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A Review of Theories for Sound Transmission through Infinite Double Panels and Identification of Asymptotic Behavior

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A REVIEW OF THEORIES FOR SOUND TRANSMISSION THROUGH INFINITE DOUBLE PANELS AND IDENTIFICATION OF ASYMPTOTIC BEHAVIOR

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PURDUE MECHANICAL ENGINEERING

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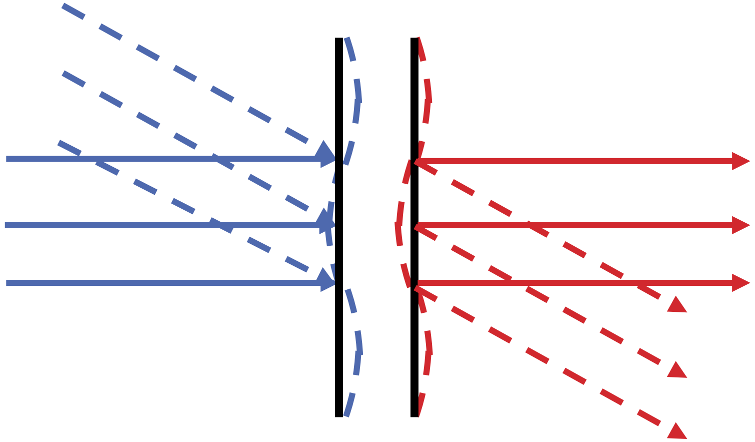


Sound transmission through an infinite double-panel systems

- **Review of Existing Theories**
  Classic models by **Beranek** and **Work**, **London**, **Mulholland et al.**, **Heckl**, **Fahy**, and **Hamada** and **Tachibana**
- **Asymptotic Behavior**
  Asymptotic behavior of double-panel systems with **stiff** panels
- **Porous Lining and Average Transmission Loss**
  Effect of **limp porous** lining and the **average** transmission loss over a range of incidence angles
- **Conclusion**

Existing Theories

Boundary Value Problem	Alternative approaches
Beranek and Work[1]	Mulholland, Parbrook, and Cummings[4] → Multiple-reflection
London[2]	Hamada and Tachibana[7] → Transfer matrix
Different media ← Mulholland, Price, and Parbrook[3]	
Locally reacting ← Heckl[5]	
Fahy[6]	

↓

Can be extended to many applications

Different panel impedance leads to,

Limp panel	Limp panel + Constant resistance	Stiff panel + Constant resistance	Stiff panel + Hysteretic resistance

Existing Theories

- **Beranek and Work’s equation for a double-limp-panel system without resistance** ($Z = j\omega m$) [1],

$$p_0/p_5 = \left(\cos kd - \frac{\omega \overset{\substack{\text{panel surface mass}\\ \uparrow}}{m_1}}{\underset{\substack{\downarrow\\ \text{air properties}}}{\rho c}} \sin kd\right) + j\left[\sin k\overset{\substack{\text{panel distance}\\ \uparrow}}{d} + \frac{\omega(m_1 + m_2)}{\rho c} \cos kd - \frac{\omega^2 m_1 m_2}{\rho^2 c^2} \sin kd\right]$$

- Normal incidence
- Different panels
- The total sound pressure ratio can be transferred to transmission coefficient[3].
- This limp panel impedance can be substituted into other models, i.e. London’s model.

Existing Theories

- **London's expression of transmission coefficient for identical panels**[2],

$$T = \frac{p_t}{p_i} = 1/\left[1 + \frac{\overset{\substack{\text{panel impedance}\\ \uparrow}}{Z_w}\cos\theta}{\rho c} + \frac{Z_w^2\cos^2\theta}{4\rho^2c^2}\left(1 - e^{-j\,2kd\cos\theta}\right)\right]$$

- **A generalized version of London's model**[8],

$$|T|^2 = \left|1/\left[1 + \frac{Z_1 + Z_2}{2\rho c}\cos\theta + \frac{Z_1Z_2\cos^2\theta}{4\rho^2c^2}\left(1 - e^{-2jkd\cos\theta}\right)\right]\right|^2$$

- Oblique incidence
- Different panels

Existing Theories

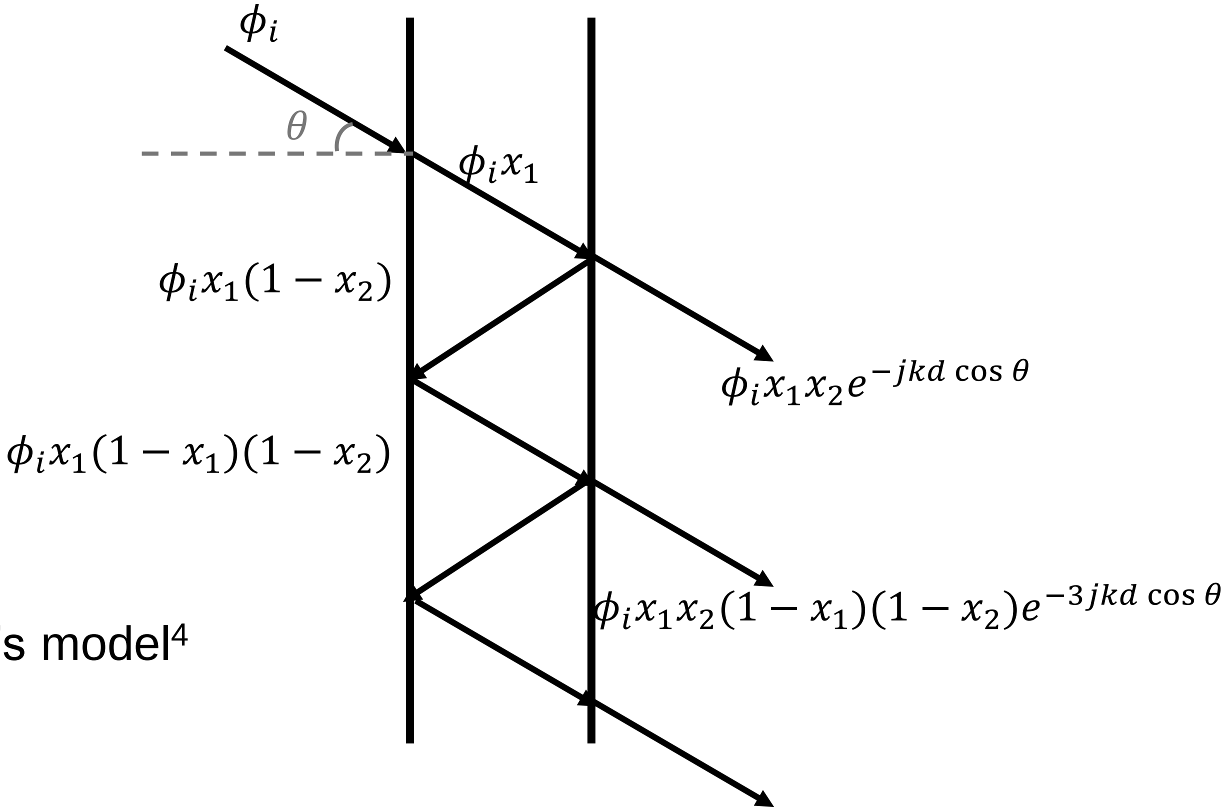
- **Mulholland et al. derived multiple-reflection theory[4],**

$$|T|^2 = \left|\frac{x^2}{1-(1-x)^2 e^{-j2kd\cos\theta}}\right|^2$$

- **A generalized version derived from[9],**

$$|T|^2 = \left|\frac{x_1 x_2}{1-(1-x_1)(1-x_2) e^{-j2kd\cos\theta}}\right|^2$$

- Equivalent to Beranek and Work's and London's model[4]



Existing Theories

- **Mulholland et al. extended Beranek and Work's method**[3],

medium between panels

$$T = \left[\frac{2\rho_2 c_2 \cos\theta_1}{\left(Z_f \cos\theta_1 + \rho_1 c_1\right)\cos\theta_2}\right]\left[\frac{\cosh\Phi}{\sinh(jk_2 d\cos\theta_2 + \Phi)}\right]\left[\frac{\rho_1 c_1}{\rho_1 c_1 + j\omega m_2 \cos\theta_1}\right]$$

medium outside

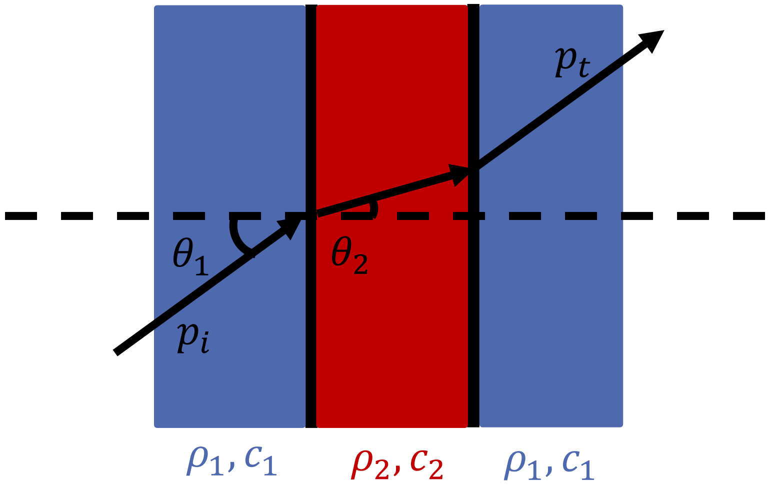
incident angle

$$\Phi = \operatorname{arccoth}\left[\frac{(j\omega m_2 \cos\theta_1 + \rho_1 c_1)}{\rho_2 c_2}\frac{\cos\theta_2}{\cos\theta_1}\right]$$

refracting angle between panels

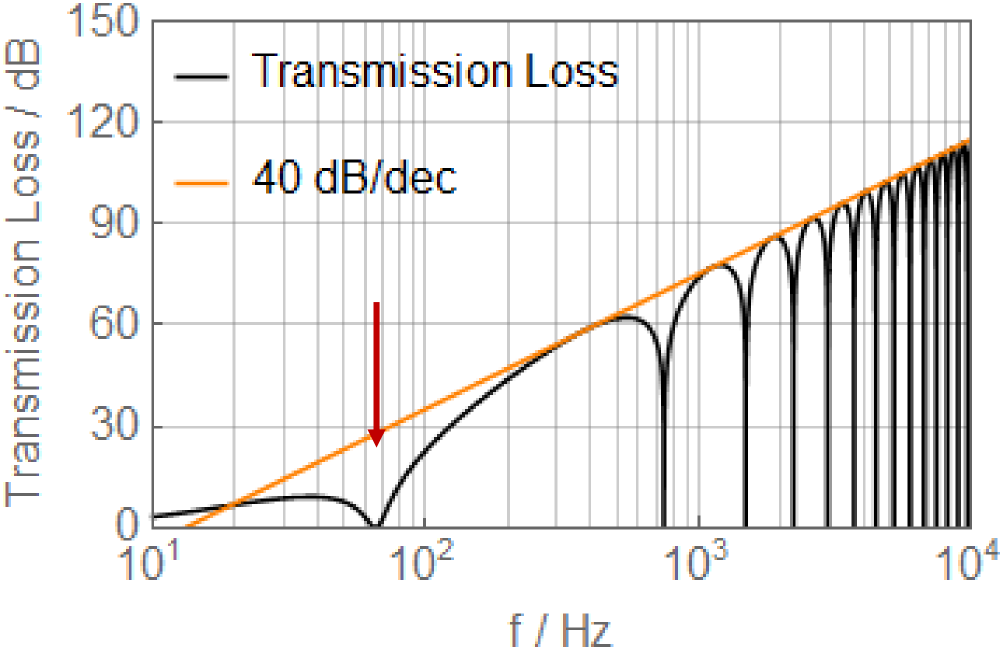
$$Z_f = \frac{\rho_2 c_2 \coth(jk_2 d\cos\theta_2 + \Phi) + j\omega m_1 \cos\theta_2}{\cos\theta_2}$$

- Oblique incidence
- Different media and panels

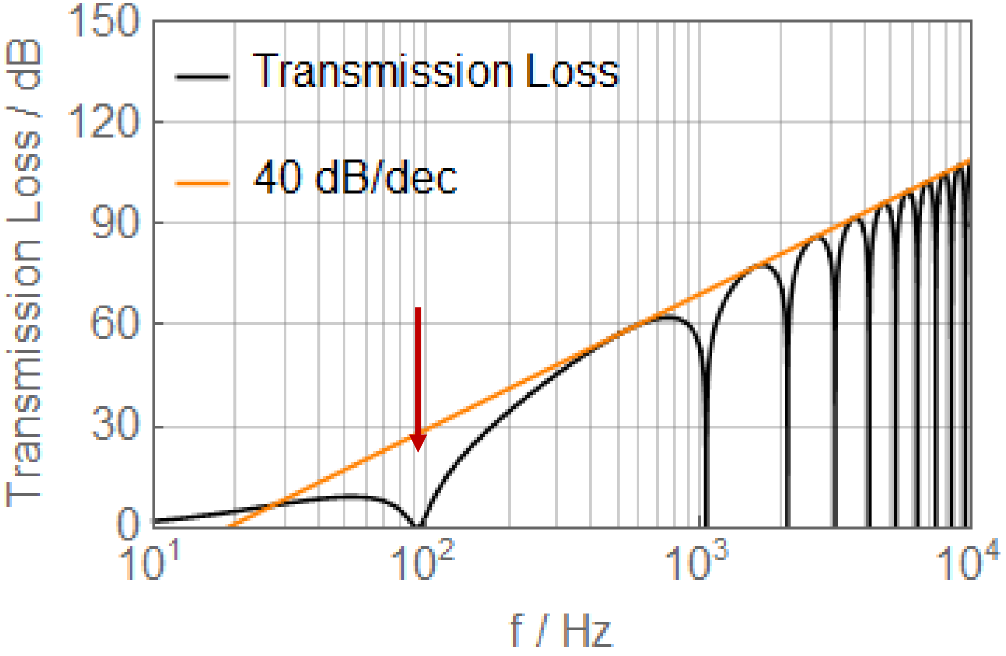


Existing Theories

→|| double-limp-panel
$m_1 = 7\ \mathrm{kg/m^2}\quad m_2 = 7\ \mathrm{kg/m^2}\quad d = 0.23\ \mathrm{m}$



↘|| double-limp-panel at $\theta_i = \pi/4$
$m_1 = 7\ \mathrm{kg/m^2}\quad m_2 = 7\ \mathrm{kg/m^2}\quad d = 0.23\ \mathrm{m}$

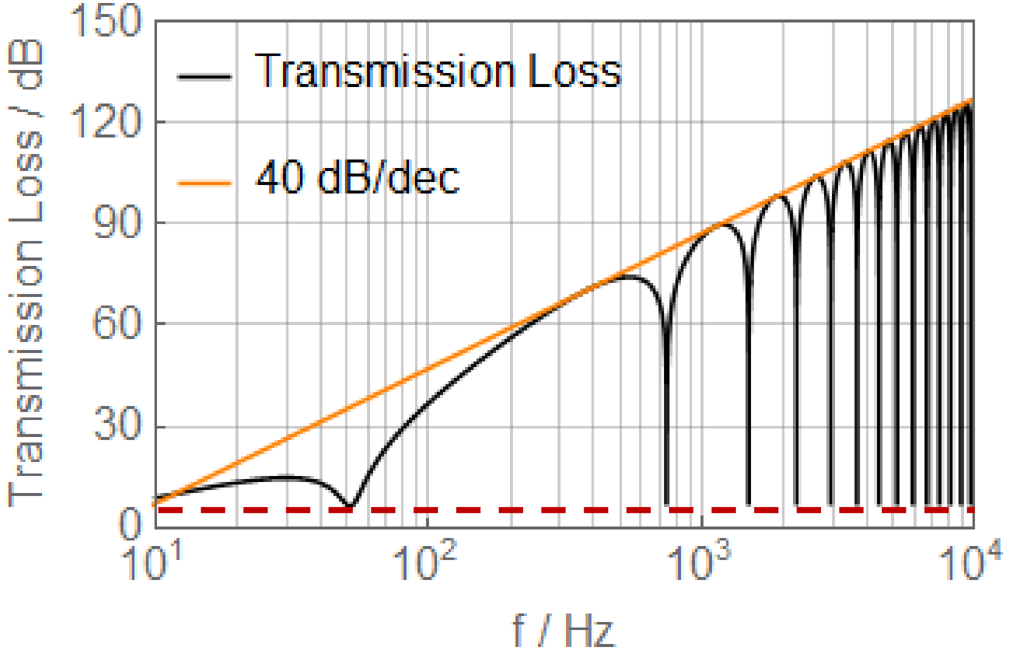


- Governed by mass law – 40 dB/dec
- Minima of transmission loss go to zero
- Resonances shift to higher frequencies at oblique incidence

Existing Theories

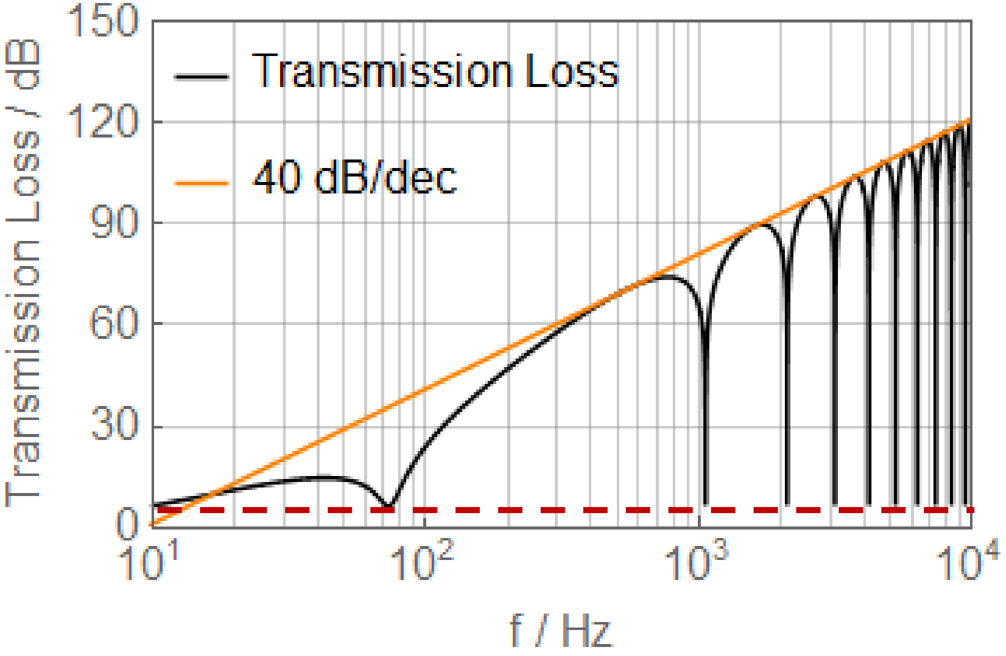
double-limp-panel

$m_1 = 28\ \mathrm{kg/m^2}$ $m_2 = 7\ \mathrm{kg/m^2}$ $d = 0.23\ \mathrm{m}$



double-limp-panel at $\theta_i = \pi/4$

$m_1 = 28\ \mathrm{kg/m^2}$ $m_2 = 7\ \mathrm{kg/m^2}$ $d = 0.23\ \mathrm{m}$



- Governed by mass law – 40 dB/dec
- Minima of transmission loss do not go to zero
- Resonances shift to higher frequencies at oblique incidence

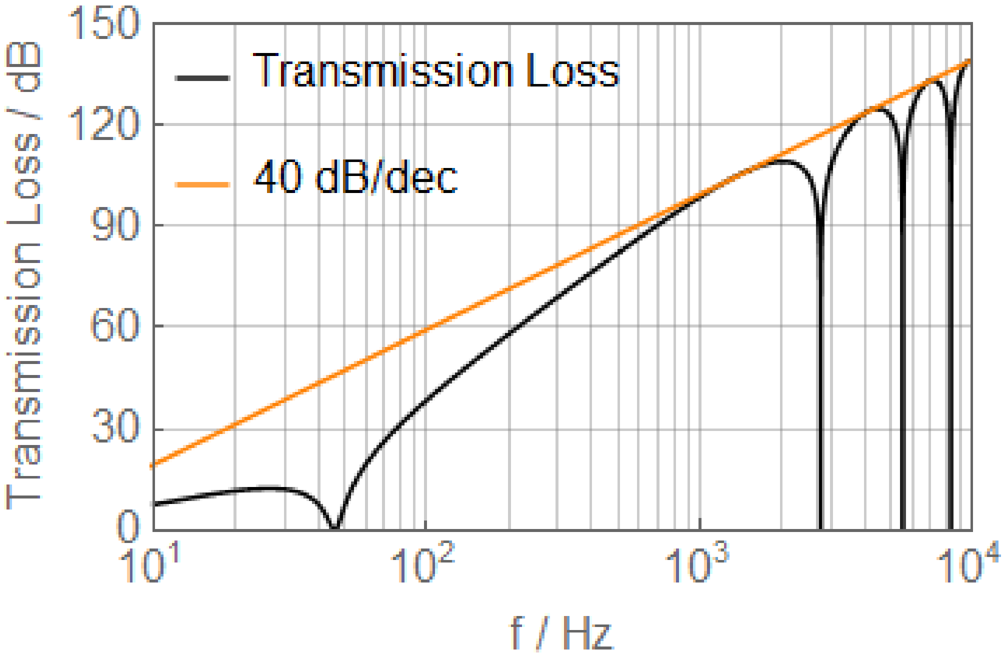
Existing Theories

With hydrogen between panels and air outside

$\rho_2 = \ 0.08988\ \mathrm{kg/m^3}\quad c_2 = 1270\ \mathrm{m}/s$

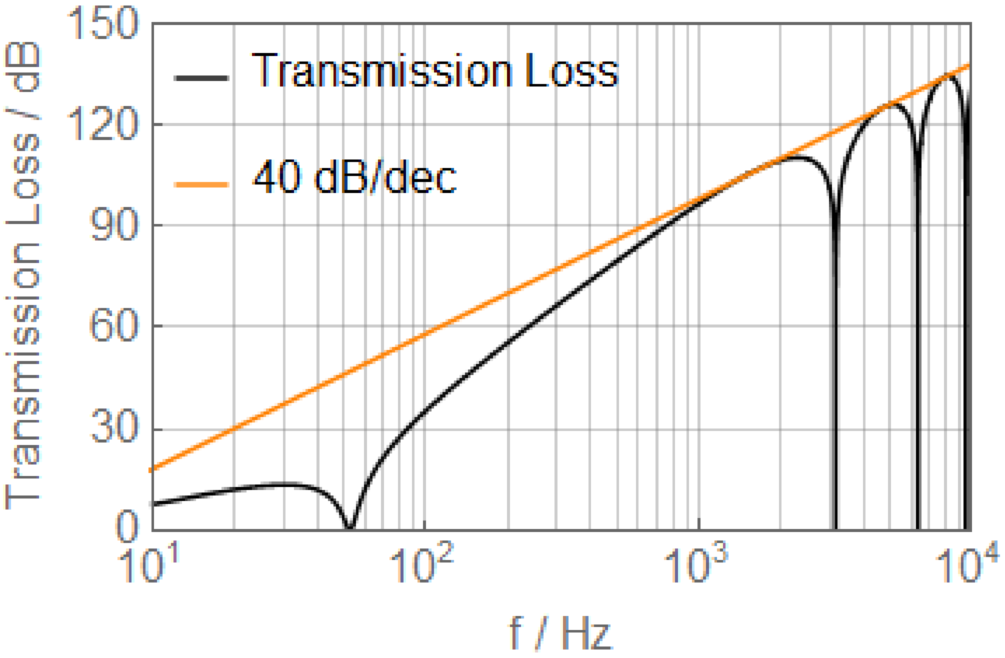
→|| double-limp-panel

$m_1 = 15\ \mathrm{kg/m^2}\quad m_2 = 15\ \mathrm{kg/m^2}\quad d = 0.23\ \mathrm{m}$



↘|| double-limp-panel at $\theta_i = \pi/24$

$m_1 = 15\ \mathrm{kg/m^2}\quad m_2 = 15\ \mathrm{kg/m^2}\quad d = 0.23\ \mathrm{m}$



- Higher sound speed shifts resonances to higher frequencies

Existing Theories

- **Fahy’s expression of transmission coefficient**[6],

$$T = -\frac{2j\rho^2 c^2 \sec^2\theta \sin(kd\cos\theta)}{z_1' z_2' \sin^2(kd\cos\theta) + \rho^2 c^2 \sec^2\theta}$$

$$z' = \underbrace{j\omega m + r}_{\text{panel impedance}} + \rho c \sec\theta\, [1 - j\cot(kd\cos\theta)]$$

limp panel + constant resistance

- **London’s panel impedance**[2]

$$Z_w = \frac{2r}{\cos\theta} + j\omega m\left(1 - \frac{f^2}{\underset{\downarrow}{f_c^2}}\sin^4\theta\right)$$

critical frequency

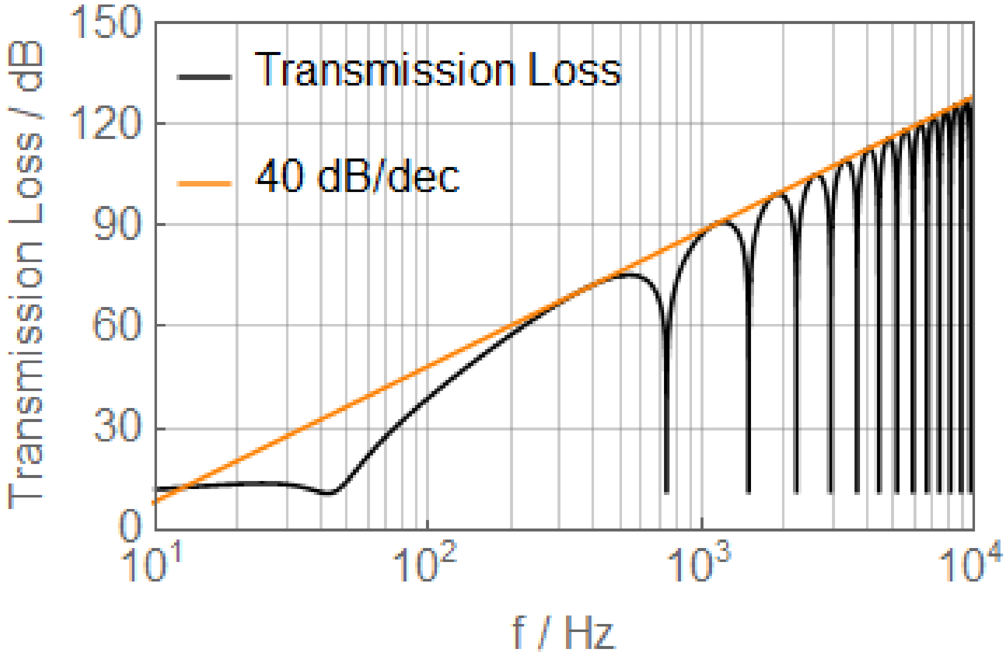
stiff panel + constant resistance

Existing Theories

With Fahy’s model and panel impedance

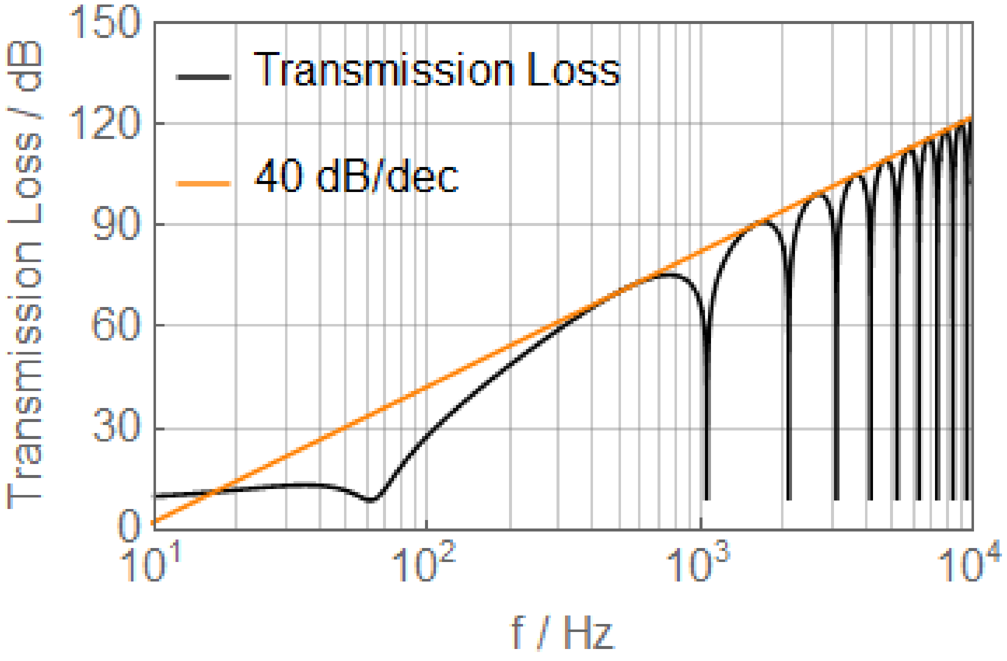
→|| double-limp-panel

$m = 15\ \mathrm{kg/m^2}$ $r = 1000\ \mathrm{kg/m^2 s}$ $d = 0.23\ \mathrm{m}$



↘|| double-limp-panel at $\theta_i = \pi/4$

$m = 15\ \mathrm{kg/m^2}$ $r = 1000\ \mathrm{kg/m^2 s}$ $d = 0.23\ \mathrm{m}$

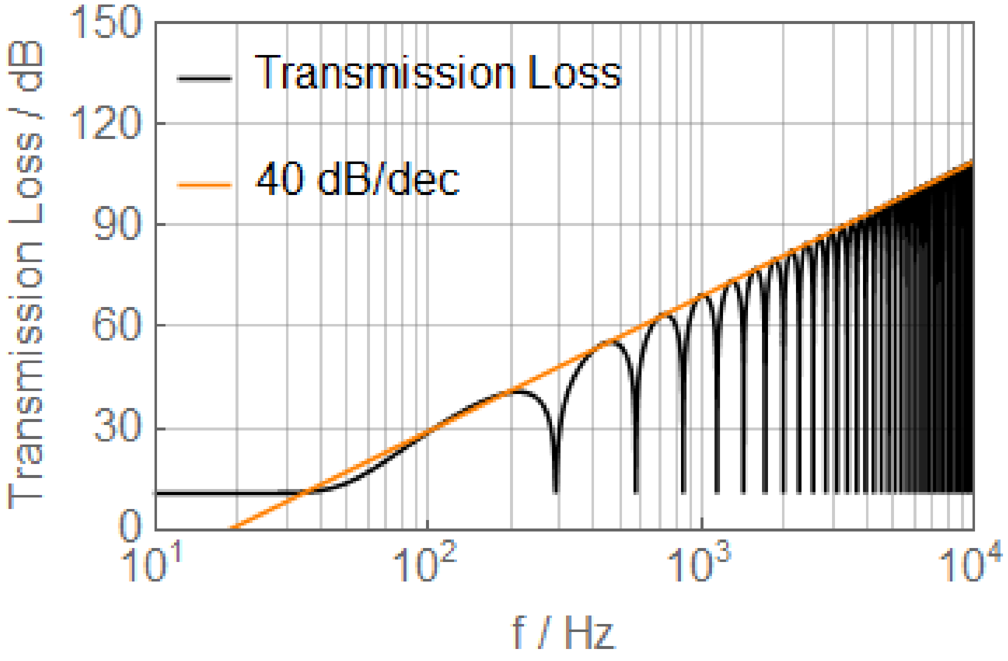


- Resistance in formulations means that minima do not go to zero

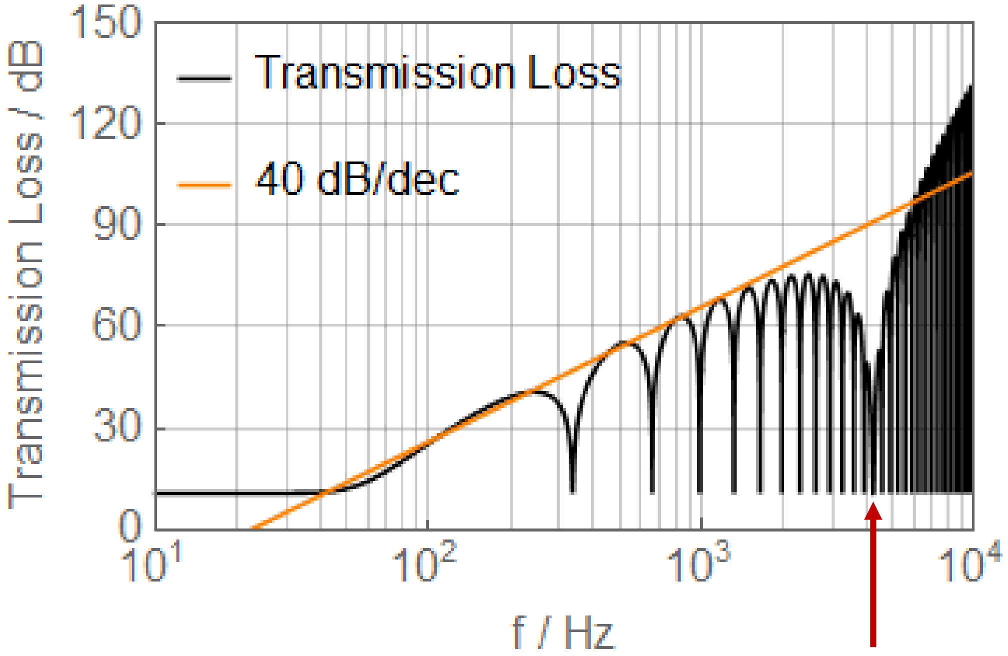
Existing Theories

With London’s model and panel impedance

→|| double-stiff-panel
$m = 5\ \mathrm{kg/m^2}\ \ r = 500\ \mathrm{kg/m^2s}\ \ d = 0.6\ \mathrm{m}$
$f_c = 1062\ \mathrm{Hz}$



↘|| double-stiff-panel at $\theta_i = \pi/6$
$m = 5\ \mathrm{kg/m^2}\ \ r = 500\ \mathrm{kg/m^2s}\ \ d = 0.6\ \mathrm{m}$
$f_c = 1062\ \mathrm{Hz}$



coincidence

- Minimum at coincidence frequency at oblique incidence
- Mass law no longer applies at frequencies higher than coincidence frequency

Existing Theories

- **Heckl's model with locally-reacting material between panels**[5],

$$Z = \rho c / \cos\theta$$

$$|T|^2 = \left| 1 \Big/ \left[1 - \omega^2 \frac{m_1' + m_2'}{2s} + j\omega \frac{m_1' + m_2'}{2Z} \left(1 - \omega^2 \frac{m_1' m_2'}{s(m_1' + m_2')} + \frac{Z^2}{s(m_1' + m_2')} \right) \right] \right|^2$$

stiffness per unit area in between

$$\left.\begin{aligned} m' &= m[1 - (k^4 D/\omega^2 m)\sin^4\theta] \\ D &= D'(1 + j\eta) \end{aligned}\right\}$$

stiff panel + hysteretic resistance

loss factor

Existing Theories

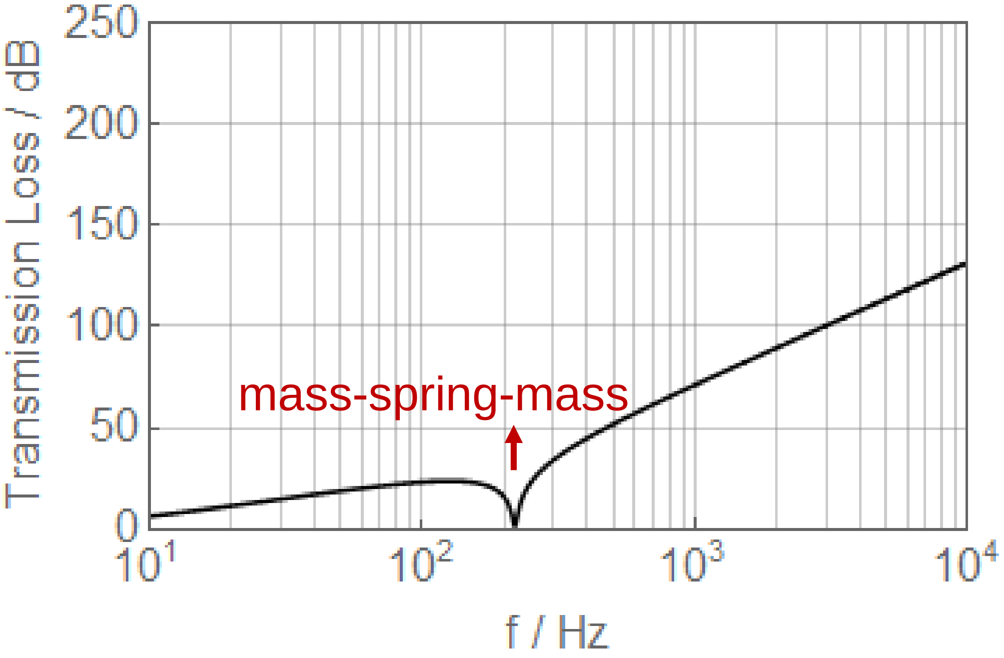
With Heckl's model

→|| double-stiff-panel

$m_1 = 8\ \mathrm{kg/m^2}\ \ D_1 = 20000\ \mathrm{kg \cdot m^2/s^2}\ \ \eta_1 = 0.0015$

$m_2 = 16\ \mathrm{kg/m^2}\ \ D_2 = 10000\ \mathrm{kg \cdot m^2/s^2}\ \ \eta_2 = 0.001$

$d = 0.6\ \mathrm{m}\ \ s = 1 \times 10^7\ \mathrm{kg/s^2m^2}$

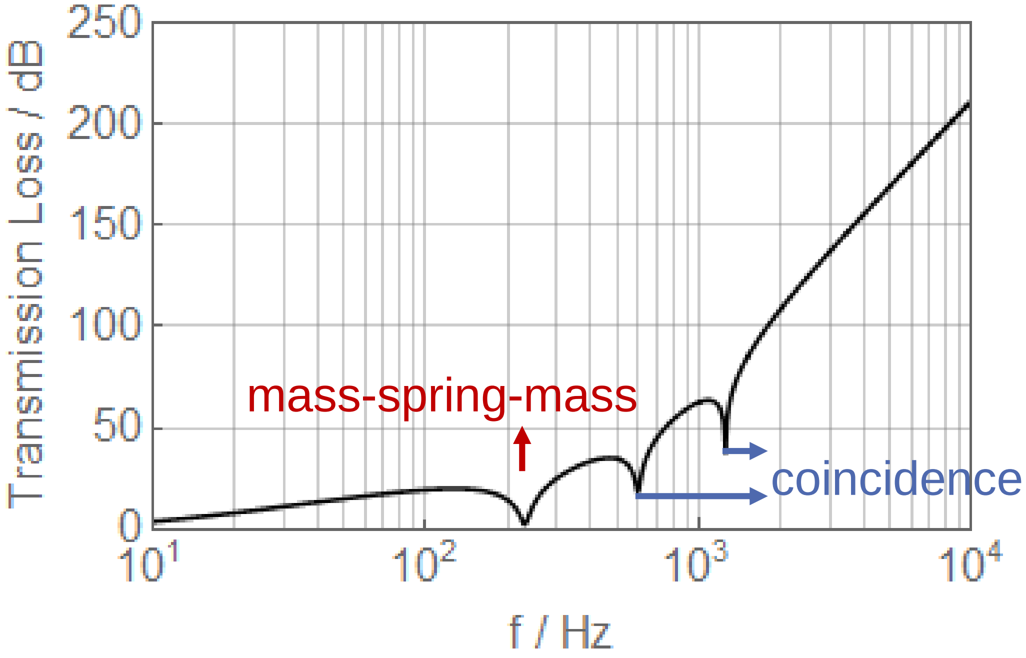


↘|| double-stiff-panel at $\theta_i = 50°$

$m_1 = 8\ \mathrm{kg/m^2}\ \ D_1 = 20000\ \mathrm{kg \cdot m^2/s^2}\ \ \eta_1 = 0.0015$

$m_2 = 16\ \mathrm{kg/m^2}\ \ D_2 = 10000\ \mathrm{kg \cdot m^2/s^2}\ \ \eta_2 = 0.001$

$d = 0.6\ \mathrm{m}\ \ s = 1 \times 10^7\ \mathrm{kg/s^2m^2}$



- No wave propagation between panels, so inter-panel resonances are absent

Existing Theories

- **Hamada and Tachibana's transfer matrix method**[7],

$$\mathbf{F}_{\boldsymbol{\theta}} = \begin{bmatrix} A_\theta & B_\theta \\ C_\theta & D_\theta \end{bmatrix} = \mathbf{F}_{1\theta}\mathbf{F}_{A\theta}\mathbf{F}_{2\theta}$$

The matrices for panels,

$$\mathbf{F}_{i\theta} = \begin{bmatrix} 1 & Z_{i\theta} \\ 0 & 1 \end{bmatrix} \rightarrow \text{panel impedance}$$

The matrix for the air gap,

$$\mathbf{F}_{A\theta} = \begin{bmatrix} \cos(kd\cos\theta) & \frac{j\rho c}{\cos\theta}\sin(kd\cos\theta) \\ \frac{j\cos\theta}{\rho c}\sin(kd\cos\theta) & \cos(kd\cos\theta) \end{bmatrix}$$

Asymptotic Behavior

- **The stiffness + hysteretic damping impedance introduced by Cremer**[10],

loss factor ←

$$Z_w = \underbrace{\frac{\eta D}{\omega} k^4 \sin^4\theta}_{\Re Z_w} + \underbrace{j\left(m\omega - \frac{D}{\omega} k^4 \sin^4\theta\right)}_{\Im Z_w}$$

Substitute into generalized London's model,

$$\left|\frac{1}{T}\right|^2 = \{1 + \alpha(\Re Z_1 + \Re Z_2) + \alpha^2[(1 - \cos 2\beta)(\Re Z_1 \Re Z_2 - \Im Z_1 \Im Z_2) - \sin 2\beta\,(\Re Z_1 \Im Z_2 + \Re Z_2 \Im Z_1)]\}^2$$

$$+\{\alpha(\Im Z_1 + \Im Z_2) + \alpha^2[(1 - \cos 2\beta)(\Re Z_1 \Im Z_2 + \Re Z_2 \Im Z_1) + \sin 2\beta\,(\Re Z_1 \Re Z_2 - \Im Z_1 \Im Z_2)]\}^2$$

with $\alpha = \cos\theta\,/2\rho c$, $\beta = kd\cos\theta$

Asymptotic Behavior

- when $\cos 2\beta = -1$ (maxima of transmission loss)

$$\left|\frac{p_i}{p_t}\right|^2 \approx 4\alpha^4\left[\left(\frac{\eta_1 D_1 k_x^4}{\omega}\right)^2 + \left(\omega m_1 - \frac{D_1 k_x^4}{\omega}\right)^2\right]\left[\left(\frac{\eta_2 D_2 k_x^4}{\omega}\right)^2 + \left(\omega m_2 - \frac{D_2 k_x^4}{\omega}\right)^2\right] = \mathrm{O}(\omega^{12}) \quad 120\ \mathrm{dB/dec}$$

$$k_x = k \sin\theta$$

corresponding to $d/\lambda = 1/4,\ 3/4,\ 5/4$, etc. at normal incidence

- when $\cos 2\beta = 1$ (minima of transmission loss)

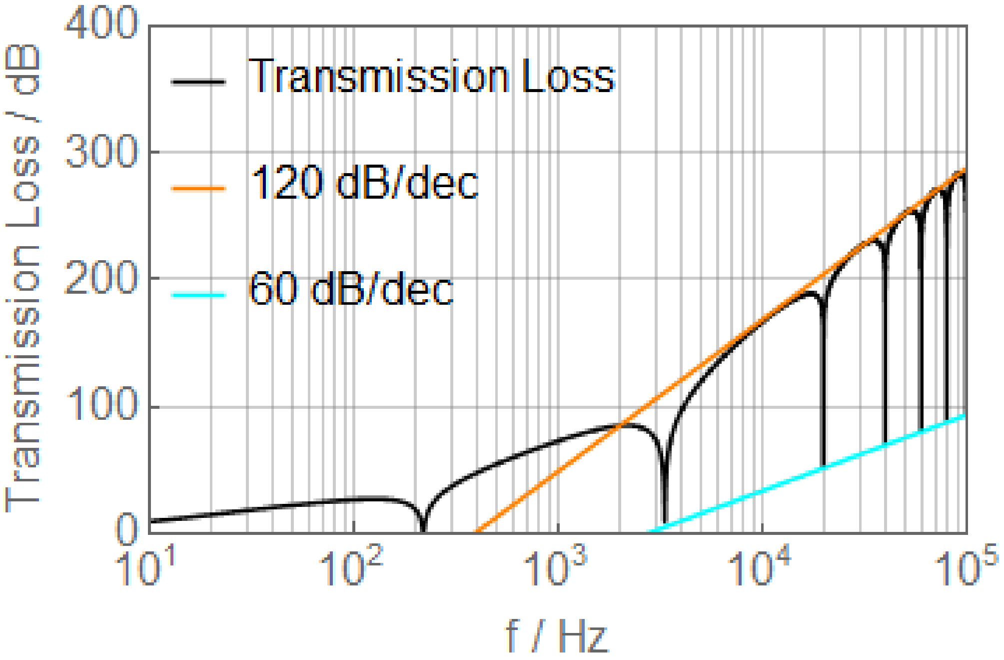
$$\left|\frac{p_i}{p_t}\right|^2 = \left[1 + \frac{\alpha k_x^4}{\omega}(\eta_1 D_1 + \eta_2 D_2)\right]^2 + \alpha^2\left[\omega(m_1 + m_2) - \frac{k_x^4}{\omega}(D_1 + D_2)\right]^2 = \mathrm{O}(\omega^6) \quad 60\ \mathrm{dB/dec}$$

corresponding to $d/\lambda = 1/2,\ 1,\ 3/2$, etc. at normal incidence

Asymptotic Behavior

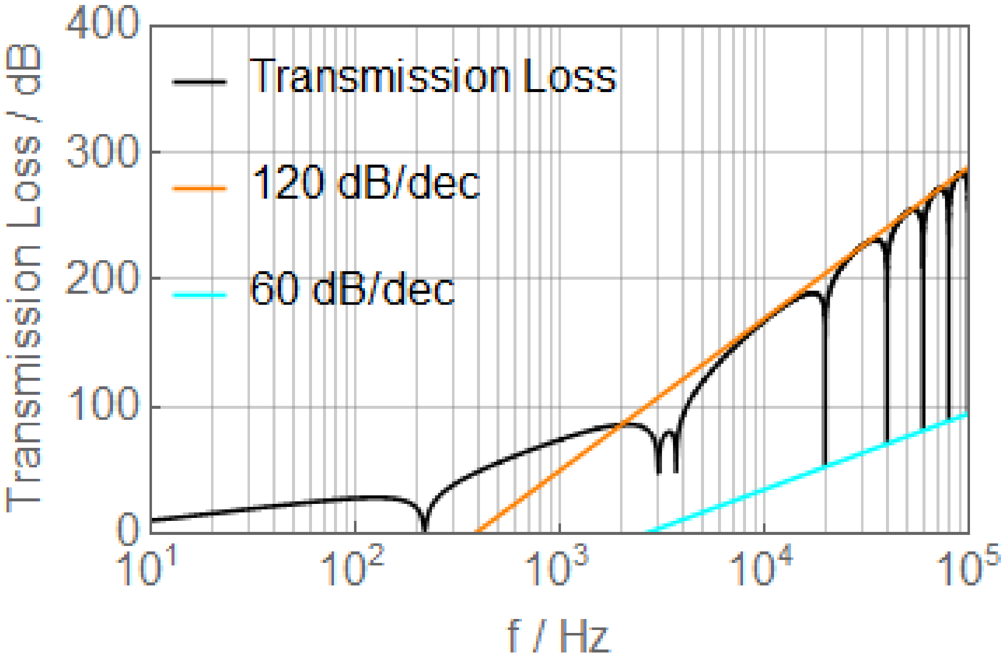
↘|| at $\theta_i = \pi/6$

$m_1 = 20\ \mathrm{kg/m^2}\quad D_1 = 10000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_1 = 0.002$
$m_2 = 20\ \mathrm{kg/m^2}\quad D_2 = 10000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_2 = 0.002$
$d = 0.01\ \mathrm{m}\quad f_{c1} = f_{c2} = 823\ \mathrm{Hz}$



↘▌| at $\theta_i = \pi/6$

$m_1 = 15\ \mathrm{kg/m^2}\quad D_1 = 9000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_1 = 0.003$
$m_2 = 30\ \mathrm{kg/m^2}\quad D_2 = 12000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_2 = 0.001$
$d = 0.01\ \mathrm{m}\quad f_{c1} = 751\ \mathrm{Hz}\ f_{c2} = 920\ \mathrm{Hz}$



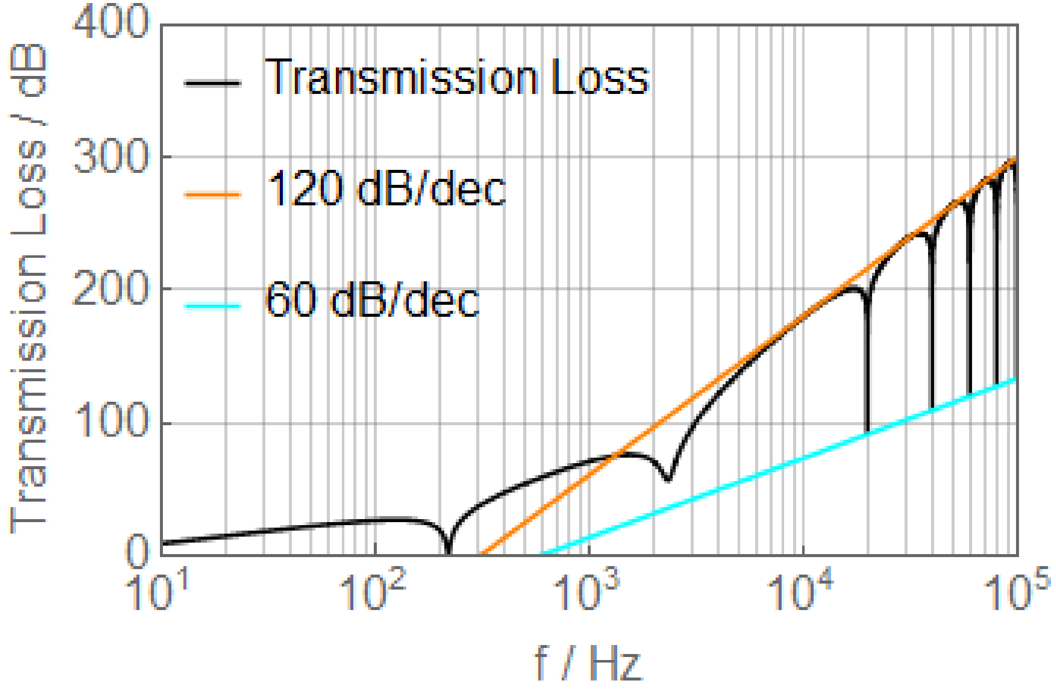
Asymptotic Behavior

$\searrow||$ at $\theta_i = \pi/6$

$m_1 = 20\ \mathrm{kg/m^2}\quad D_1 = 20000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_1 = 0.1$

$m_2 = 20\ \mathrm{kg/m^2}\quad D_2 = 20000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_2 = 0.1$

$d = 0.01\ \mathrm{m}$



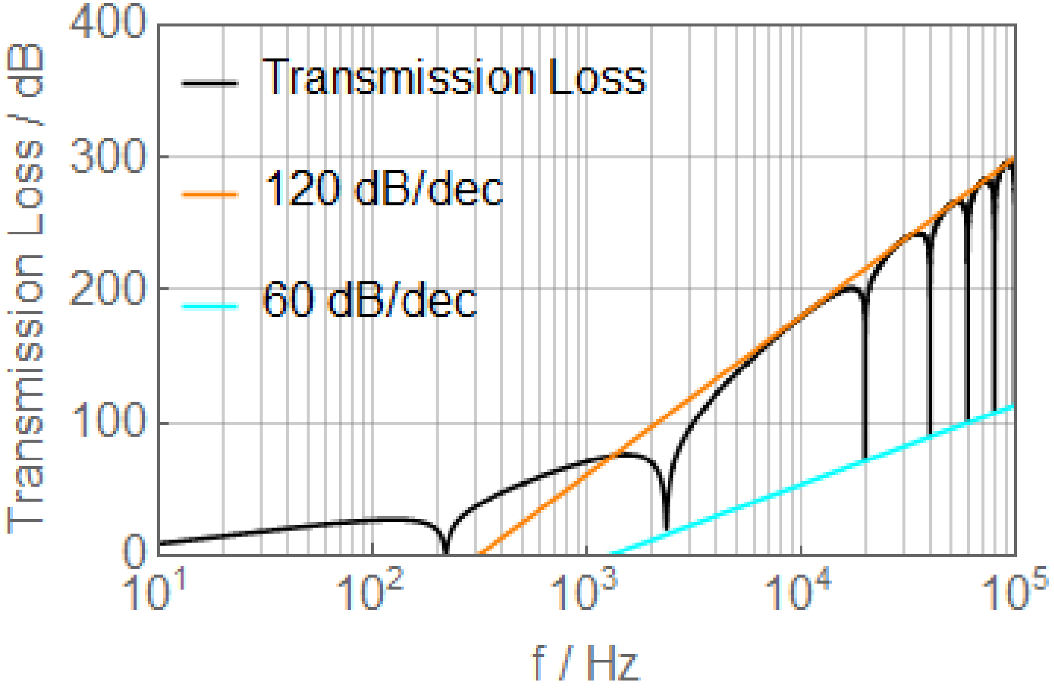
Asymptotic Behavior

$\searrow||$ at $\theta_i = \pi/6$

$m_1 = 20\ \mathrm{kg/m^2}\quad D_1 = 20000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_1 = 0.01$

$m_2 = 20\ \mathrm{kg/m^2}\quad D_2 = 20000\ \mathrm{kg\cdot m^2/s^2}\quad \eta_2 = 0.01$

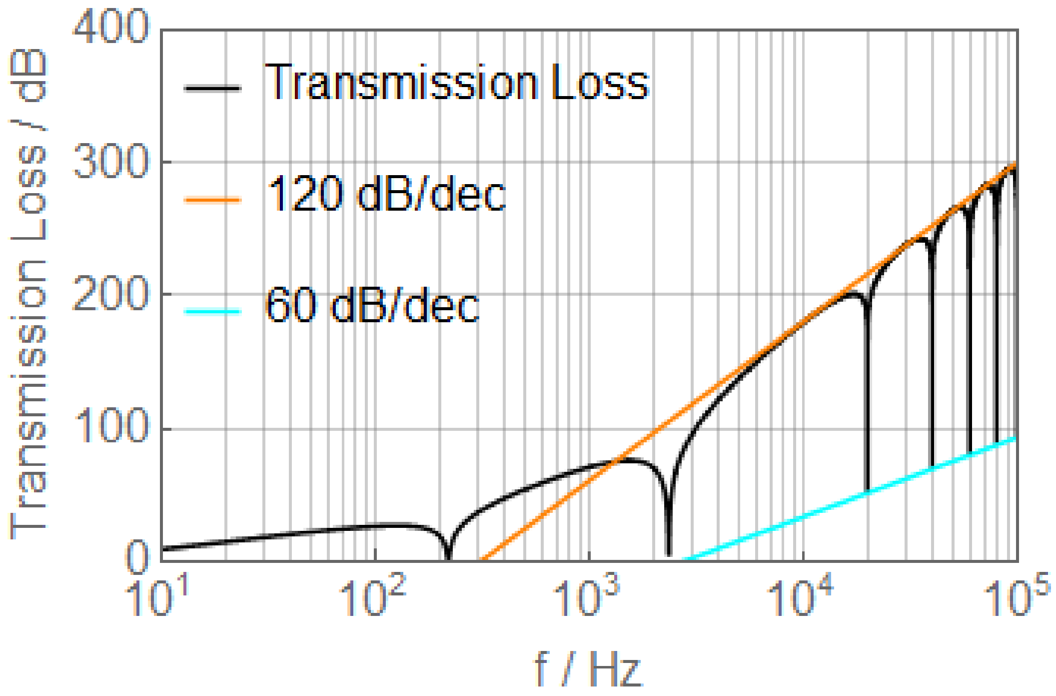
$d = 0.01\ \mathrm{m}$



Asymptotic Behavior

↘$||$ at $\theta_i = \pi/6$

$m_1 = 20\ \mathrm{kg/m^2}$ $D_1 = 20000\ \mathrm{kg \cdot m^2/s^2}$ $\eta_1 = 0.001$
$m_2 = 20\ \mathrm{kg/m^2}$ $D_2 = 20000\ \mathrm{kg \cdot m^2/s^2}$ $\eta_2 = 0.001$
$d = 0.01\ \mathrm{m}$



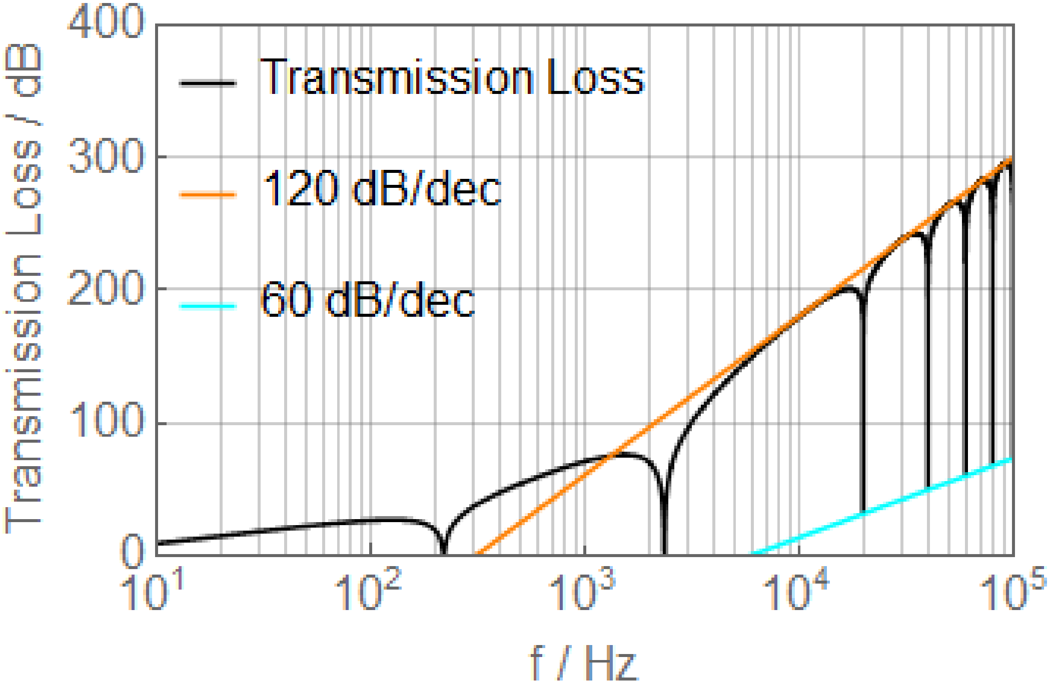
Asymptotic Behavior

↘|| at $\theta_i = \pi/6$

$m_1 = 20\ \mathrm{kg/m^2}\quad D_1 = 20000\ \mathrm{kg \cdot m^2/s^2}\quad \eta_1 = 1 \times 10^{-4}$

$m_2 = 20\ \mathrm{kg/m^2}\quad D_2 = 20000\ \mathrm{kg \cdot m^2/s^2}\quad \eta_2 = 1 \times 10^{-4}$

$d = 0.01\ \mathrm{m}$



The 60 dB/dec line shifts toward high frequencies as loss factor decreases

Porous Lining

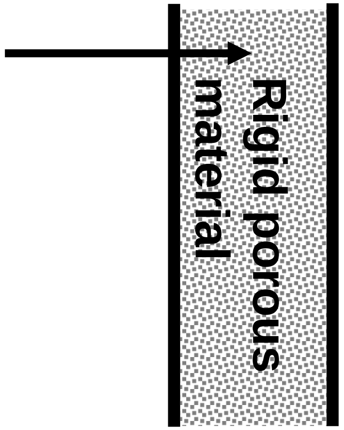
- **The resistance in system will suppress the dips in transmission loss**

↘|| at $\theta_i = \pi/6$

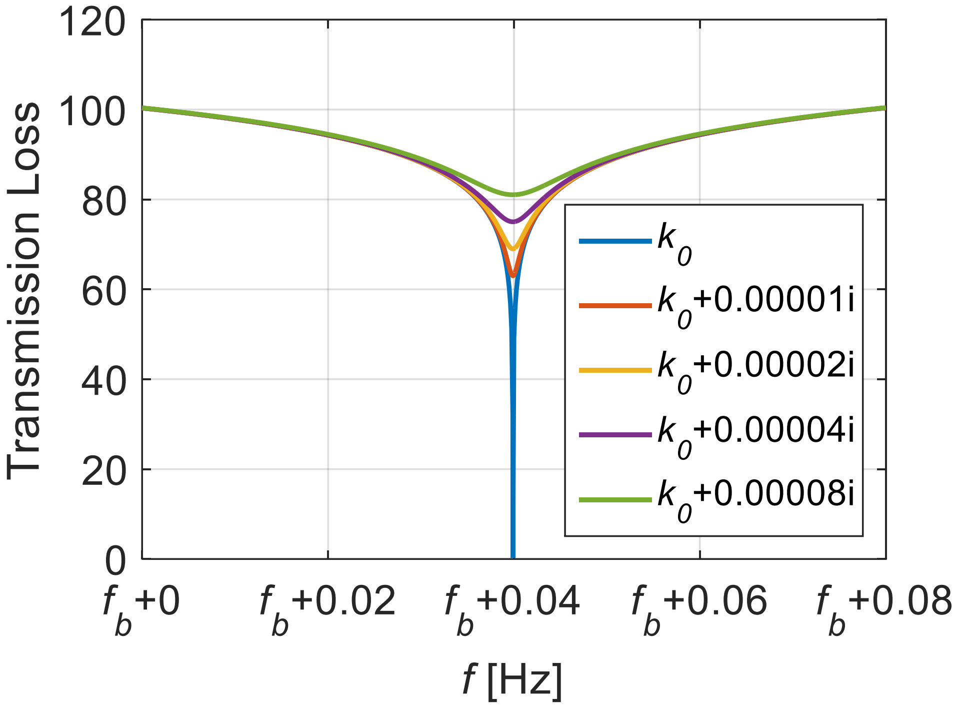
$m = 20\ \mathrm{kg/m^2}$ $D = 10000\ \mathrm{kg \cdot m^2/s^2}$ $\eta_1 = 0$

$d = 0.01\ \mathrm{m}$

$f_b = 19629.8\ \mathrm{Hz}$



resistance brought by imaginary part of wavenumber



So we are primarily interested in the maximum transmission loss behavior

Porous Lining

- A layer of porous material described with,

Flow Resistivity	Porosity	Tortuosity	VCL	TCL	Solid Density
1.5×10^5 rayls/m	0.97	1.5	$20\ \mu\text{m}$	$40\ \mu\text{m}$	$2000\ \text{kg/m}^3$

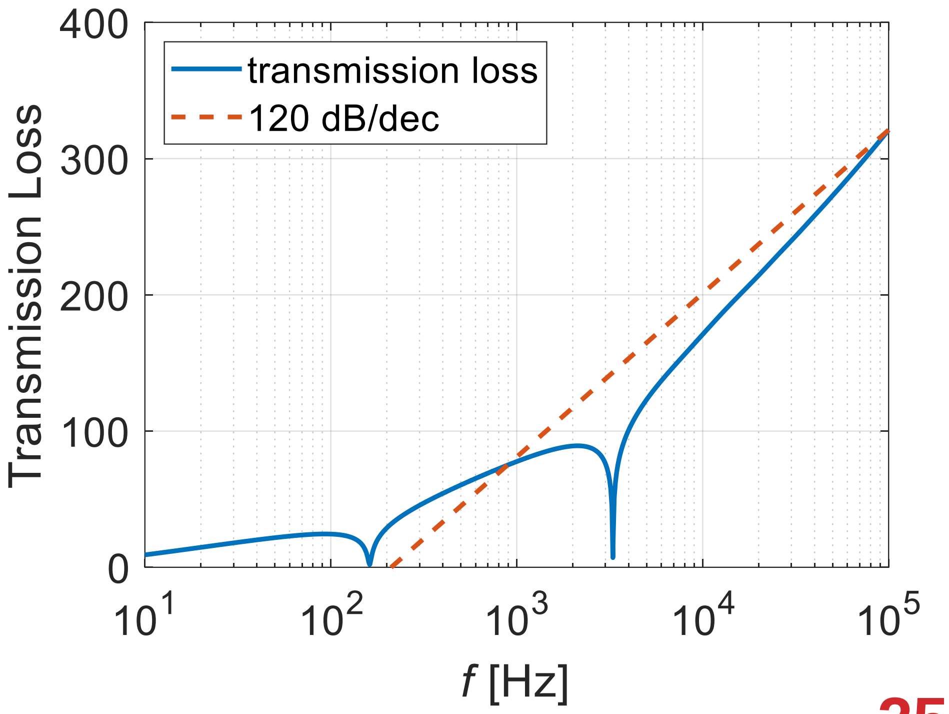
- Effective density and wavenumber calculated with JCA-Limp model[11, 12]


at $\theta_i = \pi/6$

$m = 20\ \text{kg/m}^2 \quad D = 10000\ \text{kg}\cdot\text{m}^2/\text{s}^2 \quad \eta_1 = 0$

$d = 0.01\ \text{m}$

- The increase rate is now greater than 120 dB/dec



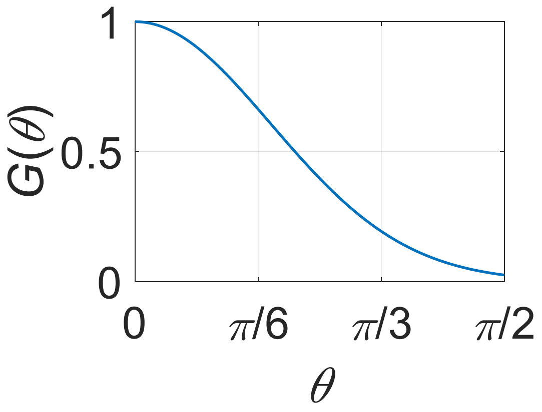
Average Transmission

- **Kang et al. proposed an approach of calculating average transmission loss**[12]

$$\tau(\omega) = \frac{\int_0^{\pi/2} G(\theta)\,|T(\omega,\theta)|^2 \sin\theta\cos\theta\,d\theta}{\int_0^{\pi/2} G(\theta)\sin\theta\cos\theta\,d\theta}$$

- **A distribution function for incident energy versus incidence angle is applied**

$$G(\theta) = e^{-\zeta\theta^2}$$

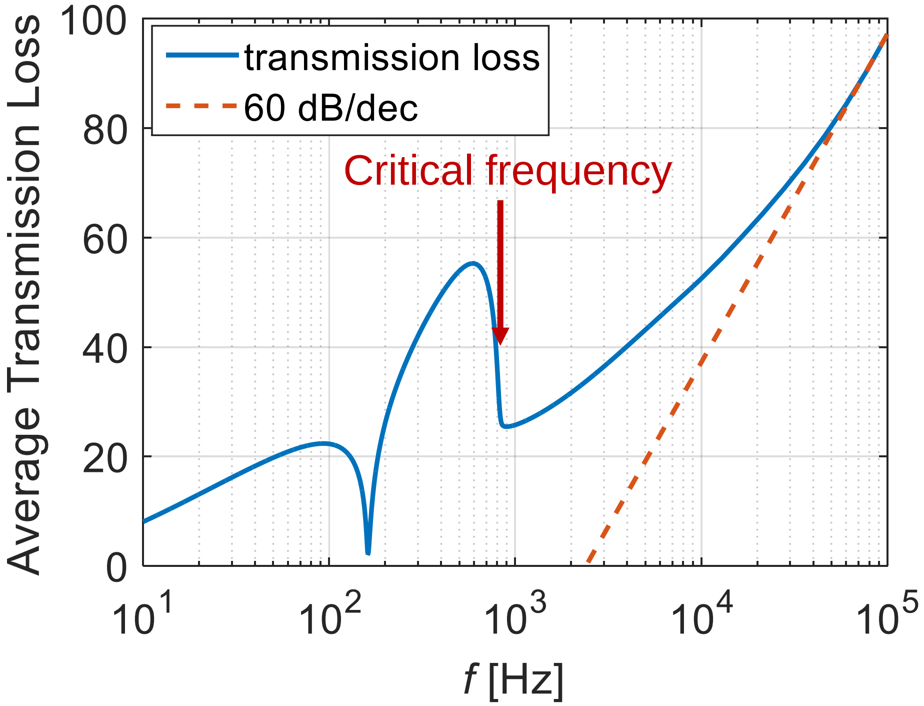


- **The average transmission loss**

$$\mathrm{TL}(\omega) = 10\log_{10}\left[\frac{1}{\tau(\omega)}\right]$$

Average Transmission

- With $\zeta = 1.5$, the average transmission loss of the double panel system with porous material inside in the previous case was calculated,



- A drop of transmission loss occurs at critical frequency $f_c = 823\ \mathrm{Hz}$

Conclusions

- Classic models were reviewed

- Asymptotic behavior of double-stiff-panel systems at oblique incidence were studied
  - The peaks of the transmission loss increases at 120 dB/dec
  - The minima of the transmission loss increases at 60 dB/dec
  - The minima shift to higher frequencies as hysteretic damping decreases

- Porous lining between panels will suppress the resonance pattern of double panels and change the transmission loss increase rate

- Average transmission loss was obtained with Gaussian distribution applied

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