Analysing Fish Behaviours Using Three-Dimensional Qualitative Trajectory Calculus

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Abstract. Fish swim freely in water and their 3D spatial interactions may carry important information for biological study. The two-dimensional Qualitative Trajectory Calculus (QTC_{2D}), a spatiotemporal calculus, is a method for representing and reasoning about movements of objects in a qualitative framework. This emerging technique encodes the spatial geometric relationships between distinct objects, such as machines, humans or animals. This paper presents a method for generalising QTC_{2D} into 3D space, called $3DQTC$, as a work-in-progress. The $3DQTC$ method is based on a geometrical analysis method of estimating the 3D orientation of fish. Initial results indicate the potential of representing and analysing the fish behaviours in 3D space. $3DQTC$ is also demonstrated as a means of extracting useful information (e.g. *follow* and *non-follow* behaviours) which cannot be achieved by QTC_{2D}. We conclude by discussing further work and development.

1 Introduction

The development of computer vision as a method for automatically analysing human activities is a well-established research area. However, applications to the analysis of animal behaviours are relatively rare. The three-spined stickleback fish (*Gasterosteus aculeatus*) is a model species, used for behavioural studies in laboratories around the world [1]. These species are widely used for studies in behavioural ecology [1] as they are relatively easy to collect, to house and manipulate in the laboratories. Stickleback fish have similar shape, colour and size which makes the tracking and identification process a challenge. Current methods of studying their social behaviours rely on manual observation and recording, which is very time-consuming, error prone, limiting on the amount of data which can be collected, and applicable only to small number of fish.

Qualitative Trajectory Calculus (QTC) is "a calculus for representing and reasoning about movements of objects in a qualitative framework" [2]. The QTC was developed to represent the relative movements between two disjoint objects in 1D and 2D space. In our current work, QTC_{2D} provides a basis of representing and analysing fish movements and interactions.

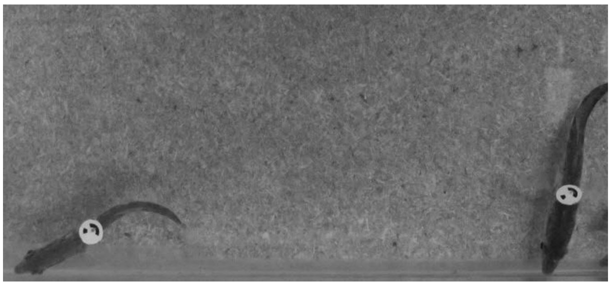


Fig. 1. Pair of three-spined sticklebacks with a unique circular tags.

Existing methods apply the QTC to represent the spatial interactions between moving objects in 2D space. In this paper, we present our new method for generalising Qualitative Trajectory Calculus into 3D Space (called $3DQTC$) based on the geometric method of estimating the 3D orientation, as presented in [3]. We combine the geometrical analysis method [4] and our Reprojection method [3] to define the three-dimensional interactions between a pair of fish. Then, a new variant of QTC ($3DQTC$) was constructed based on the 3D information. The aim of this work is to extract useful information which cannot be achieved by QTC_{2D} such as: pairs of fish swim on same or different water surface. Our system, and for the first time, represent and capture the fish relative movements in 3D space and their behaviours using qualitative representations. We also present a novel fish identification, tracking and analysis method in 3D space which automatically provides accurate measurements for biologists studying behaviour of these fish, providing much larger and more robust data sets than can be gathered manually. We use a circular marker system (tagging system) presented in [1, 3] to identify individuals; these tags are attached to the dorsal spine, and our system estimates their position and orientation over a period of time. Each circular marker (tag) has a unique pattern. Fig. 1 shows a sample image of three-spined sticklebacks.

2 Related Work

2.1 Fish Monitoring

There has been some previous work describing video-based systems for automated (or semi-automated) processing of fish behaviour. Zhu [5] studied a stereo-vision based real-time tracking method to monitor the 3D behaviour of aquatic animals. However, this method is used over relatively short time periods only. A model-based approach, which estimates the dimensions of free-swimming fish, was developed in [6], but requires manual operator inputs. A machine vision system that automatically analyses underwater videos for counting fish was presented in [7], while Kane et al. [8] developed a movement analysis system to measure variables such as velocity, distance and space utilisation of fish in tanks. An online monitoring system for fish behaviour was presented in [9]; the system detected abnormal behaviour. However, the methods in [8, 9] are either limited to the tracking a single fish, or are only applicable to relatively short time periods (< 15 minutes). Recently, the method in [10] investigated individual level interactions between shoaling sticklebacks. The method manually estimates the 2D positions of sticklebacks to analysis their interactions. This method is time-consuming, error prone, and the amount of data which can be collected is limited.

2.2 Qualitative Spatial and Temporal Reasoning

Qualitative spatial-temporal reasoning is an approach for dealing with knowledge on which human perception of relative interactions is based without using numerical computation [2]. There has been previous work introduced in qualitative spatial and temporal calculi such as Qualitative Trajectory Calculus (QTC) [2]. The QTC describes and encodes the interactions between Moving Point Objects (MPOs) in a qualitative way. It reduces the continuum to three qualitative values (or symbols) -, 0 and +. In the case where the changing in the distance between two MPOs is considered; the symbol - means a decrease in distance between both objects, + an increase in distance and 0 if the distance remains the same.

Different types of QTC have been proposed depending on the level of details and the number of spatial dimensions: QTC Basic (QTC_B) considers only the changing distance between two MPOs and QTC Double Cross (QTC_C) consider the direction in which an object is moving with respect to the segment line connecting between the two objects. Given the positions of two moving objects (called Obj_1 and Obj_2); the QTC represents the relative motion between the two objects at instant time as follow:

1. $Code_1$: movement of Obj_1 with respect to Obj_2
   - : Obj_1 approaching Obj_2
   + : Obj_1 moving further away from Obj_2
   0 : Obj_1 is stable with respect to Obj_2
2. $Code_2$: movement of Obj_2 with respect to Obj_1
   similar to $Code_1$ but with the Obj_1 and Obj_2 swapped.
3. $Code_3$: Relative speed of Obj_1 with respect to Obj_2 (which dually represents the relative speed of Obj_2 with respect to Obj_1):
   - : Obj_1 slower than Obj_2
   + : Obj_1 faster than Obj_2
   0 : Obj_1 and Obj_2 move with the same speed
4. $Code_4$: Movement of Obj_1 with respect to the reference line L
   - : Obj_1 moves to the left of L
   + : Obj_1 moves to the right of L
   0 : Obj_1 move a long the line L
5. $Code_5$: Movement of Obj_2 with respect to the reference line L
   similar to $Code_4$ but with the Obj_1 and Obj_2 swapped.

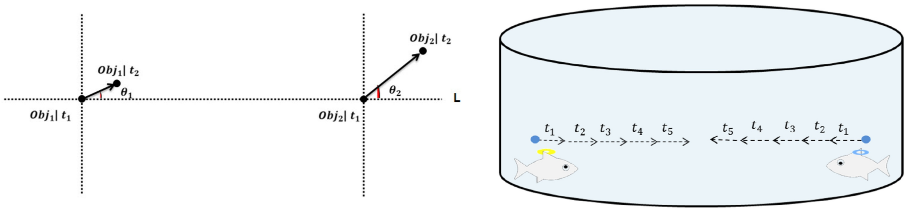
6. $Code_6$: θ_1 the minimal angle between the velocity vector of Obj_1 and vector L, and θ_2 the equivalent for Obj_2
   - : $\theta_1 < \theta_2$
   $+$: $\theta_1 > \theta_2$
   0 : $\theta_1 = \theta_2$

where $Code_i$ represents the qualitative relations in QTC and L is the line connecting between the two objects at time t_1. Fig. 2(a) illustrates the concept of qualitative relations for two disjoint objects. The two types, QTC_B and QTC_C, have been defined into different subsets as follow: QTC_{B21} contains ($Code_1$ and $Code_2$) and QTC_{B22} contains ($Code_1$, $Code_2$ and $Code_3$). QTC_{C21} contains ($Code_1$, $Code_2$, $Code_4$ and $Code_5$) and QTC_{C22} contains $Code_1$ through $Code_6$. The combinations of the six codes results in $3^6 = 729$ QTC states, whereas QTC_{C22} has only 305 possible states [2]. There has been some applications for QTC such as: moving vehicles [11] and human-robot interaction [12]. A detailed description for QTC relations in the domain of fish interactions is presented in Section 3.

Most of the existing methods and applications of QTC are in 2D space. However, one existing work has been presented for QTC in 3D space by Mavridis et al. [13]. The method developed in [13] generalises QTC into 3D space based on transformations of the Frenet-Serret frames. Two Frenet-Serret frames (t_1, t_2) are used to represent the two moving points. Each frame consists of *tangent (t)*, *normal (n)*, and *binormal (b)* vectors. The method calculate the Euler angles $A_{ng} \in (yaw, pitch$, and $roll)$. Then, the three Euler angles are mapped into qualitative symbols $\{-, 0, +\}$. In [13], this process is applied for modeling bird flight in 3D space. However, this method has a limitation in real applications and from mathematical point of view: the Fernet frame has an inherent drawback in that it is undefined at points where the curvature is zero [14]. Therefore, when the object moves in a straight-line (collinear curve) or remains stationary, the method will fail in representing the relative movement. Moving in a straight-line or staying stationary is a common state in object behaviours such as fish. For example, the "Approach" fish behaviour, where one fish moves toward another fish while the other fish standing still.

3 Representing Fish Behaviour Using QTC_{2D}

The use of qualitative spatial representations is an adequate and powerful method to abstract a large number of possible objects interactions (e.g. fish) or scenarios such as "one fish follow another fish". In this section we introduce QTC_{2D} as a method for representing the spatial interactions between a pair of fish. We have developed an automated visual tracking method which estimates the pose of the tags (Fig. 1), fish position and orientation, from monocular video. Our method is fully described in [3], and comprises the following components:
Camera Calibration: The camera parameters (focal length f, principle point of the image plane and lens distortion factors) are estimated using [15].
Image Enhancement: An adaptive background mixture model with shadow



(a) (b)

Fig. 2. (a) Example of QTC relations between Obj_1 and Obj_2. (b) Example of spatial interactions between two fish (converge behaviour) during the time interval t_1 to t_5.

detection [16] is used to locate moving objects and eliminate the image noise.
Circular Feature Edge Detection: The Canny edge detector method [17] is used to detect elliptical shapes of the circular tag as a set of pixel edge points.
Lens Distortion Compensation: Radial lens distortion factors obtained in camera calibration process [15] are used to find accurate positions of edge points.
2D Position Estimation: The direct least square ellipse fitting method [18] is applied to estimate the basic parameters of the elliptical projection of the tag. The center of the tag represents 2D fish position.
3D Orientation Estimation: Our Reprojection method [3] is applied to estimate the 3D orientation of the circular tag. Fig. 3, shows the schematic representation of 3D orientation estimation of fish swimming.
Tag Identification: The method in [1] is used to read the tag pattern (fish ID).

We adopt QTC_{2D} to represent and reason about fish movements in a free Euclidean space. The spatial position of the fish movement is abstracted to a single point, which represents the tag centroid. As an example, consider the interaction between two fish during a given time interval as shown in Fig. 2(b), which represents a section of real data from our dataset. The interaction is captured using QTC representations: both fish are converging during the time interval $[t_1, t_5]$. This interaction is described as follow using QTC state sequences. Using QTC_{B21}: $(-\ -)_{t1} \rightsquigarrow (-\ -)_{t2} \rightsquigarrow (-\ -)_{t3} \rightsquigarrow (-\ -)_{t4} \rightsquigarrow (-\ -)_{t5}$. The QTC_{C21}: $(-\ \ -\ \ 0\ 0)_{t1} \rightsquigarrow (-\ \ -\ \ 0\ 0)_{t2} \rightsquigarrow (-\ \ -\ \ 0\ 0)_{t3} \rightsquigarrow (-\ \ -\ \ 0\ 0)_{t4} \rightsquigarrow (-\ \ -\ \ 0\ 0)_{t5}$. The interaction may also be described with more details using QTC_{C22} by including the speed and angle features, but the expansion is omitted.

4 Three-Dimensional QTC

This section presents our novel method (called $3DQTC$) of generalising QTC into 3D space. In Section 2.2, we defined four QTC_{2D} features which map onto the six codes: distance:$\{Code_1, Code_2\}$; side:$\{Code_4, Code_5\}$; speed:$\{Code_3\}$; and angle:$\{Code_6\}$. The distance, speed and angle features have equivalents in 3D space, and can be easily generalised by defining these features in 3D space. However, there is no equivalent side feature $(Code_4, Code_5)$. The main challenge of generalising 2D QTC into 3D QTC is that in 2D space a unique line connecting

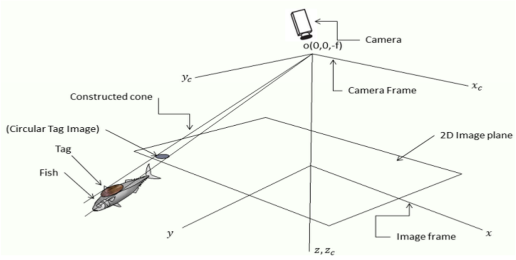


Fig. 3. Schematic representation of 3D orientation estimation of fish swimming.

the two moving points can be drawn, and a plane which define the left/right regions (Fig. 2(a)). However, such a unique plane cannot be defined between two points in 3D space. We propose to use the geometrical analysis method [4] and our Reprojection method [3] to define the direction movement between a pair of fish in 3D space. We use our Reprojection method [3] to estimate the 3D orientation (α_j, β_j, and γ_j) of each fish. This orientation values are then used to represent the analogous *side* feature (called *orientation feature*) in 3D space. The *orientation feature* will encode the relative change in the three angles α_j, β_j, and γ_j of a pair of fish.

Definition: Given the 3D orientations (V_1, V_2) and positions (P_1, P_2) of two moving objects in 3D space (called Obj_1 and Obj_2), where $V_1 = [\alpha_1, \beta_1, \gamma_1]^T$ and $V_2 = [\alpha_2, \beta_2, \gamma_2]^T$ are the 3D orientations of Obj_1 and Obj_2, respectively. The $3DQTC$ represents the relative motion between Obj_1 and Obj_2 at time instant t as follows:
First, we define the distance:$\{Code_1, Code_2\}$; speed:$\{Code_3\}$; and angle:$\{Code_6\}$ features similarly to QTC_{2D} generalised into 3D space. We calculate the symbols $\{-, 0, +\}$ for all three features. Secondly, we define a new feature called "*orientation feature*" as follows:
orientation feature: The relative change of the orientation between Obj_1 and Obj_2 can be defined as the differences $d_\alpha = (\alpha_1 - \alpha_2); d_\beta = (\beta_1 - \beta_2)$; and $d_\gamma = (\gamma_1 - \gamma_2)$ between V_1 and V_2 components. This can be described using new three codes $(Code_7, Code_8, Code_9)$ as follows:

7. $Code_7$:	8. $Code_8$:	9. $Code_9$:
$- : d_\alpha < -\kappa$	$- : d_\beta < -\kappa$	$- : d_\gamma < -\kappa$
$+ : d_\alpha > \kappa$	$+ : d_\beta > \kappa$	$+ : d_\gamma > \kappa$
$0 : -\kappa \leq d_\alpha \leq \kappa$	$0 : -\kappa \leq d_\beta \leq \kappa$	$0 : -\kappa \leq d_\gamma \leq \kappa$

where $Code_i$ represents the qualitative relations in $3DQTC$, and κ is a threshold. In the new variant of QTC ($3DQTC$) there exist 7 codes $(Code_1, Code_2, Code_3, Code_6, Code_7, Code_8, Code_9)$ represented in $3^7 = 2187$ combinations of symbols, and $3^3 = 27$ possible combinations of symbols for the *orientation feature*.

Note that the codes: $Code_1$, $Code_2$, $Code_3$, $Code_6$ are analogous to the ones in QTC_{2D}. On the other hand, the three codes: $Code_7$, $Code_8$, $Code_9$ are the ones differentiate between $3DQTC$ from QTC_{2D}. Therefore, in our fish experiment, the focus will be on using these triple features to capture fish interactions which can not be captured using QTC_{2D}.

Table 1. QTC_{B21} relations occurrences and duration for two fish in the video.

QTC Relation	Occurrences	Duration(s)
(- +)	12170	811.4
(+ -)	4604	307
(0 -)	4943	198.3
(- -)	3564	237.7
(- 0)	4943	329.6
(0 +)	2742	182.9
(+ +)	3200	213.4
(+ 0)	3695	246.4
(0 0)	16109	1074

Table 2. Fish behaviours classifications based on QTC_{B21} relations.

group #	QTC Relation	Behaviour
group 1	(- +), (+ -)	Follow
group 2	(0 +), (+ +), (+ 0)	Diverge
group 3	(0 -), (- -), (- 0)	Converge
group 4	(0 0)	Stationarity

5 Experiments

We evaluate the effectiveness of our proposed system and our novel $3DQTC$ in this section. The experiment setup utilised the following hardware imaging components: A canon camera (PowerShot SX200) with resolution 1280 × 720 pixels and focal length: 5-60mm f/3.4-5.3. A $30cm \times 20cm$ calibration board containing 6 × 6 equal squares used for camera calibration process. A black circle tank ($30cm$ diameter and $15cm$ water depth) is used to house the fish. An Intel Core i5-2450M laptop, CPU@2.50GHz was used to run the experiments.

5.1 Experiment I: Relative Fish Movements

Our dataset includes one hour of video for two fish captured (with rate 30 frames per second) using the hardware imaging components described in section 5. This dataset was used as an input for our system to analyse fish spatial behaviours. In this experiment, we focus on the QTC_{B21} relations between two fish. The computational procedures of our framework in Section 3 were applied to estimate the position and orientation for each circular tag (fish position and orientation). Then, the QTC_{B21} codes were extracted.

Table 1 displays results pertaining to the relations and interactions between the two fish in the whole dataset. It summarises the total number of occurrences and the total duration for each relation for all fish interactions. It also shows that all nine QTC_{B21} relations do have at least one occurrence, and the occurrences

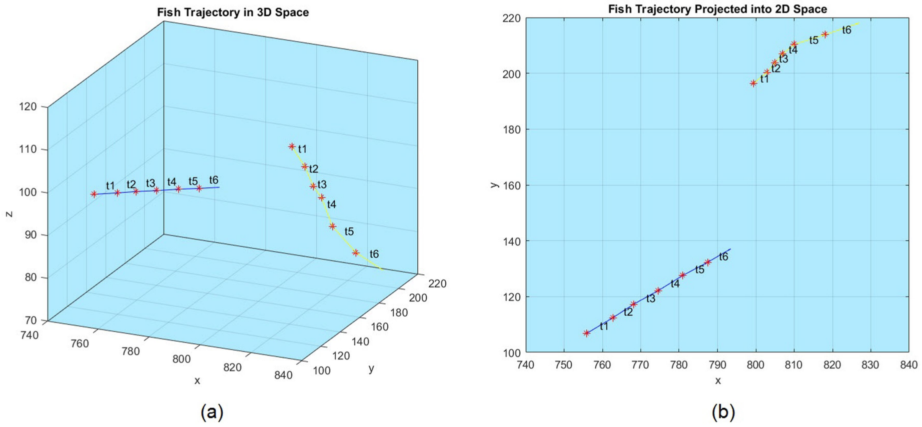


Fig. 4. (a) Representation of a pair of fish trajectory in 3D space. (b) Same trajectory in (a) projected into 2D space.

of these relations in the dataset are not equally distributed. Based on the biologist classifications for these nine fish interactions, four groups of behaviours (Table 2), were extracted as follows: The first group represent the $(-\ +)_{B21}$ and $(+\ -)_{B21}$ relations. It is the most common relations occur between the two fish in the dataset, lasting for around 0.33 of the cumulative time. This indicates that the two fish perform a *follow* behaviours; where the $(-\ +)_{B21}$ relation occurs at 0.24; Obj_1 moves toward Obj_2 and Obj_2 moves away. On the other hand, the relation $(+\ -)_{B21}$ represents the case where Obj_2 moves toward Obj_1 and Obj_1 moves away, occurs at just 0.09. The second group represent the $(0\ +)_{B21}$, $(+\ +)_{B21}$ and $(+\ 0)_{B21}$ relations. These relations occurs 0.18 of the time and represent the *diverge* behaviours. The relations $(0\ -)_{B21}$, $(-\ -)_{B21}$ and $(-\ 0)_{B21}$ represent the third group, where the two fish perform a *converge* behaviours; both fish are approaching each other or one of them approaching and the other is stationarity. Finally, the fourth group is represented by the symmetric $(0\ 0)_{B1}$ relation; which imply that the two fish are *stationarity.* Table 2 shows the biological categorisations of the QTC_{B21} relations.

5.2 Experiment II: Fish "Follow" Behaviour in 3D Space

To test the feasibility of our $3DQTC$, the *follow* behaviour have been chosen from our dataset described in Section 5.1. We selected a subset of our data where Obj_1 *follow* Obj_2 (group 1: $(-\ +)_{B21}$). We estimated the 3D orientation of each individual in this subset. The fish orientation represents the rotation angles that the surface normal vector of the circular tag makes with x, y and z axes (camera frame), respectively. Note that the z axes of the camera frame is perpendicular to the water surface. Then, we used the estimated $\alpha_j, \beta_j, \gamma_j$ pairs for each fish as input and a threshold $\kappa = 5°$, and constructed corresponding $3DQTC$ relations (*orientation feature*: $\{Code_7, Code_8, Code_9\}$) using our method described in

Section 4. The threshold κ has been defined experimentally, and we have found $\kappa = 5°$ is adequate. We used the combinations of three codes ($3^3 = 27$ relation) as a feature to derive a meaningful qualitative representation for fish interaction. Finally, we extracted the total number of occurrences and the total duration for each *orientation feature* relations. This results in splitting group 1 (*follow* behaviour) into two groups, the first group 1.1 represents the (0 0 0) relation and shapes 91% of the dataset. This represents the cases where the two fish swim on same or parallel water surface (insignificant variations in α, β, γ angles). While the second group 1.2 (shapes 9% of the dataset) contains all other cases where the two fish swim in different water surface.

An example of two sequences of $3DQTC$ relations (*orientation feature*) in 6 consecutive observations are as follow: $S_1\{0\,0\,0\,,0\,0\,0\,,0\,0\,0\,,0\,0\,0\,,0\,0\,0\,,0\,0\,0\}$; $S_2\{-+-\,,-+-\,,-+-\,,-+0\,,-+-\,,-+-\}$. Note that in S_1 both fish swim on same or parallel surface and according to biological experts, the two fish perform *follow* behaviour. While the fish swim in different water surface in S_2, which is *non-follow* behaviour. Fig. 4 shows that the two fish swim on different water surface and our $3DQTC$ capture this information. However, QTC_{2D} in Experiment I classify these relations as *follow* behaviour. This information is very important for biological study which we could not capture using QTC_{2D}.

6 Conclusion and Discussion

A new method for generalising 2D QTC into 3D space called $3DQTC$ was presented. Our method uses the geometrical analysis, the Reprojection methods and our fish recognition and tracking system to define the three-dimensional interactions between pairs of fish. We applied our method on a real-word fish dataset (*follow* behaviour). We show that our $3DQTC$ coding scheme can provide rich representations of spatial interactions between pairs of fish in 3D space, and capture information of high value for biological study which can not be represented using QTC_{2D}. Our method solves the cases where the objects are stationary or moving in straight-line. A QTC calculi for representing and analysing the spatial behaviours of fish in 2D space has been also presented. QTC_{2D} gave us the benefit of qualitative abstraction defined the fish states.

Encouraged by our initial results, we plan to extend our work in a number of ways. We intend to further to implement the full $3DQTC$ (seven codes: $Code_1$, $Code_2$, $Code_3$, $Code_6$, $Code_7$, $Code_8$, $Code_9$). This results in $3^7 = 2187$ states, and require to define the possible states in real-life. We plan to encode the fish interactions as a single trajectory of $3DQTC$ states. Then, apply standard time-series analysis methods to group the fish behaviours. We also intend to develop a method to analyse differences in behaviour between pairs of fish in 3D space, over long-run datasets.

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