Development of a Modified Material Balance Equation for Complex Reservoirs with the Inclusion of Fluid Velocity

By

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A Thesis

Submitted to the School of Graduate Studies

in partial fulfillment of the requirements for the degree of

Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

May 2018

St. John's **Newfoundland**

To the departed soul of my father….

To my mom and my siblings

Abstract

Material Balance Equation (MBE) is among the most widely used techniques to estimate the reserve of any types of hydrocarbon reservoir. Complex reservoirs, such as fractured oil reservoir and shale gas reservoir have been recognized as a potential energy sources where a substantial amount of reserve are trapped. Modified MBE for complex reservoir enhances the accuracy and assists production engineers to take a proper decision regarding the selection of the recovery techniques. The one of the aims of this study is to review the existing knowledge and find out the significant gap in previous research on numerous recently developed material balance equation. The central goal of this work is to develop two modified MBE for the mentioned reservoir types. These equations include the velocity of the fluid and other rocks and fluid properties: viscosity, permeability and length of the reservoir. The results showed that incorporating the velocity and other fluid properties increases the recovery of the reserves. An important factor to derive an MBE is effective compressibility which is ultimately influenced by fluid velocity, viscosity and permeability. A sensitivity analysis has been conducted that shows the influence of the change of these properties on effective compressibility. Performing reserve estimation procedure without shut in is a big challenge for the production engineer. No researcher has introduced any parameter of fluid velocity in MBE which are so far reviewed in this study. Because of the absence of velocity in the established MBE, production is needed to be shut in to estimate the reserve. The developed material balance model includes this important velocity term which enhances the recovery of reserves and allows to calculate the reserve without shut in. The results of the modelling and sensitivity analysis can be a future guideline to develop an impressive material balance model for fractured reservoir.

Acknowledgement

First of all, I would like to thank my former supervisor Dr. M Enamul Hossain for his valuable guidance and generous support. He gave me the freedom to conduct the research on different material balance techniques and showed me the right way when I got derailed from right track. I thank him for suggesting me the appropriate course work which helped me to conduct my research properly.

I would like to express my special thanks to my supervisor Dr. Salim Ahmed for accepting me as a one of his student without asking my status. I thank him for providing me proper guideline to finish my research within the assigned time frame.

I thank my close friend and group mate Tareq Uz Zaman for his help with Matlab and for providing me valuable research idea. I also thank another group mate Md Shad Rahman for his cooperation with the thesis arrangement. I thank the former academic program administrator Moya Crocker, Tina Dwyer and Mahoney Collen for making a friendly and welcoming atmosphere at the university.

I express my ultimate gratitude to my mother and deceased father for being a source of inspiration, love and affection. I dedicate this thesis to my parents and siblings. I thank all my teachers from primary level to graduate level for their guidance and encouragement.

Finally, I would like to thank the School of Graduate Studies (SGS); Research & Development Corporation of Newfoundland and Labrador (RDC), funding no. 210992; and Statoil Canada Ltd., funding no. 211162 for providing financial support to accomplish this research under Statoil Chair in Reservoir Engineering at the Memorial University of Newfoundland, St. John’s, NL, Canada.

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$\Delta_k G$ cumulative gas production, *scf*

$\Delta_k N$ cumulative oil production, *stb*

$\Delta_k W$ cumulative water production, *stb*

ρ_b bulk density, g/cm^3

B_g gas formation volume factor, *rb/scf*

B_{gf} gas formation volume factor for fracture, *rb/scf*

B_{gi} Bg at initial reservoir pressure, *rb/scf*

B_{gm} gas formation volume factor for matrix, *rb/scf*

B_o oil formation volume factor, *rb/stb*

B_{of} oil formation volume factor for fracture, *rb/stb*

B_{oi} initial oil formation volume factor, *b/stb*

B_{om} oil formation volume factor for matrix, *rb/stb*

B_t total formation volume factor, *rb/stb*

B_{ti} B_t at initial reservoir pressure, *rb/stb*

B_w water formation volume factor, *rb/stb*

B_{wi} B_w at initial reservoir condition, *bbl/stb*

B_{wm} water formation volume factor for matrix

C'_{epm} modified dimensionless parameter

C_T total compressibility, psi^{-1}

C_{epm} parameter of effective compressibility due to residual fluid, dissolved gas and formation for the proposed MBE, dimensionless

C_f rock compressibility, psi^{-1}

c_f Average fracture compressibility, psi^{-1}

C_g gas compressibility, psi^{-1}

C_m matrix compressibility, psi^{-1}

c_m average matrix compressibility, psi^{-1}

c_s reservoir rock formation compressibility at a reduced pressure p, psi^{-1}

C_o oil compressibility, psi^{-1}

C_{rock} rock compressibility, psi^{-1}

C_w water compressibility, psi^{-1}

C_w water compressibility, psi^{-1}

C_φ pore compressibility, psi^{-1}

E_i initial expansion, RB/STB

E_{o1} Net expansion of the original oil-phase in the matrix system, RB/STB

E_{o2} Net expansion of the original oil-phase in the fracture network, RB/STB

G_{2i} Initial gas in the secondary-porosity, scf

G_i Initial gas in the reservoir, scf

G_{inj} cumulative gas injection, m^3

G_p produced wellhead gas, *scf*

G_t total produced gas, *scf*

M_α Marangoni number

N_1 Original oil in-place in the rock matrix, *stb*

N_2 Original oil in-place in the fractures, *stb*

N_p Cumulative produced oil, *stb*

P_L Liquid pressure, psi

P_i Initial pressure, psi

Q_k Cumulative production at average pressure point time,

R_p Cumulative gas oil ratio

R_{sf} Solution gas oil ration in fracture

R_{si} initial solution gas oil ratio

R_{sm} solution gas oil ratio in matrix

R_{soi} initial solution gas oil ratio

R_{swi}	Initial solution oil water ratio
S_{Hcl}	saturation of HCl
S_{gi}	initial gas saturation
S_{oi}	initial oil saturation
S_{wf}	final water saturation
S_{wi}	initial water saturation
S_{wm}	water saturation in matrix
T_T	Tank temperature, °R
T_{sc}	Temperature in standard condition, °R
V_L	liquid volume, bbl
V_{b2}	Bulk volume of secondary layer, ft^3
W_e	water influx (encroachment), cumulative, bbl
W_{inj}	water injected, cumulative, cm^3
W_p	water produced, cumulative, cm^3
Z_c	critical compressibility factor
Z_i	initial gas compressibility factor
Z_{sc}	gas compressibility factor in standard condition
$\frac{du_x}{d_y}$	velocity gradient along y-direction, 1/s
r_i	inner radius of well, ft
t_k	time at average pressure point time, hrs
w_f	fluid influx, bbl
y_i	initial mole fraction
α_D	thermal diffusivity, ft^2/s
μ_o	oil viscosity, cp
τ_T	shear stress at temperature T, Pa
ϕ_g	porosity of fracture
ϕ_g	granular porosity
ϕ_i	initial porosity
ϕ_m	porosity of matrix
ϕ_n	noneffective porosity
ω_a	fraction of the original gas in place
ω_m	fraction of original gas in place in matrix
ΔP	pressure difference, psi
ΔT	Temperature difference
C'	effective matrix compressibility, 1/psi
C''	effective fracture compressibility, 1/psi
E	Net expansion function
F	Underground recoverable function
K	Reservoir Permeability, mD
M	Molecular weight, g/mol
P	Reservoir pressure, psi
$P(t)$	Pressure as a function of time, psi
R	gas-law constant, J/ (g mol-K)
Z	gas deviation factor
m	ratio of initial reservoir free-gas volume to initial reservoir oil volume
$r(k)$	radius of reservoir as a function of permeability, ft
y	mole fraction
Γ	gamma function
α	fractional order of differentiation, dimensionless
η	ratio of the pseudo permeability of the medium with memory to fluid viscosity, ft^3 $S^{1+\alpha}$/lbm
ξ	a dummy variable for time i.e. real part in the plane of the integral, s
ω	fraction of original gas in place

Chapter 1
Introduction

1.1 Background and motivation

Estimation of hydrocarbon-in-place is a vital factor for the development of a field and its associated production strategies as well as the design of necessary facilities. The fundamental techniques used for estimating as well as calculating hydrocarbon-in-place are volumetric method, decline curve analysis, material balance method and numerical simulation. MBE is recognized as one of the most efficient techniques to estimate the hydrocarbon reserves, recovery factors, etc., since this method considers the actual production performance data and is applicable for all types of reservoirs. Literature shows that there are three potential types of material balance are available. These are: 1) conventional material balance, 2) flowing material balance (Shahamat and Clarkson, 2017), and 3) dynamic material balance (Ojo *et* al. 2006).
The conventional material balance method represents the relationship between the average reservoir pressure and the cumulative volumes of reservoir fluids produced. This method is commonly applied during the later stages of development in a field. The overall idea of this method is to measure the average reservoir pressure before and after a certain portion of hydrocarbon fluids is produced. The appropriate change of the PVT properties with pressure is used to estimate the remaining reserves. A long shut-in is required to acquire the average reservoir pressure which makes a big loss of production expenses.

The flowing material balance approach disregards the shut-in procedures and permits performing of material balance calculations at dynamic reservoir conditions which is done by using the bottom hole flowing pressures and constant flowrates. The dynamic material balance equation can be applied for almost all kind of oil and gas reservoir as it is not limited by static pressure measurements (Mattar and Anderson 2005).

A substantial amount of hydrocarbon reserves is trapped in fracture carbonate reservoir. Exploration shows that more than 50 percent of the world's proven hydrocarbon are available in the fractured formation. That's why during the last few years, research on material balance has been conducted for the fractured reservoir to improve the reservoir analysis. However, all previous works are applicable to limited ranges of data. Porosity and permeability throughout the reservoir are assumed uniform in case of conventional MBE. As dual porosity system is generated for the naturally fractured reservoirs (NFR), the assumption is not valid. The compressibility of fractures is much higher than the matrix. In addition, the porosity of fracture and matrix changes when there is a change in pressure (Nelson, 1985). Walsh (1994) developed a comprehensive straight line method to estimate hydrocarbon reserve for the conventional reservoir and this method is applicable to a full range of reservoir fluids.

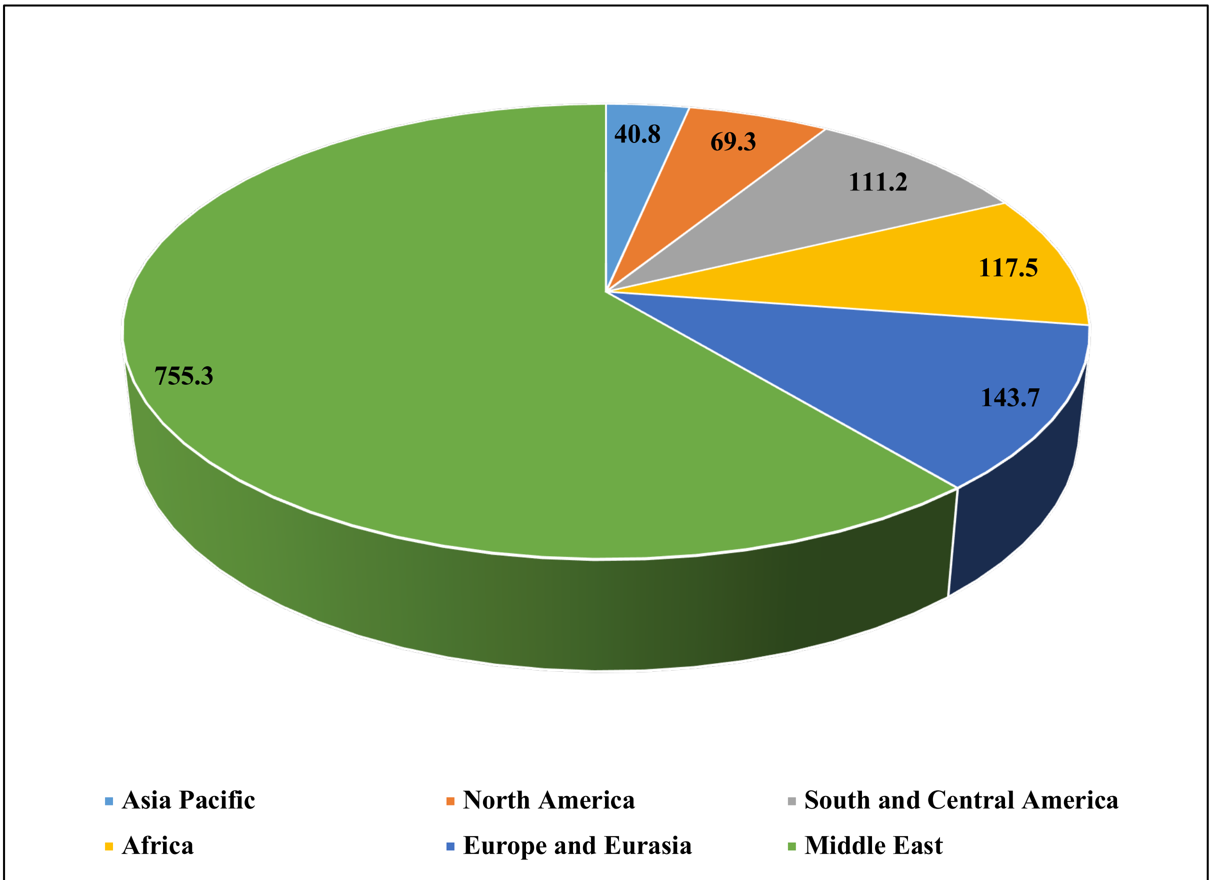

Figure 1.1: Statistical data for the reserve in fractured formation (BP Statistical Review, 2008)

1.2 Objectives

The goal of the research is to develop a modified material balance equation that would be applicable to apply either during production or shut in condition. To maximize the applicability, fluid velocity and water influx parameters will be incorporated. Prior to develop the final model, a compressibility model will also be derived to analyze the sensitivity of the developed material balance equation. Two separate MBE model for complex reservoir will be developed where one model will applicable to estimate the reserve in the oil reservoir and another one will be applicable for gas reservoir. For both model, fractured formation will be considered as a complexity of the reservoir. Although the limitations of all types of material balance equation will be discussed in the literature review, however, a focus will be given in the static material balance equation.

1.3 Significance of this work

Material balance method is a well-established tool to estimate the hydrocarbon reserve. Numerous material balance equation have been derived by the researcher based on the reservoir condition. Almost all the established equation cover a few particular rocks and fluid properties. In other words, if an equation is suitable for regular reservoir, it may not be an effective tools for the fractured reservoir. If an equation can be developed by considering maximum rocks and fluid properties, it would be a wonderful contribution to the oil and gas industry. In this work, an effort has been made to derive a material balance equation with highest number of rocks and fluid properties. Velocity of the fluid, permeability of the matrix and fracture and mobility ratio have been incorporated with the established material balance equation. Hopefully, this equation will be utilized for different types reservoir.

1.4 Organization of Thesis

The rest of the thesis is organized as follows: Chapter 2 provides a critical review on material balance equation; major contributions to this field are summarized and the gaps in research are identified. Chapter 3 presents the development of a compressibility model for the sensitivity analysis of material balance equation. A modified material balance equation for naturally fractured reservoir is developed in chapter 4. Chapter 5 is all about a material balance equation for fractured gas reservoir where the water influx is incorporated. Finally, chapter 5 concludes this thesis by highlighting the contributions of this thesis and few future recommendations are made in the end.

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Chapter 2
A Critical Review on Material Balance Equation

2.1 Abstract

Engineers have been using the material balance equation (MBE) for almost last five decades to estimate cumulative production. However, it still is an effective tool to estimate the original hydrocarbon (oil and gas) available in the reservoir. The conventional material balance method has been successfully applying for the regular structure of a typical reservoir. By this conventional technique, P/Z versus cumulative hydrocarbon production (G_P) curve is finally extrapolated to the zero value of P/Z for obtaining the original hydrocarbon in place (G). The method was modeled for a 'volumetric' hydrocarbon reservoir. In this method, all formation properties are assumed constant. However, it is the very important to take care the alteration of rock and fluid properties with respect to space and time during the production history of the reservoir. Therefore, there is an immense need to understand how porosity, permeability, compressibility, and other rock and fluid properties changes with space and time. In this paper, a thorough review and critical analysis on MBE are presented so that researcher can find a solution why and how the incorporation of continuous alteration phenomena is needed to be considered during the development of new and dynamic MBE. In addition, the need for incorporation of all unconventional properties is detailed in this review research. For example, the fractured condition which is the one significant complexity of the reservoir should be considered to obtain a reasonable estimate of a reserve. In the conventional MBE technique, only regular formation permeability and porosity are considered which may lead an erroneous result for a complex reservoir. In such case for example if the pore volume of the fractured formation is employed, the reserve can be estimated accurately. This review will help to the new researcher to get a guideline for starting further research on material balance equation. It is challenging to get a linear plot of P/Z versus G_P. This study makes a crucial scope for carrying a research to make a way out for the nonlinear behavior of the reservoir. This article also shows how different unconventional properties of the reservoir have been overlooked in many research works. Finally, a guideline is provided to overcome the previous challenges in estimating hydrocarbon reserve and a workflow is presented to develop a new dynamic MBE.

Keywords
Complex Reservoir, Fracture Reservoir, Dual Permeability, Dual Porosity, Reserve Estimation.

2.2 Introduction

MBE is recognized as one of the most efficient techniques to estimate the hydrocarbon reserve. In the case of a conventional hydrocarbon reservoir, a graphical representation of P/Z versus G_P can be made. If there is no water influx, it gives a linear trend and this method is used to estimate the original-gas-in-place (OGIP) (Dake, 1978). For using the conventional MBE, fracture and other unconventional properties should be considered. To analyze the reservoir performance, several endeavors have been accomplished by the material balance method. Schilthuis (1936) was the first who formulated the material balance analysis. And later, several MBE has been offered for single porosity reservoir (Muskat 1949, Pirson 1958, Amyx *et al.,* 1960, Craft *et al.,* 1991, Dake 1994, Walsh 1995). A graphical representation of MBE as a straight line was recommended by Havlena and Odeh (1963). Likewise, Campbell (1978) offered a proposal to identify the new method of depletion mechanisms, e.g. gas cap or water drive. However, in the case of the complex reservoir, the scenario becomes completely different.

2.2.1 Complex Reservoir

Structurally complex reservoirs are a specific class of reservoir in which fault and fracture play an important role in petroleum trapping and production behavior. There is an increasing technical challenge to handle these behaviors accurately (Moller-Pedersen & Koestler 1997; Coward *et al.,* 1998; Jones *et al.,* 1998; McClay 2004; Swennen *et al.,* 2004; Sorkhabi and Tsuji 2005; Lonergan *et al.,* 2007). A significant number of hydrocarbons are trapped in these complex reservoirs. Due to the fault and fracture, the flow path of the hydrocarbon becomes complex. Sometimes, the permeability of the matrix is too low to flow the hydrocarbon and fractures are needed to be created artificially. That's why production engineers face huge challenges to extract these remaining trapped hydrocarbons. The updated production tools are providing sufficient technology to produce hydrocarbon from these faulted and fractured reservoirs. However, improved analytical models are needed to optimize field development, rates of production and ultimate recovery (Jolley, 2007). This analytical model usually begins with imaging and mapping from the 3D seismic survey. A 3D structural framework model can be developed by using newly developed modeling technique which can easily investigate the fault and fracture conditions of the reservoir (Badley *et al.,* 1990; Needham *et al.,* 1996; Rutten and Verschuren 2003). Figure 2.1 shows a geological formation of the structurally complex reservoir. A structurally non-complex reservoir also showed in figure 2.2.

Figure 2.1: A sample of structurally complex formation

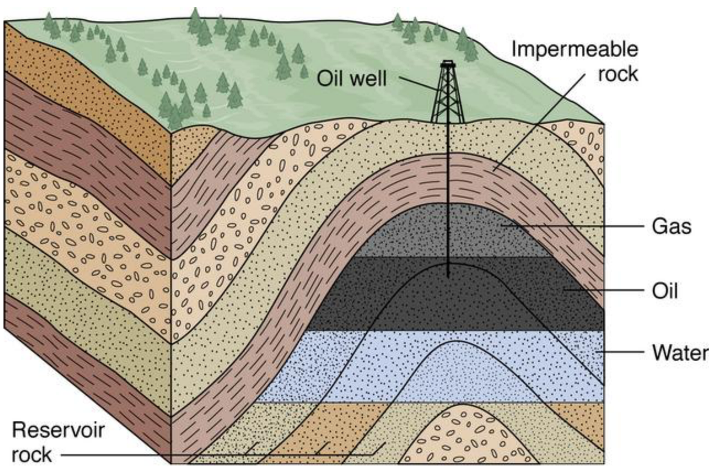

Figure 2.2: A sample of structurally non-complex formation

2.2.1.1 Fractured Reservoir

During the last few years, research on material balance has been conducted for the fractured reservoir to improve the reservoir analysis. However, all previous works are applicable to limited ranges of data. Porosity and permeability throughout the reservoir are assumed uniform in case of conventional MBE. As dual porosity system is generated for the naturally fractured reservoirs (NFR), the assumption is not valid. The compressibility of fractures is much higher than the matrix. In addition, the porosity of fracture and matrix changes when there is a change in pressure (Nelson, 1985). Walsh (1994) developed a comprehensive straight line method to estimate hydrocarbon reserve for the conventional reservoir and this method is applicable to a full range of reservoir fluids. This paper presents all previous works on the conventional reservoir in an organized way so that readers can capture the missing criteria for unconventional reservoir without difficulty.

2.2.1.2 Tight Gas Reservoir

To produce natural gas at an economic rate from low permeability reservoir rock, massive hydraulic fracturing is required. This type of natural gas is known as tight gas. The matrix permeability of tight gas reservoir is less than 0.1 mD and the porosity of the matrix is less than 10% (Ben *et al.,* 1993, Sharif 2007). Some productive work were conducted on tight gas reservoir by using MBE. Application of MBE to the tight gas reservoir is not straightforward, however it can misinterpret the results (Hagoort *et al.,* 2000). P/Z vs G_p graph exhibits the nonlinear behavior in the case of tight gas reservoir whereas conventional reservoir shows the linear trend (Engler, 2000). The nonlinearity is related to the pressure measurement technique and the reservoir characteristics. Nobakht *et al.*(2010) introduced a simplified method to predict the production for the tight gas reservoir which exhibits extended linear flow periods. The advantages of this prediction method are: (i) only initial rate and original gas in place (OGIP) are the required parameters, and (ii) there is no need to forward time step to calculate cumulative gas production (Morgan 2010).

2.3 A Critical Literature Survey

Material balance method is a great practice for reservoir engineers in order to find out the original-hydrocarbon-in-place (OHIP) (Moghadam *et al.,* 2009). However, the same equation is not applicable for all reservoirs. Due to the diversity of the hydrocarbon reservoir, researchers developed numerous model based on the conditions of the reservoirs. For unconventional reservoir a simplified MBE was proposed by Jensen and Smith(1997).

2.3.1 General Material Balance Equation (GMBE)

Schilthuis (1936) primarily presented a general MBE for the homogeneous reservoir. In hydrocarbon reservoir, to determine drive mechanism and estimate their performance Schilthuis' MBE was the only means until 1950. A very simple and spontaneous form of MBE is:

$$\left[(B_t - B_{oi}) + \frac{mB_{oi}}{B_{gi}}(B_g - B_{gi}) + (1+m)B_{oi}C_T\Delta P\right] + W_e = N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right] + B_wW_p - G_{inj}B_{ginj} - W_{inj}B_w \quad (2.1)$$

$$B_t = \left[B_o + (R_{si} - R_s)B_g\right] \quad (2.2)$$

$$C_T = \frac{c_wS_{wi} + c_{rock}}{1 - S_{wi}} \quad (2.3)$$

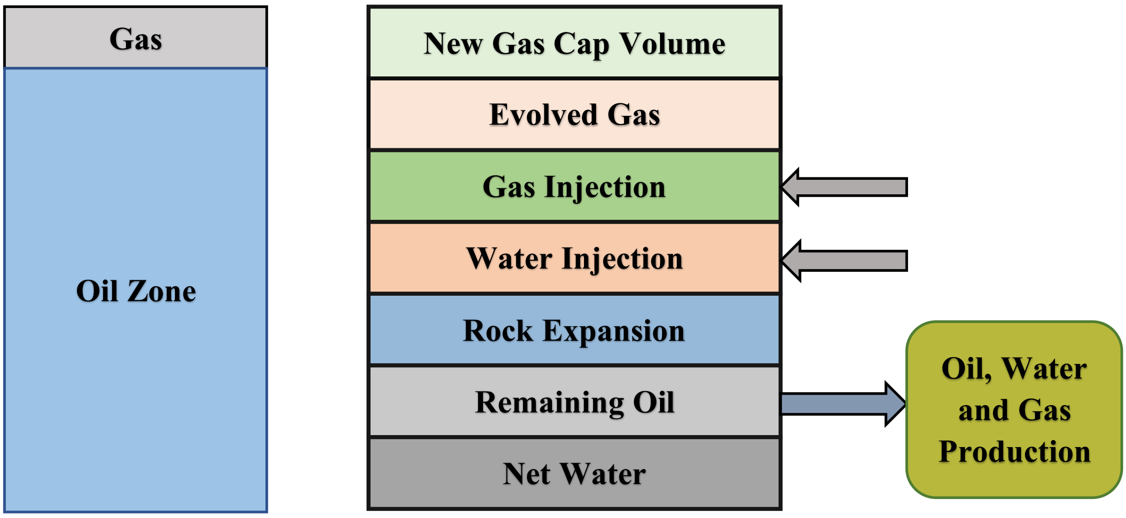

Figure 1.3: Pore volume balance for material balance equation (redrawn from Ahmed et al., 2005)

MBE is the simplest presentation of the mass conservation in a hydrocarbon reservoir. Mass conservation theory can be applied for the prediction of hydrocarbon in the reservoir which is known as “material balance equation”(Havlena and Odeh, 1963). The equation developed by Havlena and Odeh is the expression of the constant behavior of the reservoir. For developing this basic and fundamental equation, they considered some assumptions. The straight line method of MBE requires to create the plot of a group of variables vs. other variable groups. With the increasing production from the different unconventional reservoir, industries are inclined with modified MBEs. The model developed by Havlena and Odeh (1963) has a number of limitations. It is established and proven fact that the reservoir shows a linear behavior only for some instances. In the majority of the cases, the reservoir shows a nonlinear behavior. Walsh (1995) addressed that this nonlinearity appears when the properties of the reservoir alter during any change of the natural phenomena, and/or production. He presented a generalized MBE applicable to a reservoir where rock/fluid properties change. Buduka *et al.*

(2015) showed the limitation of straight line method and provided a solution. Based on matching pressure and production data, they developed a mathematical model which is referred to the history matching. Therefore, there is an immense need for developing a comprehensive dynamic MBE where an option of considering the alteration of rock/fluid properties exists.

2.3.1.1 MBE for Gas Reservoirs

For the general material balance equation, some major assumptions were made (Ahmed *et al.,* 2005): (i) the temperature, pressure, and porosity are constant within the reservoir, (ii) water is present in the water phase only, (iii) total thermodynamic balance i.e. uniformity of PVT data, (iv) production rate independent fluid recovery, and (v) available production data which are reliable as well. The GMBE was formulated on a black oil reservoir and consequently, is not directly applicable for volatile oil or gas-condensate reservoirs. In addition, this model is not able to cover naturally fractured reservoir as uniform porosity was considered. (Bashiri *et al.,* 2011).

2.3.1.1.1 Shale Gas Reservoir

The gas which is trapped in shale formation is known as shale gas. With the increasing interest in USA and rest of the petroleum world, shale gas has become an important resource (Stevens, 2012). Some researchers are expecting that this type of gas will increase the energy supply throughout the world. Multiple porosities are the important characteristics of Shale gas reservoirs (Orozco and Aguilera, 2017). These multiple porosities are: (i) adsorbed porosity, (ii) organic porosity, (iii) inorganic matrix porosity, (iv) natural fractures porosity, and (v) hydraulic fractures porosity (Aguilera and Lopez, 2013). Ignoring the gas dissolved in shale formation results in an uncertain estimate in MBE method.

The conventional gas MBE was modeled for a volumetric reservoir. However, p/Z vs. cumulative gas plot gives some unrealistic results in the case of some abnormal situations such as over-pressured condition (e.g. coal bed methane), and desorption condition (e.g. shale formation). Figure 2.4 shows p/Z vs. cumulative production (G_p) plot for different reservoir conditions. From the figure, it is observed that all plots are nonlinear except for the volumetric one. This is because in the straight-line method, only gas expansion was incorporated as a drive mechanism. Basically, there are different drive mechanisms involves based on different reservoir categories. In water drive reservoir, water influx acts as a drive mechanism, formation and residual fluid expansion acts as driving force in an over-pressured reservoir. Singh *et al.* (2013) reported that gas desorption has a significant role on shale or CBM reservoir as a driving force. Dotted line of all four types of reservoir show the trend of production behavior. By extrapolating the solid line, original gas in place can be calculated. In this case, the original gas in place is G. Figure 2.5 shows a P/Z schematic for a normally-pressured volumetric gas reservoir.

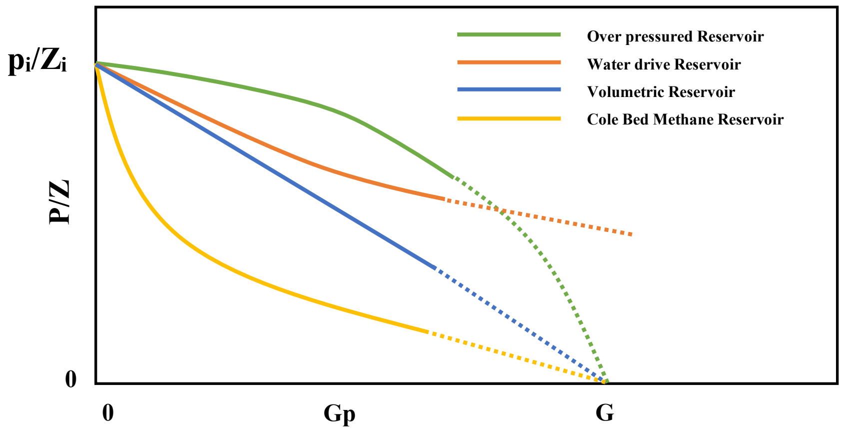

Figure 2.4: Conventional P/Z vs. cumulative production plot (Singh et al., 2013, redrawn)

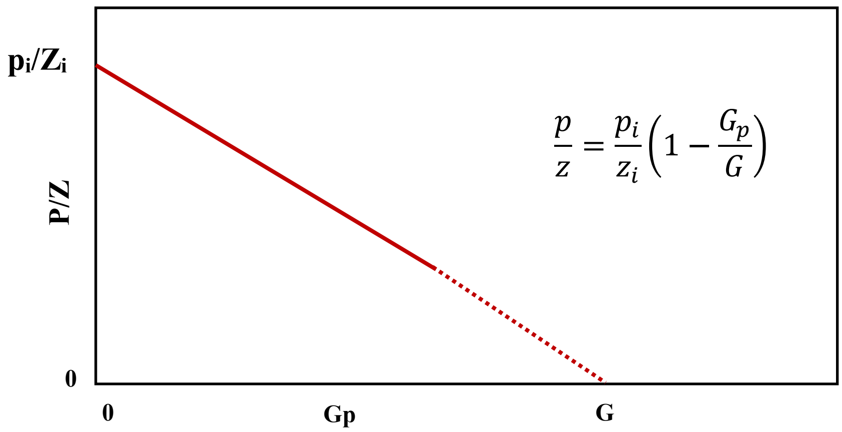

Figure 2.5: P/Z schematic for a normally-pressured volumetric gas reservoir

2.3.1.1.2 Abnormally Pressured Reservoir

To calculate OGIP in the volumetric reservoir, the frequently recognized method is the MBE. Many production engineers follow this technique for abnormally pressured reservoirs and eventually ended up with a huge percentage of error in calculating the production estimation.

The solution to lessen the error is to adjust rock and water compressibility between conventional p/z plot and the plot of the abnormally pressured reservoir (Ramagost and Farshad, 1981). There are significant differences of reservoir properties between normally and abnormally pressured reservoirs. The changes of these properties have a significant impact on the accuracy of hydrocarbon reserve estimation. These variable pressure conditions are needed to consider for the improvement of accuracy in reserve estimation.

The main statement of MBE for an abnormally pressured reservoir is the OGIP which is equal to the hydrocarbon withdrawals divided by the gas formation volume factor and water expansion. Mathematically, this can be written as;

$$G = \frac{G_p B_g}{B_g - B_{gi} + \frac{B_{gi} \Delta p \left(C_w S_w + C_f\right)}{(1 - S_w)}} \tag{2.4}$$

On the other hand, Gonzalez and Blasingame (2008) developed a quadratic model of MBE for the abnormally pressured gas reservoir. In that model, they mainly focused on developing (i) a quadratic MBE model, (ii) plotting functions for the analysis of reservoir performance based on rigorous quadratic MBE, and (iii) a dimensionless type curve solution. Mathematically, the model is written as:

$$\frac{p}{z} \approx \frac{p_i}{z_i}\left[1 - \left[\frac{1}{G} - \omega\right] G_p - \frac{\omega}{G} G_p^2\right] \tag{2.5}$$

Where ω is defined as a function of cumulative gas production G_p.

2.3.1.2 MBE for Oil Reservoir

Fattah *et al.* (2009) recommended a set of comprehensive relationships for material balance oil (MBO) model variables based on more than 2,000 PVT data points. He also incorporated the gas-oil ratio. England (2002) identified complexity for such approximation during improvement of some correlations. Figure 4 shows the concept of the gas-oil ratio for MBE.

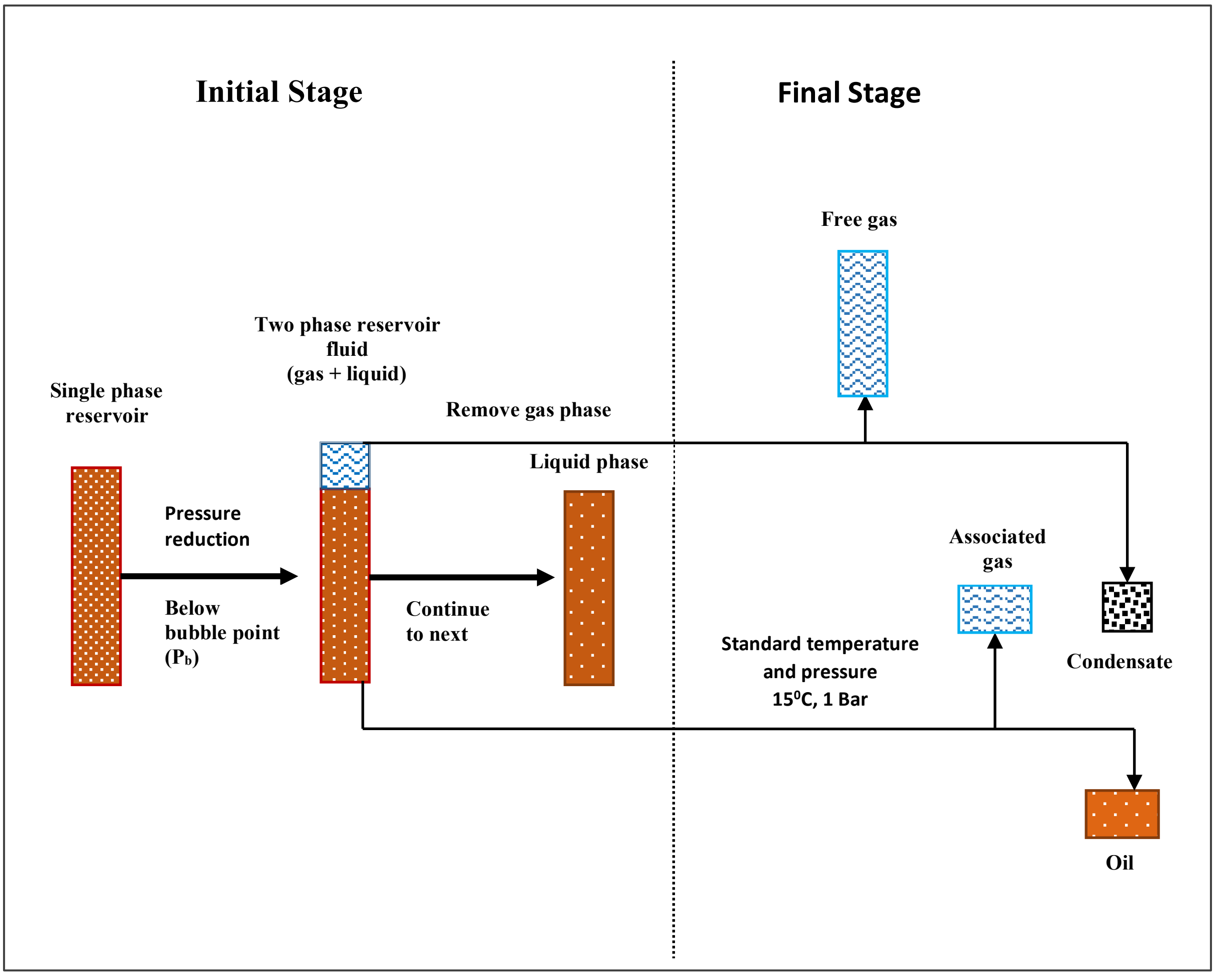

Figure 2.6: Concept of Oil-gas ratio for generalized material balance equation (Graas *et* al., 2000) (redrawn)

The figure explained how the oil, gas and condensate are separated from single-phase reservoir fluid. Based on reservoir condition, different types of oil reservoirs are available in the world. The two most common oil reservoir types are (i) saturated reservoir, and (ii) under-saturated reservoir. Approved and accepted material balance method are available for these two types of the reservoirs.

2.3.1.2.1 Saturated Reservoir

A reservoir is said to be saturated when its temperature goes equal or below bubble-point. This type of reservoir is also known as a multi-phase reservoir. Mosobalaje (2015) developed a method to estimate hydrocarbon for this type of reservoir. This method has been applied to two reservoirs model and found very effective to predict hydrocarbon reserve through a numerical simulation study. Two profiles are correlated to get production profile as a time function. Whenever well performance prediction is completed using inflow performance relationship (IPR), cumulative oil production were attained from the applicable MBE. To verify this model, the author applied it to the solution gas drive reservoir model published by Camacho and Raghvan (1987) and Frederick and Kelkar (2005).

2.3.1.2.2 Under-saturated Reservoir

When the reservoir temperature goes above bubble point is known as an undersaturated reservoir. As the gas exists in dissolved condition with oil, this type of reservoir is also called single-phase reservoir. As there is a temperature difference between under-saturated and saturated reservoir, a modified MBE should be used. Barry (1963) developed a model of MBE for better estimation of an undersaturated reservoir. He incorporated the water drive condition during the development of his model. Walsh and Raghavan (1994) proposed a generalized material balance model which is applicable to the under-saturated volumetric reservoir. The author tried to eliminate assumptions which were considered by Havlena and Odeh (1963). Havlena and Odeh (1963) considered only gas expansion as a driving force. But in the majority of the cases, another mechanism also responsible for driving force which has elaborately been explained in section 2.1.1.1.

2.3.2 MBE for Fractured Formation

In many reservoirs, the primary pathways for hydrocarbon migration and production are natural fractures and faults. Sixty percent of the world's remaining oil reserves exist in fractured formation (Harris and Weber, 2006). Natural fracture is the macroscopic discontinuity of reservoir rock which affects the multiphase flow within the reservoir (Fig 5).

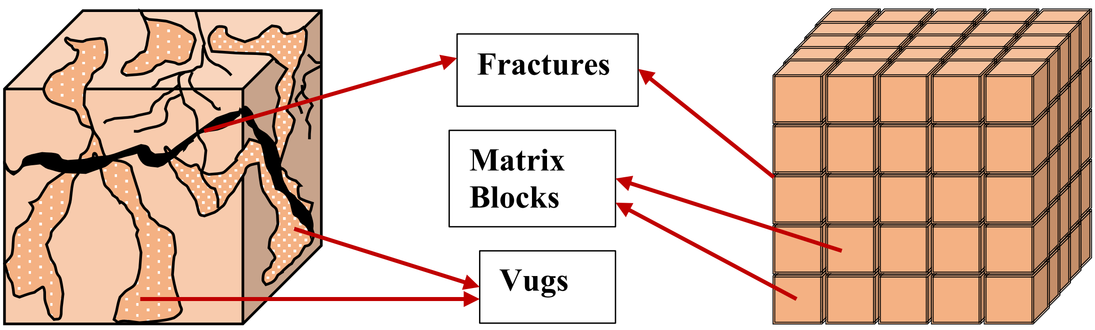

Figure *2.7*: Physical structure of the dual porosity model (Redrawn from Warren and Root, 1963; Kazemi, 1969)

Despite the presence of fractures in all reservoirs from geological and reservoir engineering point of view, a formation can be defined as a fracture only when it has an effect (i.e. either positive or negative) on the flow of the fluid within the reservoir (Aguilera, 1995). Sometimes, a small amount of hydrocarbon contained in the matrix can be easily produced where there is high permeability of the surrounding fractures which is frequently encountered in the Middle East. Aguilera used MBE for a saturated and undersaturated reservoir by considering effective compressibility of matrix and fractures. A critical summary is given below about the research works conducted on the fractured reservoirs.

2.3.2.1 Significance of Fractured Formation

There is a huge impact of fractured formation on hydrocarbon production. By applying the role of fracture condition of a reservoir, production rate and cumulative production can be increased. A plethora research is going on to estimate the reserve by material balance technique considering the fractured condition. The importance of fracture network is shown in Figure 6. Total four cases of permeability (k) and porosity (ϕ) are shown in this figure. The figure shows that fault and fracture have a great effect on porosity and permeability of a reservoir. From the figure, it is also clear that only the presence of fractures and faults is not enough to occur migration and storage of hydrocarbons. There should have a good combination of porosity and permeability in the fracture to increase the storage capacity and to facilitate the flow of hydrocarbon. In case of very poor porosity and permeability of the matrix, fractures provide both storage and flow pathways. The matrix of high porosity and low permeability contribute significantly on production and this type of combination is suitable for secondary and tertiary recovery.

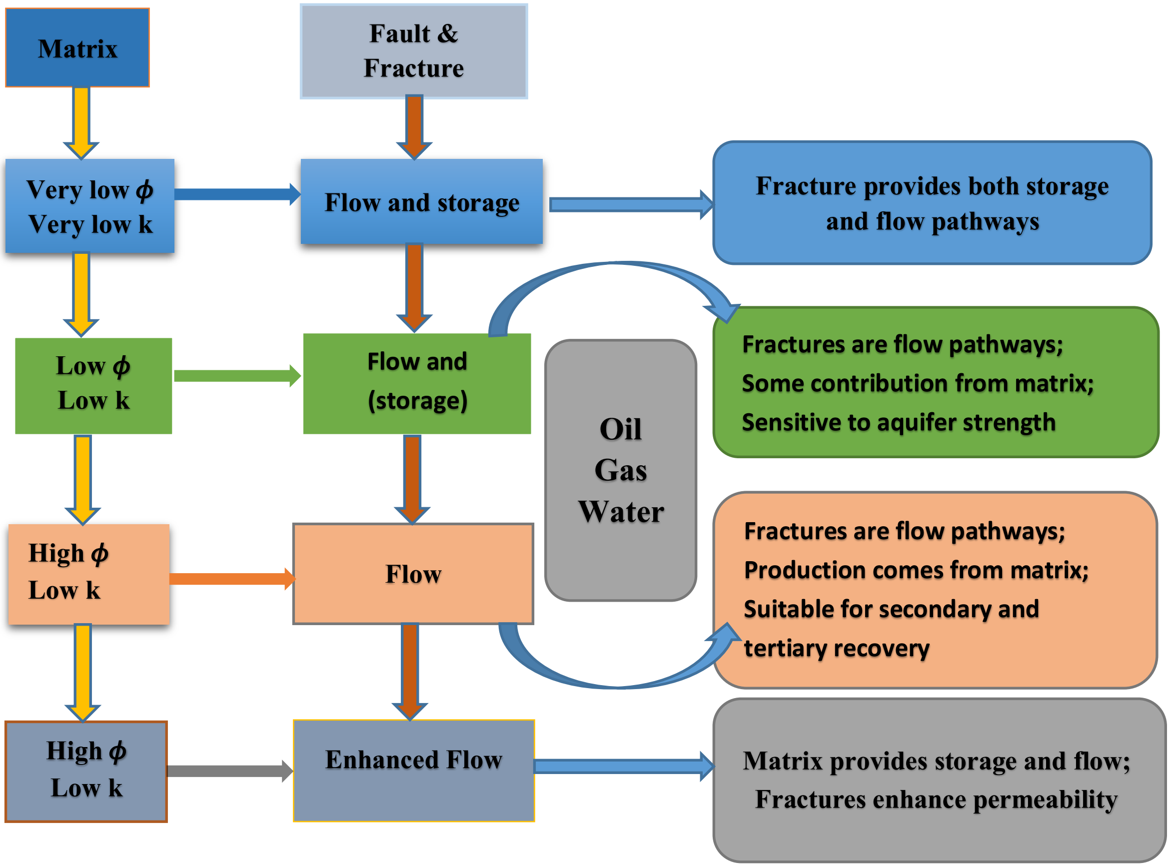

Figure 2.*8*: A diagram showing the importance of fracture and matrix

There are three different types of fractures available in the reservoir. These are type I, II and III and all the three fractures are created naturally. Table 1 and figure 7 show the classification and role of the naturally fractured reservoir. Type I reservoir provides essential porosity and permeability whereas type II provides only permeability which is also important for the flow of hydrocarbon. There is no direct contribution of the fracture of type III but it assists the other permeable path in the reservoir.

Table 2.1: Classification of naturally fractured reservoir

Type of fracture	Characteristics	Contribution in production
Type I	Primary porosity (ϕ) Primary Permeability (k)	Cover large drainage area
Type II	Low permeability (k)	Good initial production rate
Type III	Complex directional permeability (k)	Sustained production rate
Type IV	Negative permeability (k)	Not good

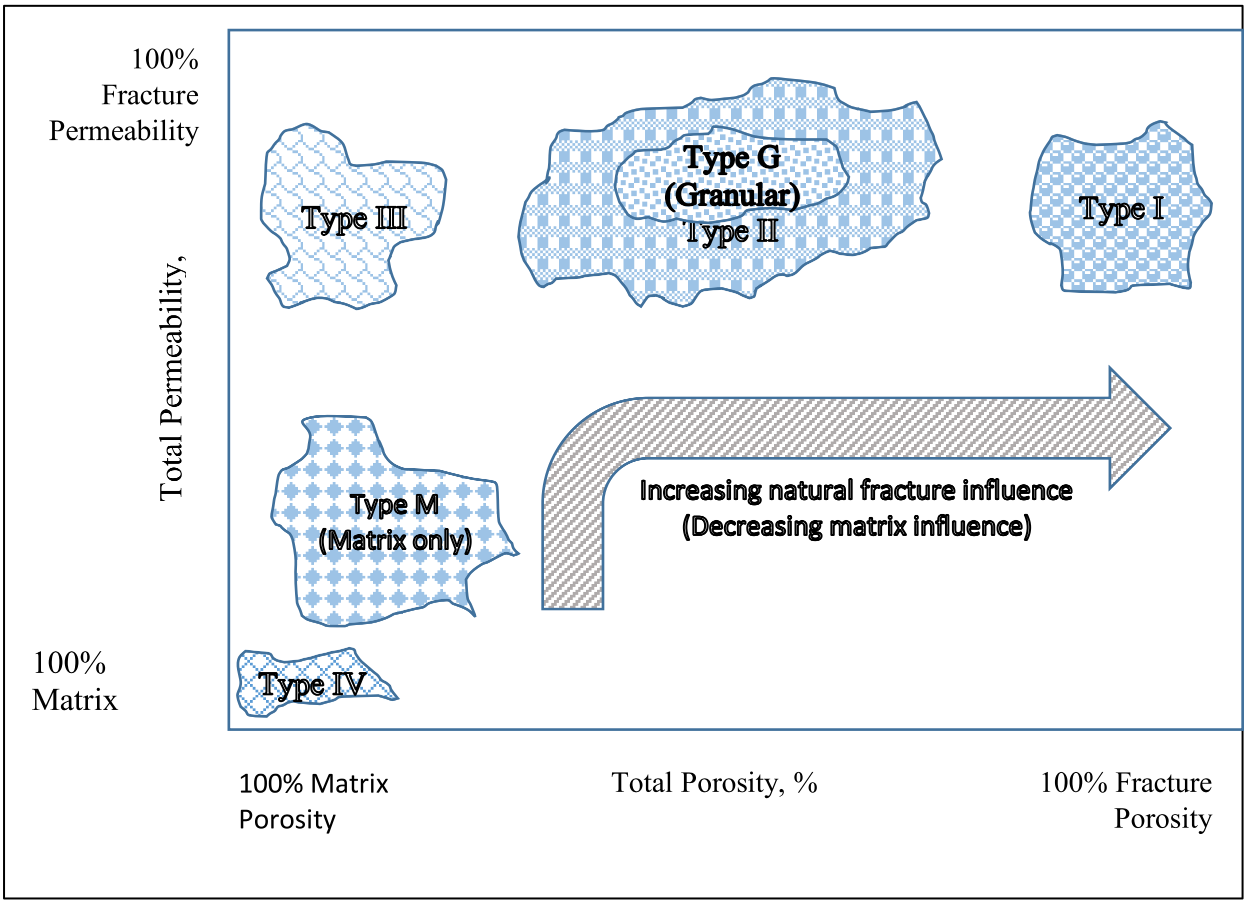

Figure 2.9: Influence of matrix and fracture on porosity and permeability in a reservoir (Nelson, 1982) (Redrawn)

The presence of natural fracture in shale gas reservoir is universal. In fact, their presence is the critical factors to estimate a prospective reservoir (Walton and McLennan, 2013). A common mechanism of production from shale is to use the naturally fractured network as the formation is severe tight (Carlson and Mercer, 1991). The effects of fractures and matrix compressibilities are considered for both saturated and undersaturated reservoirs. Hall (1953) and van der Knaap (1959) modeled some correlation to calculate that compressibility.

Aguilera (1999) used his correlation for the fractured reservoir to estimate the recovery factor based on the different drive mechanism. However, his model has some challenges and he provided some future guidelines. Most of the naturally fractured reservoirs have low matrix permeability and porosity. For this kind of reservoirs, volumetric reserve calculation is a difficult task and hence reserve estimation. The author suggested to categorize this estimation as a possible reserve. Early material balance calculation provides the probable reserves but with the cumulative production and good pressure data, the reserve should be considered as a possible reserve. For the unproved reserves, decline curve analysis is a good approach from short reservoir history. However, decline curve is not suitable for proved reserve unless the well is in the late production stage.

Cherif *et al.* (2014) published an observation of the relationship between natural fracture and production. They showed that, how the fracture affects the oil production in the unconventional reservoir. Fracture type has also the influence on oil production. Average production from a reservoir depends on not only intensity of fractures but also the type of fracture. Table 2 shows that the well GS-04 has a strong intensity of fracture but the average production is bad as the fracture is closed. GS-07 has also the strong fracture but closed and therefore, the average production is bad. The information from Table 2 proves that there should have a good combination between the intensity of fracture and the type fracture. Well no. GS-14, GS-15, and GS-17 show a good combination. These well have the strong and open fracture, eventually these wells have a good production.

Table 2.2: Quality of production based on fracture condition (Cherif et al., 2014)

Well	Proximity of faults	Intensity of fractures	Type of fracture	Average production
GS-03	E-W	average	Closed	Bad
GS-04	E-W	strong	Closed	Bad
GS-21	N120	strong	Partially open	Good
GS-08	E-W	average	Partially open	Bad
GS-07	E-W	strong	Partially open	Bad
GS-15	N120	strong	Open	Good
GS-17	N120	strong	Open	Good
GS-14	N120	strong	Open	Good
GS-11	E-W	average	Partially open	Bad

2.3.2.2 Artificial Fractured Formation (Hydraulic Fracturing)

The United States and Canada have up to 780 TCF and 1100 TCF shale gas reserves respectively (Frantz and Jochen, 2005). These possible reserves were produced by the combination of horizontal well technology and hydraulic fracturing. After drilling the horizontal well, fracturing fluids are injected into the well at high pressure to create the fracture, which increases the permeability of the shale zone largely. These fracturing fluids will be recovered immediately after opening the well during a post-stimulation 'flow-back'. The flow back data helps significantly to design the fracture model and production forecasting. One of the main natural resources in the world is the shale gas reservoir of Argentina (Duarte *et al.,* 2014). The dual porosity system is available in those reservoirs. The primary porosity is associated to the matrix, and fracture contains the secondary porosity. High matrix porosity but low permeability of the reservoir blocks the movement of the fluids. So, the fracture is very important for smooth permeable ways. In general, adsorption mechanism works to store the gas to this kind of rock, even sometimes 85% volume is occupied (Watson, 1989).

2.3.2.2.1 Aguilera Approach

Various authors have been facing huge challenges on material balance method for the years. Effect of fracture compressibility on gas reservoir has been neglected. Researchers make some assumption for conventional MBE such as: (i) the effects of water influx is negligible, (ii) there is no change in reservoir formation, and (iii) the compressibility of water and formation are neglected. Although these assumptions have a no significant effect for some cases, however there are some cases where the effects are significant (e.g. fracture, compressible rock etc.). In such situation, conventional MBE has failed to give a proper estimation of the reserve. These types of challenges are also observed in the geo-pressured reservoir (Aguilera, 2003 and 2004). A part of Aguilera (2008) model development is shown in Appendix 1.
In the case of storage capacity, fractured reservoirs have much more influence on production engineering. Three types of storage can be identified in this kind of reservoir. Matrix blocks, which is the main storage for hydrocarbon, is denoted by Type A. Fracture networks are included with the matrix in Type B storage. The storage capacity of fracture networks is Type C (Aguilera, 1995). In a reservoir of Type A, the matrix contains a significant portion of hydrocarbon whereas very small amount in fractures (McNaughton 1975).

2.3.2.3 Naturally Fractured Reservoir

Uniform porosity and compressibility assumption of the conventional MBE is no longer valid for naturally fractured reservoirs. The conventional MBE is effectively applicable for Type A and C, whereas modified MBE is suitable for Type B (Penuela *et al.,* 2001).

Penuela *et al.,* (2001) and Sandoval *et al.,* (2009) were proposed some modified models for MBE within the naturally fractured reservoir (NFR). In those models, matrix OHIP and fracture OHIP were shown instead of overall OHIP. However, some hidden limitations are found in such modifications where matrix and fracture system were supposed to have an individual effect on reservoir pore volume which is not an accurate assumption for NFR.

For NFR only average pressure and compressibility should be modified which is indicated in Eq. (1). Bashiri *et al.,* (2010) used a more logical modification which was the compressibility and porosity definition to derive following equation. Gerami *et al.* (2007) showed more simplified formulation of existing model where effective compressibility of NFR can be reduced.

$$C_T = \frac{\phi_m(c_m+c_w S_{wi})+c_f\phi_f}{\phi_m\,(1-S_{wi})+\phi_f} \qquad (2.6)$$

The compressibilities of formation fluid, and reservoir rock have a great effect on hydrocarbon production. For calculating total compressibility of the reservoir, the porosity of the fracture and matrix has the equal role. The majority of the authors incorporated only the porosity of matrix but avoided the porosity of fracture in their developed model. The porosity of the fracture has been neglected for a long time by the researcher. To characterize the reservoir more accurately, all types of porosities should be considered. Due to the negligence of fracture porosity, hydrocarbon reserve was not properly estimated. Gerami et al. (2007) proved that the total compressibility of the reservoir depends on the porosity of matrix and fracture, the compressibility of matrix, and fracture and initial water saturation. The right-handed side of equation 6 has fracture porosity as a denominator. If this porosity is not considered in the above equation, the total compressibility will be increased which finally will affect the reserve estimation.

2.3.3 Dynamic Material Balance Equation

The dynamic material balance (DMB) is an additional feature of flowing material balance equation and this equation is applicable to both constant and variable flow rate. Both flowing material balance method and DMB method are suitable for oil and gas reservoir. DMB is nothing but a systematic way that alters the flowing pressure at any point to the average reservoir pressure. Once the average reservoir pressure is calculated, the classical material balance method becomes eligible to apply and then the traditional P/Z vs G_p plots are generated. Although, the author described the procedure in an effective way, but still there are some common limitations like others. Unable to treat with transient flow data is one of the major limitation of this method. Pressure dependent permeability and variable skin factor have not been considered during the development of the equation (Mattar *et al.,* 2006).

DMB is also an effective method to determine the initial gas-oil ratio (m), initial-oil-in-place (N), reservoir permeability (K), skin factor (S) and average pressure decline history. An estimation of the original-oil-in-place (OOIP) and the determination of average pressure decline history can be obtained through the inclusion of time variable into the classical MBE. Average pressure decline history directly helps to calculate reservoir permeability and skin factor. The model was developed by assuming no flow existence in the bounded reservoir. This dynamic method has been developed based on a visual basic program (Ojo *et al.,* 2003).

2.3.4 Time Dependent MBE

Huge research works on MBE have been conducted for the last five decades (Havlena and Odeh, 1963; Havlena and Odeh, 1964; Ramagost and Farshad, 1981; Fetkovich *et al.,* 1991; Fetkovich *et al.,* 1998; Rahman *et al.,* 2006a). All these previous researchers developed MBE for gas reservoir by using expansion drive mechanism. Hossain *et al.,* (2009) incorporated time-dependent rock/fluid properties into the previous model. Expansion of oil, water, rock and dissolved gas are included in their model. In addition, the authors incorporated the time-dependent rock/fluid properties which are named as memory function. This concept is defined as "the properties of rock and fluid that help to account for changes in rock properties (such as permeability and porosity) and fluid properties (such as pressure dependent fluid properties and viscosity) with time and space" (Hossain 2016). In addition, a simple definition of memory concept is also proposed by Hossain (2016) as “the system can remember its previous state”.

According to Bruce (1983), “The past is the key to the future”. Irrespective of the research area, a variety of studies are going on to know about the future. With the technological advancement, these future-predicting studies are going to be easier. Scientists or the researchers are now predicting the future trend of the respective field with more accuracy. But one exclusive way to predict the future is the study of the past what is known as memory.

Predicting future geologic trend through the study of past events has been initiated in the 1970’s. The geologic prediction has been established through three major lines. These are- climate change, element migration and geotectonic (Bruce 1983).

In the fundamental sense, the thinking ability of mankind and animal is known as memory. In recent times, scientists have included the nonliving things into the definition of memory. A computer has also the memory which is called storage. This type of memory is also known as indirect signal memory. Rock also has the memory which includes long-term history memory, behavior- reproducing memory and stress memory. The main theme of stress memory of rock is: “every preceding step of excavation must give effects to all of the subsequent steps; i.e., rocks can remember all of the stresses they underwent in the past” (Xuefu *et al.,* 1995). For example, $y = x^3$, $\frac{dy}{dx} = 3x^2$; $y^1 = 3x^2$; $y^2 = 6x$. In the derivative, the order is an integer. What will happen if the order is a fraction such as ½? To give the answer in engineering aspect, Du *et al. (2013)* published an article on “measuring memory with the order of fractional derivative.” In their observation, there are two stages of memory process. One is a fresh stage and another one is working stage. Fractional derivative is an index of memory. That’s why the order with integer number cannot give the proper idea of memory. The critical point between fresh stage and the working stage is needed to be considered to get the accurate index of memory.

As a result, the researcher should consider the system’s the previous history for the future forecast of the outcome. Table 3 shows how the time variable is considered in different disciplines. Du *et al.* (2013) conducted a research on mechanics and he successfully incorporated the time variable as a fractional derivative in his works. He showed that there must be a change of properties after the change of gradient within a time interval. Xuefu *et al. (*1995) used the time variable in his works as a memory term where he explained that every previous step of excavation will give the effect to immediate step. Bruce (1983) conducted a good research on time factor for the geological change and he summarized that the past explains what will happen next which we can designate as memory in our current subject research.

Table 2.3: A summary of some research on time factor

<table>
<tr><th>Researcher</th><th colspan="2">Field of research</th><th>Considered terminology</th><th>Theme of the research</th></tr>
<tr><td>Hassan et al., 2016</td><td colspan="2">Reservoir engineering</td><td>Memory based fluid viscosity, velocity and pseudo-permeability</td><td>Reservoir rock and fluid properties affects the pressure response with the effect of memory</td></tr>
<tr><td>Du et al. (2013)</td><td colspan="2">Mechanics</td><td>Fractional derivative (Continuous time function)</td><td>The change of a property after change of gradient of that property within a time interval</td></tr>
<tr><td rowspan="2">Hossain et al., 2009</td><td>Reservoir Engineering</td><td>Enhanced oil recovery (EOR)</td><td rowspan="2">Time-dependent permeability and viscosity</td><td rowspan="2">Diminution of permeability with time due to the reduction of pore size</td></tr>
<tr><td>Chemical Engineering</td><td>Polymer Manufacturing</td></tr>
<tr><td>Hossain et al., 2008</td><td>Reservoir engineering</td><td>Fluid flow through porous media</td><td>Memory
(time and space)</td><td>During the geothermal action and chemical reactions in reservoir, permeability and viscosity act as time dependent parameter</td></tr>
<tr><td>Xuefu et al. (1995)</td><td colspan="2">Rock mechanics</td><td>Memory</td><td>Every previous step of excavation will give the effect to immediate step</td></tr>
<tr><td>Bruce R. Doe (1983)</td><td colspan="2">Geology</td><td>Memory</td><td>The past explains what will happen next</td></tr>
<tr><td>Caputo, 1999</td><td colspan="2">Geothermics</td><td>Time-dependent permeability</td><td>effect of decreasing permeability with a memory formalism</td></tr>
<tr><td>Caputo, 2000</td><td colspan="2">Water resource</td><td>Pressure and density variations with memory formalism</td><td>Permeability varies with time when there is a change of pressure gradient and flow</td></tr>
<tr><td>Hossain et al., 2008</td><td colspan="2">Reservoir engineering; Reservoir characterization</td><td>Memory as a stress-strain relationship</td><td>A nonlinear and chaotic behavior of stress-strain relationship can be observed if memory is considered</td></tr>
<tr><td>Hossain et al., 2008</td><td colspan="2">Reservoir engineering (EOR; Thermal recovery)</td><td>Temperature variation with the change of time and distance</td><td>Time, formation fluid velocity and steam injection velocity play a vital role on temperature profile behavior.</td></tr>
</table>

2.3.4.1 Hossain et al. Approach

Hossain *et al*. (2009) developed a generalized MBE based on the expansion drive mechanism to explore the effect. They introduced a new dimensionless parameter, C_{epm} to illustrate the whole expansion drive mechanism which is written as:

$$C_{epm} = \frac{S_{oi}C_o + S_{wi}C_w + S_{gi}C_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + C_s + M(C_w + C_s)}{1 - S_{wi}}\Delta p \qquad (2.7)$$

Equation (7) is applicable when average reservoir pressure is considered.

A dimensionless term, C_{epm} in the above equation can be counted as an important energy source for production of oil in an expansion drive. Compressible residual fluids and expansion of rock are the two main drivers for their model equation. Referring to other researchers, the value of C_{epm} is not considered only as oil/gas compressibilities (Dake, 1978; Fetkovich *et al.,* 1991; Fetkovich *et al.,* 1998; Ahmed, 2000; Rahman *et al.,* 2006b), rather C_{epm} is the function of present reservoir pressure, compressibilities, initial saturation, dissolved gas properties and associated volume fraction. Appendix 2 shows the derivation of the Eq. (7).

2.3.4.2 Buduka et al. Approach

The straight-line model of Havlena and Odeh is based on the existing reservoir drive. Estimation of initial-oil-in-place and cumulative oil produced by Havlena and Odeh method do not consider the time function of the average production of the field life. Buduka *et al.* (2015) presented an alternative Havlena and Odeh method in which underground recoverable functions F are plotted against oil plus gas expansion function E per cumulative time so that reservoir engineers can get the updated information in each time limit. Warner *et al.* (1979) identified that, though material balance method used as a pre-processing tool to estimate the hydrocarbon-in-place, but it still has some limitations.

2.3.4.2.1 Introduction of Time dimension

By using Havlena and Odeh model for general MBE, Buduka *et al.* (2015) introduced an alternative time function model. They defined all the term in general MBE by incorporating time function. They developed a model where a plot constructed on the average production rate of reservoir vs. cumulative production. The time was set as an independent variable.

$$N_p = \sum_{k=1}^{n} Q_k t_k \qquad (2.8)$$

K represents the time at the point of each reservoir average pressure and the total point of average pressure is n. A flowchart is provided in Appendix 3 to show the development of the model. Anderson and Mattar (2003) showed that time function is mandatory to convert the general production condition into an equivalent constant rate solution. The time is a superposition function when the depletion is volumetric. For the bounded flow regime, material balance with time function provides an exact conversion of constant pressure data to type curves of a constant rate (Blasingame *et al.,* 1991, and Agarwal *et al.,* 1998). Poe (2002) showed the usefulness of using a material balance with time variable for transient flow regime. Therefore, time-dependent MBE is needed to characterize the rock/fluid alteration during the production life of the reservoir in addition to get a reliable reserve estimate.

2.3 Critical Analysis

After a long review of the existing work on material balance method, some critical analyses should be provided. All the researchers tried to develop a model with more accuracy. The reviewed article proves that maximum effort in terms of knowledge and experiment was delivered to develop these innovative models. As a beginner in the research area, it is difficult and challenging to analyze the established work critically. However, a small attempt should be taken to improve the research skill. Most the cases, single porosity is considered instead of dual porosity. Cabrapan *et al.* (2014) developed a model to estimate original-gas-in-place where he assumed the presence of dual porosity. Thus, he got a better estimation of hydrocarbon for a specific field. But on the contrary, the author didn't incorporate the water influx which affects the production badly. So, this is a scope to conduct further research where an MBE model will be developed with the inclusion of dual porosity and water influx. In Table 4, only five authors are included where four of them neglected the inclusion of water influx in their research. Some critical review is shown in Table 4.

Table 2.4: Some recent research summary on material balance method

Authors	Assumptions	Findings	Limitations	Inclusion of water influx, (We)
Cabrapan *et al.*, (2014)	Presence of dual porosity	Additional way to estimate OGIP	No incorporation of water influx (We)	×
Ismadi et. al (2011)	Homogeneous reservoir with radial geometry	Combined static and dynamic method	Different conditions of the reservoir are missing	×
Sandoval *et al.*, (2009)	Existence of four faces: oil, water, gas and naturally fractured rock	Calculate IOIP for fracture containing saturated and under-saturated reservoir	Some critical assumptions were made. e.g. $\frac{d\phi_f}{dr} = 0$	×
Peron *et al.*, (2007)	Tri- phase flow	Contribution of matrix to the production	Applicable only for high permeability fractures	×
Penuela *et al.*, (2001)	instantaneous flow of hydrocarbons from the matrix to the fracture media.	Simultaneous estimation of oil stored both in the matrix and fracture	$\frac{d\phi_f}{dr} = 0$	√

To increase the scope of further research, a critical review conducted in terms of considered parameter. Ibrahim *et al.* (2013) developed an MBE where he considered separator conditions. Singh (2013) incorporated desorption term, G_d in his model. Penuela *et al.* (2001) added the net expansion of the matrix, E_{o1} and net expansion of the fracture, E_{o2}. Table 5 shows a summary of some works based on considered parameter. Pennuela (1998) considered dual porosity in his developed model where he considered the secondary porosity. It is already stated that, most of the authors avoided the formation compressibility in their developed model. Ambastha (1990) followed another approach to reduce the error created by neglected compressibility. He used a correction factor in his model to increase the accuracy. Table 5 also shows that, some important parameters which are neglected by some researcher although some of them incorporated those parameters. For instance, Nader (1964) developed a model on two phase reservoirs, where he incorporated gas formation volume factor but didn't consider the parameter water influx and solution gas oil ratio. This table also shows the development of some models with the inclusion of special parameters.

Table 2.5: A comparative study on the developed model for material balance equation

Reservoir Type		Authors	Equations	Formation Volume factor (B)	Water influx (W_e)	Solution gas oil ratio (R_s)	Newly added parameter
Oil Reservoir	Saturated Oil Reservoir	Hurst (1974)	$N(B_t - B_{ti}) = N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right]$	√	×	√	Total formation volume factor, B_t
	Single phase reservoir	Tracy (1955)	$n = \frac{1 - \left(n_{i-1}\phi_n + G_{i-1}\phi_g\right)}{\phi_n + \left(\frac{r_{i-1} + r_i}{2}\right)\phi_g}$	×	×	×	Instantaneous produced gas-oil ratio, dimensionless ratio, r_i
	Active oil reservoir	Schilthuis (1936)	$\left[(B_t - B_{oi}) + \frac{mB_{oi}}{B_{gi}}\left(B_g - B_{gi}\right) + (1+m)B_{oi}C_T\Delta P\right] + W_e = N_p\left[B_t + \left(R_p - R_{si}\right)B_g\right] + B_wW_p - G_{inj}B_{ginj} - W_{inj}B_w$	√	√	×	Water Influx, W_e
Gas Reservoir	Shale gas and unconventional reservoir	Singh (2013)	$G_p = \frac{V_{b2}\phi_i Z_{sc}T_{sc}}{p_{sc}T}\left\{\frac{(1 - S_{wi})p_i}{Z_i} - \frac{\left[1 - C_\phi(p_i - p)\right](1 - S_{wi})p}{Z}\right\} + G_d$	×	×	×	Desorption Term, G_d
	Coal seam gas reservoir	Penuela (1998)	$G_p = \frac{G_{2i}\left(B_g - B_{gi}\right) + 5.615\left(W_e - W_pB_w\right)}{B_g}$	√	√	×	Initial gas in the secondary porosity, G_{2i}
	Gas condensate reservoir	Humphrey (1991)	$\frac{G_p}{G_i} = 1 - \left[\frac{E}{E_i}\right]\left[\frac{1-y}{1-y_i}\right]\left[\frac{1 - S_w - S_{Hcl}}{1 - S_{wi}}\right]\left(1 - C_f\Delta p\right)$	×	×	×	Mole fraction of vapor phase, y
	Normally pressured gas reservoir	Ambastha (1990)	$\frac{p}{z}\left[1 - C(p_i - p)\right] = \frac{p_i}{z_i} - \frac{p_i}{z_i}\frac{G_p}{G}$		×	×	Effects of formation and water compressibility, correction factor, C
Mixed type reservoir	Volatile oil and gas condensate reservoir	Ibrahim *et al.,* (2013)	$N_p\left[B_o + \left(R_p - R_s\right)B_g\right] = NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + \left(\frac{C_wS_{wc} + C_f}{1 - S_{wc}}\right)\Delta p\right] + \left(W_e - W_p\right)$	√	√	√	Separator conditions
	Multiphase Reservoir	Penuela *et al.,* (2001)	$\frac{F}{E_{o1}} = N_1 + N_2\frac{E_{o2}}{E_{o1}}$	×	×	×	Net expansion of the matrix, E_{o1}; Net expansion of the fracture, E_{o2}
	Two phase reservoir	Nader (1964)	$N = \frac{\Delta_kN\left[B_t(k) - B_g(k)[r(k) - R_s(i)]\right] - CF_g(k) + \Delta_kW}{mB_t(i)\left[\frac{B_g(k)}{B_g(i)} - 1\right] + B_t(k) - B_t(i)}$	√	×	×	Cumulative oil, gas and water production, Δ_kN, Δ_kG and Δ_kW

Table 2.6: A summary of some important works based on dynamism, applicability and limitations

Authors	Dynamic/ static	Reservoir type: (Complex/conventional)	Application	Limitations
Istayeva and King (2014)	Dynamic	Conventional	Applicable for pre- and post-well operations.	Only suitable for carbonate reservoir
Ismadi et. al (2011)	Static and dynamic	Conventional	Applicable for layered system reservoir, pseudo-steady state approach	Homogeneous reservoir with radial geometry
Ojo et. al (2006)	Dynamic	Conventional	Applicable for saturated and primarily depleted reservoir	No guideline for fractured reservoir are suggested
Mattar et. al (2006)	Dynamic	Conventional	Applicable for either $q = constant$ or variable flowrate. P_r can be obtained without shut in the production	Fractured conditions of the reservoir are not considered
Tian and Zhao (2004)	Static	Conventional	N/A	1. Phase equilibrium throughout the reservoir 2. $\bar{T}_{res} = constant$
Ojo et. al (2004)	Dynamic	Conventional	Applicable even with limited pressure data	Visual basic based program was used
Ambastha (1990)	Static	Conventional	1. Applicable for normally pressured gas reservoir 2. also for abnormally pressured gas reservoir	1. Shows the non-linearity for p/z vs G_P graph due to the pressure squared term(p^2/z) 2. Not applicable for oil reservoir
Miranda and Raghavan (1975)	Static	Conventional	1. for determining oil in place and 2. the ratio of gas cap to oil column volume	Not suitable water drive reservoir
Hurst (1974)	Static	Conventional	Developed a relationship between oil saturation in place versus reservoir pressure.	Insensitive to establish OIP
Tracy (1955)	Static	conventional	Determination of instantaneous GOR	Unreliable OIP in early life of a reservoir

One of the most important purposes of this review is to establish a theoretical framework for my new model development. Therefore, this review has been critically reviewed different articles in terms of the dynamism, applicability and limitations. Istayeva and King (2014) developed a dynamic material balance equation (DMBE) for the conventional reservoir. This equation is applicable for pre-and post-well operations. However, this model is suitable only for carbonate reservoir. The model may be extended to develop a model for sandstone and limestone reservoir. Ismadi *et al.,* (2011) offered a static and dynamic model for the conventional reservoir which is applicable for layered system reservoir. He followed pseudo-steady state approach to develop his model. He considered that the reservoir is homogeneous and the shape of the reservoir is radial. So far it very rare to find a truly radial shape reservoir. Many of the reservoirs has the irregular shape. To overcome this limitation, a new DMBE can further be developed for the irregularly shaped reservoir. A summary of previous works on dynamic material balance method is given in Table 6.

2.4 Research Challenges and Guidelines

Material balance method is the basis of analyzing reserve estimation as well as reservoir performance. The combination of mass balance and energy balance is the governing equation for material balance technique. To develop a rigorous MBE, some simplified assumptions are considered such as the reservoir is assumed to be homogeneous and isotropic, rock compressibility is assumed to be negligible, and the flow through porous media are considered as a steady state. These unsound assumptions will lead to a result of less accurate when reservoir properties are implemented in field condition.

Disregarding time factor in the equation is one of the most vital causes of increasing error of result. Based on the conducted research, now it is clear that almost all the reservoir properties are time-dependent. There is a significant alteration of these reservoir properties while it undergoes a change with time due to production, in-situ stresses, mineralization, precipitation etc. Therefore, there is a need to incorporate time dimension during the development of a DMBE.

One of the important challenges for the shale gas reservoir is that the production mechanism is totally affected by the condensation. Artificial fracturing and gas injection may decrease this effect and improve the performance. Low pressure is the most challenge for saturated oil reservoir. Sufficient PVT and viscosity data can be used to overcome this challenge.

Sometimes, in the dry gas reservoir, getting flowing bottom-hole data is a troublesome issue. In such case, all types of data consideration would be a good solution.

Sometimes, CO_2 is injected in the depleted reservoir to get a driving force for production. However, other impurities with injected CO_2 create a major problem during production. The production engineer should be very careful during the injection. There are lots of multilayered reservoirs in the world. These multilayered reservoirs have different skin factor for each layer. This skin factor affects the response of pressure buildup. Multi-rate testing is a good solution for this situation. Some other challenges and guidelines are reported in Table 7.

Table 2.7: Some guidelines based on current challenges

MBE for different reservoir conditions	Author	Current development	Challenges	Guidelines
Shale gas reservoir	Orozco and Aguilera, 2017	Method of estimating OGIP and OCIP	Production mechanism affected by condensation	Artificial fracturing and gas injection are needed to improve the performance of this method
Saturated oil reservoir	Mosobalaje *et al.*, 2015	A material balance equation for multiphase flow	Approximation of gas oil ratio (R_p) and low pressure	Accuracy of PVT and viscosity data should be increased
Dry gas reservoir	Guzman *et al.*, 2014	Flowing gas material balance equation	Insufficient flowing data	Controllable and uncontrollable flow data are needed to consider
Depleted gas reservoir	Frailey, 2004	A material balance equation with CO_2 sequestration	Impurities in injected CO_2	Monitoring reservoir and CO_2 during injection
Tight gas reservoir	Kuppe *et al.,* 2000	• Layered material balance equation • A diagnostic tool to determine OGIP	• Water influx are not considered • The pressure buildup response is affected by layer skin factor	• Multi-rate tests may be conducted to verify layering • Advanced decline curve analyzed should be used
Over-pressured gas reservoir	Wang *et al.,* 1999	Method of detecting aquifer influence, water influx and OGIP	The availability of laboratory measured fluid compressibility (C_f)	C_f, in the order of 10^{-5} 1/psi suggested avoiding overestimation
Under-pressured gas reservoir	Wang, 1998	A method of MBE for normal and abnormal pressure gradient	Absence of aquifer	Water injection may give a good estimation
Water drive reservoir	Sills, 1996	Water drive material balance with using CARET	Constant water influx	Variable aquifer compressibility may be added
Undersaturated oil reservoir	Barry, 1963	A modification of standard MBE for the reservoir of above bubble point pressure.	Consideration of circular reservoir	Should be applied to some other field to justify the result

2.5 Conclusions

The comprehensive literature review provides that a plethora of research works was conducted on the material balance method for both conventional and unconventional reservoirs. In these studies, many of mathematical models were developed to estimate hydrocarbon reserves. Most of those models were developed based on some assumptions which don't reflect the real behavior of the reservoir. The MBE cannot be used in prospective reserve estimations if the reservoir shows unconventional circumstances. In fact, the considered assumptions restrict the extensive use of the model.

This analysis shows that in almost every work, few parameters are incorporated to develop a model. For instance, continuous alteration of rock and fluid properties is ignored during the model development for the fractured reservoir. Without considering the time variable, the model of material balance method cannot estimate the prospective reserve accurately. For example, in much of the study, porosity and permeability of the reservoir were considered uniform throughout. However, as porosity and permeability are the parameters that change with time, the reserve predictions are no longer effective, and accurate. The dual porosity of the reservoir was also ignored in some research on the fractured reservoir. Considering single porosity instead of dual porosity will not give a reliable result on a reserve estimation. Without a proper estimation of the reserve, feasibility study of production will be interrupted and economic viability of the project will be questionable. This critical review will help to understand how to modify the current MBEs for different reservoir types including unconventional reservoirs. In addition, this research will help to guide the development of DMBE. In such case, the developed model can be applied for all the discussed conditions to accurately estimate the reserves of unconventional reservoirs too.

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Appendix 1

For fractured reservoir,

$$\frac{G_p}{G_t} = 1 - \frac{\frac{P}{Z}}{\frac{P_i}{Z_i}}\{1 - [(1-\omega)C' + \omega C'']\Delta P \qquad (2.9)$$

Where, ω is the fraction of the OGIP. ΔP , is the difference of initial pressure and average reservoir pressure. C' and C'' are the compressibilities defined by,

$$C' = \frac{C_{pm}+C_w S_{wm}}{1-S_{wm}} \qquad (2.10)$$

$$C'' = \frac{C_f+C_w S_{wf}}{1-S_{wf}} \qquad (2.11)$$

For economically viable production, hydraulic fracturing is needed to increase the permeability. Then considering stimulated reservoir volume (SRV), a new MBE is presented.

$$\frac{G_p}{G_t} = 1 - \frac{\frac{P}{Z_c}}{\frac{P_i}{Z_i}} \qquad (2.12)$$

Where, Z_c is defined by:

$$Z_c = Z\left[1 - \omega_a - (\omega_m C' + \omega C'')\Delta P + \omega_m \frac{\rho_b V_L B_g}{35.315\phi(1-S_{wm})}\frac{P}{P_L+P}\right]^{-1} \qquad (2.13)$$

Appendix 2

If time-dependent variable is considered, Hossain *et al.,* (2009) showed for equation (2.7) as follows:

$$C_{epm} = \frac{\left[S_{oi}C_o + S_{wi}C_w + S_{gi}C_g\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi} + C_s + M(C_w+C_s)\right]*[P_i - P(t)]}{1-S_{wi}}\Delta p \qquad (2.14)$$

Hossain *et al.,* (2007) described the stress-strain formulation to derive the mathematical explanation,

$$\tau_T = (-1)^{0.5} * \left(\frac{\partial\sigma}{\partial T}\frac{\Delta T}{\alpha_D M_\alpha}\right) * \left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial\xi\,\partial x}\right)d\xi}{\Gamma(1-\alpha)}\right]^{0.5} * \left(\frac{6K\mu_o\eta}{\frac{\partial p}{\partial x}}\right) * e^{\left(\frac{E}{RT_T}\right)}\frac{du_x}{d_y} \qquad (2.15)$$

Equation (14) reduces to (Hossain, 2008):

$$P_i - P(t) = -\frac{6K\mu_o\eta\left(\frac{\Delta T}{\alpha_D M_\alpha}\frac{\partial\sigma}{\partial T}\right)^2 * \left[\frac{\int_0^t (t-\xi)^{-\alpha}\left(\frac{\partial^2 p}{\partial\xi\,\partial x}\right)d\xi}{\Gamma(1-\alpha)}\right] * e^{2\left(\frac{E}{RT_T}\right)} * \left(\frac{du_x}{d_y}\right)^2}{\tau_T^2} u_x\Delta t$$

(16)

$$\frac{1}{B_o}\left(1 - C_{epm} - \frac{W_e - W_p B_w}{NB_{oi}}\right) = \frac{1}{B_{oi}}\left(1 - \frac{N_p}{N}\right) \tag{2.17}$$

When dissolved gas saturation and associated volume expansion are neglected, equation (2.11) becomes:

$$C_{epm} = \frac{S_{oi}C_o + S_{wi}C_w + C_s}{1 - S_{wi}}\Delta p \tag{2.18}$$

Appendix 3

$$\boldsymbol{F = \sum^{n} Q_k t_k \left[B_o + (R_p - R_s)B_g + W_p B_w\right]}$$

Introducing time function

$$\boldsymbol{F = \sum^{n} Q_k t_k \left[B_{ok} + (R_{pk} - R_{sk})B_{gk} + W_{pk} B_{wk}\right]}$$

$$\boldsymbol{F = \frac{\sum_{k=1}^{n} Q_k \sum_{k=1}^{n} t_k}{n}\left[B_{ok} + (R_{pk} - R_{sk})B_{gk} + W_{pk} B_{wk}\right]}$$

$$\boldsymbol{F = \sum^{n} Q_k \left[B_{ok} + (R_{pk} - R_{sk})B_{gk} + W_{pk} B_{wk}\right]\frac{\sum_{k=1}^{n} t_k}{n}}$$

Introducing underground withdrawal function, F´

$$\boldsymbol{F = F' \frac{\sum_{k=1}^{n} t_k}{n}}$$

The new model

$$\boldsymbol{F' \frac{\sum_{k=1}^{n} t_k}{n} = N\left[E_o + mE_g + E_{fw}\right] + W_e B_w}$$

Where,

$$F' = \sum_{k=1}^{n} Q_k \left[B_{ok} + \left(R_{pk} - R_{sk}\right)B_{gk} + W_{pk}B_{wk}\right] \tag{2.19}$$

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g \tag{2.20}$$

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \tag{2.21}$$

$$E_{fw} = (1 + \mathrm{m})\mathrm{B_{oi}}\left[\frac{\mathrm{C_w S_{wi} + C_f}}{1 - \mathrm{S_{wi}}}\right]\Delta\mathrm{P} \tag{2.22}$$

Chapter 3
Development of a Compressibility Model for the sensitivity analysis of Material Balance Equation

3.1 Abstract

Oil and gas reserves in the North Atlantic region have major commercial and strategic significance to the nation's future. The central goal of the proposed research is to develop a dynamic material balance equation (MBE) for this potential zone. The traditional MBE has its own limitations. Several assumptions are considered for conventional MBE that is not always validated. Such assumptions were required because of the limitations in computational procedures. The present study involves determining a way to avoid the traditional assumptions. Temperature, permeability and porosity are always considered as a constant parameter for the conventional reservoir. But for the fractured reservoir those parameters are completely variable. An effort has been conducted to incorporate those variable parameters into the established MBE. None of these three parameters are directly incorporated in the general MBE but they have the significant effect on the equation as well as reserve estimation. To include this weighty effect, an effective compressibility term has been used. By using continuity equation, a model has been developed which expresses the relationship between effective compressibility and pressure. A sensitivity analysis is carried on by using this relationship. Finally, sets of oil field data are used to analyze the sensitivity of the established MBE. Detailed mathematical formulations are presented to get this model. Numerical solutions are offered to estimate the reserve and cumulative oil production. and the reserve estimation. This study is important because it offers a wide-ranging means of the feasibility of the production in the fractured formations especially for a high potential offshore basin of North Atlantic region.

3.2 Introduction

During the last few years, research on material balance has been conducted for the fractured reservoir to improve the reservoir analysis. However, all previous works are applicable to limited ranges of data. Porosity and permeability throughout the reservoir are assumed uniform in case of conventional MBE. As dual porosity system is generated for the naturally fractured reservoirs (NFR), the assumption is not valid. The compressibility of fractures is much higher than the matrix. In addition, the porosity of fracture and matrix changes when there is a change in pressure (Nelson, 1985). Walsh (1994) developed a comprehensive straight line method to estimate hydrocarbon reserve for the conventional reservoir and this method is applicable to a full range of reservoir fluids. This paper presents all previous works on the conventional reservoir in an organized way so that readers can capture the missing criteria for unconventional reservoir without difficulty.

MBE is recognized as one of the most efficient techniques to estimate the hydrocarbon reserve. In the case of a conventional hydrocarbon reservoir, a graphical representation of P/Z versus GP can be made. If there is no water influx, it gives a linear trend and this method is used to estimate the original-gas-in-place (OGIP) (Dake, 1978). For using the conventional MBE, fracture and other unconventional properties should be considered. To analyze the reservoir performance, several endeavors have been accomplished by the material balance method. Schilthuis (1936) was the first who formulated the material balance analysis. And later, several MBE has been offered for single porosity reservoir (Muskat 1949, Pirson 1958, Amyx et al., 1960, Craft et al., 1991, Dake 1994, Walsh 1995). A graphical representation of MBE as a straight line was recommended by Havlena and Odeh (1963). Likewise, Campbell (1978) offered a proposal to identify the new method of depletion mechanisms, e.g. gas cap or water drive. However, in the case of the fractured reservoir, the scenario becomes completely different.

Therefore, it is important to find out the necessary change needed for the general MBE. There are some parameters in the general MBE which have very smooth value for convention reservoir. But in the case of fractured reservoir, they give anomalous value. Therefore, it is not logical to use the general MBE for fractured reservoir. In this research, an endeavour has been carried out to find out the variable behaviour of different rocks and fluid properties. A further research will be carried out based on this research to modify the established MBE.

3.3 Model Development

The relative change in oil volume per unit change in pressure is called oil compressibility. Oil compressibility is a driving force for fluid through porous media. It is a governing drive mechanism for an undersaturated reservoir. But for a saturated reservoir, gas compressibility has more dominance over oil compressibility because of producing dissolved gas. Oil compressibility is a part of total compressibility, which directly assists to calculate the skin, material balance and dimensionless time.

Oil compressibility versus pressure plot has a noticeable discontinuity at the bubble point pressure. For the under-saturated reservoir when pressure goes above bubble point, the oil exist in single-phase liquid which is the combination of oil and dissolved gas.

Mathematically,

$$c_o = -\frac{1}{V_o}\frac{\partial V_o}{\partial p}]_T \tag{3.1}$$

$$\Rightarrow c_o \partial p = -\frac{1}{V_o}\,\partial V_o$$

$$\Rightarrow c_o \int_{p_i}^{p} \partial p = -\int \frac{1}{V_o}\,\partial V_o$$

$$\Longrightarrow c_o(p_i - p) = lnV_o$$

$$V_o = e^{c_o(p_i-p)} \tag{3.2}$$

When the above exponential function $e^{c_o(p_i-p)}$ is expanded, for the constant compressibility we can write;

$$e^{c_o(p_i-p)} \approx 1 + c_o(p_i - p) \tag{3.3}$$

where c_o indicates the compressibility of oil

$$e^{c_w(p_i-p)} \approx 1 + c_w(p_i - p) \tag{3.4}$$

where c_w indicates the compressibility of water

$$e^{c_g(p_i-p)} \approx 1 + c_g(p_i - p) \tag{3.5}$$

Where c_g indicates the compressibility of gas

$$e^{-c_s(p_i-p)} \approx 1 + c_s(p_i - p) \tag{3.6}$$

Where c_s indicates the compressibility of solid (rock matrix)

It is recognized that there is a proportional relationship between pressure drop through a granular bed and fluid velocity at low flow rates, and which is square of the velocity at high flow rates. Osborne Reynolds (Osborne O., 1900) first formulated this relationship which is as follows-

$$\frac{\Delta p}{L} = au + b\rho u^2 \tag{3.7}$$

As it is assumed that, the flow is laminar, so velocity of the fluid is very tiny. The range of the fluid flow is 0.0334 to 0.6676 ft/s. Eventually the square of the value of velocity will be negligible. That's why the term $b\rho u^2$ can be considered as negligible. Mathematically,

$b\rho u^2 \rightarrow 0$

So, the equation (3.7) becomes

$\frac{(p_i-p)}{L} = au$

$$\Rightarrow \ (p_i - p) = auL \tag{3.8}$$

Here "a" is the coefficient. The value of **"a"** depends on the value of pressure difference, fluid velocity and length of the reservoir.

$So, a = \frac{(p_i-p)}{ul}$

When a fluid flow with a velocity of 1 in/s through a 1 in bed with 1 psi pressure difference, then $a = \frac{1psi}{in/_s} in$.

In other word, $a = 1\frac{psi}{in/_s} in$ is meant that, when a fluid flow in a 1 in porous path with a velocity of 1 in/s, the pressure difference will be 1psi.

For the simplicity, we can use $a = 1\frac{psi}{in/_s} in$ for the next derivation.

$$\text{Then, } (p_i - p) = uL \tag{3.9}$$

When there is a pressure difference and constant velocity in the porous media;
ρ is the density of respective phase.
Therefore, the equations (3.3), (3.4), (3.5) and (3.6) become,

$$e^{c_o(p_i-p)} \approx 1 + c_o uL \tag{3.10}$$

where ρ_o is the density of oil

$$e^{c_w(p_i-p)} \approx 1 + c_w uL \tag{3.11}$$

where ρ_w is the density of water

$$e^{c_g(p_i-p)} \approx 1 + c_g uL \tag{3.12}$$

where ρ_g is the density of gas

$$e^{-c_s(p_i-p)} \approx 1 + c_s uL \tag{3.13}$$

where ρ_s is the density of solid (rock matrix)
Parameter of effective compressibility from Hossain and Islam (2011)-

$$C_e = \frac{S_{oi}\left(e^{c_o(p_i-p)}-1\right)+S_{wi}\left(e^{c_w(p_i-p)}-1\right)+S_{gi}\left(e^{c_g(p_i-p)}-1\right)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+\left(1-e^{-c_s(p_i-p)}\right)+ M\left[\left(e^{c_w(p_i-p)}-1\right)+\left(1-e^{-c_s(p_i-p)}\right)\right]}{1-S_{wi}} \tag{3.14}$$

For the compressibility of solid and the average pressure drop of the reservoir,

$$c_s \to 0$$

Therefore,

$$e^{-c_s(p_i-p)} \approx 1;$$

So, equation (3.14) can be written as:

$$C_e = \frac{S_{oi}\left(e^{c_o(p_i-p)}-1\right)+S_{wi}\left(e^{c_w(p_i-p)}-1\right)+S_{gi}\left(e^{c_g(p_i-p)}-1\right)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+M\left(e^{c_w(p_i-p)}-1\right)}{1-S_{wi}} \quad (3.15)$$

Substituting equation (3.3) to (3.5) into equation (3.15), the equation becomes;

$$C_e = \frac{S_{oi}(1+c_o uL-1)+S_{wi}(1+c_w uL-1)+S_{gi}(1+c_g uL-1)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+M(1+c_w uL-1)}{1-S_{wi}} \quad (3.16)$$

$$C_e = \frac{S_{oi}(c_o uL)+S_{wi}(c_w uL)+S_{gi}(c_g uL)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+M(c_w uL)}{1-S_{wi}} \quad (3.17)$$

$$C_e = \frac{uL\left\{S_{oi}c_o+S_{wi}c_w+S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+Mc_w\right\}}{1-S_{wi}} \quad (3.18)$$

$$C_e = \frac{\left\{S_{oi}c_o+S_{wi}c_w+S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+Mc_w\right\}}{1-S_{wi}}(uL) \quad (3.19)$$

Again, according to Darcy's law;

$$Q = -\frac{kA}{\mu L}(p_a - p_b) \quad (3.20)$$

$$\Rightarrow uA = -\frac{kA}{\mu L}(p_a - p_b)$$

$$\Rightarrow u = -\frac{k}{\mu L}(p_a - p_b) \quad (3.21)$$

The above equation is applicable for single phase (fluid) flow. The negative sign indicates that fluid flows from high pressure region to low pressure region. For the negative change of pressure (where $p_b > p_a$), the flow will follow the positive direction.

If p_i is the initial pressure of the reservoir, then equation (3.18) becomes;

$$u = -\frac{k}{\mu L}(p - p_i) \quad (3.22)$$

where p is the average reservoir pressure.

As most of the cases; $p_i > p$;

We can rewrite the equation (3.20) as:

$$u = \frac{k}{\mu L}(p_i - p) \tag{3.23}$$

Replacing equation (3.23) into equation (3.19), we get-

$$C_e = \frac{\left\{S_{oi}c_o + S_{wi}c_w + S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + Mc_w\right\}}{1 - S_{wi}}\left[\frac{k}{\mu L}(p_i - p)L\right] \tag{3.24}$$

$$\boldsymbol{C_e = \frac{\left\{S_{oi}c_o + S_{wi}c_w + S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + Mc_w\right\}}{1 - S_{wi}}\left[\frac{k}{\mu L}(p_i - p)L\right]} \tag{3.25}$$

The above equation is the final model for compressibility which shows a correlation between compressibility and reservoir pressure.

Again, recalling equation (3.20)

$$Q = -\frac{kA}{\mu L}(p_a - p_b)$$

With the previous explanation, the above equation can be written as;

$$Q = \frac{kA}{\mu L}(p_i - p) \tag{3.26}$$

$$\Rightarrow (p_i - p) = \frac{Q\mu L}{kA}$$

$$\Rightarrow (p_i - p) = \frac{uA\mu L}{A}$$

So, $(p_i - p) = u\mu L$ (3.27)

Therefore, the equations (3.1), (3.2), (3.3) and (3.4) become,

$$e^{c_o(p_i - p)} \approx 1 + c_o\mu_o L \tag{3.28}$$

where μ_o is the viscosity of oil.

$$e^{c_w(p_i - p)} \approx 1 + c_w\mu_w L \tag{3.29}$$

where μ_w is the viscosity of water.

$$e^{c_g(p_i - p)} \approx 1 + c_g\mu_g L \tag{3.30}$$

where μ_g is the viscosity of gas.

$$e^{-c_s(p_i - p)} \approx 1 + c_s\mu_s L \tag{3.31}$$

where μ_s is the viscosity of solid (rock matrix).

Recalling equation (3.15)

$$C_e = \frac{S_{oi}\left(e^{c_o(p_i - p)} - 1\right) + S_{wi}\left(e^{c_w(p_i - p)} - 1\right) + S_{gi}\left(e^{c_g(p_i - p)} - 1\right)\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + M\left(e^{c_w(p_i - p)} - 1\right)}{1 - S_{wi}}$$

Substituting the equation (26) to (29) into above equation, above equation can be written as;

$$C_e = \frac{S_{oi}(1+c_o\mu_o L-1)+S_{wi}(1+c_w\mu_w L-1)+S_{gi}(1+c_g\mu_g L-1)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+M(1+c_w\mu_w L-1)}{1-S_{wi}} \quad (3.30)$$

$$C_e = \frac{S_{oi}(c_o\mu_o L)+S_{wi}(c_w\mu_w L)+S_{gi}(c_g\mu_g L)\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+M(c_w\mu_w L)}{1-S_{wi}} \quad (3.31)$$

$$C_e = \frac{\left\{S_{oi}c_o\mu_o+S_{wi}c_w\mu_w+S_{gi}c_g\mu_g\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+Mc_w\mu_w\right\}}{1-S_{wi}}L \quad (3.32)$$

Recalling equation (3.9):

$p_i - p = uL$

$$\Rightarrow L = \frac{p_i - p}{u} \quad (3.33)$$

Applying equation (3.33) into equation (3.32);

$$\boldsymbol{C_e = \frac{\left\{S_{oi}c_o\mu_o+S_{wi}c_w\mu_w+S_{gi}c_g\mu_g\left(\frac{R_{soi}}{B_{oi}}+\frac{R_{swi}}{B_{wi}}\right)B_{gi}+Mc_w\mu_w\right\}}{1-S_{wi}}\frac{(p_i-p)}{u}} \quad \textbf{(3.34)}$$

The above equation is another model compressibility which relates a correlation between the compressibility and velocity of the fluid.

3.4 Significance of effective compressibility (C_e)

A dimensionless term, C_e in the equation (3.34) can be counted as an important energy source for production of oil in an expansion drive. Compressible residual fluids and expansion of rock are the two main drivers for the equation. Referring to other researchers, the value of C_e is not considered only as oil/gas compressibility (Dake, 1978; Fetkovich *et al.,* 1991; Fetkovich *et al.,* 1998; Ahmed, 2000; Rahman *et al.,* 2006b), rather C_e is the function of present reservoir pressure, compressibility, initial saturation, dissolved gas properties, viscosity, phase density and permeability. Ce is an effective parameter as it defines the relationship between compressibility and permeability. Finally, this significant parameter dictates how the modified MBE should be derived.

3.5 Numerical Simulation

By solving the equation (22), the numerical results of the dimensionless parameter can be obtained. A volumetric reservoir which is under-saturated and has no gas cap is considered for the simulation. $p_i = 4000\ psi$ is the initial pressure of the reservoir. For solving the mentioned equation some rocks and fluid properties has been used which is shown in table 1. Matlab_R2016a software has been used for all computation. To generate the correlation between compressibility and pressure, the Virginia Hills Beaverhill Lake field [Ahmed, 2002] data and an additional data of Table 1 and Table 2 are considered.

Table 3.1: Reservoir rocks and fluid properties for numerical simulation

Rock and fluid properties [Hall, 1953; Dake, 1978; Ahmed, 2000]	
$B_{gi} = 0.00087\ rb/scf$	$C_w = 3.62 \times 10^{-6}\ psi^{-1}$
$B_{oi} = 1.2417\ rb/stb$	$R_{soi} = 510.0\ scf/stb$
$B_{wi} = 1.0\ rb/scb$	$R_{swi} = 67.5\ scf/stb$
$C_g = 500.0 \times 10^{-6}\ psi^{-1}$	$S_{gi} = 20\%$
$C_o = 15.0 \times 10^{-6}\ psi^{-1}$	$S_{oi} = 60\%$
$C_s = 4.95 \times 10^{-6}\ psi^{-1}$	$S_{wi} = 20\%$

Table 3.2: The field production and PVT data (Example 11-3: of Ahmed, 2002)

Volumetric average pressure (psi)	No. of producing wells	B_o rb/stb	N_p $mstb$	W_p $mstb$
3685	1	1.3120	0	0
3680	2	1.3104	20.481	0
3676	2	1.3104	34.750	0
3667	3	1.3105	78.557	0
3664	4	1.3105	101.846	0
3640	19	1.3109	215.681	0
3605	25	1.3116	364.681	0
3567	36	1.3122	542.985	0.159
3515	48	1.3128	841.591	0.805
3448	59	1.3130	1273.530	2.579
3360	59	1.3150	1691.887	5.008
3275	61	1.3160	2127.077	6.500
3188	61	1.3170	2575.330	8.000

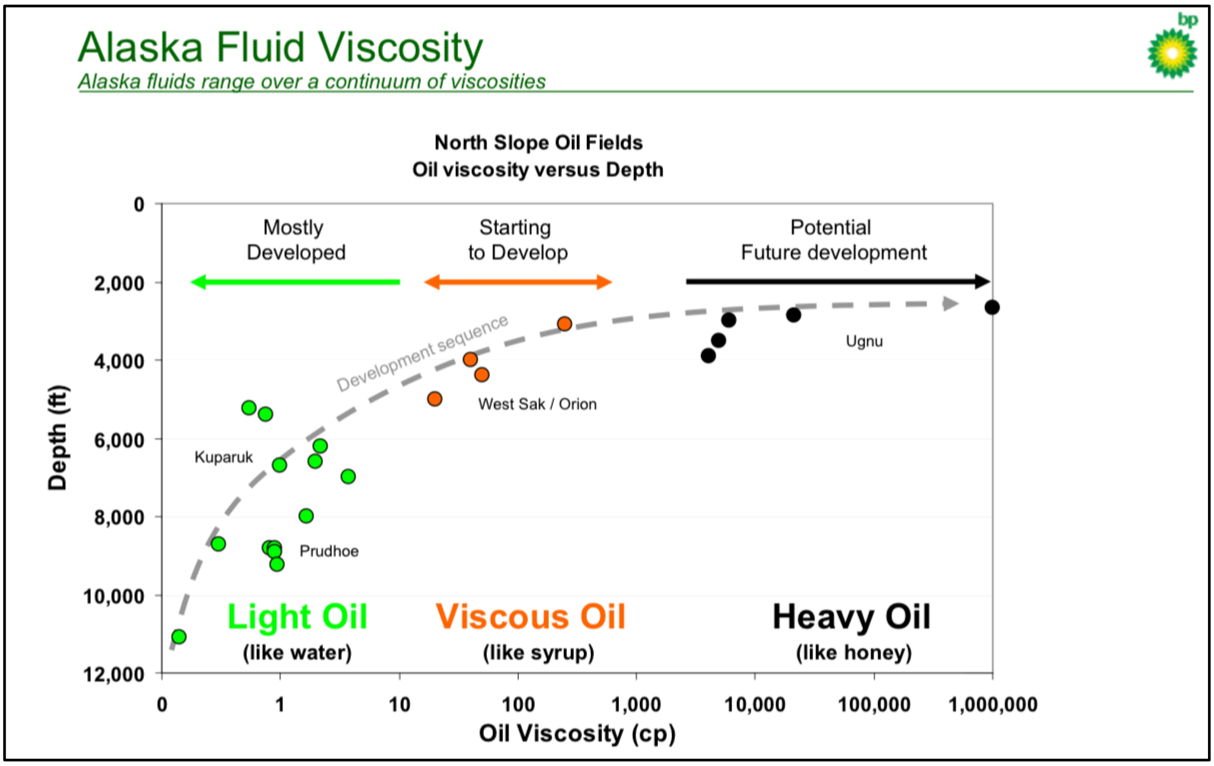

Figure 3.1: Oil viscosity used for numerical simulation (Heavy oil vs. light oil; a survey of bp, March 2011)

3.6 Results and discussion

3.6.1 Effects of reservoir properties on compressibility

Modified material balance equation will be developed for oil and gas reservoir in next two chapters. Modification will be done by including some important parameter into the established material balance equation. These important parameters are viscosity (μ), pressure (P), mobility ratio (M), and velocity (u). An explanation of mobility ration may help to understand the effect obviously.

Mobility ratio of a fluid is defined as its relative permeability divided by its viscosity. Mobility combines a rock property e.g. permeability with a fluid property e.g. viscosity. Gas-oil relative permeabilities are assumed to be dependent on the saturations of the two fluid phases and independent of fluid viscosity. In mathematical expression:

$$M_i = \frac{kk_{ri}}{\mu_i}$$

Where, k_{ri} is the relative permeability to the respective fluid.

But all the parameters don't have the same effect on material balance calculation. Some of the parameter might have significant effect and some of the parameter may have very negligible effect on reserve estimation. Which parameter have much effect on material balance calculation will be identified in this chapter. And that parameters will be used for the modification.

For all types of material balance equation, compressibility is a significant parameter. Hydrocarbon in place calculation is depends highly on this parameter. However, the change of compressibility totally depends on the changes of other rocks and fluid properties. To check the sensitivity of compressibility with other parameter, different plots are generated by using Matlab software.

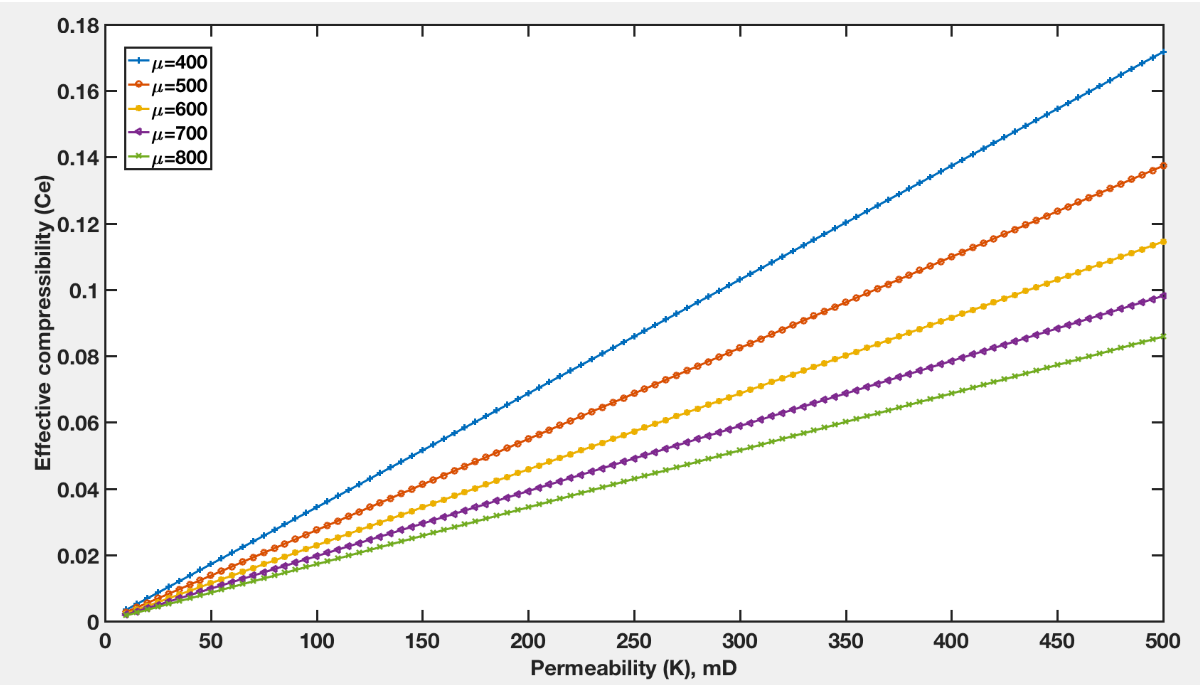

Figure 3.2: Effect of the change of permeability on effective compressibility for different viscosity

Permeability and effective compressibility also showing the proportional relationship for different viscosity. When the value of viscosity is 800 cP, the slope value is 0.00017. But when the value of viscosity is decreased by half, the slope value increases in a significant amount. That means the increment rate is more when the viscosity is decreased. This phenomena can be explained physically as well. Viscosity is an important characteristics of any fluid. It refers how much a fluid can resist itself when pressure is applied. The more the viscosity, the more a fluid can resist itself from external pressure. In other way, the lower the viscosity, the higher the compressibility.

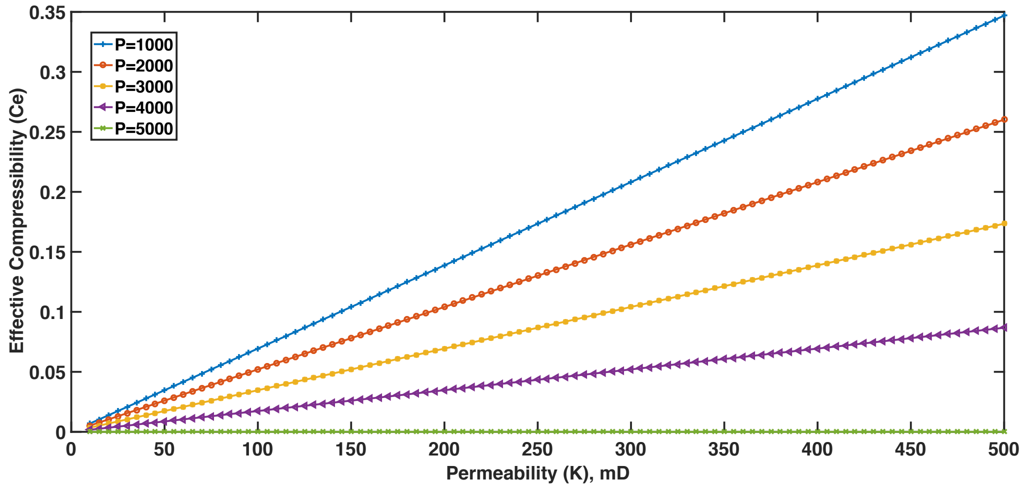

Figure 3.3: Change of compressibility with the change of permeability for different reservoir pressure

Figure 3.2 shows that, reservoir pressure has a significant effect on compressibility value. During high reservoir pressure, the increment of compressibility is less with the increment of permeability. But during the low reservoir pressure, the scenario is opposite. This phenomenon can be explained physically. When the reservoir pressure is high, the rocks and fluids are already in a compressed state. So for the reservoir with high pressure, even if the permeability becomes higher, there is no significant change in compressibility value.

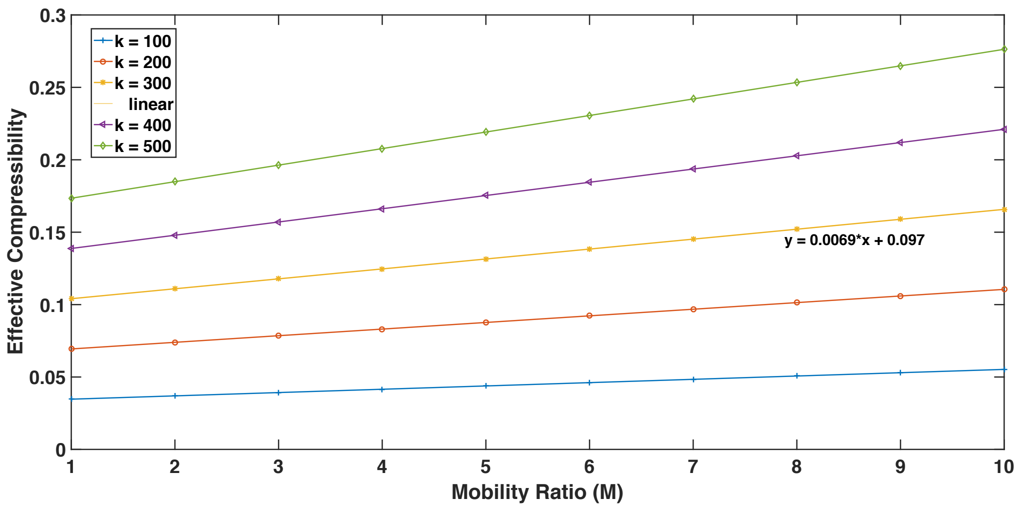

Figure 3.4: Dependency of compressibility on mobility ration when different permeability value considered

Mobility ratio is an important fluid property which may have good effect on material balance calculation. But figure 3.4 shows that, mobility ratio does not apply that much role on material balance calculation.

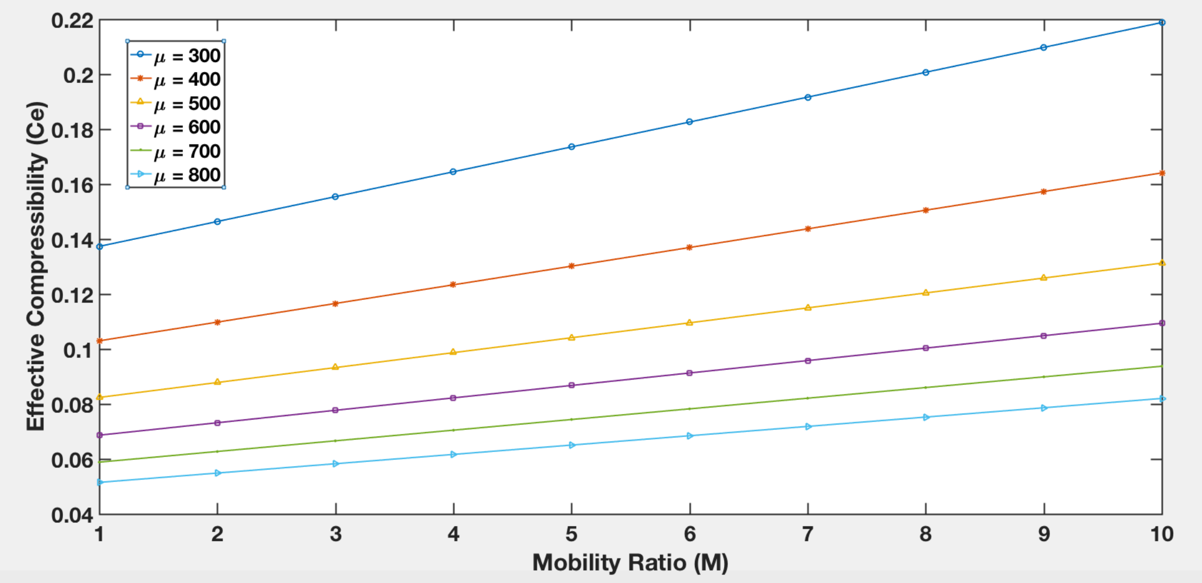

Figure 3.5: Change of compressibility with the change of mobility ratio for different values of viscosity

Figure 3.4 shows that, though the permeability is increased, the mobility ratio did not apply much effect on compressibility. Figure 3.5 shows that, with the change of viscosity, mobility ratio also changes however, the change still negligible.

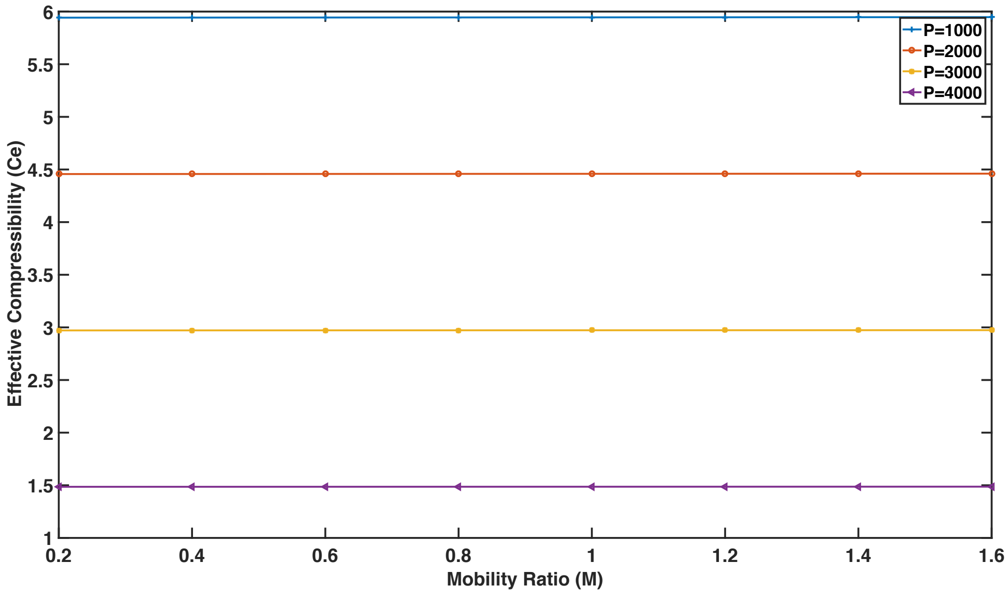

Figure 3.6: The relationship between effective compressibility and mobility ratio for different reservoir pressure

Figure 3.6 also shows the negligible effect of mobility ratio. However, there is some effect available when reservoir pressure is decreased.

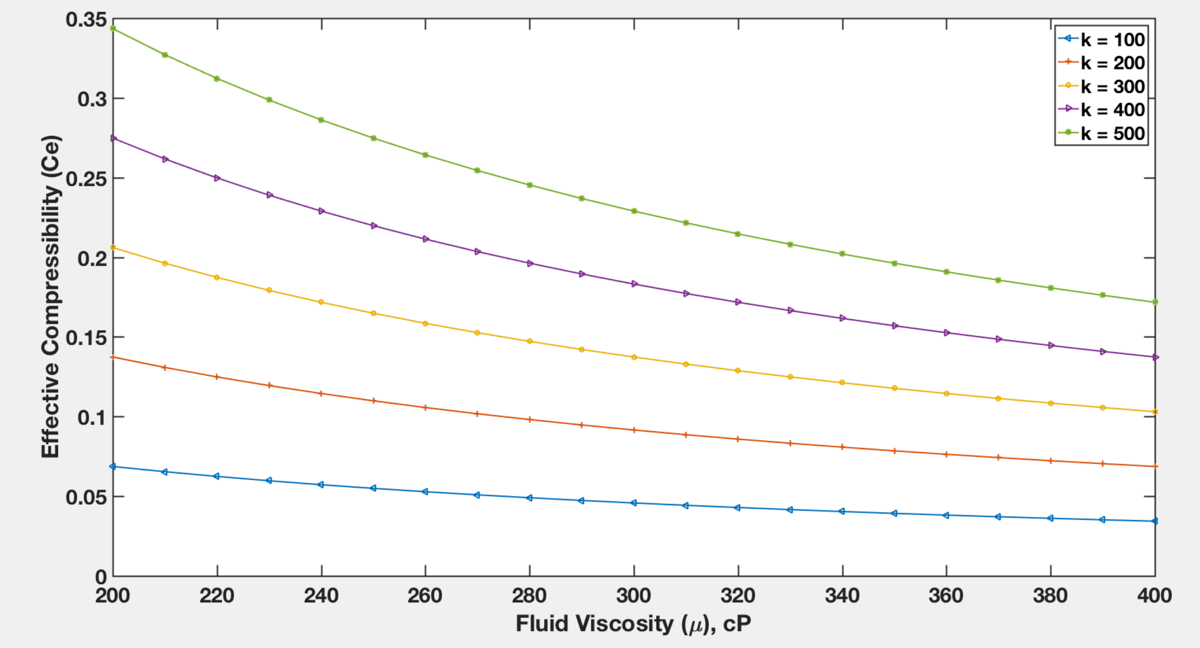

Figure 3.7: Correlation of effective compressibility and fluid viscosity for variable permeability

Figure 3.7 shows that, when fluid viscosity increases, the effective compressibility decreases. However, the decrement rate increases a bit when permeability increases.

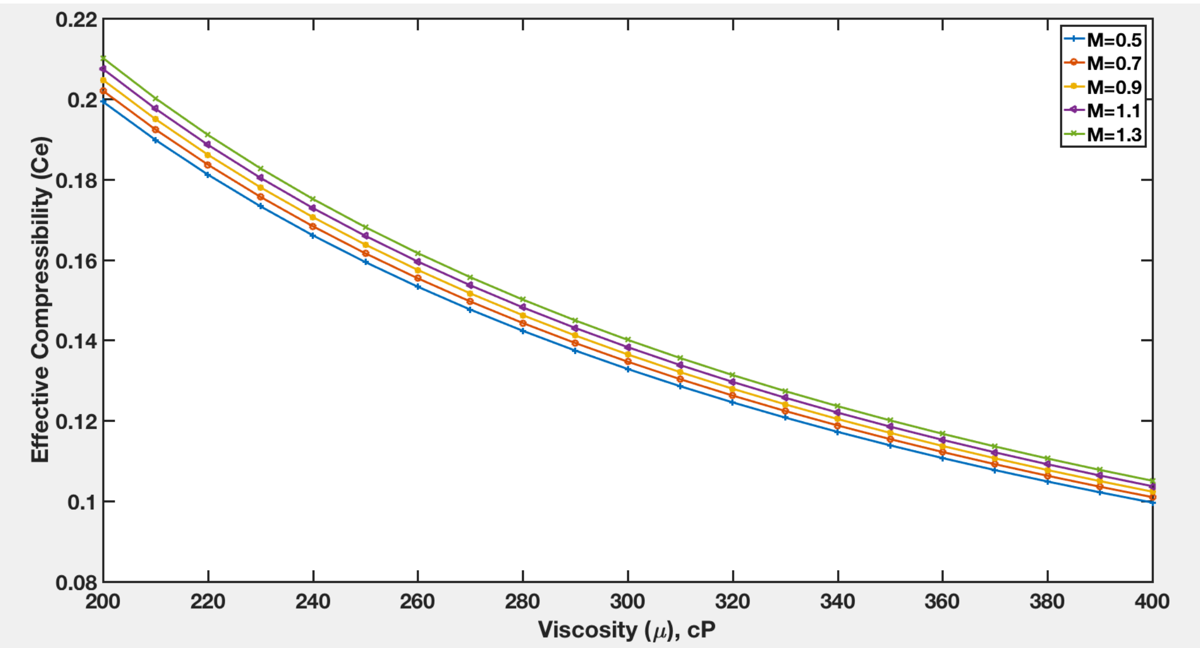

Figure 3.8: Change of effective compressibility with the change of fluid viscosity when mobility ratios are changed

The previous couple of figures showed the low response of mobility ratio. Figure 3.8 also proves that, there is a very less contribution of mobility ratio on material balance calculation.

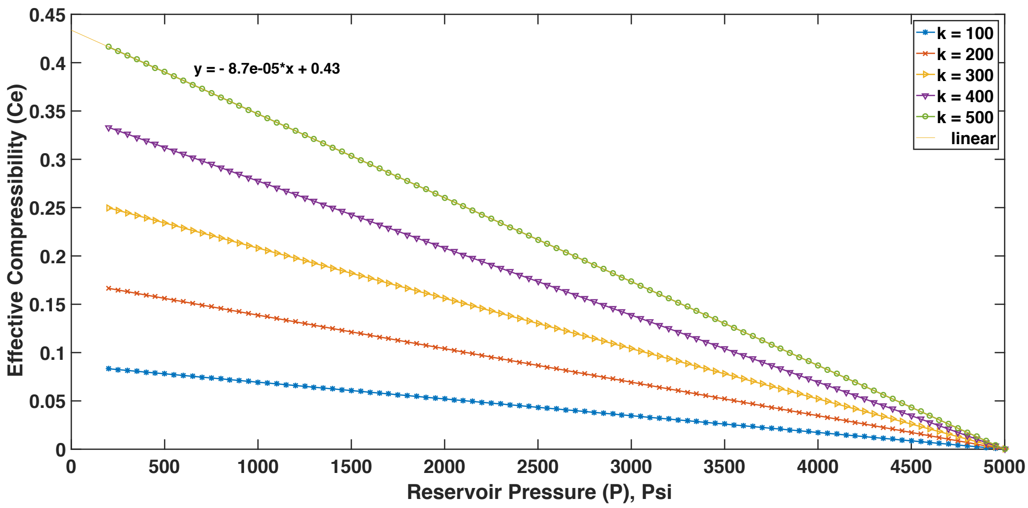

Figure 3.9: The relationship between effective compressibility and reservoir pressure with the change of permeability for the reservoir

From the literature, it is known that, permeability and reservoir pressure have a very significant role on reserve estimation. Above figure also showing that, there is a rapid change of compressibility with the change of reservoir pressure. For the highly permeable reservoir, the change is more rapid.

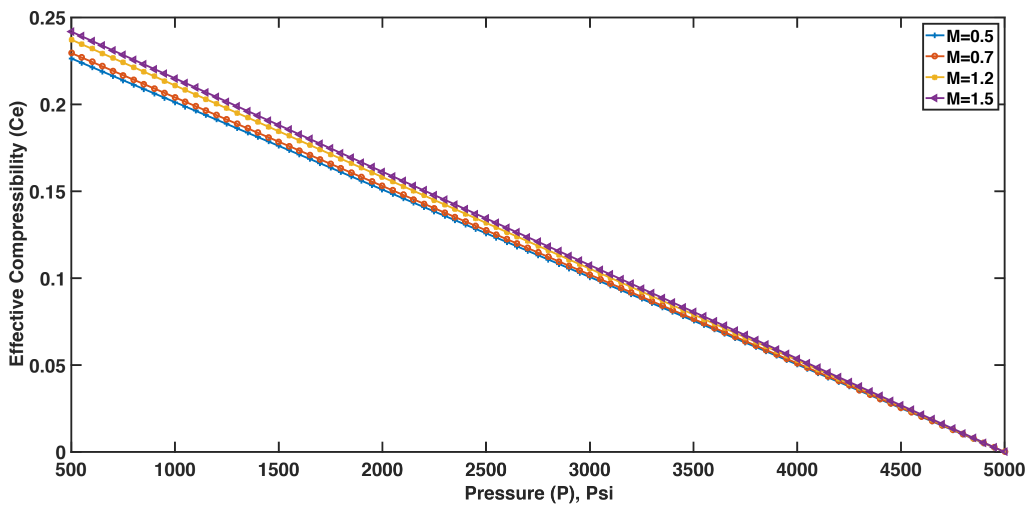

Figure 3.10: Relationship between effective compressibility and reservoir pressure when the mobility ratios are increased

From the previous figure, it is obvious that there is a big change of compressibility when the reservoir pressure changes. However, same as before there is a very negligible change of compressibility with the rapid change of mobility ratio.

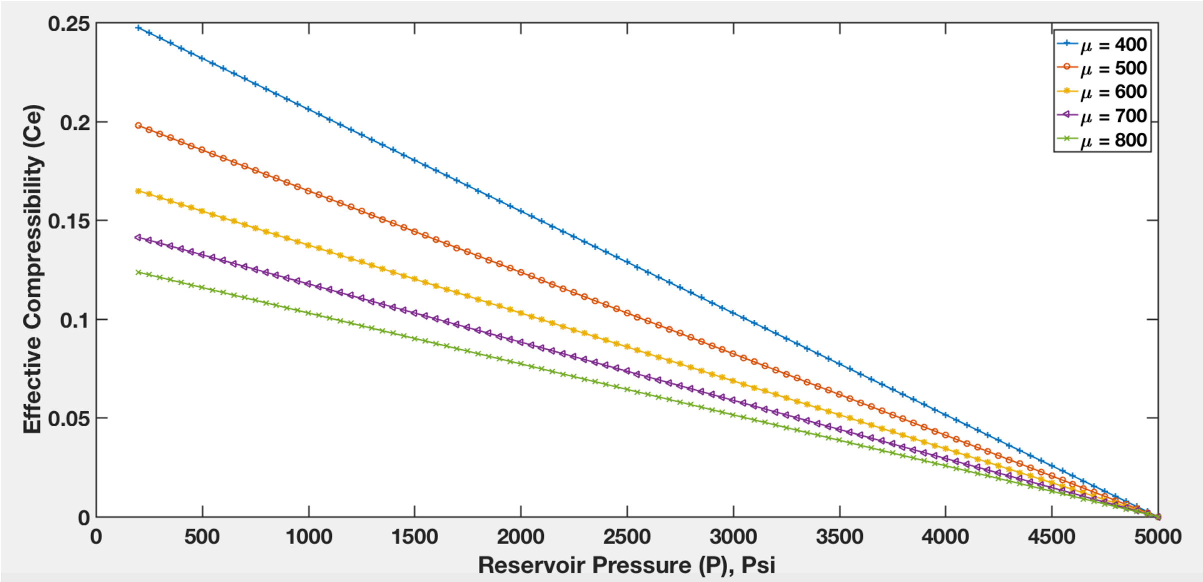

Figure 3.11: Change of effective compressibility with the increment of reservoir pressure when fluid viscosity is also increased

Viscosity also has a significant role on material balance calculation. From above figure, it is clear that, the slope value for the viscosity of 400 cP is higher than the slope value for the viscosity of 800 cP.

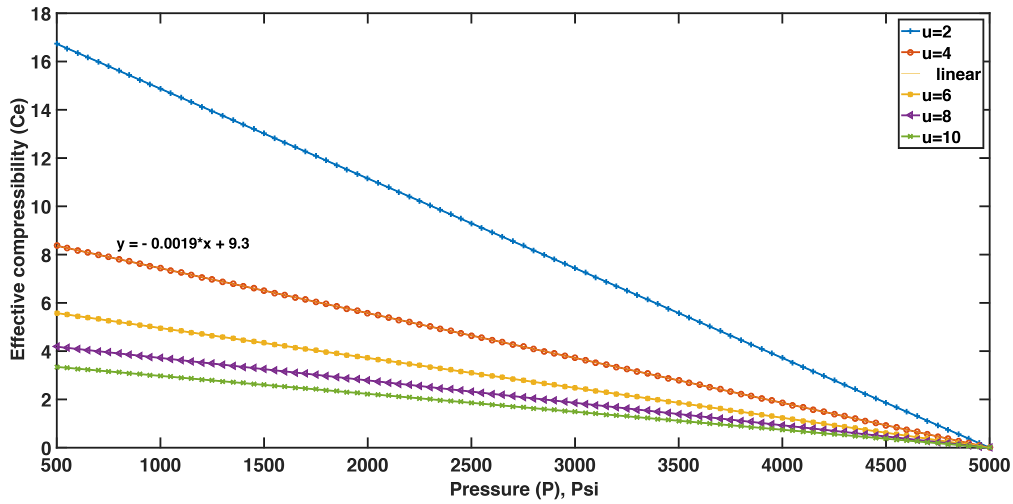

Figure 3.12: Effect of velocity on the relationship of effective compressibility and pressure

The response of different parameter is clearer when the velocity parameter is utilized. Above figure, when u=10, the compressibility reduction is negligible with the increment of reservoir pressure. However, for u=2, the deduction rate of compressibility is much higher.

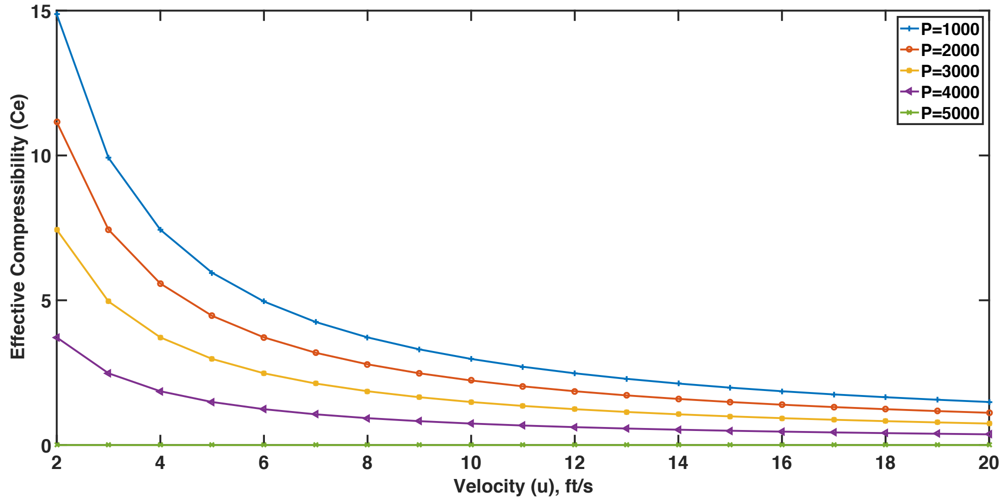

Figure 3.13: Relationship between effective compressibility and velocity when pressure effect is considered

Above figure, the graph shows the better response for lower velocity. Velocity from 2 to 10 shows a significant reduction of compressibility but after 10 it is negligible. However, for the lower reservoir pressure the response is high.

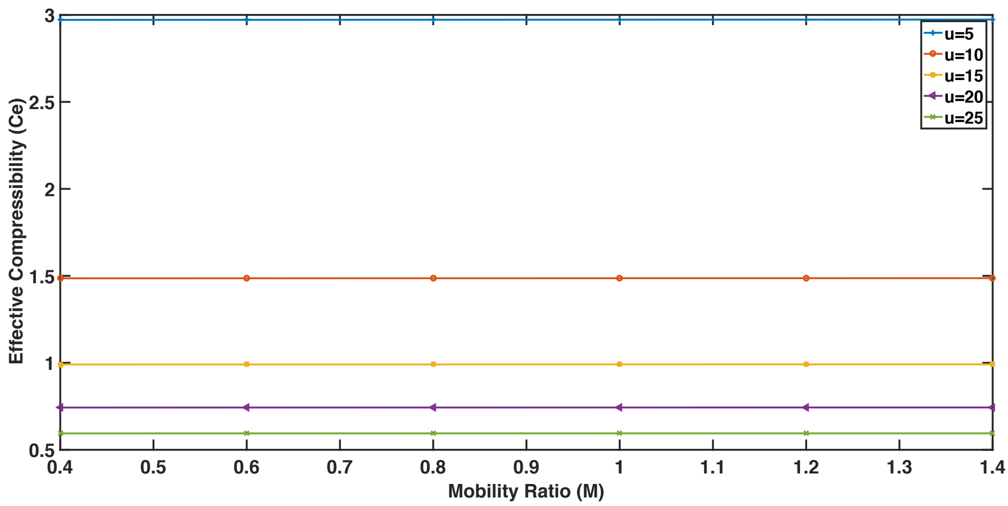

Figure 3.14: Effect of mobility ratio on effective compressibility for different fluid velocities

Figure 3.14 and figure 3.15 will be the decision-making figure. Figure 3.14 shows that, there is no change of compressibility with the change of mobility ratio. For different value of velocity, the result is same.

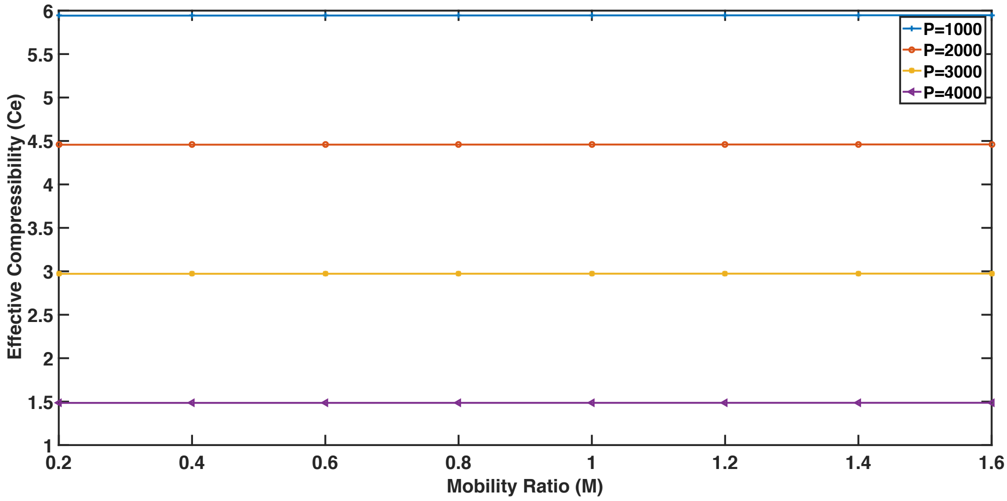

Figure 3.15: Relationship between effective compressibility and mobility ratio for different reservoir pressure

Mobility ratio is showing no effect also in the figure 3.15. Even for different reservoir pressure, the change of compressibility is almost zero.

Equation 3.25 and equation 3.34 have been derived to analyze the sensitivity of established material balance equation. The main purpose of this thesis work is to incorporate some valuable parameters into the established material balance equation. This chapter is the preliminary steps to proceed this work. The importance of incorporating these valuable parameters has been explained in this chapter through equation 3.25 and equation 3.34. To discuss the sensitivity, some established equation can be recollected. One very common material balance equation for oil reservoir is-

$$N_p\left[B_o + \left(R_p - R_s\right)B_g\right] = NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + \left(\frac{C_w S_{wc} + C_f}{1 - S_{wc}}\right)\Delta p\right] + \left(W_e - W_p\right)$$

Here, the compressibility term is $\left(\frac{C_w S_{wc}+C_f}{1-S_{wc}}\right)$.

Compressibility and other reservoir properties have a very significant relationship. For example, viscosity, permeability and velocity of the fluid in the reservoir have a substantial relationship with compressibility. If the value of any of these properties is changed, the compressibility is changed which leads a big change in the calculation of original hydrocarbon in place. To modify the established material balance equation, it is necessary to prove this fact. The modification will be done in the next two chapters. However, a trial has been made in this chapter to prove the importance of the modification.

Figure 3.1 to figure 3.13 have been generated based on the equation 3.25 and equation 3.34. Viscosity, mobility ratio, pressure and permeability parameters have been considered to show the graphical correlation with effective compressibility. Apparently, one can think that, it is not mandatory to show the graphical correlation as the equation itself express the relationship between parameters. But the reason to show the graphical correlation is to check which parameters are giving more responses. To explain it clearly, equation 3.25 can be recalled.

$$\boldsymbol{C_e = \frac{\left\{S_{oi}c_o + S_{wi}c_w + S_{gi}c_g\left(\frac{R_{soi}}{B_{oi}} + \frac{R_{swi}}{B_{wi}}\right)B_{gi} + Mc_w\right\}}{1 - S_{wi}}\left[\frac{k}{\mu L}(p_i - p)L\right]}$$

It is clear from the equation that if the viscosity of the fluid increases, the effective compressibility is decreased. If the permeability of the reservoir increases, the effective compressibility also increases. But which parameter has more response is not clear. If the permeability is increased from 1mD to 10 mD, there will have an increment with the value of effective compressibility. And if the viscosity is increased from 1 cP to 10 cP, there will have a decrement of the value of effective compressibility. The increment value in first case and the decrement value in second case are not same.

After the analyses of all above figures, it is possible to say that, pressure, viscosity, permeability and velocity has a significant role on material balance calculation whereas mobility ratio has very negligible effect on the reserve estimation. Therefore, an effort will be carried on in the next two chapters to incorporate these four parameters into the established material balance equation.

3.8 Conclusion

In this study a model has been developed to analyze the response of different reservoir properties on material balance calculation. Darcy's law and another velocity dependent relationship has been implemented to derive the equation. Reliable data from different literature has been utilized to verify the model. Five parameters have been considered to check the response with effective compressibility. Graphical presentation for each parameter has been generated by using the derived model. All the responses have been checked with respect to the effective compressibility. A third parameter has been considered to see the effect of that parameter on other two. Four parameters showed a very significant response though the sensitivity of the response is not same for all parameters. Viscosity gives the maximum response whereas the mobility ratio gives a negligible response. Mobility ratio has not showed any effect on effective compressibility and thus on material balance calculation. Therefore, mobility ratio will not be taken care during the modification of established material balance equation. Other four parameters will be incorporated to modify the traditional material balance equation.

3.9 References

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Chapter 4
Development of a Modified Material Balance Equation for Naturally Fractured Reservoir

4.1 Abstract

Reservoir engineers are no more fascinated to use the simplified Material Balance Equation (MBE) for naturally fractured reservoir. Because of the complexity of naturally fractured reservoir, reservoir engineers are trending to a modified version of the material balance equation for a good estimation of hydrocarbon. This study presents a new material balance equation for naturally fractured reservoir considering laminar fluid flow. The proposed model will be a reliable tool to estimate the initial hydrocarbon in place for fracture and matrix. A general material balance equation has been derived for the naturally fractured reservoir. By using the fundamentals of fluid flow, a velocity term has been incorporated to derived model. This velocity term defines the condition of flow such as laminar and turbulent. Velocity for the laminar flow has been used to validate the model. By using field data, fracture compressibility versus original oil in place plot has been generated. The plot shows some deviation with one established model where no velocity was considered. A big number of hydrocarbons is left because of a significant pressure drop in later stage of production. By using the material balance equation for low velocity fluid flow reservoir, the estimation of reserve can be optimized.

Keywords

Naturally Fractured Reservoir; Laminar Flow; Fractured System; Matrix System; Reserve Estimation

4.2 Introduction

Material balance method is nothing but an application of conservation of mass to the reservoir engineering. Schilthuis (1936) primarily presented a general MBE for the homogeneous reservoir. In hydrocarbon reservoir, to determine drive mechanism and estimate their performance Schilthuis' MBE was the only means until 1950. And later, several MBE has been offered for single porosity reservoir (Muskat 1949, Pirson 1958, Amyx *et al.,* 1960, Craft *et al.,* 1991, Dake 1994, Walsh 1995). A graphical representation of MBE as a straight line was recommended by Havlena and Odeh (1963). Likewise, Campbell (1978) offered a proposal to identify the new method of depletion mechanisms, e.g. gas cap or water drive. However, in the case of the complex reservoir, the scenario becomes completely different.

During the last few years, research on material balance has been conducted for the fractured reservoir to improve the reservoir analysis. However, all previous works are applicable to limited ranges of data. Porosity and permeability throughout the reservoir are assumed uniform in case of conventional MBE. As dual porosity system is generated for the naturally fractured reservoirs (NFR), the assumption is not valid. The compressibility of fractures is much higher than the matrix. In addition, the porosity of fracture and matrix changes when there is a change in pressure (Nelson, 1985).

In the case of storage capacity, fractured reservoirs have much more influence on production engineering. Three types of storage can be identified in this kind of reservoir. Matrix blocks, which is the main storage for hydrocarbon, is denoted by Type A. Fracture networks are included with the matrix in Type B storage. The storage capacity of fracture networks is Type C (Aguilera, 1995). In a reservoir of Type A, the matrix contains a significant portion of hydrocarbon whereas very small amount in fractures (McNaughton 1975).

The proposed equation is formulated for the initially undersatureate black oil reservoir by considering both porous media (matrix) and fracture network. Solution of Havlana and Odeh (1963) is applied to both fracture and matrix for calculating initial oil in place.

4.2.1 Model Assumptions

The derivation of the model follows some logical assumptions. These are:

1. The reservoir condition is isothermal
2. The reservoir has four components: a) stock tank oil, b) produced surface gas, c) produced water and d) naturally fractured rock
3. Four phases are available in the reservoir: oil, gas, water and solid (rock).
4. The stock tank oil does not have any dissolved gas or water.
5. Fracture and matrix of the reservoir are compressible.
6. The water production is negligible and there is no water encroachment.
7. No water or gas has been injected to the reservoir.
8. Porosity of fracture and matrix is almost same throughout the reservoir.

9. The reservoir has the uniform water saturation.
10. No horizontal or vertical pressure gradient is present in the reservoir.

4.3 General MBE for Fractured Reservoir

The general form MBE for fractured reservoir is different from the MBE for converntional reservoir. Conventional reservoir has only the hydrocarbon storage of matrix whereas the fractured reservoir has both the matrix and fracture as storage. That's why MBE derived for fractured reservoir calculate the material balacne for both matrix and fracture.

According to the assumed condition, the derivation of MBE for naturally fractured reservoir is made based on the idealistic model shown in figure 1.

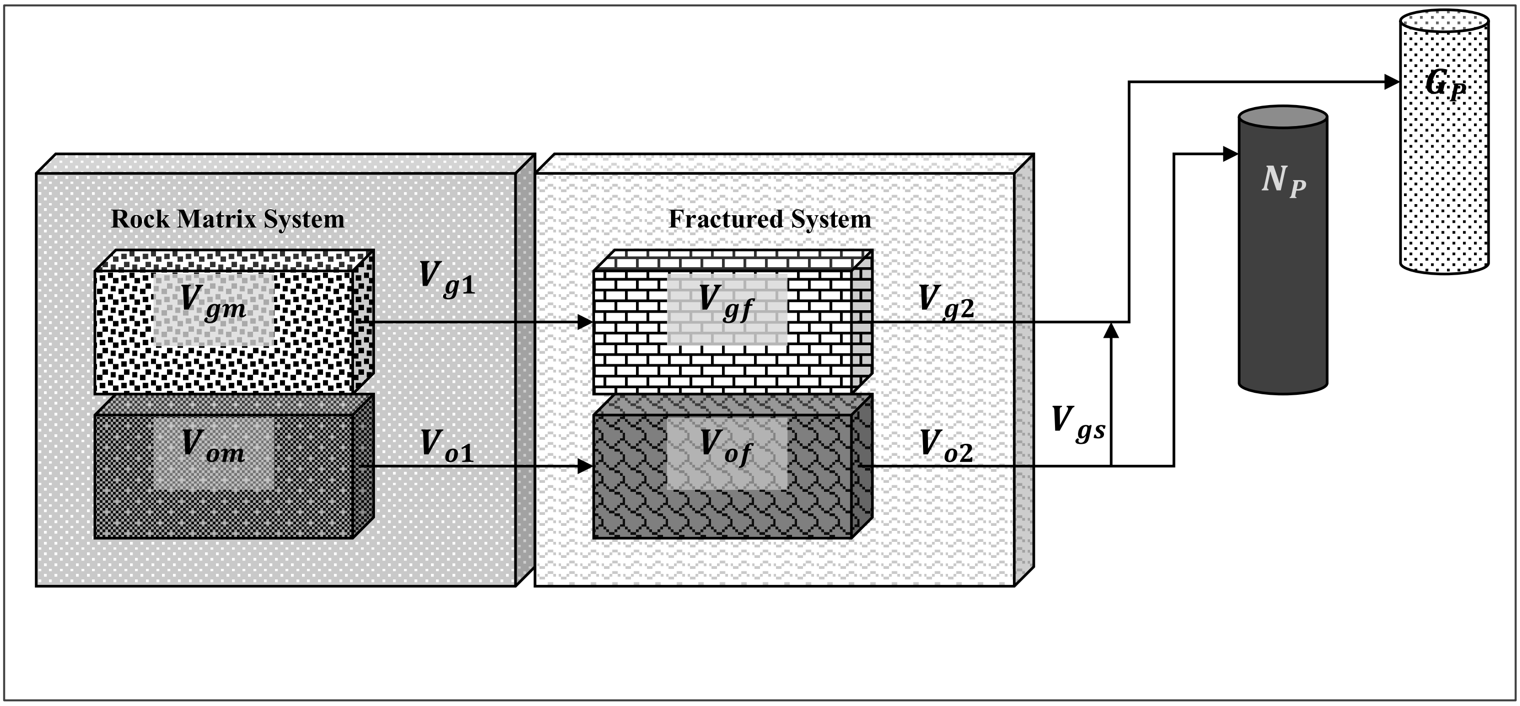

Figure 4.1: Volumetric material balance equation for naturally fractured reservoir

A volumetric material balance for the fractured reservoir shows

$$V_{ofi} + V_{gfi} = V_{of2} + V_{gf2} + V_{o1} + V_{g1} - V_{o2} - V_{g2} + \Delta V_{fw} - \Delta V_f + \left(W_e - W_p\right) \quad (4.1)$$

The initial oil in the fractured structure is

$$V_{ofi} = N_2 B_{oi} \quad (4.2)$$

The initial gas in the fractured structure is

$$V_{gfi} = 0 \tag{4.3}$$

as there is no free gas since an initially-undersaturated condition was assumed.

After a pressure drop, the volume of oil in the fracture system-

$$V_{of2} = N_2 B_o \tag{4.4}$$

After a pressure drop, the volume of free gas in the fracture system-

$$V_{gf2} = N_2 (R_{si} - R_s) B_g \tag{4.5}$$

where the original oil in place is calculated as

$$N_2 = \frac{V_b \phi_{fi} (1 - S_{wfi})}{B_{oi}} \tag{4.6}$$

The volume of oil that released by matrix is

$$V_{o1} = N_1 B_o + \Delta V_p + \Delta V_w - N_1 B_{oi} \tag{4.7}$$

where

$$N_1 = \frac{V_b \phi_{mi} (1 - S_{wi})}{B_{oi}} \tag{4.8}$$

$$\Delta V_p = V_b \phi_{mi} C_m \Delta p \tag{4.9}$$

$$\Delta V_w = V_b \phi_{mi} S_{wi} C_w \Delta p \tag{4.10}$$

Replacing equation (4.8) to (4.10), equation (4.7) becomes

$$V_{o1} = N_1 \left[B_o - B_{oi} + \left(\frac{S_{wi} C_w + C_m}{1 - S_{wi}} \right) \Delta p B_{oi} \right] \tag{4.11}$$

Because of the pressure reduction, the free gas will be evolved from the matrix and it is assumed that this free gas flows directly to the fracture. The volume of this free gas-

$$V_{g1} = N_1(R_{si} - R_s)B_g \tag{4.12}$$

The volume of the released oil from fracture is-

$$V_{o2} = N_pB_o \tag{4.13}$$

The volume of the produced gas from fracture is-

$$V_{g2} = N_p\left(R_p - R_s\right)B_g \tag{4.14}$$

The expanded pore volume of the fracture due to pressure drop is-

$$\Delta V_f = V_b\phi_f C_f\Delta p \tag{4.15}$$

Due to pressure drop in the fracture system, the net expansion of the connate water volume is-

$$\Delta V_{fw} = V_b\phi_f S_w C_w\Delta p \tag{4.16}$$

Recalling equation (4.1),

$$V_{ofi} + V_{gfi} = V_{of2} + V_{gf2} + V_{o1} + V_{g1} - V_{o2} - V_{g2} + \Delta V_{fw} - \Delta V_f + \left(W_e - W_p\right)$$

Replacing the previous equations into equation (4.1), the material balance equation for naturally fractured reservoir can be found. Which is-

$$N_2B_{oi} + 0 = N_2B_o + N_2(R_{si} - R_s)B_g + N_1\left[B_o - B_{oi} + \left(\frac{S_{wi}C_w + C_m}{1 - S_{wi}}\right)\Delta pB_{oi}\right] +$$
$$N_1(R_{si} - R_s)B_g - N_pB_o - N_p\left(R_p - R_s\right)B_g + V_b\phi_f S_w C_w\Delta p - V_b\phi_f C_f\Delta p + \left(W_e - W_p\right) \tag{4.17}$$

$$N_p\left[B_o + \left(R_p - R_s\right)B_g\right] = N_1\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wi} + C_m}{1 - S_{wi}}\right)\Delta pB_{oi}\right] +$$
$$N_2\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wfi} + C_m}{1 - S_{wfi}}\right)\Delta pB_{oi}\right] + V_b\phi_f\Delta p\left(S_wC_w - C_f\right) + \left(W_e - W_p\right) \tag{4.18}$$

where oil initially in place in the fracture by N_2 and oil initially in place in the matrix is denoted by N_1, N_p represents the cumulative whereas R_p represents the produced gas-oil ratio. Average matrix compressibility and average fracture compressibility are denoted by C_m and C_f respectively. The other notations are briefly described in the nomenclature section.

As it is assumed that there is no water encroachment as well as no water production, we can write as:

$W_e = 0$ (4.19)

$W_p = 0$ (4.20)

Hence, equation (4.18) becomes:

$$N_p\left[B_o + \left(R_p - R_s\right)B_g\right] = N_1\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_w S_{wi} + C_m}{1 - S_{wi}}\right)\Delta p B_{oi}\right] +$$
$$N_2\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_w S_{wfi} + C_m}{1 - S_{wfi}}\right)\Delta p B_{oi}\right] + V_b \phi_f \Delta p\left(S_w C_w - C_f\right) \quad (4.21)$$

It is recognized that there is a proportional relationship between pressure drop through a granular bed and fluid velocity at low flow rates, and which is square of the velocity at high flow rates. Osborne Reynolds (1900) first formulated this relationship which is as follows-

$\frac{\Delta p}{L} = au + b\rho u^2$ (4.22)

As it is assumed that, the flow is laminar, so velocity of the fluid is very tiny. Eventually the square of the value of velocity will be negligible. That's why the term $b\rho u^2$ can be considered as negligible. Mathematically,

$b\rho u^2 \rightarrow 0$

So, the equation (22) becomes

$\frac{\Delta p}{L} = au$ (4.23)

$\Rightarrow \Delta p = auL$ (4.24)

Here a is the coefficient. The value of **"a"** depends on the value of pressure difference, fluid velocity and length of the reservoir.

$So, a = \frac{(p_i - p)}{ul}$

When a fluid flow with a velocity of 1 in/s through a 1 in bed with 1 psi pressure difference, then $a = \frac{1psi}{in/_s} in.$

$a = 1\frac{psi}{in/_s} in$ is meant that, when a fluid flow in a 1 in porous path with a velocity of 1 in/s, the pressure difference will be 1psi.

For the simplicity, we can use $a = 1\frac{psi}{in/_s} in$ for the next derivation.

So, $\Delta p = uL$ (4.25)

Therefore, equation (4.21) becomes-

$$\boldsymbol{N_p\left[B_o + \left(R_p - R_s\right)B_g\right] = N_1\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}uL\right] + N_2\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}uL\right] + V_b\phi_f\left(S_wC_w - C_f\right)uL} \quad \textbf{(4.26)}$$

Equation (4.26) is the proposed material balance equation for the naturally fractured reservoir when the flow is considered as laminar.

Again, according to Darcy's law;

$$Q = -\frac{kA}{\mu L}(p_a - p_b) \quad (4.27)$$

$$\Rightarrow uA = -\frac{kA}{\mu L}(p_a - p_b)$$

$$\Rightarrow u = -\frac{k}{\mu L}(p_a - p_b) \quad (4.28)$$

The above equation is applicable for single phase (fluid) flow. The negative sign indicates that fluid flows from high pressure region to low pressure region. For the negative change of pressure (where $p_b > p_a$), the flow will follow the positive direction.

If p_i is the initial pressure of the reservoir, then equation (4.28) becomes;

$$u = -\frac{k}{\mu L}(p - p_i) \quad (4.29)$$

where p is the average reservoir pressure.

As most of the cases; $p_i > p$;

We can rewrite the equation (4.29) as:

$$u = \frac{k}{\mu L}(p_i - p) \quad (4.30)$$

Replacing equation (4.30) into equation (4.26), we get-

$$N_p\left[B_o+\left(R_p-R_s\right)B_g\right]=N_1\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}\frac{k}{\mu L}(p_i-p)L\right]+N_2\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}\frac{k}{\mu L}(p_i-p)L\right]+V_b\phi_f\left(S_wC_w-C_f\right)\frac{k}{\mu L}(p_i-p)L \qquad (4.31)$$

$$\boldsymbol{N_p\left[B_o+\left(R_p-R_s\right)B_g\right]=N_1\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}\frac{k}{\mu}(p_i-p)\right]+N_2\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}\frac{k}{\mu}(p_i-p)\right]+V_b\phi_f\left(S_wC_w-C_f\right)\frac{k}{\mu}(p_i-p)} \qquad \textbf{(4.32)}$$

Equation (4.32) is another proposed model for the naturally fractured reservoir when the laminar flow condition is considered.

4.4 Results and Discussions

Recalling equation (4.26),

$$\boldsymbol{N_p\left[B_o+\left(R_p-R_s\right)B_g\right]=N_1\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}uL\right]+N_2\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}uL\right]+V_b\phi_f\left(S_wC_w-C_f\right)uL}$$

$$\boldsymbol{\Rightarrow N_p\left[B_o+\left(R_p-R_s\right)B_g\right]-V_b\phi_f\left(S_wC_w-C_f\right)uL=N_1\left[B_o-B_{oi}+(R_{si}-R_s)B_g\left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}uL\right]+N_2\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}uL\right]} \qquad \textbf{(4.33)}$$

Now, one of the goal to derive the previous equation is to calculate the oil volume trapped in the matrix and fracture. To do it easily and avoid the complexity, let;

$$\boldsymbol{N_p\left[B_o+\left(R_p-R_s\right)B_g\right]-V_b\phi_f\left(S_wC_w-C_f\right)uL=F} \qquad \textbf{(4.34)}$$

$$\boldsymbol{\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}uL\right]=E_{o1}} \qquad \textbf{(4.35)}$$

$$\boldsymbol{\left[B_o-B_{oi}+(R_{si}-R_s)B_g+\left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}uL\right]=E_{o2}} \qquad \textbf{(4.36)}$$

Therefore, the equation (4.33) can be rewritten as:

$$\boldsymbol{F=N_1E_{o1}+N_2E_{o2}} \qquad (4.37)$$

Where $\boldsymbol{E_{o1}}$ the net expansion is term of the matrix system in oil phase and $\boldsymbol{E_{o2}}$ is the net expansion term of the fracture network in original oil phase.

Equation (4.37) can be rearranged according to the established MBE model of Havlena and Odeh (1963).

$$\frac{F}{E_{o1}} = N_1 + N_2 \frac{E_{o2}}{E_{o1}} \quad (4.38)$$

Table 4.1: PVT data used for model validation

Pressure (P) Psi	Produced oil (N_p), MM STB	Produced gas oil ratio(R_p), scf/STB	Oil Viscosity (μ_o), cP	Oil Formation Volume Factor(B_o), rb/STB	Solution gas oil ratio (R_{so}), scf/STB	Gas Viscosity (μ_g), cP	Oil Formation Volume Factor(B_g), rb/SCF
3150	3.295	1050	1.1252	1.2353	477	0.01582	0.001991
3000	5.903	1060	0.9241	1.2222	450	0.01726	0.001467
2850	8.852	1160	0.7866	1.2122	425	0.01890	0.001171
2700	11.503	1235	0.8270	1.2022	401	0.02066	0.000987
2550	14.513	1265	0.8763	1.1922	375	0.02247	0.000865
2400	17.730	1300	0.9333	1.1822	352	0.02427	0.000779

Table 4.2: Rocks and fluid properties used in the synthetic example (Penuela, 2001)

Reservoir properties		**Fluid Properties**	
Productive area	826 acres	Oil gravity	30^0 API
Reservoir thickness	50 ft	Gas specific gravity	0.7 (air=1)
Matrix permeability	10 md	Oil density	54.64 lb/ft^3
Matrix compressibility	$3 * 10^{-6} psi^{-1}$	Water compressibility	$3 * 10^{-6} psi^{-1}$
Fracture permeability	300 md	Initial reservoir pressure	3810 psia
Fracture compressibility	$3 * 10^{-5} psi^{-1}$	Bubble point pressure	2000 psia
Initial water saturation in the water system	20%		
Initial water saturation in the fractured system	5%		
Fracture porosity	$3 * 10^{-5}$		
Matrix porosity	0.2		

To calculate the value of F, bulk volume needs to be calculated. Then using above production data and reservoir rocks and fluid properties, $\boldsymbol{F, E_{o1}}$ ***and*** $\boldsymbol{E_{o2}}$ can be calculated.

Bulk volume of productive area,

$$\boldsymbol{V_b = 826\, acre * 50\, ft}$$

$$= \mathbf{3.598 * 10^7 ft^2 * 50\ ft}$$
$$= \mathbf{1.8 * 10^9\ ft^3}$$

Now, to validate the model, the matrix and fracture reserve needed to be calculated separately. Velocity is the important parameter of the equation (4.34), (4.35) and (4.36). It is assumed that, the flow is laminar. So, the Reynolds number equation should be taken care of in this regard. The established equation for Reynolds number is:

$$R_e = \frac{\rho u L}{\mu}$$

$$\Rightarrow u = \frac{\mu R_e}{\rho L} \tag{4.39}$$

There is also a range for Reynolds number for different types of flow. The laminar flow is taken into account when the Reynolds number is less than 2100 whereas the turbulent flow is considered when the Reynolds Number is more than 4000. Transitional flow exists between these values. Therefore, calculating fluid velocity for different Reynolds Number will be reasonable instead of using mean fluid velocity. To avoid the complexities, the average fluid density and viscosity can be considered.

Table 4.3: Calculation of fluid velocity for different Reynolds Numbers

Reynolds Number,	Fluid Viscosity (μ), cP	Fluid Density (ρ), lb/ft3	Reservoir thickness (L), ft	Velocity (u), ft/s
100	0.912	54.64	50	0.0334
200	0.912	54.64	50	0.0668
300	0.912	54.64	50	0.1001
400	0.912	54.64	50	0.1335
500	0.912	54.64	50	0.1669
600	0.912	54.64	50	0.2003
700	0.912	54.64	50	0.2337
800	0.912	54.64	50	0.2671
900	0.912	54.64	50	0.3004
1000	0.912	54.64	50	0.3338
1100	0.912	54.64	50	0.3672
1200	0.912	54.64	50	0.4006
1300	0.912	54.64	50	0.4339
1400	0.912	54.64	50	0.4673
1500	0.912	54.64	50	0.5007
1600	0.912	54.64	50	0.5341
1700	0.912	54.64	50	0.5675
1800	0.912	54.64	50	0.6009
1900	0.912	54.64	50	0.6343
2000	0.912	54.64	50	0.6676

Now, using table 4.1, table 4.2, table 4.3 and other relevant data on Matlab software, the value of all the important parameters can be calculated.

Table 4.4: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=100 and u=0.033382138 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
10.4793	9.27e-06	6.837e-06	1.129e+06	0.7368
15.1469	0.0265	0.0265	571.187	0.9999
20.9991	0.0378	0.0378	555.511	0.9999
25.9476	0.0419	0.0419	618.959	0.9999
31.1251	0.0451	0.0451	689.535	0.9999
36.7037	0.0443	0.0443	828.821	0.9999

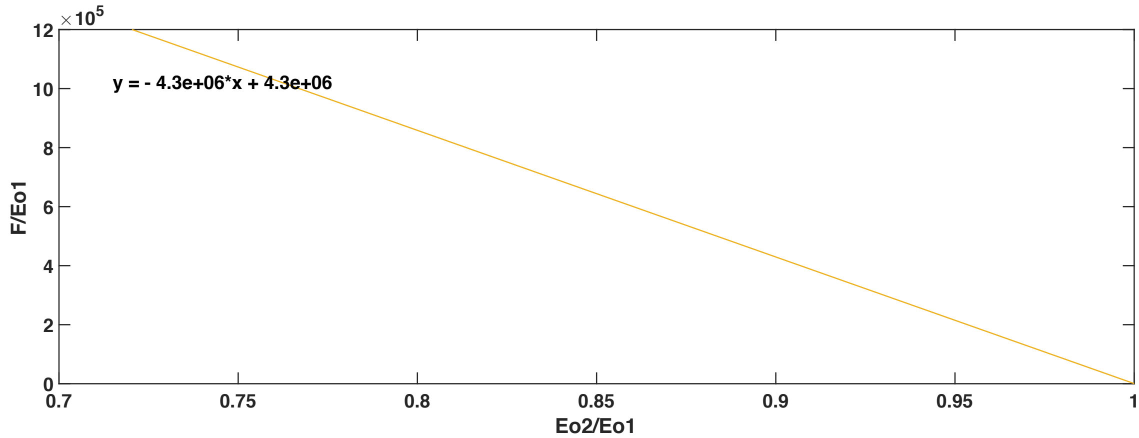

Figure 4.2: Reserve estimation plot for NFR (for Re=100 and u=0.033382138 ft/s)

The above straight line equation of the plot can be compared with equation (4.38). Recalling equation and produced straight line equation,

$$\frac{F}{E_{o1}} = N_1 + N_2 \frac{E_{o2}}{E_{o1}}$$

$$y = -4.3e6 * x + 4.3e6 \qquad (4.40)$$

From above two equation, it is clear that,

$N_1 = 4.3e6$ and $N_2 = -4.3e6$

In equation (4.40), the value -4.3e6 is the slope of the straight line where the negative sign is indicating that, with the increase of the value of X-axis, the value of Y-axis is increasing.

If the negative sign is ignored, then

$N_1 = 4.3e6$ and $N_2 = 4.3e6$

As the unit of produced oil is MMSTB, so;

$\boldsymbol{N_1 = 4.3e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 4.3e6\ MMSTB}$

Therefore, the original oil in matrix is 4.3e6 MMSTB and the original oil in fracture is also 4.3e6 MMSTB.

Again, for Re = 200 and u= 0.066764275 ft/s, table (4.4) can be recalculated.

Table 4.5: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=200 and u=0.066764275 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
13.1291	1.8557e-05	1.3673e-05	707517.4579	0.7369
17.7968	0.0265	0.0265	670.8802	0.9999
23.6489	0.0378	0.0378	625.4582	0.9999
28.5974	0.0419	0.0419	682.0191	0.9998
33.7749	0.0451	0.0451	748.0854	0.9999

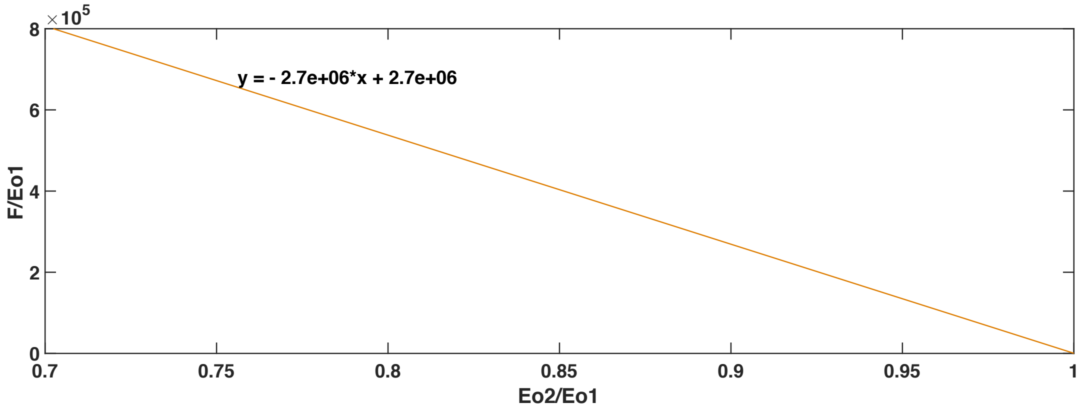

Figure 4.3: Reserve estimation plot for NFR (for Re=200 and u=0.066764275 ft/s)

According to the previous discussion, the original oil in matrix is 2.7e6 MMSTB and the original oil in fracture is also 2.7e6 MMSTB.

Table 4.6: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=400 and u= 0.133528551 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
18.4289	3.7113e-05	2.7346e-05	496558.0474	0.7368
23.0966	0.0265	0.0265	870.0543	0.9996
28.9487	0.0378	0.0378	765.2486	0.9997
33.8972	0.0419	0.0419	808.0549	0.9997
39.0747	0.0451	0.0451	865.1145	0.9997

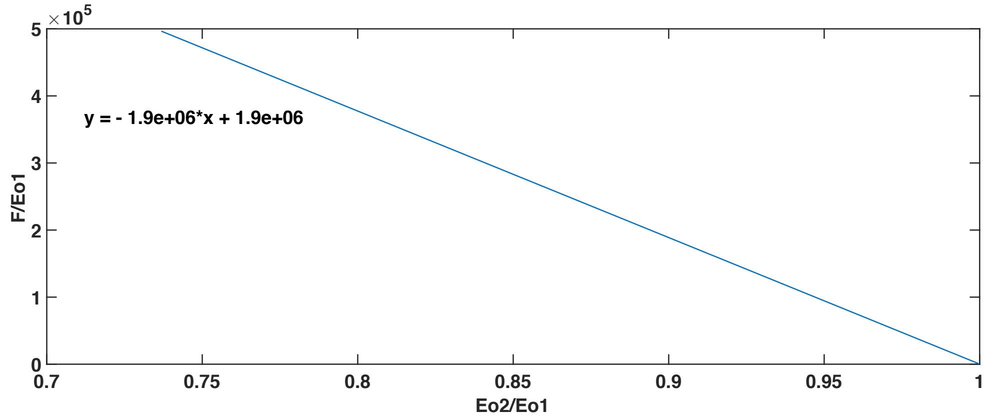

Figure 4.4: Reserve estimation plot for NFR (for Re=400 and u= 0.133528551 ft/s)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.9e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.9e6\ MMSTB}$

Table 4.7: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=600 and u= 0.200292826 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
23.7286	5.5669e-05	4.1019e-05	426238.2452	0.7368
28.3963	0.0265	0.0265	1068.9501	0.9994
34.2484	0.0378	0.0378	904.9017	0.9996
39.1969	0.0419	0.0419	933.9794	0.9996
44.3745	0.0451	0.0451	982.0475	0.9997

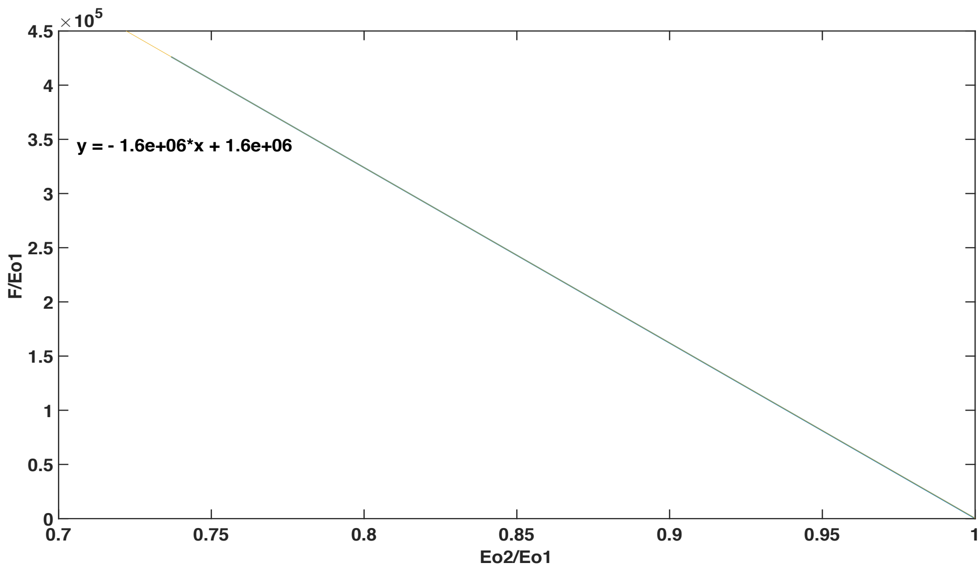

Figure 4.5: Reserve estimation plot for NFR (for Re=600 and u= 0.200292826 ft/s)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.6e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.6e6\ MMSTB}$

Table 4.8: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=1000 and u= 0.333821376 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
34.3281	9.2783e-05	6.8366e-05	369982.4033	0.736846
38.9958	0.0266	0.0265	1465.9095	0.9991
44.8479	0.0379	0.0378	1183.7977	0.9994
49.7964	0.0420	0.0419	1185.4943	0.9994
54.9739	0.0452	0.0452	1215.6256	0.9995

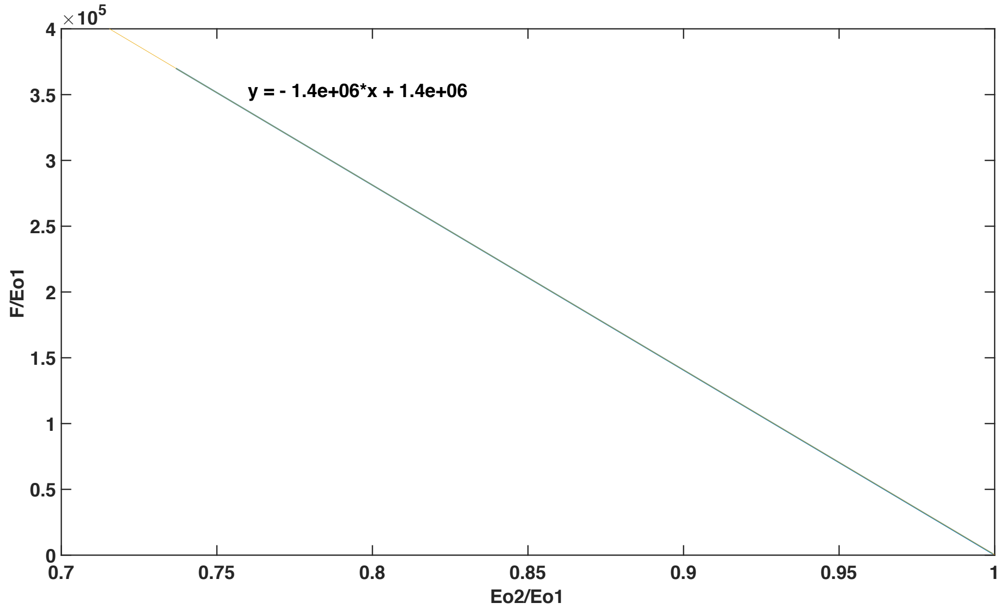

Figure 4.6: Reserve estimation plot for NFR (for Re=1000 and u= 0.333821376 ft/s)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.4e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.4e6\ MMSTB}$

Table 4.9: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=1400 and u= 0.467349927 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
44.9276	0.00012989	9.5713e-05	345872.7565	0.73686
49.5953	0.02663889	0.02660	1861.7626	0.9988
55.4474	0.03792189	0.03789	1462.1477	0.9991
60.3959	0.04204189	0.04200	1436.5653	0.9992
65.5734	0.04525989	0.04523	1448.8206	0.9993

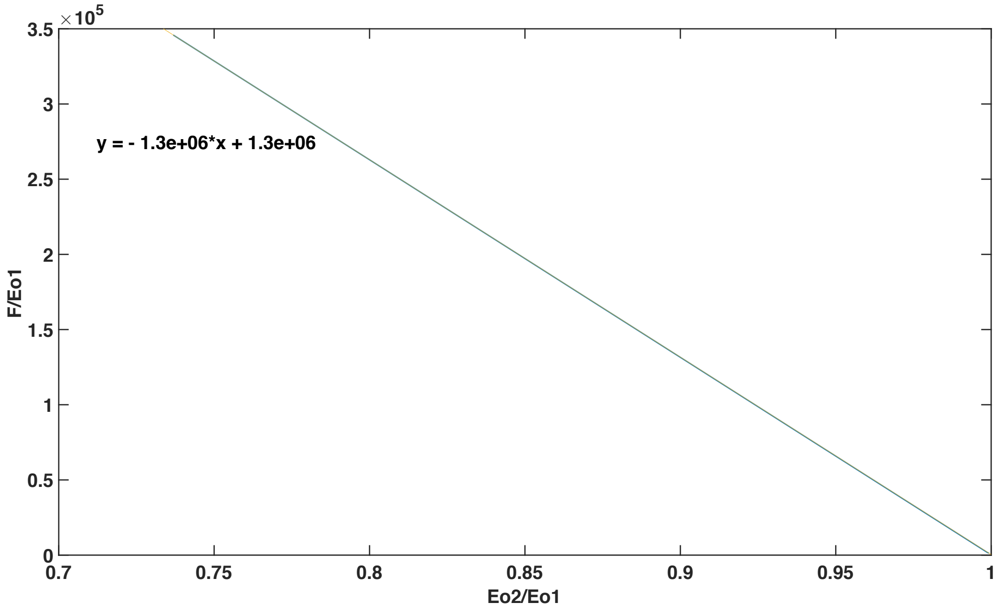

Figure 4.7: Reserve estimation plot for NFR (for Re=1400 and u= 0.467349927 ft/s)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.3e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.3e6\ MMSTB}$

Table 4.10: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=1700 and u= 0.56749634 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
52.8773	0.00015773	0.0001162231	335236.1478	0.7368
57.5449	0.02666673	0.026625223	2157.9294	0.9984
63.3970	0.03794973	0.037908223	1670.5529	0.9989
68.3456	0.04206973	0.042028223	1624.5778	0.9990
73.5231	0.04528773	0.045246223	1623.4659	0.9990

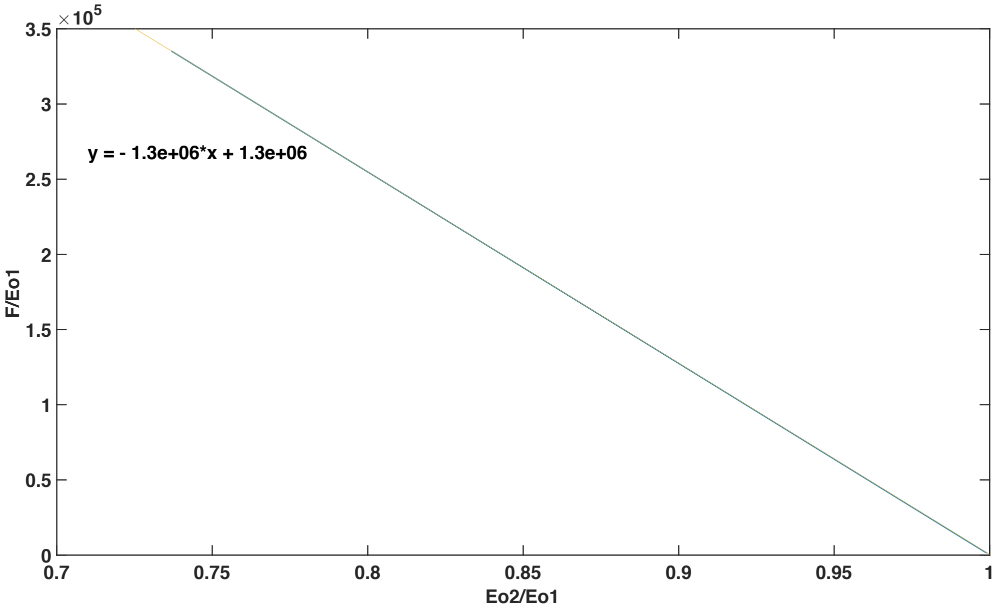

Figure 4.8: Reserve estimation plot for NFR (for Re=1700 and u= 0.56749634 ft/s)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.3e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.3e6\ MMSTB}$

Table 4.11: Calculation of fluid withdrawal and expansion parameter to generate the graph (for Re=2000 and u= 0.667642753 ft/s)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
60.8269	0.00018556	0.0001367330	327790.5216	0.7368
65.4946	0.02669456	0.026645733	2453.4786	0.9982
71.3467	0.03797756	0.037928733	1878.6528	0.9987
76.2952	0.04209756	0.042048733	1812.3417	0.9988
81.4727	0.04531556	0.045266733	1797.8969	0.9989

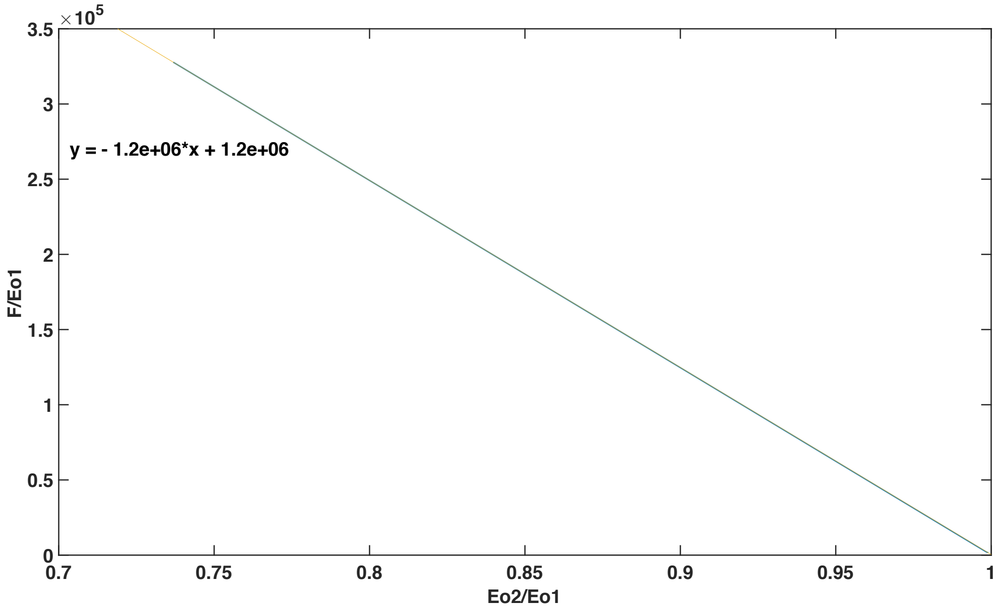

Figure 4.9: Reserve estimation plot for NFR (for Re=2000 and u= 0.667642753 ft/s)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.2e6\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.2e6\ MMSTB}$

Now a graph can be generated to see how the reserve is changing with the change of velocity.

Table 4.12: Estimated reserve for different velocities

Velocity (ft/s)	Total reserve (MMSTB)
0.0334	8.60E+06
0.0668	5.40E+06
0.1335	3.80E+06
0.2003	3.20E+06
0.3338	2.80E+06
0.4673	2.60E+06
0.5675	2.60E+06
0.6676	2.40E+06

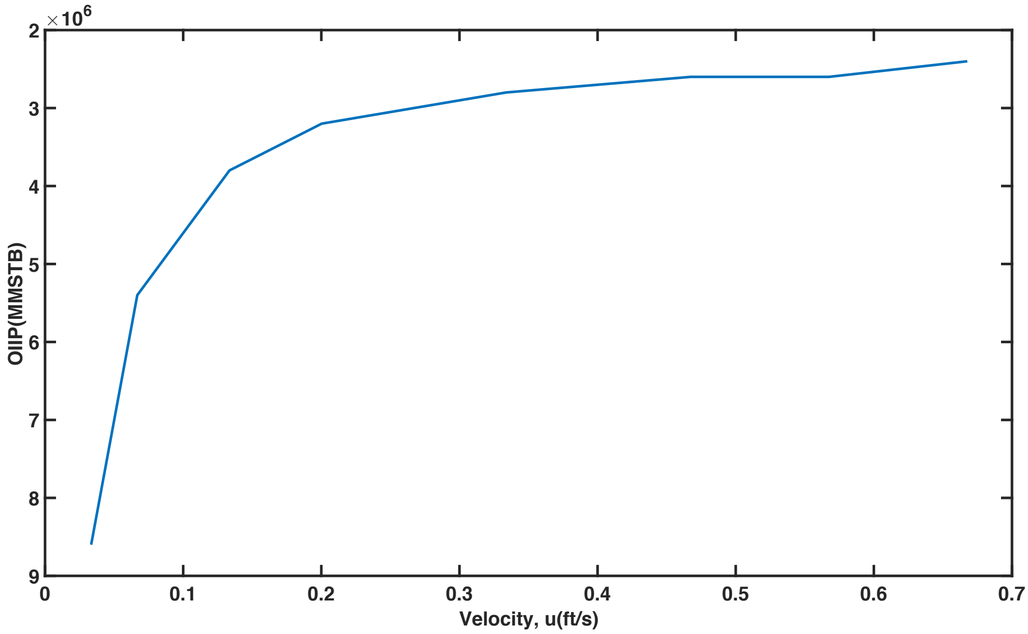

Figure 4.10: Effect of increasing fluid velocity on Original Oil in Place

The mentioned objective of this chapter is to develop a modified material balance equation by which hydrocarbon reserve can be estimated when velocity term is considered. The reserve has already been estimated. Now, it is crucial to see how the velocity term affecting on the estimated reserve. Table 4.4 to table 4.11 has been generated to make the plot. Figure 4.2 to figure 4.9 has been generated by using table 4.4 to table 4.11. The estimated reserve from figure 4.2 to figure 4.9 has been summarized in table 4.12. The plot in figure 4.10 has been produced by using table 4.12.

Now, by explaining figure 4.10, the effect of velocity can be figured out. The figure shows that, when the fluid velocity increases, the OIIP decreases which means because of the higher velocity of fluid, accumulation of hydrocarbon is being disturbed. Therefore, incorporating velocity parameter in the material balance equation, the estimated reserve can be optimized. If no velocity effect is considered, the overestimation might hamper on the proper economics of the production.

Now, recalling equation 4.32:

$$\boldsymbol{N_p\left[B_o + \left(R_p - R_s\right)B_g\right] = N_1\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}\frac{k}{\mu}(p_i - p)\right] + N_2\left[B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}\frac{k}{\mu}(p_i - p)\right] + V_b\phi_f\left(S_wC_w - C_f\right)\frac{k}{\mu}(p_i - p)}$$

Now, one of the goal to derive the previous equation is to calculate the oil volume trapped in the matrix and fracture. For equation 4.32, the reserve calculated when fluid velocity is considered. This time it is being calculated when permeability, viscosity and pressure terms are incorporated. To do it easily and avoid the complexity, let;

$$\boldsymbol{N_p\left[B_o + \left(R_p - R_s\right)B_g\right] - V_b\phi_f\left(S_wC_w - C_f\right)\frac{k}{\mu}(p_i - p) = F} \tag{4.41}$$

$$\boldsymbol{B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wi}+C_m}{1-S_{wi}}\right)B_{oi}\frac{k}{\mu}(p_i - p) = E_{O1}} \tag{4.42}$$

$$\boldsymbol{B_o - B_{oi} + (R_{si} - R_s)B_g + \left(\frac{C_wS_{wfi}+C_m}{1-S_{wfi}}\right)B_{oi}\frac{k}{\mu}(p_i - p) = E_{O2}} \tag{4.43}$$

Therefore, the equation (4.32) can be rewritten as:

$$\boldsymbol{F = N_1E_{o1} + N_2E_{o2}} \tag{4.44}$$

Where $\boldsymbol{E_{o1}}$the net expansion is term of the matrix system in oil phase and $\boldsymbol{E_{o2}}$ is the net expansion term of the fracture network in original oil phase.

Equation (4.41) can be rearranged according to the established MBE model of Havlena and Odeh (1963).

$$\boldsymbol{\frac{F}{E_{o1}} = N_1 + N_2\frac{E_{o2}}{E_{o1}}} \tag{4.45}$$

Now, using table 4.1, table 4.2 and other relevant data on Matlab software, the value of all the important parameters can be calculated.

Table 4.13: Calculation of fluid withdrawal and expansion parameter to generate the graph (for p=2400 psi)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
7.82939	5.00E-05	4.00E-05	158479.379	0.73684
10.76867	0.01361	0.01359	791.28105	0.99884
20.27502	0.05325	0.05324	380.71815	0.99965
22.12729	0.03271	0.03269	676.5261	0.99946
32.4277	0.07641	0.07639	424.41617	0.99978
34.05387	0.04433	0.04432	768.11111	0.99965

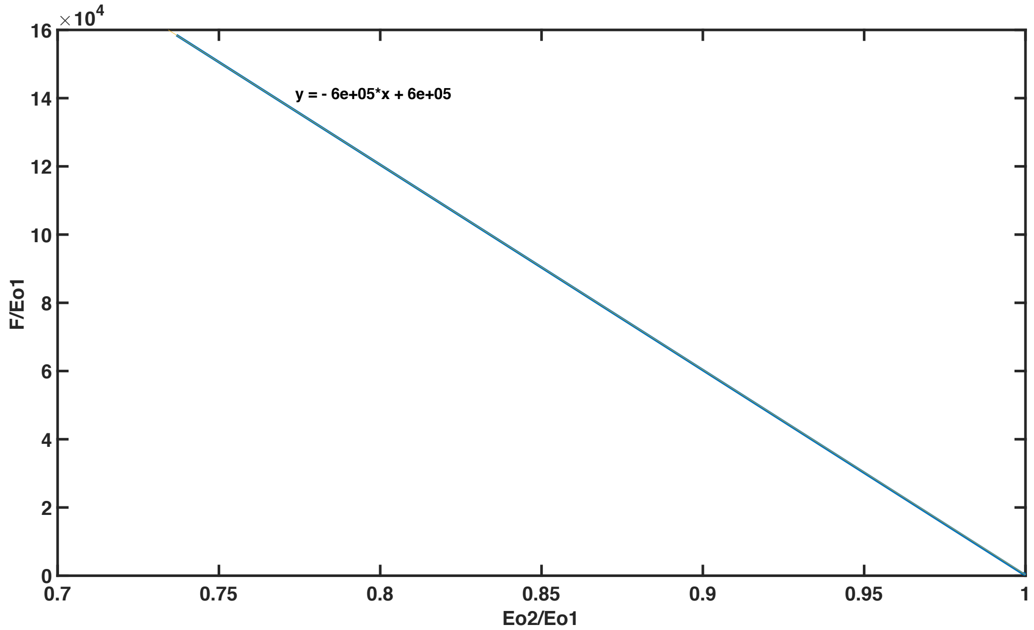

Figure 4.11: Reserve estimation plot for NFR (for p=2400 psi)

From the straight line of above figure,

$N_1 = 6e5\ MMSTB$ **and** $N_2 = 6e5\ MMSTB$

Table 4.14: Calculation of fluid withdrawal and expansion parameter to generate the graph (for p=2550 psi)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
2124.25314	5.00E-05	4.00E-05	42998275.4	0.73684
2587.7625	0.01361	0.01359	190148.665	0.99884
3047.73497	0.05325	0.05324	57229.4413	1.00E+00

2901.69198	0.03271	0.03269	88717.1774	0.99946
2749.99018	0.07641	0.07639	35992.0752	0.99978
2585.645	0.04433	0.04432	58321.2042	0.99965

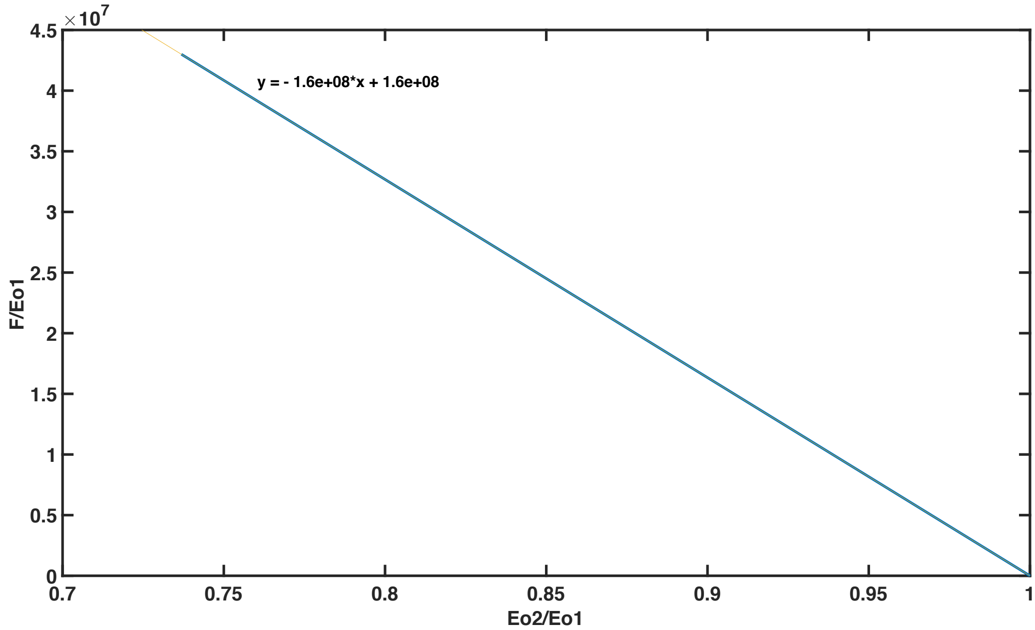

Figure 4.12: Reserve estimation plot for NFR (for p=2550 psi)

From the straight line of above figure,

$\boldsymbol{N_1 = 1.6e8\ MMSTB}$ **and** $\boldsymbol{N_2 = 1.6e8\ MMSTB}$

Table 4.15: Calculation of fluid withdrawal and expansion parameter to generate the graph (for p=2700 psi)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
4240.67688	5.00E-05	4.00E-05	85838071.4	0.73684
5164.75633	0.01361	0.01359	379506.048	0.99884
6075.19493	0.05325	0.05324	114078.164	0.99965
5781.25667	0.03271	0.03269	176757.829	0.99946
5467.55266	0.07641	0.07639	71559.7343	0.99978
5137.23613	0.04433	0.04432	115874.297	0.99965

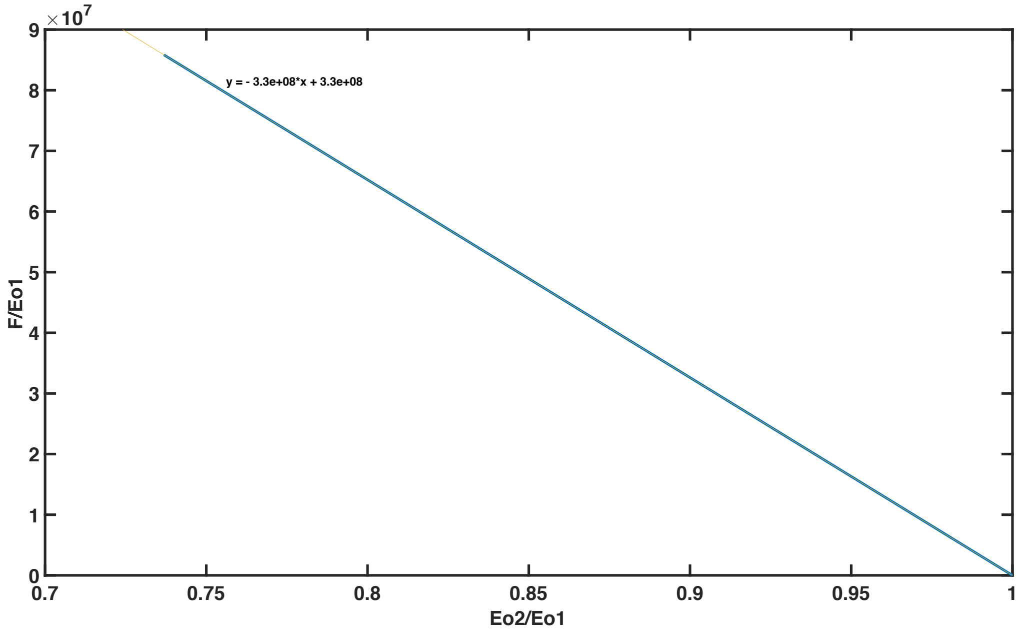

Figure 4.13: Reserve estimation plot for NFR (for p=2700 Psi)

From the straight line of above figure,

$\boldsymbol{N_1 = 3.3e8\ MMSTB}$ **and** $\boldsymbol{N_2 = 3.3e8\ MMSTB}$

Table 4.16: Calculation of fluid withdrawal and expansion parameter to generate the graph (for p=2850 psi)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
6357.10063	5.00E-05	4.00E-05	128677867	0.73684
7741.75016	0.01361	0.01359	568863.432	0.99884
9102.65488	0.05325	0.05324	170926.888	0.99965
8660.82136	0.03271	0.03269	264798.48	0.99946
8185.11514	0.07641	0.07639	107127.393	0.99978
7688.82725	0.04433	0.04432	173427.39	0.99965

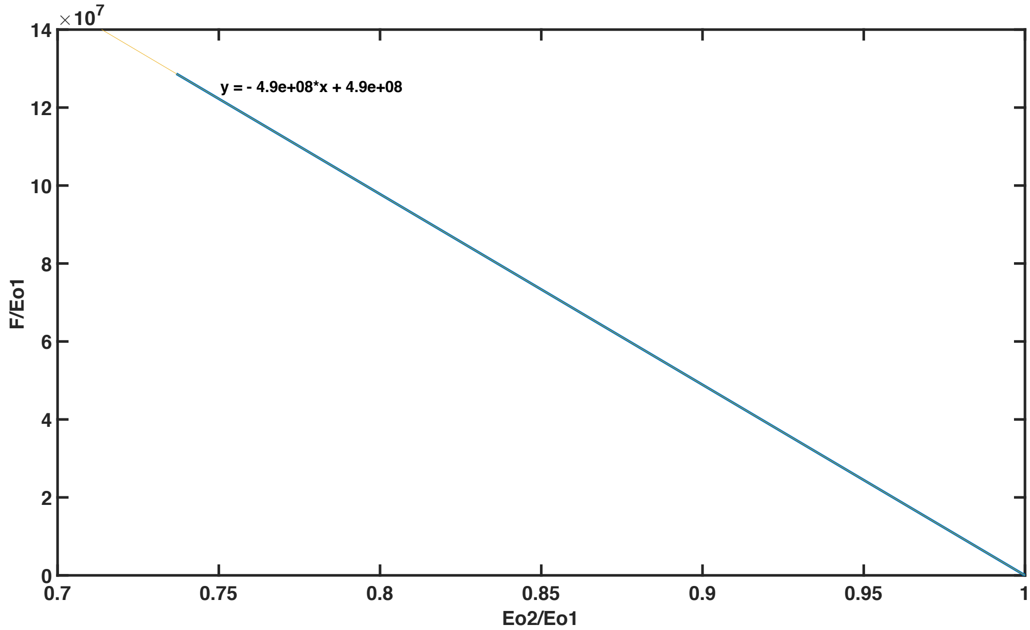

Figure 4.14: Reserve estimation plot for NFR (for p=2850 psi)

From the straight line of above figure,

$\boldsymbol{N_1 = 4.9e8\ MMSTB}$ **and** $\boldsymbol{N_2 = 4.9e8\ MMSTB}$

Table 4.17: Calculation of fluid withdrawal and expansion parameter to generate the graph (for p=3000 psi)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
8473.52438	5.00E-05	4.00E-05	171517663	0.73684
10318.744	0.01361	0.01359	758220.815	0.99884
12130.1148	0.05325	0.05324	227775.611	0.99965
11540.3861	0.03271	0.03269	352839.131	0.99946
10902.6776	0.07641	0.07639	142695.052	0.99978
10240.4184	0.04433	0.04432	230980.483	0.99965

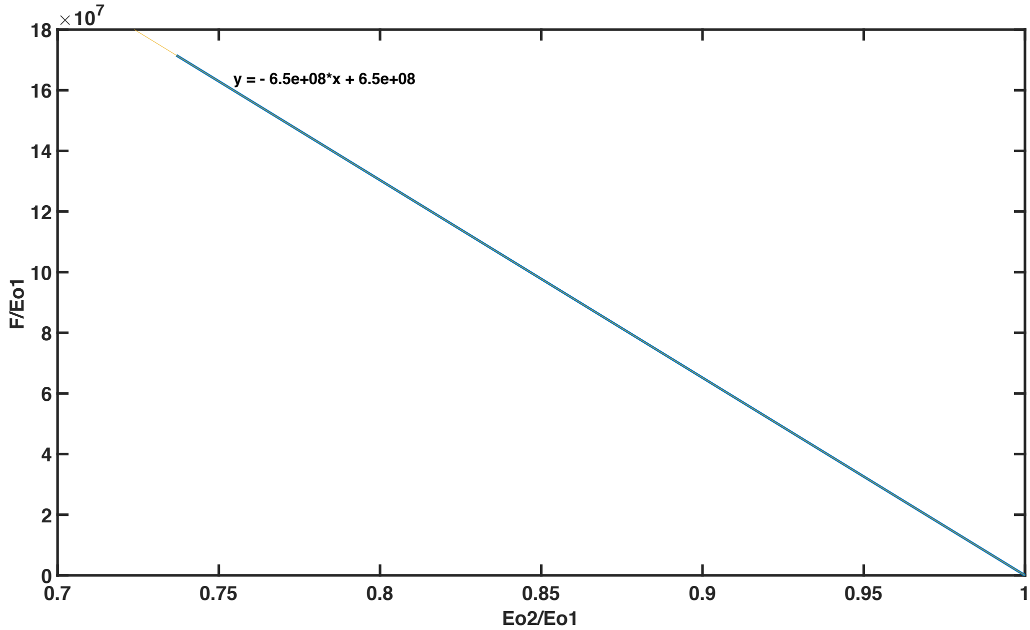

Figure 4.15: Reserve estimation plot for NFR (for p=3000 psi)

From the straight line of above figure,

$N_1 = 6.5e8\ MMSTB$ **and** $N_2 = 6.5e8\ MMSTB$

Table 4.18: Calculation of fluid withdrawal and expansion parameter to generate the graph (for p=3150 psi)

F	Eo1	Eo2	F/Eo1	Eo2/Eo1
10589.9481	5.00E-05	4.00E-05	214357459	0.73684
12895.7378	0.01361	0.01359	947578.199	0.99884
15157.5748	0.05325	0.05324	284624.334	0.99965
14419.9507	0.03271	0.03269	440879.783	0.99946
13620.2401	0.07641	0.07639	178262.711	0.99978
12792.0095	0.04433	0.04432	288533.576	0.99965

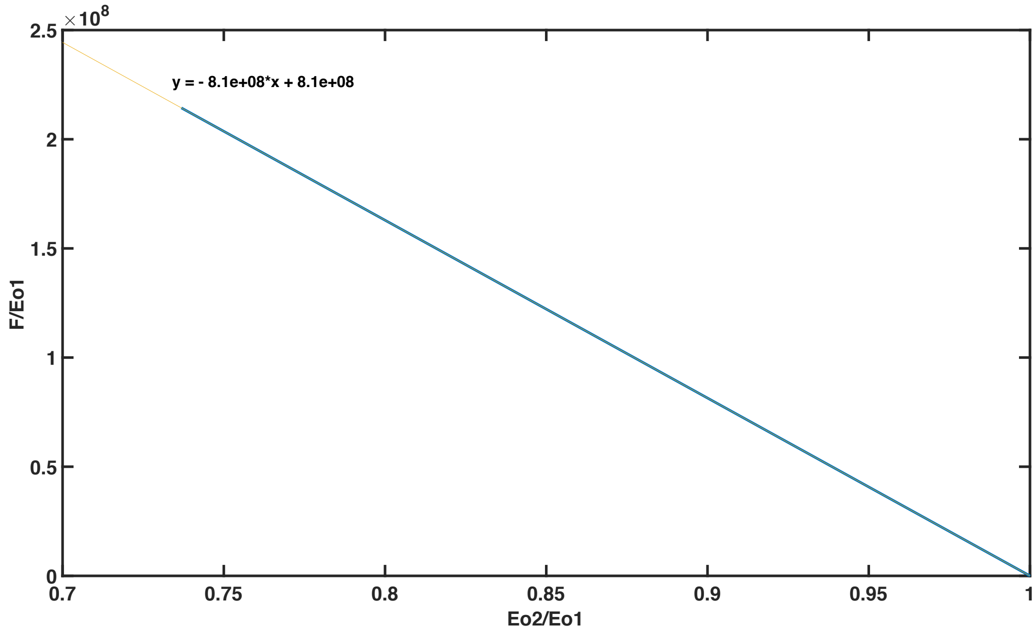

Figure 4.16: Reserve estimation plot for NFR (for p=3150 psi)

From the straight line of above figure,

$\boldsymbol{N_1 = 8.1e8\ MMSTB}$ **and** $\boldsymbol{N_2 = 8.1e8\ MMSTB}$

Now a graph can be generated to see how the reserve is changing with the change of pressure.

Table 4.17: Estimated reserve for different pressures

Pressure (Psi)	Total reserve (N1+N2), MMSTB
3150	16.2E+08
3000	13E+08
2850	9.8E+08
2700	6.6E+08
2550	3.2E+08
2400	12E+05

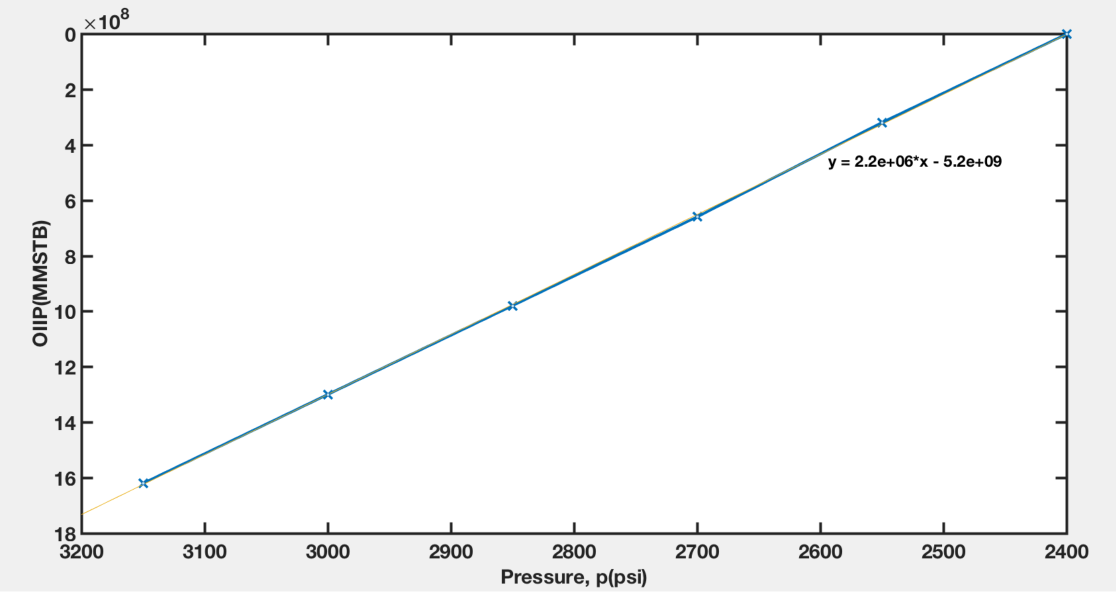

Figure 4.17: Effect of increasing pressure on Original Oil in Place

The mentioned objective of this chapter is to develop a modified material balance equation by which hydrocarbon reserve can be estimated when Darcy flow is considered. The reserve has already been estimated. Now, it is crucial to see how pressure term affecting on the estimated reserve. Table 4.13 to table 4.18 has been generated to make the plot. Figure 4.11 to figure 4.16 has been generated by using table 4.13 to table 4.18. The estimated reserve from figure 4.11 to figure 4.16 has been summarized in table 4.17. The plot in figure 4.17 has been produced by using table 4.17.

Now, by explaining figure 4.17, the effect of pressure can be figured out. The figure shows that, when the reservoir pressure decreases with the production, the OIIP decreases which means because of continuous production, the remaining hydrocarbon is going to decrease. This is the common trend for all other established material balance equation. So, in one sense it can be stated that, the model is showing the normal behaviour which is expected. The figure is also showing that, though 2400 psi is a sufficient pressure to lift the hydrocarbon up but maybe within this pressure the reservoir has already depleted. That's why at 2400 psi, the OIIP is 0. Finally, using this pressure and OIIP relationship, a straight-line equation can be proposed to calculate the reserve for any pressure of low-pressured reservoir. Which is:

$$\boldsymbol{OIIP = 2.2e6 * P - 5.2e9}$$

The above equation provides the original oil in place for any reservoir pressure. But this model is applicable only when the reservoir fluid flow is laminar and the reservoir pressure is under 3500 psi. More studies and experiments are needed to validate this equation.

4.5 Model Validation

In the previous section, the reserve for matrix and fracture has been calculated separately. In almost all cases, the reserve of matrix and fracture is same. Someone might think that the calculation has some error and that's why the reserve is same for all cases. To clarify the issue, the result can be compared to previous research. Penuela (2001) derived some similar model and calculated the reserve for both matrix and fracture. To test the accuracy of his proposed MBE, three cases were designed. In first case, he assumed that the oil in fracture media is twice the matrix. Secondly, it was assumed that the initial oil in both systems is equal. In third cases, he assumed that, the OIIP in the matrix system is twice the oil in fracture media.

Because of the deficiency of field data, the reserve calculated in this study is not same with Penuela (2001) but the reserve of matrix and fracture support the second case of Penuela (2001). Table 4.18 shows the summary of the reserve of Penuela (2001).

Table 4.18: Original Oil in place calculation (Penuela, 2001)

MMSTB	Simulator		MBE for NFR	
	N1	N2	N1	N2
Case 1	2.38	4.75	2.49	4.60
Case 2	4.75	4.75	4.79	4.54
Case 3	9.50	4.75	9.80	4.58

4.6 Conclusion

From the analyses described above, the following conclusions are offered:

1. Two modified material balance equation has been derived. In the first model, velocity term has been incorporated and in the second one, Darcy's law has been applied. For both cases, the result calculated for matrix and fracture.
2. The literature and simulation indicate the same reserve for matrix and fracture in this study. But the amount of total reserve depends on the velocity of the fluid and reservoir pressure. Within the laminar flow range, when the fluid velocity increases, the original oil in place decreases. When the reservoir pressure increases, the OIIP decreases. That means both model shows the inverse relationship between respective parameters.

3. A straight-line model has been proposed to calculate the oil reserve in any pressure of the reservoir. When the fluid flow is laminar and the pressure is under 3500 psi, the model could be applied properly but more research needed to validate it completely
4. Finally, because of the lacking of proper field data, the result might have a small percentage of error which can be easily minimised with accurate field data.

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Chapter 5

Development a Modified Material Balance Equation for Fractured Gas Reservoir considering Water Influx

5.1 Abstract

Material Balance Equation for fractured gas reservoir can be a widespread model if the factor of water influx is considered. However, industries are also concentrating on the development and optimization of the methods which focus the abnormal behavior of the reservoir. Because of the low permeability, artificial or natural fractures are needed to produce gas from shale gas reservoir. This study presents a modified Material Balance Equation (MBE) which considers the flow conditions, the water influx and the hydrocarbon storage of fractures. Flow conditions are introduced through a velocity term by using Darcy's law and Erguns's model. The inclusion of velocity term expresses the dynamic behavior of the reservoir which allows estimating the reserve during production. To get a proper guideline regarding recovery technique, both fracture and matrix storage should be considered and therefore, these both storages are considered during the development of the model. Later, water influx term is included in the derived model of Dumore by which hydrocarbon gets more driving force to flow through the producing well. The inclusion of these crucial factors into conventional MBE maximize the final recovery which is noticed from a comparative study.

5.2 Introduction

The gas which is trapped in shale formation is known as shale gas. With the increasing interest in USA and rest of the petroleum world, shale gas has become an important resource (Stevens, 2012). Some researchers are expecting that this type of gas will increase the energy supply throughout the world. Multiple porosities are the important characteristics of Shale gas reservoirs (Orozco and Aguilera, 2017). These multiple porosities are: (i) adsorbed porosity, (ii) organic porosity, (iii) inorganic matrix porosity, (iv) natural fractures porosity, and (v) hydraulic fractures porosity (Aguilera and Lopez, 2013).

The conventional gas MBE was modeled for a volumetric reservoir. However, p/Z vs. cumulative gas plot gives some unrealistic results when fractures and water drive mechanism are considered. in the case of some abnormal situations such as over-pressured condition (e.g. coal bed methane), and desorption condition (e.g. shale formation). Figure 3 shows p/Z vs.

cumulative production (G_p) plot for different reservoir conditions. From the figure, it is observed that all plots are nonlinear except for the volumetric one. This is because in the straight-line method, only gas expansion was incorporated as a drive mechanism. Basically, there are different drive mechanisms involves based on different reservoir categories. In water drive reservoir, water influx acts as a drive mechanism, formation and residual fluid expansion acts as driving force in an over-pressured reservoir. Singh *et al.* (2013) reported that gas desorption has a significant role on shale or CBM reservoir as a driving force. The paper aims to derive an equation where irregular behaviors like fractures and water influx are considered.

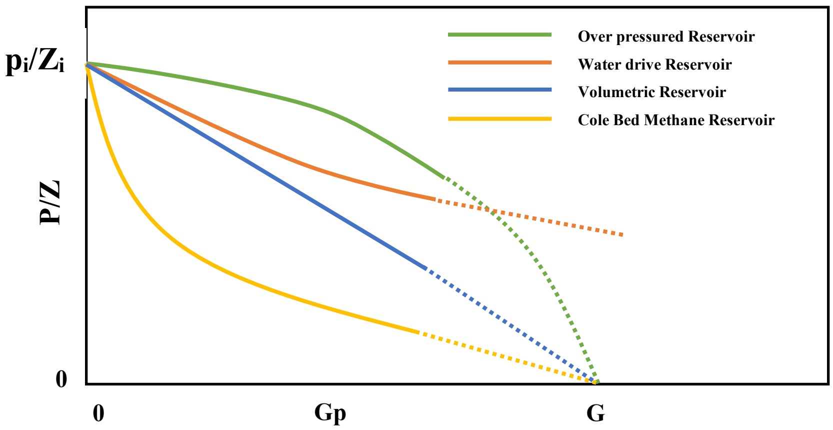

Figure 5.1: Conventional P/Z vs. cumulative production plot (Singh et al., 2013, redrawn)

The paper aims to derive an equation where irregular behaviors like fractures and water influx are considered.

5.3 General MBE for Gas Reservoirs

The established general MBE for gas reservoir is:

$$OGIP(B_g - B_{gi}) + \Delta V_m + \Delta V_{wm} + W_e = G_p B_g + W_p B_w \tag{5.1}$$

Remembering that:

According to the definition of pore compressibility,

$$C_p = \frac{1}{V_p}\frac{\Delta V_m}{\Delta P} \tag{5.2}$$

Water compressibility is defined by:

$$C_w = \frac{1}{V_w}\frac{\Delta V_{wm}}{\Delta P} \tag{5.3}$$

Using equation (5.2) and (5.3) in equation (5.1):

$$OGIP(B_g - B_{gi}) + C_p V_p \Delta P + C_w V_w \Delta P + W_e = G_p B_g + W_p B_w \tag{5.4}$$

If the reservoir is volumetric (no water encroachment or water production) and considering:

$OGIP = \frac{V_p(1-S_w)}{B_{gi}}$

$$\Rightarrow V_p = \frac{OGIP\ B_{gi}}{1-S_w} \tag{5.5}$$

$V_w = V_p * S_w$

$$\Rightarrow V_w = \frac{OGIP\ B_{gi}S_w}{1-S_w} \tag{5.6}$$

By using equation (5.5) and (5.6), we can write equation (4) as:

$$OGIP(B_g - B_{gi}) + C_p \frac{OGIP\ B_{gi}}{1-S_w}\Delta P + C_w \frac{OGIP\ B_{gi}S_w}{1-S_w}\Delta P + W_e = G_pB_g + W_pB_w \tag{5.7}$$

$$\Rightarrow OGIP(B_g - B_{gi}) + \frac{OGIP\ B_{gi}}{1 - S_w}(C_p + C_wS_w)\Delta P + 0 = G_pB_g + 0 * B_w$$

$$\Rightarrow \boldsymbol{OGIP(B_g - B_{gi}) + \frac{OGIP\ B_{gi}}{1-S_w}(C_p + C_wS_w)\Delta P = G_pB_g} \tag{5.8}$$

It is recognized that there is a proportional relationship between pressure drop through a granular bed and fluid velocity at low flow rates, and which is square of the velocity at high flow rates. Osborne Reynolds (Osborne O., 1900) first formulated this relationship which is as follows-

$$\frac{\Delta p}{L} = au + b\rho u^2 \tag{5.9}$$

As it is assumed that, the flow is laminar, so velocity of the fluid is very tiny. Eventually the square of the value of velocity will be negligible. That’s why the term $b\rho u^2$ can be considered as negligible. Mathematically,

$b\rho u^2 \rightarrow 0$

So, the equation (5.8) becomes

$$\frac{\Delta p}{L} = au \tag{5.10}$$

$$\Rightarrow \Delta p = auL \tag{5.11}$$

Here a is the coefficient. The value of **“a”** depends on the value of pressure difference, fluid velocity and length of the reservoir.

$So, a = \frac{(p_i-p)}{ul}$

When a fluid flow with a velocity of 1 in/s through a 1 in bed with 1 psi pressure difference, then $a = \frac{1psi}{in/_s} in$.

$a = 1\frac{psi}{in/_s} in$ is meant that, when a fluid flow in a 1 in porous path with a velocity of 1 in/s, the pressure difference will be 1psi.

For the simplicity, we can use $a = 1\frac{psi}{in/_s} in$ for the next derivation.

So, $\Delta p = uL$ (5.12)

So the equation (5.8) becomes

$$\boldsymbol{OGIP(B_g - B_{gi}) + \frac{OGIP\ B_{gi}}{1-S_w}(C_p + C_w S_w)uL = G_p B_g} \quad (5.13)$$

If the presence of aquifer is considered, the above model will be changed. Dumore (1973) When $t < 0$ i.e. before starting the production of gas, there is no flow of water to the gas zone. But at $t > 0$ i.e. when gas production is started, the pressure at the original gas-water contact is reduced. This pressure reduction allows the aquifer to expand and flow through the boundary with the reservoir. Therefore, the invaded cumulative volume of the water from aquifer to the reservoir is, until time t, is

$$W_e = \int_0^{h_w} \left\{1 - \left(\frac{S_{gr}}{S_{gi}}\right)\right\}.A.dh + W_p \quad (5.14)$$

where W_p is the cumulative produced water until time t. If the average value of $\frac{S_{gr}}{S_{gi}}$ is considered, equation (5.12) becomes

$$W_e = \left\{1 - \left(\frac{S_{gr}}{S_{gi}}\right)\right\}.V_r.h_w + W_p \quad (5.15)$$

Applying different conditions, Dumore concluded the model as below:

$$W_e = \frac{\left\{(1/B_g) - (1/B_g)_i\right\} V_i + G_p}{(1/B_g)} \quad (5.16)$$

Recalling equation (5.7):

$$OGIP(B_g - B_{gi}) + C_p \frac{OGIP\ B_{gi}}{1 - S_w}\Delta P + C_w \frac{OGIP\ B_{gi} S_w}{1 - S_w}\Delta P + W_e = G_p B_g + W_p B_w$$

Applying equation (5.12) and (5.16) into above equation;

$$OGIP(B_g - B_{gi}) + C_p \frac{OGIP\ B_{gi}}{1-S_w} uL + C_w \frac{OGIP\ B_{gi} S_w}{1-S_w} uL + \frac{\{(1/B_g)-(1/B_g)_i\} V_i + G_p}{(1/B_g)} = G_p B_g + W_p B_w$$

(5.17)

$$\Rightarrow \boldsymbol{OGIP(B_g - B_{gi}) + \frac{OGIP\ B_{gi}}{1-S_w}(C_p + C_w S_w) uL + \frac{\{(1/B_g)-(1/B_g)_i\} V_i + G_p}{(1/B_g)} = G_p B_g + W_p B_w}$$

(5.18)

Again, according to Darcy's law;

$$Q = -\frac{kA}{\mu L}(p_a - p_b) \quad (5.19)$$

$$\Rightarrow uA = -\frac{kA}{\mu L}(p_a - p_b)$$

$$\Rightarrow u = -\frac{k}{\mu L}(p_a - p_b) \quad (5.20)$$

The above equation is applicable for single phase (fluid) flow. The negative sign indicates that fluid flows from high pressure region to low pressure region. For the negative change of pressure (where $p_b > p_a$), the flow will follow the positive direction.

If p_i is the initial pressure of the reservoir, then equation (5.20) becomes;

$$u = -\frac{k}{\mu L}(p - p_i) \quad (5.21)$$

where p is the average reservoir pressure.

As most of the cases, $p_i > p$;

We can rewrite the equation (5.21) as:

$$u = \frac{k}{\mu L}(p_i - p) \quad (5.22)$$

Applying equation (5.22), equation (5.18) becomes:

$$\boldsymbol{OGIP(B_g - B_{gi}) + \frac{OGIP\ B_{gi}}{1-S_w}(C_p + C_w S_w)\frac{k}{\mu}(p_i - p) + \frac{\{(1/B_g)-(1/B_g)_i\} V_i + G_p}{(1/B_g)} = G_p B_g + W_p B_w} \quad (5.23)$$

Again,

$$u = \frac{k}{\mu L}\Delta p$$

$$\Rightarrow \Delta p = \frac{u\mu L}{k} \quad (5.24)$$

Applying equation (5.16) and (5.24) into equation (5.7):

$$OGIP\left(B_g - B_{gi}\right) + C_p \frac{OGIP\ B_{gi}}{1-S_w} \Delta P + C_w \frac{OGIP\ B_{gi} S_w}{1-S_w} \Delta P + W_e = G_p B_g + W_p B_w \tag{5.25}$$

$$OGIP\left(B_g - B_{gi}\right) + C_p \frac{OGIP\ B_{gi}}{1-S_w} \frac{u\mu L}{k} + C_w \frac{OGIP\ B_{gi} S_w}{1-S_w} \frac{u\mu L}{k} + \frac{\left\{\left(1/B_g\right)-\left(1/B_g\right)_i\right\} V_i + G_p}{\left(1/B_g\right)} = G_p B_g + W_p B_w \tag{5.26}$$

$$\Rightarrow \boldsymbol{OGIP\left(B_g - B_{gi}\right) + \frac{OGIP}{1-S_w} B_{gi}\left(C_p + C_w S_w\right) \frac{u\mu L}{k} + \frac{\left\{\left(1/B_g\right)-\left(1/B_g\right)_i\right\} V_i + G_p}{\left(1/B_g\right)} = G_p B_g + W_p B_w} \tag{5.27}$$

5.4 Fractured Gas Reservoir

For fractured gas reservoir, a dual porosity model can be applied, which considers two tanks, one for the matrix and one for the fracture system.
For the matrix, the general material balance equation becomes:

Recalling equation (5.7)

$$OGIP_m\left(B_g - B_{gi}\right) + \frac{OGIP_m\ B_{gi}}{1-S_{wm}}\left(C_{pm} + C_w S_{wm}\right)\Delta p + W_e = G_{pm} B_g + W_p B_w \tag{5.28}$$

For the fracture, the general material balance equation becomes:

$$OGIP_f\left(B_g - B_{gi}\right) + \frac{OGIP_f\ B_{gi}}{1-S_{wf}}\left(C_{pf} + C_w S_{wf}\right)\Delta p + W_e = G_{pf} B_g + W_p B_w \tag{5.29}$$

Adding equation (5.20) and equation (5.21):

$$OGIP_m\left(B_g - B_{gi}\right) + \frac{OGIP_m\ B_{gi}}{1-S_{wm}}\left(C_{pm} + C_w S_{wm}\right)\Delta p + OGIP_f\left(B_g - B_{gi}\right) + \frac{OGIP_f\ B_{gi}}{1-S_{wf}}\left(C_{pf} + C_w S_{wf}\right)\Delta p + 2W_e = G_{pm} B_g + G_{pf} B_g + 2W_p B_w \tag{5.30}$$

$$\Rightarrow \left(OGIP_m + OGIP_f\right)\left(B_g - B_{gi}\right) + \frac{OGIP_m\ B_{gi}}{1-S_{wm}}\left(C_{pm} + C_w S_{wm}\right)\Delta p + \frac{OGIP_f\ B_{gi}}{1-S_{wf}}\left(C_{pf} + C_w S_{wf}\right)\Delta p + 2W_e = \left(G_{pm} + G_{pf}\right) B_g + 2W_p B_w \tag{5.31}$$

Applying equation (5.12) and (5.16) into above equation;

$$\boldsymbol{\left(OGIP_m + OGIP_f\right)\left(B_g - B_{gi}\right) + \frac{OGIP_m\ B_{gi}}{1-S_{wm}}\left(C_{pm} + C_w S_{wm}\right) uL + \frac{OGIP_f\ B_{gi}}{1-S_{wf}}\left(C_{pf} + C_w S_{wf}\right) uL + \frac{2\left\{\left(1/B_g\right)-\left(1/B_g\right)_i\right\} V_i + G_p}{\left(1/B_g\right)} = \left(G_{pm} + G_{pf}\right) B_g + 2W_p B_w} \tag{5.32}$$

Again, applying equation (5.16) and (5.24) into equation (5.31);

$$\left(OGIP_m + OGIP_f\right)\left(B_g - B_{gi}\right) + \frac{OGIP_m\, B_{gi}}{1-S_{wm}}\left(C_{pm} + C_w S_{wm}\right)\frac{u\mu L}{k} + \frac{OGIP_f\, B_{gi}}{1-S_{wf}}\left(C_{pf} + C_w S_{wf}\right)\frac{u\mu L}{k} + \frac{2\left\{(1/B_g)-(1/B_g)_i\right\}V_i + G_p}{(1/B_g)} = \left(G_{pm} + G_{pf}\right)B_g + 2W_p B_w \quad \textbf{(5.33)}$$

5.5 Results and Discussion

Table 5.1: Rocks and fluid properties used in the synthetic example (Rojas, 2003)

Rocks and fluid properties	
Initial reservoir volume, V_i	$1.24 * 10^9 ft^3$
Reservoir thickness	200 ft
Matrix permeability	10 md
Matrix compressibility	$4 * 10^{-12} psi^{-1}$
Fracture permeability	10 md
Fracture compressibility	$3 * 10^{-5} psi^{-1}$
Initial water saturation in the matrix system	0.10
Initial water saturation in the fractured system	0.10
Fracture porosity	0.001
Matrix porosity	0.1
Pore compressibility, C_p	$1 * 10^{-5}\ 1/psi$
Gas specific gravity	0.815
Gas density	9.20 lbm/ft^3
Water compressibility	$3 * 10^{-6} psi^{-1}$
Initial reservoir pressure	5000 psia
Bottom-hole flowing pressure, P_{wf}	500 psia
Abandonment pressure, P_{ab}	600 psia

Table 5.2: Cumulative gas production and corresponding properties for different pressure (Dumore, 1973)

$P(psi)$	ΔP	$P/Z(psi)$	$G_p(10^6\ SCF)$	B_g	$W_p\ (10^6\ STB)$
5850.6	0	5328.75	0	0.004032258	0
5850.6	0	5328.75	7.06294	0.004032258	0
5840.31	10.29	5324.34	303.70642	0.004035513	0.0629
5830.02	20.58	5318.46	882.8675	0.004040404	0.18241
5815.32	35.28	5309.64	1599.75591	0.004046945	0.33337
5793.27	57.33	5296.41	2652.13397	0.004056795	0.55352
5771.22	79.38	5283.18	3676.26027	0.004066694	0.77367
5735.94	114.66	5262.6	5381.96028	0.004083299	1.1322
5688.9	161.7	5233.2	7695.07313	0.00410509	1.62911
5663.91	186.69	5217.03	9022.90585	0.004118616	1.91845
5640.39	210.21	5202.33	10251.85741	0.004130525	2.18263
5610.99	239.61	5183.22	11745.66922	0.004144219	2.50971
5566.89	283.71	5155.29	14062.31354	0.004168404	3.02549
5519.85	330.75	5122.95	16650.88105	0.004194631	3.60417
5477.22	373.38	5095.02	18974.58831	0.00421763	4.12624
5433.12	417.48	5065.62	21284.16969	0.004240882	4.6546
5387.55	463.05	5036.22	23738.54134	0.004266212	5.22699
5325.81	524.79	4993.59	27086.3749	0.004302926	6.01324
5275.83	574.77	4959.78	29862.11032	0.004332756	6.67369
5230.26	620.34	4928.91	32408.30019	0.004359198	7.29011
5164.11	686.49	4883.34	36095.15487	0.004399472	8.18958
5075.91	774.69	4821.6	41014.49258	0.004456328	9.36581
5003.88	846.72	4771.62	45061.5572	0.004502476	10.46656
4937.73	912.87	4726.05	48787.25805	0.004545455	11.43522
4848.06	1002.54	4662.84	53886.70073	0.004608295	14.50474

Recalling equation (5.8)

$$OGIP\left(B_g - B_{gi}\right) + \frac{OGIP\ B_{gi}}{1-S_w}\left(C_p + C_w S_w\right)\Delta P = G_p B_g$$

Using all the data, original gas in place can be calculated for different pressure.

Table 5.3: OGIP calculation for different reservoir pressure

$P(psi)$	ΔP	OGIP(MMSCF)
5850.6	0	-
5850.6	0	-
5840.31	10.29	328595.118284628
5830.02	20.58	392178.688571449
5815.32	35.28	396818.791928874
5793.27	57.33	395810.626296530
5771.22	79.38	392403.177083254
5735.94	114.66	390117.007449823
5688.9	161.7	393416.477049331
5663.91	186.69	391288.219641229

5640.39	210.21	392206.296869633
5610.99	239.61	395686.145022085
5566.89	283.71	392777.079396727
5519.85	330.75	393187.533520172
5477.22	373.38	394999.333498187
5433.12	417.48	396085.295411767
5387.55	463.05	396650.161422739
5325.81	524.79	395240.467548322
5275.83	574.77	395647.031584851
5230.26	620.34	397321.114137780
5164.11	686.49	398100.381236399
5075.91	774.69	397490.751899776
5003.88	846.72	398374.135025842
4937.73	912.87	399335.555486472
4848.06	1002.54	399044.429188663

Recalling equation (5.13)

$$OGIP\left(B_g - B_{gi}\right) + \frac{OGIP\ B_{gi}}{1-S_w}\left(C_p + C_w S_w\right)uL = G_p B_g$$

Velocity is the important parameter of the equation (4.34), (4.35) and (4.36). It is assumed that, the flow is laminar. So, the Reynolds number equation should be taken care of in this regard. The established equation for Reynolds number is:

$$R_e = \frac{\rho u L}{\mu}$$

$$\Rightarrow u = \frac{\mu R_e}{\rho L} \qquad (4.39)$$

There is also a range for Reynolds number for different types of flow. The laminar flow is taken into account when the Reynolds number is less than 2100 whereas the turbulent flow is considered when the Reynolds Number is more than 4000. Transitional flow exists between these values. Therefore, calculating fluid velocity for different Reynolds Number will be reasonable instead of using mean fluid velocity. To avoid the complexities, the average fluid density and viscosity can be considered.

Using the value $\mu_g = 0.01625, cP,\ \rho_g = 9.20 \frac{\mathrm{lb}}{\mathrm{ft}^3}\ and\ h = 200\ ft$, velocity can be calculated for different Reynolds number.

Table 5.4: Calculation of fluid velocity for different Reynolds Numbers

Reynolds Number, Re	Velocity $(u), ft/s$
100	0.0333
200	0.0668
300	0.1001
400	0.1335
500	0.1669
600	0.2003
700	0.2337
800	0.2671
900	0.3004
1000	0.3338
1100	0.3672
1200	0.4006
1300	0.4339
1400	0.4673
1500	0.5007
1600	0.5341
1700	0.5675
1800	0.6009
1900	0.6343
2000	0.6676

Using all the data, original gas in place can be calculated for different velocity.

Table 5.5: OGIP calculation for different fluid velocity using equation 5.13

u=0.0008832	u=0.0017663	u=0.0026495	u=0.0035326	u=0.0044158	u=0.0052989	u=0.0061821	u=0.0070652	u=0.0079484	u=0.0088315
0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
3.49E+06	1.75E+06	1.16E+06	8.74E+05	6.99E+05	5.82E+05	4.99E+05	4.37E+05	3.88E+05	3.49E+05
3.76E+05	3.75E+05	3.74E+05	3.73E+05	3.72E+05	3.71E+05	3.70E+05	3.69E+05	3.68E+05	3.67E+05
4.37E+05	4.37E+05	4.37E+05	4.36E+05	4.36E+05	4.35E+05	4.35E+05	4.34E+05	4.34E+05	4.34E+05
4.41E+05	4.40E+05	4.40E+05	4.40E+05	4.40E+05	4.39E+05	4.39E+05	4.39E+05	4.39E+05	4.38E+05
4.38E+05	4.38E+05	4.38E+05	4.38E+05	4.38E+05	4.38E+05	4.37E+05	4.37E+05	4.37E+05	4.37E+05
4.34E+05	4.34E+05	4.34E+05	4.34E+05	4.34E+05	4.34E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.34E+05	4.34E+05	4.34E+05	4.34E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05
4.35E+05	4.35E+05	4.35E+05	4.35E+05	4.35E+05	4.35E+05	4.35E+05	4.35E+05	4.34E+05	4.34E+05
4.31E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.32E+05

4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.30E+05	4.30E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05

Table 5.6: OGIP calculation for different fluid velocity using equation 5.13

u=0.0097 14674	**u=0.0105 97826**	**u=0.0114 80978**	**u=0.0123 6413**	**u=0.0132 47283**	**u=0.0141 30435**	**u=0.0150 13587**	**u=0.0158 96739**	**u=0.0167 79891**	**u=0.0176 63043**
0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
3.18E+05	2.91E+05	2.69E+05	2.50E+5	2.33E+05	2.18E+05	2.06E+05	1.94E+05	1.84E+05	1.75E+05
3.66E+05	3.66E+05	3.65E+05	3.64E+5	3.63E+05	3.62E+05	3.61E+05	3.60E+05	3.59E+05	3.59E+05
4.33E+05	4.33E+05	4.32E+05	4.32E+5	4.31E+05	4.31E+05	4.31E+05	4.30E+05	4.30E+05	4.29E+05
4.38E+05	4.38E+05	4.38E+05	4.37E+5	4.37E+05	4.37E+05	4.37E+05	4.36E+05	4.36E+05	4.36E+05
4.37E+05	4.37E+05	4.37E+05	4.36E+5	4.36E+05	4.36E+05	4.36E+05	4.36E+05	4.36E+05	4.36E+05
4.33E+05	4.33E+05	4.33E+05	4.33E+5	4.33E+05	4.33E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.29E+05	4.29E+05	4.29E+05	4.29E+05	4.29E+05
4.33E+05	4.33E+05	4.33E+05	4.33E+5	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.31E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.34E+05	4.34E+05	4.34E+05	4.34E+5	4.34E+05	4.34E+05	4.34E+05	4.34E+05	4.34E+05	4.34E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.32E+05	4.31E+05	4.31E+05	4.31E+5	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+5	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.33E+05	4.33E+05	4.33E+05	4.33E+5	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05	4.33E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.30E+05	4.30E+05	4.30E+05	4.30E+5	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05	4.30E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+5	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+5	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+5	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+5	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05
4.32E+05	4.32E+05	4.32E+05	4.32E+5	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05	4.32E+05
4.31E+05	4.31E+05	4.31E+05	4.31E+5	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05	4.31E+05

Recalling equation (5.18)

$$OGIP\left(B_g - B_{gi}\right) + \frac{OGIP\ B_{gi}}{1-S_w}\left(C_p + C_w S_w\right)uL + \frac{\left\{(1/B_g)-(1/B_g)_i\right\}V_i+G_p}{(1/B_g)} = G_p B_g + W_p B_w$$

Now the consideration is aquifer. Original gas in place (OGIP) is calculated when water influx parameter is considered. Initial reservoir volume also need to be used.

Table 5.7: OGIP calculation for different fluid velocity using equation 5.18

u=0.0008	u=0.0017	u=0.0026	u=0.0035	u=0.0044	u=0.0052	u=0.0061	u=0.0070	u=0.0079	u=0.0088
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
4.9951E+11	4.98266E+11	4.97027E+11	4.95795E+11	4.94569E+11	4.93349E+11	4.92135E+11	4.90926E+11	4.89724E+11	4.88528E+11
5.30915E+11	5.30384E+11	5.29855E+11	5.29327E+11	5.288E+11	5.28274E+11	5.27749E+11	5.27225E+11	5.26703E+11	5.26181E+11
5.34207E+11	5.3391E+11	5.33615E+11	5.33319E+11	5.33024E+11	5.32729E+11	5.32435E+11	5.3214E+11	5.31847E+11	5.31553E+11
5.32929E+11	5.32752E+11	5.32575E+11	5.32398E+11	5.32222E+11	5.32045E+11	5.31869E+11	5.31693E+11	5.31517E+11	5.31341E+11
5.32063E+11	5.31937E+11	5.31811E+11	5.31686E+11	5.3156E+11	5.31434E+11	5.31309E+11	5.31183E+11	5.31058E+11	5.30932E+11
5.29257E+11	5.29173E+11	5.29088E+11	5.29004E+11	5.28919E+11	5.28835E+11	5.28751E+11	5.28666E+11	5.28582E+11	5.28498E+11
5.31141E+11	5.31082E+11	5.31022E+11	5.30963E+11	5.30903E+11	5.30844E+11	5.30785E+11	5.30725E+11	5.30666E+11	5.30607E+11
5.29621E+11	5.29571E+11	5.29521E+11	5.29471E+11	5.29421E+11	5.29371E+11	5.29321E+11	5.29271E+11	5.29221E+11	5.29171E+11
5.29588E+11	5.29544E+11	5.295E+11	5.29457E+11	5.29413E+11	5.29369E+11	5.29325E+11	5.29281E+11	5.29237E+11	5.29193E+11
5.31641E+11	5.31602E+11	5.31563E+11	5.31525E+11	5.31486E+11	5.31447E+11	5.31408E+11	5.3137E+11	5.31331E+11	5.31293E+11
5.29712E+11	5.29681E+11	5.29649E+11	5.29617E+11	5.29585E+11	5.29554E+11	5.29522E+11	5.2949E+11	5.29459E+11	5.29427E+11
5.29462E+11	5.29435E+11	5.29409E+11	5.29382E+11	5.29356E+11	5.29329E+11	5.29303E+11	5.29276E+11	5.29249E+11	5.29223E+11
5.30089E+11	5.30066E+11	5.30042E+11	5.30019E+11	5.29996E+11	5.29973E+11	5.29949E+11	5.29926E+11	5.29903E+11	5.29879E+11
5.30609E+11	5.30588E+11	5.30567E+11	5.30547E+11	5.30526E+11	5.30505E+11	5.30484E+11	5.30464E+11	5.30443E+11	5.30422E+11
5.30921E+11	5.30903E+11	5.30884E+11	5.30866E+11	5.30847E+11	5.30829E+11	5.3081E+11	5.30792E+11	5.30773E+11	5.30755E+11
5.29667E+11	5.29651E+11	5.29635E+11	5.29619E+11	5.29603E+11	5.29587E+11	5.29571E+11	5.29555E+11	5.29539E+11	5.29523E+11
5.29593E+11	5.29579E+11	5.29565E+11	5.2955E+11	5.29536E+11	5.29521E+11	5.29507E+11	5.29493E+11	5.29478E+11	5.29464E+11
5.30487E+11	5.30474E+11	5.3046E+11	5.30447E+11	5.30434E+11	5.30421E+11	5.30407E+11	5.30394E+11	5.30381E+11	5.30368E+11
5.30528E+11	5.30516E+11	5.30504E+11	5.30492E+11	5.3048E+11	5.30469E+11	5.30457E+11	5.30445E+11	5.30433E+11	5.30422E+11
5.28365E+11	5.28355E+11	5.28345E+11	5.28335E+11	5.28325E+11	5.28314E+11	5.28304E+11	5.28294E+11	5.28284E+11	5.28274E+11
5.301E+11	5.30091E+11	5.30082E+11	5.30073E+11	5.30064E+11	5.30054E+11	5.30045E+11	5.30036E+11	5.30027E+11	5.30018E+11
5.30335E+11	5.30326E+11	5.30318E+11	5.3031E+11	5.30301E+11	5.30293E+11	5.30284E+11	5.30276E+11	5.30267E+11	5.30259E+11
5.59314E+11	5.59306E+11	5.59298E+11	5.59291E+11	5.59283E+11	5.59275E+11	5.59267E+11	5.59259E+11	5.59251E+11	5.59243E+11

Table 5.8: OGIP calculation for different fluid velocity using equation 5.18

u=0.0097	u=0.01059	u=0.01148	u=0.0123	u=0.01324	u=0.01413	u=0.01501	u=0.01589	u=0.01677	u=0.017663
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
4.87337E+11	4.86152E+11	4.84973E+11	4.838E+11	4.82633E+11	4.81471E+11	4.80314E+11	4.79163E+11	4.78018E+11	4.76878E+11
5.2566E+11	5.2514E+11	5.24622E+11	5.24104E+11	5.23587E+11	5.23072E+11	5.22557E+11	5.22043E+11	5.21531E+11	5.21019E+11
5.3126E+11	5.30967E+11	5.30674E+11	5.30382E+11	5.3009E+11	5.29799E+11	5.29507E+11	5.29216E+11	5.28926E+11	5.28635E+11
5.31165E+11	5.30989E+11	5.30814E+11	5.30638E+11	5.30463E+11	5.30287E+11	5.30112E+11	5.29937E+11	5.29762E+11	5.29587E+11
5.30807E+11	5.30682E+11	5.30556E+11	5.30431E+11	5.30306E+11	5.30181E+11	5.30056E+11	5.29931E+11	5.29806E+11	5.29681E+11
5.28413E+11	5.28329E+11	5.28245E+11	5.28161E+11	5.28077E+11	5.27993E+11	5.27909E+11	5.27824E+11	5.2774E+11	5.27656E+11
5.30547E+11	5.30488E+11	5.30429E+11	5.3037E+11	5.3031E+11	5.30251E+11	5.30192E+11	5.30133E+11	5.30073E+11	5.30014E+11
5.29121E+11	5.29072E+11	5.29022E+11	5.28972E+11	5.28922E+11	5.28872E+11	5.28822E+11	5.28772E+11	5.28723E+11	5.28673E+11
5.29149E+11	5.29106E+11	5.29062E+11	5.29018E+11	5.28974E+11	5.2893E+11	5.28886E+11	5.28843E+11	5.28799E+11	5.28755E+11
5.31254E+11	5.31215E+11	5.31177E+11	5.31138E+11	5.31099E+11	5.31061E+11	5.31022E+11	5.30983E+11	5.30945E+11	5.30906E+11
5.29395E+11	5.29364E+11	5.29332E+11	5.293E+11	5.29269E+11	5.29237E+11	5.29205E+11	5.29174E+11	5.29142E+11	5.2911E+11
5.29196E+11	5.2917E+11	5.29143E+11	5.29117E+11	5.2909E+11	5.29064E+11	5.29037E+11	5.29011E+11	5.28984E+11	5.28957E+11
5.29856E+11	5.29833E+11	5.2981E+11	5.29786E+11	5.29763E+11	5.2974E+11	5.29716E+11	5.29693E+11	5.2967E+11	5.29647E+11
5.30402E+11	5.30381E+11	5.3036E+11	5.30339E+11	5.30319E+11	5.30298E+11	5.30277E+11	5.30257E+11	5.30236E+11	5.30215E+11
5.30736E+11	5.30718E+11	5.30699E+11	5.30681E+11	5.30662E+11	5.30644E+11	5.30625E+11	5.30607E+11	5.30588E+11	5.3057E+11
5.29508E+11	5.29492E+11	5.29476E+11	5.2946E+11	5.29444E+11	5.29428E+11	5.29412E+11	5.29396E+11	5.2938E+11	5.29364E+11
5.2945E+11	5.29435E+11	5.29421E+11	5.29407E+11	5.29392E+11	5.29378E+11	5.29364E+11	5.29349E+11	5.29335E+11	5.29321E+11
5.30355E+11	5.30341E+11	5.30328E+11	5.30315E+11	5.30302E+11	5.30289E+11	5.30275E+11	5.30262E+11	5.30249E+11	5.30236E+11
5.3041E+11	5.30398E+11	5.30386E+11	5.30374E+11	5.30363E+11	5.30351E+11	5.30339E+11	5.30327E+11	5.30316E+11	5.30304E+11
5.28264E+11	5.28253E+11	5.28243E+11	5.28233E+11	5.28223E+11	5.28213E+11	5.28203E+11	5.28193E+11	5.28182E+11	5.28172E+11
5.30008E+11	5.29999E+11	5.2999E+11	5.29981E+11	5.29972E+11	5.29963E+11	5.29953E+11	5.29944E+11	5.29935E+11	5.29926E+11
5.30251E+11	5.30242E+11	5.30234E+11	5.30225E+11	5.30217E+11	5.30208E+11	5.302E+11	5.30192E+11	5.30183E+11	5.30175E+11
5.59235E+11	5.59227E+11	5.59219E+11	5.59211E+11	5.59204E+11	5.59196E+11	5.59188E+11	5.5918E+11	5.59172E+11	5.59164E+11

Recalling equation (5.27)

$$OGIP\left(B_g - B_{gi}\right) + \frac{OGIP}{1-S_w}\, B_{gi}\left(C_p + C_w S_w\right)\frac{u\mu L}{k} + \frac{\left\{(1/B_g)-(1/B_g)_i\right\}V_i + G_p}{(1/B_g)} = G_p B_g + W_p B_w$$

Table 5.10: OGIP calculation for different fluid velocity using equation 5.27

u=0.0008	u=0.00176	u=0.00264	u=0.00353	u=0.004415	u=0.005298	u=0.006182	u=0.0070652	u=0.007948	u=0.0088315
0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.62E-08	1.31E-08	8.73E-09	6.55E-09	5.24E-09	4.37E-09	3.74E-09	3.27E-09	2.91E-09	2.62E-09
3.08E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.07E+11	3.07E+11	3.07E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11

Table 5.11: OGIP calculation for different fluid velocity using equation 5.27

u=0.0097	u=0.01059	u=0.01148	u=0.01236	u=0.013247	u=0.014130	u=0.015013	u=0.015896	u=0.0167798	u=0.0176630
0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.38E-09	2.18E-09	2.01E-09	1.87E-09	1.75E-09	1.64E-09	1.54E-09	1.46E-09	1.38E-09	1.31E-09
3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.08E+11	3.08E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.07E+11	3.07E+11	3.07E+11	3.07E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11
3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11	3.08E+11

Again, recalling equation 5.33

$$\left(OGIP_m + OGIP_f\right)\left(B_g - B_{gi}\right) + \frac{OGIP_m\, B_{gi}}{1-S_{wm}}\left(C_{pm} + C_w S_{wm}\right)\frac{u\mu L}{k} + \frac{OGIP_f\, B_{gi}}{1-S_{wf}}\left(C_{pf} + C_w S_{wf}\right)\frac{u\mu L}{k} + \frac{2\left\{(1/B_g)-(1/B_g)_i\right\}V_i + G_p}{(1/B_g)} = \left(G_{pm} + G_{pf}\right)B_g + 2W_p B_w$$

By using above equation, original gas in place for matrix and fracture can be calculated easily. Unavailability of the production data for fracture and matrix don't allow to validate the model. If reliable data can be get from any fractured gas field, the original gas in place for matrix and fracture can be estimated. Future research can be carried on to validate this model with proper data.

Now, to make a comparative study for four derived equation, a summary of OGIP value can be created.

Table 5.12: Summary of the OGIP value calculated by different equations

$P(psi)$	OGIP(MMSCF) Calculated by eq. 5.8	OGIP(MMSCF) Calculated by eq. 5.13	OGIP(MMSCF) Calculated by eq. 5.18	OGIP(MMSCF) Calculated by eq. 5.27
5850.6	-	0.00E+00	0	0.00E+00
5850.6	-	5.82E+05	0	4.37E-09
5840.31	3.2860E+05	3.71E+05	4.93349E+11	3.07E+11
5830.02	3.9218E+05	4.35E+05	5.28274E+11	3.07E+11
5815.32	3.9682E+05	4.39E+05	5.32729E+11	3.08E+11
5793.27	3.9581E+05	4.38E+05	5.32045E+11	3.08E+11
5771.22	3.9240E+05	4.34E+05	5.31434E+11	3.08E+11
5735.94	3.9012E+05	4.30E+05	5.28835E+11	3.08E+11
5688.9	3.9342E+05	4.33E+05	5.30844E+11	3.08E+11
5663.91	3.9129E+05	4.30E+05	5.29371E+11	3.08E+11
5640.39	3.9221E+05	4.31E+05	5.29369E+11	3.08E+11
5610.99	3.9569E+05	4.35E+05	5.31447E+11	3.08E+11
5566.89	3.9278E+05	4.30E+05	5.29554E+11	3.08E+11
5519.85	3.9319E+05	4.30E+05	5.29329E+11	3.08E+11
5477.22	3.9500E+05	4.32E+05	5.29973E+11	3.08E+11
5433.12	3.9609E+05	4.33E+05	5.30505E+11	3.08E+11
5387.55	3.9665E+05	4.33E+05	5.30829E+11	3.08E+11
5325.81	3.9524E+05	4.31E+05	5.29587E+11	3.08E+11
5275.83	3.9565E+05	4.30E+05	5.29521E+11	3.08E+11
5230.26	3.9732E+05	4.32E+05	5.30421E+11	3.08E+11
5164.11	3.9810E+05	4.32E+05	5.30469E+11	3.08E+11
5075.91	3.9749E+05	4.31E+05	5.28314E+11	3.08E+11
5003.88	3.9837E+05	4.31E+05	5.30054E+11	3.08E+11
4937.73	3.9934E+05	4.32E+05	5.30293E+11	3.08E+11
4848.06	3.9904E+05	4.31E+05	5.59275E+11	3.08E+11

The principal objective of deriving modified material balance equation is to get a better estimation of the original gas in place by using all known properties of rocks and fluid. Aguilera and Orozco (2008) also derived the similar type of correlation where the velocity and water influx term is missing. With the inclusion of these two important parameters, estimation of gas reserve will be more accurate. The viscosity and permeability parameters are also incorporated to increase the accuracy of the model.

Above table summarizes the OGIP value for different pressure. Second column is the OGIP value calculated by equation 5.8. Minimum reservoir properties are incorporated in that equation and consequently the estimated reserve is comparatively less. The average OGIP value for this column 2 is 4*10e5 MMSCF.

Third column of the table summarizes the OGIP value calculated by equation 5.13. Velocity parameter has been introduced in that equation. That's why equation 5.13 is showing better estimation than equation 5.8. The average OGIP value for third column is 4.30*10e5 MMSCF.

Fourth column of the table shows the OGIP value calculated by equation 5.18 when water encroachment has been considered. This equation calculated the maximum value of OGIP. The average OGIP value calculated by equation 5.18 is 5.30*10e11 MMSCF.

The OGIP value calculated by equation 5.27 has been summarized in column 5 of the above table. All the important reservoir parameters have been incorporated in this equation. That's why, this estimation can be said as a best estimation. The average OGIP value in the column is 3.08*10e11 MMSCF.

It would be a good validation, if these results could be compared with others result. In the literature, no similar work has been found out. It can be recommended for the future research to validate the model extensively.

5.7 Conclusion

This research work proposes a new material balance equation applicable to fractured gas reservoir. Darcy's law and another velocity dependent correlation has been implanted to derive all the equations. Equation 5.8 has been derived only with the traditional parameters. The OGIP calculated with this equation shows an under estimation of the reserve. With the incorporation of fluid velocity, second equation has been modelled. As fluid velocity has an important role on material balance calculation, equation 5.13 gives a better estimation than first one. Equation 5.18 has been derived with the consideration of water encroachment. Including fluid velocity, the water encroachment parameter gives the equation an additional strength to estimate the reserve in a better way. Almost all the important reservoir parameters have been covered in equation 5.27. Along with other traditional parameters, permeability, viscosity, fluid velocity and the water encroachment parameter have been incorporated in this equation. This equation gives the maximum estimation of the reserve. Equation 5.33 derived to calculate the reserve for fracture and matrix. It would be a wonderful addition for this thesis if this equation can be validated with the production data from any fractured gas reservoir. Due to unavailability of produced matrix and fracture gas amount, the reserve calculation has not been done for the fractured model.

5.8 References

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Chapter 6
Conclusions and Recommendation for Future Work

This chapter discusses the main conclusions obtained from this study, as well as proposes the direction for future work. The main objective of this research was to develop two modified mathematical model for material balance calculation. Standard rocks and fluid properties and production data collected from literature have been used to compare the model to other industry accepted techniques.

6.1 Summary of Conclusions

Conclusions answering the objectives of this research are presented below:
A) A compressibility model has been developed to make a sensitivity analysis of the established material balance equation. This technique incorporates additional physics into the original formulations (Hossain and Islam, 2011), such as a comprehensive material balance equation with the inclusion of memory. This model helps to identify the actual parameter to be included for the modification.

B) A modified material balance equation has been developed for a naturally fractured oil reservoir where dynamic condition has been incorporated. Fluid velocities have been calculated for different Reynolds number. The range of Reynolds number is 0 to 100 in this regard as the flow has been considered as laminar. Permeability and viscosity have also been taken care for the derivation to characterize the reservoir. Using field data, the oil in place in fracture and matrix has been calculated. Finally, the calculated reserve has been used to validate the model with another established model. Incorporation of velocity, permeability and viscosity optimize the reserve estimation.

C) A modified material balance equation for naturally fractured gas reservoir has been developed. A strong aquifer has been considered around the gas zone. Therefore, water influx term has been incorporated to the developed model. Original gas in place (OGIP) has been calculated by using standard rocks and fluid properties. Finally, the developed model has been validated with another established model by using calculated OGIP. A better result of OGIP has been generated with the incorporation of water influx.

6.2 Recommendations for Future Work

This study offers a set of evidences that proves the applicability of modified material balance equation for reservoir estimation. This study developed several ideas that are beyond the goals of the research and are recommended for future development of this topic. These recommendations are outlined as follows:

A) Develop a dynamic material balance equation for oil reservoir that will consider the velocity of the fluid in the reservoir and well. Optimizing a velocity will improve the quality of the reserve estimation.

B) Likewise, develop a dynamic material balance equation for gas reservoir considering similar condition as previous one.

C) Apply the developed material balance equation for other types of reservoir, such as coal bed methane, two-phase reservoir, volatile oil etc. and get what adjustment are required for each type of reservoir.

D) Try to collect all the rocks and fluid and production data from a specific reservoir so that all data have the consistency.