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Teaching and learning the concept of area and perimeter of polygons without the use of formulas

Jamie Robin Anderson Mickens

TEACHING AND LEARNING THE CONCEPT OF AREA AND PERIMETER OF POLYGONS WITHOUT THE USE OF FORMULAS

A Project

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Teaching Mathematics

by

Jamie Robin Anderson Mickens

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ABSTRACT

This study examined teaching and learning the concept of area and the concept of perimeter of polygons without the use of numbers. The purpose of this study was to increase the students' understanding of the measures of area and perimeter of polygons. The goal of this project was to create a supplemental geometry unit to develop the concept of the area and perimeter of a polygon without the use of formulas and numbers and to measure the effectiveness of this unit on student understanding.

Although this study was a small pilot study done with a fairly small sample group (the two geometry classes participating in the project had less than twenty-eight students each) and even though the treatment group's sample data only supported significant growth on two problems, the project's impact on the students' assessment results were noticeable. The treatment group showed growth on twelve of the eighteen problems on the post-test.

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CHAPTER ONE

INTRODUCTION

Context of the Study[1]

The objective of this project was to establish a supplemental geometry unit that develops the concept of the area and the perimeter of a polygon without the use of formulas. The purpose of this study was to improve the students' understanding of the measure of area and perimeter of polygons. A pre- and post-test was designed to measure the effects of this project's unit on the students' comprehension of the concept of area and perimeter.

It is important for future references and comparisons to know about the community in which the school is situated in order to understand the context of teaching and learning at that high school. Burdett High School (pseudonym) is the sample high school for this project.

Description of Community

My master's degree project was implemented in a school in a Southern California community within the

[1] *To preserve anonymity, the websites and other material used to collect information about the city, school district, and schools are not cited.*

Inland Empire. The community in which the school is situated is ethnically diverse, and is one of the fastest growing areas in the state. According to the California Department of Finance (2006), the approximate population of the community is 99,200 residents as of January 2006. The ethnic makeup of the community breaks down as 51.2% Hispanic or Latino (of any race), 39.4% White, 22.3% Black or African American, and 12.9% are listed as "others" reported by the U.S. Census Bureau (Census 2000). The median age of people living in the community is 26.4 years old and nearly 60% of them speak English with another 40% speaking Spanish.

Economically, of 24,659 total households, 83.2% were family households with a median household income of $41,254 (U.S. Census, 2000). The percent of owner-occupied housing units from the above total households is 68.4%, leaving 31.6% as renter-occupied housing units. The unemployment rate for those over 16 years old in this community was just over 6%. The school districts, with 2,251 employees, and FedEx Ground Distribution and Packages, with 1,750 employees were the two major employers in this region. Most people traveled just over 30 minutes when going to their jobs.

Most of the people living in this community, according to the U.S. Census, have had no college education (U.S. Census, 2000). Only 2.3% were reported as having received a graduate or professional degree and 6.4% had received a bachelor's degree. Just over 6% of the people were shown as receiving an associate's degree, and 23.6% had some college though they had not gone on to receive degrees.

In 2000, nearly 28,000 students were enrolled in the local K-12 school district (U.S. Census, 2000). The district included 13 elementary schools, 5 middle schools, and 5 high schools. A sizable number of students were raised by their grandparents, as 38.7% of grandparents living with children less than 18 years old were primary caregivers.

<u>Burdett High School's Demographics</u>

As published by School Wise Press, in the school accountability report card for 2004-2005, there were 2,245 students enrolled in the Burdett High School. The ethnic makeup of the high school students mimicked that of the community with 51% Latino/Hispanic and 34% African American. However, there was a lower percentage of white students attending the sample high school, 12% compared to the nearly 40% White population in the community as a whole. Ten percent of these high school students were English learners and primarily spoke Spanish at home. Over 60% of the students came from households whose adults have attended some college and another 28% came from homes in which the caregivers had a college degree. Thirty-eight percent of these students were on free or reduced-price meal, which was subsidized for low-income students whose families earn less than $34,873 a year for a family of four.

In 2006, the district employed eighty-three teachers at Burdett High School. The average teaching experience within the high school was ten years, with 25% of the staff having less than two years experience. About half of the teachers had only a bachelor's degree while slightly more than half of the teachers also held a graduate degree. While most of the teachers held single subject credentials, some of the teachers who taught at the Burdett High School held a multiple subject credential. Only 87% of Burdett's teaching faculty held the secondary (single-subject) credential, below the statewide average of 90%. The math department had an even lower percentage, with 70% of the faculty holding a secondary (single-subject) credential with the remaining math teachers holding a supplemental credential. Ninety percent of the faculty held a full, clear authorization to teach either at the elementary level with a supplemental credential or the secondary level with a single subject credential while 8% held an internship credential, and so were still taking university courses to complete their preliminary credential. Two percent of the high school teachers held an emergency permit.

Instructional Information

Burdett High School was the newest school within the district. In 2004, when the school opened, Burdett had only a junior class. The school initially was on a modified block schedule, meaning that on Monday and Friday students would attend all their classes for fifty-five minutes, and on Tuesday, Wednesday, and Thursday they would attend four classes for eighty minutes. Each class would meet four times per week, Monday and Friday, and then twice during Tuesday, Wednesday, or Thursday. The average class size was thirty students compared to statewide average of twenty-nine students. The school was staffed with a career counselor and two career technicians. Students had an opportunity to be part of

Advancement via Individual Determination (AVID) Program to prepare them for college. Some students were able to enroll in courses that were more challenging than the required courses such as honors, Advanced Placement (AP), and International Baccalaureate (IB). In 2005, 13% of the students took and passed Advanced Placement (AP) exams, below the 15% county average recorded in the school accountability report card.

Purpose of the Project

Burdett's geometry students encountered difficulties in high school geometry. In 2005, according to the school accountability report card, only 9% of the students scored at the proficient or advanced levels on the State's standardized testing program for geometry, which was far below the average California high school at 24%. The California Standards Test (CST) sub-score analysis in geometry was broken up into four clusters, logic and geometric proofs, volume and area formulas, trigonometry, and angle relations, constructions, and lines. Students at Burdett scored the least overall within the cluster of volume and area formulas at 42% correct, which was below 70%, the cutoff score for minimal proficiency. Anecdotal evidence gathered from student work and teacher comments indicated that students struggled to memorize geometric definitions, theorems, and formulas.

Scope of the Project

Throughout this project's unit students compared the areas and lengths of polygons by using two-dimensional constructions. The project addressed the definitions and concepts of area and perimeter of polygonal regions and challenged students to know, derive, and solve problems involving perimeter and area without the use of formulas. Two geometry classes participated in the project, one using traditional instruction via the textbook and direct instruction, and the other class using the project's unit. Students in small groups from the class using the project's curriculum unit were challenged to find the area and perimeter of a polygon in as many ways as possible. The length of the project's unit was ten days.

The participants in the project's new unit were from a geometry class consisting of ninth, tenth, and eleventh grade students. The project was to supplement Burdett's geometry textbook, Glencoe's "Geometry: Concepts and Applications" (Cummins, Kanold, Kenney, Malloy, & Mojica, 2001). The expected results of this project were that students using the new unit that emphasized exploration and discovery would gain a better grasp of the concepts of area and perimeter and thus be able to accurately complete test questions at a higher rate of success than the students in the traditional geometry class. The study shows that modifications in teaching curriculum can improve student learning.

CHAPTER TWO

LITERATURE REVIEW

Geometry

When people look around in nature and their natural environment they encounter geometric concepts: in architecture, art, advertising, nature, neighborhoods, their homes, landmarks, and the streets they travel. Geometry is a natural place to develop students reasoning. In most geometry classes in the United States, this is not occurring. Geometry is the area of mathematics on which most elementary and middle school teachers spent the least time (Van de Walle, 2004). This lack of attention created a ripple affect reaching the college level. Clements and Stephan reported that several research reports revealed that college-level students had difficulties with area measurement (Clements & Stephan, 2003). It has been noted that even though geometry is a natural place to develop students' reasoning, it is the area of mathematics where students perform poorly (Kenney & Silver, 1997; Beaton et al., 1996).

Geometry's Place

"Geometry is a natural place for the development of students' reasoning and justification skills, culminating in work with proof in the secondary grades" (The National Council of Teachers of Mathematics [NCTM], 2000, p. 41). Geometric ideas and spatial reasoning are useful tools in solving problems. The National Council of Teachers of Mathematics (NCTM) (2000) reminds teachers that through geometric ideas, students are provided with important tools to describe and interpret their physical environment and to solve real-world situations. The state of California echoes the NCTM's statement about the need for students to develop good reasoning skills through the venue of geometry. The Mathematics Framework for California Public Schools standards states: "The main purpose of the geometry curriculum is to develop geometric skills and concepts and the ability to construct formal logical arguments and proofs in a geometric setting" (Curriculum Development and Supplemental Materials Commission [CDE], 2000, p. 162). Developing informal to more formal thinking in geometry across the grades is consistent with the thinking of theorists and researchers (Fuys, Geddes, & Tischler, 1988; NCTM, 2000; van Hiele, 1999). "The Geometry Standard includes a strong focus on the development of careful reasoning and proof, using definitions and established facts" (NCTM, 2000, p. 41). Despite the benefits for understanding geometry and spatial sense, research suggests "many teachers do not consider geometry and spatial relations to be an important topic, which gives rise to feelings that geometry lacks firm direction and purpose" (Pegg & Davey, 1998, p. 109). Dina van Hiele and Pierre van Hiele are credited with "improving teaching geometry by organizing instruction to take into account students' thinking" (Pegg & Davey, 1998, p. 110). A closer examination of the van Hiele framework is discussed later in this paper.

Geometry Teachers

Since mathematics educators, such as Van de Walle (as cited in Menon, 1998), have called for teaching mathematics with understanding, researchers' attention have focused not only on the mathematical competency of the students, but have also on "how much mathematics the pre-service teachers themselves understand" (Menon, 1998, p. 361). Menon states that there are very few studies on pre-service elementary teachers' understanding of perimeter and area (1998). Menon (1998) conducted a limited study on fifty-four pre-service elementary teachers which shows that these teachers' conceptual understanding of perimeter and area is less than satisfactory. It is reasonable to assume this weak foundation at the elementary level impacts the ability of high school students to grasp a more in-depth understanding of area and perimeter. In order to increase adequate mathematical competency of students, further research needs to ascertain pre-service teachers' conceptual understanding of primary school mathematics curriculum, in order to develop more effective pre-service mathematics education courses (Menon, 1998).

Geometry Curricula

There are educators designing curricula to increase conceptual understanding of geometry and spatial sense. One such study was performed by Lehrer et al. (1998). Their "approach to geometry with young children begins with students' informal knowledge about situations, followed by progressive mathematical reinterpretation of these experiences" (p. 169). The instructional design for the teaching and learning of geometry is established through the components of research models of student thinking, professional development workshops and teacher authoring, parents as partners, and classroom-based collaborative research.

Three teachers, who participated in the same workshops and summer institutes devoted to curriculum design, each developed a unique curriculum to teach geometry and spatial sense (Lehrer et al., 1998). One teacher's predominant themes for her students to know space is through measuring it. "Students designed their own tape measures for length, investigated and invented units for area," and designed containers that will hold the most popcorn and the least popcorn (p. 176). The second teacher's predominant themes for her students to know space is "to experiment with form, and many of the tasks she posed to students involved contrasting and comparing different two- and three-dimensional forms, finding and constructing the Platonic solids, and designing quilts and other patterns" (p. 176). The third teacher's predominant themes for her students to know space was through mapping, graphing, and way finding. During the three year study, each teacher progressively designed more interconnected tasks and used them to revisit important ideas through their continually growing understanding of student thinking. The most noticeable change is in the class' communication about space. When students spoke about space, they usually talked about drawing or building a measuring tool. In each year of the three year study they noted significant transitions in student thinking in all classrooms, and significant growth in children's number sense as well as their spatial sense.

Geometry Students

Weak subject matter knowledge in geometry along with the United States' students practicing routine procedures during 96% of their seatwork time is problematic for students across the nation (Cass, Cates, Jackson, & Smith, 2003). United States students spend less time on geometric measurement than other countries; as a result, United States students perform poorly on assessments of measurement (Clements & Bright, 2003). Clements and Battista's study (as cited in Clements & Bright, 2003) found that students, because of inadequate mathematical competency, "use measurement instruments or count units in a rote fashion and apply formulas to attain answers without meaning." Clements states that less than 50% of seventh graders can calculate the measurement of a line segment given a broken (Clements & Bright, 2003). Students fail to develop clear understandings of measurement because they lack the ability to partition space into equal linear units or arrays of two dimensional units (Clements & Stephen, 2003). To counteract this, they suggest that teachers should encourage students to measure items with standard and nonstandard units.

One measure to assess a high school student's level of understanding of geometry is the van Hiele Levels of Geometric Thought. "The model is a five-level hierarchy of ways of understanding spatial ideas" (Van de Walle, 2004, p. 348). These five levels include:

Level 0: Visualization

Level 1: Analysis

Level 2: Informal Deduction

Level 3: Deduction

Level 4: Rigor

Pierre M. van Hiele (1999) states, that the types of

> …instruction intended to foster development from one level to the next should include sequences of activities, beginning with an exploratory phase, gradually building concepts and related language, and culminating in summary activities that help students integrate what they have learned into what they know (p. 311).

This concept of developing students' thinking from one level to the next is the impetus for the present project.

Evidence shows that across the nation students lack the understanding of geometry concepts. The Third International Mathematics and Science Study (TIMSS) revealed the fact that the United States' eighth grade students' geometry achievement is below average among the forty-five countries involved in the study (Beaton et al., 1996). The report covered more than thirty languages at five grade levels, and revealed that eighth grade students from the United States showed little understanding of the properties of perimeter and area (Beaton et al., 1996). The TIMSS report asked the students this question for perimeter: "What is the ratio of the length of a side of a square to its perimeter? A. 1/1 B. 1/2 C. 1/3 D. 1/4 (p. 78)." The correct answer is D. Fifty-five percent of eighth grade students from the United States answered this question correctly which is far below the 80% successful answers from eighth grade students living in Japan and Singapore, and just below the forty-five countries' average of 56%. To test students understanding of area, they are given a rectangle and asked a two part question. The first task asked students to draw a rectangle "whose length is one and one-half times the length of the given rectangle and whose width is half the width of the rectangle (p. 95)." The next part of the question asked the students to state "the ratio of the area of the new rectangle to the area of the first one" and they are asked to show their work (p. 95). The level of difficulty of this problem proved to be far above the seventh and eighth grade students participating in this study. On average, 31% of the eighth grade students in the forty-five countries drew the correct rectangle compared to 16% in the United States. The second part of the question proved to be even more difficult. On average, in the forty-five countries the number of students answering that part of the question correctly is just 6%, while the United States did better with 10%.

Conclusion

Geometry is important in terms of developing students' mathematical reasoning skills (NCTM, 2000). The literature shows there is a need to meet the students at their knowledge and develop their knowledge through everyday experiences (Fuys et al., 1988; Lehrer et al., 1998; van Hiele, 1999). There is a need to develop curricula to increase the students' spatial awareness and understanding (Lehrer et al., 1998; Menon, 1998). The development of pre-service elementary teachers will assist in developing students' conceptual understanding instead of computational knowledge without understanding (Menon, 1998). Rote memorization of formulas has proven ineffective (Cass et al., 2003; Clements & Stephan, 2004; de Villiers, 1998; Malloy, 1999; Ridgway & Healy, 1997). American students and teachers are not proficient in their understanding of area and perimeter (Addington, 2006; Malloy, 1999; Menon, 1998; TIMSS, 1996). Area and perimeter is one field in geometry that needs more attention. The goal of this project was to create a supplemental geometry unit to develop the concept of the area and perimeter of a polygon without the use of formulas and numbers, thereby increasing students' understanding of these concepts.

CHAPTER THREE

METHODOLOGY

Understanding Area and Perimeter

The purpose of this study was to test whether hands-on activities can increase the student's understanding of the measures of area and perimeter of polygons. The goal of this project was to create a supplemental geometry unit to develop the concept of the area and perimeter of a polygon without the use of formulas and numbers, and test its effect on student understanding of these concepts. Michael de Villiers argues "that students should be actively engaged in the defining of geometric concepts" while actively participating in the construction and the development of the content (1998, p. 248). Studying area and perimeter through two dimensional constructions gives the students a visual meaning of the definition of area and perimeter, which increases their interest while helping them with their understanding (Murrey & Newton, 2007).

Problem Description

According to the California mathematics standards, students begin learning about area and perimeter in elementary school (CDE, 2000). Malloy (1999) states that by the time students enter the middle grades they should have a concept of what area and perimeter are. She argues that, although many students may be able to compute the area and perimeter of given figures, few have fully conceptualized the meaning of area and perimeter.

Perimeter and area are concepts that are usually learned by formulas. Students often become "confused by the formula and find area when they are asked for perimeter and perimeter when they are asked for area" (Malloy, 1999, p. 87). Other research shows that students and some teachers try to compare perimeter and area even though these quantities have different units (Addington, 2006). However, if meaning is attached to perimeter and area, then "confusion can be eliminated because the measures are obviously different: one is the number of length units that fits around the figure, and the other is the number of square units enclosed by the figure" (Moyer, 2001, p. 52).

Common Student Difficulties

This project addresses several areas highlighted by research as problematic for geometry students. Research conducted by Fuys, Geddes, and Tischler (1998) found, "Experiences of secondary school mathematics teachers indicate that many students encounter difficulties in high school geometry" (p. 4). One of the causes for these difficulties was traditional instruction in which students were taught rote memorization of formulas (Cass et al., 2003; Clements & Stephan, 2004; de Villiers, 1998; Malloy, 1999; Ridgway & Healy, 1997). Another cause for these difficulties was that traditional instruction needs to account for the different phases of the learning process: the instruction must foster development from one level of understanding to the next (Fuys et al., 1988; Lehrer et al., 1998; van Hiele, 1999). Students may also encounter difficulties in high school geometry through their textbook's inability to account for the various phases of the learning process (Fuys et al., 1998).

Pierre M. van Hiele believed that secondary school geometry requires a high level of thinking while many secondary geometry students did not have sufficient experience in thinking at lower levels. A gap exists between students' level of thinking and the required level of thinking necessary for geometry success (van Hiele, 1999). Dina van Hiele and Pierre van Hiele "observed that teachers often talked about geometry using language that students could not understand," placing the teacher and students at different levels of thought about geometry (Malloy, 1999, p. 1). This lack of communication between teachers and students is an added obstacle to the students' understanding of secondary school geometry.

Burdett High School

Can a student gain a solid understanding of area and perimeter of polygons without the use of numbers and formulas? The project's unit was taught to a geometry class at Burdett High School, supplementing the textbook's chapter ten, while another geometry teacher taught the traditional geometry class. Both classes were given a pre-test prior to the start of chapter ten, and both teachers gave a post-test following the conclusion of the chapter or the project's unit.

The first goal of the project was to deepen the student's understanding of the concept of area of a polygon. Students were challenged to find the area of a polygon in as many ways as they could. Within small groups and with the class, students shared and discussed their results and strategies. The second goal of the project was to define and estimate the perimeter of polygons. Working in small groups, students were challenged to find two figures with the same area and different perimeters (Addington, 2005). In small groups the students discussed and shared their results with the class.

The following sections will detail the project's scope and sequence.

The Project's Pre/Post-Test

The project's pre- and post-test were broken up into two types of problems. The first area of the test dealt with the concept of geometric area of polygons. The second area of the test dealt with the concept of length measurement with units and perimeter, mostly without units. Both the pre- and post-test consisted of twelve problems, with four of the problems having two parts. Five of the remaining twelve problems dealt with two different figures, with one of the figures having two questions, and the other figure having three questions.

The first question of the pre- and post-test dealt with the concept of dissection: taking a polygon and separating it into pieces, and then comparing the original polygon's area and perimeter to the new figure created. The students were required to recognize that the two figures were constructed with congruent pieces, and then analyze whether the figures' area, then perimeter, were the same, less than, or greater than. Then the students were to explain their reasoning.

Problem number two and three had the same concept. In problem two there was a story and a picture about a rope tied to make a loop; it was thrown on the ground twice making two different figures. The students were required to recognize that the rope was the same each time creating two different figures. Students were to analyze whether the figures' area, then perimeter, were the same, less than, or greater than. Then the students were asked to explain their reasoning. In problem number three the students were given a triangle with the side lengths marked. Two parallelograms were created with two copies of the triangle. Students were to find the perimeter and the area of each parallelogram, and if they could not find the area, then they were to compare the areas of the two parallelograms.

Problem number four was a labeled triangle with three line segments drawn in the interior from two vertices. The students were to recognize and list the triangle's altitudes.

Problem number five simulated a bricklayer building a patio. The outline of the patio was given, and one of the brick units was labeled. The students were to find the area and perimeter of the finished patio.

Problem number six and seven pertained to a figure that was inscribed in another figure. The inside figure's interior was shaded. Problem six asked the students to determine which figure had the greater perimeter and why. Problem seven asked the students to determine which figure had the greater area and why.

Problems eight, nine, and ten pertained to two figures on a square grid. The figure created had a side labeled and a hypotenuse labeled with letters. Students were asked to compare the two figures' area and perimeter. Problem ten asked students to write an expression for the perimeter by using the labeling of the side and the hypotenuse.

Problem eleven asked students to interpret the length of a line drawn under a partial ruler that started at zero and ended at one, and was marked with binary fraction subdivisions. The unit was nonstandard, and the students were told that the basic unit was called an elbo. The students were asked how long (in elbos) the heavy line segment beneath the ruler was.

Problem twelve was a labeled parallelogram. The students were asked if the height of the parallelogram was greater than, less than, or equal to the side.

The Control Group

The control group had a class of less than thirty students, but only twenty students were in attendance for both pre- and post-test. The control group used Burdett High School's textbook, "Geometry: Concepts and Applications," and direct instruction (Cummins et al., 2001). This group covered all seven sections of chapter 10, "Polygons and Area," in the textbook. The class took one day for the pre-test, ten days to cover the material, and one day for the post-test. Burdett High School was on a modified block schedule meaning that on Monday and Friday students would attend all their classes for fifty-five minutes, and on Tuesday, Wednesday, and Thursday they would attend four classes for eighty minutes. Each week each class would meet four times per week, Monday and Friday, and then twice during Tuesday, Wednesday, or Thursday.

The Treatment Group

The treatment group had a class of less than thirty students with twenty-four students in attendance to take both the pre- and post-test. The treatment group was on the same modified block schedule as the control group which met four times a week, two days for fifty-five minutes and two days for eighty minutes. This class took one day for the pre-test, ten days for the project's unit, and one day for the post-test. The treatment group also used Burdett High School's textbook, but, covered only three sections of chapter ten: 1) Naming Polygons, 2) Area of Polygons, and 3) Areas of Triangles and Trapezoids. These sections were sprinkled throughout the project's unit. The four sections of chapter ten of Burdett High School's textbook that was not covered were Diagonals and Angle Measure, Areas of Regular Polygons, Symmetry, and Tessellations. The other lessons in the project's unit took material from the textbook "Geometry: Seeing, Doing, Understanding (Jacobs, 2003), and the textbook "Discovering Geometry: An Investigative Approach" (Serra, 2003).

The Project's Lessons

The routine each day in which the project was implemented was to discuss the previous day's concepts and assignment. I checked for understanding and assisted with clarity as needed. Concepts that were unclear were re-visited. Concepts that led into the day's topic were reviewed.

Lesson One. In the first lesson of the project, students were asked to compare the area of polygons without the use of formulas. Students were asked to state the definition of the "area of a polygon." Students were given a laminated map of the United States to compare the areas of different states on the map (Jacobs, 2003). Normally, to compare sizes, we usually use numbers. Without numbers such comparisons are not always as easy. There was class discussion of how to approach this without numbers. Numbers were usually used as a measuring tool to compare sizes. We discussed other possible measuring tools that could be used to compare sizes. Then students were challenged to use the map and pick a measuring tool like rice, white beans, "CHEEZ-IT", or any tool which they chose to calculate the next three largest states following Texas. Students were to state their measuring tool; they were to explain why they chose that particular measuring tool; and they were to explain why their tool was accurate. The students were asked to name the next three largest states following Texas and state how they arrived at that result. Students were asked what would be the unit of measurement, and why? Then for homework the students were asked to use two other measuring tools to calculate the area without formulas and record what they used and their results. Next, students were to discuss which measuring tool they preferred and explain why. Finally, the students discussed the fact that if someone else were to use their measuring tool would they arrive at the same result, and briefly explain why they would or would not.

Lesson Two. In the second lesson students discussed what measuring tools they used during class and why? Students were asked about the measuring tools used when they were at home and were the results the same? Students were asked what would be the unit of measurement, and why? Finally, each group was asked to state in their own words the definition of the area of a polygon which prompted the lesson's discussion on polygonal regions. Students were asked to define a polygonal region (Jacobs, 2003). I presented several copies of the same polygonal region. I took one copy of the polygonal region and cut it up into several pieces. I reassembled the pieces with overlapping pieces, and discussed whether or not it had the same area as the original polygonal region. A complete definition of the area of a polygonal region was established and discussed. The class practiced estimating the area of polygonal regions (Jacobs, 2003). Students were asked to compare areas of polygonal regions for homework and justify their answers (Jacobs, 2003).

Lesson Three. In the third lesson I expanded their homework problem with the flags of Thailand and Panama. Each flag had three colors. The areas of the flags were described by a variable or variables, and the students were asked to write an expression for the area of a particular part or parts of the flag. I used these flags to introduce perimeter to the class. We discussed what the definition of perimeter was and how it differed from area. Students were challenged to come up with a variable expression for the perimeter of the flags if given variables for the sides. Because of the different color strips on the flag of Thailand, we were able to vary the variables for the side of this flag creating different scenarios leading to different expressions for the same perimeter. While in their small groups, students were asked to compare and contrast the meaning of the area of the flags with the perimeter of the flags. Then we used Burdett High School's textbook, chapter ten section one, to define a regular polygon, a convex polygon, and a concave polygon (Cummins et al., 1998). Through guided practice we determined if figures were polygons or not, and if the figure was a polygon, then it was determined if it was concave or convex. Polygons were classified by the number of sides by using prefixes. Homework was assigned where students were asked to identify each polygon by its sides, to classify each polygon as convex or concave, and then find the perimeter of each regular polygon with the given side lengths (Cummins et al., 1998).

Lesson Four. In the fourth lesson we reviewed the definition of polygonal region and the area of a polygon. We discussed how area can be used to describe, compare, and contrast polygons. We classified congruent polygons as having equal areas. We discussed the area addition postulate which states, "The area of a given polygon equals the sum of the areas of the non-overlapping polygons that form the given polygon" (Cummins et al., 1998). We looked at similar polygons and determined whether or not they were congruent. In small groups the students investigated the relationship between the areas of a polygon drawn on rectangular dot paper and the number of dots on the figure (Cummins et al., 1998). These figures were drawn with no dots in the interior of the polygons. On the overhead projector I drew a polygon with a dot in its interior so students could see how not to draw their polygons. The students were asked to draw polygons that go through three dots, four dots, five dots, and 6 dots, having no dots in the interior, and they were given examples. Next, they were asked to copy a table which had a row for the number of dots on the figure and a row for the corresponding area in square units of that polygon which was created by that number of dots. Given the number of dots on the figure, the students were required to fill in the area in square units of the corresponding polygon. Students were asked to predict the area of a figure whose sides go through twenty dots and verify it. Finally, students were asked to choose the correct relationship that exists between the number of dots on the figure and the area of the figure. Homework was given which mimic this hands-on geometry activity using rectangular dot paper with polygons having no dots in the interior to polygons having one dot in the interior (Cummins et al., 1998).

<u>Lesson Five</u>. In the fifth lesson we discussed the homework and compared the polygon area patterns found in the hands-on geometry activity when polygons had no dots in the interior to polygons having one dot in the interior. We discussed whether or not we could use these patterns in the future, and under what circumstances will they work. Caution was given to the students not to apply these patterns to any polygon drawn on rectangular dot paper, but only to those polygons with no dots or one dot in the interior. I provided guided practice on estimating areas of polygons. Students were reminded of the area addition postulate, and shown that one way to find the area of a polygon was to divide it into shapes such as squares, rectangles, and triangles. Students were asked to use rectangular dot paper to draw a polygon with the same area as a given polygon, but not congruent to that polygon. We also discussed how to sketch two polygons that both had the same perimeter, but had different areas. One of their homework problems required them to do this. Homework from Burdett's textbook, chapter 10 section

three, was given that allowed students to practice estimating the area of given polygons on grid paper and rectangular dot paper (Cummins et al., 1998).

Lesson Six. In lesson six we began by discussing the previous lesson's homework problem which asked the students to sketch two polygons that both had a perimeter of twelve units, but that had different areas. I had several students present their results to this problem on the board, and as a class, we discussed their results. After that discussion, I had students draw a rectangle ten units by four units on grid paper. I asked them to find the area of the rectangle. Then I directed the students to draw a diagonal to divide the rectangle into two congruent triangles. I then asked the students to find the area of one of those triangles. We discussed the relationship of the area of the rectangle to the area of the triangle. After they found the area of one of those triangles, I drew three more triangles on grid paper on the overhead. One of the triangles had the altitude in the interior of the triangle, another one of the triangles had the altitude outside the triangle, and the last triangle had one of the sides of the triangle as the altitude. The students were instructed to copy these triangles on their grid paper and to find each triangle's altitude. As a class we discussed the definition of altitude and its attributes. Then we discussed the definition of height, and whether or not it meant the same as the altitude. For homework the students were given three different triangles drawn on dot paper with equal areas. The students were asked to explain why the triangles had equal areas.

Lesson Seven. In lesson seven the students were asked to draw two unequal parallel line segments on their grid paper, and they were to draw a line segment connecting the left endpoints together, and then a line segment connecting the right endpoints together. The students were asked if they knew the name of this shape. We discussed the features of the trapezoid, and the students were challenged to find the area of the trapezoid that they had drawn. They were reminded of the Area Addition Postulate, and we discussed how the trapezoid could be separated into pieces to estimate the area. For homework Burdett High School's textbook, chapter 10 section four, asked the students to make a conjecture about how the area of a trapezoid changes if the lengths of its bases and altitude are doubled (Cummins et al., 1996). I encouraged the students to draw both trapezoids on rectangular dot paper, and estimate both areas before they make their conjectures. Finally in the students' homework they were given an isosceles trapezoid separated into four right triangles. On rectangular dot paper, the students were to draw three isosceles trapezoids. The students were to separate one of the isosceles trapezoids into three isosceles triangles, another one into two congruent trapezoids, and the last one into five polygonal regions (name the regions) (Cummins et al., 2001). I asked the students to estimate the area of those three isosceles trapezoids, the area of the interior regions that they created, and show that all the parts are equal to the whole.

Lesson Eight. In lesson eight we reviewed the definition of an altitude. In small groups each student was given heavy grid paper to construct a parallelogram (Serra, 2003). From the vertex of the obtuse angle adjacent to the base, the students were to draw an altitude to the side opposite the base. The students were shown how to label their parallelogram. Next, students were to cut out the parallelogram and then cut along the altitude leaving them with two pieces, a triangle and a trapezoid. They were challenged to try to arrange the two pieces into other shapes without overlapping them. The students were asked whether or not the areas of each of the new shapes are the same as the area of the original parallelogram (Serra, 2003). First in small groups, and then as a class, we discussed why. Next we discovered whether or not anyone created a rectangle as their new shape and how it compared to the original parallelogram. Students were asked how the parallelogram and the rectangle were the same, and how they were different (Serra, 2003). We discussed as a class what the students knew about area. We discussed the idea that area often means a number associated with the region enclosed by the shape. Then the students were asked to state a conjecture for the area of a parallelogram (Serra, 2003).

Lesson Nine. Students were taught how to read a ruler. I drew a line on the board and randomly asked students to measure it with a ruler in centimeters, millimeters, and inches while noting each time the unit of measurement. I then showed how an inch could be broken down into fourths, eighths, and sixteenths. As a class we practiced measuring different lengths of lines using different scales of rulers. I drew a line on each student's paper and asked them to measure it in inches and centimeters. Next students were asked to create a triangle with sides of four centimeters, seven centimeters, and nine centimeters (Addington, 2005). The students were asked to draw the altitude in the interior from the vertex created by the four centimeter and the seven centimeter sides of the triangle, and then measure and label the altitude. Students were to label the sides of the triangle in the interior, so that the triangle could be cut out. With two copies of this cut-out triangle, students were to construct three parallelograms with different side lengths. They were to calculate the area and perimeter of each parallelogram, and state their findings. We discussed the students' findings as a class and shared why their results were attained.

Lesson Ten. In lesson ten students were posed with the question, "Do all rectangles with the same perimeter have the same area" (Serra, 2003)? Students were given rectangle dot paper, geoboards, and strings to assist in their investigation. Students were asked to document their examples along with their conclusion, and discuss their findings. Then students were asked to investigate and document their findings concerning these questions:

Is it possible to have two plane figures with the same area and different perimeter? Is it possible to have two plane figures with the same perimeter and different area? Find two shapes that are not congruent but have the same area and perimeter (Addington, 2005).

Students were asked to document their examples along with their findings, and conclusions.

CHAPTER FOUR

DATA ANALYSIS

Results

The following sections analyze the pre- and post-tests for both the treatment group and the control group students. In each group, problems which showed statistically significant growth were analyzed along with problems which showed decline.

Treatment Group's Results

When comparing pre- and post- tests, I found that the students involved in the research project showed nominal growth in some areas and declines in others. Perimeter was a challenging concept for most of the students. The intention of this project was to engage the students while deepening their knowledge of area and perimeter.

Table 1 (see Table 1 below) shows that out of the nine problems dealing with perimeter and ruler measurement, students showed growth on six of the nine problems, and declines in three problems. Two of the six problems that showed growth had a significant gain while another problem just missed showing a significant growth from the pre-test to the post-test.

Table 1. Class Data: Proportion Correct

	Question Number	Treatment Group's Data (24 students)			Control Group's Data (20 students)		
		Pre-Test proportion right ($prop_1$)	Post-Test proportion right ($prop_2$)	$prop_2$-$prop_1$	Pre-Test proportion right ($prop_1$)	Post-Test proportion right ($prop_2$)	$prop_2$-$prop_1$
Perimeter and Ruler Measure Problems	Circle to Strip (1b)	6/24=.25	10/24=.417	.167	4/20=.2	4/20=.2	0
	Rope (2b)	10/24=.417	11/24=.458	.041	1/20=.05	5/20=.25	.200
	Triangles to Parallelogram (3a1)	9/24=.375	17/24=.708	.333	8/20=.4	8/20=.4	0
	(3a2)	7/24=.292	16/24=.667	.375*	7/20=.35	8/20=.4	.050
	Brick Patio (5b)	9/24=.375	6/24=.25	-.125	6/20=.3	2/20=.1	-.200
	Inscribed Star (6)	10/24=.417	5/24=.208	-.209	8/20=.4	3/20=.15	-.250
	Geo-board Shapes (9)	14/24=.583	5/24=.208	-.375	12/20=.6	3/20=.15	-.450
	(10)	3/24=.125	4/24=.167	.042	0/20=0	0/20=0	0
	Ruler (11)	8/24=.333	17/24=.708	.375*	1/20=.05	2/20=.10	.050
Area Problems	Hexagon to Strip (1a)	17/24=.708	17/24=.708	0	16/20=.8	9/20=.45	-.350
	Rope (2a)	14/24=.583	10/24=.417	-.166	12/20=.6	10/20=.5	-.100
	Triangles to Parallelogram (3b1)	7/24=.292	11/24=.458	.166	1/20=.05	6/20=.3	.250
	(3b2)	7/24=.292	9/24=.375	.083	1/20=.05	6/20=.3	.250
	Altitude of Triangle (4)	3/24=.125	8/24=.333	.208	2/20=.1	1/20=.05	-.050
	Brick Patio (5a)	14/24=.583	15/24=.625	.042	9/20=.45	6/20=.3	-.150
	Inscribed Star (7)	11/24=.458	14/24=.583	.125	10/20=.5	9/20=.45	-.050
	Geo-board Shapes (8)	21/24=.875	19/24=.792	-.083	17/20=.85	14/20=.7	-.150
	Height of Parallelogram (12)	7/24=.292	12/24=.5	.208	4/20=.2	9/20=.45	.250

* Post-test proportion correct shows a 1% significant level of growth from the pre-test proportion correct.

The sample data supports the claim that problem three (Part a, parallelogram 2)and problem eleven's post-test shows significant growth from the pre-test. Problem three (Part a, parallelogram 2) asked the students to find the perimeter of the parallelogram made with two copies of the given triangle. The lengths of all sides of the triangle were given. The next problem that showed a significant gain was problem eleven. Problem eleven asked students to measure the length of a line drawn under a partial ruler that started at zero and ended at one. The unit was nonstandard, and the students were told that the basic unit was called an elbo. The students were asked how long (in elbos) the heavy line segment beneath the ruler was. The problem that just missed showing a significant gain was problem number three (part a, parallelogram 1).

Problem number nine showed a significant loss among the three problems that showed declines within the problems dealing with perimeter and ruler measurement. In problem number nine the students were asked to find the perimeter of the two figures drawn on a square grid with one side and one hypotenuse labeled. Another problem that showed a nominal decline was problem six. In problem six one figure was inscribed in another figure. The inside figure's interior was shaded. Problem six asked the students to determine which figure had the greater perimeter.

In the nine problems dealing with area, students showed nominal growth on six of the nine problems, no change in one problem, and declines in two problems. No sample data showed any significant growth from the pre-test to the post-test. The problem that dealt with the tied rope created a slight problem when it came to area, but they showed growth when it came to perimeter.

<u>Control Group's Results</u>

In the nine problems on perimeter and ruler measurement, the control group showed nominal increase in three problems, no change in three problems, and declines in three problems. In the nine problems on area, the control group showed nominal increase in three problems, and declines in six problems. There was no significant growth in both perimeter and ruler measurement or area problems. Problem nine, like the treatment group, showed a significant loss from the pre-test to the post-test. The students were asked to find the perimeter of the two figures drawn on a square grid with one side and one hypotenuse labeled. The control group also was unable to compare the lengths of the perimeter when there were unequal number of sides and hypotenuses. The next problem that showed a nominal decline was problem one (part a) which asked students to compare the area of two figures whose pieces were rearranged into a different figure.

Analysis of Assessment Results

The following sections analyzed the problems that showed statistically significant growth. Problems that showed significant decline were also analyzed along with problems that just missed showing significant growth.

Treatment Group's Assessment Results. Problem three (part a, parallelogram 1 & 2) showed that students were capable of finding the perimeter of the parallelogram made with two copies of the given triangle (see Figure 1 below) on the post-test. The reason parallelogram 1 did not show a significant gain was probably because of the information gathered on the pre-test problems. On the pre-test the students were capable of calculating the perimeter for parallelogram 1, but struggled to calculate the perimeter for parallelogram 2; therefore producing a significant gain for parallelogram 2, and just missing a significant gain for parallelogram 1. On the pre-test the students explained that the two figures were made with the same sides, but turned in different directions. The students may not have checked to see that the length of the sides for parallelogram 1 were different than parallelogram 2. Therefore, more students missed the perimeter for parallelogram 2 on the pre-test.

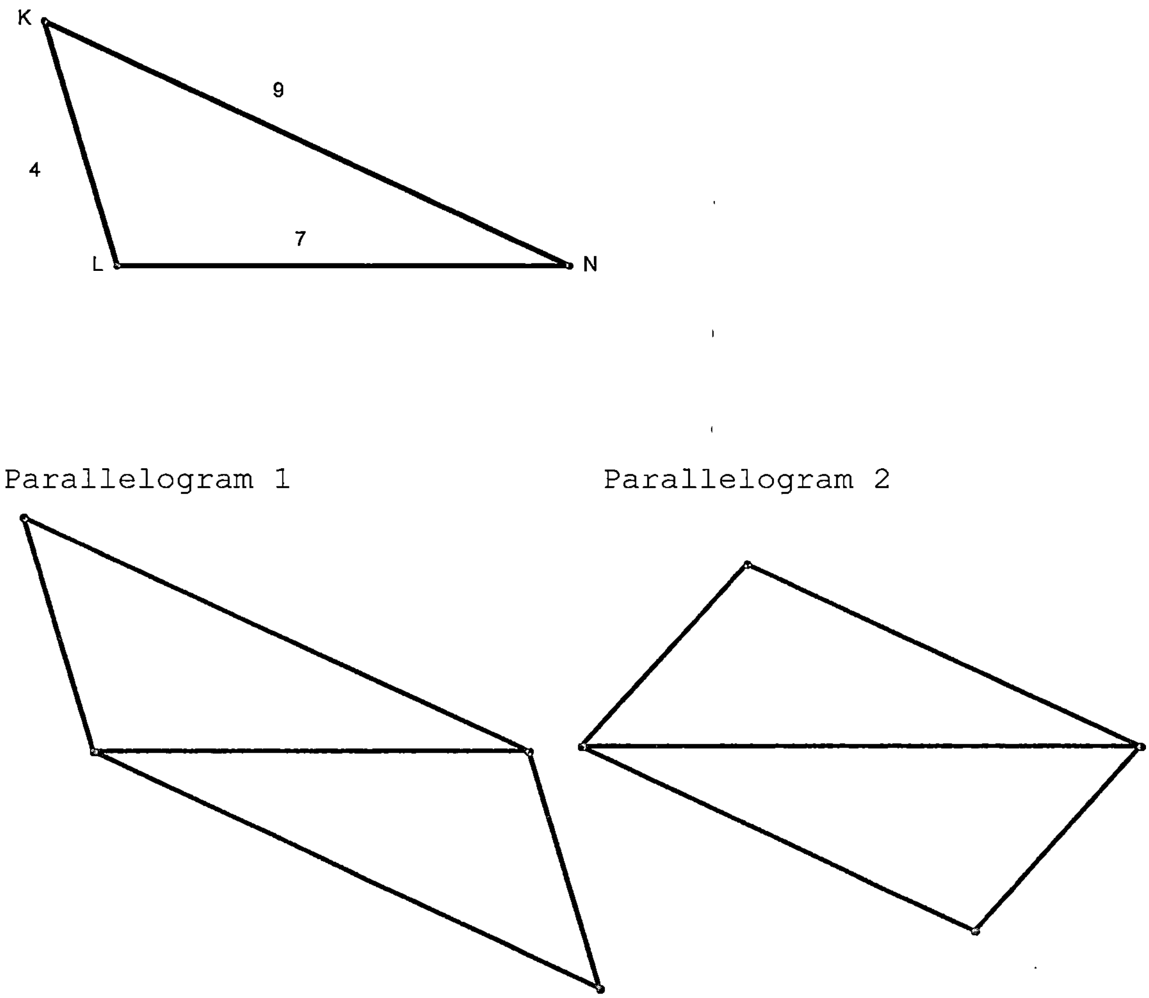


Figure 1. Triangle to Parallelogram

In the treatment group's lesson the students were asked to use two copies of the same triangle cut out, and arrange the two triangles' creating two different polygons. The two triangles sides were labeled in the interior. When the polygons were formed with the two triangles the students labeled the polygons. The students were asked to find the perimeter and area of the polygons created by the triangles. This activity seemed to have increased the treatment group's understanding of how two polygons can be made with different sides of two copies of the same triangle, thereby creating different perimeters.

In problem eleven (see Figure 2 below) the students were asked to measure how long (in elbos) the length of the heavy line beneath the ruler was.

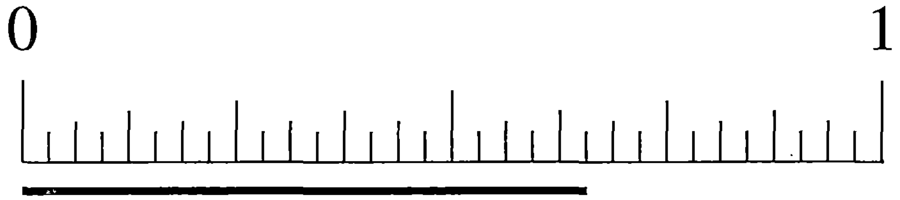


Figure 2. Ruler

Within the project's unit lesson the students were asked to measure a line using different units consisting of standard units (inches and centimeter) and nonstandard units (a created basic unit of length). The students were asked to measure the line using a ruler in inches by using fourths, eighths, and sixteenths, and then in centimeters. Once the students grasped the fact that the same line could be measured by using different units they seemed to have less challenges with the use of rulers. This activity seemed to have increased the treatment group's understanding of ruler measurement.

In problem number six the students were asked to determine which figure had the greater perimeter. One figure was inscribed in the other figure. The inside figure's interior was shaded (see Figure 3).

Students' written explanations made it clear that the treatment group could not see that the inside figure were created by curving the arc of the circle inside, therefore giving the same perimeter. No lesson within the project's unit seemed to have addressed this concept.

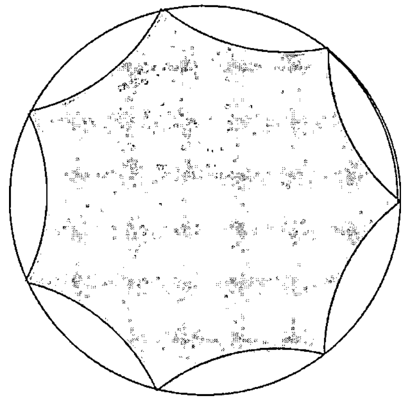


Figure 3.   Inscribed Star

In problem number nine the students were asked to compare the perimeter of the two figures drawn on a square grid with one side and one hypotenuse labeled (see Figure 4).

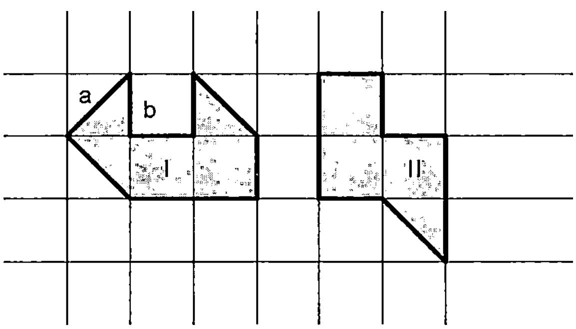


Figure 4.   Geo-board Shapes

The students did not take into consideration that the hypotenuse was longer than a side. Figure I had two sides that were longer than those in Figure II because they were hypotenuses. The students who missed this problem stated in their explanations that Figure II had the greater perimeter or they stated that the perimeter for both figures were the same. The project's unit-lessons did not deal with comparing the length of a side to the hypotenuse; therefore, the treatment group's students did not have this concept in mind even though the treatment group was aware that the hypotenuse was longer than its side. Parts of a right triangle were taught in earlier lessons, but should have been reviewed.

The students struggled with the concept of an altitude on the pre-test. The students showed nominal growth on the post-test when it came to finding the altitude of the triangle, which was problem four, and finding the area of the parallelogram created with two triangles in problem three.

The students judged the rope problem by its appearance which was van Hiele's lowest level, level 0: visualization. The students explained that since the rope was the same each time it was thrown down that the perimeter and area were the same also. The students did not reason about what makes one area greater than another.

The average number of problems correct on the pre-test dealing with area was higher than the average number of problems correct on the pre-test dealing with perimeter. The students' pre-test scores in area were higher with problems dealing with dissection of polygons and its area, comparing two polygons and their area, and counting the area of a polygon with square units. Where the students struggled in area problems was on the problems that dealt with triangles and estimating their area using altitudes.

In sum, the treatment group showed a statistically significant gain on two of the eighteen problems dealing with area or perimeter and ruler measurement. This class showed nominal change from the pre-test to the post-test on the other problems.

<u>Control Group's Assessment Results</u>. Problem number nine showed a significant loss in both classes (see Figure 4 above). The control group also failed to recognize that the hypotenuse's length was longer than the side's length. They too thought that the two figures either had the same

perimeter or that Figure II had a larger perimeter. The control group was also taught the parts of a right triangle in earlier lessons; therefore these students were aware that the hypotenuse was longer than its side, but were unable to put that information together on this problem. The next problem that showed a nominal loss was problem number one (part a) (see Figure 5 below).

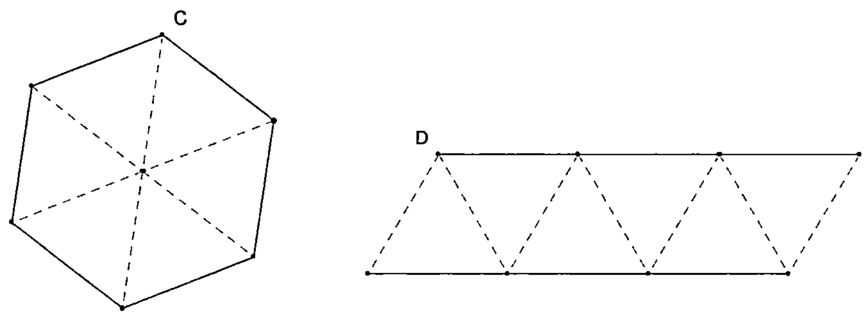


Figure 5. Hexagon to Strip

Most of the students in the control group thought that the area in Figure C was larger than the area of the dissected Figure D. They both had the same area. The control group was familiar with the dissection of the circle on the pre-test and was unfamiliar with the dissection of the hexagon on the post-test. The students in the control group were thinking on van Hiele's level 0: visualization. The students did not analyze the pieces that made up both figures; the students explained that they thought that Figure C looked larger. Lastly, the control group also struggled with problem number six (see Figure 3). The students were asked to determine which figure had the greater perimeter. One figure was inscribed in the other figure. The inside figure's interior was shaded. The students could not see that the inside figure was created by curving the arch of the circle inward, therefore having the same perimeter.

The control group did not show a statistically significant gain on any of the eighteen problems dealing with area or perimeter and ruler measurement. Statistically, the control group showed a significant loss on one of the eighteen problems.

Comparison of Groups. There was not significant data to support the claim that the project's unit increased the conceptual understanding of area and perimeter in the treatment group compared to the control group. The treatment group did show growth on eleven of the eighteen parts of the test where the control group showed growth on six of the eighteen parts.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

Summary

Analysis of Burdett's textbook showed an inability to account for various phases of the learning process. Burdett's textbook did not challenge students to analyze figures in terms of their components or have the students prove or establish the definitions of area and perimeter. The textbook, "Geometry Concepts and Applications," spread out the concept of area and perimeter throughout the book (Cummins et al., 2001). There was no concentration of area and perimeter within the textbook to analyze, compare and contrast those concepts. Students were not given an opportunity to investigate and develop the formulas presented. Burdett's textbook did not challenge the students to analyze the differences between the units of perimeter and area.

Area and perimeter were introduced in chapter one under the section titled "A Plan for Problem Solving" (Cummins et al., 2001). In that chapter the textbook defined and provided the formulas and definitions for the perimeter P of a rectangle as $P = 2l + 2w$ where the length is l and width is w, area A of a rectangle as A = l w, and area A of a parallelogram as A = b h where base is b and height is h. Twenty-two of the thirty-two problems for that section had problems which only required the students to find the area or perimeter of polygons by substituting numbers for variables given in the appropriate formulas. The difficulty with these problems was that teaching computational and procedural skills may create an absence of understanding mathematics content standards (CDE, 2000). Anecdotal evidence suggested that most students did not learn the concepts of area and perimeter, but instead memorized and used formulas. Murrey and Newton (2007) state, "Students may have a difficult time remembering and applying formulas because they have not had the opportunity to investigate and develop these formulas" (p. 36).

After presenting the two ideas in chapter one, the text used at Burdett High School then reviewed area and perimeter by including one problem per chapter that dealt with the topic until chapter ten, "Polygons and Area," which this project supplements (Cummins et al., 2001). The textbook used by Burdett's geometry classes assumed that the students would memorize the definitions, theorems and formulas in order to apply them to problems. The application of the formulas within the area and perimeter unit was the primary means for students to gain an understanding of the concept of area and perimeter, which was to know the meaning of area and perimeter and how those mathematical ideas could be used in the real world.

The National Council of Teachers of Mathematics noted that many students had difficulty understanding how the formulas for perimeter and area related to the attributes being measured and the measuring unit to use (NCTM, 2000). The purpose for this project was to investigate teaching and learning mathematics, specifically with the concept of area and perimeter, without memorizing formulas.

Significance of the Project

Many students struggle to make the meaningful connections necessary for understanding the concept of area and perimeter (Murrey & Newton, 2007). They search their memory bank for the right key, a formula, to open the door. Rarely do the students look at a polygon and have a mental reference as to what the area or perimeter might be. Often students try to memorize formulas without an understanding of the concept. "Vinner and many others have presented arguments and empirical data that just knowing the definition of a concept does not at all guarantee understanding of the concepts" (de Villiers, 1998, p.249). The present project attempted to develop geometric thinking by providing the students an opportunity to explore and discover mathematical formulas for area and perimeter.

"The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase" (NCTM, 2000, p.4). Greater opportunities are afforded to students who comprehend and perform well in mathematical computations because we live in a dynamic world. Evidence has made it clear that many students are not learning the mathematics necessary to reshape their future (Kenney & Silver, 1997; NCTM, 2000). Low performing students in mathematics tend to have a strong dislike toward mathematics where as high achieving students in mathematics tend to have a strong liking of mathematics (Beaton et al., 1996). The confidence level of high school students toward their mathematics ability tends to have a direct relationship between their mathematics achievements in college (House, 2001). "It is crucial for students to realize that math is an integral part of everyday life rather than just a series of problems to be solved in a textbook (Cass et al., 2003, p. 112). Unfortunately, many students struggle in their mathematical studies, producing low mathematical achievements.

This project attempted to develop geometric thinking by providing the students an opportunity to make the learning experience more personal by requiring the students to take more responsibility for their own learning. Students developed and constructed the area and perimeter so that it was more comprehensible to them. The project provided students with the knowledge of area and perimeter without the use of formulas and numbers. The purpose for not using formulas and numbers was to create a deep understanding along with a visual picture of what was meant by area and perimeter. This project was to mitigate the difficulties of memorizing and understanding the usage of definitions, theorems, and formulas.

Limitations of the Project

One limitation with this project was class time. To develop the deep understanding of area and perimeter required the students to develop and construct their meaning in as many ways as they can. Students were asked to explain and compare what they had developed about area and perimeter. The students were then asked to share their findings with the class. Fifty minutes of class time only permitted a limited amount of this work to be completed during a class period, requiring the class to finish the next day. This break in time limited the flow of an idea, making it difficult to pick up where we left off. Another limitation with this project was the extended time it took to finish that particular chapter. This meant we were behind the district's timeline; therefore the students struggled on the benchmark test from the district because of the information not provided to them due to the time spent on the project. The final limitation with this project was that it was a small pilot study done with a fairly small sample group. The two geometry classes participating in the project had less than twenty-eight students each.

Recommendations

The students in the treatment group struggled with the concept that the perimeter of shapes on a geo-board has to take into consideration both the number of horizontal and vertical sides and the number of hypotenuses when comparing with other shapes.

Modification to lesson five of the project's unit, which asked students to use rectangular dot paper to draw a polygon with the same area as a given polygon, but which is not congruent, may clarify this concept. The teacher should then have the students analyze the perimeters of both polygons. In analyzing both perimeters the students should first label a side and a hypotenuse with a variable. Next, the students should write an expression in terms of the variables (without numbers for length) for each perimeter. Then the students should compare the two perimeters to determine which one was greater, and explain why it is greater by using what they knew about sides and hypotenuses. To assist students with comparing perimeters, add activities that require the students to compare the distance around their head to the length of their forearm, or compare the length of their waist to the length of their leg, or the length of their hand spread out to the length around their foot.

To assist the students with finding the altitude of a triangle, in lesson six of the project's unit, the teacher should have the students construct the altitudes of a triangle from each vertex by using the corner of a three by five card and a straightedge. Then the students should label the triangle and its bases. Following this, have the students name the altitude and its corresponding base. Finally, the students should explain how many altitudes a triangle has and why it is important to have the correct altitude with the correct base.

To get the students to understand that the perimeter of the inscribed star was created by the arc of the circle turned inward, the teacher should create an activity that uses the geo-board and a loop of string. Have the students create a shape on the geo-board with the loop of string. Next, have the students move only pieces of the string to create another shape. Ask the student to compare the original shape, area and perimeter, with the new shape created. The students are to draw each shape and document their findings.

Discussion of Project Results

Area and perimeter are difficult concepts for students to conceptually understand. Burdett High School students are introduced to the concepts of area and perimeter in elementary school. In the third grade the students are introduced to area and perimeter of a square and a rectangle by looking at pictures in the textbook with formulas. In the fourth and fifth grade the students continue with area and perimeter of a square and a rectangle. By seventh grade they have a chapter dedicated to area of parallelograms, triangles, and trapezoids. However, the concept of perimeter is embedded throughout the chapters in a few problems. The seventh grade's pre-algebra textbook treats perimeter as if the students have mastered this concept and only need a few problems for review. The perimeter problems that are given throughout the seventh grade pre-algebra textbook were used to have the students practice writing and solving equations. In the eighth grade, the algebra I textbook assumes that the students have mastered both area and perimeter. In the algebra I textbook, the concept of area and perimeter is sprinkled through out the chapters in different sections. In the ninth grade the students are taking geometry. Burdett High School's textbook refreshes the concept of area and perimeter in chapter one in a section titled "A Plan for Problem Solving," but does not spend much time on these concepts (Cummins et al., 2001). In this section the textbook reviewed the definition and formula for the area and perimeter of a rectangle and parallelogram. The difficulties with the problems in this section are that these problems only require the students to find the area or perimeter of polygons by substituting numbers for variables into the appropriate formulas. Rote memorization of formulas has proven ineffective (Cass et al., 2003; Clements & Stephen, 2004; de Villiers, 1998; Malloy, 1999; Ridgway & Healy, 1997).

The honors geometry class at Burdett High School uses "Discovering Geometry: An Investigative Approach" as their textbook (Serra, 2003). The honors geometry teacher stated that even though Serra covers geometry in a discovery fashion, he is unable to use it for the whole year because some required topics are missing from this book. He mixes discovery with direct teaching to get through the material. He uses two textbooks for his honors students. This project's supplemental unit used Serra's textbook for a couple of the lessons. The approach used in Serra's textbook provided the students with a discovery approach to understanding the concepts taught. The disadvantage to Serra's approach to teaching geometry, as stated by the honors geometry teacher, is the time needed to cover the material and trying to complete the schedule set by the district.

During Math 632, "Geometry from a Teaching and Problem Solving Perspective," at California State

University, San Bernardino, the participants in the class brought their districts' geometry textbooks to class one night to analyze them. Comparing Burdett High School's textbook to other district's textbook revealed a few interesting facts. First, the students at Burdett are using a textbook that avoids two-column proofs until chapter 15, which is rarely reached during the year. Secondly, the textbook is written on a lower level to meet the students' abilities, but does a poor job at developing the knowledge learned. Finally, the textbook fails to stretch the students or challenge them to develop and apply conceptual understanding of topics learned.

Conclusion

This project's unit attempted to stretch the students' understanding of the concept of area and perimeter while supplementing Burdett High School's textbook. This study is done with a fairly small sample group. The treatment group's results showed gain in the students' understanding of twelve of the nineteen problems on the post-test. Even though the sample data only supported statistically significant growth on two problems, the project's unit impact on the students' result is noticeable. Developing a project's unit to increase students' conceptual understanding of area and perimeter must be a learning process. Implementing the recommendations suggested by this paper along with this unit will develop curricula closer to establishing an understanding of area and perimeter. Maybe these topics, area and perimeter, were harder than I thought, and a two week lesson is not enough time to accomplish statistical success. More time should be given to these topics at the beginning of the school year with two dimensional constructions. Then when this project supplements the textbook's chapter ten it will provide reinforcement and continual conceptual understanding of area and perimeter. This project is designed to develop a high school unit in mathematics that will provide the students with an opportunity to construct understanding of area and perimeter. I look forward to doing more research in this area.

APPENDIX A

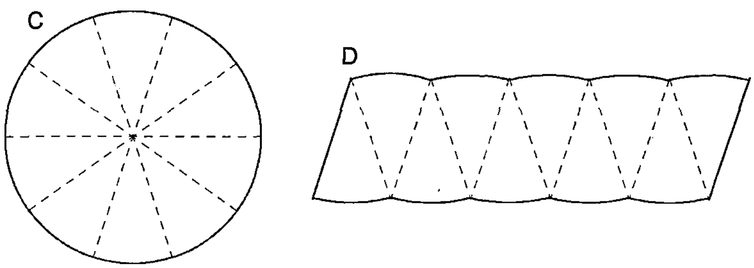
POLYGONS AND AREA PRE-TEST

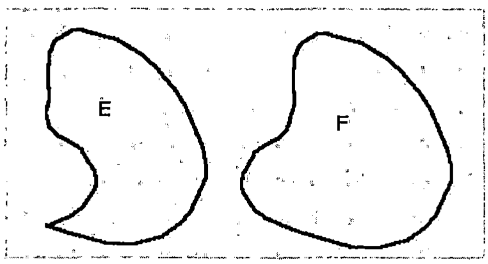
Polygons and Area Pre-Test

Directions: You may use a ruler or protractor on all parts except 1 and 2.

In problems 1 and 2 fill in the blanks with >, =, or < . Use your experience and intuition; no measuring tools or numbers allowed.

1a. Area of C _____ Area of D
b. Perimeter of C _____ Perimeter of D
Briefly explain your reasoning for both questions.





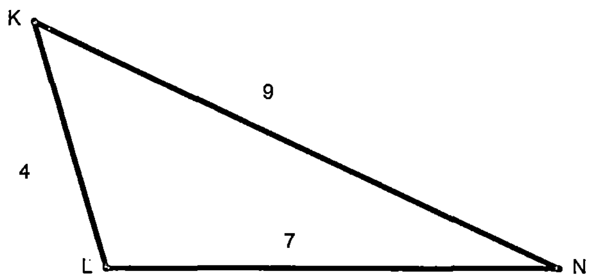
2. Fernando made a loop of rope and threw it down on the ground twice. These are the shapes he got (E and F):
a. Area of E _____ Area of F
b. Perimeter of E _____ Perimeter of F
Briefly explain your reasoning for both questions.

Polygons and Area Pre-Test

3. The two parallelograms below were each made with two copies of the triangle KLN. Measures on the triangle are in centimeters.

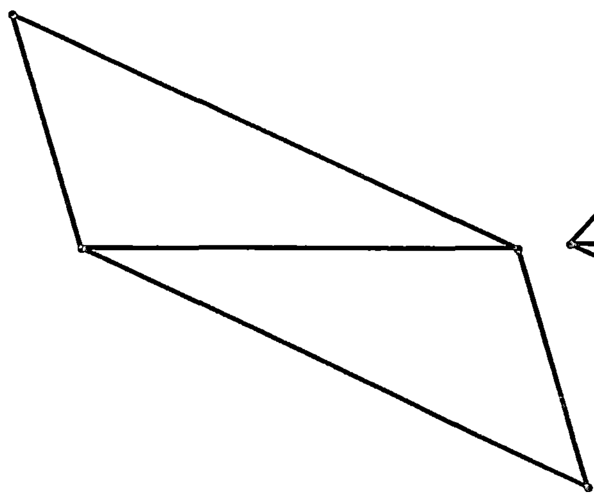
   a. Find the perimeter of each parallelogram, and give the units.

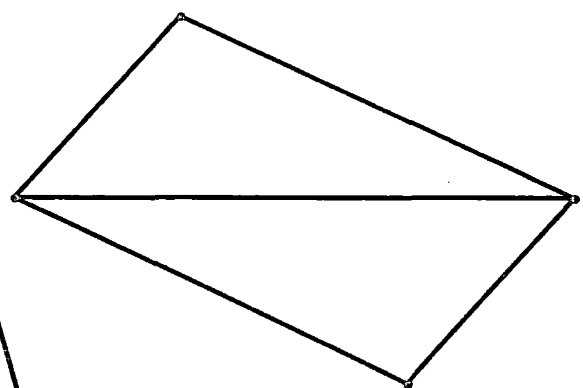
   b. Find the area of each parallelogram, and give the units. If you can not find the exact areas, at least compare the areas of the parallelograms.



Parallelogram 1

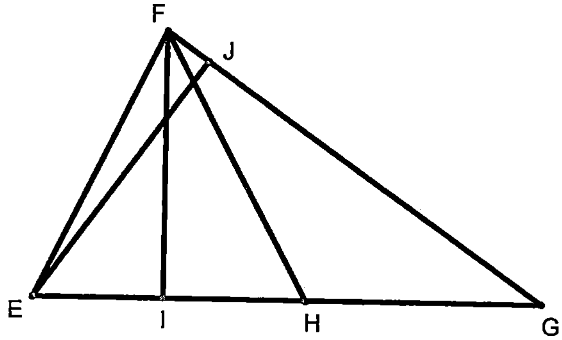
Parallelogram 2



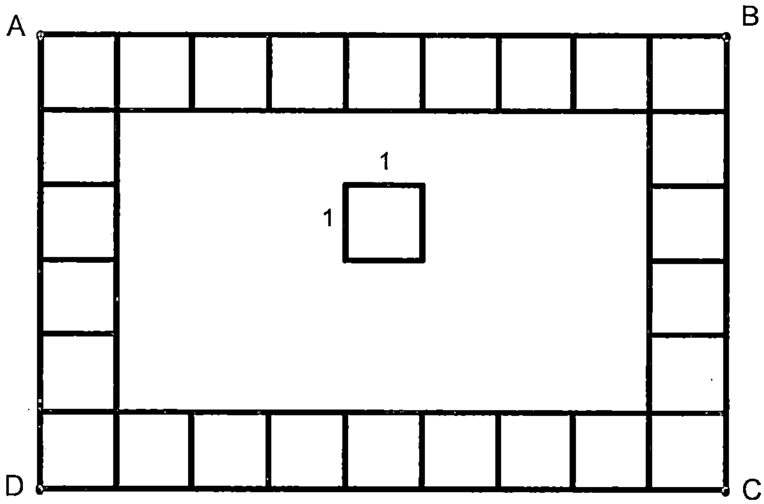


Polygons and Area Pre-Test

4. List all segments that are altitudes of triangle EFG.

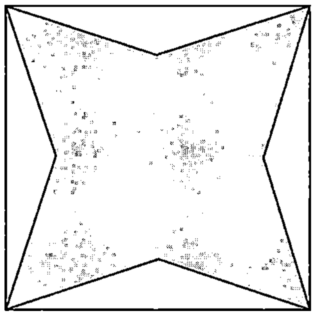


5. A bricklayer is building a patio. The outline of patio ABCD is below.
   a. Find the area of patio ABCD when it is finished.
   b. Find the perimeter of patio ABCD when it is finished.



Polygons and Area Pre-Test

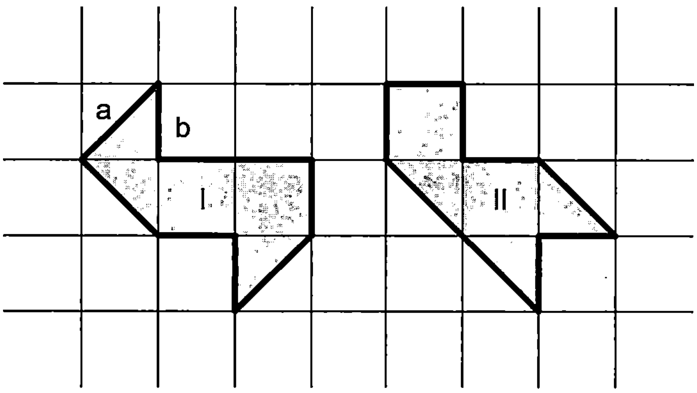
Star in Square. The figure below shows a "star" drawn inside a square.



6. Which has the greater perimeter, the "star" or the square? Why?
7. Which has the greater area? Why?

Polygons and Area Pre-Test

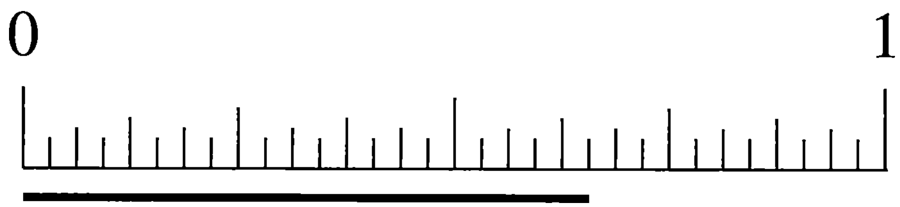
Drum Polygons. You might expect identical drums to sound alike. Surprisingly, mathematicians Carolyn Gordon and David Webb have discovered that drumheads with these two shapes sound alike.



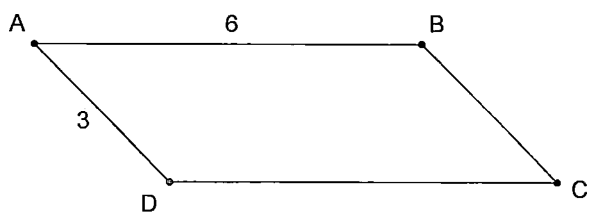
(For convenience, the shapes have been drawn on a square grid.)

8. How do the polygons compare in area?
9. How do their perimeters compare?
10. Write an expression in terms of a and b for each perimeter.

11. In the imaginary country of Elbonia, the basic unit of length is the elbo. Here is a piece of a ruler in elbos. How long (in elbos) is the heavy line segment beneath the ruler?



12. Is the height of the parallelogram ABCD greater than 3, less than 3, or equal to 3?



APPENDIX B

POLYGONS AND AREA POST-TEST

Polygons and Area Post-Test

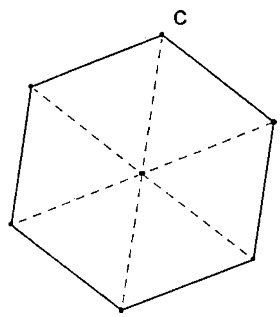
Directions: You may use a ruler or protractor on all parts except 1 and 2.

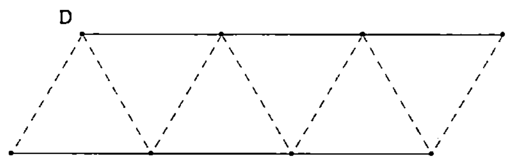
In problems 1 and 2 fill in the blanks with >, =, or < . Use your experience and intuition; no measuring tools or numbers allowed.

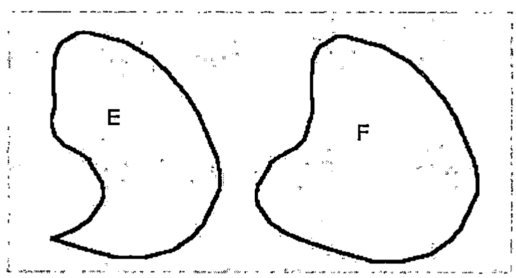
1a. Area of C _____ Area of D

b. Perimeter of C _____ Perimeter of D

Briefly explain your reasoning for both questions.







2. Fernando made a loop of rope and threw it down on the ground twice. These are the shapes he got (E and F):

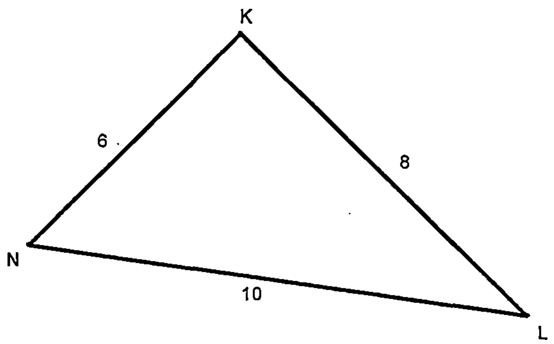
a. Area of E _____ Area of F

b. Perimeter of E _____ Perimeter of F

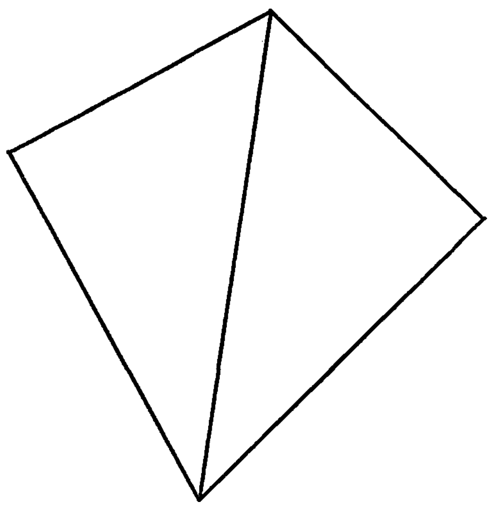
Briefly explain your reasoning for both questions.

Polygons and Area Post-Test

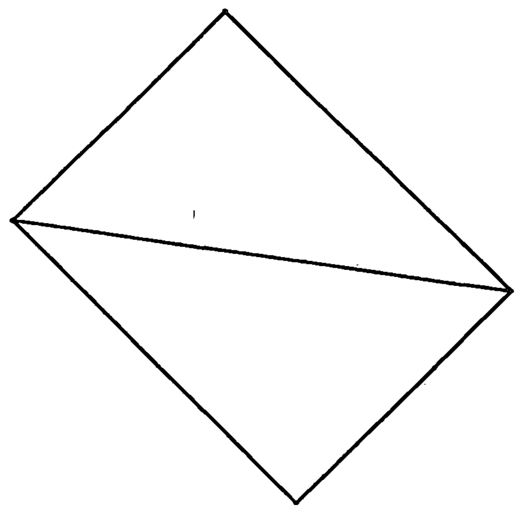
3. The two parallelograms below were each made with two copies of the triangle KLN. Measures on the triangle are in centimeters.
   a. Find the perimeter of each parallelogram, and give the units.
   b. Find the area of each parallelogram, and give the units. If you can not find the exact areas, at least compare the areas of the parallelograms.



Parallelogram 1

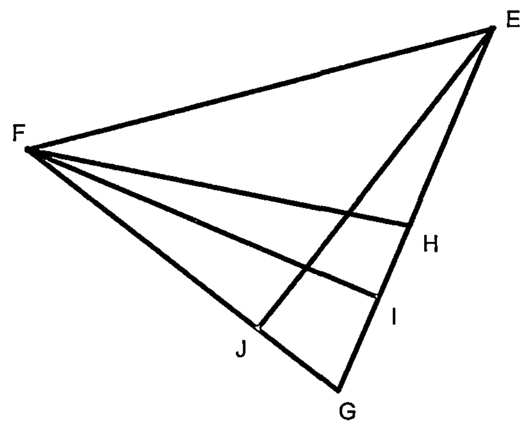


Parallelogram 2

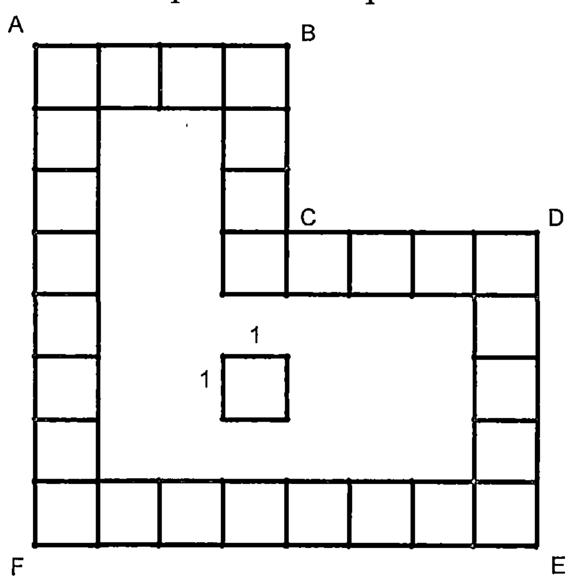


Polygons and Area Post-Test

4. List all segments that are altitudes of triangle EFG.

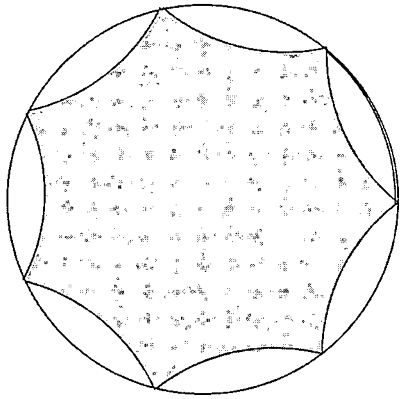


5. A bricklayer is building a patio. The outline of patio ABCDEF is below.
   c. Find the area of patio ABCDEF when it is finished.
   d. Find the perimeter of patio ABCDEF when it is finished.



Polygons and Area Post-Test

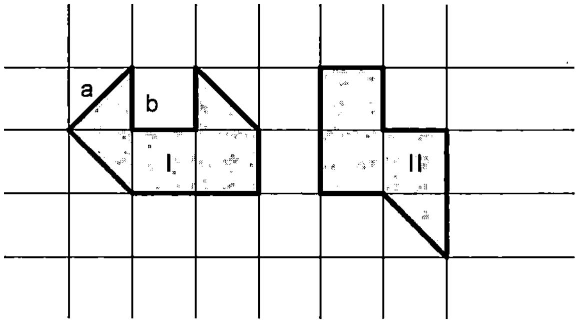
The figure below shows a shaded area drawn inside the circle.



6. Which has the greater perimeter, the circle or the shaded area? Why?
7. Which has the greater area? Why?

Polygons and Area Post-Test

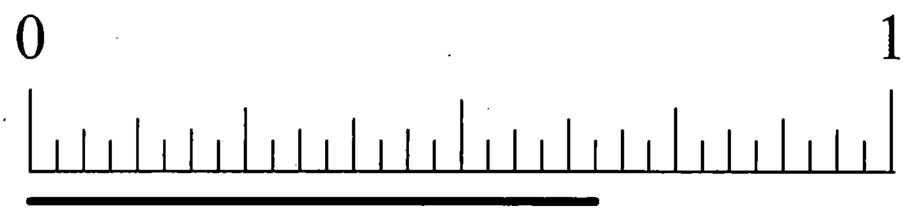
Swimming pool Polygons. You have designed two swimming pools. You need to decide which backyards will be able to accommodate your swimming pool designs.



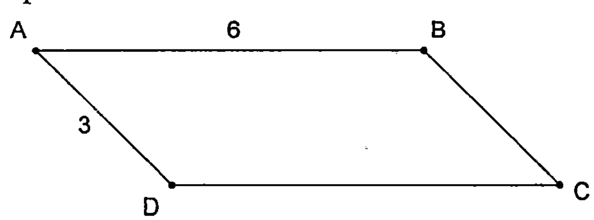
(For convenience, the shapes have been drawn on a square grid.)

8. How do the polygons compare in area?
9. How do their perimeters compare?
10. Write an expression in terms of a and b for each perimeter.

11. In the imaginary country of Elbonia, the basic unit of length is the elbo. Here is a piece of a ruler in elbos. How long (in elbos) is the heavy line segment beneath the ruler?



12. Is the height of the parallelogram ABCD greater than 3, less than 3, or equal to 3?



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