effects of the separation flow on the heat transfer performance, based on the configurations and the boundary conditions. Based on the above discussion, clearly, very few studies have investigated the heat transfer of the aligned MHD magnetic and non magnetic nanofluids flowing over the flat or inclined plates and wedges. More investigation needs to be carried out by varying the magnetic nanoparticles, geometries and the conventional base fluids to determine their effects on the velocity, heat transfer as well as skin friction and Nusselt number.

1.3 Problem Statement

Fluid heating and cooling holds lot of significance in several sectors, including manufacturing, power, electronics and transportation. Effectual ways of cooling are extremely essential for cooling any high-energy apparatus. The usual heat transfer fluids like ethylene glycol, water and engine oil have inadequate or substandard heat transfer competences because of their limited heat transfer attributes. On the other hand, metals demonstrate thermal conductivities almost thrice more compared to such fluids. Thus, it is obvious that a blend of the two substances is sought to create a heat transfer medium which performs like a fluid but exhibits a thermal conductivity like that of a metal. As mentioned earlier, nanofluids can overcome these issues. It is a fluid which comprises tiny volumetric amounts of nanometre-sized particles known as nanoparticles. It is basically engineered colloidal suspensions of nanoparticles within a base fluid. The nanoparticles utilised in nanofluids are mostly formulated from oxides, metals, carbides or CNTs.

Nanofluids are often a topic of research due to their heat transfer attributes. These nanofluids improve convective properties as well as thermal conductivity over the base fluid's properties. Normally, the heat transfer coefficient improves by almost 40% and thermal conductivity improvements have been found in the range of 15 to 40% over the base fluid (Yu *et al.*, 2008). Such levels of increase in thermal conductivity cannot be completely attributed to the added nanoparticles' higher thermal conductivity. Attribution behind performance enhancements must have come from other mechanisms as well. Besides, there are many potential applications that use the interaction between nanofluids and the magnetic field to address issues such as liquid sodium that results in cooling of nuclear reactors and induction flow meter that relies on the potential difference in the fluid, which is in the direction perpendicular to the magnetic field and to the motion. Therefore, this study has explored the following research questions:

1. How is the mathematical model of the steady and unsteady aligned MHD free convection boundary layer flows over three different geometries which are a flat plate, an inclined plate and a wedge formulated?

2. How do the mathematical models describing the nature of steady and unsteady aligned MHD free convection boundary layer flows over three different geometries which are a flat plate, an inclined plate and a wedge?

3. How does the presence of magnetic and non magnetic nanoparticles in the base fluid together with aligned MHD and other parameters affect the fluid motion, heat transfer, skin friction and Nusselt number?

1.4 Research Objectives

This study numerically investigates the steady and unsteady of aligned MHD free convection of boundary layer flows of magnetic and non magnetic nanofluids over a flat plate, an inclined plate and a wedge. This includes the constitution of suitable mathematics models by formulating the appropriate governing equations with some physical conditions and solving the resulting governing equations numerically. The specific objectives of this research are the following:

1. To derive the mathematical models of the problems which consist of continuity, momentum and energy equations.

2. To carry out mathematical formulation and simplification.

3. To solve the dimensionless governing equations numerically by using Keller box method.

4. To develop computational algorithm for solving the problem.

5. To obtain the numerical results of velocity and temperature profiles as well as skin friction and Nusselt number for each of the problem.

6. To analyse the results obtained graphically and tabulated for different physical condition namely magnetic field inclination angle parameter, magnetic interaction parameter, plate inclination angle parameter, nanoparticles volume fraction parameter, wedge angle parameter, moving wedge parameter, unsteadiness parameter, Grashof number and thermal buoyancy parameter.

1.5 Scope of Research

This thesis is focused on the steady and unsteady aligned MHD free convection flows, incompressible and two dimensional laminar boundary layer flow of magnetic and non magnetic nanofluids past along three geometries such as wedge, vertical an inclined plates. The constant wall temperature and no slip velocity condition are considered. Nanoparticles are suspended inside regular fluids where water and kerosene are chosen for this purpose. The selected nanoparticles are magnetic nanoparticles (Fe_3O_4) and non magnetic nanoparticles (alumina oxide (Al_2O_3)) (Sheikholeslami and Ganji, 2014, Sheikholeslami and Gorji, 2014). The base fluids and the selected nanoparticles are assumed to be in thermal equilibrium. By following Tiwari and Das (2007), several nanofluids models are derived. Bearing this scope in mind and apart from Chapters 1, 2 and 3, the following problems have been considered in Chapters 4, 5, 6, 7, 8 and 9 of the thesis.

1. Steady aligned MHD free convection flow of magnetic and non magnetic nanofluids along a vertical flat plate.

2. Steady aligned MHD free convection flow of magnetic and non magnetic nanofluids along an inclined plate.

3. Steady aligned MHD free convection flow of magnetic and non magnetic nanofluids along a static and moving wedge.

4. Unsteady aligned MHD free convection flow of magnetic and non magnetic nanofluids along a vertical flat plate with leading edge accretion.

5. Unsteady aligned MHD free convection flow of magnetic and non magnetic nanofluids along an inclined plate with leading edge accretion.

6. Unsteady aligned MHD free convection flow of magnetic and non magnetic nanofluids along a wedge.

The problems are formulated and transformed using similarity transformation and solved numerically by Keller box method with the help of FOTRAN software. The scope and research framework of this study are shown in Figures 1.1 and 1.2 respectively.

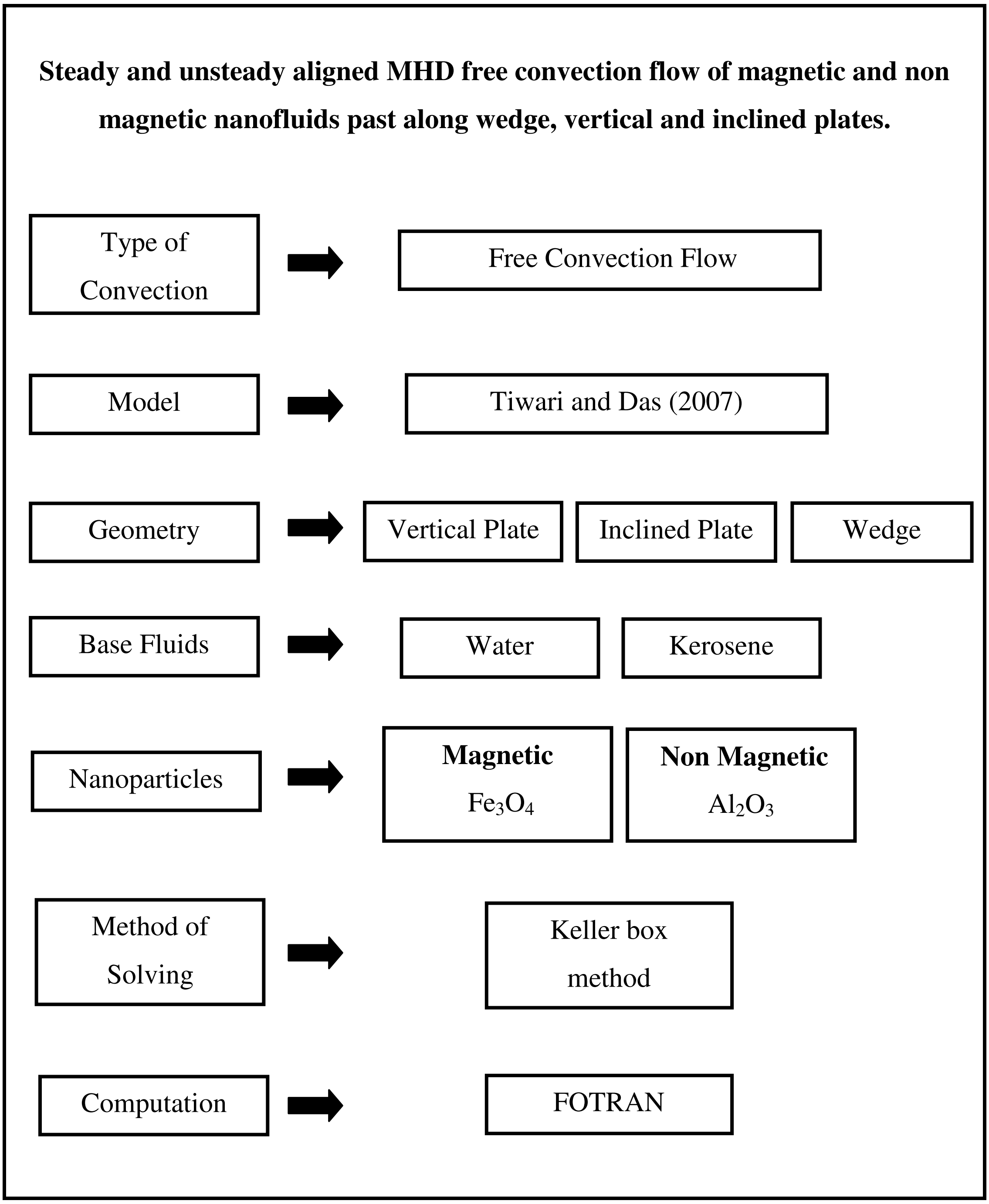

Figure 1.1 Scope of research

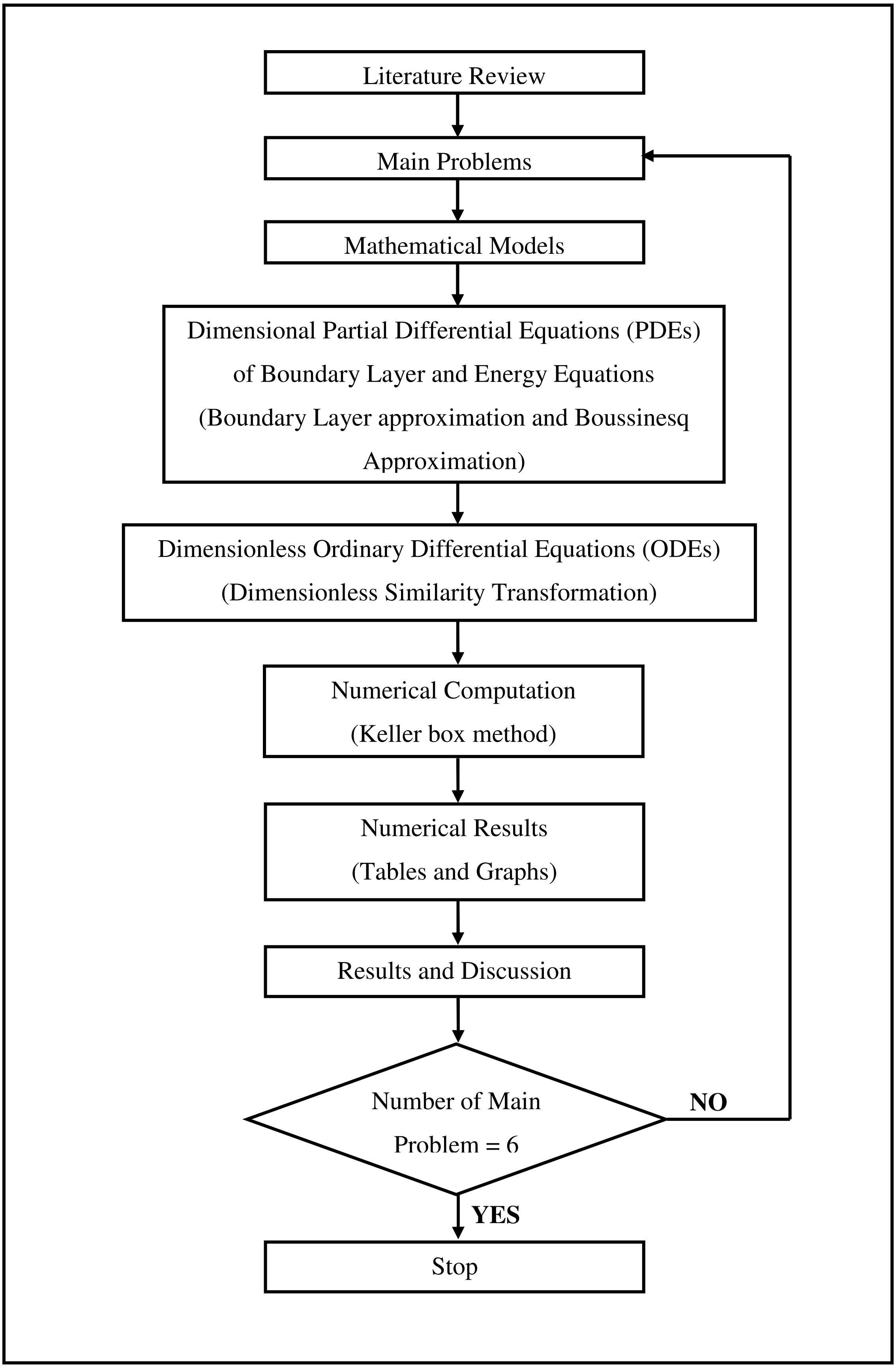

Figure 1.2 Research framework

1.6 Significance of Research

Cooling is essential for maintaining the stability and the performance of many devices like power electronics, computers, car engines and the high-power lasers or x-rays. Thermal conductivity is an important process in many of the consumer and industrial processes. However, the inherent poor thermal conductivity of the conventional base fluids fundamentally limits their heat transfer process and they are unable to satisfy the current industrial and technological demands (Sheikholeslami and Ganji, 2014). Hence the significance of this thesis are:

1. To develop a theory of magnetic and non magnetic nanofluids and explain how nanoparticles change the thermal properties of magnetic and non magnetic nanofluids and present different effects and conditions that provide the best option needed in the flow and heat transfer characteristics.

2. The results obtained from this research enable to enhance knowledge of the steady and unsteady aligned MHD free convection flow and heat transfer past along a vertical flat plate, an inclined plate and a wedge.

3. The results obtained can be used as bases for complex flow problems frequently occuring in engineering and applied sciences. This idea can be extend for other fluids.

1.7 Thesis Organization

This thesis consists of ten chapters in which Chapter 1 consist of the introduction of the research, Chapter 2 discuses on the literature review, Chapter 3 deliberated the derivation of the governing equations which represent the problem, Chapters 4 to 9 represent six research problems of this study and lastly, Chapter 10 represents the main conclusion of the overall problems.

Chapter 1 which represents introductory part of the main research contains the background of research, problem statement of research, objective of research, scope of research and significance of research. Then, in Chapter 2, previous research work regarding the research area concerning the proposed problems is reviewed and discussed.

In Chapter 3, the derivation of the basic governing equations for nanofluids are discussed in detail. Further, the formulation of the main problem which involves the derivation of the volume forces, Boussinesq and boundary layer approximations are employed.

Chapters 4 to 6 present the problem on the effect of steady aligned MHD on free convection flow of magnetic and non magnetic nanofluids past along a vertical flat plate, an inclined plate and a wedge. The nonlinear PDEs that govern the problem are transformed into nonlinear ODEs by using similarity transformation. Physical quantities are also included in this chapter to analyze. The full numerical processes of Keller box method is also explained in Chapter 4.

Further, Chapters 7 to 9 discuss the unsteady problem on the effect of aligned MHD on free convection flow of magnetic and non magnetic nanofluids past along a vertical flat plate, an inclined plate and a wedge. For vertical and inclined plate the effect of leading edge accretion is highlighted.

In each chapter, the content begins with the introduction, followed by mathematical formulation continued with solution procedures and will end up with the details on results and discussion.

Finally in Chapter 10, concluding remarks of the thesis, proposed future work and implications are presented. At the end, all the references used in this thesis are listed in the references.

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