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Spectrally Efficient Iterative MU-MIMO Receiver for SC-FDMA Based on EP

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***Abstract*—A novel multiuser multiple-input multiple-output (MU-MIMO) receiver for single-carrier frequency-division multiple-access (SC-FDMA) is proposed within the expectation propagation (EP) framework. Recent advances in iterative receiver design show the ability of low complexity EP-based iterative algorithms to achieve performance close to maximum a posteriori (MAP) detection. In this paper, based on the exact derivation of frequency-domain EP-based turbo receivers, we propose a novel spectrally and computationally efficient soft interference canceller (IC) with a doubly-iterative architecture. Asymptotic and finite-length analysis show that our proposal outperforms existing schemes with comparable complexity, in various spatial-multiplexing configurations.**

I. Introduction

In future wireless communications systems, embracing interference has become a necessity to increase throughput, while saving resources. Indeed, scarce frequency bands and strict time constraints encourage the use of *spatial multiplexing* [1] and *non-orthogonal* sharing of resources by multiple users [2] for increasing the overall system load. Moreover, in frequency-selective channels, *inter-symbol interference* (ISI) has to be jointly handled by *multi-user detectors* (MUD), along with *multi-antenna interference* (MAI) and *multi-user interference* (MUI). In this context, 3GPP *Long Term Evolution* (LTE) and LTE-Advanced (LTE-A) have chosen SC-FDMA as their uplink technology, since it has various advantages ranging from low peak-to-average power ratio, to reduced sensitivity to time and frequency synchronization errors [3].

Even if computationally-efficient *frequency domain* (FD) receivers designed with the *minimum-mean square error* (MMSE) criterion have proven their value [4], the aforementioned challenges push the research community towards the exploration of more advanced techniques. With the advances in turbo equalization in FD [5], [6], recent proposals make use of soft MUD to improve the spectral efficiency. In particular *space-time bit-interleaved coded modulation* (STBICM) scheme [7] allows the use of turbo design methods, such as *extrinsic information transfer* (EXIT) charts [8], to design high performance MUD, as in [6], [9].

Turbo FD MUDs for STBICM, referenced above, use *linear equalization* (LE) with IC either based on *extrinsic* (EXT) information from conventional turbo principle, or with *a posteriori* (APP) feedback, which empirically yields improved performance [6], [10]. However, novel more efficient equalization proposals exist, having complexity similar to the LE, e.g. self-iterated FD LE-IC [11], which is a hybrid between the turbo LE-IC [5], and the APP-based receiver in [12]. This receiver carries out an initial IC with EXT outputs from the decoder, and then it executes another inner IC loop, that we call a *self-iteration*, with the detector's APP estimations.

Self-iterated turbo receivers, based on an expectation propagation (EP) formulation, have been recently investigated for time-domain equalization [13] and iterative MIMO detection [14], [15]. EP is an approximate Bayesian inference method, that generalizes belief propagation (BP) to random variables following exponential distributions [16]. When used in an iterative message passing based receiver [17], unlike BP, it is able to compute an *extrinsic message on symbols from the demapper*, in accordance with extrinsic principle for turbo feedback, which had been ignored in solutions involving APP [10], [11]. Previous works designing iterative receivers inside the EP framework showed both impressive and close to capacity performance [13]–[15], [18]. However its usage in FD remains limited, [19], with either excessive complexity, or high sensitivity to spectral nulls.

This paper addresses the derivation of low-complexity MUD for non-orthogonal multiple-access in the presence of ISI and MAI. A bin-wise adaptive FD-MUD is derived, i.e. a receiver which processes each sub-carrier independently, with a double-loop architecture. Detection is handled by a parallel interference cancellation of MAI and ISI by iterating an EP-based soft-demapper and the equalizer (self-iteration). Channel decoding successively improves MUI removal, by iterating with the EP-based soft-demapper. This receiver is shown to have encouraging performance even when users have similar path-loss, a case where IC is limited by decoding errors. It is also shown to outperform alternative bin-wise FD MUDs, while having comparable computational costs.

The remainder of this paper is organized as follows. Section II presents the system model, whereas the derivation of the proposed receiver takes place in Section III. Section IV carries out asymptotic analysis and finite-length simulations, before concluding in Section V.

II. MU-MIMO SC-FDMA System Model

We consider a synchronous, open-loop, multi-user spatial multiplexing system with U independent users, each employing a space-time bit-interleaved coded modulation (STBICM) SC-FDMA modulation (illustrated in Fig. 1 in [20]). Each user u in $\{1,\dots,U\}$ has T_u transmit antennas, with the total number of antennas being $T \triangleq \sum_{u=1}^{U} T_u$, and all users share K sub-carriers and N time samples. The STBICM scheme for the u^{th} user applies channel encoding on the information block $\mathbf{b}_u \in \mathbb{F}_2^{K_{b,u}}$ to produce the codeword $\mathbf{c}_u \in \mathbb{F}_2^{K_{c,u}}$, with rate $R_{c,u} = K_{b,u}/K_{c,u}$, which is interleaved into the block $\mathbf{D}_u \in \mathbb{F}_2^{K_{d,u}\times T_u}$, with $K_{c,u} = K_{d,u}T_u$. Here $\mathbb{F}_2$ denotes the binary field and users' interleavers may be different, but it is not necessary in general. Each column $\mathbf{d}_{t,u} \in \mathbb{F}_2^{K_{d,u}}$ is processed by a memoryless modulator φ_u to yield the data block $\mathbf{x}_{t,u} \in \mathcal{X}_u^K$, with a M_u^{th} order constellation $\mathcal{X}_u \subset \mathbb{C}$, with zero mean and average power $\sigma_x^2 = 1$. Note that $K_{d,u} = q_u K$, with $q_u = \log_2 M_u$ and the value of $d_{kq_u+j,t,u}$ is also denoted by $\varphi_{j,u}^{-1}(x_{k,t,u})$ or by $d_{k,j,t,u}$. The q_u-word associated to the symbol $x_{k,t,u}$ is also denoted by $\mathbf{d}_{k,t,u} = [\mathbf{d}]_{q_u k:q_u(k+1)-1,t,u}$. Finally, $\mathbf{x}_{t,u} \in \mathbb{C}^K$, denotes u^{th} user's symbols for antenna t.

User symbols are mapped to K sub-carriers among N, using a K-point discrete Fourier transform (DFT), and a sub-band mapper $\mathbf{P} \in \mathbb{C}^{N\times K}$. We have $[\mathbf{P}]_{n,k} = \delta(n - f_{\text{sb}}(k))$, where δ is the Kronecker delta function, and f_{sb} is an injective function from $\{1,\dots,K\}$, to $\{1,\dots,N\}$. Finally, after the application of a N-point inverse DFT (IDFT) to go back in the TD, a cyclic prefix (CP) is appended. We denote the K-DFT matrix by $[\mathcal{F}_K]_{l,k} = \exp(-2j\pi kl/K)/\sqrt{K}$, such that $\mathcal{F}_K\mathcal{F}_K^H = \mathbf{I}_K$.

The receiver is equipped with R antennas, it is ideally synchronized both in time and frequency and perfect channel knowledge is available. Using a CP of sufficient length, the received signal at the r^{th} antenna is

$$\mathbf{y}_r = \sum_{u=1}^{U}\sum_{t=1}^{T_u} \mathbf{H}_{r,t,u}\mathcal{F}_N^H \mathbf{P}\mathcal{F}_K \mathbf{x}_{t,u} + \mathbf{w}_r, \quad (1)$$

where the $N \times N$ matrix $\mathbf{H}_{r,t,u}$ is the circulant matrix generated by the L-tap channel impulse response $[h_{1,r,t,u},\dots,h_{L,r,t,u}]$ (with $\sum_l \mathbb{E}[|h_l|^2] = 1$) between u^{th} user's t^{th} antenna, and the receiver's r^{th} antenna, and where the noise $\mathbf{w}_r \sim \mathcal{CN}(0, \sigma_w^2\mathbf{I}_N)$, i.e. a circularly symmetric additive white Gaussian noise (AWGN) with covariance $\sigma_w^2\mathbf{I}_N$.

The SC-FDMA baseband receiver uses N-DFT, and a subband demapper $\mathbf{P}^T$, such that the FD samples are

$$\underline{\mathbf{y}}_r = \sum_{u=1}^{U}\sum_{t=1}^{T_u} \underline{\mathbf{H}}_{r,t,u}\mathcal{F}_K \mathbf{x}_{t,u} + \underline{\mathbf{w}}_r, \quad (2)$$

where $\underline{\mathbf{H}}_{r,t,u} = \mathbf{P}^T\mathcal{F}_N\mathbf{H}_{r,t,u}\mathcal{F}_N^H\mathbf{P}$ is the equivalent FD channel, and $\underline{\mathbf{w}}_r = \mathbf{P}^T\mathcal{F}_N\mathbf{w}_r$ is the equivalent noise. $\underline{\mathbf{H}}_{r,t,u}$ is a $K \times K$ diagonal matrix, whose diagonals are

$$\underline{h}_{k,r,t,u} = \sum_{l=1}^{L} h_{l,r,t,u} \exp(-2j\pi f_{\text{sb}}(k)l/N). \quad (3)$$

Stacking receiver antennas to form $\underline{\mathbf{y}} = [\underline{\mathbf{y}}_1;\dots;\underline{\mathbf{y}}_R]$, transmit antennas for $\mathbf{x}_u = [\mathbf{x}_{1,u};\dots;\mathbf{x}_{T_u,u}]$, and users $\mathbf{x} = [\mathbf{x}_1;\dots;\mathbf{x}_U]$ we have

$$\underline{\mathbf{y}} = \underline{\mathbf{H}}\mathcal{F}_{K,T}\mathbf{x} + \underline{\mathbf{w}}, \quad (4)$$

where $\underline{\mathbf{H}}$ is a $RK \times TK$ K-partitioned-diagonal matrix, $\mathcal{F}_{K,T} = \mathbf{I}_T \otimes \mathcal{F}_K$ is the T-block DFT matrix, and $\underline{\mathbf{w}} = [\underline{\mathbf{w}}_1;\dots;\underline{\mathbf{w}}_R]$ with $\underline{\mathbf{w}} \sim \mathcal{CN}(\mathbf{0}_{RK}, \sigma_w^2\mathbf{I}_{RK})$.

III. MUD-IC Design with Factor Graphs

A. Message Passing Model for EP

This section aims to approximate the optimum MAP FD joint detection and decoding problem, by using an EP-based message passing formulation.

The joint posterior probability density function (PDF) of data bits is given by $p(\mathbf{b},\mathbf{c},\mathbf{d},\mathbf{x}|\underline{\mathbf{y}})$, where user bits are stacked for convenience. STBICM simplifies derivations, by asymptotically (ie. when assuming sufficiently long interleavers) enforcing an independence on transmitted bits with

$$p(\mathbf{D}_u) = \prod_{t=1}^{T_u} p(\mathbf{d}_{t,u}) = \prod_{t=1}^{T_u}\prod_{j=1}^{K_{d,u}} p(d_{j,t,u}) \quad (5)$$

which serves as a prior constraint provided by the decoder, from the detector's point of view, and any soft-input soft-output decoder can be used. The joint PDF $p(\mathbf{d},\mathbf{x}|\underline{\mathbf{y}})$ can be further factorized, with the memoryless mapping assumption, as

$$p(\underline{\mathbf{y}}|\mathbf{x}) \prod_{u=1}^{U}\prod_{t=1}^{T_u}\prod_{k=1}^{K} p(x_{k,t,u}|\mathbf{d}_{k,t,u}) \prod_{j=1}^{q_u} p(d_{k,j,t,u}). \quad (6)$$

In a message passing based decoding algorithm over a factor-graph, variable nodes (VN) $x_{k,t,u}$ and $d_{k,j,t,u}$ are updated iteratively, using the constraints imposed by factor nodes (FN), corresponding to the factorization of the posterior PDF in (6). More specifically, denoting the equalization factor

$$f_{\text{EQU}}(\mathbf{x}) \triangleq p(\underline{\mathbf{y}}|\mathbf{x}) \propto \exp(\sigma_w^{-2}\|\underline{\mathbf{y}} - \underline{\mathbf{H}}\mathcal{F}_{K,T}\mathbf{x}\|^2), \quad (7)$$

where dependence on $\underline{\mathbf{y}}$ is omitted, as it is fixed during iterative detection. The FN for resolving the mapping constraints is $f_{\text{DEM}}^{(u)}(x_{k,t,u},\mathbf{d}_{k,t,u})$, and it is given by

$$p(x_{k,t,u}|\mathbf{d}_{k,t,u}) \propto \prod_{j=1}^{q_u} \delta(d_{k,j,t,u} - \varphi_{j,u}^{-1}(x_{k,t,u})), \quad (8)$$

where δ denotes the Dirac delta function. And finally, prior constraints given by channel coding are handled by the factor $f_{\text{DEC}}^{(u)}(d_{k,j,t,u}) \triangleq p(d_{k,j,t,u})$.

The factor graph of the STBICM MU-MIMO SC-FDMA is given by Fig. 1, where DEC and DEM FNs are regrouped by user for convenience. A message passing algorithm based on EP is an extension of BP, where messages and the involved VN lie in a distribution from the exponential family [17].

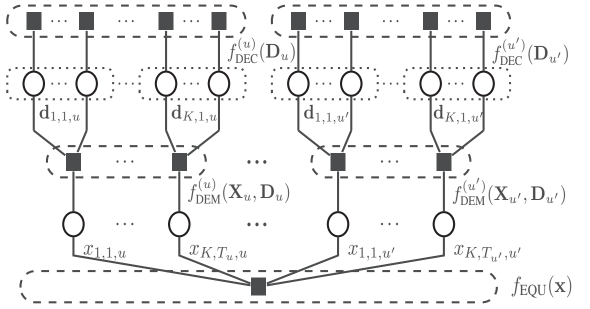

Fig. 1. MU-MIMO factor graph of $p(\mathbf{d}, \mathbf{x}|\underline{\mathbf{y}})$, with users u and u'.

Updates at a FN F connected to VNs $\mathbf{v}$ is given by messages exchanged between VN v_i, i^{th} component of $\mathbf{v}$, and F are

$$m_{v\to\text{F}}(v_i) \triangleq \textstyle\prod_{\text{G}\neq F} m_{\text{G}\to v}(v_i), \tag{9}$$

$$m_{\text{F}\to v}(v_i) \triangleq \frac{\text{proj}_{\mathcal{Q}_v}\left[\int_{\mathbf{v}^{\setminus i}} f_{\text{F}}(\mathbf{v}) \prod_{v_j} m_{v\to\text{F}}(v_j) \mathbf{dv}^{\setminus i}\right]}{m_{v\to\text{F}}(v_i)}, \tag{10}$$

where $\mathbf{v}^{\setminus i}$ are VNs without v_i, and $\text{proj}_{\mathcal{Q}_{v_i}}$ is the Kullback-Leibler projection towards the probability distribution $\mathcal{Q}_{v_i}$ of v_i [17]. This process yields a fully-factorized approximation of $p(\mathbf{v})$, which is marginalized to obtain estimations on v_i.

In our case with $p(\mathbf{d}, \mathbf{x}|\underline{\mathbf{y}})$, the transmitted symbol blocks follow uncorrelated multivariate Gaussians and the projection is equivalent to *moment matching*. Hence a message involving these VNs can be fully characterized by a *mean* and a *variance*. On the other hand code bits follow Bernoulli distributions, thus, the involved messages are characterized by bit *log-likelihood ratios* (LLRs), as in the BP algorithm.

B. Exact Derivation of the Receiver Structure

Here the exact MUD architecture is derived by computing explicit expressions of EP message parameters. The messages arriving on $x_{k,t,u}$'s VN are denoted

$$m_{\text{EQU}\to x}(x_{k,t,u}) \quad\propto\quad \mathcal{CN}\left(x^e_{k,t,u}, v^e_{k,t,u}\right), \tag{11}$$

$$m_{\text{DEM}\to x}(x_{k,t,u}) \quad\propto\quad \mathcal{CN}\left(x^d_{k,t,u}, v^d_{k,t,u}\right). \tag{12}$$

Note that $x_{k,t,u}$ is approximated by a unique mean, and the estimation reliability is given by a unique variance.

1) Messages between DEM and DEC: First, for the sake of brevity, we omit derivation details concerning the messages exchanged between DEC and DEM FNs. These are unchanged for any BICM scheme using memoryless modulation, and derivations are already available in [14], [18]. DEC and DEM exchange LLRs on coded bits, and we define $L_a(.)$ as the *a priori* LLRs provided by the decoder, which yields a prior probability mass function (PMF) on symbols, $\forall\alpha \in \mathcal{X}_{u'}$,

$$\mathcal{P}_{k,t,u}(\alpha) \propto \textstyle\prod_{j=1}^{q_u} e^{-\varphi_j^{-1}(\alpha) L_a(d_{k,j,t,u})}. \tag{13}$$

DEM node can computes a posterior PMF, $\forall u, \forall\alpha \in \mathcal{X}_u$,

$$\mathcal{D}_{k,t,u}(\alpha) \propto e^{-|x^e_{k,t,u}-\alpha|^2/v^e} \mathcal{P}_{k,t,u}(\alpha). \tag{14}$$

Messages from DEM to DEC are *extrinsic* LLRs

$$L_e(d_{k,j,t,u}) = \ln \frac{\sum_{\alpha\in\mathcal{X}^0_{j,u}} \mathcal{D}_{k,t,u}(\alpha)}{\sum_{\alpha\in\mathcal{X}^1_{j,u}} \mathcal{D}_{k,t,u}(\alpha)} - L_a(d_{k,j,t,u}),$$

where $\mathcal{X}^b_{j,u} = \{\alpha \in \mathcal{X}_u, \varphi^{-1}_{j,u}(\alpha) = b\}$. Extrinsic LLRs from DEM are used by DEC to compute $L_a(d_{k,j,t,u})$.

2) Updates of messages from DEM to EQU: DEM computes $m_{\text{DEM}\to x}(x_{k,t,u})$ using (10); where the argument of the projection is $\mathcal{D}_{k,t,u}(\alpha)$, and

$$\mu_{k,t,u} \triangleq \mathbb{E}_{\mathcal{D}}[x_{k,t,u}], \quad \gamma_{k,t,u} \triangleq \text{Var}_{\mathcal{D}}[x_{k,t,u}]. \tag{15}$$

Moreover as $m_{x\to\text{DEM}}(x_{k,t,u}) = m_{\text{EQU}\to x}(x_{k,t,u})$, the mean and the variance of DEM's message are given by

$$x^d_{k,t,u} = \frac{\mu_{k,t,u} v^e_{k,t,u} - x^e_{k,t,u}\gamma_{k,t,u}}{v^e_{k,t,u} - \gamma_{k,t,u}}, \tag{16}$$

$$v^d_{k,t,u} = v^e_{k,t,u}\gamma_{k,t,u}/(v^e_{k,t,u} - \gamma_{k,t,u}). \tag{17}$$

This feedback is one of the major benefits in using EP. With BP, computing an extrinsic feedback from DEM results in $\mathcal{P}_{k,t,u}$. Some solutions by-pass the extrinsic principle for the turbo feedback by using $\mathcal{D}_{k,t,u}$ to improve performance [10]. Note that the occurrence of $v^e_{k,t,u} \leq \gamma_{k,t,u}$ in eqs. (16)-(17) might cause errors in the feedback [16]. Erroneous moments are replaced by posterior statistics $\mu_{k,t,u}$ and $\gamma_{k,t,u}$, as in [14].

3) Updates of messages from EQU to DEM: A detailed derivation of messages from EQU factor node is available in [20], and $m_{\text{EQU}\to x}(x_{k,t,u})$ satisfies

$$x^e_{k,t,u} = x^d_{k,t,u} + \mathbf{f}_{k,t,u}{}^H(\underline{\mathbf{y}} - \underline{\mathbf{H}}\mathcal{F}_K\mathbf{x}^{\mathbf{d}}), \tag{18}$$

$$v^e_{k,t,u} = 1/\xi_{k,t,u} - v^d_{k,t,u}, \tag{19}$$

where $\mathbf{v}^{\mathbf{d}}$, $\mathbf{x}^{\mathbf{d}}$ are $TK \times 1$ stacked vectors of $x^d_{k,t,u}$, $v^d_{k,t,u}$ following antennas and users. Moreover

$$\mathbf{f}_{k,t,u} \triangleq \xi^{-1}_{k,t,u}\boldsymbol{\Sigma}^{\mathbf{d}\,-1}\underline{\mathbf{H}}\mathcal{F}_{K,T}\mathbf{e}_{k,t,u}, \tag{20}$$

$$\xi_{k,t,u} \triangleq \mathbf{e}^H_{k,t,u}\mathcal{F}^H_{K,T}\underline{\mathbf{H}}^H\boldsymbol{\Sigma}^{\mathbf{d}\,-1}\underline{\mathbf{H}}\mathcal{F}_{K,T}\mathbf{e}_{k,t,u}, \tag{21}$$

$$\boldsymbol{\Sigma}^{\mathbf{d}} \triangleq \sigma^2_w\mathbf{I}_{RK} + \underline{\mathbf{H}}\,\underline{\mathbf{V}}^{\mathbf{d}}\underline{\mathbf{H}}^H, \tag{22}$$

with $\underline{\mathbf{V}}^{\mathbf{d}} \triangleq \mathcal{F}_{K,T}\mathbf{Diag}(\mathbf{v}^{\mathbf{d}})\mathcal{F}^H_{K,T}$, and $\mathbf{e}_{k,t,u}$ is a $KT\times 1$ zero vector except $K(\sum_{v=1}^{u-1} T_v + t) + k^{\text{th}}$ index is 1.

One can emphasize the similarity of these expressions to those of conventional MMSE receivers; indeed, as MAP and MMSE estimators are identical for Gaussian distributions [21], and as EP approximates MAP, such similarities are expected. However, EP provides a *different* extrinsic feedback $\mathbf{x}^{\mathbf{d}}$ and $\mathbf{V}^{\mathbf{d}}$ for IC, than those of conventional turbo receivers [5], [6].

These equations have similar expressions with previous work on EP [13], [14], [19], as any other linear estimation model would do so, but underlying problems are quite different. As far as we know, none of the prior work on EP studied non-orthogonal FD MUD where same frequency and time resources are shared.

Besides, the structure described by eqs. (18)-(22) involves intensive computations with limited practical interest. In the following we use simplifying assumptions to propose a robust structure with affordable complexity.

C. Novel Low-Complexity Bin-wise Receiver Structure

1) White multivariate Gaussian symbol assumption: Computational bottleneck of the exact receiver is the $RK \times RK$ matrix inversion in (20). Nevertheless, if all involved matrices were to be K-partitioned-diagonal, complexity would reduce drastically. This is used in the literature for bin-wise MMSE FD-LE design [5], [6], [11], by using an invariant covariance on each antenna. Indeed, if the EP messages of the previous section are derived with symbols VNs assumed to be *temporally-white* Gaussian distributed

$$m_{\text{EQU}\to x}(x_{k,t,u}) \quad \propto \quad \mathcal{CN}\left(x^e_{k,t,u}, \bar{v}^e_{t,u}\right), \quad (23)$$

$$m_{\text{DEM}\to x}(x_{k,t,u}) \quad \propto \quad \mathcal{CN}\left(x^d_{k,t,u}, \bar{v}^d_{t,u}\right). \quad (24)$$

Note that unlike in eqs. (11)-(12), $x_{k,t,u}, \forall k$, have a unique reliability measure, which yields a a bin-wise receiver. In detail, the message to EQU from DEM is simplified, (16)-(17) are replaced by

$$x^d_{k,t,u} = \frac{\mu_{k,t,u}\bar{v}^e_{t,u} - x^e_{k,t,u}\bar{\gamma}_{t,u}}{\bar{v}^e_{t,u} - \bar{\gamma}_{t,u}}, \quad (25)$$

$$\bar{v}^d_{t,u} = \bar{v}^e_{t,u}\bar{\gamma}_{t,u}/(\bar{v}^e_{t,u} - \bar{\gamma}_{t,u}). \quad (26)$$

where $\bar{\gamma}_{t,u} \triangleq \sum_{k=1}^{K} \gamma_{k,t,u}$. This approach is experimentally verified to improve performance, compared to (16)-(17) with invariant filters (applying sample average on (17)). Consequently, $\mathbf{v^d}$ is replaced by

$$\bar{\mathbf{v}}^d = \left[\bar{v}^d_{1,1}\mathbf{1}_{K\times 1}; \ldots; \bar{v}^d_{T_U,U}\mathbf{1}_{K\times 1}\right], \quad (27)$$

and the matrices used in detection ((20)-(22)), become partitioned-diagonal, as $\underline{\mathbf{V}}^{\mathbf{d}} = \textbf{Diag}(\bar{\mathbf{v}}^{\mathbf{d}})$. In detail, using the partitioning $\underline{\mathbf{H}} = \left[\underline{\mathbf{H}}_{1,1}, \ldots, \underline{\mathbf{H}}_{T_1,1}, \ldots, \underline{\mathbf{H}}_{T_U,U}\right]$, with $\underline{\mathbf{H}}_{t,u} \in \mathbb{C}^{RK\times K}$, the covariance matrix in (22) is

$$\mathbf{\Sigma^d} = \sigma_w^2\mathbf{I}_{RK} + \sum_{u=1}^{U}\sum_{t=1}^{T_u} \bar{v}^d_{t,u}\underline{\mathbf{H}}_{t,u}\underline{\mathbf{H}}^H_{t,u}. \quad (28)$$

Partitioned-diagonal structure of $\mathbf{\Sigma^d}$ ensures that its inverse also keeps the same property, and detection ((18)-(21)) costs become $O(K(TR^2 + (T+R)\log K))$ instead of $O(K^3R^2T + K(T+R)\log K)$. The covariance matrix inversion remains the computational bottleneck, but with the recursive method proposed in Appendix A, its computational complexity is reduced down from $O(K^3R(R^2+T^2))$ to $O(KR^2T)$.

Using these simplifications, EQU's extrinsic message can be rewritten in the frequency domain as

$$\underline{x}^e_{k,t,u} = \underline{x}^d_{k,t,u} + \sum_{r=1}^{R} \underline{f}_{k,r,t,u}{}^* \, \underline{\tilde{y}}_{k,r}, \quad (29)$$

$$\bar{v}^e_{t,u} = 1/\bar{\xi}_{t,u} - \bar{v}^d_{t,u}, \quad (30)$$

where $\underline{\tilde{y}}_{k,r} = \underline{y}_{k,r} - \sum_{u=1}^{U}\sum_{t=1}^{T_u} \underline{h}_{k,r,t,u}\underline{x}^d_{k,t,u}$, and

$$\underline{f}_{k,r,t,u} = \bar{\xi}^{-1}_{t,u}\sum_{r'=1}^{R} \lambda^d_{k,r,r'}\underline{h}_{k,r',t,u}, \quad (31)$$

$$\bar{\xi}_{t,u} = K^{-1}\sum_{r=1}^{R} \underline{h}^*_{k,r,t,u}\sum_{r'=1}^{R} \lambda^d_{k,r,r'}\underline{h}_{k,r',t,u}, \quad (32)$$

where $\lambda^d_{k,r,r'}$ is the k^{th} diagonal of $\mathbf{\Sigma^{d-1}}$'s $(r,r')^{\text{th}}$ partition. Then $\mathbf{x^e}$ is given by using the IDFT matrix $\mathcal{F}^H_{K,T}$ on the FD vector $\underline{\mathbf{x}}^{\mathbf{e}}$. These operations are carried out using fast Fourier transforms (FFTs) for reduced computational complexity.

2) Further improvements on EP feedback robustness: EP corresponds to a first order approximation of the computationally-intensive expectation consistency approximation, used for approximate Bayesian inference [22]. While this algorithm provably converges towards MAP performance, EP can converge towards undesirable fixed-points. In [17], the use of a feature-based damping of messages is suggested to improve the convergence of EP, and [22] verified its usage on a MIMO detector.

Below, an alternate approach that uses a linear filter with dynamic coefficients is proposed. DEM message mean and variance, given in (25)-(26), are replaced by

$$x^{d(\text{next})}_{k,t,u} \quad = \quad (1-\beta)x^*_{k,t,u} + \beta x^{d(\text{prev})}_{k,t,u}, \quad (33)$$

$$\bar{v}^{d(\text{next})}_{t,u} \quad = \quad (1-\beta)\bar{v}^*_{t,u} + \beta\bar{v}^{d(\text{prev})}_{t,u}, \quad (34)$$

where $(x^*_{k,t,u}, \bar{v}^*_{t,u})$ are given by (25)-(26), and β denotes the dynamic smoothing parameter, configuring the damping across the iterations of the algorithm.

D. Scheduling

Although the messages exchanged over the factor graph are defined, due to the multiple cycles present on the graph, a scheduling scheme is required. A robust MUD is proposed via a flexible structure, achieved by exploiting the presence of two loops: the first one refers to turbo-iterations (TI) between the DEC and the DEM FNs, while the second one refers to self-iterations (SI) between the DEM and the EQU FNs.

Within each TI $\tau = 0, \ldots, \mathcal{T}$, each user data is decoded *successively*, with the information provided by VNs $\mathbf{d}$ to FN DEC (i.e. $L_e(d_{k,j,t,u})$). A natural ascending decoding order is used, for notation convenience, but in practice, a reliability-based ordering could be used.

In detail, while decoding a user u, FN EQU uses *parallel* scheduling, for joint ISI, MAI and MUI cancellation, and detection (eqs. (29)-(32)). This is enabled by FN DEM's extrinsic outputs (eqs. (25)-(26)), computed with prior FN DEC inputs (eq. (13)) of users $u' < u$ decoded in the current TI, and prior inputs of users $u' \geq u$ decoded in the previous TI. Messages between EQU and DEM FNs (eqs. (29)-(30) and (25)-(26)) are self-iterated for improving u^{th} user's detection by processing all users, with $s = 0, \ldots, \mathcal{S}$ denoting self-iterations. When computing posteriors in eq. (14), at $s = 0$, EQU FN's messages are reset with $x^{e(s=0)}_{k,t,u} = 0$

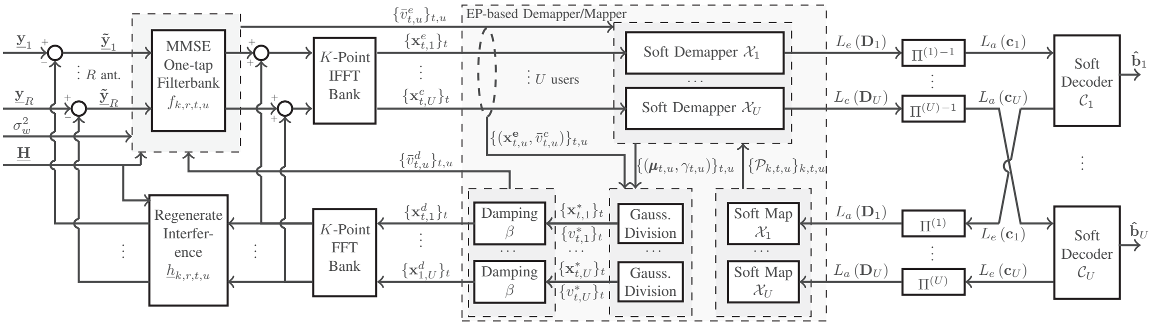

Fig. 2. Proposed iterative MU-MIMO SC-FDMA receiver with bin-wise filterbank and interference cancellation based on EP.

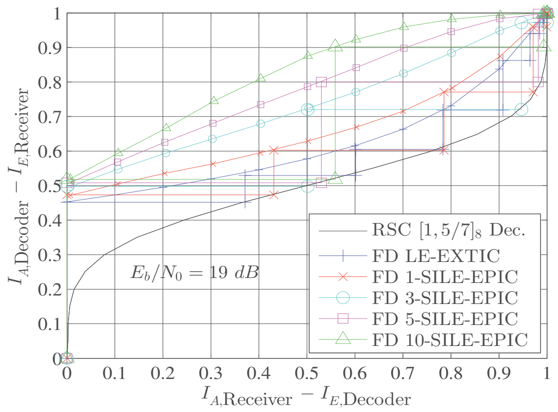

Fig. 3. EXIT Charts of the proposed FD SILE-EPIC in single-user AWGN Proakis C channel with 16-QAM modulation.

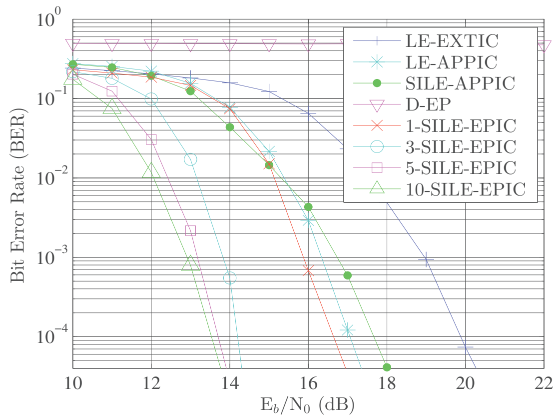

Fig. 4. BER in AWGN Proakis C with coded 16-QAM, single-user with $T = R = 1$, and with 10 turbo-iterations.

and $v_{k,t,u}^{e(s=0)} = +\infty$. When $s = \mathcal{S}$, current SI's messages on users other than u are discarded, and their VNs are reinitialized to their previous values from FN DEC, whereas the user u's data are given to DEC by DEM.

Selection of $\mathcal{S}$ and $\mathcal{T}$ allow for a control over the processing latency and the required computational complexity. As SIs of a FD MUD consist of demapping, they are computationally less complex than decoding, and this makes TIs less desirable.

Combination of the simplified messages from the previous subsection with this schedule, yields the proposed low-complexity MUD with hybrid IC, displayed on Fig. 2. In the following, we refer to this solution as frequency-domain self-iterated linear equalizer with EP-based interference cancellation, i.e. *FD SILE-EPIC*.

IV. Performance Evaluation

A. Single-User Asymptotic Analysis

Here, the asymptotic performance of the proposed receiver is studied in a single user case, through an EXIT chart analysis [8]. The receiver is described by the transfer function $I_E = \mathcal{T}_R(I_A, \underline{\mathbf{H}}, \sigma_w^2)$, dependent on the channel, where I_A and I_E are the mutual information (MI) between coded bits, and respectively the receiver's a priori and extrinsic LLRs.

A single-antenna receiver is considered in the highly-selective Proakis C channel $[1, 2, 3, 2, 1]/\sqrt{12}$. Considering SC-FDE transmission with recursive systematic convolutional (RSC) encoder $[1, 5/7]_8$, and with 16-QAM modulation, EXIT charts of the proposed receiver are plotted, with plain lines, in Fig. 3, for $\mathcal{S} = 0, 1, 3, 5, 10$ and $\beta = 0.75$. For $\mathcal{S} = 0$, the proposal is equivalent to the conventional MMSE linear receiver *LE-EXTIC* [5], [6]. Use of SIs ($\mathcal{S} > 0$) brings significant increase in the area beneath the EXIT curve, and hence in the receiver's achievable rates. Besides with dashed lines, finite-length MI evolution with packets of $K_1 = 1024$ bits are superimposed in Fig. 3, and SI is seen to reduce the number of required TI to achieve the matched filter bound limit. Finally, MI increases when $I_A = 0$, implying possible improvements on the waterfall threshold depending on the decoder's transfer function.

B. Comparisons with Prior Works

In this subsection, our proposal's finite-length bit error rate (BER) performance is compared with similar bin-

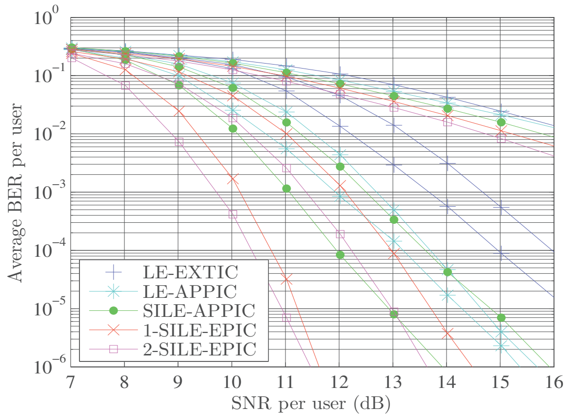

Fig. 5. BER in AWGN Proakis B with 16-QAM, $U = 2$, $R = 2$ and $T_1 = T_2 = 1$ (0 TIs: dash-dotted, 1 TI: dashed, 4 TIs: plain).

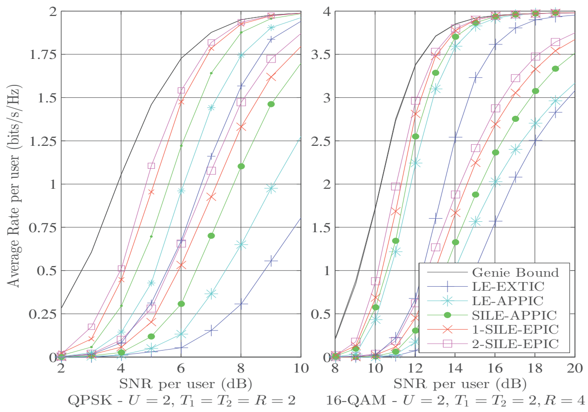

Fig. 6. Throughput in Rayleigh fading EQU4, $U = 2$ (for QPSK, 1 TI: dashed, 3 TIs: plain; for 16-QAM 0 TIs: dashed, 1 TI: plain).

wise FD receivers from the literature, which all the same order of complexity for filter computation, per SI and TI.

As references, *LE-EXTIC*, and its APP-based variant *LE-APPIC* [6], [10] are considered. Among recent structures, we consider the self-iterated LE with APP feedback *SILE-APPIC* [11], [12] (denoted "Subopt SD-FDE-SIC-III" in [11]), and the EP-based low-complexity receiver, with memory-based demapper, *D-EP* in [19]. Note that, *for fair comparison, SILE-APPIC is extended* to multi-antenna users, and all receivers use our proposed hybrid scheduling with random interleavers. In this paragraph we consider SC-FDE signalling with $K = N$.

First, the previous section's EXIT analysis is completed with finite-length bit error rate (BER) simulations, using 1024-bit information blocks, in Fig. 4. Note that the proposed *SILE-EPIC* with $\mathcal{S} = 0$ corresponds to *LE-EXTIC*. Self-iterations bring significant improvement in the waterfall threshold, with around 3dB for the first SI, and up to 6dB with $\mathcal{S} = 5$, for BER=10^{-3}. *D-EP* cannot decode up to a signal-to-noise ratio of 40 dB, as it uses a least-squares-like criterion in [19, eq. (48)], making it sensitive to channel nulls.

Next, we consider the scenario from [11] with two single-antenna users, where a 2-antenna MUD, and a non-systematic convolutional encoder $[17, 13]_8$ are used, in a generalized AWGN Proakis B channel. Up to two SI are considered with $\beta = \max(0.3, 0.5 \times (0.8)^{s+\tau})$, and BER performance per user is plotted in Fig. 5. Our proposal outperforms concurrent structures for all TI, with over 2 dB margin for BER $= 10^{-5}$. Moreover, *SILE-APPIC* with its original scheduling (parallel interference cancellation for the MUI), at $\mathcal{T} = 4$, shown in [11, Fig. 2], is outperformed by *1-SILE-EPIC* with $\mathcal{T} = 1$. This suggests increasing the number of SI can be used to reduce the number of required TI; as demapping costs less than decoding, our proposal improves the overall computational costs. Note also that APP-based receivers show some loss of diversity.

TABLE I
COMPUTATIONAL COSTS FOR THE QPSK SCENARIO IN FIG. 6

Receiver	Costs per TI	SNR (dB) 1 TI	SNR (dB) 3 TIs	Total Costs 1 TI	Total Costs 3 TIs
LE-EXTIC	$4.739e^5$	10.8	6.7	$2.1e^6$	$4.0e^6$
LE-APPIC	$4.746e^5$	9.1	6.1	$2.1e^6$	$4.0e^6$
SILE-APPIC	$9.070e^5$	7.8	5.6	$2.9e^6$	$5.8e^6$
0-SILE-EPIC	$4.452e^5$	10.8	6.7	$2.0e^6$	$3.9e^6$
1-SILE-EPIC	$8.559e^5$	7.2	5.1	$2.8e^6$	$5.6e^6$
2-SILE-EPIC	$12.665e^5$	6.8	4.8	$3.7e^6$	$7.2e^6$

C. Spatial Multiplexing for MU-MIMO with SC-FDMA

Finally, we evaluate the throughput of two MU-MIMO spatial multiplexing schemes. Two users, with two spatially uncorrelated antennas each, transmit over the EQU4 channel, which has four equal power taps following Rayleigh statistics. Localized-FDMA is used, with $K = 256$ and $N = 512$.

In the first scenario, an under-determined system with a MUD with $R = 2$ is considered. Thanks to IC with STBICM, such situations become resolvable [7]. User throughput, using QPSK with a $[1, 5/7]_8$ encoder, is plotted on the left side of the Fig. 6. Our proposal, with $\beta = 0$, requires 0.5 - 1 dB less energy than *FD-SILE-APPIC* for achieving 1 bit/s/Hz.

At the right side of the figure, a more conventional situation with $R = 4$, and 16-QAM constellation, using the same channel encoder is considered. In this case, the interest of our receiver is more prevalent for $\mathcal{T} = 0$ with over 1 dB gain at 2 bits/s/Hz. Note that, in both schemes, MUDs without SIs have a significant disadvantage.

In Table I, the computational costs are given in the number of required floating-point operations (FLOPs). The required SNR for reaching 1 bits/s/Hz, and the total detection and decoding FLOPs at the reported TI are given in the latter columns. It is assumed the proposed

MUD uses the matrix-inversion scheme of Appendix A, whereas prior works use the one of [9]. First, *0-SILE-EPIC*, equivalent to *LE-EXTIC*, shows 6% complexity savings thanks to our update scheme alone. Moreover, it is seen that SIs do not cost much (relative to decoding), and *2-SILE-EPIC* with 1 TI, is 10% less expensive but offers similar performance as *LE-EXTIC* with 3 TIs.

V. Conclusion

A novel multi-user detector is proposed that exploits the expectation propagation framework, to jointly mitigate MAI and ISI. Building upon the exact FD receiver, given by the factor-graph of the overall system, a low-complexity scheme is obtained, with several simplifications. In particular, the key element of our proposal, is an alternative approach to EP-based message passing with statistically whitened Gaussian distributions, which yields simple bin-wise equalization filterbanks. Moreover, an efficient matrix inversion scheme from the literature is optimized for iterative MUDs.

Through finite-length and asymptotic analysis, it is seen that the proposed structure outperforms other bin-wise FD MUD, and attests the importance of a feedback computed by using EP. Improvements are seen to be remarkable in especially highly selective channels. It is shown that exploiting a self-iteration loop between demapping device and the MUD can improve performance at lesser costs than decoding, and provides novel options for MUD design. Moreover EP-based SIs are seen to be more efficient than APP-based alternatives.

Future work will explore the use of EP in other communication systems dealing with significant interference.

Appendix

A. Computationally-efficient matrix inversion

Covariance matrix inversion can be simplified by exploiting the structure of equation (28). Inspired from a method originally used with CDMA receivers in Table 1 of [23], and considered for FD MUD in [9], we propose a recursive update strategy for handling TI and SI.

The main steps of the procedure are as follows. Let $\mathbf{\Psi}^{\text{old}}$ denote an initially available inverse of (28). When the covariance on t^{th} antenna of the user u changes to $\bar{v}_{t,u}^{\text{new}}$, using the matrix inversion lemma, the new inverse is given by

$$\mathbf{\Phi}^{(t,u)} = \mathbf{\Psi}^{\text{old}}\underline{\mathbf{H}}_{t,u}, \quad (35)$$

$$\mathbf{\Lambda}^{(t,u)} = \Delta\bar{v}_{t,u}^{-1}\mathbf{I}_K + \underline{\mathbf{H}}_{t,u}{}^H\mathbf{\Phi}^{(t,u)}, \quad (36)$$

$$\mathbf{\Psi}^{\text{new}} = \mathbf{\Psi}^{\text{old}} - \mathbf{\Phi}^{(t,u)}\mathbf{\Lambda}^{(t,u)-1}\mathbf{\Phi}^{(t,u)H}, \quad (37)$$

where $\Delta\bar{v}_{t,u} = \bar{v}_{t,u}^{\text{new}} - \bar{v}_{t,u}^{\text{old}}$ denotes the change in symbol covariance. This algorithm is initialized with $\mathbf{\Psi}^{\text{old}} = \sigma_w^{-2}\mathbf{I}_{RK}$ at the $s = 0$, $\tau = 0$ (see scheduling). Existing schemes [23] did not store updated values of the covariance inverse between TIs, and had to recompute the whole procedure for all antennas/users at each TI. The impact of using this scheme is greater in a hybrid schedule architecture such as ours, as seen in Table I.

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