A COMPARATIVE STUDY OF THE INTRODUCTION OF ADVANCED PLACEMENT AND INTERNATIONAL BACCALAUREATE MATHEMATICS COURSES IN NINGBO, CHINA

by

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ABSTRACT

A COMPARATIVE STUDY OF THE INTRODUCTION OF ADVANCED PLACEMENT AND INTERNATIONAL BACCALAUREATE MATHEMATICS COURSES IN NINGBO, CHINA

Xingchi Lu

Recently, in an attempt to promote globalization and internationalization, the Chinese government has introduced AP (Advanced Placement) and IB (International Baccalaureate) programs in Chinese high school. Although it is one of the biggest countries to introduce international programs to its secondary education in an effort to prepare more students to go overseas to pursue their higher education, China does not have much research focusing on introduced foreign academic programs. The purpose of this study is to fill in some gaps in the research while providing a better understanding of the depth behind the phenomenon of introducing the AP program and the IB diploma program in China and that introduction's impact on the existing general Chinese high school program in mathematics. Multiple sources of data were collected and used to make various kinds of analysis such as contextual analysis, cross-curricula comparisons and statistical analysis. The findings illustrated the differences and similarities between the AP program and the IB program in their respective schools in Ningbo, China. It further examined the differences and similarities between the AP program/ IB program and the intended Chinese high school program in mathematics education. It also explored mathematics instructors' perceptions of the imported AP and IB programs in China.

Limitations of this study include the absence of some test scores, the relative small sample size and the circumscribed selection of interviewees. This study provides a guide

to help Chinese students and their parents decide on a learning program based on individual preferences. Also, the results of this study indicate that a considerable gap exists between secondary education and higher education in Chinese mathematics, and also points to possible limitations for individualized learning. The findings imply the need to consider curricular reform, and suggest that local teachers and non-local teachers who teach in the imported programs consider reinforcing their teaching by learning from each other. Policymakers need to make adjustments to consider local conditions when introducing international programs so as to offer the most suitable program possible to native students.

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X. L.

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INTRODUCTION

Need for the Study

Constant improvements in global communications have made studying abroad a significant trend during the globalization and internationalization of education; this trend occurred even where well-established indigenous academic traditions already existed, as in China (Altbach, Reisberg, & Rumbley, 2019; Bashir, 2007; Bhandari & Blumenthal, 2010; Gonzalez, O'Connor, & Miles, 2001; Mazzarol, Soutar, & Seng, 2003). According to Altbach (2003), globalization includes the broad, largely inevitable economic, technological, political, cultural, and scientific trends that directly affect higher education. Internationalization includes policies and programs adopted by governments, and by academic systems and subdivisions, to cope with or exploit globalization.

Davis (2000) provides a comprehensive information resource on the more than 514,000 international students in the United States. Asian students constitute more than half of the international enrollments. In particular, China and India account for 24.3% of the foreign scholars in the United States (Davis, 2000). Three years later, China and India became two of the largest countries to send students to the United States. A mutual relationship caused this situation. Besides the demand of the market, some universities, predominantly in the United States and in the United Kingdom, see themselves as operating in a global higher education market. They try to channel effective purchasing power to their institutions by advertising their products in demand-intensive societies, mostly in East Asia (Odin & Manicas, 2004). From these data, it is clear that globalization influences higher education in developing countries, in which future expansion will occur (Altbach, 2003). Researchers suggest various rationales or motivations for internationalization, such as political rationale, economic rationale, academic rationale, and cultural and social rationale (Knight & de Wit, 1997). In addition, people generally regard the internationalization of higher education as one of the ways a country responds to the impact of globalization, yet at the same time respects the individuality of the nation (Zha, 2003).

Besides traditional private and public higher education institutions, some of the ‘new providers,’ such as corporate universities, professional associations, and international conglomerates, seek to establish a physical presence through teaching and testing centers, independent institutions, branch campuses, and acquisitions or mergers with local education institutions (Altbach & Knight, 2007). Many studies investigate the importance of international or cross-cultural education by conducting comparative studies. Such studies not only help us to understand the differences among institutions and countries but also give us more information when considering the future development of introducing foreign curricula in different countries.

Although all countries regard education as one of their most important issues, China especially puts mathematics education in a crucial position in its education system. Bhandari and Blumenthal (2010) mentioned that studying abroad activities in China are strongly supported by Chinese government policy. Moreover, with the boosting pace of economic development, more higher education Chinese students have enough financial resources to afford the expense of studying abroad in foreign countries. Therefore, studying abroad is an inevitable trend in the Chinese education world, which, in turn, attracts more foreign educational institutions to collaborate with Chinese schools and enroll more Chinese students to go to school overseas. Recently, one of the most remarkable phenomena is the introduction of AP (Advanced Placement) and IB

(International Baccalaureate) programs in Chinese high school classrooms where schools employ well-organized foreign AP and IB teams to conduct instruction (Cheng & Deng 2006; Gu, 2006a; Weng, 2008; Xu, 2001).

The original intent of the Advanced Placement Program, administered by the College Board since 1955, was to offer rigorous, college-level courses and assessments at high schools across the United States and the world, and provide students the opportunity to take college-level coursework and earn college credit during high school (Geiser & Santelices, 2004; Liang, 2012; Mattern, Shaw, & Xiong, 2009; Song & Wang, 2015; Wang & Wang, 2016). In 1968, the International Baccalaureate Office (IBO) was legally established in Geneva (Fox, 1985). As an internationally accepted pre-university qualification, the AP Program is now offered in more than 70 countries worldwide. As a university entrance qualification accepted internationally, the IB Program aims not only to provide an appropriate academic curriculum, but also to support geographic and cultural mobility and to promote international understanding (Hayden & Wong, 1997). After the 1980s, admission personnel generally view the presence of AP or IB courses on a transcript as an indicator of the applicant's willingness to confront academic challenges (National Research Council, 2002a). On AP's official website, published by the College Board, the newest data show that 276 authorized AP schools are in China, compared to 124 schools in 2013; the rate has been sharply doubled. And about 70% of AP schools in China offer AP calculus and statistics, which are the most popular AP courses in this country (AP Course Ledger, 2017). Meanwhile, until 2018, China already had 142 authorized IB schools, and 102 of them are IBDP that could provide an IB diploma. And the number is still increasing.

Many researchers and scholars devoted themselves to exploring the AP program and the IB course. Mattern et al. (2009) mentioned that given the purpose of the AP program, most of AP research has focused on the relationship between AP examination performance and course placement (Burnham & Hewitt, 1971; Dodd, Fitzpatrick, De Ayala, & Jennings, 2002; Morgan & Crone, 1993; Morgan & Ramist, 1998). Researchers, for instance, note that advanced mathematics placement has a positive effect on students' grade level standardized test scores and the school's overall performance rating (Lawson, 2005; Morgan & Ramist, 1998; Soares, 2001); Geiser and Santelices (2004) found that performance on the AP examination is a valid indicator of a student's likelihood of college success (Dougherty, Mellor, & Jian, 2006; Geiser & Santelices, 2004; Hargrove, Godin, & Dodd, 2008; Mattern et al., 2009).

Some researchers also notice the positive effect of taking the IB course. The IB can clearly facilitate mobility and can contribute to the development of international understanding, and it can support the preservation of individual cultures and national identities at the same time, provided contextual factors are appropriately arranged (Hayden & Wong, 1997; Xu & Gao, 2012). According to Taylor (2006), IB graduates reported positive experiences with the program; they felt the rich curriculum to which they were exposed, and the critical thinking and time management skills that they developed, were well worth the extra effort required to earn an IB diploma. In addition, "IB has been implemented in competition with local curricula, being offered to local students as well as to internationally mobile students" (Doherty, 2009, p. 77). In the USA, the introduction of the IB was used to draw middle-class students back to the school (Mathews & Hill, 2005). In the UK, the former Prime Minister Tony Blair announced that from 2008, the IB would be offered in UK government schools as an alternative to a renovated 'A' level curriculum (Phillips & Pound, 2003).

Callahan (2003) states that the rapid growth of Advance Placement and International Baccalaureate programs in the United States is based on filling a gap in providing of high level, challenging courses at the high school level. Some researchers also mention that "today's AP and IB courses and exams demonstrate that independent entities can and do make programs and assessments that are rigorous, fair, and intellectually richer than almost any state standard and exam for high school that we have seen" (Byrd, 2007, p. 18). Chinese introduction of AP and IB is a good example of this phenomenon. However, the booming development of AP and IB programs in China is not the same as in the United States or anywhere else. Chinese students who are learning the AP program or the IB program are required to learn the general Chinese high school curriculum at the same time, which means they have to do two different curricula together. And those students who choose to learn the AP program or the IB program, automatically have to give up their right to take the national Chinese college entrance examination, so even if they change their minds and want to go to a Chinese college after high school graduation, it is not workable. Therefore, for the Chinese international students, their choice of the AP program or the IB program can have a profound influence on their academic lives.

Overall, although China becomes one of the biggest countries to introduce more foreign programs to its secondary education in order to prepare more students to go overseas to pursue their higher education, not much research is focused on the comparison of introduced foreign academic programs in China. In general, scholars and researchers have conducted pairwise comparative studies of one foreign program and their native high school programs (Chai, 2006; B. Chen, 2006; J. Chen, 2013; Cheng & Deng, 2006; Gu, 2006a, 2006b; Gu & Yang, 2002; Jia, 2015; Jin, 2008; Kong & Ma, 2007; Lai, 2013; J. Li & Yao, 1998; X. Li, 2009 ; Y. Li, 2006; Liang, 2016; Liu, 2008; Liu & Fan, 2014; Liu & Liu, 2008; F. Lu, 2013; H. Lu, 2000; Ma, 2009, 2010, 2012, 2017; Ren, 2007, 2008; H. Song, 2008; J. Song, 2012; Tang, Wen, & Liu, 1999; Tao, 2012; Y. Wang, 2016; Z. Wang, 2017; Wu, 2014; Xiwei, 2014; D. Xu, 2017; H. Xu, 2001; Yang, 2013; Yang & Dai, 2012a, 2012b; Zhang, 2014; Zhao, 2010; Zhao & Zhang, 2009a). Some of them compared two subjects horizontally to explore the similarities and differences of curricula and assessment, and some chose a vertical comparison to mainly analyze students' performance (Li & Zhang, 2006; Liu, 2008; Ma, 2008; Weng, 2008; Xie & Shen, 2013). A very few of the former studies focused on a horizontal comparison that includes AP programs, IB diploma programs, and the native high school programs while making a horizontal comparison and several vertical comparisons (Ma, 2011; Ma, Liu, & Liu, 2012; Liu & Zhang, 2016).

Moreover, Chinese educators started late in exploring AP programs and IB programs. With the progress of the reform and opening-up policy in the 1990s, people began to notice these introduced programs (Song, 2012). In addition, the study of these programs in China is relatively superficial; most of them remain without comprehensive analysis. Based on a review of literature, much research was published between 2000 to 2015, which means some results might be outdated given the rapid growth of AP and IB diploma programs (Song, 2012). Inevitably, AP programs keep changing as well. For example, in the 2016-2017 school year, AP Calculus AB changed its time format and added the concept of L'Hôpital's rule to evaluate the indeterminate forms of limits; and AP Calculus BC added limit comparison tests, absolute and conditional convergence, and the alternating series (AP Calculus AB and BC Course and Exam Description, 2018). For another instance, the concept of "function and limit" has been changed to "limit" as the first big idea of four ideas in AP Calculus curriculum. In addition, besides the change of the AP program itself, starting from 2016, newly enrolled students who decide to study the AP program or the IB diploma program in Ningbo, China, are required to study the domestic high school curriculum and take the High School Academic Proficiency Test (the General Examination) due to the government policy that expects international school students to grow up with patriotism and global vision. Therefore, it could be meaningful to explore in depth the phenomenon of introducing the AP program, the IB diploma program, and the existing general Chinese high school program in mathematics, incorporating the latest data to generate an updated result so that more students and their parents can have a clearer picture of the unprecedented trend of learning the AP program or the IB diploma program in China, and can gain a better understanding of the globalization and internationalization of mathematics education.

Purpose of the Study

The purpose of this case study research is to compare and analyze the AP and the IB programs in Ningbo, China. Ningbo is a typical second-tier city that does not have the type of educational resources available in top-tier cities like Beijing and Shanghai. However, it has more resources available than those cities in West China, which lack educational resources. Thus, a Ningbo study would be representative of the general situation in China. In order to achieve the purpose of the research, the following specific questions were addressed:

1. What are the significant differences and similarities between the AP (Advanced Placement) program and the IB (International Baccalaureate) program in their respective schools in Ningbo, China?
2. What are the differences and similarities in curriculum and student performance between the AP (Advanced Placement) program and the intended Chinese high school program in mathematics education?
3. What are the differences and similarities in curriculum and student performance between the IB (International Baccalaureate) program and the intended Chinese high school program in mathematics education?
4. What are mathematics instructors' perspectives on teaching imported AP (Advanced Placement) and IB (International Baccalaureate) programs in China; also, what are their expectations and suggestions for curriculum innovations in Chinese mathematics education?

To answer these questions, this study investigated two sister high schools in Ningbo: Ningbo Xiaoshi high school, the top high school in Ningbo, provides an IB program, and Ningbo Foreign Language School, another top-tier high school in the city, offers the AP program. Getting authorization to run the AP program and the IB program is based on the quality of the school and the economic level of the city. Also, both high schools belong to the same student recruiting system, which means students in both programs need to pass the same Entrance Examination.

Procedure

To address the first research question, I conducted comparative research on both the AP (Advanced Placement) and the IB (International Baccalaureate) programs. I retrieved related program information from their official websites. Then, I collected information sources such as recruitment handbooks with detailed program descriptions directly from the school administration.

To address the second and third research questions, I first collected data from the general Chinese high school curriculum in mathematics education, including topics, contents, and course descriptions established by the Ningbo Education Bureau (Appendix G). Then I analyzed the curricular difference between the general Chinese high school program and the AP program, as well as the curricular difference between general Chinese high school programs and the IB program. Also, in order to understand students' learning outcomes, I collected data from the mathematics scores on the General Examination, SAT examination, AP examination (both AB and BC), and IB examination (both Standard Level and Higher Level). The Chinese government has required that every student enrolling in these programs must also complete the General Examination that every high school student needs to pass to get a diploma. Thus, I used the AP students' mathematics scores on the General Examination and compared those with their grades on SAT mathematics, AP Calculus AB, and AP Calculus BC. Doing so enabled me to understand the relationship between students' learning outcomes on these two kinds of tests, which are based on two different curricula. It is to be noted that the IB school has followed the government policy of requiring students to take the General Examination since 2016. Since the year used to identify data in this study refers to students' graduation year, IB students' mathematics scores on the General Examination will not be available until June, 2019. I also compared students' SAT mathematics scores with their mathematics scores on the AP examination and the IB examination so that I could further determine if there is any correlation between these data.

To address the fourth research question, I conducted interviews with mathematics instructors to view things from the educators' perspective. Some questions focused on foreign mathematics instructors' observation of the specialty of teaching mathematics in China and student learning outcome to learn mathematics in the bilingual environment. Other questions sought to uncover anything that can help innovation in Chinese education, especially in mathematics teaching and learning. Based on some policy changes in the Chinese education system, I also investigated the reasons and likelihood of any future development of foreign curricula from teachers' perspective.

Positionality

As an international student who has been studying abroad for almost seven years in the United States, my position gives me a more profound understanding of international curricula. I finished my high school in mainland China, then I spent four years in Seattle for my undergraduate program; after that, I moved to New York to continue my learning of mathematics education. My perspective on mathematics education keeps changing. When I was in China for my high school study, I barely had any interest in mathematics, even if I was good at memorizing theorems and formulas and doing quick calculations. Later in Seattle, when I was learning standard mathematics, I received an entirely different mathematics education, not only because the studying environment in college is freer and more relaxing, but I also had more flexibility to choose what I want to learn based on my personal academic preferences. Of course, secondary education is different from higher education, and that is the reason I made up my mind to study mathematics education at Teachers College, Columbia University. During years of studying, observing, and experiencing, I noticed the differences and similarities between various curricula; therefore, I wanted to go further, to focus on the adaption of different curricula, especially the development of introduced international curricula in China.

Chapter II

REVIEW OF LITERATURE

Introduction

Currently, with the development of international education under the backdrop of globalization and internationalization, the popularity of international curricula has captured the interest of scholars and researchers from all around the world. In particular, the Advanced Placement Program (AP) and the International Baccalaureate Program (IB) have received much attention. During my review of the relevant literature, I noticed that much current work focused on vertical comparisons, on the development process of either the AP program or the IB program. Some horizontal comparisons offer overviews of the AP program and the IB program, their assessment, their curriculum, and so on. A few scholars conducted comparative studies of local curriculum standards and either the AP program or the IB program. These are horizontal comparisons with two subjects. However, few of them compared three or more of the AP program, the IB program, and other programs.

I will separate this chapter into two parts: Advanced Placement and International Baccalaureate. First, I will give an introduction to the AP program and then the IB program by starting with their history, their modes of assessment and so on to provide some basic knowledge of what AP and IB are. Next, I will talk about the specific academic area I am interested in, mathematics. I will describe the mathematics course and mathematics assessment in each program to give a sense of their curricula. I will then mention the benefits, the current situation and development (worldwide and in China), and each program's influence on Chinese high school curriculum based on other scholars' and researchers' works.

Since my research includes the general Chinese high school mathematics program as one of its comparison subjects, I spent much time searching related works that have been done by Chinese researchers. Within the literature I found, most of the Chinese researchers focused on a comparison of the general Chinese high school program and either the AP program or the IB program. However, there were a few studies conducted by Chinese scholars that focused on a comparison of the AP program and the IB program and some others that compared the AP program, the IB program, and the general Chinese high school program. I will discuss the findings of those studies. Last, I will talk about the suggestions offered by some Chinese scholars.

Advanced Placement

The Advanced Placement program, administered by the College Board since 1955, was designed for high school students and provides them the opportunity to take college-level courses while in high school and receive college credit (Chai, 2006; Geiser & Santelices, 2004; Liang, 2012; Lu, 2013; Mattern et al., 2009; Nugent & Karnes, 2002; Song & Wang, 2015; Wang, 2017; Wang & Wang, 2016; Yang, 2013; Zhao & Zhang, 2009a). The Advanced Placement (AP) program can be best understood by examining its history and its methods of assessment.

History

The College Board is an American non-profit organization based in New York City that was formed in 1899 as the College Entrance Examination Board (CEEB) to expand access to higher education (Jia, 2015; Mestre & Lochhead, 1990).

After the end of World War II, the Ford Foundation created the Fund for the Advancement of Education to explore the overlap of high school and college course contents. It sponsored the first study across three private high schools (Phillips Academy Andover, Phillips Exeter Academy, and the Lawrenceville School) and three universities (Harvard University, Princeton University, and Yale University) (Freedman & Krugman, 2001; Nugent & Karnes, 2002; Tian, 2004; Zhao, 2010). In 1952, members of those schools' faculties issued the report *General Education in School and College: A Committee Report*, which mentioned the waste involved in the transition from secondary to higher education due to overlapping class contents (Blackmer et al., 1952; Harrington, 2005; Katz, 2006; Zhao, 2010; Zhao & Zhang, 2009a). In order to avoid repetitive coursework, high schools and colleges needed to work together to provide a concrete plan for allowing high school seniors to engage in independent study and take college-level courses (Blackmer et al., 1952; Chai, 2006; Liang, 2012; Nugent & Karnes, 2002; Song & Wang, 2015; Tian, 2004; Wang, 2017; Wang & Wang, 2016; Yang, 2013; Zhao, 2010; Zhao & Zhang, 2009a). In the second study, the Committee on Admission with Advanced Standing developed a college-level curriculum that formed the basis for Advanced Placement courses. It aimed to create comparable introductory college courses in each subject and accompany them with standardized assessments that could be regarded as rigorous enough to merit college credit (AP at Glance, 2018; Liang, 2012; Nugent & Karnes, 2002; Song & Wang, 2015; Wang & Wang, 2016; Zhao & Zhang, 2009b).

In 1952, a pilot program was conducted that involved eleven subjects including English (composition, literature), foreign languages (French, Latin, German, Spanish), history, science (biology, chemistry, physics) and mathematics (Nugent & Karnes, 2002; Tian, 2004; Xu, 2017; Zhao, 2010). The College Board has run the AP program since 1955 (Freedman & Krugman, 2001; Jia, 2015; Nugent & Karnes, 2002; Tian, 2004; Zhao, 2010; Zhao & Zhang, 2009a).

Assessment

Generally, AP courses and examinations are developed by committees of college faculty members and experienced high school AP teachers. The collaboration of the high school and college faculties balances the transition from secondary education to higher education with regard to the expectations of the courses, the curriculum framework, the requirement of students' knowledge and skills, and examination format and scoring criteria (Ren, 2008; Tian, 2004).

AP examinations are given each May at testing locations all over the world (AP at a Glance, 2018; Kong & Ma, 2007; Lu, 2013; Nugent & Karnes, 2002; Song & Wang, 2015; Tao, 2012). AP examinations include three question types: multiple-choice questions, free-response questions, and through-course performance tasks. Most AP examinations are a mix of multiple-choice and free-response questions designed to measure skills and knowledge using the "evidence-centered approach" to assessment design. Several courses, such as AP Computer Science Principles, AP Research, AP Seminar, and AP Studio Art, require through-course performance tasks, either in place of or in addition to multiple-choice or free-response questions (Example Development, 2018; Kong & Ma, 2007; Zhao & Zhang, 2009a).

The scoring of AP examinations combines student scores on the multiple-choice and free-response sections. Table 1 displays the final scores and their corresponding meanings, with 5 being the highest numerical grade and 1 being the lowest.

Table 1. AP Exam Scores and Corresponding College Course Grade Equivalents

AP Examination Score	Recommendation	College Course Grade Equivalent
5	Extremely well qualified	A+ or A
4	Very well qualified	A-, B+, or B
3	Qualified	B-, C+, or C
2	Possibly qualified	
1	No recommendation	

AP examination outcomes indicate how qualified students are to meet college-level expectations and receive college credit or placement. A score higher than or equal to 3 points fulfills the admission requirement in most higher education institutions. Some top colleges and universities set their standards at 4 or 5 points (Discover the Benefits of AP, 2018; Kong & Ma, 2007; Score Setting and Scoring, 2018; Song & Wang, 2015; Tian, 2004; Wang & Wang, 2016; Yang, 2013; Zhao & Zhang, 2009a).

AP Mathematics

The following two sections offer a view of the AP mathematics program by detailing its courses and its methods of assessment.

Courses

AP Mathematics includes AP Calculus and AP Statistics (AP Calculus AB the Course, 2018; AP Calculus BC the Course, 2018; AP Statistics the Course, 2018). AP Statistics is a non-calculus-based course in statistics that is equivalent to a one-semester introductory college course with four main topics including exploring data, sampling and experimentation, anticipating patterns, and statistical inference. Students are expected to understand the concepts and to use proper tools for data collection, analysis, and conclusion (AP Statistics the Course, 2018).

AP Calculus is separated into two courses: AP Calculus AB and AP Calculus BC (AP Calculus AB the Course, 2018; AP Calculus BC the Course, 2018). Both the courses are college-level calculus courses (AP Calculus AB the Course, 2018; AP Calculus BC the Course, 2018; Kong & Ma, 2007; Lai, 2013; Lu, 2013; Song & Wang, 2015; Wang, 2017; Wang & Wang, 2016). To be specific, AP Calculus AB is roughly equivalent to a first semester calculus course at most colleges and universities. It is devoted to topics in differential and integral calculus including concepts and skills of limits, derivatives, definite integrals, and the fundamental theorem of calculus. AP Calculus BC is roughly equivalent to both first and second semester calculus courses at most colleges and universities. It extends the content learned in AB to include parametric, polar, and vector equations, and introduces the topic of series (AP Calculus AB the Course, 2018; AP Calculus BC the Course, 2018; Chai, 2006; Kong & Ma, 2007; Wang, 2017). In mathematical terms, AP Calculus AB is a subset of AP Calculus BC (Ma, 2011). Both AP Calculus AB and BC require the same level of understanding: that students can approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and that they can make connections amongst these representations (AP Calculus AB the Course, 2018; Chai, 2006; Kong & Ma, 2007; Lai, 2013; Wang, 2017). Those four forms of representations are also called the "Rule of Four" to describe concepts and problems (Chai, 2006; Lai, 2013; Wang, 2017). It is obvious that AP Calculus aims to build students' conceptual understanding of the content, and to offer them various methods to solve relative problems and to apply the knowledge into real practice. AP Calculus emphasizes the extensive usefulness of its methods and the breadth of its concepts, which means the key point of the course is not about calculation or memorization, but, rather, concerns application and making connections between each topic as an organic whole (Chai, 2006; Lai, 2013; Wang, 2017).

In addition, for all AP Mathematics courses, students must learn how to use technology to help with question investigation, problem solving, result interpretation and conclusion generation in order to build conceptual understanding (AP Calculus AB the Course, 2018; AP Calculus BC the Course, 2018; AP Statistics the Course, 2018; Chai, 2006; Lai, 2013; Wang, 2017). Thus, using a graphing calculator becomes another important part of learning AP Mathematics (Chai, 2006; Lai, 2013; Kong & Ma, 2007; Wang, 2017). The College Board provides a list of approved graphing calculators online for students to bring with them to the examination (AP Students Calculator Policy, 2018;

Kong & Ma, 2007). Although different types of calculators have different capabilities, students should be able to use at least four built-in capabilities: plotting the graph of a function within an arbitrary viewing window, finding the zeros of functions, numerically calculating the derivative of a function, and numerically calculating the value of a definite integral (AP Students Calculator Policy, 2018; Lai, 2013).

Assessment

AP Calculus examinations include two question types: multiple-choice questions (Section I) and free-response questions (Section II). Table 2 shows the assessment details of AP Calculus examinations.

Both AP Calculus AB and BC have the same examination format (AP Calculus AB the Exam, 2018; AP Calculus BC the Exam, 2018; Kong & Ma, 2007).

Table 2. AP Calculus Assessment

<table>
<tr><th>Type of Assessment</th><th>Format of Assessment</th><th>Weighting of Final Grade</th></tr>
<tr><td rowspan="2">Section I: Multiple Choice
(45 questions; 105 mins)</td><td>Part A:
30 questions; 60 minutes
(calculator not permitted)</td><td rowspan="2">50%</td></tr>
<tr><td>Part B:
15 questions; 45 minutes
(graphing calculator required)</td></tr>
<tr><td rowspan="2">Section II: Free Response
(6 questions; 90 mins)</td><td>Part A:
2 questions; 30 minutes
(graphing calculator required)</td><td rowspan="2">50%</td></tr>
<tr><td>Part B:
4 questions; 60 minutes
(calculator not permitted)</td></tr>
</table>

Using technology certainly plays an important role in the examination. Both sections I and II have parts that require students to use graphing calculators. Also, in section II (free-response), students are expected to show enough work so that the reasoning and explaining process can be followed by AP readers (the high school teachers and college faculty who score the AP examination); they must not simply write down the solutions obtained. For example, if students are asked to find a relative maximum value of a given function, it is necessary to show the mathematical process step by step to get to the answer (AP Students Calculator Policy, 2018; Kong & Ma, 2007).

AP Concept

As a university entrance qualification accepted internationally, the Advanced Placement program was initially focused on providing opportunities for students to achieve excellence through independent study. The selection of AP courses greatly depends on individual personality and interest (Curry, MacDonald, & Morgan, 1999; Lu, 2013; Zhao, 2010; Zhao & Zhang, 2009a).

Moreover, especially from the end of 1960s, Advanced Placement also aimed equity in education, which means providing the support and resources for every student to help them achieve at consistently high levels of education. And “The College Board’s Equity and Access Policy Statement” exactly demonstrates that principle. Therefore, equity and excellence are the essential concept of the Advanced Placement program (AP Exam Instructions, 2018; Zhao, 2010).

AP Benefits as Identified in Literature

There are three major ways in which students may benefit from participating in the AP program: the gaining of heightened skills and confidence, better prospects for college admission and academic success, and savings of time and money.

Student Skills and Confidence

Studying in the AP program will allow students to develop essential time management and study habits necessary for tackling rigorous coursework. In addition, it can broaden their intellectual horizons by encouraging them to explore the world from a variety of perspectives. Also, choosing the difficulty level allows students to exercise choice and can help them dig deeper into subjects that interest them, encouraging them to develop creativity and sharpen their problem-solving techniques to address course challenges (Discover the Benefits of AP, 2018; Tian, 2004).

College Admission and Student Success

Taking AP classes can also help students stand out in the competitive contest that is the college admissions process. It not only demonstrates maturity and readiness for college and career, but also emphasizes commitment to academic excellence (Discover the Benefits of AP, 2018; Lai, 2013; Xu, 2017; Yang, 2013; Zhao & Zhang, 2009a).

Research shows that 90% of colleges and universities in the United States participate in the AP program (Tian, 2004); according to the record published by College Board, 4,287 colleges admit AP scores (Annual AP Program Participation 1956-2018, 2018; Program Summary Report, 2018). Also, 85% of selective colleges and universities report that a student's AP experience favorably impacts admission decisions because these institutions believe students who take AP courses send a signal that they are serious about their education and that they are willing to take the challenge of rigorous coursework (Discover the Benefits of AP, 2018; Lai, 2013; Xu, 2017; Yang, 2013; Zhao & Zhang, 2009a).

Mattern et al. (2009) mentioned that given the purpose of the AP program, most AP research has focused on the relationship between AP examination performance and course placement (Burnham & Benjamin, 1971; Dodd et al., 2002; Morgan & Crone, 1993; Morgan & Ramist, 1998). Researchers note, for instance, that Advanced Placement mathematics has a positive effect on students' grade level, standardized test scores, and the school's overall performance rating (Lawson, 2005; Morgan & Ramist, 1998; Soares, 2001); researchers also found that performance on the AP examination is a valid indicator of a student's likelihood of college success (Dougherty et al., 2006; Geiser & Santelices, 2004; Hargrove, Godin, & Dodd, 2008; Mattern et al., 2009).

Research consistently demonstrates that students who receive scores of 3 or higher on AP examinations typically experience greater academic success in college than those who do not. They tend to earn higher GPAs in college, perform as well as or better in subsequent college courses in the examination discipline than their non-AP peers who took the corresponding introductory college courses, and take more challenging, rigorous coursework in college (AP Program Results Overview, 2018; Discover the Benefits of AP, 2018; Morgan & Crone, 1993; Morgan & Klaric, 2007; Ren, 2008; Tian, 2004). For instance, Simms (1982) concluded that students with 3 or better on AP examinations were prepared in high school to take upper-level college courses and did not experience any negative effects from doing so in college (Nugent & Karnes, 2002; Simms, 1982). Morgan and Crone (1993) determined that students receiving a 3 or higher on AP examinations received higher grades than did non-AP students who first took an introductory course, and the AP students continued to pursue knowledge more intensively in that subject than other students (Morgan & Crone, 1993; Morgan & Klaric, 2007; Nugent & Karnes, 2002). In another study, Morgan and Ramist (1998) scrutinized AP students who took upper-level college courses and noticed that the average grades were better than for those students who took the prerequisite courses.

Savings of Time and Money

Research consistently shows that academic preparation is positively correlated to rates of college completion; students who take AP courses and examinations, and especially those who score a 3 or higher on an AP examination, are much more likely than their matched peers to graduate in 4 years, and they have higher graduation rates as well (AP Program Results Overview, 2018; Discover the Benefits of AP, 2018). For example, a study performed in 2003 showed that the passing of an AP examination is a strong indicator of the likelihood of bachelor's degree attainment. Sixty-one percent of students who took two or more AP examinations in high school obtained their degrees within 4 years; 45% of students who passed one AP examination in high school attained degrees within 4 years; and only 29% of students who never passed an AP course graduated with a degree within 4 years of college study (Camara, 2003; Xu, 2017). Therefore, most students who take AP classes in high school can avoid paying for a fifth year. Most college and universities accept qualifying AP examination scores for a college credit, Advanced Placement or both. In this way, students can skip introductory courses or required general-education courses allowing them to fulfill graduation requirements early and possibly graduate under 4 years (Discover the Benefits of AP, 2018; Lai, 2013; Morgan & Klaric, 2007; Zhao & Zhang, 2009a). The fee for each AP Examination is $94, and for non-U.S. regions, it is $124 per examination. Usually, one AP course yields three to six college credits; at $1000 per credit, students can save $3000 to $6000 by taking only one AP Examination and getting a qualifying score. Such savings is very attractive for students, especially for international students, who face higher tuition costs (Camara, 2003; Gallagher, 2009; Kong & Ma, 2007; Lai, 2013; Malkus, 2015; Tian, 2004; Yang, 2013; Zhao & Zhang, 2009a).

Current Situation and Development

The current situation and development of AP programs in China can best be understood within the context of international AP programs. The literature concerning Chinese AP programs sheds light on five important aspects: localized features, challenges to Chinese students, benefits for Chinese students, differences between AP and general Chinese education, and differences between AP mathematics and general Chinese mathematics.

International AP Programs

Worldwide university recognition of the AP program is steadily increasing with the development of globalization and internationalization in higher education. Nowadays, qualifying AP examination scores can earn credit, placement or both in over 90% of colleges and universities in the United States and Canada, as well as many institutions in over 100 other countries (AP International, 2018; Discover the Benefits of AP, 2018; Tian, 2004; Xu, 2017; Zhao, 2010; Zhao & Zhang, 2009b).

The overall AP data, such as the number of schools that provide AP classes, colleges and universities that grant AP credit or placement, and students who attend AP examinations, demonstrate a rapid expansion of the AP program (Malkus, 2015; Overview, 2018; Zhao & Zhang, 2009b). The 38 AP courses in seven subject categories in total have more than tripled the original offerings founded in 1952 (AP at a Glance, 2018; Kong & Ma, 2007; Liu & Fan, 2014; Ma, 2012; Wang, 2017; Zhao & Zhang, 2009a). In the first year, 104 high schools and 130 colleges participated in the College Board's AP program, with 1,229 students taking 2,199 examinations across the 11 disciplines. By the 2017-18 school year, 22,612 high schools and 4,287 colleges participated in the AP program. The number of students ballooned to 2,808,990 taking a total of 5,090,324 tests across 38 subjects (Annual AP Program Participation 1956-2018, 2018; AP Student and Exam Volume per School, 2018; Number of Schools Offering AP Exams, 2018; Program Summary Report, 2018). And the largest number of examinations taken by each school increased from 168.64 in 2009 to 225.12 in 2018 (AP Student and Exam Volume per School, 2018; Number of Schools Offering AP Exams, 2018).

In 2000, approximately 60% of the 22,000 high schools in the U.S. provided AP course work to more than 750,000 students participating in the AP program (College Entrance Examination Board & Educational Testing Service, 2000; Nugent & Karnes, 2002; Russo, 2000). The 2017 AP program results show that, with the class of 2017, more U.S. students than ever before took at least one AP examination in high school and more earned a qualifying score of 3 or higher on an AP Examination. And according to Malkus, although AP participation increased by 35% between 2000 and 2009, the national data did not suggest a watering down of AP courses (AP Program Results Overview, 2018; Malkus, 2015).

According to the AP Exam Volume by Region table (2018), non-U.S. regions rose from 39,568 examinations given in 2008 to 132,191 in 2018, which shows a more than quintuple increase in the past ten years (Table 3).

Table 3. AP Examination Volume by Region

AP Examination Volume by Region		
	2008	2018
National Total	2,674,296	4,923,072
Canada	20,273	30,662
Non-U.S.	39,568	132,191
U.S. Territories	2,308	4,399
Grand Total	2,736,445	5,090,324

Chinese AP Programs

As part of the development of globalization and internationalization, the AP program has been widely welcomed in China (Lu, 2013; Yang, 2013). Chinese scholars and researchers have spent a lot of time in conducting comparative studies of the AP program and the general Chinese high school program.

Some of them talked about the history of the AP program's development in China. In 2006, the College Board set up an office in Shanghai, China; in May 2007, the Beijing 21st Century International School became one of the first schools in China to offer AP courses (Xu, 2017; Zhao & Zhang, 2009a). Since 2010, the College Board has officially begun cooperating with the Chinese National Education Examinations Authority (NEEA) to hold AP examinations; every school must be authorized by the NEEA before offering AP courses (Xu, 2017; Yang, 2013). A few private international schools in China introduced the AP program first, and then the public schools opened the AP Center, also called the “Sino-US joint international high school AP program” (Yang, 2013). On AP’s official website, published by the College Board, the newest data shows that 276 authorized AP schools are in China, compared to 124 schools in 2013; the number has doubled within five years (AP Course Ledger, 2018). The AP program in China is continuing to grow (Lu, 2013; Xu, 2017). At present, Chinese students tend to choose subjects such as mathematics and science based on their overall academic strength (Yang, 2013). 225 schools offer AP Calculus AB, 199 schools offer AP Calculus BC and 192 schools offer AP Statistics, which means that AP mathematics courses are the most popular AP courses in the country (AP Course Ledger, 2018).

Localized features. Because the College Board encourages that AP courses be taught in such a way that students are allowed to explore knowledge, Chinese traditional educational methods, which favor infusing knowledge into students, cannot be used. Therefore, Sino-US cooperation became necessary for the successful incorporation of AP courses into Chinese schools (Yang, 2013).

Students in the Chinese AP schools need to learn the Chinese high school curriculum and the AP curriculum at the same time. This is true not only because of the existence of overlapping contents between both curricula, each with its strength in terms of breadth and depth of knowledge, but also because of the government policy of adjusting the curriculum to suit the regional context. Therefore, AP schools in China cover both the general Chinese educational requirements and the AP curriculum and approach so that their students receive Chinese high school diplomas, because the AP program itself is not a diploma program (Yang, 2013; Zhao & Zhang, 2009a).

On the other hand, mixing traditional Chinese instructor-centered teaching and the inspiring AP-style student-centered learning can balance knowledge and quality. Students will have a solid foundation of basic knowledge and improve their communication skills, collaboration spirit, research ability, independent thinking capacity and organizational leadership (Zhao & Zhang, 2009a).

Challenges to Chinese students. Compared to general high school students, students in the AP Center face more pressures (Yang, 2013). Language is one of the toughest obstacles. Chinese students need to have enough English ability to handle the studying in a bilingual environment. This is especially true since the examination is in English. Although students have strong calculation and memorization skills, correctly understanding questions and accurately representing ideas in English still takes time and practice. As Chinese AP students work in both the Chinese secondary education system and the U.S. higher education system, mastering a foreign language becomes their most significant challenge. The intense schedule is also a challenge to every student. Learning two curricula at same time requires students to be prepared for the SAT test, the TOEFL, the Chinese General Examination, and AP examinations. Their workload is double that of general Chinese high school students (Liang, 2012; Yang, 2013).

Benefits for Chinese students. Teaching methods in the AP program position the teacher as a guide and focus on student subjectivity. The design of the questions and problems tends to encourage students to think independently, and various in-class activities also make the instruction more attractive to students and, thus, help to motivate them, enhancing classroom engagement (Lu, 2013; Ma, 2012; Yang, 2013).

The relationship between teachers and students in the Chinese AP classroom is different from that of Chinese general education. It becomes more harmonious, with mutual respect and equal communication (Yang, 2013).

Difference between AP and general Chinese education. The AP program builds a positive connection between secondary education and higher education, whereas general Chinese secondary education is more distinct from higher education and there exists a significant gap (Liang, 2012; Lu, 2013; Tian, 2004; Xu, 2017). According to Xu (2017), pursuing entry-level college courses in high school is not only a sublimation of high school knowledge but also a preliminary experience of college knowledge. Also, students can begin to understand majors so that they will not choose a major and a future career field blindly.

The general Chinese high school approach is exam-oriented because of the competitiveness of the College Entrance Examination. Therefore, Chinese students have a purposeful and utilitarian attitude toward their secondary education. Most Chinese high schools plan to finish all the coursework in the first two years of study (Grade 10 and 11), and the last year (Grade 12) is used to review and practice repeatedly to prepare for the College Entrance Examination (Xu, 2017).

In general, traditional Chinese education values knowledge accumulation and infusion, fosters students' respect for knowledge and authority, and emphasizes construction of a knowledge system. Comparatively, the AP program emphasizes students' ability to apply acquired knowledge, encourages students to question knowledge and authority, and highlights the importance of creation and expansion of knowledge (Tao, 2012).

The autonomy of Chinese students is lacking. The course schedule is basically fixed, and they have no means of selecting either course variety or intensity. In contrast, the AP program is open to every student featuring independent choice in line with personal interest and academic advantage (Xu, 2017).

Difference between AP mathematics and general Chinese mathematics. According to Tao (2012), who conducted research in an AP Center of a public high school in Anhui, China, AP mathematics emphasizes the application of knowledge compared to general Chinese mathematics. Also, Tao (2012) mentioned that AP mathematics not only focuses on building connections among different mathematic topics and relating mathematical ideas to real-life situations, but also regards the integration of mathematics and technology, which leads to the use of a calculator. In China, it is believed that a conceptual framework and basic operations constitute the foundation of mathematics. Therefore, although the textbook does have some explanation of how to use a calculator, teachers worry that students might become excessively dependent on it. Therefore, using a calculator in the General Examination or the College Entrance Examination is not permitted. Research shows that most Chinese students are good at calculation; however, when doing real practice with various technologies, Chinese students lag far behind Western students.

Jia (2015) also compared AP mathematics and general Chinese mathematics and found out that AP calculus puts emphasis on the integrity of the knowledge system, while general Chinese calculus is concerned with operations. It was further discovered that AP statistics focuses more on the understanding of statistical concepts and basic ideas while general Chinese statistics highlights the value of calculation (Lu, 2013).

To be specific, Tao (2012) compared the “function” part of both curricula, and found out that AP mathematics involves an “inverse function” that is not part of the Chinese mathematics curriculum; likewise, Chinese mathematics refers to a “set” that is not in the AP mathematics curriculum. Jia (2015) listed major differences and similarities by comparing each of the four main topics of AP calculus.

As a result, Tao (2012) and Jia (2015) both stated that AP mathematics follows the integrity of the knowledge system and expands its details while Chinese mathematics starts from a specific question as an entry point and expands around that point.

Especially for calculus, Chai (2006) mentioned that the Chinese system has two different notions: for mathematics majors, it is called “mathematical analysis” which emphasizes the theoretical foundation and the mathematical reasoning of calculus; for non-mathematics majors such as science and engineering, it is called “advanced mathematics,” which underlines computing technique and the application of relative calculus knowledge in other subjects. He also noticed that there are two major issues in Chinese calculus teaching and learning. First, it is excessively exam-oriented. Second, it guarantees teaching quality through its mass approach to higher education.

Moreover, Tao (2012) suggested the necessity of using language consistently under a bilingual studying environment. For instance, the term “whole number” does not have a fixed corresponding meaning in Chinese; instead, it corresponds to three separate terms under different conditions: non-negative integer, positive integer, and integer. Many teachers translate “whole number” into “integer” in the Chinese language which is only partially correct, and it is possible to cause misunderstanding.

Influence of the AP Program on the Chinese High School Program

The introduction of the AP program in China has had an influence on the regular Chinese high school program, causing certain innovations to occur. I will examine Chinese high school program innovations as well as Chinese high school mathematics innovations in detail.

Chinese High School Program Innovation

The Chinese high school program innovations that have been driven by the presence of the AP program can be placed into 6 categories: bridging the gap between secondary and higher education, college admission criteria, curriculum design, depth and breadth, flexible schooling and the credit system, and teacher training.

Bridging the gap between secondary and higher education. As Chai (2006) said in his research, although Chinese high schools provide optional courses for students to select, the credits for those courses are not admitted as official college credits or for Advanced Placement, which means that there exists a disconnection between secondary and higher education. However, scholars have demonstrated that the AP program gives students an opportunity to learn college-level courses in advance, and it is recognized by almost every college and university in the United States. Also, Kong and Ma (2007) add that the program avoids the repetition of the teaching of same knowledge in both high school and college, which adds to the efficiency of teaching and learning. Thus, the expansion of the AP program makes for a continuous and integrated curriculum structure from secondary to higher education, which is definitely worth adopting by Chinese higher education (Chai, 2006; Kong & Ma, 2007; Lu, 2013; Wang, 2017).

College admission criteria. Zhao and Zhang (2009b) mention that the only way to get into higher education in China is to take the College Entrance Examination, which recruits a lot of talented students for this country but also it limits students' diversified development. Therefore, scholars also recommend introducing more elective programs such as the Advanced Placement program to provide an alternative reference index for college admission criteria that may also help fully develop all students' potential abilities (Wang, 2017; Zhao & Zhang, 2009a).

Curriculum design. Zhao and Zhang (2009b) also noticed that in the Advanced Placement program, students have the opportunity to select preferred courses, which greatly helps to increase their diversification. However, in China, the entire curriculum only lists a few kinds of electives, and the quantity of these optional courses is far behind the total 38 subjects provided by the AP program. The Chinese curriculum could give students more options to cultivate their autonomous learning so that they might take more interest and responsibility toward the classes they choose on their own. Eventually it will help them to become more independent learners instead of learning passively (Lu, 2013; Zhao & Zhang, 2009a).

Depth and breadth. Chinese secondary education tends to be more focused on skills training and, does not foster creativity and learning confidence. However, the AP program seeks to increase understanding and facilitate application of knowledge; also, its students illustrate excellence, confidence, and creative thought (Kong & Ma, 2007; Lu, 2013). In addition, Chinese optional courses have extensive content but do not encourage deep thought, which causes a "mile-wide inch-deep" phenomenon. Teachers sometimes roughly handle the optional content because of the limited time available to them, so students do not develop a complete understanding of the basic concept. Because students choose an elective based on their desire to explore the subject, it is critical to increase their depth of knowledge in that subject; therefore, it is important to find a balance between the depth and breadth of optional classes (Zhao & Zhang, 2009b).

Flexible schooling and the credit system. Kong and Ma (2007) point out that most Chinese high schools use competition as a method of encouraging advanced students to take challenges. Although there are various patterns in Western secondary education, most schools' aim is to offer different education for students with different learning abilities. Western schools utilize a flexible credit system which allows some advanced students to choose programs such as the AP program and earn college credit while in high school, and to even graduate early (Zhao & Zhang, 2009b). Thus, introducing the Advanced Placement program in Chinese high schools is an alternative way to give advanced students more opportunities. Chinese schools could even borrow AP ideas to build flexible schooling as well as the credit system to allow students to study according to their abilities and be more successful in their future studies and careers (Kong & Ma, 2007; Wang & Wang, 2016; Zhao & Zhang, 2009b).

Teacher training. Tian (2004) mentions the importance of teacher training as it relates to the success of the AP program. The College Board created the AP Teacher Community to support teachers with an extensive resource library, and the Board provides sponsored workshops and online professional development opportunities (Professional Development, 2018). In this way, teachers have sufficient instructional supports.

Chinese High School Mathematics Innovation

The innovations in Chinese high school mathematics that have resulted from the presence of the AP program in China can be grouped into three sorts: mathematics teaching mode, understanding and application, and language and technology.

Mathematics teaching mode. Chai (2006) and Wang (2017) both suggest using the "Rule of Four," that approaches calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and can reveal connections amongst these representations to open instructors' minds to new ideas in the teaching of mathematics. In Chinese mathematics teaching, the description of mathematical questions usually lays particular stress on formula and reasoning. But graphical, numerical or verbal representations can give students a direct impression, which helps with the understanding of mathematical thoughts and linking mathematical knowledge to real-life situations. Thus, the "Rule of Four" is an effective method to help innovate Chinese mathematics teaching.

Understanding and application. AP mathematics values students' full understanding of mathematical concepts and problems not only in its curriculum goals but also in its the examinations (Chai, 2006; Lu, 2013). Some scholars also recognize that many of the examination questions, especially the free-response questions in AP calculus, are aimed at testing students' ability to apply acquired mathematical knowledge. Those questions closely integrate with real-life practice and are heavily related to other subjects such as physics and economics. Students should not only be able to solve these problems but also be able to provide logical reasoning. However, Chinese students are trained in calculating and memorizing for a long time, which makes them better at doing math but not applying math. Furthermore, their exam-oriented education makes this situation worse, which causes a poverty of conception that is fostered by overly rigid thoughts and which may limit students' imagination and learning efficiency. For example, students lack the creative ability to explore mathematical problems. Thus, it is necessary to emphasize the understanding and application of mathematics in real practice in both teaching and learning (Chai, 2006; Lai, 2013; Lu, 2013; Zhang, 2012).

Language and technology. Liang (2012) and Lai (2013) mention that since the entire AP program, including both AP courses and AP examinations, requires all students to read and respond in English, it is inevitable that students will face language-related challenges. Students need to be able to have an accurate understanding of mathematical problems, and particularly of mathematical terms represented in English. Li, Chu, Lai, and Chen (2018) encourage the use of modern educational technology; Kong and Ma (2007) agree, saying that Chinese students should have the opportunity to use graphing calculators because AP calculus examinations allow their use in some questions.

International Baccalaureate

The International Baccalaureate (IB), formerly known as the International Baccalaureate Office (IBO), was founded in Geneva, Switzerland in 1968 and was developed to create a rigorous and universal curriculum standard to meet the educational needs of "Global Citizens": geographically mobile students who required academic credentials that were accepted worldwide, such as children of diplomats, students living abroad, native students returning from abroad, and children likely to travel extensively abroad. The IBO was a chartered, private, nongovernmental international educational foundation (Cheng & Deng, 2006; Fox, 1985; Gu, 2006a; Li, 2006; Li & Yao, 1998; Liu, 2008; Ma, 2009; Nugent & Karnes, 2002; Poelzer & Feldhusen, 1997; Song, 2012; Xu, 2001; Zhang, 2014).

IB holds consultative status as a non-governmental organization (NGO) at the United Nations Educational, Scientific, and Cultural Organization (UNESCO) and is recognized by the Council of Europe and the Organization Internationale de la Francophonie (OIF) (Cheng & Deng, 2006; Hong, 1994; Gu & Yang, 2002; Li & Yao, 1998; Nugent & Karnes, 2002; Song, 2012).

IB offers an education for students aged 3 to 19, and is comprised of four educational programs: the IB Primary Years Programme (PYP) for students aged 3 to 12, the IB Middle Years Programme (MYP) for students aged 11 to 16, the IB Diploma

Programme (IBDP) and the IB Career-Related Programme (CRP) for students aged 16 to 19 (Clayton, 1998; Gu & Yang, 2002; IB Programmes, 2018; Li & Yao, 1998; Liang, 2016; Nugent & Karnes, 2002; Xu, Xia, & Chen, 2015; Xu & Xiang, 2015; Zhang, 2014). The International Baccalaureate (IB) program can be best understood by examining its history, the IB Diploma Programme (IBDP) curriculum outline, and its methods of assessment.

History

In 1951, the United Nations Educational, Scientific, and Cultural Organization (UNESCO) founded the International Schools' Association (ISA) and gave it three successive contracts to study practical ways of harmonizing curricula and methods for the development of international understanding. In August 1964, the Twentieth Century Fund offered ISA a grant of $75,000 for an investigation into a common curriculum and examination program for international schools, which would facilitate admission to any college or university (Jonietz & Harris, 2012; Liu, 2008; Peterson, 2003; Poelzer & Feldhusen, 1997; Renaud, 1974). In 1965, the International Schools' Examination Syndicate (ISES) was founded; it changed its name to the International Baccalaureate Office (IBO) in 1968 (Jonietz & Harris, 2012; Nugent & Karnes, 2002; Poelzer & Feldhusen, 1997; Renaud, 1974).

With IBO's inception in 1965, IB Diploma Program became the first offered program that has been offered in 1968. It aims to develop students who have excellent breadth and depth of knowledge, students who flourish physically, intellectually, emotionally, and ethically (Gu, 2006b; IB Programmes, 2018; Key Facts about the DP, 2018; Liu & Liu, 2008; Song, 2012). The first conference in 1965 decided on the framework of the system and discussed the five subjects considered to have priority: Language A, Language B, History, Geography, and Biology (Renaud, 1974). The years 1964-1970 were devoted to experimenting with the new system of teaching and assessment, and the 1970 session saw the program's first graduates receiving their diplomas or certificates (Hong, 1994). Also in that year, the first 29 students sat for the official IB Examinations for college entrance (Fox, 1985; Peterson, 2003; Renaud, 1974; Song, 2012; Xu, 2001). Later, the original IBDP expanded to offer MYP, which started in 1994; PYP, which was piloted in 1997; and CRP, which began in 2012 (Clayton, 1998; IB Programmes, 2018; Nugent & Karnes, 2002; Song, 2012). To teach all these programs, schools are required to obtain authorization from the International Baccalaureate Organization (IBO) (How to Become an IB School, 2018; Song, 2012).

According to Peterson (2003), International Baccalaureate North America (IBNA) was established in 1975; International Baccalaureate Africa, Europe and Middle-East (IBAEM) was founded in 1986; and International Baccalaureate Asia Pacific (IBAP) was also set up during the same period.

IBDP Curriculum Outline

The International Baccalaureate (IB) Diploma Programme (DP) was established to provide students with a balanced education, to facilitate geographic and cultural mobility, and to promote international understanding; it was the first program offered by the IB and is available for students aged 16-19 (Gu, 2006b; IB Programmes, 2018; Key Facts about the DP, 2018; Liu & Liu, 2008; Liu, Liu, & Jiang, 2015). The IBDP curriculum is made up of three DP cores and six subject groups (Gu, 2006b; Cheng & Deng, 2006; IB Curriculum, 2018; Jonietz & Harris, 2012; Li, 2006; D. Li, 2009; Liu & Liu, 2008; Song, 2008; Song, 2012; Yang & Dai, 2012a, 2012b; Zhang, 2014).

Figure 1 shows the curriculum outline of the International Baccalaureate Diploma Program (IBDP).

Figure 1. IB Diploma Program Curriculum (Retrieved from https://www.ibo.org/programmes/)

The DP core includes three required components designed to broaden students' educational experience and challenge them to apply their knowledge and skills (see Table 4.) The three core elements are: Theory of Knowledge (TOK); Extended Essay (EE); and Creativity, Action, Service (CAS) (Cheng & Deng, 2006; Gu, 2006b; Hong, 1994; IB Curriculum, 2018; Li, 2006; Li & Yao, 1998; Liu & Liu, 2008; Liu et al., 2015; Nugent & Karnes, 2002; Song, 2012; Song & Wang, 2015; Wang & Wang, 2016; Yang & Dai, 2012a; Zhang, 2014).

Table 4. Three IBDP Cores

Three IBDP Cores	
Theory of Knowledge (TOK)	A reflection on the nature of knowledge and on how we know what we claim to know, finishing with an oral presentation and a 1,600-word essay
The Extended Essay (EE)	An independent, self-directed piece of research, finishing with a 4,000-word paper
Creativity, Activity, Service (CAS)	A project related to those three concepts

The TOK course is mandatory; it encourages critical thinking about knowledge and helps students to understand the content. It involves various activities and discussions that encourage students to express their thoughts and emphasize their logical thinking. It can prevent students from making subjective assumptions and engaging in perspective bias while sharing ideas and listening to others (Cheng & Deng, 2006; Cox & Daniel, 1983; Hong, 1994; Li, 2006; Nugent & Karnes, 2002; Peterson, 2003; Poelzer & Feldhusen, 1997; What is TOK, 2018; Yang & Dai, 2012a; Zhang, 2014). The EE course comes from one of the student's six chosen subjects from the IB Diploma Program, and students are required to make an in-depth study of the chosen topic. It aims to increase students' research and writing skills and is very important in their future studies. In higher education, each student is expected to possess the ability to do research independently. Therefore, the EE course requires them to select topics of their own interest; to conduct research under the guidance of a supervisor; and to assemble a formally presented, reasonably structured, and coherently expressed 4000-word paper (Cheng & Deng, 2006; Cox & Daniel, 1983; Hong, 1994; Li, 2006; What is EE, 2018; Yang & Dai, 2012a; Zhang, 2014). The CAS course is intended to enhance students' personal and interpersonal development through experiential learning. "Creativity" includes arts and other activities that can help enhancing creative thinking. For example, a group of students in New York produced and performed a play to raise awareness of a real-world issue. "Activity" means physical exertion contributing to a healthy lifestyle that complements academic work. For example, a student in Australia achieved his dream of becoming a youth soccer coach through CAS. “Service” is a voluntary exchange that can improve students’ personal and interpersonal development. For example, students in Indiana organized a large-scale recycling drive to help an IB School in Michigan (Cheng & Deng, 2006; Li, 2006; Liu & Liu, 2008; What is CAS Project, 2018; Yang & Dai, 2012a; Zhang, 2014). Moreover, the CAS project can address any single strand of CAS, or it can combine two or three of them (What is CAS Project, 2018). Researchers such as Yang & Dai (2012a) believe that the DP Core shows the comprehensiveness and uniqueness of the IBDP.

The six subject groups are: studies in language and literature, language acquisition, individuals and societies, sciences, mathematics, and the arts (see Table 5.) There are different courses within each subject group, and students have a choice within the six (Cheng & Deng, 2006; Gu, 2006b; IB Curriculum, 2018; Li, 2006; Li & Yao, 1998; Liu & Liu, 2008; Liu et al., 2015; Nugent & Karnes, 2002; Xu, 2001; Yang & Dai, 2012b; Zhang, 2014). In addition, students may choose to study an additional science, individual and society, or language courses, instead of taking a course in the arts (IB Curriculum, 2018; Xu, 2001).

Table 5. Six IBDP Subject Groups

Six IBDP Subject Groups
Studies in Language and Literature
Language Acquisition
Individuals and Societies
Sciences
Mathematics
The Arts

The courses are available in two levels: standard level (SL) and higher level (HL). SL subjects require 150 teaching hours whereas HL comprises 240 teaching hours (IB Curriculum, 2018; Nugent & Karnes, 2002). The difference between SL and HL course is a matter of scope; students are measured by the same grade descriptors, with HL students expected to demonstrate a greater body of knowledge, understanding, and skills (IB Curriculum, 2018; Koetzsch, 1997; Nugent & Karnes, 2002). Each student is required to select one subject from each of the six subject groups (Cheng & Deng, 2006; Li, 2006; Nugent & Karnes, 2002; Xu, 2001; Yang & Dai, 2012b). At least three (but not more than four) must be subjects at the HL level, and the remaining at the SL level (Cheng & Deng, 2006; IB Curriculum, 2018; Li, 2006; Song & Wang, 2015; Wang & Wang, 2016; Xu, 2001; Zhang, 2014). Researchers such as Gu and Yang (2002), Cheng and Deng (2006), and Yang and Dai (2012b) thought the many choices in terms of disciplines and course levels reveal a curriculum design that effectively balances breadth, depth, and student interest in academic subjects.

IBDP Assessment

Generally, each program has a program committee, which is responsible for supervising quality and development, and so does the IBDP (What You Need to Know about IB, 2018). For the IBDP, there are two examination sessions, one in May and the other in November (Getting Results of IBDP, 2018; Ma, 2009; Song, 2012). DP assessment mainly measures students' academic skills, but it also encourages an international outlook and intercultural skills (Liu et al., 2015). The assessment measures academic skills including basic skills such as retaining knowledge, understanding key concepts, and applying standard methods as well as advanced skills like analyzing and presenting information, evaluating and constructing arguments, and solving problems creatively (Assessment and Exams of IBDP, 2018).

The IBDP uses both external and internal assessments to measure student performance (Assessment and Exams of IBDP, 2018; Cheng & Deng, 2006; Hong, 1994; Jonietz & Harris, 2012; Li, 2009; Liu & Liu, 2008; Liu et al., 2015; Song, 2008, 2012; Understanding DP Assessment, 2018). Written examinations form the basis of the external assessment for most courses because of their high levels of objectivity and reliability (Assessment and Exams of IBDP, 2018; Understanding DP Assessment, 2018; What You Need to Know about IB, 2018). They include structured problems, short-response questions, data-response questions, text-response questions, case-study questions, multiple-choice questions, and essays (Assessment and Exams of IBDP, 2018). Essays on the theory of knowledge (TOK) and the extended essay (EE) are parts of the externally-assessed coursework that are completed by students over an extended period under authenticated teacher supervision (Understanding DP Assessment, 2018). In most subjects, internal assessment is also used, and it is marked by teachers according to IB criteria and then moderated by the IB (Assessment and Exams of IBDP, 2018; Understanding DP Assessment, 2018). This kind of assessment by teachers includes oral work in languages, fieldwork in geography, laboratory work in the sciences, investigations in mathematics, and artistic performance (Assessment and Exams of IBDP, 2018).

Every subject is scored on a 1 to 7 scale, with 7 being the highest numerical grade (Cheng & Deng, 2006; Li & Yao, 1998; Liu et al., 2015; Nugent & Karnes, 2002; Song, 2012; Song & Wang, 2015; Wang & Wang, 2016; Understanding DP Assessment, 2018; Xu, 2001). Scores in the higher level (HL) and standard level (SL) courses have the same number of points, reflecting the IB's belief in the importance of achievement across a broad range of academic disciplines. Therefore, as Table 6 shows, scores from a student's six chosen subjects total 42 points. The TOK essay and the EE essay contribute up to 3 additional points in total. And the remaining element in the DP core, CAS, does not count for points but is regarded as participation, which is part of the requirement for getting the diploma (Understanding DP Assessment, 2018). So, the overall possible score in this program is 45 points (Li & Yao, 1998; Liu et al., 2015; Song, 2012; Song & Wang, 2015). A student's final diploma score is a combination of scores for each subject, and student who gains at least 24 points and meets certain minimum levels of performance including completion of the 3 DP cores will be awarded the diploma (Cheng & Deng, 2006; Liu et al., 2015; Nugent & Karnes, 2002; Song, 2012; Song & Wang, 2015; Understanding DP Assessment, 2018; Xu, 2001). Results will be issued on July 6 for the May session and January 3 for the November session. In each examination session, about 80% of DP students are awarded the diploma (Getting Results of IBDP, 2018).

Table 6. IBDP Score Combinations and Their Corresponding Maximum Scores

IBDP Score Combination		Maximum Score
Six Subject Groups	Six Chosen Subjects	6 subjects * 7 points = 42 points
Three DP Cores	Theory of Knowledge (TOK)+ The Extended Essay (EE)	3 points
	Creativity, Activity, Service (CAS)	0 points (participation)
Total IBDP Maximum Score		45 points

IB Mathematics

The following two sections offer a view of the IB mathematics program by detailing its courses and its methods of assessment.

Courses

As one of the 6 subject groups, group 5, it is a requirement that students choose at least one course in mathematics (IB Mathematics, 2018a; Liu, 2008; Yang & Dai, 2012b). The four courses in mathematics are designed to develop students' mathematical knowledge, concepts, and principles; to develop their logical, critical, and creative thinking; and to employ and refine their powers of abstraction and generalization. The courses also endeavor to encourage students' appreciation of the international dimensions of mathematics and the multiplicity of its cultural and historical perspectives (IB Mathematics, 2018a; Liu & Liu, 2008; Ma, 2009; Xu et al., 2015; Xu & Xiang, 2015; Yang & Dai, 2012b). To be specific, mathematical studies standard level (SL), mathematics standard level (SL), mathematics higher level (HL), and a further mathematics higher level comprise the mathematics domain, and each course is designed to accommodate individual students' different needs, interests and abilities as well as to fulfill the requirements of various university and career aspirations (Hong, 1994; IB Mathematics, 2018a, 2018b; Liu, 2008; Lu, 2000; Ma, 2009; Song, 2012; Wang & Wang, 2016; Xu & Xiang, 2015; Yang & Dai, 2012b). Moreover, these four courses are set up in order of increasing difficulty (Liu, 2008; Liu & Liu, 2008; Lu, 2000; Song, 2012; Xu & Xiang, 2015).

Mathematical studies SL is designed for students whose main interests lie outside the field of mathematics, and for many of them this course will be their final experience of being taught formal mathematics. It focuses on important interconnected mathematical topics by placing emphasis on student understanding of fundamental concepts and helping students to develop their mathematical reasoning and to relate acquired mathematical knowledge to real-life situation (IB Mathematics, 2018b; Song, 2012; Yang & Dai, 2012b). Moreover, a feature unique to this course is the project that is based on students' own research and is guided and supervised by their teachers. This specific project provides an opportunity for students to carry out a mathematical study of their choice using their own experience as well as knowledge and skills acquired during the course (IB Mathematics, 2018b).

Mathematics SL is a course for students with a good background in mathematics and strong analytical and technical skills. It aims at introducing students to important mathematical concepts in a comprehensible and coherent way through the development of mathematical techniques (IB Mathematics, 2018b; Song, 2012; Yang & Dai, 2012b). Some students will be expecting to include mathematic courses in their university studies or will be taking courses such as chemistry, economics, and business management that contain elements of mathematics (Song, 2012; Yang & Dai, 2012b). SL emphasizes the application of the acquired mathematical knowledge to solve realistic problems set in an appropriate context (IB Mathematics, 2018b). The core topics of this course are algebra, functions and equations, circular functions and trigonometry, vectors, statistics and probability, and calculus (see Table 7).

Table 7. IB Mathematics Standard Level (SL) Components

Component	Recommended Teaching Hours
Topic 1: Algebra	9
Topic 2: Functions and equations	24
Topic 3: Circular functions and trigonometry	16
Topic 4: Vectors	16
Topic 5: Statistics and probability	35
Topic 6: Calculus	40
Mathematical exploration: A piece of individual written work that involves investigating an area of mathematics.	10

Mathematics SL does not have the depth found in the HL course. So those who want to pursue subjects with a high degree of mathematical content should opt for a mathematics HL course that focuses on developing important mathematical concepts in a comprehensible, coherent, and rigorous way (IB Mathematics, 2018b; Yang & Dai, 2012b).

Some students will be expecting to include mathematics courses in their university studies, or courses that contain mathematics content and application such as physics, engineering and technology (Song, 2012; Yang & Dai, 2012b). It requires students to study a broad range of mathematical topics through many approaches and to varying degrees of depth. Besides the emphasis on applying mathematical knowledge to solve problems in a variety of meaningful contexts, this course devotes more time to the justification and proof of results, the insight into mathematical form and structure, the connection between concepts in different topic areas, and the skills needed to help build mathematical growth in other learning environments (IB Mathematics, 2018b).

From Table 8, it is clear that the core topics of the Mathematics HL course including algebra, functions and equations, circular functions and trigonometry, vectors, statistics and probability, and calculus. Also, students must choose one optional topic out of four alternatives (IB Mathematics, 2018b; Ma, 2009; Wu, 2014).

Further mathematics HL is intended for students with strong interests in mathematics who want to take an even more rigorous and demanding course; it focuses on several branches of mathematics to encourage students to appreciate the diversity of the subject and to reach an equivalent level of understanding across all topics (IB Mathematics, 2018b; Yang & Dai, 2012b). According to the official statement, students may register for mathematics HL only or for further mathematics HL only or for both. However, students who take this course will be presumed to already know the topics in the core syllabus of mathematics HL and to have studied one of the optional topics in the mathematics HL course. The difficulty of examination questions is comparable with those set on the four optional topics in the mathematics HL course. Unlike the others, this course has no internal assessment component (IB Mathematics, 2018b).

Table 8. IB Mathematics Higher Level (HL) Components

Component	Recommended Teaching Hours
Topic 1: Algebra	30
Topic 2: Functions and equations	22
Topic 3: Circular functions and trigonometry	22
Topic 4: Vectors	24
Topic 5: Statistics and probability	36
Topic 6: Calculus	48
Optional syllabus content (one of the following) Topic 7: Statistics and probability Topic 8: Sets, relations and groups Topic 9: Calculus Topic 10: Discrete mathematics	48
Mathematical exploration: A piece of individual written work that involves investigating an area of mathematics.	10

Assessment

Tables 9 and 10 demonstrate that in IB mathematics students' work is assessed externally and internally (Assessment and Exams of IBDP, 2018). Both mathematics SL and mathematics HL courses have external examinations weighted for 80% of the final

Table 9. Assessment of IB Mathematics (SL)

Type of Assessment	Format of Assessment	Weighting of Final Grade
External (*3 hours*)		80%
Paper 1 (non-calculator) *1.5 hours*	Section A: Compulsory short-response questions based on the whole syllabus. Section B: Compulsory extended-response questions based on the whole syllabus.	40%
Paper 2 (graphing calculator required) *1.5 hours*	Section A: Compulsory short-response questions based on the whole syllabus. Section B: Compulsory extended-response questions based on the whole syllabus.	40%
Internal		20%
Mathematical exploration	Internal assessment in mathematics SL involves individual exploration. This is a piece of written work that requires investigating an area of mathematics.	

Table 10. Assessment of IB Mathematics (HL)

Type of Assessment	Format of Assessment	Weighting of Final Grade
External (*5 hours*)		80%
Paper 1 (non-calculator) *2 hours*	Section A: Compulsory short-response questions based on the *core* syllabus. Section B: Compulsory extended-response questions based on the *core* syllabus.	30%
Paper 2 (graphing calculator required) *2 hours*	Section A: Compulsory short-response questions based on the *core* syllabus. Section B: Compulsory extended-response questions based on the *core* syllabus.	30%
Paper 3 (graphing calculator required) *1 hour*	Compulsory extended-response questions based mainly on the syllabus options.	20%
Internal		20%
Mathematical exploration	Internal assessment in mathematics HL involves individual exploration. This is a piece of written work that requires investigating an area of mathematics.	

grade and internal assessment weighted for 20%. The external examinations include two types of questions: short-response questions (Section A) and extended-response questions (Section B).

Assessment in mathematics SL involves two papers that need to be finished in 3 hours in its external assessment, whereas assessment of mathematics HL has three papers that take 5 hours. Graphing calculators cannot be used in paper 1, but they are required in papers 2 and 3.

From 2006 to 2014, the internal assessment component, portfolio, consisted of two parts including mathematical investigation (Type I) and mathematical modeling (Type II) (Ma, 2010, 2017; Wu, 2014; Xie & Shen, 2013). Starting in May 2014, it changed to only one form, the mathematical exploration; the change was intended to give students the opportunity to develop their independent mathematical learning through various mathematical activities and ideas (Ma, 2017; Wu, 2014). It also allows them to work without the time constraints and to enhance their skill in communication of mathematical ideas (IB Mathematics, 2018b; Ma, 2017; Wu, 2014).

According to the IB assessment criteria (2018), each exploration should be assessed against the following five criteria (see Table 11).

Table 11. IB Assessment Criteria

Criterion A (max mark 4)	Communication
Criterion B (max mark 3)	Mathematical presentation
Criterion C (max mark 4)	Personal engagement
Criterion D (max mark 3)	Reflection
Criterion E (max mark 6)	Use of mathematics

The descriptions of the achievement levels for each of five criteria follow; each achievement level represents the minimum requirement for that level to be awarded (see Table 12).

Table 12. IB Achievement Level Description (Criterion A to D)

Achievement Level	Descriptor of Criterion A	Descriptor of Criterion B	Descriptor of Criterion C	Descriptor of Criterion D
0	The exploration does not reach the standard described by the descriptors below.	The exploration does not reach the standard described by the descriptors below.	The exploration does not reach the standard described by the descriptors below.	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence.	There is some appropriate mathematical presentation.	There is evidence of limited or superficial personal engagement.	There is evidence of limited or superficial reflection.
2	The exploration has some coherence and shows some organization.	The mathematical presentation is mostly appropriate.	There is evidence of some personal engagement.	There is evidence of meaningful reflection.
3	The exploration is coherent and well organized	The mathematical presentation is appropriate throughout.	There is evidence of significant personal engagement.	There is substantial evidence of critical reflection.
4	The exploration is coherent, well organized, concise and complete.		There is abundant evidence of outstanding personal engagement.	

The final mark for each exploration is the sum of the achievement levels awarded for each criterion A to E. The maximum possible mark is 20 (IB assessment criteria, 2018).

The descriptors for criterion E are noticeably different for mathematics SL and mathematics HL (see Table 13).

Table 13. IB Achievement Level Description (Criterion E)

Achievement Level	Descriptor of Criterion E (SL only)	Descriptor of Criterion E (HL only)
0	The exploration does not reach the standard described by the descriptors below.	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. Limited understanding is demonstrated.	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and reflects the sophistication expected. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and reflects the sophistication and rigor expected. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and reflects the sophistication and rigor expected. Thorough knowledge and understanding are demonstrated.

IB Concept

In short, the IB is motivated by its mission to create a better world through education (IB Mission, 2018; What You Need to Know about IB, 2018).

The goal of the IB was to create an advanced and rigorous pre-university curriculum that emphasizes critical thinking and international awareness in order to foster inquiring, knowledgeable, and caring young people from varying cultural, economic, and societal backgrounds (Chen & Hou, 2011; Cheng & Deng, 2006; Clayton, 1998; IB Mission, 2018; Koetzsch, 1997; Lateer, 1999; Li, 2006; Liang, 2016; Liu et al., 2015; Nugent & Karnes, 2002; Song, 2012; Tookey, 1999; What You Need to Know About IB, 2018; Xu & Gao, 2012). Those young people are active, compassionate, and lifelong learners across the world who can help to create a better and more peaceful world through intercultural understanding and respect (Chen & Hou, 2011; Gu & Yang, 2002; IB Mission, 2018; Li, 2006; Liu et al., 2015; Song, 2012; What You Need to Know About IB, 2018; Xu & Gao, 2012; Xu et al., 2015), Renaud (1974) named people with these qualities as "whole men." In detail, IB learners strive to be inquirers, reflective thinkers, communicators as well as principled, open-minded, caring, risk-taking, balanced, and reflective (Li, 2006; Liu et al., 2015; What You Need to Know About IB, 2018; Yamamoto et al., 2016).

To this end, the IB also focuses on working with schools, governments, and international organizations to develop challenging international programs featuring rigorous assessment (IB Mission, 2018).

IB Benefits as Identified in Literature

There are three major ways in which students may benefit from participating in the IB program. They are the gaining of heightened skills and college readiness, better prospects for college admission and academic success, and savings of time and money.

Student Skills and College Readiness

According to Dr. David Conley (2007), college readiness is understood as the level of preparation a student needs to be able to enroll and succeed in college. Researchers suggested that students who participate in the IB program during high school have a more successful adjustment to college with its rigor and expectations (Beckwitt, Van Camp, & Carter, 2015; Chen, 2013; Cheng & Deng, 2006; Conley, McGaughy, Davis-Molin, Farkas, & Fukuda, 2014; IB Students' Preparedness for Success, 2018; Li & Yao, 1998; What is the DP, 2018). More specifically, after experiencing the pressure of IB program's rich curriculum, students reported that they felt better prepared for college's exam-based grading structure and its coursework. It also helped them practice efficient time management helped them to cope with the demanding workloads, while non-IB students indicated that they felt less adept at managing their time or studying for culminating examinations (Conley et al., 2014; IB Students' Preparedness for Success, 2018; Taylor & Porath, 2006; What is the DP, 2018; What You Need to Know About IB, 2018). Also, for non-academic preparation, findings suggest that IB students tend to use risk and experimentation strategies to overcome challenges and to show more independent learning while non-IB students may rely more on family or others for academic support (Beckwitt et al., 2015; Conley et al., 2014; What You Need to Know About IB, 2018).

Furthermore, findings illustrated many aspects in which the IBDP provides a more comprehensive and effective approach to readying students for college than other types of curricula (Conley et al., 2014; IB Students' Preparedness for Success, 2018). Conley (2013) stated that the IBDP perhaps becomes the single most important factor in students' success, namely the degree to which they take ownership of their learning and are allowed to do so. Some DP students reported that they could have benefited from stronger content knowledge before entering the college, particularly in mathematics or other STEM-related fields (Lee et al., 2017). To be specific, the three IBDP cores provides an important grounding in developing student skills (Aulls, Lemay, & Peláez, 2013; Billig & Good, 2013; Conley et al., 2014; IB Students' Preparedness for Success, 2018; Inkelas et al., 2013; Lee et al., 2014; Liu et al., 2015; Saavedra, 2016; What is the DP, 2018; Wray, 2013). For example, researchers suggested that IBDP students carried strong critical thinking skills and adopted multiple perspectives to solve the same problems. They also demonstrated a deeper understanding of the structure of knowledge, large concepts, and the content connections across disciplines than their non-IB peers. That understanding helped a lot with their cognitive development and fostered an ability to make connections across academic disciplines, to present in front of a class, and to critically evaluate knowledge, after learning the Theory of Knowledge (TOK) core (Beckwitt et al., 2015; Bergeron & Rogers, 2015; Cole et al., 2014; Conley et al., 2014; IB Students' Preparedness for Success, 2018; Lee et al., 2014). Analysis of IBDP students in Canada, the UK, and the USA revealed that the IBDP's Extended Essay (EE) core makes students better able to write their college theses due to the experience of working with mentors, conducting in-depth research, determining the credibility of relevant sources, and producing coherent extended essays with cited references (Aulls et al., 2013; Conley et al., 2014; IB Students' Preparedness for Success, 2018; Inkelas et al., 2013; Lee et al., 2014; What is the DP, 2018; What You Need to Know About IB, 2018; Wray, 2013). The study done at the University of Virginia even compared IB students with former Advanced Placement (AP) students and noticed that IB students were significantly more likely to show preparedness for college-level coursework involving research; it also found their research skills to be important to future success (IB Students' Preparedness for Success, 2018; Inkelas et al., 2013). Moreover, research conducted in 2016 made a complementary statement that IB students graduate with a sense of civic responsibility and engage in service activities to help the community (Billig & Good, 2013; IB Students' Preparedness for Success, 2018). Conley et al. (2014) and Hayden et al. (2017) agreed that the Creativity, Activity, Service (CAS) core contributed mostly to this aspect. Besides the civic responsibility, Saavedra (2016) found that IBDP students also demonstrate academic civic mindedness due to their sufficient knowledge of government, public policy, and effective advocacy techniques (IB Students' Preparedness for Success, 2018).

Besides fostering life-time learners, the IB also helps students study topics from an international perspective, which becomes its most unique feature (Doherty, 2009; What You Need to Know about IB, 2018). Exploring their own cultures and learning how to respect other cultures can help students better communicate with people from around the world; the organization is more dedicated than ever to developing international education that will help to create a better world (About the IB, 2018).

College Admission and Student Success

The IB program fosters international perspectives and intercultural understanding while supporting the maintenance and development of its students' cultural identities. A growing number of colleges and universities worldwide recognize the IB credential, and it provides IB students with more chances to attend top colleges and universities (Gu, 2006b; Gu & Yang, 2002; D. Li, 2009; Li & Yao, 1998; Liu et al., 2015; Lu, 2000; Song, 2012; Xu, 2001; Xu & Gao, 2012; Zhang, 2014). For example, according to a 2013 study conducted by researchers from the University of Hong Kong, about 72% of students who graduated from IB programs in China between 2002 and 2012 were later enrolled in the world's top 500 universities (Lee et al., 2014; What is the DP, 2018). In 2008, the former Prime Minister Blair in the UK announced that the IB would be offered in UK government schools as an alternative to the renovated A-Level curriculum (Philips & Pound, 2003). And it turns out that more than twice the number of IB students attended a top 20 university than did the average A-Level students (IB Diploma Compared with Other Qualifications, 2018). The breadth of its subjects allows students to be much better for life beyond school (Jenkins, 2003; What You Need to Know about IB, 2018). Overall findings from research studies have been very favorable for the IB Diploma Programme

(IB Diploma Compared with Other Qualifications, 2018). In the US, four IBDP standard level (SL) courses including mathematics were compared with similar Advanced Placement (AP) courses and the assessors assigned the DP SL courses equal or higher grades than the AP courses (Byrd, 2007; IB Diploma Compared with Other Qualifications, 2018). In the UK, a report compared about 20 curricula/examinations including the IB Diploma Programme, and the result shows that the DP materials were highly regarded in many areas (IB Diploma Compared with Other Qualifications, 2018). The DP courses allow for careful differentiation between students' achievements by grade, which helps to identify the highest levels of achievement for competitive higher education admission purposes (Bergeron & Alcantara, 2015).

The IB works hard to focus on personal, professional, and academic development. It is globally recognized by colleges and universities for the holistic and rigorous education it offers, with its students gaining competitive offers for further studies (Cheng & Deng, 2006; Gu, 2006b; Gu & Yang, 2002; Li, 2006; Li & Yao, 1998; Liu & Liu, 2008; Recognition of the IB, 2018; Xu, 2001; Zhang, 2014). Based on the newest record, there are IB World Schools in nearly 150 countries worldwide, and students send their examination results to higher education institutions in about 90 countries annually (Recognition of the IB, 2018). An analysis of the recognition policies of the top universities in the US reveals that most of these top institutions value IBDP students' outstanding academic performance and their abilities to handle the challenges of independent work (IB Diploma Compared with Other Qualifications, 2018).

Also, surveys of college and university admission officers in the US, the European Union, and Australia showed that these professionals are familiar with the program's reputation and regard the IB credential with high esteem and a symbol of greater accomplishment at the high school level compared to other qualifications (IB Diploma Compared with Other Qualifications, 2018; Kyburg, Hertberg-Davis, & Callahan, 2007).

In addition, the introduction of the IB in the US was used to draw middle-class students back to school (Matthews & Hill, 2005), and minority and low income IB students were shown to go on to university at significantly higher rates than a matched control group of their non-IB peers of similar academic ability (Coca et al., 2012; IB Diploma Compared with Other Qualifications, 2018).

Research demonstrates a positive relationship between indicators of high school IB participation and performance and college performance (Shah, Dean, & Chen, 2010b). Several studies commissioned by the IB have concluded that IB students tend to go to university at higher rates, go to more selective universities, and perform better once there (IB Diploma Compared with Other Qualifications, 2018). Conley and a team of researchers (2014) conducted a study and found out that IB students scored higher on university mathematics placement tests than their non-IB peers, and students who had completed four or more IB courses in high school were more likely to show persistence through college in comparison with non-IB students (IB Students' Preparedness for Success, 2018).

Research also indicated the IB program was key in preparing students for success in college (Conley et al., 2014). Recent studies indicate that the results of the DP examination are not just a predictor of gaining college admission, but also of continued success. Teachers and administrators also report that students who do well in the DP often "coast" through the first year of college (Lee et al., 2014). IB itself has also increased research efforts to better understand the impact of its programs, inform the global conversation on student success, and continually contribute to the improvement of education (IB Students' Preparedness for Success, 2018). In 2014, the IB released a series of research studies primarily exploring the impact of the Diploma Programme (DP), which is designed to prepare high school students for success at higher education and life beyond and also elicit high levels of intellectual, behavioral, and emotional engagement than non-IB students (IB Students' Preparedness for Success, 2018; Shah, Dean & Chen, 2010a).

Savings of Time and Money

One of the oft-cited benefits of completing the IB program is the opportunity to finish the bachelor's degree in a shorter time, which helps students save money on tuition because they may be awarded college credit from colleges and universities in the United States and other countries (Bragg, Kim, & Barnett, 2006; Chmelynski, 2005; Fowler & Luna, 2009; Hertberg-Davis, Callahan, & Kyburg, 2006; Hoffman, 2003; Kyburg et al., 2007; Plucker, Chien, & Zaman, 2006). For instance, some colleges and universities allow students who have outstanding IB Examination results to skip most or all of the freshman-year courses (Li & Yao, 1998; Zhang, 2014). Some colleges and universities such as San Sebastian University in Chile even have a standard tuition discount for students with higher IB results (Li & Yao, 1998).

Research studies show that IB students have higher graduation rates than their non-IB peers. For example, Caspary (2011) mentioned that of students who enrolled full-time at a 4-year college, 69% graduated within 4 years, and 84% within 6 years. Furthermore, DP students have notably higher graduation rates (83%) than the 2009 national average of 56% (Bergeron, 2015).

Current Situation and Development

The current situation and development of IB programs in China can best be understood in relation to international IB programs. The literature concerning Chinese IB programs yields 5 important aspects: localized features, challenges to Chinese students, benefits for Chinese students, differences between IB and general Chinese education, and differences between IB mathematics and general Chinese mathematics.

International IB Programs

As a universally recognized qualification, the IB program's participation has increased dramatically since its inception (Cheng & Deng, 2006; Gu, 2006b; Gu & Yang, 2002; Kyburg et al., 2007; Li & Yao, 1998; Lu, 2000; Renaud, 1974; Xu, 2001; Zhang, 2014). IB was first available in 1971 in only 7 schools from 10 countries with an estimated 749 students, and it was only offered in private schools (IB Presentation of History, 2018; Kyburg et al., 2007). Between 2012 and 2017, the number of IB programs offered worldwide has increased by 39.3% (IB Annual Review 2016-17, 2018; IB Facts and Figures, 2018). Its flexibility and reputation have made it suitable for use in a various regional and national education systems; the IB program continues to grow each year (About the IB, 2018; Cheng & Deng, 2006; Gu, 2006b; Gu & Yang, 2002; Li & Yao, 1998; Lu, 2000; Xu, 2001; Zhang, 2014). Based on the latest data, on November 6, 2018, more than one million IB students attended nearly 4,954 schools in 153 countries and 6,425 programs were being offered worldwide (About the IB, 2018; IB Facts and Figures, 2018; IB Students' Preparedness for Success, 2018; IB Presentation of History, 2018). In addition, a May 2018 statistical report shows that about 49.39% of IB schools are state schools, whereas the other 50.61% are private schools (IB Statistical Bulletin, 2018). To be specific, there are 2,931 (59.1%) IB world schools with 3,470 (54%) programs in the Americas; 1,149 (23.2%) schools with 1,656 (25.8%) programs in Africa, Europe, and the Middle East; and 880 (17.7%) schools with 1,299 (20.2%) programs in the Asia-Pacific region (IB Facts and Figures, 2018).

Furthermore, as a very popular program, the IBDP is already widely accepted as a qualification for access to higher education in all continents of the world (Gu, 2006b; Gu & Yang, 2002; D. Li, 2009; Li & Yao, 1998; Lu, 2000; Xu, 2001; Zhang, 2014). As of February 2018, there are 3,182 schools offering the DP in 153 different countries worldwide (Key Facts about the DP). In 1971, there were only 681 DP examination candidates (IB Presentation of History, 2018); however, in May 2018, the number of DP candidates had increased to 163,173 and 615,074 examinations were taken (IB Statistical Bulletin, 2018). Also, according to the newest IB Annual Review (2018), the number of diploma candidates increased from 55,743 in 2010 to 92,746 in 2017, with an approximate 66.38% growth, and the number of diploma examinations taken increased by about 63.02%, from 412,139 to 671,874, during the same time period (IB Annual Review 2016-17, 2018).

Chinese IB Programs

The IB program is flourishing in Chinese society with the globalization and internationalization of Chinese education (Gu, 2006b; Li & Yao, 1998; Zhang, 2014). A lot of Chinese scholars and researchers have devoted themselves to exploring the effects of introducing the IB program in China, especially since the 1990s. The International School of Beijing became the very first IB school in this country 1991 (IB China, 2018; Zhang, 2014). Interestingly, at the beginning, the growth of IB schools in China was very slow: during the five-year period from 2001 to 2005, only 12 more schools joined the community; from 2006 to 2010, 29 more schools entered the market. However, since 2012, the growth has accelerated, with 65 more schools by 2017, and the average increase in the number of schools is about 13 per year (IB China, 2018). Therefore, after less than 30 years, there are 142 IB schools in China right now: the PYP is offered by 74 schools, the MYP is offered by 39 schools, the CRP is only offered by 2 schools, and the DP is the most popular program, being present in 102 schools (IB China, 2018). IB fits well in China mainly because traditional Chinese high school runs from grade 10 to 12, and the IB Diploma Programme is implemented during grades 11 and 12. Therefore, IB schools in China usually teach Pre-IBDP content in grade 10 to prepare students for the 2-year IBDP learning which follows (Liang, 2016).

Localized features. The IB's primary priority is to provide internationalized education (Liu et al., 2015; Zhang, 2014). However, the IB regards each region's national culture as its core, which means that IB seeks to use cultural fusion to promote educational integration. Doherty (2009) mentions that IB has been implemented in collaboration with local curricula, and that it is not only offered to local students but also to internationally mobile students. Zhang (2014) believes it is also the crucial reason that so many countries have become very supportive of its popularization and have actively introduced the IB program into their education system. The IB program is meant to be localized. For instance, students in the Chinese IB schools are required to take the General Examination, which means they need to learn Chinese high school curriculum and the IB curriculum at the same time (Yang, 2013). It is a government adjustment to accommodate regional situations. Therefore, Sino-US cooperation in instruction is a common paradigm in the IB schools in China. Students who take the IBDP program in China can earn two diplomas: one from the IBO and another from the local high school that they attended.

Challenges to Chinese students. As Xu (2001) and Song (2012) mention in the articles, the IB program requires students to learn at least two languages, which is listed in its six objects. Since their native language is not English, Chinese students have to deal with significant challenges from the bilingual learning environment. In addition, Ma (2009) notices that the time commitment of the IBDP's higher level courses is greater than the overall studying time required by the Chinese courses, which means that students are facing an intense schedule and heavy workload. These pressures sometimes frustrate Chinese students.

Benefits for Chinese students. Some researchers also notice the positive effect of IB courses for Chinese students. The IB can clearly facilitate mobility and can contribute to the development of international understanding; it can also support the preservation of individual cultures and national identities at the same time, provided contextual factors are appropriately arranged (Gu, 2006b; Hayden & Wong, 1997; Liu et al., 2015; Song, 2012; Xu & Gao, 2012). Finally, the IB program provides a solid foundation for students who want to continue learning languages, social sciences, natural sciences, and mathematics (Xu, 2001). Gu (2006b) and Song (2012) also mention that the IB curriculum, which covers both depth and breadth, can specifically improve Chinese students' critical thinking skills. Some scholars believe that the mixed assessment, especially the internal assessment, helps Chinese students achieve more comprehensive development (Gu, 2006b; Jin, 2008; Song, 2008).

Difference between IB and general Chinese education. According to Li and Yao (1998) and Song (2012), the standard level of the IB program is similar in difficulty level to the Chinese high school curriculum but focuses more on students' creative abilities, experimental abilities, and application abilities. The higher and further levels of the IB program is similar to first-year college level difficulty. Therefore, in general, the IB program is a pre-college level curriculum that helps students transition from secondary education to higher education (Chen, 2013; Li & Yao, 1998; Song, 2012; Song & Wang, 2015; Wang & Wang, 2016). Cheng and Deng (2006) also mention that the IB program helps international students to prepare themselves for their following studies in colleges or universities overseas and bridge the gap between secondary education and higher education. Furthermore, scholars believe that high school should assist students with higher education and their future careers: that is achieved by the design of IB program but is not reflection in general Chinese high school education (Cheng & Deng, 2006; Wang & Wang, 2016). For example, Chinese high school does not offer many elective courses, minor courses, or interdisciplinary courses, not to mention pre-college level courses; and it causes students to have a narrow scope of knowledge. Furthermore, Song (2012) mentions the IBO's emphasis on the regular update of the IB curriculum in every five-year.

Besides the curriculum, there also exists significant difference in concept (Cheng & Deng, 2006; Li & Yao, 1998). Scholars and researchers specifically emphasize that the IB program devotes itself to fostering responsible global citizens that consider global issues based on a deep understanding of diverse values and global vision as well as a genuine interest in and ability to solve real-world problems that affect human and environmental sustainability, eventually making the world a better and more peaceful place (Chen & Hou, 2011; Cheng & Deng, 2006; Fullan, 2001; Li & Yao, 1998; Song, 2012). These goals require students to experience comprehensive development both mentally and physically to acquire sufficient knowledge as well as the skills and traits of critical thinking, smooth communication, persistence, dedicated exploration, timely reflection, and positive willingness to attempt and innovate (Cheng & Deng, 2006; Liang, 2016).

In addition, under the current examination-oriented education system, Chinese high schools expend a lot of effort in homework and remediation (Zhang, 2014). Chen (2013) also points out that Chinese high school teaching is still over-focused on passive learning, rote memorization, and stuffing exercises. However, Chinese curriculum does offer detailed and defined standards for each of its teaching requirements, so teachers have a specific and explicit plan with which to conduct instruction (Ma, 2009). However, the IB program does not provide adequate guidance for teaching, which offers teacher freedom to some extent but sometimes leads to superficial explanation of certain topics (Chen, 2013; Ma, 2009).

Chinese high school education overvalues general principles but overlooks the individual applications so as to conduct a highly unified teaching module; it causes restraint of the students' personalized development (Cheng & Deng, 2006; Song, 2012). In contrast, the IB program recognizes the diversity of students and provides flexible and diverse course contents for students to choose depending on their personal academic interests (Chen, 2013; Ma, 2017; Zhang, 2014). Moreover, instead of using a single standard to measure student performance as Chinese high schools do, the IB uses both internal and external assessment, which includes various reference standards such as assignments, experiments, portfolios, group work, and presentations; teachers also evaluate through a consultative process (Cheng & Deng, 2006; Liu & Liu, 2008; Ma, 2017; Song, 2012; Song & Wang, 2015; Wang & Wang, 2016; Zhang, 2014).

Difference between IB mathematics and general Chinese mathematics. In 1999, scholars noticed that the difference between Chinese mathematics and Western mathematics is in the different ways of managing materials. Chinese mathematics usually starts from a real problem and then introduces the corresponding mathematical idea later on, along with the introduction of another new mathematics idea. But Western mathematics often begins with a real problem and introduces the relative mathematics idea; then it applies the idea to another real problem. Thus, the former has as its teaching purpose the deepening of content, but the latter's purpose is in the practical application of the mathematical concept (Song, 2012; Tang et al., 1999; Xu & Xiang, 2015).

Liang (2016) finds that general Chinese mathematics focuses more on the basis of knowledge, requires that students build up their foundation of mathematics, and encourages them to have excellent mathematical thinking habits. Under the pressure of an examination-orientation education system, students face great stress in attempting to enter higher education. General Chinese mathematics appears to have some representative traits like teacher-centered classrooms, a cramming-style approach to education, excessive assignments, and mass memorization of formulas.

IB mathematics regards application of mathematics as the crucial part, and it also stresses the importance of international communication, interdisciplinary connection, and relationship to real-life. And explorative spirit and critical thinking skills are greatly encouraged (Song, 2012; Xu et al., 2015; Xu & Xiang, 2015). On the other hand, this difference can also mean that IB students do not have solid fundamentals, which can lead to relatively weak calculation skills, for example (Liang, 2016). Ma (2009) makes a complementary statement that the exams in IB mathematics and general Chinese mathematics demonstrate their different priorities. Usually, IB mathematics exams elicits student understanding of conceptual framework by using word problems, while general

Chinese mathematics sets up a high standard for investigating students' operational capability and proving skills (Ma, 2009; Wang, 2016).

In addition, there is a large difference between assessment in the two types of mathematics courses. IB mathematics assessment uses a combination of both external and internal assessments, which provides a full measuring of students' mastery of mathematics knowledge and relative skills, abilities and attitudes, results and processes (Liang, 2016; Liu, 2008; Song, 2012; Song & Wang, 2015; Wang & Wang, 2016; Xu & Xiang, 2015). In other words, test scores are not the only method used to determine student performance (Liang, 2016; Liu & Liu, 2008; Song & Wang, 2015; Wang & Wang, 2016; Xu & Xiang, 2015). IB assessment focuses more on the evaluation of process. The internal assessment, which is used differently by IB than it is by other programs, measures the entire learning process and can better evaluate acquired mathematical abilities than the external assessment can (Liang, 2016; Liu, 2008; Liu & Liu, 2008; Song & Wang, 2015). In addition, in order to guarantee the objectivity of the internal assessment, IBO conducts a sampling survey in each IB school to make sure of the fairness of the results given by teachers (Liu & Liu, 2008; Song, 2012). Correspondingly, Liu (2008) also mentions that the assessment described in the Chinese high school mathematics curriculum standard (published in 2003) does not reveal sufficient objectivity. The IB mathematics internal assessment wants students to find a problem, ask questions, think about it like a mathematician, and then choose ideal strategy to solve the problem; general Chinese mathematics has less emphasis on these elements (Ma, 2017).

IB mathematics posits the idea of mathematical culture to emphasize mathematics' internationalization and its links to other subjects as a common academic language in the world. It can be shown via the change of the IB mathematics curriculum description; in the 2006 version, it added mathematical internationalization and the diversity of mathematical culture and history as its first aim which marked a change from the 2001 version (Liu & Liu, 2008; Song, 2012; Xu & Xiang, 2015). Mathematics plays an important role in basic education worldwide, but students often ask questions about why the purpose of mathematical study. By creating an atmosphere of mathematical culture and with the guidance of teachers to illustrate the scientific value, application value, and humanistic value of mathematics, students can better understand the meaning and value of learning mathematics, and they are more likely to be motivated and interested in mathematics (Liu & Liu, 2008; Xu & Xiang, 2015).

Scholars mention that the continuous development of modern technology have had a profound influence mathematics (Li et al., 2018; Liu & Liu, 2008). One example is the "four color theorem," or the "four color map theorem," which is a theorem that states that given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map such that no two adjacent regions have the same color. It was proven in 1976 by Kenneth Appel and Wolfgang Haken and was the first major theorem to be proved using a computer. IB mathematics certainly places great importance on the application of technology in its curriculum, and it believes that understanding and judging the rationality of the results generated from technology represents a significant mathematical ability (Liu & Liu, 2008). According to the IBO official support materials, one of the objectives for all group 5 (mathematics) subjects is to "use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems." Although for external assessment the use of technology is limited to the graphing calculator, for internal assessment, students are expected to utilize technology in one or more ways while producing their explorations. For example, they can use any kind of calculator, the internet, data-logging devices, word processing packages, spreadsheets, graphics packages, statistics packages, or computer algebra packages (IB Use of Technology, 2018). Many Chinese scholars notice that as a major component of human civilization, the development of technology can be positively applied to help mathematical learning as a life-long subject that is bound up with everyone's daily life (Li et al., 2018; Liang, 2016; Liu, 2008; Liu & Liu, 2008; Wang, 2016). General Chinese mathematics seems set up to work against excessive dependence of using a on calculators and to provide training in manual computation skills; the IB is also concerned about this hidden trouble, so the IB set a standard in 2008 that 30% of each IB mathematics exam must require students to calculate manually, without any electronic aid (Liu & Liu, 2008; Xu & Xiang, 2015).

In 2008, Liu and Liu analyzed the curriculum of IB higher level mathematics and noticed the breadth of IB mathematics; not only did IB HL math include many college-level topics such as calculus, but it also embodied many optional topics. One year later, Ma (2009) made a comparison between the IB mathematics and general Chinese mathematics, specifically on the topic of vector. He found out that IB mathematics has a wider range of content but a shallower depth than general Chinese mathematics. Wang (2016) conducted research on the probability content of IB mathematics and also observed the extensive content of the IB mathematics. For example, in the probability content, the IB mathematics involve some ideas about "Poisson distribution" and "Bayes theorem"; neither of them are contained in the probability content of general Chinese mathematics.

Besides, Ma (2009) noticed that IB mathematics tends to deemphasize definitions. For example, IB higher level mathematics does not include the definition of vector in the 2D plane and its coordinate representation, whereas general Chinese mathematics regards this content indispensable. For another example, the IB does not mention vector projection in its curriculum. So this kind of difference requires IB teachers to explain these uncovered terms so that students will not have a knowledge deficit. Moreover, there are other differences in design in the two curricula: IB mathematics does not talk about 3D geometry while the general Chinese mathematics lists it as an individual chapter in its curriculum (Ma, 2009).

Furthermore, IB mathematics provides autonomy by letting students choose the difficulty of the subject from four available courses, select their ideal elective classes, and determine their personal topics for the mathematical exploration part (Liu & Liu, 2008; Ma, 2017; Song, 2012). Ma (2017) believes that during this process, students cultivate abilities to finish a project independently and obtain risk awareness and evaluation skills, which are beneficial to their future development.

Influence of the IB Program on the Chinese High School Program

The introduction of the IB program in China has had an influence on the regular Chinese high school program, causing certain innovations to occur. I will examine Chinese high school program innovations as well as Chinese high school mathematics innovations in detail.

Chinese High School Program Innovation

The Chinese high school program innovations that have been spurred by the introduction of the IB program in China fall into six categories: bridging the gap between secondary and higher education, changes to the assessment system, curriculum design, global vision and international understanding, comprehensive development and quality education, and teacher training.

Bridging the gap between secondary and higher education. As Cheng and Deng (2006) mention in their research, it is very important to provide opportunities for students to undertake more challenges, especially for talented students. Because excellent learners always look for broader learning content and a faster teaching processes to prove their capabilities and motivation, they might need to learn higher level courses during secondary education so that they can become familiar with more advanced course materials. Also, Li (2006) states that taking college-level courses in advance helps to build a more solid foundation for future studies in colleges and universities. And if during that process students could also acquire college credit before officially starting their college lives, it would greatly help them efficiently transition from secondary education to higher education (Cheng & Deng, 2006; Li, 2006; Xu & Xiang, 2015). In addition, learning the three cores of the IB program plays a significant role in students' studies in higher education, and the abilities they learned from doing these cores activities have great influence on their careers (Xu & Xiang, 2015; Zhang, 2014).

Assessment system. As Jin (2008) states, the assessment of student performance is one of the most basic areas of education and is also very important. With the combination of external and internal assessment, the IB program attaches great importance to formative evaluation. Cheng and Deng (2006) believe that all factors of education should be fully considered when conducting innovation in high school education: educational objectives, curriculum installation, student background, learning environment, teaching modes, teaching methods, teaching conditions, faculty structure, assessment approach, and evaluation tools. Therefore, scholars and researchers believe that the assessment system of the IB program reveals a mature approach to dealing with student performance, and it is necessary for Chinese high schools to both learn from it and pay more attention to pursuing formative evaluation (Cheng & Deng, 2006; Jin, 2008; Li, 2006; Song, 2008). Therefore, objectively, systematically, deeply, and fully understanding this assessment method would be beneficial in promoting the innovation of Chinese students' assessment and further guaranteeing the quality of Chinese high school education (Jin, 2008; Song, 2012; Xu & Gao, 2012; Xu & Xiang, 2015).

Curriculum design. According to Song (2012)'s thesis, the curriculum design follows a basic principle to encourage students to have fundamental knowledge that is necessary in their future career lives despite the major of choice. In addition, some researchers also notice that the IB program designs its curriculum to achieve individualized learning by respecting each student as an individual learner (Gu & Yang, 2002; Hong, 1994; Song, 2012; Wang & Wang, 2016; Xu & Xiang, 2015). Therefore, IB courses adapt to the differences of students and leave them with elective space in the development of their studies and personalities. For example, in its designed curriculum, each IB course offers two levels: standard level (SL) and higher level (HL) to satisfy the needs of average students and advanced students. Also, Cheng and Deng (2006) think high schools should be able to offer various elective courses with rich contents, and those courses can help better prepare students for future development in content, structure, and learning experience. Moreover, Zhang (2014) makes a complementary statement that the IB program's curriculum is based on students' different interests and, especially, students' future career preferences. However, the general Chinese high school curriculum does not offer students with difficulty-based choices for classes, but insists on a unified standard. So, it is worth using these features for reference to inform Chinese curriculum reform (Cheng & Deng, 2006; Gu & Yang, 2002; Wang & Wang, 2016; Xu & Xiang, 2015; Zhang, 2014).

Global vision and international understanding. The IB program aims at creating global citizens who have international vision and understanding (Chen & Hou, 2011; Gu & Yang, 2002; Hong, 1994; D. Li, 2009; Liu et al., 2015; Song, 2012; Weng, 2008; Xu & Gao, 2012; Zhang, 2014). With the phenomenon of global economic integration, having international talents with awareness of multiple cultures and global vision to face the era of the knowledge economy is part of every country's future educational goal (Song, 2012; Weng, 2008; Xu & Gao, 2012; Zhang, 2014). Also, modern technology makes international exchange and cooperation more frequent than ever. Thus, increasing high school students' concentration on the world, enhancing their understanding of intercultural blending, and helping them to see the importance of international cooperation are appropriate educational aims (Cheng & Deng, 2006; Hong, 1994; Xu & Gao, 2012). Therefore, preparing Chinese students in both attitude and spirit to get ready for the internationalization and globalization should become a crucial part of the innovation (Song, 2008).

Comprehensive development and quality education. As Gu and Yang (2002) found in their study, Chinese traditional high school classes focus more on the knowledge objective and seek to foster future professionals, which makes mediocre students tend to feel that the courses are impractical and abstruse. However, society needs the new generation to become global citizens who have diverse knowledge about resources, environment, technology, art and so on (Cheng & Deng, 2006; Chen & Hou, 2011; Gu & Yang, 2002; Jin, 2008; Xu et al., 2015). These kinds of knowledge do not have substantive status in the Chinese high school education, which will have a profound impact on students' social adaptability in the future. However, the IB program effectively brings diverse learning into its courses and does not decrease their value as electives; it emphasizes the function and meaning of learning instead of focusing on learning itself (Gu & Yang, 2002; Xu et al., 2015). The IB program also uses an integrated curriculum instead of only paying attention to mastering a single course (Cheng & Deng, 2006; Gu & Yang, 2002). With rich and varied out-of-classroom activities such as the Creativity, Action, Service (CAS) core, each student is able to experience quality education that promotes moral traits, academic accomplishment, and also the development of potential (Cheng & Deng, 2006; Xu et al., 2015; Xu & Xiang, 2015).

The IB program also emphasizes the cultivation of humanity, morality, personality, emotion, and attitude (Gu & Yang, 2002; Weng, 2008; Zhang, 2014). It values the relationship between man and nature, and it demands a knowledge of the natural world, realizing the connection between man and nature and respecting life; its overall effect is to contribute to the consciousness of environmental protection and the love of life and nature (Gu & Yang, 2002; Xu et al., 2015).

Teacher training. According to Zhang (2014), Chinese high schools should learn from the IB program in order to establish efficient teacher training as well as to set up a thorough training and evaluation system to stimulate faculty's enthusiasm for professional improvement. The IB program not only has a high standard for teacher qualification that requires all IB teachers to attend a regular training and take an examination to be certified for teaching with the IB, but it also encourages its teachers to join various seminars to engage in interdisciplinary discussions so as to better integrate teaching knowledge from different domains and to complement each other's teaching. Thus, these training activities positively promote teachers' mutual respect, sharing of experiences, and opportunities to grow together. Faculty eventually pass their acquired expertise to IB students (Cheng & Deng, 2006; Li, 2006; Yang, 2018; Zhang, 2014).

Chinese High School Mathematics Innovation

The innovations in Chinese high school mathematics resulting from the influence of IB programs in China can be grouped into three sorts: mathematics teaching mode, understanding and application, and language and technology.

Developmental aim and mathematical literacy. It cannot be overlooked that Chinese students are still overburdened with intense course loads, extra work, and remedial lessons. Students in Chinese high schools are taught through exercises and tests, whereas the IB program chooses a developmental aim to highlight creative thinking. IB's developmental aim is its program objective rather than just a suggestion to students (Gu & Yang, 2002).

Liu and Liu (2008) mention that Chinese high schools are trying to strengthen students' mathematical literacy, which involves mathematical knowledge, mathematical ability, mathematical language, scientific spirit, and calculator usage. This pursuit also fits the definition of mathematical literacy in Western mathematics education.

Therefore, it is important to enhance students' understanding of mathematics content and to improve their thinking habits so that they can not only have excellent theoretical knowledge and critical thinking skills, but can also expand knowledge, obtain research skill, and apply that knowledge and skill in practice (Cheng & Deng, 2006; D. Li, 2009; Liang, 2016; Song, 2008, 2012; Xu & Xiang, 2015). Song (2012) believes that being critical is a prerequisite of being creative and puts a particular emphasis on IB's cultivation of critical ability that involves the critical thinking skills and the critical thinking spirit, which is embodied in its entire curriculum design, instruction and assessment; specifically, one of the core course, TOK, mainly trains student to have critical attitude to think.

Teaching content and evaluation method. Researchers and scholars think the achievement of quality education needs the support of concrete teaching content and evaluation methods (Liang, 2016; Liu & Liu, 2008). Liang (2016) illustrates that the IB program emphasizes the connection between different subjects. For instance, when teaching statistics, teachers might ask students to collect their running data from their physical education class and then statistically analyze the data and formulate a conclusion. In this process, students not only experience the procedure of doing research, but they also understand the application of mathematics in real-life (Liang, 2016; Song, 2012; Xu & Xiang, 2015). Liang (2016) believes such an activity can better cultivate students' mathematical literacy than just doing couple of word problems.

The evaluation method of the IB program follows the program's objectives and provides a reasonable and effective review mechanism to ensure the equity of its result. At the same time, it benefits students' understanding of mathematical knowledge and increases their interests in learning mathematics (Chen, 2006; Cheng & Deng, 2006; Xu & Xiang, 2015). Ma (2017) states that in educational innovation, the kernel is curriculum reform, and evaluation methods directly affect the outcome of the curriculum reform. Today, the Chinese mathematics education community is finding good ways to avoid students becoming learning machines and is attempting to change the means of assessment to change classroom atmosphere. Ma points out that assessment is a process, but it is not the final goal of teaching. Assessment's goal should be educating people (Ma, 2017).

Chen (2006) recommends establishing formative evaluation that uses inquiry-based learning as its main part. Liang (2016) suggests formulating an all-round evaluation system with various standards. Chen (2006) and Liang (2016) agree with using formative evaluation and summative evaluation together to assess Chinese students' mathematics performance, based on local conditions. They also state that it is important to pay attention to students' learning results and also to recognize and understand their learning process.

Language and technology. With the booming development of international education, great importance has been attached to language learning (Xu, 2001). The IB program includes both studies in language and literature as well as language acquisition in its six subjects, which demonstrates its special stress on language.

Liu and Liu (2008) mention that in addition to its emphasis on using modern technology in basic education in developed countries, the IB program also regards using graphing calculators as an indispensable tool when teaching. Liu (2008) also notices that IB mathematics specifically emphasizes the use of modern technology, which is stated in its teaching purposes. Therefore, having a technology-friendly environment in which Chinese students can learn is meaningful. But the changing of IB policy from full usage of graphing calculators before 2008 to 70% usage after that is also though-provoking when considering the IB's approach and philosophy (Liu, 2008; Liu & Liu, 2008; Xu & Xiang, 2015).

Chinese Views

There are very few Chinese studies that compare the AP and IB programs and a similarly small number which deal with the AP, IB, and general Chinese high school programs.

Similarities between the AP Program and the IB Program

Both the AP and IB programs provide college-level classes which fill the gap between secondary education and higher education (Li & Zhang, 2006; Liu & Zhang, 2016; Ma, 2011; Song & Wang, 2015; Wang & Wang, 2016; Weng, 2008; Xie & Shen, 2013; Xu & Xiang, 2015). Weng (2008) writes that one of the most important goals of high school education is preparing students for continuous study in colleges and universities. Results of Ma's (2011) questionnaire (among 20 graduates of the imported programs) suggested that learning calculus systematically in high school and having a solid foundation for college calculus are positively correlated. These imported programs appear to achieve a connection between secondary and higher education. (Song & Wang, 2015; Wang & Wang, 2016; Weng, 2008; Xie & Shen, 2013; Xu & Xiang, 2015).

Some Chinese scholars note that both AP and IB programs use stratification to provide the same subject at different difficulty levels for students, a diversified feature offering individualized development (Liu, & Liu, 2012; Liu & Zhang, 2016; Ma, 2008; Ma, Wang, & Wang, 2016; Weng, 2008; Xiao, 2014; Xie & Shen, 2013; Xu et al., 2015; Xu & Xiang, 2015). To be specific, AP mathematics contains Calculus AB, Calculus BC, and Statistics, in which Calculus AB can be regarded as a proper subset of Calculus BC because the latter covers broader topics, such as "series," which is not included in Calculus AB (Ma, 2008, 2011; Xie & Shen, 2013). IB mathematics is divided into four levels: mathematical studies standard level, mathematics standard level, mathematics higher level, and a further mathematics higher level with progressive difficulty (Liu, 2008; Ma, 2008; Ma et al., 2012; Weng, 2008; Xie & Shen, 2013; Xu & Xiang, 2015).

Moreover, according to Ma (2011), IB mathematics standard level is a proper subset of IB mathematics higher level not only because the latter covers one optional topic that is absent from the former but also because the higher level offers more depth than the standard level. With different options, students can choose the course that fits their academic abilities and future career plans, which will eventually help to achieve personalized development (Liu, 2008; Ma, 2011; Wang & Wang, 2016; Weng, 2008; Xu et al., 2015; Xu & Xiang, 2015).

The calculus part of both the AP program and the IB program are much the same (Ma, 2008, 2011; Xie & Shen, 2013). Both programs choose the idea of limits and continuity as the introduction to calculus, followed by differentiation, integrals, series, and so on. The sequence not only fits the structure of knowledge and cognitive pattern of high school students but also accords with high school classes' time arrangement (Ma, 2008; Xie & Shen, 2013; Zhang, 2012; Zhuang, 2017).

Furthermore, studies suggest that both the AP program and the IB program attach great importance to the integration of mathematics knowledge and technology, which is embodied in the explicitly stipulated requirement of using graphing calculators in examinations (Liu & Zhang, 2016; Ma, 2008; Ma et al., 2012; Xie & Shen, 2013). Beyond that, the programs encourage students to embrace various kinds of modern technology such as smart boards, MATLAB, and SPSS (Ma et al., 2012; Xie & Shen, 2013).

The AP program and the IB program both adopt the system of converting raw scores to scaled scores (a 1 to 5 scale for AP and 1 to 7 scale for IB) (Liu & Zhang, 2016; Ma, 2008; Song & Wang, 2015; Wang & Wang, 2016; Xie & Shen, 2013).

Xie and Shen (2013) observe that both the AP program and the IB program encourage student exploration and application. Students are required in the IB mathematics curriculum to finish a piece of individual written work that involves investigating an area of mathematics. This project facilitates mathematical exploration.

AP mathematics also has a free response section in its examination which tests students' abilities to apply specific mathematics knowledge to solve real problems (X. Li, 2009; Xie & Shen, 2013). Zhuang (2017) finds that both AP Calculus BC and IB mathematics Higher Level (HL) contain examination questions related to real-life problems. Moreover, the AP and IB examinations do not require students to memorize formulas but provide them during the examination instead (Liang, 2016; Liu & Zhang, 2016; Xie & Shen, 2013; Zhang, 2012).

Differences between the AP Program and the IB Program

Xiao (2014) mentions that the AP program aims at educating students who want to challenge themselves and become specialists of specific subjects of interest while the IB program reflects the educational goal of cultivating well-rounded lifelong learners with multi-cultural understanding. In terms of curriculum, some Chinese scholars mentioned that the AP program tends to be more practical, emphasizing the application of mathematical knowledge in real-life situations, while the IB program pays more attention to deduction and the understanding of specific mathematical ideas (Liu & Zhang, 2016). According to Ma (2008), the most apparent difference is that IB mathematics tests high-school level mathematics knowledge. For instance, some IB content includes pre-calculus material, whereas AP mathematics investigates college-level calculus contents directly. As another example, some ideas in AP calculus, such as polynomial approximation, series, and differential equations, are present only in the contents of the IB mathematics higher-level option topic (Ma, 2011).

There also exists a difference between rules for sitting for the two examinations. Students can choose either to take the AP courses at school or to self-study the AP courses at home and then attend the AP examination, whereas the IB students have to complete a two-year IB program of courses at school to be eligible to take the IB examination (Song & Wang, 2015; Wang & Wang, 2016).

Questions on the AP examination are mostly multiple choice while the IB examination has more open-ended questions. The AP examination is more standardized with fewer opportunities for grading errors, but it cannot shed light on students' thinking processes. There also exists a certain probability of students just guessing a correct answer (Liu & Zhang, 2016; Ma, 2008; Xie & Shen, 2013).

For assessment, the AP program uses test scores along with students' performance; however, the IB program combines both external and internal assessments, offering a more rounded picture of student learning outcome (Liu & Zhang, 2016; Ma, 2008; Song & Wang, 2015; Wang & Wang, 2016; Xiao, 2014). At the same time, the subjectivity of the internal assessment causes some controversies (Liu & Zhang, 2016).

Comparison among the AP, the IB, and the General Chinese Programs

A few Chinese scholars have compared the AP program, the IB program and the general Chinese program in order to use the advantages of the foreign programs for reference while considering Chinese high school curriculum reform (Liu & Zhang, 2016; Ma, 2011; Ma et al., 2012; Xu & Gao, 2012).

First, scholars noticed that unlike the AP program and the IB program, which involve college-level contents to help bridge secondary and higher education, the general Chinese high school education curriculum has only a small proportion of college-level contents and usually assigns them to optional topics; many schools decided not to teach these contents in real practice, which makes adapting to college more difficult for Chinese students (Liu & Zhang, 2016; Ma, 2011; Song & Wang, 2015; Wang & Wang, 2016; Xu & Xiang, 2015).

Second, although the AP program and the IB program offer courses at different levels, the general Chinese program separates students only into arts or science, which makes the classification too broad to satisfy different students' needs (Liu & Zhang, 2016). Weng (2008) noticed that general Chinese high school education is a mass education which values the group higher than the individual, which causes a blindness to students' personalized development. The AP program and the IB program offer various optional courses for students to choose from (Liu & Zhang, 2016; Wang & Wang, 2016; Xu & Gao, 2012). Currently, the Chinese high school program's optional course accounts for only 10% of a student's total coursework, even after many curricular reforms. In reality, it is sharply limited by teaching resources and teaching methods; elective courses are chosen by the school instead of by the students, making the term "optional" meaningless (Liu & Zhang, 2016).

There exists a considerable gap in breadth between the general Chinese high school mathematics and the two imported programs (Liu & Zhang, 2016; Ma, 2011). For example, the Chinese high school mathematics curriculum covers only the idea of derivatives while the two imported programs contain most of the topics in calculus (Ma, 2011).

Regarding calculators, scholars noted that a few advanced cities like Shanghai allow students to use ordinary calculators during examinations, but graphing calculators are still universally prohibited (Ma et al., 2012; Xu & Xiang, 2015).

Unlike the AP program and the IB programs' use of scaling grades, the general Chinese grading system uses absolute standards to score tests. AP and IB students' final grades are unlikely to be affected by a slight difference in raw scores, but for Chinese students sometimes a one-point difference might cause a widely different ranking (Liu & Zhang, 2016; Song & Wang, 2015; Wang & Wang, 2016).

Finally, research shows that general Chinese high school mathematics focuses more on theoretical knowledge, neglecting its application, which is different from the AP program and the IB program where applications play a vital role in teaching and learning (Liu & Zhang, 2016; Ma et al., 2012). Some scholars also mentioned that the general Chinese mathematics program places emphasis on students' ability to accurately memorize formulas while both the AP and IB programs hold a contrary view concerning the value of such memorization (Liang, 2016; Liu & Zhang, 2016; Zhang, 2012).

Suggestions

Some Chinese researchers make suggestions towards the future development of the AP program in China. For example, directed at the language issue, Zhao and Zhang (2009a) advise an effective way to transit from bilingual to full-English teaching. According to them, it is better to implement bilingual teaching in the first year of high school and then gradually change to full-English teaching. Doing so will ensure that students can have enough time to learn the language while maintaining their learning initiative and learning quality (Liang, 2012).

Actively introducing advanced foreign programs, fostering more international education cooperation, and sending more students to study abroad are necessary if China is to be involved in a process of educational globalization and internationalization. This process also reveals that Chinese education, especially higher education, has certain gaps between itself and Western education which require long-term efforts to bridge (Song & Wang, 2015; Xu, 2017). However, instituting the imported programs will eventually convey a lot of qualified talents to Western higher education. Although some students will come back to China after studying abroad, there still a risk of a brain drain. Therefore, developing a better higher education system and trying to build a smoother transition between Chinese secondary education and higher education are necessary for a sustainable improvement (Song & Wang, 2015; Xu, 2017).

It is also important to localize the imported programs because both the AP program and the IB diploma program fit Western countries' education systems. However, when conducting them in real practice in China, the difference between those programs' ideals and Chinese cultural reality generates a distinction between effect and result; the origin of that distinction involves internal and external systems. The internal system in this case is the educational system, and external system is the social system, which includes physical, economic, and cultural elements. This essential differences in the two systems mean that the imported programs cannot be copied but must be adjusted in accordance with Chinese national conditions (Xu, 2017).

Chapter III

METHODOLOGY

Case study research is an investigative approach used to describe specific phenomena, such as recent events, important issues, or programs, and to reveal deeper understandings of these phenomena (Mertens, 2014). Specifically, case study focuses on examples from a group of events, issues, or programs, and peoples' interaction about them (Mertens, 2014; Moore, Lapan, & Quartaroli, 2012). According to Cherryholmes (1992), it provides individual researchers the freedom to use multiple methods, different assumptions, and different forms of data collection and analysis to best fit their needs and purposes (Creswell & Creswell, 2017; Rossman & Wilson, 1985). To better address my research questions, I chose the mixed method, which includes both quantitative and qualitative approaches as my primary methodology. My purpose for using the mixed method is to collect and analyze information of mathematics curriculum and data of students' performance from two Chinese high schools (one that has adopted the Advanced Placement program and one that has adopted the International Baccalaureate program) and also to gather and conceptualize teacher's perspectives from the interviews about teaching students who are studying the two different curricula at the same time in preparation for future overseas studies.

I will discuss the methodology used to ascertain the differences and similarities between the imported AP and IB programs in China, to assess students' mathematical performance in two different international curricula, and to relate in detail teachers' views of teaching mathematics with different curricula. My hope is that the discussion here will evoke more interest in research in international curricula and mathematics in the future.

Settings

"Chinese heart, Global vision, National spirit and World model." The director of Ningbo Education Bureau mentioned that with the globalization and internationalization, nowadays, parents and students are expecting a diversified education and a superior educational resource. "Today's education should possess the foreseeability and sustainability. It is a responsibility of schools to prepare our students better to enter the international community, to have a global view and acknowledgment and to participate in international events actively." The city of Ningbo always devote itself to providing students multiple choices for their education, and the education internationalization exactly meets that demand. Therefore, Ningbo city introduced the Advanced Placement program and the International Baccalaureate program to give students more options under this trending background. As a typical second-tier city that does not have the type of educational resources available in top-tier cities such as Beijing and Shanghai, but with more resources than those cities in West China, which lack educational resources, Ningbo became my ideal choice to reveal a general situation of these two imported programs in China.

AP Center in Ningbo

Ningbo Foreign Language School started the Sino-US joint international high school AP program called "Ningbo Foreign Language School Advanced Placement (AP) Center" from September 2011. It aims to combine the advantages of the senior secondary education from both the United States and China. Based on finishing the general Chinese high school curriculum, students who want to pursue their further education overseas in Western countries such as the United States, Canada, the United Kingdom, and so on could have an opportunity to learn a rigorous and academically challenging curriculum that will prepare them to enroll and graduate from accredited western universities. Except the preparedness to be successful in a college-level AP course with unexplored knowledge in a traditional high school curriculum, students are also affirming that they possess the mental and emotional maturity expected of a student at the college level. Also, by passing AP examinations, students can earn credit or advanced standing at most colleges and universities in Western countries.

The program in this AP school in Ningbo, China is localized with the general Chinese curriculum (see Figure 2). It is a three-year program, and AP students here are required to learn the general Chinese high school courses along with the pre-AP courses at the same time in the first year. The AP school specifically sets up the pre-AP course in order to equip Chinese students with better academic abilities to pursue the further courses in the following two years. The pre-AP courses end at the end of the first year; the AP courses themselves start in the beginning of the second year. More details of the curriculum are included in the results section of this dissertation. On the other hand, students usually finish the general Chinese courses after the first 1.5 years, and all the required subjects of the General Examination are tested continuously before the middle of the second year. Therefore, after that time, AP students only need to learn the material taught in their AP courses.

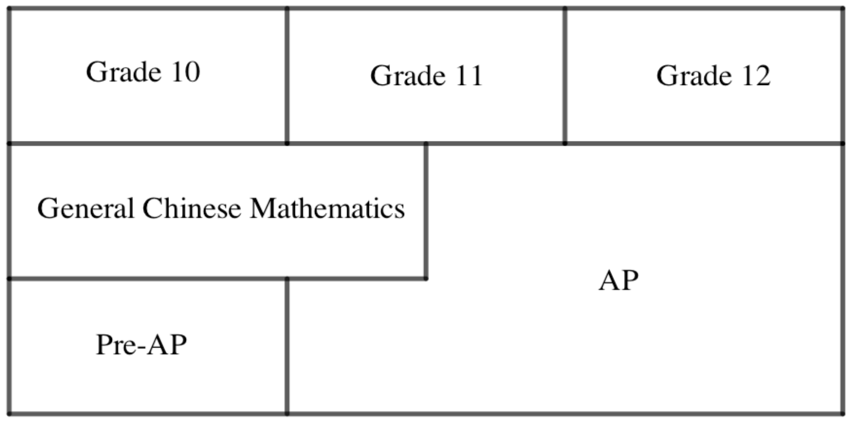


Figure 2. Guiding Figure of Program Outline in the AP School in Ningbo, China

The program has about 240 students from grades 10 to 12. The first graduating class, in 2013, had about a 68% acceptance rate of getting enrolled in the Top 50 colleges and universities in the United States. In 2014, the rate was increased to 82%, and it reached 85% in 2015. In 2016, about 70% of graduates were accepted into the Top 50 schools in the United States. Almost 100% of students achieved offers from the Top 100 colleges and universities in the States. Notably, together, the 53 students got over ten million Chinese Yuan in scholarships; the highest individual award was around ninety thousand dollars.

IB Center in Ningbo

In September, 2012, the International Baccalaureate Program (IBDP) officially launched at Ningbo Xiaoshi High School, also called the Ningbo Xiaoshi IB Center. It is a three-year full-time educational program. With full credits and a pass from IB global examinations, students will be awarded with IB diploma. The mission of the Ningbo Xiaoshi High School IB Center is to empower students to become lifelong learners and "Become creative, curious, responsible and resilient global citizens who can maximize their potential in their lives and their future careers, equipping them with an attitude that enables them to excel beyond the university level" (Appendix B). Also, the school aims to cultivate students with a global perspective and respect other cultures with peace and understanding, while retaining a deep patriotism of Chinese culture. To better localize the international education, the IB Center is managed by Ningbo Xiaoshi High School, but a non-Chinese center principal (vice-president) will take charge of its academic and daily management.

The IB program combines the best of both Chinese and introduced foreign education with high-quality educational resources, including small-size classes, certified foreign teachers with rich international teaching experience, and well-trained and professional college counselors with remarkable overseas educational backgrounds.

Like the program of the AP center in Ningbo, China, the program in this IB school in the same city is also adjusted to fit with the general Chinese curriculum (see Figure 3). The three-year program includes learning two curricula (both the general Chinese high school curriculum and the pre-IB curriculum) in the first year. The preliminary courses share the same purpose as those at the AP school, to better prepare their students for future studies, especially for learning a program that requires English language proficiency to deal with various types of writing. In the second year of study, students begin to learn the IB courses. After completing 1.5 years of learning the general Chinese courses and passing required general examinations in all subjects, IB students can focus their learning primarily on the IB courses beginning in the middle of the second year.

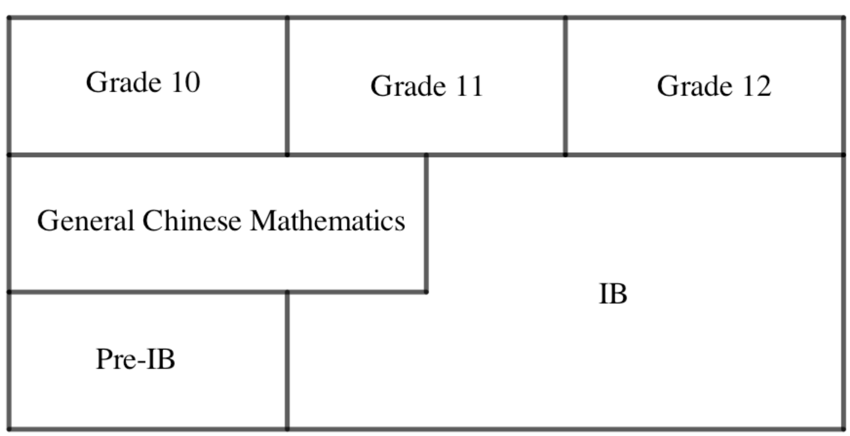


Figure 3. Guiding Figure of Program Outline in the IB School in Ningbo, China

The IB Center currently has 190 students enrolled. In 2015, 51 students, as the first graduate of the Ningbo Xiaoshi High School IB Center reaches an extremely high (96.1%) acceptance rate of getting enrolled in the Top 50 colleges and universities in the United States. The following graduates also showed an outstanding academic performance and overall ability, up to 80% of students from class of 2015 to 2017 went to the Top 50 American universities (same ratio in other countries), received accumulated scholarship more than $3 million.

Mixed (Quantitative and Qualitative) Research Design

In this study, my goal is to study the comparability of the introduction of Advanced Placement and International Baccalaureate courses in Chinese mathematics education. I focused on the differences and similarities between the imported Advanced Placement program, the International Baccalaureate program, and the intended Chinese high school program in mathematics education.

After deciding the specific subject of my case study, I started to plan my research design. Based on my research questions, I started to find relative resources such as books, articles, and online resources to see what other scholars and researcher chose to use as their targeted approaches.

Many research questions and combinations of questions are best and most fully answered through mixed methods (Johnson & Onwuegbuzie, 2004). In 2003, Teddlie and Tashakkori provided a comprehensive overview of the mixed method approach. It has been called the “third methodological movement” following the developments of first quantitative and then qualitative research (Creswell & Clark, 2017; Teddlie & Tashakkori, 2003), and the “third research paradigm” as a new third chair with qualitative research sitting on the left and quantitative research sitting on the right, and can also obtain the complementary strength of drawing on both quantitative and qualitative research and the non-overlapping weaknesses of minimizing the limitations of both approaches (Brewer & Hunter, 1989; Creswell & Clark, 2017; Johnson & Onwuegbuzie, 2004; Onwuegbuzie & Leech, 2004).

In addition, mixed methods also demand that the researchers use both qualitative and quantitative approaches or methods in the design, data collection, finding integration, inference drawing, and data analysis (Mertens, 2014; Teddlie & Tashakkori, 2009). Using quantitative and qualitative techniques together, mixed methods research will be able to incorporate the advantages of both methodologies (Sechrest & Sidana, 1995). It is formally defined as the class of research that includes both qualitative (open-ended) and quantitative (close-ended) features such as mixed research techniques, approaches, concepts or language into a single study, as well as those studies that are part of a larger research program and are designed as complementary to provide information related to several research questions, each answered with a different methodological approach (Creswell & Clark, 2017; Johnson & Onwuegbuzie, 2004; Mertens, 2014; Teddlie & Tashakkori, 2009).

Those features and advantages best fit my research needs. Therefore, mixed research methods became my choice to provide the best way to pursue my research questions.

Data Sources and Data Collection Method

Mixed methods researchers face the challenges that come with quantitative and qualitative studies plus the complexity of mixed methods. Under these circumstance, several sampling strategies that are unique to mixed methods research have been identified (Onwuegbuzie, Leech, & Collins, 2010; Onwuegbuzie & Collins, 2007). In mixed methods research, data collection strategy decisions should be guided by the purpose of the research (Greene, 2007; Mertens, 2014). My purpose of doing this research study is to compare and analyze the AP and the IB programs in Ningbo, China. After addressing my research questions, the design of data collection and data sources became the next part. I started with a quantitative phase in which I had a subsample of various types of test scores, including students' General Examination scores, SAT mathematics scores, AP calculus scores, and IB mathematics scores of two contiguous years of graduates from the two high schools in Ningbo. The results were clarified by examining statistical correlations between the measures. Also, from the curricula, I collected localized program settings of the AP program and the IB program in order to identify the similarities and differences between each program in Ningbo, China. Moreover, by comparing the corresponding detailed requirements and contents, I was able to see the similarities and differences between each program and the general Chinese program as well, specifically in the area of mathematics. And for the qualitative phase I decided to see things from teachers' perspectives. To get their insights, I planned to visit all the mathematics instructors of both programs and conduct interviews. During the meetings, open-ended data were gathered through this approach since the question protocol that I made involves multiple perspectives. I believe this process provided a more productive explanation of the teachers' experiences with their students.

According to the *fundamental principle of mixed research* referred by Johnson and Turner (2003), researchers should collect multiple data using different types of questions, approaches, data collection strategies, research methods and analysis procedures (Johnson & Onwuegbuzie, 2004; Johnson & Turner, 2003; Teddlie & Tashakkori, 2009). It is an expansive and creative form of research that can legitimate the use of multiple approaches in answering research questions, and not limiting researchers' choices (Johnson & Onwuegbuzie, 2004). For answering the first three of my research questions, I wanted different types of scores, curricula, and other data and information from various sources. And for addressing the last research question, I thought that conducting in-depth interviews would be an efficient way to understand teachers' perspectives. One type of data gives greater breath and the other gives greater depth, together as a mixed method, it is hoped that they yield a more thorough result (Teddlie & Tashakkori, 2009).

After deciding on a quantitative approach to answer my research question one to three, and using a qualitative approach to answer my last research question, I reached out to the two top-tier sister schools in Ningbo, China: Ningbo Xiaoshi high school that provides an IB program and Ningbo Foreign Language School that offers the AP program. Luckily, I am an alumnus of Ningbo Xiaoshi high school of class 2011. Based on some personal connections, I asked my high school teachers to refer me to the school's vice president who runs that IB center. I successfully scheduled our first meeting in the summer of 2016.

At the same time, I also got a referral from the Chinese vice president of Ningbo Xiaoshi high school to the Chinese vice president of Ningbo Foreign Language School. They are sister schools so it did not take much time for me to build connections.

As a reminder, each school has two vice presidents in charge of the administration, one is from the Chinese administration system and another is from the corresponding cooperative non-Chinese administration system. In general, the Chinese vice president mainly deals with all the Chinese instructors in the program and with all the academic requirements that every student needs to fill during the first 1.5 years of high school learning to pass the General Examination stipulated by the Chinese Education Bureau. The non-Chinese vice president is sent by the cooperative AP or IB program representative organizations. He or she manages all the non-Chinese instructors in the program and all the academic requirements that each student has to meet to pass the AP tests or get the IB diploma. The Chinese vice presidents have more authority in supervising the entire program since they need to communicate with the school's president and update that person about the progress of those imported programs.

At that time, I talked to the Chinese vice presidents who run those international programs at these two sister schools. They briefly introduced basic information about the programs explained the recruitment procedures to me so that I could have a clear overview of the programs and their features. After these meetings, I received the introductory brochures that the schools use to recruit their incoming students and to show parents (Appendices A and B).

Before officially conducting my interview and gathering my data, to ensure that the research complies with institutional and government-mandated guidelines for the study that involves human subjects (Blee & Currier, 2011), I applied for approval to the Institutional Review Board (IRB) of Teachers College, Columbia University. And I prepared informed consent forms for my interviewees to protect their rights and privacies. In addition, I submitted my interval protocols for two kinds of interviews (AP and IB) to the board as well. The Institutional Review Board (IRB) of Teachers College approved my proposal and my interval protocol in July 2017. Then, I formally started conducting my research and collecting my data during the beginning of the Chinese high school year in September 2017.

I first went to visit the Chinese vice presidents of these two high schools. I set up a meeting with the Chinese vice president of Ningbo Xiaoshi high school that provides the IB program. Because I had already talked with that Chinese vice president the previous summer, it became easier for me to get more information. She told me a lot about the detailed enrollment procedures when recruiting students. Because the two sister schools share the same recruiting system, every May they organize a co-sponsored enrollment test called the Entrance Examination, which is harder than the senior high school entrance examination that each Chinese student should pass before choosing high schools. The two schools have a joint recruitment committee to design the test questions. The Entrance Examination is composed of two parts, the written examination that tests students' academic abilities and the interview that evaluates students' English speaking and communication skills. The written examination has been divided into Art and Science; each takes two hours. The Art part includes Chinese and English; the Science part covers mathematics and basic science. The Entrance Examination covers 70% of the final assessment and the senior high school entrance examination accounts for the other 30%. Eventually, based on those assessments, the recruitment committee will extend offers to approximately the top 80 students. I received the updated program brochure for the year 2017 and then scheduled another meeting with the administrative director of the IB program for further data information.

Since the old Chinese vice president of the AP school had been transferred to another high school, so I met with the successor, who tried to help me find out students' scores on the Entrance Examination. Unfortunately, it turned out that the former vice president who takes care of these records kept them in her laptop, and because of her leaving, the technology specialist mistakenly deleted those files when preparing the new Chinese vice president's administrator system updates. My contact also searched for the paper file, but sadly they kept only the electronic file. Perhaps they did not regard those past data as an important part of program's development because the same situation happened at the other high school. Initially, I wanted to collect data from the mathematics scores of students' entrance examinations from both high schools who use the same version of the entrance examination. I also wanted to analyze the test rankings among all new students and collect data from students' SAT scores, especially the mathematics subject part, and then look at the test rankings for that exam. I wished to compare the two rank tables because the first rank reflects performance of students who have not been exposed to the curricula of either AP or IB, and the second is after learning AP or IB. However, with the missing data, I had to make some changes to my approach to analysis of student learning outcomes. Luckily, the Chinese vice president at the AP school referred me to his administrative director so that I could get the other scores that I needed from the system. I went to see the individual and, thanks to her help, I successfully got students' scores on the General Examination, AP examination, and SAT test since 2016. Then I reorganized those data and made an integrated analysis.

While waiting for all these data, I started conducting interviews to answer my last research question. I did some relevant research, and I found out that mixed methods can also involve multilevel design, where quantitative data are collected at one level of an organization (e.g., student level) and qualitative data are collected at another level (e.g., administrator level, or, in my research, teacher level) (Leech & Onwuegbuzie, 2009; Mertens, 2014; Onwuegbuzie & Collins, 2007; Teddlie & Tashakkori, 2009). In short, I planned my entire data section in two different levels: students and teachers. According to Mertens (2014), a variety of types of persons must be sought and information about the backgrounds of the participants should be provided by researcher, so I included all available student performance data, and I interviewed both non-Chinese mathematics teachers and Chinese mathematics teachers in both AP and IB programs in my targeted schools. At first, I went to see all the mathematics instructors who teach mathematics in Ningbo Xiaoshi high school. It should be noted that within the AP program, since it does not offer a diploma, students should learn both the general Chinese high school program and the introduced AP international program to get a general Chinese high school diploma. In other words, for mathematics specifically, students have both Chinese instructors who teach them general Chinese high school mathematics and non-Chinese instructors who give them AP mathematics-related instructions such as AP calculus. I visited two Chinese mathematics instructors first, showed them my IRB approval letter and the participant's consent forms. After carefully explaining to all my interviewees the purpose of my research, the potential risks, benefits, the anticipated scope of time for participation, and emphasizing that all interviews will be audio recorded to ensure accuracy, and asking about their willingness for further follow-ups, they agreed to sign the consent form and participated in my interviews, one after another. Therefore, all the information was gathered in accordance with IRB guidelines and involved informed permission from each participant. I designed my interval protocols for Chinese instructors and non-Chinese instructors. Most of those questions are the same, but some specific questions are different.

As stated in Appendix C, there are 22 interview questions in total, and I separated them into two versions for non-Chinese instructors and Chinese mathematics instructors because the differences in their education and teaching backgrounds might cause an apparent difference in the result, especially in the teachers' perspectives on teaching imported AP or IB programs in China. Each version has 19 questions, and among those 19 questions, non-Chinese instructors and Chinese instructors have 16 questions in common as well as 3 questions that are made for each group of instructors individually. Since in the IB Center, non-Chinese instructors take the entire responsibility of teaching both IB mathematics curriculum and the general Chinese high-school mathematics curriculum to pass the General Examination. Therefore, unlike the AP's interview that has both versions of interview questions for non-Chinese instructors and Chinese instructors, for the IB's interview, we only have one version for the non-Chinese instructors.

I asked the appropriate questions of Chinese instructors of the AP program during the interviews. The interviews were audio recorded. Then I scheduled appointments with four non-Chinese mathematics instructors the following days and repeated the steps just outlined. Later I did the same procedure with two non-Chinese mathematics instructors belonging to the IB program in Ningbo Xiaoshi high school. Because the IB program intends to train students under a complete English language environment to get the diploma, all the mathematics instructors are non-Chinese with an international background. There is one more mathematics instructor of the IB program who is also the new non-Chinese vice president of this program. I went to see her but she had just arrived to assume her new responsibilities, and although she is a very experienced educator with outstanding international education backgrounds, she said that she might not have a thorough understanding of the teaching in a Chinese international school. She did suggest that I could conduct my interview with her after a semester, and she believed that at that time she would have a more precise perspective to offer me.

In October 2017, the administrative director of the IB program finally sent me a message that saying that she had found all the data that she could and requesting me to pick up the material. Hence, we set up a convenient time for each other, and I went to her office to get those data. As noted previously, the data for the Entrance Examination were missing, but I received students' SAT total scores and SAT mathematics scores for 2016 and 2017 and students' IB mathematics scores for 2016 and 2017. Moreover, at the same time, one Chinese mathematics instructor of the AP program in Ningbo Xiaoshi high school got in touch with me to lend me the Chinese high school mathematics text content book as I had requested months ago after our interview. I made a copy of that text and returned it to him.

After another semester, I went back to China during the winter break. After the Christmas and New Year holiday, I paid a visit to meet the non-Chinese vice president of the IB program as we had agreed during the previous summer and interviewed her following my interview procedure. I added this previous interview part to my interview analysis Excel file. The responses of each participant are too detailed to explicitly organize in a couple of Excel tables, but eventually they became a complete file from which I could extract specific information. I received the newest brochure of the year 2017, and the administrative director of the IB program also helped me obtain the Chinese high school mathematics text content in the English version, translated and slightly adjusted by IB programs' administration department. Because the IB program offers students an IB diploma, but the Chinese government requires each student to at least pass the General Examination, the non-Chinese IB mathematics instructors are also responsible for covering the general Chinese high school mathematics curricula. Therefore, this English version of the general Chinese high school mathematics content is specifically prepared for letting the IB mathematics instructors know what knowledge of general Chinese high school mathematics they ought to cover when teaching the fundamental IB mathematics.

Data Analysis

Analytic and interpretive issues in mixed methods research are influenced by the researchers' paradigm and the design of the study (Mertens, 2014). As I noted earlier, mixed methods research includes rigorous methods such as data collection, data analysis and interpretation of both quantitative and qualitative data. And the two kinds of data will be integrated by merging the data, explaining the data, building from one database to another, or embedding the data within a larger framework (Creswell & Clark, 2017). It is possible that researchers might want to use the data from both types of data collection to inform their conclusions, and this would be especially true in mixed methods designs (Mertens, 2014).

Based upon data collected, I first integrated my data to compare both the AP program and the IB program by drawing on the information from recruitment handbooks containing detailed program descriptions (Appendices A and B). I also retrieved the most current information concerning both programs' curricula from their official website, and this part is also mentioned in my literature review in Chapter II. I described both programs and made a few comparative tables to illustrate significant differences and similarities.

I especially focused on the curriculum part, which will also be used while answering research question two and three. To address parts of these two questions, I also interpreted the general Chinese high school curriculum based on the bilingual version of the Chinese high school mathematics text content book. Then, by looking at the mathematics requirements and curriculum contents, I compared the intended Chinese high school curriculum with that of AP program and IB program. The Chinese government has required that every student enrolling in these programs must complete the General Examination that every high school student needs to pass to get a diploma. Thus, to analyze students' performance data, I started comparing AP students' mathematics scores on the General Examination with their grades on SAT mathematics, AP Calculus AB, and AP Calculus BC. This enabled me to look for any correlation with students' learning outcomes in these two kinds of tests based on two different curricula. It should be noted that the IB school has implemented the government policy of requiring students to take the General Examination since 2016. The year used to label the data in this study refers to students' graduation year, so IB students' mathematics scores on the General Examination will not be available until June, 2019. I also compared students' SAT mathematics with their AP mathematics grades or IB mathematics grades so that I could look for correlation between these data.

To be specific, for the AP data, I first converted the General Examination math grades from letter grades to numeric grades (A=4, B=3, C=2, D=1) to simplify the comparison. Then I chose the most recent grade of students who took the SAT test multiple times to clarify the data. Also, students who take the AP Calculus BC test will have an AP Calculus AB sub-score since about 60% of the AP Calculus BC content has the same topics as AP Calculus AB. And according to the College Board, although each college and university sets its own policy for awarding credit and/or placement for AP examination scores, it is recommended that institutions apply the same policy to the Calculus AB sub-score that they apply to the Calculus AB score. Use of sub-scores in this manner is consistent with the philosophy of the courses, since common topics are tested at the same conceptual level in both Calculus AB and Calculus BC. Therefore, I added one more comparison between AP Calculus BC and AP Calculus AB for each year to see if there was any further information that I could obtain.

Before using the statistical software to analyze my data, I set my confidence interval at 95%. Because I only obtained the 2 years of corresponding data from 2016 to

2017. Also, with the small class sizes, the data base is not big enough to handle a higher confidence interval. Thus, the commonly used 95% confidence interval was the ideal choice. First, I made tables to record the simplified data. With ordinal data, I chose Goodman and Kruskal's gamma to measure the rank correlation, which measures the similarity of the orderings of the data when ranked by each of the quantities. It enabled me to measure the strength of association between each pair. Values range from -1 (100% negative association, or perfect inversion) to +1 (100% positive association, or perfect agreement). A value of zero indicates the absence of association. The estimate of gamma, G, depends on two quantities: N_s and N_d, where N_s represents the number of pairs of cases ranked in the same order on both variables (number of concordant pairs) and N_d stands for the number of pairs of cases ranked in reversed order on both variables (number of reversed pairs). And "ties" (cases where either of the two variables in the pair are equal) are dropped. Then the gamma coefficient can be calculated manually by the following formula: $G = \frac{N_s - N_d}{N_s + N_d}$. Critical values for the gamma statistic are sometimes found by using an approximation, whereby a transformed value, t, of the statistic refers to the Student t distribution, where $t \approx G\sqrt{\frac{N_s + N_d}{n(1 - G^2)}}$. Then I used the t table to see the statistical correlation between each possible pairing of AP and IB data points.

I did the analysis separately for the AP and IB programs, and after the individual analysis I also made a table to simplify and summarize the results and to compare them by year (same kind of variables but in different years). I acquired a deeper understanding based upon the filtered results.

Subsequently, after successively conducting qualitative interviews with the mathematics teachers from both high schools, I reorganized and summarized my data by considering my research question, filled that information into tables, and attempted to draw conclusions that I found interesting and meaningful.

After translating the audio recording of two of the Chinese mathematics instructors from the AP program, I broken-down all transcriptions into short phrases or keywords and filled them into my tables. After reviewing and reorganizing all transcripts, making notes about the keywords, and labeling relevant words, I extracted frequently repeated answers or same comments and highlighted the same opinion of each question. Meanwhile, I also noted contradictions or different opinions, so I marked the different ideas towards that corresponding question as well. As such, I categorized my interview data by diverse variables such as gender, nationality, targeted program and perceived mathematics teaching experiences.

I made comparisons between the Chinese instructors and non-Chinese instructors from the AP program to find hidden connections between their answers. I also constantly compared data from the non-Chinese AP instructors with data from non-Chinese IB instructors to demonstrate any correlations between responses. To ensure consistency, I always checked against transcription tables.

While managing the teachers' interview data, I was also able to identify some possible factors that might provide explanation for the students' scores. So I went back to add this relevant information to the analysis of students' performance data.

Summary

Chapter III first described the method that I chose to approach my data collection and answer my research questions. I justified the reason that I decided to use a mixed method approach and multiple sources for data collection to validate my research based on the research purpose of my study. Then the chapter explains, in detail, the processes and procedures that I used to collect my data.

On the one hand, the curriculum analysis is designed to show the similarities and differences among the AP program, the IB program and the intended Chinese high school programs in mathematics education. Students' performance data is aimed to reveal the situation of students who were receiving the imported AP program, or the IB program, or neither of them, in China.

On the other hand, I elected to use focus groups' interviews to comprehend what teachers, specifically, mathematics instructors of AP program and IB program, observed and perceived about these two international courses in Chinese mathematics education. Moreover, this chapter also clarified how I interpret and analyze the data collected. I explained the analysis method of processing both quantitative and qualitative data (students' performance data and teachers' interview information) I captured.

Chapter IV

RESULTS

This chapter contains the results of the mixed methodology studies conducted to answer the research questions. Additionally, included in the chapter are tables and graphics used to present complementary details in curriculum analysis, data comparison, and interview results.

Research Question 1

What are the significant differences and similarities between the AP (Advanced Placement) program and the IB (International Baccalaureate) program in their respective schools in Ningbo, China?

AP in Ningbo, China

The AP Center in Ningbo devotes itself to combining the advantages of both US and Chinese secondary education. It provides an opportunity for Chinese students who want to continue their higher education overseas in Western countries, especially in the United States. They follow a rigorous curriculum that will prepare them for and connect them to enroll and succeed in an accredited western college or university. Besides the preparedness for college success, it also equips students with mental and emotional readiness.

There are four parts to the entire Sino-US joint international high school AP program in Ningbo Foreign Language School. First is the Basic Course designed upon the general Chinese high school curriculum requirements needed for passing the General Examination, and Chinese instructors will be responsible for the teaching. Second, it is the introduced AP program that is taught by non-Chinese instructors. Third, it is Academic English and SAT courses that target students' language and culture needs for studying abroad. Last, it is extracurricular practice involving community service, speech debate, various competitions and so on. In general, the second part (the AP program) and the third part (Academic English and SAT courses) take over half of the class hour in the three-year high school studies.

In short, there are two curricula for AP students to learn in Ningbo Foreign Language School: Chinese curriculum for a corresponding Chinese high school diploma and the AP curriculum.

Chinese curriculum. For the Basic Course, here are the table of the general Chinese high school curriculum arrangement and the table of the agenda of the General Examination (see Tables 14 and 15).

Table 14. Chinese Curriculum Provision and Class Distribution

Subject	Grade 10		Grade 11		Grade 12	
	Semester 1	Semester 2	Semester 1	Semester 2	Semester 1	Semester 2
Chinese	3	3	4	-	-	-
Mathematics	**4**	**4**	**4**	-	-	-
English	2	2	2	-	-	-
Physics	3	3	-	-	-	-
Chemistry	3	3	-	-	-	-
Biology	-	-	3	3	-	-
Politics	2	2	3	-	-	-
History	2	2	-	-	-	-
Geography	2	2	-	-	-	-
Computer	2	2	-	-	-	-
Technology	-	-	2	2	-	-
Music & Arts	1	1	1	1	-	-
Class Meeting	1	1	1	1	1	1
P.E.	2	2	2	2	2	2
Club Activity	1	1	1	1	1	1
Electives	1	1	1	1	1	1
TOTAL	29	29	24	11	5	5

Note: The number here indicates the amount of teaching hours per week.

Notably, since 2015, except for the subject of English, all the General Examinations will be held on every October and April. Also, all classes of the Basic Course will follow the formative evaluation that including students' performance on participation, homework, test and project, and will be shown as grades (A, B, C, D, F) in the transcript.

Table 15. Sino-US Joint International High School Agenda for the General Examination

Grade	Time	Subject	Note
Grade 11	June	English	*The General Examination mostly ends in Grade 11.*
	April	Chinese, **Mathematics,** Politics	
	October	Physics, Chemistry, History, Geography	
Grade 12	October	Biology, Technology	

AP curriculum. The Ningbo Foreign Language School AP Curriculum currently offering twenty-five AP courses, and it is the most comprehensive AP Curriculum offered in a public school in China (Appendix A). The structure of the curriculum intends to give students more options and select courses that allow them to explore their interests. To ensure that all students are seriously considering their major and/or desired field of study with maximized potential in the long-term academic future, the administration team of the AP Center will provide individual consultation with students and parents while selecting courses, especially since eleventh grade.

First, it is the Year 1 Pre-AP as a foundation year designed to help students adapt to the new bilingual learning environment in a joint international program. It emphasizes on fostering student's Academic English abilities, study skills, and studying habits. And the effort spent on the improvement of their overall English proficiency will be necessary when entering Year 2 with a full load of AP courses (see Table 16).

Table 16. Grade 10: Pre-AP Program

<table>
<tr><th colspan="3">Group 1</th><th colspan="3">Group 2</th><th colspan="3">Group 3</th></tr>
<tr><td colspan="3">English for Academic Purpose I [8]</td><td colspan="3">English for Academic Purpose I [6]</td><td colspan="3">English for Academic Purpose I [4]</td></tr>
<tr><td colspan="3">Global Perspectives [4]</td><td colspan="3">Global Perspectives [4]</td><td colspan="3">AP Human Geography [5]</td></tr>
<tr><td colspan="3">Pre-AP Calculus [4]</td><td colspan="3">Pre-AP Calculus [4]</td><td colspan="3">Pre-AP Calculus [4]</td></tr>
<tr><td colspan="3">SAT Science [4]</td><td colspan="3">AP Physics I [6]</td><td colspan="3">AP Physics I [5]</td></tr>
<tr><td>Pre-AP Art</td><td>Pre-AP Comp Sc.</td><td>Critical Thinking</td><td>Pre-AP Art</td><td>Pre-AP Comp Sc.</td><td>Critical Thinking</td><td colspan="3">German I [4]</td></tr>
<tr><td colspan="6">*Rotation taught on 4 periods per week</td><td>Pre-AP Art</td><td>Pre-AP Comp Sc.</td><td>Critical Thinking</td></tr>
</table>

Note: The number here indicates the amount of teaching hours per week.

To be noted, an English baseline vocabulary test will be conducted at the beginning of the academic year, which tests students' written receptive vocabulary knowledge and becomes a primary measurement when determining the belongingness of Grade 10 students to three different groups listed in Table 16. The admission office then combines the result of this vocabulary test with other reading, writing and speaking tests as the student's linguistic ability, and makes the final decision of assigning students to the Grade 10 groups. In addition, as you can see, English for Academic Purpose I and Pre-AP Calculus are mandatory. In mathematics part, Pre-AP Calculus is highlighted in Table 16 as well.

In Year 2, students have opportunities to select different AP courses for themselves. They may choose any course available based on their own levels. But they are required to choose an English course from AP English Language & Composition A or English for Academic Purpose II. It is said that "they must meet certain requirements to continue through the AP English course but students are given assistance in meeting these requirements should they decide to take this challenging course" (Appendix A). Moreover, as highlighted in the table, students are required to choose AP Calculus AB or AP Calculus ABC for mathematics. To be specific, AP Calculus ABC lasts from Grade

11 to 12. Students who take AP Calculus ABC do not need to take the Calculus AB examination. They will receive both a Calculus BC score and a Calculus AB sub-score. Taking an AP Science course is also recommended but not mandatory. Then they must take either 3 or 4 courses from the available options shown in the table (see Table 17).

Table 17. Grade 11

<table>
<tr><th>ENGLISH</th><th>MATH</th><th>Block 1</th><th>Block 2</th><th>Block 3</th><th>Block 4</th></tr>
<tr><td>AP English Language A</td><td>AP Calculus AB</td><td colspan="4" rowspan="2">Option Blocks will be made up of the following subjects:
• Sciences: AP Physics I and II; AP Chemistry
• Humanities: AP Economics; AP World History; AP US History; AP Human Geography
• Language: German II
• Fine Arts: Studio Art 2D; Studio Art Drawing; Computer Science A and Applied Computer Science</td></tr>
<tr><td>English for Academic Purpose II</td><td>AP Calculus ABC (AB & BC)</td></tr>
</table>

In Year 3, while academically rigorous, students should spend more time on preparing for the admission application process of getting in to top-notch universities. Students are required to take an English course and can choose either AP English Language & Composition B or AP English Literature. Also, they need to choose one from AP Statistics, AP Calculus BC and AP Computer Science A as shown in the table (see Table 18). Therefore, unlike those students who take AP Calculus ABC will have two scores in the end (Calculus BC score and Calculus AB sub-score), students who take AP Calculus AB in Grade 11 and then take AP Calculus BC in Grade 12 will receive three scores in total that includes Calculus AB score, Calculus BC score and Calculus AB sub-score. Then just like Year 2, they must take either three or four courses from the available options shown in the table (see Table 18).

Notably, Art/Humanities classes in Grade 11 and Grade 12 may be combined if numbers in classes are too low or schedule requires it. Also, AP courses are taught on 5 periods a week unless otherwise stated.

To receive full credit for an AP course, students must register and take the AP examination. If students fail to register or do not take the AP examination in the course they are studying, the credit points will default to that of an honor course.

Table 18. Grade 12

<table>
<tr><th>ENGLISH</th><th>MATH</th><th>Block 1</th><th>Block 2</th><th>Block 3</th><th>Block 4</th></tr>
<tr><td>AP English Language B</td><td>AP Statistics</td><td colspan="4" rowspan="3">Option Blocks will be made up of the following subjects:
• Sciences: AP Physics C; AP Chemistry; AP Biology; AP Environmental Science
• Humanities: AP Economics; AP World History; AP US History; AP Psychology
• Language: German III
• Fine Arts: Studio Art 2D; Studio Art Drawing; AP Art History and Applied Computer Science</td></tr>
<tr><td>AP English Literature</td><td>AP Calculus BC</td></tr>
<tr><td></td><td>AP Computer Science A</td></tr>
</table>

Assessment. The AP center is using a standard continuous and formative assessment in line with a typical American secondary school. The grade on a summative examination will not take the full weight while assessing students' performance. Instead, various types of assessments such as participation, homework, unit tests, quizzes, classwork and projects are included. Each course will have a detailed explanation of the weighting of each component on its syllabus. Credits are based on the number of semesters a course is studied over.

Besides, there are two types of examinations in the AP curriculum: Trimester examinations and AP examination offered by the College Board. Trimester examinations are administered at the end of each trimester (end of October; early January; and mid-April). And they count for between 25% and 45% of the grade for a course (in accordance with weighting outlined in the syllabus). While AP Examinations are taken in May of each year and the examinations will be graded in the United States. As mentioned before, students must register and take the AP examination in order to get full credit for an AP course, and these courses are scored on a scale of 1 to 5 with 5 being the highest mark as shown in the table as well (see Table 19). Students who receive a qualified AP Examination score are recognized by most universities and colleges in the United States.

Table 19. Course Credit

Grade Awarded	Course Average	Grade Points Awarded			Zhejiang Province Grades Breakdown
		Non-AP Course	Honors Course*	**AP or Language Course**	
90-100	A	**4**	4	**5**	85-100
80-89	B	**3**	3	**4**	70-94
70-79	C	**2**	2	**3**	55-69
60-69	D	**1**	1	**2**	40-54
0-59	F	**0**	0	**0**	0-39

IB in Ningbo, China

The IB Center in Ningbo commits itself to combining the best of both Chinese and introduced foreign programs with high quality educational resources such as small-sized classes and experienced foreign instructors. The mission of the IB Center in Ningbo is to empower students to become lifelong learners and to become comprehensive global citizens. It also aims to produce students with an international perspective and intercultural respect, while retaining a patriotism with regard to their Chinese cultural identity.

The program in the IB Center in Ningbo has two major parts: Pre-IB and the IB Diploma Program. The Year 1 Pre-IB provides preparation courses such as English, Additional Mathematics, Physics, Chemistry, Biology, Geography, Business, Economics, Arts, Music, PE and IB Skills for students' next stage of study in Year 2 and 3. After that, students officially start the IB Diploma Program learning in grade 11 and 12. They need to choose 6 courses, 3 of them are higher level (HL), and the other 3 are standard level (SL) courses, which means they can explore some subjects in depth and some more broadly within the two-year period. But students are highly encouraged to select subjects they interested and based on their academic advantages, especially for HL courses. After finishing three-year studying and passing the IB examinations, students can get an IB diploma. It should be noted that, starting from 2016, due to the government policy that expects international school students with patriotism and global vision, newly enrolled IB Center students are required to study domestic high school curriculum just like students in the AP Center, and take the Zhejiang provincial High School Academic Proficiency Test (the General Examination). Therefore, since 2016, students in the IB Center can have both Chinese high school diploma and the IB diploma instead of receiving only the IB diploma after they reached graduate standards. Besides, the IB Center also began offerings advanced extension courses, such as Critical Reading and Writing, Academic Debate, and 3D Printing in 2016.

Three cores. The IB Diploma Program has three cores as follows: Theory of Knowledge (TOK), Extended Essay (EE) and Creativity, Action, Service (CAS). The TOK aims to make students aware of the interpretative nature of knowledge, including personal ideological biases, and it offers students and their teachers the opportunity to reflect critically on diverse ways of knowing and on areas of knowledge; and to consider the role and nature of knowledge in their own culture, in the cultures of others, and in the wider world. Moreover, TOK prompts students to be aware of themselves as thinkers, encouraging them to become more acquainted with the complexity of knowledge; and to recognize the need to act responsibly in an increasingly interconnected but uncertain world. TOK also provides coherence for the student, by linking academic subject areas as well as transcending them (What is TOK, 2018).

The EE course comes from one of the students' six chosen subjects from the IB Diploma Program, and students are required to make an in-depth study of that chosen topic. It provides practical preparation for undergraduate research and it is an opportunity for students to investigate a topic of their individual interest. Because in higher education, each student is supposed to be equipped with an efficient ability to do research independently. And through the research process for the EE, students can prepare themselves in formulating an appropriate research question, engaging in a personal exploration of the topic, communicating ideas and developing an argument. Participation in this process also help students developing the capacity to analyze, synthesize, and evaluate knowledge (What is EE, 2018).

CAS involves students in a range of activities alongside their academic studies and has no prescribed specific projects or activities but requires students to engage with real-life experiences and at least one project that initiated by themselves including purposeful activities with significant outcomes; personal challenge; thoughtful consideration like planning, reviewing progress, reporting; and reflection on outcomes and personal learning through the experiences. A creativity project should involve arts, and other activities that can help enhancing creative thinking; an activity project could help a student overcome a personal fear, such as rock climbing but it does not have to be sports-related or competitive; a service project must be beneficial for the community as well as providing a learning opportunity for students. The CAS project can address any single strand of CAS, or combine two or all three of them (What is CAS Project, 2018).

Six subjects. For the six subjects, Table 20 shows the schedule of the IB Center in Ningbo.

Table 20. Subjects (2015-2016)

Subjects	Contents
Language A1	Chinese & Literature, HL/SL
Second Language	English B, HL/SL
Individuals &Societies	Business & Management, HL/SL
	Economics, HL/SL
	History, HL/SL
Experimental Science	Biology, HL/SL
	Chemistry, HL/SL
	Physics, HL/SL
Mathematics and Computer Science	Mathematics, HL/SL
	Further Mathematics, HL
The Arts	Music, HL/SL
	Visual Arts, HL/SL

However, since 2017, the school made a slight change to the subjects. “History” has been replaced by “Geography,” and “Further Mathematics” has been deleted (see Table 21).

Assessment. The assessment system of the IB Diploma Program is composed of two parts: internal and external assessment. The internal assessment is issued by subject teachers referring to assessment standards and the result is examined by the IBO, which makes up 20%-50% of the final grade. After finishing two years of the IB diploma courses, for most subjects, students need to take written examinations held and examined by the IBO as well. External assessment includes the theory of knowledge (TOK) essay and the extended essay (EE) of the three IBDP cores that mentioned before. The IB examination is held twice a year in every May and November. Also, every subject is graded on a scale of 1 to 7 points, with 7 being the highest. Then it makes up 42 points from six subjects. The TOK essay and the EE essay counts for 3 additional points in total. So, the overall score is 45 points.

Table 21. Subjects (2017)

Subjects	Contents
Language A1	Chinese & Literature, HL/SL
Second Language	English B, HL/SL
Individuals &Societies	Business & Management, HL/SL
	Economics, HL/SL
	Geography, HL/SL
Experimental Science	Biology, HL/SL
	Chemistry, HL/SL
	Physics, HL/SL
Mathematics	**Mathematics, HL/SL**
The Arts	Music, HL/SL
	Visual Arts, HL/SL

There are certain requirements to successfully get the IB diploma (see Table 22). Students need to gain at least 24 points, meet all CAS requirements, have grades in all six subjects and the TOK and EE essays, get at least got a grade D in TOK or EE; and there is no grade of 1 in any subject, no more than two grades of 2 and no more than three subjects grade 3 or below. Recipients with a qualified IB diploma are accepted by higher education institutions in many English speaking countries worldwide.

Table 22. IBDP Requirements

If 24-27 Points	If 28+ Points
An average of 4 points in each HL subject	*Same*
An average of 3 points in each SL subject	*Same*
No grade 2 in any HL subject	No more than one grade 2 in any HL subject
No more than one grade 2 in any SL subject	No more than two grade 2s in any SL subject

Similarities Between Imported AP Program and IB Program in Ningbo, China

Overall, both programs are articulated with the general Chinese curriculum, especially since 2016. Therefore, students from both centers are required to learn two curricula (the general Chinese curriculum and the introduced AP/IB curriculum) at the same time, which means every student in both AP center and IB center must take the Chinese general examination. In their first year, students in both schools are required to take the pre-program courses to prepare themselves for their following studies. For each year, both schools have two mandatory subjects: English and mathematics. To be specific, for the AP center, students are required to study English and AP mathematics every year (or choose another mathematics related course: Computer Science in the third year), and for the IB center, students have to study English and IB mathematics every year as well. Therefore, two possible factors might be generated from this finding: language challenge and localized feature. Since both programs require a certain facility with the English language, Chinese students are required to learn English to better prepare themselves for further AP and IB studies. And research also shows that mathematics became the most selected subject of the AP and IB program in China based on Chinese students' overall academic strength. For example, the most recent AP data show that currently about 81.5% (225 out of 276) authorized Chinese AP schools offer

AP Calculus AB, 72.1% (199 out of 276) schools offer AP Calculus BC and 69.6% (192 out of 276) schools offer AP Statistics (AP Course Ledger, 2018). Some mathematics instructors' responses in the interview also suggested that Chinese students have more advanced calculation skills than students from other countries. Thus, a mandatory mathematics course is required, probably to let student academic advantages lead to higher achievement while also lessening their burden of learning two curricula at the same time. To balance the difference, academically and culturally, between the general Chinese education and the imported foreign programs, each school has a non-Chinese vice principal in charge of the introduced program and a Chinese vice principal to cooperate with the communication of any imported programs' affairs with the school's administration office. Both schools also established college counseling teams, which helps with personalized academic plans, SAT and TOEFL test preparation and out-of-class activities recommendations.

For the positive sides, both programs are highly competitive with rigorous contents and high performance standards to achieve educational excellence and build a strong academic foundation before entering the higher education. And they both value critical and independent thinking skills, and regard international understanding as an important part of fostering their students. For college admission, both programs have great reputation and are widely acknowledged around the world, which can help students saving expenses and time in colleges or universities.

For the negative sides, both programs have intense course loads for Chinese student with English as a second language.

Differences Between Imported AP Program and IB Program in Ningbo, China

So, to answer research question 1 we see the similarities, and according to the information of imported AP and IB program in Ningbo, Table 23 shows the differences.

Table 23. Differences Between the AP Program and IB Program in Ningbo

	Advanced Placement	IB Diploma
Number of schools in Ningbo	6	3
Targeted colleges or universities	Mostly in the US	English speaking countries worldwide
Goal	To prepare and connect students to college success and opportunity, with mental and emotional readiness	To develop students as lifelong learners and global citizens with international perspective and intercultural respect, while retaining the patriotism
Content	All college-level; Cafeteria style with individual work	Mostly high school level, some with college-level; Comprehensive and systematic program
Emphasis of subject	Enhance students' abilities in a/some specific academic area(s)	Foster students' comprehensive abilities
Curriculum	More Content-Centered	More Student-Centered
Limitation of choosing course difficulty	No limit, can choose whatever you want	3 HL (Higher Level) and 3 SL (Standard Level)
Assessment	External	External and internal
Examination Time	Once a year (May)	Twice a year (November and May)
Score Scale	1 to 5	1 to 7
Grading Place	Graded in the United States	Graded world-wide
Requirement of student's English level	High	Extremely High
Certificate	Only 1 diploma: Chinese high-school diploma	2 diplomas (since 2016): Chinese high-school diploma; IB diploma

To be specific, the number of AP schools in Ningbo is triple the number of IB schools. Because of the differences as to which higher education institutions accept and recognize the AP and IB examinations, the AP center targets mainly colleges and universities in the United States for its graduates while the IB center sends students to many English-speaking higher education institutions worldwide.

The goal of the AP program is to prepare and connect students for college success and opportunity, with mental and emotional readiness. The IB program's goal is to develop students as knowledgeable lifelong learners and caring global citizens with intercultural understanding and respect, while retaining their patriotism toward China and its culture.

The AP center provides college-level courses with a cafeteria style so that students can choose their preferred courses, and most courses in the IB center are taught at a high-school level, with some at the college level. The IB center offers a more comprehensive and systematic diploma program not only covering various subjects but also containing three cores to specifically broaden students' thinking skills, research abilities and real-life experiences so that they can apply what they learned from school.

With different contents, the programs have different emphases when it comes to subject. The AP program tends to enhance students' abilities in one or some specific academic subject(s) while the IB program is apt to foster students' comprehensive abilities. Therefore, the curriculum of the AP center is more content-centered with more lectures and in-depth learning, while the IB center provides inquiry-based learning and more activities alongside their academic studies, which makes its curriculum more student-centered.

Also, unlike the AP center which gives students free choice when it comes to the difficulty of their courses, the IB center requires students to choose three higher level (HL) courses and three standard level (SL) courses.

A difference in the programs also appears in their methods of assessment. The IB uses both external and internal assessment to evaluate students' learning outcome while the AP only use external assessment. The examination times of the IB are twice per year in every May and November with score scale from 1 to 7, and the grading is done by members of the IBO from around the world. The AP holds its examinations once per year in each May with a score scale from 1 to 5, and exams are graded only in the United States.

In addition, given the three cores and the internal assessments that include a lot of written work, IB students are expected to be very advanced in their English language abilities to better express their thoughts. In contrast, AP students do not have much writing to do if they select courses like mathematics or science.

After graduation, AP students receive only the Chinese high school diploma because the AP program is not a diploma program, whereas IB students have earned two diplomas, from both IBO and Chinese high school, since 2016.

Research Question 2

> *What are the differences and similarities in curriculum and student performance between AP (Advanced Placement) program and the intended general Chinese high school program in mathematics education?*

Curriculum Comparison in the AP Center

In the AP Center, almost every AP student takes AP Calculus and a very few take AP Statistics. Because the sample size of students taking AP Statistics was so small, I did not use either student performance or the class's curriculum for comparison in this study. I conducted the comparison based on the following materials downloaded from College Board: the "concept outline" of the AP Calculus AB/BC courses as well as the exam description (Appendix D). At the same time, I collected the following textbooks that are used by the AP Center: *Precalculus: Graphical, Numerical, Algebraic*, 9th Edition published by Pearson Higher Education (for Pre-AP Calculus) (Appendix I) and *Calculus: Graphical, Numerical, Algebraic*, 5th Edition published by Pearson Higher Education (for AP Calculus AB & BC) (Appendix J). I spoke with instructors in the AP Center to find out what elements of the written curriculum are not covered in the active curriculum, and I only retained the contents used in real teaching practice to narrow down the variables and to create a more accurate analysis in my research. In total, there are 3 content materials for AP Mathematics, and I compared them with the General Chinese high school mathematics test contents (Appendix G).

Difference 1: Topics only in AP Calculus (AB & BC). From the concept outline and the exam description of the AP Calculus AB/BC course (Appendix D), there are four main ideas: limits, derivatives, integrals, the fundamental theorem of calculus, and series. Among these four major topics, the first three are included in the entire AP calculus curriculum with some subject matter that is included only in the BC course; the last topic (series) is only for AP Calculus BC students.

Based on the concept outline and the general Chinese high school mathematics test contents, there are four major differences (see Table 24).

Table 24. Four Major Differences between AP and General Chinese Mathematics

Topics	AP Calculus (AB & BC)	General Chinese Mathematics
Limits	Yes	-
Derivatives	Yes	- (Optional 2-2)
Integrals and the Fundamental Theorem of Calculus	Yes	-
Series	Yes (BC only)	-

From this table, it is clear that students in the AP Center they have four extra mathematical topics to learn to prepare for their AP calculus test. In other words, the major ideas from the AP calculus curriculum are excluded from the intended Chinese mathematics curriculum for AP students. One of them, derivatives, is listed as an optional topic for general Chinese students, which means this topic is included in the College Entrance Examination. But AP students only need to meet the requirements of the General Examination, which excludes the relative content of derivatives, so they just cover the content from “required 1 to option 2-1.”

While making the comparison between the AP calculus curriculum and the general Chinese high school mathematics curriculum, I used the general Chinese high school mathematics test contents as a reference. Overlapped contents are marked with colors (Appendix H).

Differences 2: Topics only in General Chinese Mathematics. From Appendix H, there are some topics that are only covered in the Chinese curriculum and not in the AP calculus curriculum (see Table 25).

Table 25. Topics Covered Only in the Chinese High School Curriculum

Topics
Sets
Monotonicity and Monotone Interval
Structure of 3D Geometry
Three-View Drawing and Trimetric Drawing of 3D Geometry
Surface Area and Volume of 3D Geometry
Position Relations between Point, Line and Plane in 3D Plane
Test and Properties of Parallel
Test for Orthogonal Lines and Planes
General Form of a Function of a Circle
Position Relations between Lines and Circles
Coordinates in 3D Space
Relations between Sin, Cos, and Tan of an Angle
Arithmetic Series
Application of Sequences
Linear Inequalities with Two Unknown Variables and Simple Liner Programming Questions
Fundamental Inequalities
Inequalities with Absolute Values
Propositions and Their Relations
Sufficient and Necessary Condition
Curve and Equation
Ellipse
Hyperbola
Parabola
Space Vector and Its Operations
Vectors in 3D Geometry

Similarities. Beside the four major differences that are only covered in AP Calculus (AB & BC) curriculum and some minor differences that are only covered in the general Chinese high school mathematics curriculum, there exist some similarities (see Table 26).

The overlapping contents between Pre-AP Calculus and general Chinese high school mathematics are marked in italics in Appendix H. Moreover, most of them concern functions. In addition, as can be seen in the contents of the textbook used by the AP Center (Appendix I and J), only a very superficial introduction of “definition and notation of sequence,” “arithmetic and geometric sequences,” and “geometric series” from the General Chinese high school mathematics curriculum is covered in AP Calculus BC. I marked these overlapped contents in white letters on Appendix H. Since teachers only mention these concepts in class, that insignificance could easily be ignored. Therefore, in general, the AP Calculus (AB & BC) contents are unlike the general Chinese high school mathematics contents. And many of the contents in general Chinese high school mathematics are similar to the contents in Pre-AP Calculus.

On the one hand, AP Calculus (AB & BC) is totally different from general Chinese high school mathematics, especially for students in the AP Center. It is easy to understand this difference because AP calculus itself represents college-level mathematics content, which certainly exceeds the general high school level. Therefore, AP calculus has a higher level of academic hardness. According to the similarities in Table 26, the overlapping contents occur in the Pre-AP class in Grade 10 to prepare students to learn AP calculus in grades 11 and 12. However, in the design of the syllabus, contents are extended in depth. For instance, those following ideas such as parametric equations, limits of functions, continuity of functions, asymptotic and unbounded behavior, implicit functions and polar functions are not covered in general Chinese high school mathematics. Even for Chinese students who need to take the College Entrance Examination, which includes the general Chinese high school mathematics test contents and adds optional sections 2-2 and 2-3 (which includes derivatives and complex numbers) the depth is still superficial relative to AP calculus. AP calculus puts the idea of complex numbers in its Pre-AP calculus classes, which means that complex numbers are pre-requisites for learning AP Calculus (AB & BC). Also, although the idea of derivative is one of the four big ideas in the AP calculus concept outline (Appendix D), it consists of more detailed contents than that subject does in general Chinese high school mathematics. For example, ideas such as differentiability and continuity, estimating a derivative (numerically and graphically), derivative of the inverse of a function and implicit differentiation, L'Hôpital's Rule are not covered in general Chinese high school mathematics.

Table 26. Similarities Between Pre-AP Calculus and the Chinese High School Curriculum

Topics
Functions and Notations
Properties of Functions
Exponential Functions
Logarithmic Functions
Power Functions
Functions and Equations
Models of Functions and Their Applications
Angle of Inclination and Slope
Linear Function
Intercept Coordinates and Distance Formula
Standard Form of a Function of a Circle
Angles and Radian Measure
Trigonometric Functions of Any Angles
Induced formula of Trigonometric Functions
Graphs and Properties of Trigonometric Functions
Graph of y=$Asin(\omega x + \varphi)$
Basic Application of Trigonometric Functions
Definition of Vector
Vector Operation
Theorems and Coordinates of Vector
Dot Product
Application of Vector
Compound Angle Identities
Basic Identities
Sine Rule and Cosine Rule
Solving Triangles Case Study
Unequal Relation and Inequalities
One-Variable Quadratic Inequalities and Their Solutions
Fundamental Inequalities: $\sqrt{ab} \leq \frac{a+b}{2}$

Moreover, there is a difference in the philosophy of both curricula: AP aims at exploring the depth of mathematics allowing students to give full play to the specific subject they are good at, while the general Chinese high school mathematics' approach is more like a mass education that is designed for everyone. Additionally, different attitudes to using technology appeared here because the use of a graphing calculator in AP Calculus is considered an integral part of the course (see Table 2). But students are only allowed to use standard scientific calculators when taking the General Examination.

In short, the curriculum of AP calculus is more advanced, deeper, and contains more applications than the curriculum of general Chinese high school mathematics.

Also, we can conclude that general Chinese high school mathematics is a foundation for learning AP Calculus (AB &BC) because many elements of its curriculum are included in Pre-AP Calculus. Just as the Chinese instructor W mentioned in an interview, learning general Chinese high school mathematics can help AP students to build a solid mathematical background in Grade 10, which will strongly benefit them in learning AP Calculus in the their later high school studies, and even in their college lives later.

Students' Performance Comparison in the AP Center

Data comparisons 2016.

AP GE (General Examination) mathematics 2016 vs AP SAT mathematics 2016. Table 27 summarizes the AP 2016 graduates' scores on the General Examination and the SAT test. There are no students who earned an "A" on their General Examination in mathematics, but the majority (41 out of 51, about 80.39%) of them scored in the high range (700-800) of the SAT mathematics test.

A gamma coefficient of 0.286 implies a moderate correlation between AP students' GE mathematics 2016 and AP students' SAT mathematics 2016.

By using the gamma coefficient, t ≈ 1.148. The t-table gives the critical value for t (2-tailed) as 2.009. Because 1.148 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that AP students' GE mathematics 2016 is positively correlated with AP students' SAT mathematics 2016.

Table 27. Data Summary of AP Comparison 1 for 2016

AP SAT mathematics	AP GE Mathematics 2016				Total Number of Students
	D	C	B	A	
580	1	-	-	-	1
670	1	-	-	-	1
680	4	-	1	-	5
690	1	1	-	-	2
700	-	1	-	-	1
710	-	3	2	-	5
720	1	1	-	-	2
730	-	6	-	-	6
740	-	1	-	-	1
750	2	2	-	-	4
760	3	2	4	-	9
770	2	4	5	-	11
780	-	1	-	-	1
790	1	-	-	-	1
800	-	1	-	-	1
Total	16	23	12	-	51

AP GE (General Examination) mathematics 2016 vs AP Calculus AB 2016. Table 28 summarizes the AP 2016 graduates' mathematics scores on the General Examination and on the AP Calculus AB exam.

A gamma coefficient of 0.487 implies a relatively strong correlation between AP students' GE mathematics 2016 and AP students' AP Calculus AB 2016.

By using the gamma coefficient, t ≈ 2.031. The t-table gives the critical value for t (2-tailed) as 2.009. Because 2.031 is greater than the critical value, we can reject the null hypothesis. We expect that AP students' GE mathematics 2016 and AP students' Calculus AB 2016 are positively correlated.

Table 28. Data Summary of AP Comparison 2 for 2016

AP Calculus AB 2016	AP GE mathematics 2016				Total Number of Students
	D	C	B	A	
1	8	2	-	-	10
2	-	1	2	-	3
3	4	6	3	-	13
4	3	6	1	-	10
5	1	8	6	-	15
Total	16	23	12	-	51

AP GE (General Examination) mathematics 2016 vs AP Calculus BC 2016. Table 29 summarizes the AP 2016 graduates' scores on the General Examination and on the AP Calculus BC test. Although no students earned an "A" on the General Examination in mathematics, no one scored less than 3 on the AP Calculus BC exam.

A gamma coefficient of 0.086 implies a very weak correlation between AP students' GE mathematics 2016 and AP students' Calculus BC 2016.

Table 29. Data Summary of AP Comparison 3 for 2016

AP Calculus BC 2016	AP GE Mathematics 2016				Total Number of Students
	D	C	B	A	
1	-	-	-	-	-
2	-	-	-	-	-
3	-	2	2	-	4
4	1	-	2	-	3
5	-	3	4	-	7
Total	1	5	8	-	14

By using the gamma coefficient, $t \approx 0.136$. The t-table gives the critical value for t (2-tailed) as 2.160. Because 0.136 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that AP students' GE mathematics 2016 is positively correlated with AP students' Calculus BC 2016.

AP Calculus AB 2016 vs AP SAT mathematics 2016. Table 30 summarizes the AP 2016 graduates' scores on the AP Calculus AB exam and their mathematics grades on the SAT test.

A gamma coefficient of 0.409 implies a relatively strong correlation between AP students' Calculus AB 2016 and AP students' SAT mathematics 2016.

By using the gamma coefficient, t ≈ 1.890. The t-table gives the critical value for t (2-tailed) as 2.009. Because 1.890 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that AP students' Calculus AB 2016 is positively correlated with AP students' SAT mathematics 2016.

Table 30. Data Summary of AP Comparison 4 for 2016

AP SAT mathematics	AP Calculus AB 2016					Total Number of Students
	1	2	3	4	5	
580	1	-	-	-	-	1
670	-	-	-	1	-	1
680	3	-	2	-	-	5
690	2	-	-	-	-	2
700	-	-	-	1	-	1
710	-	2	1	1	1	5
720	1	-	-	-	1	2
730	1	-	3	-	2	6
740	-	-	-	1	-	1
750	2	-	-	1	1	4
760	-	-	3	3	3	9
770	-	1	2	1	7	11
780	-	-	1	-	-	1
790	-	-	1	-	-	1
800	-	-	-	1	-	1
Total	10	3	13	10	15	51

AP Calculus BC 2016 vs AP SAT mathematics 2016. Table 31 summarizes the AP 2016 graduates' grades on the AP Calculus BC exam and their mathematics grades on the SAT test. Notably, no one among the sampled students scored less than a 3 on AP

Calculus BC, and those who took the AP Calculus BC test all scored at the highest range (higher than 700) on SAT mathematics. Also, half of these students (50%) achieved a score at 5 (the highest possible score), on their AP Calculus BC exams.

A gamma coefficient of 0.333 implies a moderate correlation between AP students' Calculus BC 2016 and AP students' SAT mathematics 2016.

By using the gamma coefficient, t ≈ 0.612. The t-table gives the critical value for t (2-tailed) as 2.160. Because 0.612 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that AP students' Calculus BC 2016 is positively correlated with AP students' SAT mathematics 2016.

Table 31. Data Summary of AP Comparison 5 for 2016

AP SAT mathematics	AP Calculus BC 2016					Total Number of Students
	1	2	3	4	5	
710	-	-	-	-	1	1
730	-	-	1	-	-	1
750	-	-	1	-	-	1
760	-	-	-	2	1	3
770	-	-	2	1	5	8
Total	-	-	4	3	7	14

AP Calculus BC 2016 vs AP Calculus AB 2016. Table 32 summarizes the scores of the AP 2016 graduates on their AP Calculus BC and AP Calculus AB exams. As we can see from Table 32, no student who took the AP Calculus BC test scored a 1 on the AP Calculus AB test. Only one student received a 2, and the majority (64.29%) of them received 5s on the AP Calculus AB test.

A gamma coefficient of 0.913 implies a very strong correlation between AP students' Calculus BC 2016 and AP students' AP Calculus AB 2016.

By using the gamma coefficient, t ≈ 4.058. The t-table gives the critical value for t (2-tailed) as 2.009. Because 4.058 is greater than the critical value, we can reject the null hypothesis. We expect that AP students' Calculus BC 2016 and AP students' Calculus AB 2016 are positively correlated.

Table 32. Data Summary of AP Comparison 6 for 2016

AP Calculus AB 2016	AP Calculus BC 2016					Total Number of Students
	1	2	3	4	5	
1	-	-	-	-	-	-
2	-	-	1	-	-	1
3	-	-	2	-	-	2
4	-	-	-	2	-	2
5	-	-	1	1	7	9
Total	-	-	4	3	7	14

Data comparisons 2017.

AP GE (General Examination) mathematics 2017 vs AP SAT mathematics 2017. Table 33 summarizes the scores of AP 2017 graduates on the General Examination and SAT tests. There are no students who earned an "A" or "B" on the General Examination in mathematics. However, two thirds (18 out of 27, about 66.67%) of them scored within the high range (700 to 800) on the SAT mathematics test. Also, no one scored less than 600 on the SAT mathematics test.

A gamma coefficient of 0.476 implies a relatively strong correlation between AP students' GE mathematics 2017 and AP students' SAT mathematics 2017.

By using the gamma coefficient, $t \approx 1.254$. The t-table gives the critical value for t (2-tailed) as 2.056. Because 1.254 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that AP students' GE mathematics 2017 is positively correlated with AP students' SAT mathematics 2017.

Table 33. Data Summary of AP Comparison 1 for 2017

AP SAT Mathematics	AP GE Mathematics 2017				Total Number of Students
	D	C	B	A	
650	2	-	-	-	2
680	1	1	-	-	2
690	2	-	-	-	2
700	2	1	-	-	3
710	1	-	-	-	1
720	2	-	-	-	2
730	3	-	-	-	3
740	1	2	-	-	3
750	2	-	-	-	2
760	1	-	-	-	1
770	2	1	-	-	3
790	-	2	-	-	2
800	-	1	-	-	1
Total	19	8	-	-	27

AP GE (General Examination) mathematics 2017 vs AP Calculus AB 2017. Table 34 summarizes the scores of the AP 2017 graduates on the General Examination and the AP Calculus AB test.

A gamma coefficient of 0.810 implies a very strong correlation between AP students' GE mathematics 2017 and AP students' AP Calculus AB 2017.

Table 34. Data Summary of AP Comparison 2 for 2017

AP Calculus AB 2017	AP GE mathematics 2017				Total Number of Students
	D	C	B	A	
1	3	-	-	-	3
2	3	-	-	-	3
3	7	2	-	-	9
4	4	-	-	-	4
5	2	6	-	-	8
Total	19	8	-	-	27

By using the gamma coefficient, $t \approx 2.979$. The t-table gives the critical value for t (2-tailed) as 2.056. Because 2.979 is greater than the critical value, we can reject the null hypothesis. We expect that AP students' GE mathematics 2017 and AP students' Calculus AB 2017 are positively correlated.

AP GE (General Examination) mathematics 2017 vs AP Calculus BC 2017. Table 35 summarizes AP 2017 graduates' grades on the General Examination and the AP Calculus BC exam. No students earned an "A" or "B" on their General Examination mathematics, and no one scored less than 2 on the AP Calculus BC test.

A gamma coefficient of 0.833 implies a very strong correlation between AP students' GE mathematics 2017 and AP students' Calculus BC 2017.

By using the gamma coefficient, t ≈ 1.846. The t-table gives the critical value for t (2-tailed) as 2.365. Because 1.846 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that AP students' GE mathematics 2017 is positively correlated with AP students' Calculus BC 2017.

Table 35. Data Summary of AP Comparison 3 for 2017

AP Calculus BC 2017	AP GE Mathematics 2017				Total Number of Students
	D	C	B	A	
1	-	-	-	-	-
2	1	-	-	-	1
3	3	1	-	-	4
4	1	-	-	-	1
5	-	2	-	-	2
Total	5	3	-	-	8

AP Calculus AB 2017 vs AP SAT mathematics 2017. Table 36 summarizes AP 2017 graduates' scores on the AP Calculus AB exam and their mathematics grades on the SAT test.

A gamma coefficient of 0.640 implies a strong correlation between AP students' Calculus AB 2017 and AP students' SAT mathematics 2017.

By using the gamma coefficient, t ≈ 2.589. The t-table gives the critical value for t (2-tailed) as 2.056. Because 2.589 is greater than the critical value, we can reject the null hypothesis. We expect that AP students' Calculus AB 2017 and AP students' SAT mathematics 2017 are positively correlated.

Table 36. Data Summary of AP Comparison 4 for 2017

AP SAT mathematics	AP Calculus AB 2017					Total Number of Students
	1	2	3	4	5	
650	1	1	-	-	-	2
680	-	1	1	-	-	2
690	-	-	2	-	-	2
700	-	1	1	-	1	3
710	-	-	-	-	1	1
720	1	-	1	-	-	2
730	1	-	1	1	-	3
740	-	-	2	-	1	3
750	-	-	1	1	-	2
760	-	-	-	1	-	1
770	-	-	-	1	2	3
790	-	-	-	-	2	2
800	-	-	-	-	1	1
Total	3	3	9	4	8	27

AP Calculus BC 2017 vs AP SAT mathematics 2017. Table 37 summarizes the scores of AP 2017 graduates on the AP Calculus BC exam and their mathematics grades on the SAT test. Not one of sampled students scored less than 2 on the AP Calculus BC exam, but those who took the AP Calculus BC test scored higher than or equal to 700 on SAT mathematics. Also, half (50%) of these students attained a score of 3 on the AP Calculus BC exam.

A gamma coefficient of 0.905 implies a strong correlation between AP students' Calculus BC 2017 and AP students' SAT mathematics 2017.

By using the gamma coefficient, t ≈ 3.442. The t-table gives the critical value for t (2-tailed) as 2.365. Because 3.442 is greater than the critical value, we can reject the null hypothesis. We expect that AP students' Calculus BC 2017 and AP students' SAT mathematics 2017 are positively correlated.

Table 37. Data Summary of AP Comparison 5 for 2017

AP SAT mathematics	AP Calculus BC 2017					Total Number of Students
	1	2	3	4	5	
700	-	1	-	-	-	1
730	-	-	1	-	-	1
740	-	-	1	-	-	1
750	-	-	1	-	-	1
760	-	-	-	1	-	1
770	-	-	1	-	-	1
790	-	-	-	-	2	2
Total	-	1	4	1	2	8

AP Calculus BC 2017 vs AP Calculus AB 2017. Table 38 summarizes the scores of the AP 2017 graduates on the AP Calculus BC and AP Calculus AB tests. As we can see from Table 38, not one of the students who took the AP Calculus BC test scored lower than 3 on the AP Calculus AB test. 50% of them received a moderate score of 3 on the AP Calculus AB test.

Table 38. Data Summary of AP Comparison 6 for 2017

AP Calculus AB 2017	AP Calculus BC 2017					Total Number of Students
	1	2	3	4	5	
1	-	-	-	-	-	-
2	-	-	-	-	-	-
3	-	1	3	-	-	4
4	-	-	1	1	-	2
5	-	-	-	-	2	2
Total	-	1	4	1	2	8

The gamma coefficient here is 1, implying an absolute correlation between AP students' Calculus BC 2017 and AP students' AP Calculus AB 2017. Therefore, we can reject the null hypothesis. We expect that AP students' Calculus BC 2017 and AP students' Calculus AB 2017 are positively correlated.

Comparison summary. To summarize the findings, four score pairs are compared for the years 2016 and 2017 and show the same result while two reveal a contrasting result (see Tables 39 and 40).

Table 39. Comparisons of AP 2016

Comparisons 2016	Results
AP GE mathematics 2016 vs AP SAT mathematics 2016	*- moderate correlation* *- not statistically significant*
AP GE mathematics 2016 vs AP Calculus AB 2016	- relatively strong correlation - statistically significant
AP GE mathematics 2016 vs AP Calculus BC 2016	*- very weak correlation* *- not statistically significant*
AP Calculus AB 2016 vs AP SAT mathematics 2016	- relatively strong correlation - not statistically significant
AP Calculus BC 2016 vs AP SAT mathematics 2016	- moderate correlation - not statistically significant
AP Calculus BC 2016 vs AP Calculus AB 2016	- very strong correlation - statistically significant

The following explanations shed some light on the comparison of the 4 pairs that yielded the same result:

I compared each possible pair of grades from each student's four exams (General Examination mathematics grade, SAT mathematics grade, AP Calculus AB, and AP Calculus BC), expecting that there could be a correlated relationship between these variables for the years 2016 and 2017.

Within these four pairs, two of them (the second pair and the third pair) are statistically significant except for the first pair and the third pair.

Table 40. Comparisons of AP 2017

Comparisons 2017	Results
AP GE mathematics 2017 vs AP SAT mathematics 2017	- *relatively strong correlation* - *not statistically significant*
AP GE mathematics 2017 vs AP Calculus AB 2017	- very strong correlation - statistically significant
AP GE mathematics 2017 vs AP Calculus BC 2017	- *very strong correlation* - *not statistically significant*
AP Calculus AB 2017 vs AP SAT mathematics 2017	- strong correlation - statistically significant
AP Calculus BC 2017 vs AP SAT mathematics 2017	- strong correlation - statistically significant
AP Calculus BC 2017 vs AP Calculus AB 2017	- absolute correlation - statistically significant

For the two pairs that are statistically significant, the statistical significance between AP students' GE mathematics and AP students' AP Calculus AB might imply that the general Chinese mathematics helps students build a solid foundation for learning the AP Calculus AB course. Moreover, Chinese instructor W's interview question 2 comment that "general Chinese mathematics is a great foundation for learning the AP mathematics course" also helps to clarify the connection between AP students' General Examination mathematics grades with their corresponding and AP Calculus AB grades.

Secondly, to understand the last pair, which includes students' AP Calculus BC grades and their corresponding AP Calculus AB grades, we note that AP Calculus BC is an extension of AP Calculus AB with some difference in scope (AP Calculus AB Frequently Asked Questions, 2018). Therefore, if students can handle a test with richer topics like the AP Calculus BC exam, then we can expect them to perform better in AP Calculus AB, which contains approximately 60% of the same topics as AP Calculus BC.

The lack of statistical significance in the correlation between AP students' SAT mathematics and AP students' GE mathematics might be because these two kinds of examinations focus on different aspect of the acquired mathematical knowledge. The lack of statistical significance in the correlation between AP Calculus BC and AP students'

GE mathematics might be because AP Calculus BC is an extension of AP Calculus AB with some difference in scope:

> AP Calculus AB includes techniques and applications of the derivative, the definite integral, and the Fundamental Theorem of Calculus. It is equivalent to a semester of calculus at most colleges and universities. AP Calculus BC includes all topics in AP Calculus AB, plus others such as parametric equations, polar, and vector functions, and series. It is equivalent to one year of calculus at most colleges and universities. (AP Calculus AB Frequently Asked Questions, 2018)

Also, only 14 students (27.45%) took the AP Calculus BC test in 2016, and only 8 students (29.63%) took the AP Calculus BC test in 2017, leading to a small sample size. Without further study, we cannot be certain as to why there is no demonstrable and significant correlation between AP Calculus BC and AP GE mathematics.

It was surprising to discover while comparing both sets of data from the third pair (AP General Examination mathematics with AP Calculus BC) that no one achieved a grade of A on the General Examination. The average level of students' AP General Examination mathematics score also decreased from 1.92 (in 2016) to 1.29 (in 2017). Both AP Chinese instructors mentioned in their answers to interview question 9 that since the general Chinese mathematics course is not very important to the students in the AP Center, students do not have positive attitudes and only want to pass the General Examination. General Chinese high school students seem to approach the exam with greater motivation. Perhaps students' lack of motivation and effort concerning the General Examination led to lower scores.

On the other hand, for the 2 pairs of comparisons for the years 2016 and 2017, which yielded contrasting results (see Tables 39 and 40), we could suggest the following explanation:

The statistical insignificance between AP Calculus AB/BC and AP students' SAT mathematics might imply that the topics in SAT mathematics are not related to the topics in AP Calculus. Probably the test scope of the SAT mathematics and AP Calculus was more similar in 2017 than it was in 2016. Also, since the entire sample size decreased from 51 in 2016 to 27 in 2017, these possible factors might help to explain the corresponding performance in the AP Calculus AB/BC examination and the SAT mathematics test.

From the data (Appendix K), the overall level of students' AP Calculus BC score decreased from an average of 4.2 (in 2016) to an average of 3.5 (in 2017). And student AP Calculus BC scores that were higher than or equal to 4 decreased from 71.43% (2016) to 37.5% (2017). The overall level of students' SAT mathematics scores decreased from an average of 735.69 (in 2016) to an average of 727.78 (in 2017). Students' SAT mathematics scores that were higher than or equal to 750 decreased from 52.94% (2016) to 33.33% (2017).

Additionally, from Table 27 and Table 33, we can see that in 2016, no student scored higher than a B on the General Examination in mathematics, but a majority (80.39%) of them scored within a high range (700 to 800) on the SAT mathematics test. However, in 2017, no student scored higher than a C on the General Examination mathematics, and only two thirds (66.67%) of them scored within a high range (700 to 800) on the SAT mathematics test. As is shown in Tables 29 and 35, in 2016, students scored higher than or equal to a 3 on the AP Calculus BC exam, but in 2017, students scored only higher than or equal to a 2 on the AP Calculus BC exam. As Tables 31 and 37 show, in 2016, students who took the AP Calculus BC test all scored higher than 700 in SAT mathematics and half of them got the highest score (5) on their AP Calculus BC test. In 2017, those who took the AP Calculus BC test scored higher than or equal to 700 on SAT mathematics and half of them achieved the moderate score at 3 on the AP Calculus BC exam. As Tables 32 and 38 reveal, a majority (64.29%) of the students who took the AP Calculus BC test in 2016 received the highest score of 5 on the AP Calculus AB test, but only 25% of students who took the AP Calculus BC test in 2017 achieved the highest score (5) on the AP Calculus AB test.

Moreover, one of the instructors from the AP Center felt that students' mathematical ability has decreased (according to the answer to interview question 17). The data from this study seem to suggest that the mathematical ability of students in the AP Center decreased from 2016 to 2017. However, further study is needed to gather enough evidence to demonstrate that students in 2016 were better overall than students in 2017 since the data base is not big enough.

Research Question 3

> *What are the differences and similarities in curriculum and student performance between IB (International Baccalaureate) program and the intended Chinese high school program in mathematics education?*

Curriculum Comparison in the IB Center

I conducted the comparison based on the following materials downloaded from the official website of the International Baccalaureate Organization (IBO): International Baccalaureate Diploma Programme Subject Brief: Mathematics–Standard Level (Appendix E), and the International Baccalaureate Diploma Programme Subject Brief: Mathematics–Higher Level (Appendix F). Also, I collected the following textbooks that are used by this IB Center: *New Additional Mathematics* published by Marshall Cavendish Education (for Pre-IB) (Appendix M), *Mathematics-Standard Level* 2012 Edition published by Pearson (for IB SL) (Appendix N), and *Mathematics-Higher Level* 2012 Edition published by Pearson (for IB HL) (Appendix O). In addition, since this IB Center chose Topic 9 (Option: Calculus) as the extra optional topic, and that includes more calculus-related topics with further depth than the calculus content in Topic 6, I also included the syllabus contents from the book used by this school: *Mathematics Higher Level Topic 9-Option: Calculus for the IB Diploma* published by Cambridge University Press (Appendix P). In total, there are 6 content materials for IB mathematics, and I compared them with the general Chinese high school mathematics test contents (Appendix G).

According to these content materials, and based upon the description from the IBO official website, both IB SL and HL consist of the same educational aim, core syllabus, curriculum, and assessment model. HL typically includes some additional contents designed to allow students to explore in more depth and have an extra optional topic (in this case: Topic 9–Calculus). Therefore, I combined them together, and Table 41 lists the curriculum model overview for both IB SL and HL of the IB mathematics.

While making the comparison between the IB Mathematics curriculum and the general Chinese high school mathematics curriculum, I used the general Chinese high school mathematics test contents as a reference as well. Overlapped contents are marked with colors in Appendix L as well.

Table 41. IB Mathematics Component (Standard Level and Higher Level)

IB Mathematics Component (SL & HL)
Topic 1: Algebra
Topic 2: Functions and equations
Topic 3: Circular functions and trigonometry
Topic 4: Vectors
Topic 5: Statistics and probability
Topic 6: Calculus
Topic 9: Option (Calculus)
Mathematical exploration

Differences: Topics only in General Chinese Mathematics. As reflected in the Appendix L, there are some topics that are covered in the Chinese curriculum but not in the IB Mathematics curriculum (see Table 42).

Similarities. In addition to the differences, there exist some similarities. Some contents are present in both Pre-IB Mathematics and general Chinese high school mathematics; I marked these overlapped contents in italics on Appendix L and listed them on Table 43.

In addition, there are many overlapping contents between IB mathematics (SL & HL) and general Chinese high school mathematics; I marked these overlapped contents in bold on Appendix L and listed them on Table 44.

Table 42. Topics Only Covered in the Chinese High School Curriculum

Topics
Table Notation of a Function
Power Functions
Examples of Application of Models
Comprehensive Application of Functions
Structure of 3D Geometry
Three-View Drawing of 3D Geometry
Trimetric View of 3D Geometry
Surface Area and Volume of 3D Geometry
Function of Circle
Positional Relations between Lines and Circles
Coordinates in 3D Space
Linear Inequalities in Two Variables and Simple Linear Programming Questions
Propositions and Their Relations
Sufficient and Necessary Condition
Curve and Equation
Ellipse
Hyperbola
Concepts of Focus and Directrix of Parabola
Simple Geometric Properties of Parabola

Table 43. Similarities between Pre-IB Mathematics and the Chinese High School Curriculum

Topics
Sets
Concepts of Functions
Maximum and Minimum Value of a Function
Angle of Inclination and Slope
Linear Function
Intercept Coordinates and Distance Formula

Table 44. Similarities between IB Mathematics (SL & HL) and the Chinese High School Curriculum

Topics
Definition of Interval and Its Notation
Notations of Functions
Monotonicity and Max (Min) Value
Parity of a Function
Increasing Models of Different Functions
Perspective Projection and Parallel Projection
Positional Relations between Point, Line and Plane in 3D Plane
Test and Properties of Parallel
Test whether Lines and Planes are Orthogonal
Distance between Two Points in 3D Space
Scalar Multiplication and Its Meaning
Theorems and Coordinates of Vector
Dot Product
Application of Vector
Compound Angle Identities
Basic Identities
Sine Rule and Cosine Rule
Solving Triangles Case Study
Definitions and Notations of Sequences
Arithmetic Sequences
Arithmetic Series
Geometric Sequences
Geometric Series
Applications of Sequences
Unequal Relation and Inequalities
One-Variable Quadratic Inequalities and Their Solutions
Fundamental Inequalities
Inequalities with Absolute Values
Definition of Parabola
Standard Equation of Parabola
Space Vector and Its Operations
Vectors in 3D Geometry

The rest of the overlapping contents are common to Pre-IB mathematics, IB mathematics (SL & HL), and general Chinese high school mathematics. I marked these overlapped contents in white letters on Appendix L and listed them on Table 45.

These tables show that the content of IB mathematics is relatively consistent with the content of general Chinese mathematics. Therefore, unlike the AP Center, which has Chinese instructors to teach the intended Chinese mathematics to AP students, the IB

Center does not specifically assign Chinese instructors to teach this part to prepare IB students for the General Examination.

Also, because IB is a diploma program, it covers various topics of mathematics, so IB instructors need only teach IB mathematics, filling in the contents that are unique to the Chinese curriculum as a supplement.

Table 45. Similarities between IB Mathematics (Pre-IB, SL, and HL) and the Chinese High School Curriculum

Topics
Exponential Functions
Logarithmic Functions
Functions and Equations
Angles and Radian Measure
Trigonometric Functions of Any Angles
Induced formula of Trigonometric Functions
Graphs and Properties of Trigonometric Functions
Graph of $y=A\sin(\omega x + \varphi)$
Basic Application of Trigonometric Functions
Definition of Vector
Vector Operation

Overall differences. Besides the similarities, the overall differences are as follows. The first difference is the limiting of calculator use. IB mathematics requires students to learn how to use graphing calculators because this is a part of the assessment format (see Tables 9 and 10). But students are only allowed to use standard scientific calculators when taking the General Examination. It shows the different attitudes toward using technology in practice, one of which was expressed by Instructor D from the IB Center who said in the interview that the IB class is supposed to be more technology friendly, but there still exist some technology restrictions in the Chinese IB program.

The next significant difference is that IB mathematics includes "Mathematics Exploration" (see Table 41), which is an individual exploration in the form of written work that involves investigating an area of mathematics, as an internal assessment. Therefore, by demanding that students write, IB mathematics fosters better student understanding and individual thinking skills in order to prepare them for doing academic research in their college lives. In general Chinese high school mathematics, there is no writing requirement, only solving problems.

The third difference is the breadth and difficulty of mathematics content. By comparing these collected materials, IB Mathematics and general Chinese high school mathematics both contain various kinds of mathematical ideas including algebra, functions and equations, circular functions and trigonometry, statistics and probability. But general Chinese mathematics includes some ideas that are not present in IB mathematics such as Geometry in 3D-Space, Common Logic Terms, and Conic sections and Equations. Also, topics are sometimes set at different levels of study. For example, IB mathematics started to introduce the binomial theorem in the Pre-IB class in Grade 10 while the general Chinese high school mathematics put the binomial theorem in its extended curriculum (Appendix G). Therefore, it was only for those general Chinese students who needed to take the College Entrance Examination. So, IB Mathematics deemed it prior knowledge or basic learning background, but general Chinese high school mathematics set it as an extended and optional part in its curriculum.

In addition, some college-level contents like matrices and integration are not included in the contents of general Chinese high school mathematics. Even the extended curricula (Appendix G) do not cover them. Moreover, although the idea of derivatives is also covered as an optional topic (Option 2-2) in general Chinese high school mathematics, IB students are only required to pass the General Examination, which excludes Options 2-2 and 2-3 (only covering the content from Requirement 1 to Option 2-1). Nevertheless, those non-IB students who need to take the College Entrance Examination must still study some detailed contents related to derivatives in Option 2-2, but that material does not cover ideas such as differentiability and continuity, estimating a derivative (numerically, graphically), derivative of the inverse of a function and implicit differentiation, and L’Hôpital’s Rule, as the IB curriculum does.

To sum up, IB Mathematics is more technology- and writing-oriented, challenging, and broad than general Chinese high school mathematics.

Students' Performance Comparison in the IB Center

Data comparisons 2016.

IB mathematics (all HL) 2016 vs IB SAT mathematics 2016. Table 46 summarizes IB 2016 graduates' scores on the IB mathematics (all higher level) examination and their mathematics grades on the SAT test. No one scored less than 2 on the IB mathematics HL test, and students' SAT mathematics scores are all above 600. A large majority (92.11%) of them achieved SAT mathematics scores higher than 700.

A gamma coefficient of 0.245 implies a moderate correlation between IB students' mathematics HL 2016 and IB students' SAT mathematics 2016.

By using the gamma coefficient, $t \approx 0.889$. The t-table gives the critical value for t (2-tailed) as 2.026. Because 0.889 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that IB students' mathematics HL 2016 is positively correlated with IB students' SAT mathematics 2016.

Table 46. Data Summary of IB Comparison 1 for 2016

IB SAT mathematics	IB mathematics (all HL) 2016							Total Number of Students
	1	2	3	4	5	6	7	
680	-	-	1	1	-	-	-	2
690	-	-	1	-	-	-	-	1
710	-	-	-	1	-	-	-	1
720	-	-	1	3	1	-	-	5
730	-	-	-	-	-	1	-	1
740	-	1	2	2	1	-	-	6
750	-	-	-	1	1	-	-	2
760	-	-	-	1	1	-	-	2
770	-	-	-	-	1	-	-	1
780	-	-	1	1	1	-	-	3
790	-	-	-	2	2	-	1	5
800	-	1	2	2	2	1	1	9
Total	-	2	8	14	10	2	2	38

Data comparisons 2017.

IB mathematics (HL) 2017 vs IB SAT mathematics 2017. Table 47 summarizes the scores of IB 2017 graduates on the IB mathematics higher level (HL) examination and their mathematics grades on the SAT test. Notably, no students scored less than a 2 on the IB mathematics HL test, and all (100%) students' SAT mathematics scores are within the highest range (700 to 800).

A gamma coefficient of 0.5 implies a relatively strong correlation between IB students' mathematics HL 2017 and IB students' SAT mathematics 2017.

By using the gamma coefficient, t = 1.6. The t-table gives the critical value for t (2-tailed) as 2.064. Because 1.6 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that IB students' mathematics HL 2017 is positively correlated with IB students' SAT mathematics 2017.

Table 47. Data Summary of IB Comparison 1 for 2017

IB SAT mathematics	IB mathematics (HL) 2017							Total Number of Students
	1	2	3	4	5	6	7	
760	-	1	2	2	1	-	-	6
770	-	-	-	2	1	1	-	4
780	-	-	1	1	-	-	-	2
790	-	-	2	2	2	1	1	8
800	-	-	-	1	1	2	1	5
Total	-	1	5	8	5	4	2	25

IB mathematics (SL) 2017 vs IB SAT mathematics 2017. Table 48 summarizes IB 2017 graduates' scores on the IB mathematics standard level (SL) test and their mathematics grades on the SAT test. From the table, we can see that no one scored less than a 2 on the IB mathematics SL test. And students who took the IB mathematics SL test have corresponding SAT mathematics scores higher than 600. A majority (75%) of them received SAT mathematics scores higher than 700.

A gamma coefficient of 0.478 implies a relatively strong correlation between IB students' mathematics SL 2017 and IB students' SAT mathematics 2017.

Table 48. Data Summary of IB Comparison 2 for 2017

IB SAT mathematics	IB mathematics (SL) 2017							Total Number of Students
	1	2	3	4	5	6	7	
690	-	1	-	-	-	-	-	1
700	-	-	-	-	1	-	-	1
740	-	-	-	-	1	-	-	1
760	-	-	-	-	-	-	1	1
770	-	-	-	1	-	1	-	2
780	-	-	-	-	-	1	-	1
790	-	-	-	-	-	1	-	1
Total	-	1	-	1	2	3	1	8

By using the gamma coefficient, t ≈ 0.923. The t-table gives the critical value for t (2-tailed) as 2.365. Because 0.923 is less than the critical value, we cannot reject the null hypothesis. There is not sufficient evidence that IB students' mathematics SL 2017 is positively correlated with IB students' SAT mathematics 2017.

Comparison summary. Thus, Table 49 and Table 50 simply compile the results.

Table 49. Comparisons of IB 2016

Comparisons 2016	Results
IB mathematics (all HL) 2016 vs IB SAT mathematics 2016	- moderate correlation - not statistically significant

Table 50. Comparisons of IB 2017

Comparisons 2017	Results
IB mathematics (HL) 2017 vs IB SAT mathematics 2017	- relatively strong correlation - not statistically significant
IB mathematics (SL) 2017 vs IB SAT mathematics 201	- relatively strong correlation - not statistically significant

From Tables 49 and 50, we can see that all pairs might be expected to be correlated, but not all the correlations are statistically significant. Therefore, the IB data might imply that the topics in SAT mathematics are not related to the topics in IB mathematics. Specifically, according to the original database (Appendix Q), 25 (75.76%) of students took the IB mathematics High Level test in 2017 while only 8 students

(24.24%) took the IB mathematics Standard Level test. Without further study, we cannot be certain as to why there is no demonstrable correlation between IB mathematics SL and IB SAT mathematics.

As the original database indicates (Appendix Q), all the students (100%) from the IB Center took the IB mathematics High Level test in 2016, and 75.76% of those students took the IB mathematics High Level test in 2017 while 8 of the students in that year (24.24%) took the IB mathematics Standard Level test. Tables 46 and 47 show that no one in this group scored less than 2 on the IB mathematics HL test in both 2016 and 2017. Also, in 2016, a majority (92.11%) of the students received SAT mathematics scores higher than 700 while all (100%) earned corresponding SAT mathematics scores higher than 700. Additionally, the data show (Appendix Q) that the level of students' IB mathematics HL score increased from an average of 4.21 (in 2016) to an average of 4.48 (in 2017). Students' IB mathematics HL scores that were higher than or equal to 5 increased from 36.84% (2016) to 44% (2017). The level of students' SAT mathematics scores increased from an average of 758.16 (in 2016) to an average of 773.33 (in 2017). Students' SAT mathematics scores that were higher than or equal to 750 increased from 57.89% (2016) to 90.91% (2017). The data from this study seem to suggest that the mathematical ability of students in the IB Center improved from 2016 to 2017. But without further study, there is not enough evidence to demonstrate that students in 2017 were actually better overall than students in 2016 since the data base is not big enough.

Research Question 4

What are mathematics instructors' perspectives on teaching imported AP (Advanced Placement) and IB (International Baccalaureate) programs in China; also, what are their expectations and suggestions for curriculum innovations in Chinese mathematics education?

Interviewees' Background

Table 51 provides the basic information of interviewees' background.

Table 51. Interviewees' Background Information

<table>
<tr><th colspan="2" rowspan="2">Interviewee Category</th><th colspan="3">Number of Interviewees</th></tr>
<tr><th>Chinese</th><th>Non-Chinese</th><th>Total</th></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>1</td><td>5</td><td>6</td></tr>
<tr><td>Female</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td rowspan="2">Secondary Education</td><td>Standard</td><td>2</td><td>6</td><td>8</td></tr>
<tr><td>International</td><td>-</td><td>1</td><td>1</td></tr>
<tr><td rowspan="3">Former Teaching Experiences</td><td>Standard</td><td>2</td><td>3</td><td>5</td></tr>
<tr><td>International</td><td>-</td><td>2</td><td>2</td></tr>
<tr><td>Both</td><td>-</td><td>2</td><td>2</td></tr>
</table>

There are 9 interviewees in total and 2/3 of the interviewees are male mathematics instructors whereas 1/3 of the interviewees are female. Nearly 90% (88.89%) of the interviewees attended a standard school for secondary education, and only one interviewee went to an international school (in Kenya with a British curriculum similar to an A-Level system). This experience also was the reason that the interviewee decided to teach in an international school and go overseas. Both Chinese instructors taught in standard school for 5 years before they started teaching in this AP Center. These non-Chinese instructors come from all around the world, including Canada, the United States, Italy, South Africa, Kenya, Pakistan, and Britain. Among those instructors, 5 of them (71.4%) have taught in standard school, and 4 of them (57.1%) have taught in an international school before. Two of the non-Chinese instructors have experiences teaching in both kinds of schools. The multi-cultural backgrounds of these instructors with their own local standard high school education experiences and their diverse teaching experiences bring global teaching resources to the AP Center and the IB Center in Ningbo.

The Reason for Teaching AP/IB in China

Table 52 demonstrates the factors that contributed to instructors' decisions to teach in the imported AP program or IB program in China.

Table 52. The Reason for Teaching AP/IB in China

Factor	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Environment	New teaching environment and new life overseas	"need new environment to work with young people"; "want to live overseas"	3	3	6
Interest	Personal interests of academic and career preferences	"learned math as a major"; "want to move to secondary education"; "met few friends who are teaching in China now, think it is exciting to teach here"	3	2	5
Finance	High salary	"my research project is waiting for funding"	2	1	3
Experience	Experience of different culture and teaching	"want to go cross culture and city"; "international education becomes more popular these years, want to see if there is any difference"	2	1	3

Chinese instructors show a high consistency in that they both came to the AP Center thinking that a new teaching environment that is different from the general Chinese high school curriculum would be very attractive. And the environmental factor was mentioned the most: 6 times out of the 9 interviewees (66.7%). They all stated that going overseas and exploring a new teaching environment motivated them to make the decision. Also, personal interest including career and academic preferences was cited by 55.6% (5 out of 9). Financial advantage and multi-cultural experience accounted for 33.3% (3 out of 9) of the answers.

Differences Noticed as Mathematics Instructors

Differences between teaching mathematics in the past and present. In question 17, interviewees shared the difference between their experiences in teaching mathematics today and years ago. The answers can be seen from three aspects: teacher, teacher-student, and student.

Teacher. Table 53 shows the differences that noticed by mathematics instructors between teaching mathematics in the past and present from teacher's aspect.

Table 53. Differences Between Teachers' Abilities in the Past and Present

Factor	Quote	Number of Interviewees		
		AP	IB	Total
Changed Teaching Methods	"try something new or experimental to teach"; "with technology, things are really changed"; "when I learned the inquiry-based constructivism teaching, my teaching changed"	4	2	6
Improved Teaching Skills	"be more efficient to finish everything in class and try not take much of students' after-class time"; "maintain more active now and proactive in the material"	4	1	5

From the first aspect, there are two categories: the improvement of teacher's professional skills and the change in teaching methods. 6 out of 9 (66.7%) instructors mentioned that their teaching methods have changed. For instance, instructors N and M both tried to use different strategies for teaching different materials; instructor B switched from "problem-based" to "inquiry-based" teaching; instructor W tended to stand on students' sides while designing teaching content and method; instructor S became more explicit in teaching because that instructor taught college students in the past and was concerned that the practice of giving hints might cause high school students to lose track. Meanwhile, 5 out of 9 (55.6%) instructors said that their teaching skills had been enhanced. For example, instructor W became more efficient in finishing everything during class instead of taking students' after-class time; and instructor M observed and then emulated many other teachers who were teaching using the internet and class discussion; instructor N became more active and proactive in the material.

Teacher-student. Regarding the second aspect, three instructors (33.3%, all AP non-Chinese) had smoother communication with students through more interaction and engagement.

Student. With respect to the last aspect, one Chinese instructor (11.1%) from the AP center felt students perspective has worsened. "First year graduates are the best, now the students' attitude has become worse since they do not pay attention to general Chinese mathematics course but only want to pass the General Examination. Even getting a B is not very common right now," said instructor J.

Differences between teaching in an international school and a standard school. In their answer to question 9, interviewees pointed out many differences that they noticed between teaching in an international school and a standard school.

Both Chinese instructors gave the same response: that there are two major differences (see Table 54).

Table 54. Chinese Instructors' Perception of Differences between Teaching in an International School and a Standard School

Difference	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Student Attitude	Student attitude of learning the general Chinese course in an international school and a standard school	"general Chinese math courses is not that important to them"; "just want to pass the General Examination"; "do not have enough good attitude"	2	0	2
Teaching Arrangement	Arrangement of teaching the general Chinese course in an international school and a standard school	"AP Chinese students spend 1.5 years until attend the General Examination, and general Chinese students have 3 years to attend College Entrance Examination"; "less homework"	2	0	2

First is student attitude. Since the general Chinese mathematics courses is not as important to AP students and since they have AP mathematics courses to pursue at the same time, students do not have a positive attitude; they only want to pass the General Examination. But students who are taking the general Chinese mathematics courses in a standard school regard the those courses as their top concern and work very hard to try to do their best on the College Entrance Examination.

Second is the teaching arrangement. For AP students, it takes them one and a half years before they take the General Examination; it is easier and the students' mathematics challenge is relatively lower; by contrast, for general high school students in China, they have three years to prepare for the College Entrance Examination and the expectations of them are higher, so their mathematics ability is better than those AP students. Table 55 indicates the thoughts of non-Chinese instructors.

Table 55. Non-Chinese Instructors' Perceptions of Differences between Teaching in an International School and a Standard School

Difference	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Class Size	Numbers of students in a class	"international school has much more smaller class size, 30 people is the largest; standard school usually has 40 to 50 people"	2	0	2
Cost	Studying expenses	"parents pay a lot of money to get into that program"	0	1	1
Student Personality	Students' Discipline	"kids in standard school are nicely disciplines but international school is for who is who"	0	1	1
Culture	Culture difference	"culture difference makes contextual link to international students very different, they have different familiar things"	1	0	1
School Aims	Emphasis of school	"In South Africa, majority do not have much interest in education and like stories such as dropping out from high school and get success. China regards education as a very important thing"	1	0	1
Language	English requirement	"in China, English is the second language, every teacher becomes a language teacher"	0	1	1
Diversity	Diversity of students' background in a school	"more in the makeup of diversity of students' body"	0	1	1

Two non-Chinese instructors said that an international school usually has a smaller class size (about 20 to 30 people) than a standard one, and that allows for more creative discussion. Instructor E talked about the high cost, but said that teachers do not pay more attention to students just because their parents paid a lot of money. Instructor D thought that unlike standard schools' students who are well disciplined, international students have more personality. Instructor N said there exists a cultural difference in students' familiar things which makes it challenging to relate mathematical principles to the daily lives of diverse students and also said that China regards education as a more important thing than some countries like South Africa. Instructor S thought international school aims at having students understand concepts while standard school usually uses rote learning which involves repetition and memorization. Instructor B said that by teaching in an international school every teacher becomes a language teacher. Also, usually the student body at an international school is more diverse than in a standard school, but it depends on the school's location. Here at the Ningbo IB Center, there is no diversity.

Differences between the general Chinese mathematics and the AP/IB mathematics. Concerning the second question, although two of the non-Chinese instructors and one of the Chinese instructors said that they do not know much about systems other than the ones that they teach in, most of them still gave some response.

***Designed curriculum*.** From the designed curriculum aspect, instructors think that the academic difficulty of mathematics in the imported programs, including the IB (HL) and the AP, is higher than or equal to general Chinese mathematics based upon the topics and contents, and the IB (SL) is easier than the best of general Chinese mathematics. Two non-Chinese instructors from each program believed that the difficulty of the mathematics programs is the same or similar. As instructor B replied,

> General Chinese mathematics education is a high-quality program and IB also produces high-quality if you take HL. IB has different levels of mathematics that can be taken, there are easier options for students are not up to the quality of the HL or the best of the general Chinese mathematics.

In addition, non-Chinese instructor S thinks AP mathematics has more advanced topics which are like those of first-year college courses for tenth or eleventh grade students. And Chinese instructor W's answer offered the opinion that general Chinese mathematics is a great foundation for learning AP mathematics, which will help the learning of SAT mathematics and AP calculus and so on; another Chinese instructor, J, also mentioned this point in his answer to question 7.

Student mathematical ability. From the student mathematics ability perspective, two non-Chinese instructors from the IB program said that Chinese mathematics seems to aim at teaching mathematical skills by having students learn by repeating, so "their algebra skills are way beyond students in other countries such as Canada" and another instructor, N, from the AP program agreed, saying, "Students' mathematics ability is more advanced than students in other systems for the same age group." However, "application is relatively weak," said IB instructor E.

Classroom style. Respecting the classroom style, one of the AP non-Chinese instructors thinks teaching is much less student-centered in general Chinese mathematics, "students are way more passive and tend to accept information rather than doing their own research." But another AP non-Chinese instructor said that is true across the board, and it is common because in aiming for the exam, even the AP program has intense curriculum, and students do not have the time needed to fully digest some ideas.

In question 15, I asked if the interviewee's classroom style is more teacher-centered or student-centered. Both Chinese instructors said it should be teacher-centered, which lent support to the AP non-Chinese instructor's idea that was mentioned before. One of them said that teachers just tell students how to solve the problem directly when they do not know, and the students barely ask questions.

All of the non-Chinese instructors chose a combination of both classroom styles. Three of them (42.86%) said it would be half and half, and two of them (28.57%) articulated a 60% teacher-centered and 40% student-centered combination preference.

Just as instructor E said in question 6, “The ideal goal is more student-centered, and participation is important, but real practice is the opposite and cannot be implemented completely, so it is half and half now.”

Although reality somehow limited the classroom style, many instructors still made efforts to make it closer to their ideal. Three of them (42.86%) wanted to try as often as possible to be more student-centered by doing group activities, talking a lot, and having strong relationships with students so they are not afraid to engage. “Even if some topics must be teacher-centered, I always try to break it down to involve students as much as possible,” said instructor M. And instructor N responded as follows: “I don’t want to be teacher-centered but students are very conservative and too shy to ask if they don’t know and speak if they do know, they are used to waiting for the next instruction.” But another instructor, B, gave the reason for a half and half combination: most students in an internal study are not prepared for a completely student-centered classroom (see Table 56).

Language challenge. The last factor pointed out by the interviewees is the language challenge. This factor also mentioned by Chinese instructor W in the answer to question 4. Surprisingly, for question 14 that is designed especially for non-Chinese instructors asking about their observation of student performance when learning mathematics in a bilingual environment, 6 out of 7 of them (85.7%) explicitly stated that the language challenge is the biggest struggle they notice.

First, weak English vocabulary ability makes some students have trouble understanding some wordy questions; they do not know what they are supposed to do. Sometimes it is not the mathematics that students do not understand but the language. An example, given by instructor E, was that when students were asked to “valuate” something, they did not know that “valuate” means; or to calculate the “sum,” but they do not know the meaning of “sum.” And instructor B also told me a story that happened when teaching trigonometry: “The problems is, you’ve given certain measurements of the sides of a shed, and the pitch of the roof. Then every student had a question: What is a shed?"

Table 56. Mathematics Instructor's Classroom Style Preference

Classroom Style	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Teacher-Centered	With more lectures and teachers asserts control over the material	"but I think it should be teacher-centered"	2	0	2
Student-Centered	With more discussion and activities to let students explore on their own	none	0	0	0
Combination of Both	Combine both teacher-centered and student-centered classroom styles depends on situation in real practice	"somewhere in between, most students in an internal study are not prepared for a completely student-centered classroom"; "half and half, most of things they have to do it on their own, I guide them, but sometimes I have to take the lead"	4	3	7

Second, academic terms can also get students confused. Since the notation or definition of specific terms is different, instructor S gave an example when talking about a constant. In AP mathematics, slope is "m," but in China, slope is "k"; this causes constant confusion in AP. This kind of conflict of content (what is used in China is not the same as in the imported program) was also mentioned by instructor M in question 6. In AP mathematics, $y = mx + c$, but in general Chinese mathematics, $y = kx + b$.

Therefore, instructor B believed there has to be a very big emphasis put on language learning, especially the academic language. Meanwhile, some instructors think it is not a big issue; instructor M thinks it is a nice experience for them to learn more and instructor E said that for those students with better language backgrounds, the situation improves after the first year.

Teachers' Thoughts on Chinese Mathematics Learning and Teaching

The general weakness in Chinese mathematics learning and teaching. Some features of general Chinese mathematics have already been mentioned. The next question for every instructor is about the general weakness in Chinese mathematics learning and teaching.

Learning. Table 57 simplifies the result of teachers' thoughts of the general weakness of Chinese mathematics learning. More than half of the non-Chinese interviewees (5 out of 7) said that general Chinese mathematics tends to be dominated by rote learning. AP non-Chinese instructor M said that the way students are taught is not the same as AP mathematics students are taught: "They know the concept but they cannot apply the concept to other problems that are related." Students in general Chinese mathematics learned to memorize formulas and use these for specific problems while AP mathematics gives students a concept and encourages them to use this idea to solve similar problems.

Instructor E and instructor N also noticed the same thing because Chinese students are very good at solving familiar problems but hesitate to try different approaches. When students encounter problems that they have seen before, they rarely make mistakes; otherwise, their performance is not as good. Instructor B replied as follows: "Current research shows that students do not remember for a long time period if they are taught by memorization. Students will learn longer if they have to figure out the process in their own minds."

Secondly, two non-Chinese instructors and one Chinese instructor noticed that students are lacking of independent thinking skills. They said that most students expect the teacher to tell them everything and to tell them what to do; they are not interested in understanding why but in knowing how to solve the problem.

Table 57. General Weaknesses in Chinese Learning

Learning	Definition	Quote	Number of Interviewees			
			AP		IB	Total
			Chinese	Non-Chinese	Non-Chinese (only)	
Rote-Learning	Memorize by repetition and cannot apply the concept to other problems that are related	“it tends to be memorization rote-learning”	0	3	2	5
Independent Thinking Skills	Students can manage their own studies autonomously without the support from teachers	“students do not have much time to think independently”; “expect teacher to tell them what to do”	1	1	1	3
Mathematical Skills	Various abilities to learn mathematics	“poor geometric intuition”; “quick fast at computing”; “good at fractioning and calculating”	0	1	1	2

Third, instructor L found out that students are quite fast at computing, and instructor E also mentioned that they are good at working with fractions and calculating. However, although students have outstanding algebra ability, instructor L thought their geometric intuition is relatively poor.

Teaching. Table 58 simplifies the result of teachers’ thoughts of the general weakness of Chinese mathematics teaching.

Five interviewees, including two Chinese instructors, thought general Chinese mathematics is generally teacher-centered with lecture-based classes. This point has been discussed before, and here both Chinese instructors described the teaching as “duck-stuffing” or infusion education; in other words, teachers “stuff students like Peking duck.” Chinese teaching style is often described as lecture-based cramming-style teaching in which students learn by passively accepting contents and then try to memorize to absorb as much as they can.

Table 58. General Weaknesses in Chinese Teaching

Teaching	Definition	Quote	Number of Interviewees			
			AP		IB	
			Chinese	Non-Chinese	Non-Chinese (only)	Total
Teacher-Centered	Lecture-based classes and teachers asserts control over the material	"lecture style in order to pass exam"; "students learn more contents in unit time than students from other countries"	2	1	2	5
Duck-Stuffing	Students learn by passively accepting contents through lecture-based cramming teaching, followed by memorization	"Duck-stuffing type of teaching"; "generally just infusion education"	2	0	0	2
Technology	Students' access to technology and classroom's facilities	"students are not supposed to bring laptop in school, afraid that they might do not use it properly"; "not enough software, low-speed internet"	0	0	1	1

Chinese instructor W said, "Students' interaction with the teacher is based on specific contents." And non-Chinese instructor S said that it is a human thing, not a Chinese thing to use the lecture style to help students pass the exam. Non-Chinese instructor D felt that students did not have enough access to technology and that the internet signal was poor, so it takes students' personal time to do some technical work at home. Instructor D also said, "students are not supposed to bring laptop in school to avoid using it improperly, but I personally believe it is useful."

Advice to help innovation in Chinese mathematics learning and teaching. Chinese instructors did not offer suggestions for change, but non-Chinese interviewees came up with various advice to help innovation in Chinese mathematics education (see Tables 59 and 60).

Learning. Instructors M and S suggested curriculum development to motivate student thinking to be able to apply mathematics concepts to other related problems and development of lesson plans that requires student interaction (although logistically there may be a challenge to that because it takes longer preparation time). Instructor L would push students to do independent research to improve their independent thinking skills, and instructor E said that "addressing more critical thinking and developing more thinking skills is important, training them not just to be good at working with fractions or calculating" and suggested that students accept and learn from their mistakes, which was also mentioned by instructor L in the response to question 7.

Table 59. Advice Concerning Innovation in Chinese Mathematics Learning

Learning	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Curriculum Development	Make some changes of the school curricula	"curriculum development to motivate students to think to be able to apply the concept to other problems that are related"; "the amount of lesson planning required is much larger, so logistically maybe there is a challenge to that"	2	0	2
Learn by Mistake	Let students accept being imperfect	"not let students think making mistakes is so bad, it's good to learn from mistakes"	1	1	2

Table 59 (continued)

Learning	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Independent Research	Let students do research individually with little or no supervision	"push students into independent research"	1	0	1
Thinking Skills	Foster students' comprehensive mental abilities to process and make connections of knowledge	"address more critical thinking and develop more thinking skills is important"	0	1	1

Teaching. Regarding teaching, IB instructor B, whose doctoral dissertation research is also about this specific topic, recommended teaching with an inquiry-based method. And instructor L also referred to student-centered classes. In addition, instructor D hoped to have a better technology-friendly teaching environment. Every interviewee agreed that their advice was inspired by the experience of teaching in imported programs. Although neither Chinese educator gave suggestions, Chinese instructor J explained that "traditional teaching has not been changed significantly for years. To expect to see a big innovation in a short time is impractical." It was also mentioned by non-Chinese instructor E that "the advice will not cause significant change to Chinese high school education in the short term, say for a few years."

Table 60. Advice Concerning Innovation in Chinese Mathematics Teaching

Teaching	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Inquiry-Based	Start teaching by giving questions, problems or scenarios instead of simply presenting knowledge	"recommend they investigate teaching with inquiry-based method"	0	1	1
Student-Centered	With more discussion and activities to let students explore on their own	"having student-centered classes"	1	0	1
Technology	Students' access to technology	"allow to use technology"	0	1	1

Supports and Activities Recommended for Students' Mathematics Learning

Supports of students' mathematics learning. During the discussion of interviewees' thoughts about support for students' mathematics learning, I could observe an obvious difference between Chinese instructors and non-Chinese instructors. Basically, both Chinese instructors mentioned that students do not pay much attention to the general mathematics courses of the AP program and instead only aim to pass the General Examination, which means getting a grade better than D. The general Chinese high school students spend less time with a negative attitude. Therefore, instructors showed a corresponding conservative support. Instructor J decided to use a pass-fail system to motivate them and encourage them that learning general mathematics course curriculum better is helpful for their future learning in calculus. And instructor W suggested "just offering help when they ask questions." Non-Chinese instructors, however, offered various support not only in class but also out of class (see Tables 61 and 62).

In-class supports. When teaching in class, two IB non-Chinese instructors said they usually differentiate instruction based on varying levels of students. For example, instructor E said that for students who show strong interests in mathematics, it is good for them to try more challenging problems but for weak students the teacher should just make sure they can handle the test.

Two instructors (28.6%) used group discussion to increase student engagement. Two instructors (28.6%) let students help each other with language and coursework. Since students are facing significant language challenge, those instructors said they would allow students with advanced English to translate specific mathematical ideas into Chinese for other students so that they can raise the class's efficiency. Instructor M made a complementary viewpoint that having stronger students explain hard ideas to weaker students is better than the teacher explaining it. Instructor N suggested visual learning to help with understanding, while instructor S chose to involve more games so that students can see the value in the mathematical concepts more clearly. In addition, IB instructor D guided students and allowed them to ask whatever questions they wanted to ask, "…and they do ask anything they want; they have no fear of asking questions … unlike AP students, IB students are directly taught by non-Chinese teachers after getting enrolled."

Table 61. In-Class Supports Recommended by Non-Chinese Instructors

In-Class Support	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Differentiating Instruction	Differentiating instruction for the levels of students	"students who really interested in math, give them more challenging problems; for weak students, make sure they can handle the test"	0	2	2
Group Discussion	Separate students into groups to bring up ideas, solve problems or give comments	"encourage group discussions, 40 minutes' full lesson, usually once a week"	2	0	2
Mutual Help	Let students help each other especially in language issues	"let students with advanced English to translate specific math idea in Chinese to other students"; "sometimes allow students to explain in Chinese to avoid language issue"	2	0	2
Visual Learning	Use graphs, charts and diagrams to give students an impressive visual presentation	"visual learning to help with translation"	1	0	1
Games	A fun way for students to review their understanding of learning content	"trying to do more games, they can see the value in it more clearly"	1	0	1
Encourage Questioning	Encourage students to ask questions	"allowing them to ask whatever they want to ask"	0	1	1
Guidance	Assist and advice students about how to solve problems	"guiding them instead of telling them what to do step by step"	0	1	1

That comment surprised me because other instructors, including another IB instructor (interviewee E), mentioned that students barely ask questions but, instead, wait for instruction. Maybe this contradiction happened because of teacher's personal charisma as well as the teaching environment.

Out-of-class supports. For out-of-class support, two instructors (28.6%) provided regular and quality feedback like talking to students individually. One mentioned good assessment and another one offered students time outside the classroom for any question, which matched the support that was offered by Chinese instructor W.

Table 62. Out-of-Class Supports Recommended by Non-Chinese Instructors

Out-of-Class Support	Definition	Quote	Number of Interviewees		
			AP	IB	Total
Feedback	Give comment of students' performance	"talk to student individually"; "provide the regular and good feedback"	1	1	2
Assessment	Provide reasonable evaluation of students' performance	"give good assessment"	0	1	1
Office Hour	Set up regular and pre-arranged time to answer students' questions	"offer them time outside the classroom"	0	1	1

Activities recommended for students' mathematics learning. Based on the results from question 7, we can see a difference between Chinese instructors and non-Chinese instructors. Questions 10 and 11 are designed to elicit detailed explanations of in-classroom and out-of-classroom activities that instructors used to improve students' participation and performance.

In-class activities. For in-classroom activities, both Chinese instructors said they do not create activities very often (see Table 63).

Table 63. In-Class Activities Recommended by Chinese Instructors and Their Effects

In-Class Activity	Definition	Quote	Number of Interviewees			Effect
			AP	IB	Total	
Exercise	Do problem solving to apply knowledge	"mainly do exercises"	2	0	2	Unapparent
Group Discussion	Separate students into groups to bring up ideas, solve problems or give comments	"not very often, once a while do group discussion and it depends on the content"	1	0	1	
Presentation	Let students to make a speech of their group findings	"ask some students to prepare a presentation before class"	1	0	1	

Instructor J mainly lets students do exercises and implements group discussion occasionally, depending on the topic. Instructor J usually uses discussion to share ideas and deepen the understanding of a specific concept when introducing a new topic. But the effect is not obvious, especially since the students do not have much time for the activities. And Instructor W sometimes selects some students to prepare a presentation, but that also depends on specific content as well. The frequency is 2 to 3 times per semester, and it lasts for about 20 mins (half a class). If the topic is easy, then students are selected at random; otherwise, some outstanding students are picked to present the lecture in front of the class. Instructor W believes that presentation is helpful for students to better understand the content, and that others will be inspired too. The effect is fair but some students are not well prepared for the presentation, which leads to repetition of the textbook content without individually extended thoughts to share during the presentation. In addition, instructor W has used a concept summary after each unit, but its effect is not good because most students choose to download some summary from online instead of doing it on their own. Therefore, the general effect of Chinese instructors' in-classroom activities is relatively unapparent not only because of the limitation of opportunity but also due to the lack of independent learning skills.

Surprisingly, according the responses of non-Chinese instructors, the in-class activities they encouraged students to participate in have a 100% positive effect (see Table 64). Instructors described the effect using such phrases as "really-engaged," "students love it," and "students are eager to do it…see them actually communicating in two to three people's group, you can even tell who is the team leader." Looking at these highly effective activities that they used, we can see many kinds. Instructors M and S talked about learner-centered ways to conduct an interactive lecture. Four instructors (57.1%) used group discussions and four (57.1%) used games to engage students; among these interviewees, three of them (75%) used both methods. For group discussion, instructor B said that changing group partners for different activities is good. Instructor N assembled five to seven groups with four people in each group. Instructor M used a full 40-minute lesson for group discussion once per week. When doing games, two instructors implemented the "matching game" as a warm-up activity for five to ten minutes. Instructor B believed games can help students' logical reasoning. For the example given by instructor M, a student was given a function and needed to find the student with that function's graph. Instructor L usually created posters to have students do this. Instructor S mentioned the value of "magic tricks" and wanted to do them every week. And there are three instructors (42.9%) who implemented practical activities such as real-world problems, story-based meaningful questions, and experiments. Instructor E shared the example of giving company's data concerning different batteries and then letting students do statistical analysis. Likewise, instructor B also mentioned "experimental probability." And two instructors (28.6%) mentioned having students make presentations.

In total, since Chinese instructors also mentioned group discussion and presentation methods in their answers, there are five interviewees (71.4%) who chose group discussion and three (42.9%) who used presentations. Therefore, even if the effect of Chinese instructors has not reached the expectation for student involvement, it is still clear that group discussion and presentation are two of the most popular means selected by both Chinese and non-Chinese instructors. Unlike Chinese instructor W, other teachers conducted discussions and presentations after another learner-centered activity. Instructor E used presentations after practical activities, and instructor M scheduled them after group discussions. Instructor M described his procedures: “Each group chooses one member to present after discussion and I encourage them to solve problem on the board to improve confidence and make them feel like they belong to the class.” There are several purposes for creating and implementing such activities. Instructor N and M said that talking can keep students awake and engaged so they do not get bored. Three instructors mentioned their intent to provoke students’ minds to think and learn independently. It is worth mentioning that instructor E specifically stressed “not giving them answers but giving them some time to make their own decisions and present or explain their justifications … everything has to be linked to learning, students can learn things better if they discover them on their own, rather than someone giving to them.” Most of the interviewees (4 out of 7) designed the activities in order to make sure students actually know what they are supposed to know and to help them absorb new concepts and construct knowledge in their minds.

In addition, instructor N mentioned the value of letting students speak with each other in Chinese during interactive activities because conceptual understanding is more important than language.

Instructor D likes to use technology in class when it is allowed. One example is a software called “Tarsia” that can generate a lot of nice activities involving many mathematics topics. Just as instructor N mentioned in question 6, students hesitate to see things from different perspectives, so showing them mathematics through media other than textbooks can be meaningful for them.

Table 64. In-Class Activities Recommended by Non-Chinese Instructors and Their Effects

In-Class Activity	Definition	Quote	Number of Interviewees			Effect
			AP	IB	Total	
Group Discussion	Separate students into groups to bring up ideas, solve problems or give comments	"I can see how they progressing during the group discussion"	3	1	4	Positive
Games	A fun way for students to review their understanding of learning content	"games that will help their logical reasoning"; "do games hopefully every week, like magic tricks"; "matching games as warm-up for five to ten minutes, like this person with a function should look for another person with that function's graph"	3	1	4	
Practical Activity	Students observe or manipulate real materials	"a real-world problem they are working on"; "create story-based and meaningful questions"	1	2	3	
Interactive Lecture	Engage students and allow them to participate in an activity to apply what they have learned	"have interactive lecture to make students more engaged"	2	0	2	
Presentation	Let students to make a speech of their group findings	"not giving them answers but giving them some time to make their own decisions and present or explain their justifications"; "each group choose one member to present after discussion and encourage them to solve problem on the board to improve confidence and make them feel like they belong to the class"	1	1	2	

Out-of-class activities. For out-of-classroom activities recommended by interviewees, Chinese instructors still showed a circumscribed execution (see Table 65).

Table 65. Out-of-Class Activities Recommended by Chinese Instructors

Out-of-Class Activity	Quote	Number of Interviewees		
		AP	IB	Total
No Extra Activity for All Students	"students can behave well in class and pass the general examination is enough"	1	0	1
No Extra Activity for Average Students	"for average students, no extra activities required"	1	0	1
Mathematics Competition for Advanced Students	"for outstanding students, I encourage them to attend math competition"	1	0	1

Instructor J said that for students to behave well in class and pass the General Examination is enough since they mainly focus on their AP mathematics courses and the requirement of the General Examination. Their priorities and focus are, therefore, somewhat different from those of general Chinese high school students. Instructor W partially agreed that for average students, no extra activities are required, but for outstanding students, mathematics competition should be encouraged.

Non-Chinese instructors provided several forms of out-of-class activities (see Table 66). It is noteworthy that one of the non-Chinese instructors, L, shared the thoughts of instructor W. Another non-Chinese instructor, S, also separated students into two levels and agreed that for students of low to medium ability preparation for the test was enough, but those of greater ability could be given real problems in mathematics.

Exploration activities can foster students' logical reasoning and mathematical application ability. Including instructor S, four interviewees (57.1%) proposed such activities for out-of-class enrichment. For example, IB instructor D recommended simple mathematically-based research, which is part of the IB evaluation (20%) and suggested that students should do it on their own. He added, "if they can do small things on small topics, it can really help them, especially, for the ones who think that they cannot speak good English; they can really improve. Also, since they will prepare for about five days and present their arguments in class in mathematical ways and they can pair up with somebody to form a two to three person group, it is good for mathematics, writing, and speaking." Instructor M advocated letting students do projects in real life. Students could choose a topic of their choice, gather data from friends and family, then analyze the data and organize them into graphs. In the end, they needed to make conclusions. This process is very much like what we do in higher education when doing formal research, so it can help them to adapt to college work.

Table 66. Out-of-Class Activities Recommended by Non-Chinese Instructors

Out-of-Class Activity	Definition	Quote	Number of Interviewees		
			AP	IB	Total
No Extra Activity for Average Student	Differentiate students depends on their mathematics abilities	"for weak students, classroom work is most effective"; "students whose performance is off test, then just prepare the test"	2	0	2
Exploration Activity	Let students experience, investigate, collect information, test, make inferences themselves to address logical reasoning	"anything would take logical reasoning"; "give them real problems in mathematics"	2	2	4
Mathematics Competition	Competitive mathematics events for advanced students to participate	"math competitions like Olympiads, Calculus ABC students get forced to attend that"; "for strong students, math competitions are good"	2	2	4
Mathematics Club	Play games involve mathematics contents or create works use mathematics as part of the solution	"chess club to help students to understand math"	0	1	1

Likewise, four interviewees (57.1%) thought mathematics competitions like the Mathematical Olympiad is good for students, especially for those with strong interests and abilities. AP instructor N said that Calculus ABC students are required to attend competitions. Instructor S explained in the answer to question 16 that mathematics competitions are recommended to advanced students because they have very good problems, but students are often not very interested in those competitions since they involve test taking. Furthermore, instructor B referred to groups such as chess clubs to help students better understand mathematics.

Thus, exploration activity and mathematics competitions are tied for the top choices among non-Chinese instructors. If we include the Chinese instructors, then mathematics competitions become the most preferred out-of-classroom activity by one vote.

Methods of Inspiring Students' Mathematics Learning

After discussing activities, interviewees shared ways to inspire students to learn mathematics in question 15 (see Table 67). Chinese instructor J chose to give students answers when they did not know. His approach contrasted with that of the other respondents. Another Chinese instructor, W, decided to use examples from daily life to let students know where they can apply specific mathematical ideas so that they can have a better understanding of application. And this opinion is as same as non-Chinese instructors E and B from the IB program.

In fact, five non-Chinese instructors (71.4%) mentioned the application of mathematics. Specifically, two of them emphasized the application of mathematics in students' future careers. For example, instructor M said that "if you want to learn engineering, then you must be good at calculus. And everybody needs statistical knowledge to make decisions. If you can make sense of the application, they will pay attention" and chose to introduce the application before teaching the corresponding concept. One instructor, L, mentioned that it can be inspiring when relating the mathematics they learn to other subjects such as physics, science and economics.

Table 67. Methods for Inspiring Students' Mathematics Learning

Method	Definition	Quote	Number of Interviewees			
			AP		IB	Total
			Chinese	Non-Chinese	Non-Chinese (only)	
Just Give Answer	Give straight answer without knowing if there is any questions students might have	"just directly tell them how to solve the problems when they do not know, and they barely ask questions"	1	0	0	1
Application of Mathematics	Relate mathematics contents to real-life examples	"make relevant to life content and let students know some math ideas are useful someday in their future career"	1	3	2	6
Share Real Stories	Real-life examples of using mathematics in academic or career fields	"relate something in history of mathematics"; "give them examples of students who were here in the past and show them exactly what is happening and what is possible"	0	1	2	3
Challenging Problems	More competitive and complex problems to advanced students	"give them challenging problems exceeds the target of curriculum"	0	1	1	2
Games	A fun way for students to review their understanding of learning content	"helping them to have fun with mathematics"	0	1	1	2

Moreover, non-Chinese instructors came up with other ideas. Three of them (42.86%) liked to share real stories with students. Instructor L preferred to give some ideas of famous mathematicians while instructor D tended to give examples of graduates and show students what is possible. Instructor E liked to share about the history of mathematics. Two of them (28.6%) gave more challenging problems to students. Another two (28.6%) tried to develop more games to help students have fun with mathematics. “But not too hard, otherwise, students will lose interest,” said instructor S.

Teachers’ Attitudes as Mathematics Instructors

Attitudes of sharing experiences with advanced students. Question 16 specifically asked about what experiences interviewees would share with students who show a strong interest in mathematics, teachers showed various attitudes (see Table 68). Two non-Chinese instructors explicitly stated that they are not interested in sharing personal experience. But one of them, instructor B, said that sharing experiences in American colleges and universities is fine and that would help students understand what they could do in the field of mathematics and how they could study in a similar situation. Including that one, four interviewees (all non-Chinese), or 57.1%, showed willingness to share experiences; three of them specifically mentioned university experiences. Instructor B talked about experience concerning the application of mathematics, which was also discussed in previous question; two non-Chinese instructors and a Chinese instructor (42.9%) said they would like to share this kind of experience. Instructor W said, “even though most students will not regard mathematics as their future career path, I try my best to let them know some mathematics related thing.” The two instructors who mentioned real stories such as mathematics history in question 15 reiterated their answers in their answers to this question. One of them also hoped to bring a few friends to share their experiences with the class if there was any chance. Two others (28.6%) thought they could help with a mathematics competition.

Table 68. Attitudes about Sharing Experiences with Advanced Students

Experience	Definition	Quote	Number of Interviewees			
			AP		IB	Total
			Chinese	Non-Chinese	Non-Chinese (only)	
No Personal Experience	No interest in sharing personal experience	"not really interested in sharing personal experience"	0	1	1	2
Personal Experience	Teachers' experience of their lives	"encourage them to do whatever they can and like, so many opportunities are there if you keep working hard"	0	1	0	1
University Experience	Experience in university	"all the good memories of own experience in college"; "help them understand how they could study in a same situation"	0	1	2	3
Application of Mathematics	Relate mathematics contents to real-life examples	"places be able to use mathematics"; "help them understand what they could do in the field of mathematics"	1	1	1	3
Real Stories	Real-life examples of using mathematics in academic or career fields	"trying to ask what they hope to do in the future and if it is possible, invite few friends to come in and talk to them"; "share history of math and life examples"	0	0	2	2

Table 68 (continued)

Experience	Definition	Quote	Number of Interviewees			
			AP		IB	Total
			Chinese	Non-Chinese	Non-Chinese (only)	
Mathematics Competition	Competitive mathematics events for advanced students to participate	"even if personally not very interested in it since it's test taking, they have very good problems"	1	1	0	2
Mathematics Blog	Regularly updated website sharing interesting mathematics questions	"last year opened an online blog to share some harder questions with students and parents"	1	0	0	1
Mathematics Game	A fun way for students to review their understanding of learning content	"use algorithm and knit it to create beautiful structures"; "program robots"	0	1	0	1
Challenging Problem	More competitive and complex problems to advanced students	"give them super-challenging problems and extra works not normally seen in high school"	0	1	0	1

It is also very interesting that Chinese instructor J, who showed a conservative traditional teaching style in the responses to other questions, runs an online blog and shares some harder questions with students and parents. He noted, "Some students will take look and come to discuss with me." In addition, instructor S also came up with very creative ideas like mathematics knitting (using algorithm and knitting it to create beautiful structures) and programming robots. Instructor L seemed not to quite understand the question of sharing experience, and decided to give students challenging problems and extra work beyond the current curriculum.

Attitudes toward future development of foreign curricula in China. The very last question asked about the likelihood of any future development of foreign curricula in China under the government policy; interviewees showed high, neutral, and low expectations (see Table 69).

Table 69. Attitudes toward Future Development of Foreign Curricula in China

Attitude	Quote	Number of Interviewees			
		AP		IB	Total
		Chinese	Non-Chinese	Non-Chinese (only)	
Positive	"will have more foreign curriculums with the internationalization"	2	2	2	6
Neutral	"more international programs already, whether they are going continue it is hard to say"	0	2	0	2
Negative	"maybe going to slow down the grade of students' studying abroad or prevent Chinese getting outside"	0	0	1	1

The majority were positive, 6 out of 9 (66.7%) thought the trend would increase with more students and more imported programs under the auspices of internationalization. Chinese Instructor J said that since 2012, class size has increased from about 20 students to 30 students. And non-Chinese instructor M agreed that "every year we are seeing more and more students enroll in international programs, which means it has a positive effect on Chinese society, even if I'm not able to say what specific effect it is." Instructor S added that for subjects such as mathematics, there is no political debate. Two interviewees (22.2%) thought that whether this development is going to continue is hard to say, but one of them believed the trend might be increasing. In contrast, one instructor (11.1%) was concerned that although the trend is increasing every year, the government may limit the number of students studying abroad. The families that can afford their children to go overseas usually have significant financial or social resources, and some outstanding students who decide to study abroad might not come back, so the government might want to prevent the loss of resources and future elites. Three instructors (33.3%) believed that international programs should adjust curriculum to match local conditions. One of them (a Chinese instructor) said that learning some Chinese courses such as history is necessary. Two of them (one from the AP Center and another from the IB Center) mentioned that the Chinese government requires every student in the program to take the General Examination, and so it is necessary for an imported program to adjust to the local requirements and to fit the local situation. They felt that it is better to adapt the imported program instead of accepting it entirely without any consideration of local conditions and requirements.

Teachers' Preferences as Mathematics Instructors

Breadth and depth. In response to question 18, "do you prefer focusing instruction on the breadth or the depth of a subject, especially for mathematics," interviewees stated their preferences for focusing instruction on the breadth or the depth of a subject, especially for mathematics. There are four types of responses: none, breadth, depth, and both (see Table 70).

One Chinese instructor, J, gave no preference but to follow the general syllabus. Three instructors (33.3%, all from the AP center) including one Chinese instructor preferred breadth because they hope students can experience more areas and find their own future paths. All three (100%) said focusing on breadth at this stage is better since if students want to continue with mathematics in college, then they can go deeper. Meanwhile, two instructors (22.2%) said they prefer depth not only because understanding both root and concept will ingrain the topic in students' minds but also because doing challenging things is more interesting. "Some students get bored of superficial things and they are ready to go deeper" said instructor D.

Moreover, three instructors (33.3%) preferred a combination of both. Two of the three (66.7%, both from the IB Center) gave the reason that their personal preference should follow the program, which is about breadth in general and depth in the HL class. One of the three (33.3%) preferred depth for strong students and breadth for weak students.

Table 70. Teacher's Preference for Breadth or Depth

<table>
<tr><th colspan="2" rowspan="3">Preference</th><th rowspan="3">Quote</th><th colspan="4">Number of Interviewees</th></tr>
<tr><th colspan="2">AP</th><th>IB</th><th>Total</th></tr>
<tr><th>Chinese</th><th>Non-Chinese</th><th>Non-Chinese (only)</th><th></th></tr>
<tr><td colspan="2">None</td><td>"only follow the general syllabus"</td><td>1</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td colspan="2">Breadth</td><td>"hope students experience more areas and find their own future path, which is good for their further development"; "at this stage breadth is better, they are not applying it right now"; "do not have much time for depth at this stage"</td><td>1</td><td>2</td><td>0</td><td>3</td></tr>
<tr><td colspan="2">Depth</td><td>"if students can know the reasons why things work like that, they will have better understand of the concept"</td><td>0</td><td>1</td><td>1</td><td>2</td></tr>
<tr><td rowspan="2">Both</td><td>Balance</td><td>"IB is about breadth, but the subject you learn, especially in HL class, must focus on depth"; "do not have much choice but follow the program, which aims at both breadth and depth"</td><td>0</td><td>0</td><td>2</td><td>2</td></tr>
<tr><td>Separate by students' level</td><td>"breadth for weak students, depth for strong students"</td><td>0</td><td>1</td><td>0</td><td>1</td></tr>
</table>

Research, teaching, pushing, mentoring. Question 20, "what would you describe as your priorities as a mathematics teacher," is designed to elicit their priorities concerning teaching, mentoring, research, and pushing as a mathematics teacher. It should be noted that the teaching-research group (*jiaoyanzu*) is a basic organizational structure for teachers in both elementary and secondary schools in China. The kind of research I hoped to hear about was not academic in higher education but, rather, teachers attending meetings frequently to plan lessons, discuss teaching, and keep up with trends in their subject areas. I also hoped to discover whether the teachers observed and comment on other teachers' practices regularly to improve instructional methods and skills. "Pushing" refers to the widely used cramming method of teaching in China that forces students to learn to perform better in their examination-oriented education system instead of encouraging or motivating them.

In general, there are three types of answers: same, similar, and opposite (see Table 71). According to the data, there are 2 pairs of interviewees whose priorities lists are exactly the same (TMRP and TMPR). For those 2 pairs, since the first 2 are the same (both TM) and the last 2 change the preference from RP to PR, I regarded all 4 as having similar answers. In total, there are 3 pairs of similar answers (TMRP and TMPR, TMPR and MTPR, TRMP and TRPM). In all the responses, there is 1 pair of opposite priorities, which means that the order of choices is completely in contrast (MRTP and PTRM).

Table 71. Priority Outcomes

Priority Outcome	Definition	Number of Interviewees			
		AP		IB	Total
		Chinese	Non-Chinese	Non-Chinese (only)	
TMRP	Teaching, Mentoring, Research, Pushing	1	0	1	2
TMPR	Teaching, Mentoring, Pushing, Research	1	1	0	2
MTPR	Mentoring, Teaching, Pushing, Research	0	1	0	1
TRMP	Teaching, Research, Mentoring, Pushing	0	0	1	1
TRPM	Teaching, Research, Pushing, Mentoring	0	0	1	1
MRTP	Mentoring, Research, Teaching, Pushing	0	1	0	1
PTRM	Pushing, Teaching, Research, Mentoring	0	1	0	1

The priority outcome analysis is showed in Table 72. 6 out of 9 (66.7%) instructors regarded teaching as their top priority, which means that the majority primarily focus on teaching itself. One of the instructors even said that all the rest are simply parts of teaching (but that respondent still consented to make a priority list). And 4 out of 9

(44.4%) instructors selected mentoring as their second choice. In addition, pushing is the least preferred choice; 4 instructors put it on their third choice and another 4 put it on their last. One instructor made a comment that if supervising students to finish their homework on time means pushing, then this option should not be excluded from the list, otherwise, pushing is not a possible choice.

Table 72. Priority Outcome Analysis

	Research	Teaching	Pushing	Mentoring
1st Choice	-	6	1	2
2nd Choice	3	2	-	4
3rd Choice	3	1	4	1
4th Choice	3	-	4	2

Special Questions

Three Questions for Chinese Instructors Only

In response to the three questions that are specifically designed for the Chinese instructors, one of the interviewees skipped the first two questions, and both interviewees gave the same response to the last question. For the first question (question 3), which asked about the adaption of curricular design, instructor J said that since Chinese instructors only teach general Chinese high school mathematics in the AP program, there is no comment to give. Instructor W agreed with the aim of AP mathematics and said "it is very helpful to students' learning, even to their mathematics learning after studying abroad." For the second question (question 4), instructor W said (instructor J did not respond to that question) that in-class teaching is suited to the AP program because many of the AP mathematics contents are extensions of the general Chinese high school mathematics; they have many things in common. The instructor added that students sometimes ask AP mathematics questions in the general Chinese mathematics class too. This interviewee also felt that the fostering of students' calculation and understanding abilities can be improved. This is especially the case with understanding, since AP mathematics poses a language challenge to Chinese students with its requirement to read and solve mathematics questions in English. In the last question (question 5), both instructors (100%) mentioned that they use “formative evaluation” and that each instructor enacts it before every semester as an assessment method. They stated that it can objectively reflect students’ performance, and they emphasized that general Chinese high school does not have this kind of evaluation but uses “summative evaluation,” which only focuses on test outcomes (midterms and finals). For example, instructor J’s evaluation assigns 40% of the grade to routine performance (in-class performance, unit tests, homework, quizzes) and 60% to test performance (midterms and finals). Instructor W weighs each component equally: 50% and 50%. Teachers have the freedom to decide their preference for valuating students’ performance.

Three Questions for Non-Chinese Instructors Only

There were three questions specifically designed for non-Chinese instructors to reveal their observations concerning student performance and learning of mathematics in a bilingual environment as well as their observations concerning the teaching of mathematics in China. The first part was already addressed in those interviewee’s answers to question 14. The second part, which concerned specialty of teaching mathematics in China, asked for comments concerning the teacher-student relationship and unusual findings. Regarding the teacher-student relationship (question 12), the majority (57.15%) of the interviewees gave positive feedback: that the relationship between teacher and student here in China is closer than elsewhere. Two of them (50%) mentioned the respect and appreciation of students for teachers. They noted that teachers spend more time with students because of the lecture schedule (more classes), and that students are more eager to learn, especially the language. On the contrast, 1 interviewee (14.3%) gave a negative response: that there is a distance between teacher and student, and students default to that. The respondent said that the teacher only interacts with students on an academic level and cannot know them from a different perspective. That instructor considered culture difference as the reason for this circumstance Two instructors (28.6%) gave a neutral opinion: that the relationship is the same because of the same math content and student behavior. One of them described trying to preserve the same relationship, while another one tried to balance friendliness and professionalism.

For unusual finding (question 13), 3 instructors (42.9%) provided positive answers. One said that students are smarter and quicker so that teachers need to be more professional to make sure to be a few steps ahead of them. Two of them (66.7%) mentioned that students are more hard-working. “Grade 10 students surprised me, they want me to check their homework every single day with every single question” said instructor E. Conversely, two instructors (28.6%) noticed some negative things. Instructor M thought the gap between good students and poor students is very wide, while in other countries students are at similar levels. And instructor S worried that intense and compressed classes do not give students enough time to absorb some of the concepts. Two instructors (28.6%) were neutral: they did not see anything unusual. One of them mentioned that in another IB Center in North China, the school asked teachers to combine an SL class with a HL class, maybe because there are fewer students. But here in Ningbo, the arrangement is the same.

Chapter V

CONCLUSIONS

Introduction

This chapter will review the study and revisit the research questions as well as the methodology. I will summarize and explain the findings to address each research question. Finally, limitations and implications of the study will be discussed, and recommendations for further research will be provided.

Summary of the Study

This study was designed to compare AP (Advanced Placement) and IB (International Baccalaureate) programs in Chinese mathematics education. In an attempt to promote globalization and internationalization, the Chinese government has introduced AP and IB programs in Chinese high schools. In those programs, well-organized foreign AP and IB teams conduct instruction (Cheng & Deng, 2006; Gu, 2006b; Weng, 2008; Xu, 2001).

Many researchers and scholars have explored the AP program and the IB course internationally. In general, scholars and researchers have conducted pairwise comparative studies of one foreign program and their native high school program. Although it is one of the biggest countries to introduce international programs to its secondary education in an effort to prepare more students to go overseas to pursue their higher education, China does not have much research focusing on introduced foreign academic programs. Some

Chinese researchers have compared two programs horizontally to explore the similarities and differences of curricula and assessment, and others have chosen a vertical comparison to mainly analyze students' performance over a period of time. Very few of the former studies have focused on comparisons that include the AP program, IB diploma program, and the native high school program altogether while making a horizontal comparison and several vertical comparisons.

Moreover, Chinese educators started late in exploring the AP program and the IB program. With the progress of the reform and opening-up policies in the 1990s, people began to notice these programs. Also, the study of these programs in China is relatively superficial; most of them remain at the simple contextual comparison without enough comprehensive analysis. Many scholars' and researchers' studies were published between 2000 to 2015, which means some of their results or conclusions might be outdated with the rapid growth of the development of the AP and IB diploma programs. Inevitably, the AP program keeps changing as it expands.

The purpose of this study is to fill in some gaps in the research while providing a better understanding of the depth behind the phenomenon of introducing the AP program and the IB diploma program in China and that introduction's impact on the existing general Chinese high school program in mathematics. The goal is to generate updated research. This will enable more students and their parents to make more knowledgeable decisions about learning in the AP program or the IB program in China.

In general, this study illustrated the differences and similarities between the AP program and the IB program in their respective schools in Ningbo, China. It further examined the differences and similarities between the AP program/IB program and the intended Chinese high school program in mathematics education. It also explored mathematics instructors' views of the imported AP and IB programs in China.

The following research questions were addressed in this study:

1. What are the significant differences and similarities between the AP (Advanced Placement) program and the IB (International Baccalaureate) program in their respective schools in Ningbo, China?
2. What are the differences and similarities in curriculum and student performance between the AP (Advanced Placement) program and the intended Chinese high school program in mathematics education?
3. What are the differences and similarities in curriculum and student performance between the IB (International Baccalaureate) program and the intended Chinese high school program in mathematics education?
4. What are mathematics instructors' perspectives on teaching imported AP (Advanced Placement) and IB (International Baccalaureate) programs in China; also, what are their expectations and suggestions for curriculum innovations in Chinese mathematics education?

In order to answer my research questions, I used a mixed method that includes both quantitative and qualitative approaches. The first research question was investigated using gathered information coming from sources like recruitment handbooks with detailed program descriptions for AP and IB schools. Those sources were used to make a contextual analysis to provide knowledge that might be important for explaining differences and similarities between the AP and IB programs in Ningbo, China. The second and third research questions were studied from two aspects: the collected mathematics curricula and the measures of student outcomes (test scores). The curricula include the general Chinese high school mathematics curriculum as well as the mathematics curricula of both the AP and IB programs in Ningbo, China. I used the textbooks adopted in these two schools as supplements to help conduct cross-curricular comparisons. Students' performance data used in this study included their test scores on the AP mathematics examination, the IB mathematics examination, the SAT mathematics examination, and the Chinese general mathematics examination. Goodman and Kruskal's gamma was applied to measure the rank correlation. Finally, to answer research question four, nine interviews were conducted with mathematics instructors from both AP and IB schools. Teacher responses are categorized and discussed further.

Conclusions of the Study

Research Question 1

In response to research question 1, the similarities between imported AP program and IB program in Ningbo, China are as follows: overall, both schools have non-Chinese center principals and a Chinese college counseling team who cooperate with each other. Especially since 2016, both programs pair with the general Chinese curriculum, which means that students from both schools are required to learn two curricula (the general Chinese curriculum and the AP/IB curriculum) at the same time. Also, students must take and pass the Chinese general examination. In the first year (Grade 10), students in both schools are required to take pre-program courses (Pre-AP/Pre-IB) to prepare for their following studies. Additionally, English and mathematics are mandatory subjects for them each year. On the one hand, both programs help to bridge secondary education and higher education with rigorous content and high performance standards that value critical and independent thinking skills. Such qualities gives the schools great reputations and wide recognition when it comes to college admission. On the other hand, since both programs possess intense course loads and a certain language standard for these non-native speakers, students face more challenges and pressures, which was pointed out by many teachers during their interviews.

The differences can be classified according to setting, curriculum, and assessment. In terms of setting, the number of AP schools in Ningbo, China, is triple the number of IB schools. Also, AP graduates target mainly higher education institutions in the United States, while the IB graduates have a broader target that aims at mostly English-speaking countries. They also have different goals: the AP program focuses on preparing and connecting students for college success and opportunity, both academically and mentally; the IB program devotes itself to fostering lifelong learners and global citizens who possess intercultural understanding and respect, while retaining their patriotism toward China and Chinese culture.

As far as curriculum is concerned, the two programs have different content: the AP school offers college-level courses only with a cafeteria style for students to select according to their preference, while the IB school provides a systematic diploma program with various subjects mainly taught at a high-school level with some at the college-level. The IB school recognizes three cores to specifically develop students' thinking skills, research abilities, and application of concepts in real practice. AP students receive only the Chinese high school diploma whereas IB students have been able to obtain two diplomas (from both IBO and Chinese high school) since 2016. In addition, unlike the AP students can choose the difficulty of their courses without restriction; the IB students are required to select three higher level (HL) courses and three standard level (SL) courses. They also vary in their emphasis of subject. The AP program tends to enhance students' abilities in one or some specific academic subject(s) while the IB program is apt to cultivate students' comprehensive abilities. Thus, the AP school provides a more content-centered curriculum with more lectures and in-depth learning, while the IB school implements inquiry-based learning and more activities alongside academic studies, which makes its curriculum more student-centered.

The difference in the programs also appears in their methods of assessment. The AP program uses only external assessment while the IB program uses both external and internal assessment to evaluate students' learning outcomes. Given the three cores and the internal assessments that require various written work, IB students are expected to be very advanced in their English language abilities. In contrast, AP students do not have much writing to do if they select courses such as mathematics. Additionally, the AP holds its examinations once per year in each May with a score scale from 1 to 5, and examinations are graded only in the United States; the IB examinations are twice per year in every May and November with a score scale from 1 to 7, and the grading is done by members of the IBO worldwide.

Research Question 2

To answer research question 2, I considered two elements: curriculum and data. There is a difference in the philosophy of the curricula: AP aims at exploring the depth of mathematics, allowing students to give full play to the specific subject they are good at, while the general Chinese high school mathematics' approach is a mass education that is designed for everyone. Different attitudes to using technology appeared here because the use of a graphing calculator in AP Calculus is considered an integral part of the course. But students are only allowed to use standard scientific calculators when taking the General Examination. The next difference is that the four major ideas (limits, derivatives, integrals and the Fundamental Theorem of Calculus, and series) from the AP calculus curriculum are excluded from the intended Chinese mathematics curriculum. One of them, derivatives, is listed as an optional topic that is tested in the College Entrance Examination for general Chinese students, but not for AP students, who only need to take the General Examination, which excludes the relative content of derivatives. Finally, there are some topics only covered in the general Chinese mathematics curriculum. Those include topics such as: sets; monotonicity and monotone interval; geometry in 3D space; positional relations between points, lines & planes; function of circle except the standard form of a function of a circle; relations between sin, cos, and tan of an angle; basic introduction of arithmetic series and application of sequences; some fundamental ideas of inequalities; common logic terms; conic sections and equations; space vector; and solid geometry.

In general, the AP Calculus (AB & BC) contents are unlike the general Chinese high school mathematics contents, which makes sense because AP calculus itself represents college-level math content, which certainly exceeds the general high-school level. Many of the contents of general Chinese high school mathematics are similar to the contents in Pre-AP Calculus. The majority of the overlapping contents concerns functions (functions and notations, properties of functions, elementary functions, application of functions, line and functions, trigonometric functions, and trigonometric identities) and some ideas about solving triangles and inequalities. Also, only a very superficial introduction to “definition and notation of sequence,” “arithmetic and geometric sequences,” and “geometric series” from the General Chinese mathematics curriculum is covered in AP calculus BC. Teachers did not go through these concepts in detail but just briefly mentioned them.

In short, the curriculum of AP mathematics is more advanced, deeper, and contains more applications than the curriculum of general Chinese high school mathematics. General Chinese high school mathematics functions as a foundation for learning AP mathematics.

In order to investigate the data, I pair-compared four kinds of student learning outcomes data for every year: AP students’ General Examination in Mathematics, AP students’ SAT mathematics, AP Calculus AB, and AP Calculus BC. To summarize the findings, four score pairs are compared for the years 2016 and 2017, and they show the same result, while two reveal a contrasting result.

Within these four pairs, two of the relationships (the second pair and the sixth pair) are statistically significant. For the two pairs that are statistically significant, the statistical significance between AP students’ GE mathematics and AP students’ AP Calculus AB might imply that the general Chinese mathematics helps students build a solid background for learning the AP Calculus AB course. Chinese instructor W’s interview response also helps to clarify the connection. And we could also expect that if students can handle a test with richer topics like the AP Calculus BC exam, then they would tend to perform better in AP Calculus AB, which contains approximately 60% of the same topics as AP Calculus BC. The lack of statistical significance in the correlation between AP students' SAT mathematics and AP students' GE mathematics might be because these two kinds of examinations have a different scope.

The lack of statistical significance in the correlation between AP Calculus BC and AP students' GE mathematics might be because AP Calculus BC is an extension of AP Calculus AB with some difference in scope. Also, only 14 students (27.45%) took the AP Calculus BC test in 2016, and only 8 students (29.63%) took the AP Calculus BC test in 2017, leading to a small sample size. It was surprising to discover while comparing this pair that no one achieved a grade of A on the General Examination, and the average level of students' AP General Examination mathematics score also decreased from 1.92 (in 2016) to 1.29 (in 2017). Both AP Chinese instructors mentioned in the interview that since the general Chinese mathematics course is not very important to the students in the AP Center, students do not have positive attitudes and only want to pass the General Examination. Perhaps students' lack of motivation and effort concerning the General Examination led to lower scores.

The two pairs that yield a contrasting result might imply that the topics in SAT mathematics are not related to the topics in AP Calculus. Also, since the entire sample size decreased from 51 in 2016 to 27 in 2017, these possible factors might help to explain corresponding performance in the AP Calculus AB/BC examination and the SAT mathematics test.

Additionally, the overall level of students' AP Calculus BC scores decreased from an average of 4.2 (in 2016) to an average of 3.5 (in 2017). Student AP Calculus BC scores that were higher than or equal to 4 decreased from 71.43% (2016) to 37.5% (2017) in that same period. The overall level of students' SAT mathematics scores decreased from an average of 735.69 (in 2016) to an average of 727.78 (in 2017). The percentage of students' SAT mathematics scores that were higher than or equal to 750 decreased from 52.94% (2016) to 33.33% (2017). Moreover, in 2016, no student scored higher than a B on the General Examination in mathematics, but a majority (80.39%) of them scored within a high range (700 to 800) on the SAT mathematics test. However, in 2017, no student scored higher than a C on the General Examination mathematics, and only two thirds (66.67%) of them scored within a high range (700 to 800) on the SAT mathematics test. Furthermore, in 2016, students scored higher than or equal to a 3 on the AP Calculus BC exam, and students who took the AP Calculus BC test all scored higher than 700 in SAT mathematics and half of them got the highest score (5) on their AP Calculus BC test; but in 2017, students scored only higher than or equal to a 2 on the AP Calculus BC exam, and those who took the AP Calculus BC test scored higher than or equal to 700 on SAT mathematics and half of them achieved the moderate score at 3 on the AP Calculus BC exam. A majority (64.29%) of the students who took the AP Calculus BC test in 2016 received the highest score of 5 on the AP Calculus AB test, but only 25% of students who took the AP Calculus BC test in 2017 achieved the highest score (5) on the AP Calculus AB test.

One of the instructors from the AP Center mentioned in his interview that students' mathematical ability has been decreasing. The data seem to suggest that the mathematical ability of students in the AP Center decreased from 2016 to 2017. However, further study is needed to gather enough evidence to demonstrate that students in 2016 were better overall than students in 2017 since the data base is not big enough.

Research Question 3

To answer research question 3, I also considered two areas: curriculum and data. To account for any differences in the curriculum part, some college-level contents, especially those of IB mathematics Higher Level (HL) courses, are not included in the contents of general Chinese high school mathematics, even in its extended curricula

(Appendix G). Also, there are some topics that are covered in the Chinese curriculum but not in the IB mathematics curriculum (table notation of a function, power functions, models of functions and their applications except the increasing models of different functions, geometry in 3D space except perspective and parallel projection, function of circle except distance between 2 points in 3D space, linear inequalities in two variables and simple linear programming questions, common logic terms, and conic sections and equations except the definition of parabola and standard equation of parabola). Other differences include the limiting of calculator use: IB mathematics requires students to learn how to use graphing calculators because this is part of the assessment format, but students are only allowed to use standard scientific calculators when taking the General Examination. There are different attitudes toward using technology in practice, one of which was expressed by Instructor D from the IB Center. He said in the interview that the IB class is supposed to be more technology friendly but that there still exist some technology restrictions in the Chinese IB program. The next significant difference is that IB mathematics includes "Mathematics Exploration," which is an individual exploration in the form of written work that involves investigating an area of mathematics, as an internal assessment. In general Chinese high school mathematics, there is no written requirement, only solving problems.

IB Mathematics and general Chinese high school mathematics both contain various kinds of mathematical ideas including algebra, functions and equations, circular functions and trigonometry, statistics and probability. Generally, the curricular content of IB mathematics is consistent with the content of general Chinese mathematics. To sum up, IB Mathematics is more technology and writing oriented, challenging, and broad than general Chinese high school mathematics.

For the data part, I compared each possible pair of grades from each student's mathematics grades on IB examination and SAT test. In the data year 2016, sample students all took the HL test. So, there is only one comparison pair in 2016. Also, because

IB conducted the government policy of requiring students to take the General Examination since 2016 and because this policy affects students who have enrolled since 2016, the year given to the data analyzed here is the students' graduation year. Therefore, IB students' mathematics scores on the General Examination will not be available until June, 2019. The findings do suggest that all comparison pairs will be correlated. However, the correlations between the pairs are not all statistically significant. Therefore, the results of the IB data might imply that the topics in SAT mathematics are not related to the topics in IB mathematics.

The finding also shows that no one in this group scored less than 2 on the IB mathematics HL test in both 2016 and 2017. Also, in 2016, a majority (92.11%) of the students received SAT mathematics scores higher than 700 while all (100%) earned corresponding SAT mathematics scores higher than 700 in 2017. The level of students' IB mathematics HL score increased from an average of 4.21 (in 2016) to an average of 4.48 (in 2017). The level of students' SAT mathematics scores increased from an average of 758.16 (in 2016) to an average of 773.33 (in 2017). Students' SAT mathematics scores that were higher than or equal to 750 increased from 57.89% (2016) to 90.91% (2017). The data from this study seem to suggest that the mathematical ability of students in the IB Center improved from 2016 to 2017. But without further study, there is not enough evidence to demonstrate that students in 2017 were actually better overall than students in 2016 since the data base is not big enough.

Research Question 4

During the interviews in this study, mathematics instructors from both the AP school and the IB school articulated the differences they noticed while teaching. First, they noticed significant difference between teaching mathematics in the past and present. Interviewees said that for teachers, improved teaching skills and changed teaching methods are typical differences in current teaching. In the area of teacher-student relationships, they mentioned smoother communication with more interaction and engagement. One also noticed that students' motivation and ability decreased. Second, significant difference was also found between teaching in an international school and a standard school. International schools have smaller class sizes, higher costs, many cultures, and a higher language requirement. Students there have more personalities and tend to lack motivation and effort concerning the local curriculum. Finally, some teachers said that the academic difficulty of mathematics in the imported programs, including IB (HL) and AP, is higher than or equal to general Chinese mathematics based upon the topics and contents, and that the IB (SL) is easier than the best of general Chinese mathematics. Some instructors think Chinese students have advanced calculation and memorization skills, but their application abilities are weak. Others said that weak English vocabulary ability makes some students have trouble understanding wordy questions, and academic terms can also confuse them. Respecting the classroom style, both Chinese interviewees preferred a teacher-centered approach, while all non-Chinese instructors chose a combination of both a teacher-centered and a student-centered classroom.

Teacher perceptions of Chinese mathematics learning and teaching provide insight into innovations in Chinese mathematics education. Concerning the general weakness in Chinese mathematics learning and teaching, most instructors mentioned rote-learning and a teacher-centered classroom. Additionally, some noted a lack of independent thinking skills and unbalanced mathematical skills while others talked about infusion education ("duck-stuffing") and the limitation of opportunities to use technology. Chinese instructors did not offer suggestions for change, but non-Chinese interviewees came up with advice to help innovate in Chinese mathematics education. For learning, they suggested to promote curriculum development, help students learn from mistakes, encourage students to do independent research, and foster critical thinking. For teaching, they recommended inquiry-based and student-centered classrooms and the implementation of technology.

During the discussion of interviewees' thoughts about support, activities, and methods of inspiring students' mathematics learning, there exists an obvious difference between Chinese instructors and non-Chinese instructors. For support, both Chinese instructors mentioned that students do not pay much attention to the general Chinese mathematics courses but only wish pass the General Examination. Therefore, those instructors showed corresponding conservative notions of support: one of them decided to use a pass-fail system to motivate them, and another suggested "just offering help when they ask questions." Non-Chinese instructors, however, offered various means of support not only in class but also out of class such as differentiating instruction, group discussion, mutual help, visual learning, games, regular feedback, and office hours. Concerning activities, both Chinese instructors said they do not create in-class activities very often, and expressed similar usage of out-of-classroom activities. Therefore, the general effect is relatively unapparent. Surprisingly, according to the responses of non-Chinese instructors, the in-class activities they encouraged students to participate in such as group discussion, games, and presentation demonstrated a 100% positive effect. They also provided several forms of out-of-class activities such as exploration activities, mathematics competitions, and mathematics clubs. In order to inspire students' mathematics learning, one Chinese instructor noted a tendency to "just give the answer to students directly," while another one mentioned the application of mathematics principles and processes, which also has been discussed by non-Chinese instructors. And non-Chinese instructors added some methods like sharing real stories or engaging students with more mathematical games.

There were a few marked points of similarity between teacher attitudes about sharing experiences with advanced students and towards future development of different curricula in China. In short, most teachers were interested in sharing the university experience and highlighting applications of mathematics. And 66.7% showed a positive attitude toward and anticipated an increasing trend in China of importing more international curricula.

The majority of the interviewees preferred focusing instruction on breadth or a combination of both breadth and depth in mathematics. They believe that it is the best option at this stage because if students want to continue with mathematics in college, they can go deeper with their studies. They also described their priorities as mathematics teachers, and the results show that 66.7% of them regard teaching as their first choice of career activity, followed by mentoring and research; pushing is the least preferred choice.

Limitations of the Study

The findings of this study are limited in a few ways. Some kinds of data concerning student outcomes could not be collected as expected, mainly because of changes in the faculty members in charge. When preparing the new staff's system updates, some old data were mistakenly erased without any back-up copy. Due to the absence of some test scores, it was difficult to follow the original data analysis plan to achieve an ideal and complete result. Another limitation was that the amount of data concerning student performance is not big enough to generate a more accurate statistical result. Because both schools are newly established, in 2011 and 2012, they have only a few years of graduates. Besides, with class sizes of 25 to 30 students on average, the number of total students per year is low. After subtracting some incomplete data, the entire valid data size was further narrowed. In addition, since it is a case study that includes only two schools, and given the fact that the imported programs have localized features both in program setting and in curriculum design, the results of this study might not be representative of the entire country. For example, some archival resources, such as textbooks used in each school, might not be chosen to implement the programs in other cities. Also, it was difficult for the investigator to interview additional mathematics instructors such as former faculty members. Many of them are not reachable or they are unavailable to participate in an interview because of work relocation. More interviews could have been conducted with more time and opportunity. Finally, without some qualitative explorations of students' perceptions, there is no solid proof of identifying the potential differences in self-selection populations of AP students and IB students. To be specific, due to the different concept and curriculum outline of the imported AP and IB programs, Chinese students who are used to a more examination-oriented education might regard the AP program as very similar to the general Chinese courses. In the meanwhile, they might deem the IB program as the more different one since it has a lot of written material, diversified subjects and various assessment methods that encourage students have not only comprehensive academic abilities but also global caring perspectives. My review of literature also showed that the growth of the IB program in China was very slow at the beginning and has significantly accelerated since 2012 (IB China, 2018), possibly due to the time needed to merge with a very different curriculum. Therefore, it is possible that students who want to look for an in-depth study with the specific academic subject they are good at might tend to select the AP program and try to "nail" the AP examination to help their college admission. On the contrast, those willing to take challenges to try more inquiry-based and student-centered learning they are not very familiar with, and face a language barrier coupled with high-standards writing in subjects like mathematics, might be more likely to choose the IB program.

Implications of the Study

This comparative study suggests the need for additional studies of AP and IB programs in China. For Chinese students and their parents, this study provides a guide to help decide on a learning program based on individual characteristics and preferences.

Also, the results of this study indicate that a considerable gap exists between secondary education and higher education in Chinese mathematics, and there is also a limit to the possibilities for individualized learning. The findings imply the need to consider curricular reform, such as adding some college-level courses to the high school curriculum. Other ideas like setting up more optional courses and courses at different levels of difficulty for students to choose might help improve the autonomy of Chinese high school education. A more comprehensive assessment might also generate a thorough understanding of student learning outcomes. For example, a combination of both formative and summative assessment might help balance traditional Chinese evaluations (that use test scores as a sole index) but not completely change them out of respect for their social and cultural roots. And internal assessments with certain writing requirements, beneficial for measuring student learning, might also be encouraged. Moreover, a more open-minded technology implementation, such as graphing calculator use, internet access and multiple software support, is worth considering to motive student learning and to cultivate their application of acquired knowledge in different ways.

The results of this study also have empirical implications. For local teachers and non-local teachers who teach in these imported programs, it is beneficial to learn from each other to reinforce teaching skills and methods of fostering students. Moreover, with the development of globalization and internationalization as educational movements, more international programs could be imported to fit different education systems. It is important to realize that imported programs will need to have localized features that might cause other changes to their curricula, course settings, and so on. Therefore, policymakers need to make adjustments to consider local conditions when introducing foreign programs so as to offer the most suitable program possible to native students.

Recommendations of the Study

Recommendations for Practitioners/Educators

Because the research did not examine the differences and similarities between AP mathematics and IB mathematics in China, future studies could explore if and how these two imported programs differ or are similar to each other. Other investigators could also explore differences and similarities between students' perceptions and teachers' perceptions of these imported programs. A qualitative approach such as conducting interviews or surveys would be helpful to answer this question. Also, there are other kinds of imported programs (such as A Levels [Advanced Levels]) with wide popularity around the world, especially in the United Kingdom. Future studies do not need to be confined to certain programs; they could extend to explore multiple programs instead of only comparing two.

Recommendations for Further Research

This study investigated the AP and IB programs in Ningbo, China, but several questions that have not been covered might be answered through further study. If this study could be repeated, it would be advantageous to have a larger data size to obtain more students' test scores. In this study, an average of 39 AP students and 32 IB students were measured. 100 students per program would yield a more representative result. I also recommend doing a longitudinal study to gather continuous data over a longer time period. This study has two years of data, but a five-year study would be more comprehensive, and ten years of data would be even better. In addition, the number of interviews of mathematics instructors could be increased with advantage. This study was limited to interviewing mathematics instructors who teach in the two subject schools. The interview could be distributed to former mathematics teachers. Further studies could also interview some students, including both alumni and currently-enrolled students to obtain the students' perspective.

Moreover, these introduced programs have localized features, which means not every city in China shares the same curriculum. Therefore, more research is needed to examine the introduced AP and IB programs in other cities in China. Alternatively, other AP schools and IB school in Ningbo, China could be studied. Exploring some comparative research across different countries is another genre of study. For example, it could be enlightening to know how the findings about these programs in China compare to the same imported programs in another Asian country, perhaps Singapore.

Using this study as a guide may help further research to answer some questions that have not yet been examined and create a more thorough understanding of the imported AP and IB programs and their importance to educational globalization and internationalization.

REFERENCES

About the IB. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/about-the-ib/

Altbach, P. G. (2003). Globalization and the university: Myths and realities in an unequal world. *Journal of Educational Planning and Administration, 17*(2), 227-247.

Altbach, P. G., & Knight, J. (2007). The internationalization of higher education: Motivations and realities. *Journal of Studies in International Education, 11*(3-4), 290-305.

Altbach, P. G., Reisberg, L., & Rumbley, L. E. (2019). *Trends in global higher education: Tracking an academic revolution.* Leiden, The Netherlands: Brill Sense.

Annual AP program participation 1956-2018. (2018, Oct 15). Retrieved from https://secure-media.collegeboard.org/digitalServices/pdf/research/2018/2018-Annual-Participation.pdf

AP at a glance. (n.d.). Retrieved October 17, 2018, from https://apcentral.collegeboard.org/about-ap/ap-a-glance

AP Calculus AB and BC course and exam description. (2018, Oct 15). Retrieved from https://apcentral.collegeboard.org/pdf/ap-calculus-ab-and-bc-course-and-exam-description.pdf

AP Calculus AB the course. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/courses/ap-calculus-ab/course?course=ap-calculus-ab

AP Calculus AB the exam. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/courses/ap-calculus-ab/exam

AP Calculus AB frequently asked questions. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/courses/ap-calculus-ab/course/frequently-asked-questions

AP Calculus BC the course. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/courses/ap-calculus-bc/course?course=ap-calculus-bc

AP Calculus BC the exam. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/courses/ap-calculus-bc/exam?course=ap-calculus-bc

AP course ledger. (2017, March 27). Retrieved from https://apcourseaudit.epiconline.org/ledger/search.php

AP exam instructions. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/pdf/ap-exam-instructions-2018.pdf

AP exam volume by region 2008-2018. (n.d.). Retrieved October 17, 2018, from https://secure-media.collegeboard.org/digitalServices/pdf/research/2018/2018-Exam-Volume-by-Region.pdf

AP International. (n.d.). Retrieved October 17, 2018, from https://apcentral.collegeboard.org/start-grow-ap/international

AP Program results overview. (n.d.). Retrieved October 10, 2018, from https://reports.collegeboard.org/ap-program-results/overview

AP Statistics the course. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/courses/ap-statistics/course?course=ap-statistics

AP student and exam volume per school. (n.d.). Retrieved October 15, 2018, from https://secure-media.collegeboard.org/digitalServices/pdf/research/2018/Student-and-Exam-Volume-Per-School-2018.pdf

AP Students Calculator Policy. (n.d.). Retrieved October 10, 2018, from https://apstudent.collegeboard.org/apcourse/ap-calculus-ab/calculator-policy

Assessment and exams of IBDP. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/assessment-and-exams/

Aulls, M. W., Lemay, D., & Peláez, S. (2013). *Exploring the learning benefits and outcomes of the IB extended essay in preparing students for university studies in Canada (two phases)*. Montreal, Canada: Learning Sciences: Department of Educational and Counseling Psychology, McGill University.

Bashir, S. (2007). *Trends in international trade in higher education: Implications and options for developing countries.* Washington, DC: World Bank Publications.

Beckwitt, A., Van Camp, D., & Carter, J. (2015). *International Baccalaureate implementation study: Examination of district-wide implementation in the US.* Germantown, MD: Asher Consulting, LLC.

Bergeron, L. (2015). *Diploma Programme students' enrollment and outcomes at US postsecondary institutions 2008–2014.* Bethesda, MD: International Baccalaureate Organization.

Bergeron, L., & Alcantara, A. (2015). *IB mathematics comparability study: Curriculum and assessment comparison.* Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/globalassets/publications/ib-research/dp/maths-comparison-summary-report.pdf

Bergeron, L., & Rogers, L. (2015). *Teaching the theory of knowledge course in IB world schools*. Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/contentassets/d1c0accb5b804676ae9e782b78c8bc1c/tok-final-report.pdf

Bhandari, R., & Blumenthal, P. (Eds.). (2010). *International students and global mobility in higher education: National trends and new directions.* New York, NY: Springer.

Billig, S. H., & Good, B. M. (2013). *International Baccalaureate Diploma Programme: Study of North and South American students' civic-mindedness*. Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/globalassets/publications/ib-research/dp/civicmindednesssummaryfinal_000.pdf

Blackmer, A. R., Bragdon, H. W., Bundy, M., Harbison E. H., Seymour, C., Jr., & Taylor, W. H. (Eds.). (1952). *General education in school and college: A committee report by members of the faculties of Andover, Exeter, Lawrenceville, Harvard, Princeton, and Yale*. Cambridge, MA: Harvard University Press.

Blee, K. M., & Currier, A. (2011). Ethics beyond the IRB: An introductory essay. *Qualitative Sociology*, *34*(3), 401.

Bragg, D. D., Kim, E., & Barnett, E. A. (2006). Creating access and success: Academic pathways reaching underserved students. *New Directions for Community Colleges*, *135*, 5-19.

Brewer, J., & Hunter, A. (1989). *Multimethod research: A synthesis of styles*. Thousand Oaks, CA: Sage.

Burnham, P. S., & Hewitt, B. A. (1971). Advanced Placement scores: Their predictive validity. *Educational and Psychological Measurement*, *31*(4), 939-945.

Byrd, S. (2007). *Advanced Placement and International Baccalaureate: Do they deserve gold star status?* Washington, DC: Thomas B. Fordham Institute.

Callahan, C. M. (2003). *Advanced Placement and International Baccalaureate programs for talented students in American high schools: A focus on science and mathematics.* Storrs, CT: National Research Center on the Gifted and Talented.

Camara, W. J. (2003). *College persistence, graduation, and remediation: Research summary. RS-09.* New York, NY: College Entrance Examination Board.

Caspary, K. (2011). *Research brief: Postsecondary enrollment patterns of IB certificate and diploma candidates from international high schools*. Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/contentassets/d74675437b4f4ab38312702599a432f1/ibsuppdomesticresearchbrief2011-04-06.pdf

Chai, J. (2006). Woguo weijifen jiaoxue gaige fangxiang de sikao – jianlun meiguo AP weijifen jihua gei women de qishi [Teaching reform of calculus in China - inspired by AP calculus]. *College Mathematics*, *22*(3), 17-20.

Chen, B. (2006). *Guoji daxue wenping yuke shuxue kecheng neibu pingjia tixi de tezheng fenxi ji qishi [The analysis of internal assessment of International Baccalaureate mathematics]* (Masters thesis). Retrieved from CNKI. (No. O-4)

Chen, J. (2013). Curriculum and instruction design principles for promoting learning with understanding. *Global Education*, *42*(1), 53-61.

Chen, Y., & Hou, Z. (2011). Review on curriculum of IB programs. *Journal of Heilongjiand College of Education*, *30*(8), 51-53.

Cheng, K., & Deng, Y. (2006). The commentary of American International Baccalaureate Program. *Studies in Foreign Education*, *33*(7), 41-44.

Cherryholmes, C. H. (1992). Notes on pragmatism and scientific realism. *Educational Researcher*, *21*(6), 13-17.

Chmelynski, C. (2005). International Baccalaureates just keep growing in the US. *The Education Digest*, *70*(5), 58.

Clayton, M. (1998). World-class rigor spreads in US high schools. *Christian Science Monitor*, *90*(38), 12.

Coca, V., Johnson, D., Kelley-Kemple, T., Roderick, M., Moeller, E., Williams, N., & Moragne, K. (2012). *Working to my potential: The postsecondary experiences of CPS students in the International Baccalaureate Diploma Programme*. Chicago, IL: Consortium on Chicago School Research.

Cole, D. R., Gannon, S., Ullman, J., & Rooney, P. K. (2014). *IB Programme: Theory of knowledge (TOK): Exploring learning outcomes, benefits, and perceptions*. Sydney, Australia: University of Western Sydney.

Conley, D. (2007). *Redefining college readiness*. Eugene, OR: Educational Policy Improvement Center.

Conley, D. (2013). *Getting ready for college, careers, and the Common Core: What every educator needs to know*. New York, NY: Wiley.

Conley, D., McGaughy, C., Davis-Molin, W., Farkas, R., & Fukuda, E. (2014). *International Baccalaureate Diploma Programme: Examining college readiness*. Eugene, OR: Educational Policy Improvement Center.

Cox, J., & Daniel, N. (1983). Options for the secondary level G/T student (Part II): The International Baccalaureate. *Gifted Child Today*, *6*(2), 24-30.

Creswell, J. W., & Clark, V. L. P. (2017). *Designing and conducting mixed methods research*. Thousand Oaks, CA: Sage.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: Sage.

Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. Thousand Oaks, CA: Sage.

Curry, W., MacDonald, W., & Morgan, R. (1999). The Advanced Placement Program: Access to excellence. *Journal of Secondary Gifted Education*, *11*(1), 17-23.

Davis, T. M. (2000). *Open Doors 1996-1997: Report on international educational exchange*. Annapolis Junction, MD: IIE Books, Institute of International Education.

Discover the benefits of AP. (n.d.). Retrieved October 17, 2018, from https://apcentral.collegeboard.org/about-ap/discover-benefits

Dodd, B. G., Fitzpatrick, S. J., De Ayala, R. J., & Jennings, J. A. (2002). *An investigation of the validity of AP® grades of 3 and a comparison of AP and non-AP student groups.* New York, NY: College Board. Retrieved from https://files.eric.ed.gov/fulltext/ED561017.pdf

Doherty, C. (2009). The appeal of the International Baccalaureate in Australia's educational market: A curriculum of choice for mobile futures. *Discourse: Studies in the Cultural Politics of Education*, *30*(1), 73-89.

Dougherty, C., Mellor, L., & Jian, S. (2006). *The relationship between Advanced Placement and college graduation.* Austin, TX: National Center for Educational Accountability.

Exam development. (n.d.). Retrieved October 15, 2018, from https://apcentral.collegeboard.org/about-ap/how-ap-develops-courses-and-exams/exam-development

Fowler, M., & Luna, G. (2009). High school and college partnerships: Credit-based transition programs. *American Secondary Education*, *38*(1), 62-76.

Fox, E. (1985). International schools and the International Baccalaureate. *Harvard Educational Review*, *55*(1), 53-69.

Freedman, O., & Krugman, J. (2001). *Access to excellence: A report of the Commission on the Future of the Advanced Placement Program.* New York, NY: College Entrance Examination Board.

Fullan, M. (2001). *Leading in a culture of change*. New York, NY: Wiley.

Gallagher, S. A. (2009). Myth 19: Is Advanced Placement an adequate program for gifted students? *Gifted Child Quarterly*, *53*(4), 286-288.

Geiser, S., & Santelices, V. (2004). The role of advanced placement and honors courses in college admissions. In P. Gandara, G. Orfield, & C. L. Horn (Eds.), *Expanding opportunity in higher education: Leveraging promise* (pp.75-113). Albany: State University of New York Press.

Getting results of IBDP. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/assessment-and-exams/getting-results/

Gonzalez, E. J., O'Connor, K. M., & Miles, J. A. (2001). *How well do Advanced Placement students perform on the TIMSS Advanced Mathematics and Physics tests.* Chestnut Hill, MA: International Study Center, Lynch School of Education, Boston College.

Greene, J. C. (2007). *Mixed methods in social inquiry* (Vol. 9). New York, NY: Wiley.

Gu, B. (2006a). International Baccalaureate: A model program for the international elitism education. *Journal of Nantong University (Education Sciences Edition)*, *22*(1), 44-46.

Gu, B. (2006b). A *study of International Baccalaureate Diploma Program* (Masters thesis). Retrieved from CNKI. (No. G649.1)

Gu, L., & Yang, Q. (2002). Guoji wenping zuzhi zhongxue xiangmu kecheng de qishi [Enlightment of the IB Middle Years Program]. *Shandong Education Research*, *5,* 22-23.

Hargrove, L., Godin, D., & Dodd, B. (2008). *College outcomes comparisons by AP® and Non-AP high school experiences*. New York, NY: College Board.

Harrington, V. S. (2005). *High school/college credit programs and university success* (Doctoral Dissertation). Retrieved from ProQuest Dissertations Publishing. (No. 3166111)

Hayden, M., Hemmens, A., McIntosh, S., Sandoval-Hernández, A., & Thompson, J. (2017). *The impact of creativity, action, service (CAS) on students and communities*. Bethesda, MD: International Baccalaureate Organization.

Hayden, M. C., & Wong, C. S. (1997). The International Baccalaureate: International education and cultural preservation. *Educational Studies*, *23*(3), 349-361.

Hertberg-Davis, H., Callahan, C. M., & Kyburg, R. M. (2006). *Advanced Placement and International Baccalaureate Programs*. Storrs, CT: National Research Center on the Gifted and Talented.

Hoffman, N. (2003). College credit in high school: Increasing college attainment rates for underrepresented students. *Change: The Magazine of Higher Learning*, *35*(4), 42-48.

Hong, G. (1994). Guoji zhongxue wenping kecheng [International Baccalaureate Diploma Program]. *Global Education*, (5), 1-6.

How to become an IB school. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/become-an-ib-school/how-to-become-an-ib-school/

IB annual review 2016-17. (n.d.). Retrieved October 17, 2018, from https://www.ibo.org/contentassets/9faa0cd4d3eb4c4ab5f239f7342d4547/annual-review-2016-2017-en.pdf

IB assessment criteria. (n.d.). Retrieved October 15, 2018, from http://xmltwo.ibo.org/publications/DP/Group5/d_5_matsl_tsm_1205_1/html/content/exist/rest/app/tsm.xql@doc=d_5_matsl_tsm_1205_1_e&part=1&chapter=8.html

IB China. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/country/CN/

IB curriculum. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/curriculum/

IB diploma compared with other qualifications. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/university-admission/recognition-of-the-ib-diploma-by-countries-and-universities/ib-diploma-compared-with-other-national-exams/

IB facts and figures. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/about-the-ib/facts-and-figures/

IB mathematics. (2018a, Oct 15). Retrieved from https://www.ibo.org/programmes/diploma-programme/curriculum/mathematics/

IB mathematics. (2018b, Oct 15). Retrieved from https://www.ibo.org/programmes/diploma-programme/curriculum/mathematics/mathematics/

IB mission. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/about-the-ib/mission/

IB presentation of history. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/globalassets/digital-tookit/presentations/1711-presentation-history-of-the-ib-en.pdf

IB programmes. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/

IB statistical bulletin. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/contentassets/bc850970f4e54b87828f83c7976a4db6/dp-statistical-bulletin-may-2018-en.pdf

IB students' preparedness for success. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/news/news-list/in-the-ibs-2014-review-of-research-results-find-ib-programmes-to-have-a-positive-impact-on-student-preparedness-for-college-career-and-civic-life/

IB use of technology. (n.d.). Retrieved October 15, 2018, from https://ibpublishing.ibo.org/live-exist/rest/app/tsm.xql?doc=d_5_matsl_tsm_1205_1_e&part=1&chapter=5

Inkelas, K. K., Swan, A. K., Pretlow, J., & Jones, J. N. (2013). *Exploring the benefits of the International Baccalaureate extended essay for university studies at the University of Virginia*. Charlottesville: Center for Advanced Study of Teaching and Learning in Higher Education, University of Virginia.

Jenkins, C. (2003). *Perceptions of the International Baccalaureate Diploma Programme: A report for in inquiry carried out in 2003 at UK universities and institutions of higher education*. Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/contentassets/d1c0accb5b804676ae9e782b78c8bc1c/perceptionsoftheibdp2003.pdf

Jia, X. (2015). Meiguo AP shuxue kecheng bijiao yu fenxi [A comparison and analysis on the mathematics courses of the Advanced Placement program]. *Kaoshi Zhoukan*, (6), 14-16.

Jin, T. (2008). *Guoji wenping xiangmu xuesheng pingjia yanjiu [A research of student assessment of International Baccalaureate Diploma Programme]* (Doctoral dissertation). Retrieved from CNKI. (No. G40)

Johnson, B., & Turner, L. A. (2003). Data collection strategies in mixed methods research. In A. Tashakkori & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 297-319). Thousand Oaks, CA: Sage.

Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, *33*(7), 14-26.

Jonietz, P. L., & Harris, D. (2012). *World yearbook of education 1991: International schools and international education*. Abington, UK: Routledge.

Katz, S. (2006). The liberal arts in school and college. *Chronicle of Higher Education*, *52*(27), 46-47.

Key facts about the DP. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/what-is-the-dp/key-facts-about-the-dp/

Knight, J., & de Wit, H. (Eds.). (1997). *Internationalisation of higher education in Asia Pacific countries*. Amsterdam, the Netherlands: European Association for International Education, in cooperation with IDP Education Australia and the Programme on Institutional Management in Higher Education of the Organisation for Economic Cooperation and Development.

Koetzsch, R. E. (1997). *The parents' guide to alternatives in education*. Boulder, CO: Shambhala Publications.

Kong, D., & Ma, S. (2007). Meiguo gaozhong shuxue weijifen AP kecheng ji kaoshi jieshao [An introduction of the United States senior high school's AP Calculus and its Achievement Examinations]. *Shuxue Jiaoxue*, (1), 3-10.

Kyburg, R. M., Hertberg-Davis, H., & Callahan, C. M. (2007). Advanced Placement and International Baccalaureate programs: Optimal learning environments for talented minorities? *Journal of Advanced Academics*, *18*(2), 172-215.

Lai, H. (2013). Meiguo AP weijifen AB/BC kecheng tixi yanjiu [Research on AP calculus AB/BC course system]. *Journal of Mathematics (China)*, *3*, 5-10.

Lateer, B. (1999). The Middle Years Programme for International Baccalaureate. *California Association for the Gifted*, *17*(2), 50-51.

Lawson, A. J. (2005). *The effects of advanced math placement on advanced math students' Arkansas middle-level Benchmark Exam performance and school performance rating* (Doctoral Dissertation). Retrieved from ProQuest Dissertations Publishing. (No. 3223261)

Lee, M., Leung, L., Wright, E., Yue, T., Gan, A., Kong, L., & Li, J. (2014). *A study of the International Baccalaureate Diploma in China: Program's impact on student preparation for university studies abroad*. Hong Kong: Education Policy Unit, Faculty of Education, Hong Kong University. Retrieved from https://ibo.org/contentassets/d1c0accb5b804676ae9e782b78c8bc1c/ibchinafullreportenglish.pdf

Lee, M., Spinks, J. A., Wright, E., Dean, J., & Ryoo, J. H. (2017). *A study of the post-secondary outcomes of IB Diploma alumni in leading universities in Asia Pacific.* Bethesda, MD: International Baccalaureate Organization.

Leech, N. L., & Onwuegbuzie, A. J. (2009). A typology of mixed methods research designs. *Quality and Quantity*, *43*(2), 265-275.

Li, D. (2009). A look at international high-school curriculum: Take IBDP as example. *Journal of Schooling Studies*, *6*(8), 39-42.

Li, J., & Yao, L. (1998). Guoji wenping zuzhi he guoji wenping gaishu [An introduction of the IBO and IBDP]. *Primary and Secondary Schooling Abroad, 6*, 37-38.

Li, J., & Zhang, J. (2006). Meiguo AP tongjixue kecheng kaoshi zhi tese fenxi [The characteristics of Advanced Placement statistics]. *Shuxue Jiaoxue*, (7), 32-35.

Li, X. (2009). Meiguo gaozhong AP jihua weijifen kecheng pingshu [Review of AP calculus]. *Journal of Mathematics (China), 10*, 1-3.

Li, Y. (2006). Cong IB wenping kecheng de sheji kan guoji jiaoyu de jiaoyu linian [The IB Diploma Programme and its globally recognized educational philosophy]. *Primary and Secondary Schooling Abroad*, *5*, 8-12.

Li, Y., Chu, N., Lai, L., & Chen, W. (2018). Xinxi jishu yu shuxue jiaoxue de shendu ronghe: Yi IB shuxue guoji kecheng weili [A further integration of information technology and mathematics teaching: Take IB mathematics as an example]. *China Information Technology Education*, *9*, 61-64.

Liang, S. (2016). IB zhongxue xiangmu yu guonei chuzhong shuxue kecheng bijiao [A comparative study of IB Middle Years Programme and Chinese middle school mathematics courses]. *Shanghai Zhongxue Shuxue*, *7*, 64-69.

Liang, Y. (2012). Guoji kecheng yinjin yu bentuhua de shishi [Introduction of international programs and their localizations]. *Beijing Education*, *6*, 56.

Liu, M., Liu, L., & Jiang, H. (2015). Key development characteristics of the International Baccalaureate Diploma Programme. *Primary and Secondary Schooling Abroad*, *11*, 16-22.

Liu, Q., & Fan, B. (2014). Ideas of American AP courses and inspiration to Chinese universities admission. *Journal of National Academy of Education Administration*, *4*, 90-94.

Liu, S. (2008). IB shuxue kecheng pingjia jiqi tixi chubu jianjie [Introduction of IB mathematics and its comments]. *Shuxue Jiaoxue*, *1*, 12-14.

Liu, S., & Liu, J. (2008). Jianping IB shuxue gaoshuiping kecheng tigang jiqi bianqian [A brief commentary on IB mathematics higher level curriculum and its development]. *Shanghai Zhongxue Shuxue*, *10*, 45-48.

Liu, S., & Zhang, S. (2016). Comparative research on the international mainstream mathematics courses in high school. *International and Comparative Education, 5*, 71-75.

Lu, F. (2013). *Meiguo AP shuxuelei kecheng yu woguo gaozhong shuxue kecheng de bijiao yanjiu [The comparative study between Advanced Placement mathematics course in the United States and high school mathematics curriculum in our country]* (Masters thesis). Retrieved from CNKI. (No. G633.6)

Lu, H. (2000). Shuxue IB kecheng shezhi [The curriculum of IB mathematics]. *Journal of Vocational Education, 5*, 58-59.

Ma, F. (2008). IB shuxue (gaoshuiping) yu AP weijifen (BC) de yitong chutan [A preliminary study of the differences and similarities between IB mathematics higher level and AP calculus BC]. *Teaching Reference of Middle School Mathematics*, *4*, 55-57.

Ma, F. (2009). Guonei gaozhong shuxue yu IB (HL) shuxue zhong xiangliang bufen de neirong bijiao yu chayi fenxi [A comparative study of "vector" between Chinese high school mathematics and IB (HL) mathematics]. *Journal of Mathematics (China)*, *48*(3), 17-20.

Ma, F. (2010). Shili jianxi IB shuxue baogao de yaoqiu he tedian [Case study of the requirements and features of IB mathematics portfolio]. *Journal of Mathematics (China)*, *49*(4), 21-24.

Ma, F. (2011). Comparative study of calculus course in high school based on real practice [J]. *Journal of Mathematics Education*, *20*(6), 59-63.

Ma, F. (2017). Jifa xingqu wajue qianneng peiyang chuangzaoli: Ping IB shuxue neibu pingjia de zuixin yanbian [Interest stimulation, potential exploration, creativity cultivation: Commentary on the latest development of the internal assessment of IB mathematics]. *Journal of Mathematics (China)*, *56*(12), 27-36.

Ma, F., Liu, S., & Liu, Q. (2012). Shuxue xueke gaozhong guoji kecheng bijiao [A comparative study of high school mathematics international curriculum]. *Shanghai Education*, *14*, 56-59.

Ma, X. (2012). *Meiguo AP weijifen kecheng xiangmu jianjie jiqi qishi [A brief introduction of the Advanced Placement program and its enlightments]* (Masters thesis). Retrieved from CNKI. (No. G423)

Malkus, N. (2015). *AP at scale: Public school students in Advanced Placement, 1990-2013*. Washington, DC: American Enterprise Institute. Retrieved from http://www.aei.org/publication/ap-at-scale-public-school-students-in-advanced-placement-1990-2013/

Mathews, J., & Hill, I. (2005). *Supertest: How the International Baccalaureate can strengthen our schools*. Chicago, IL: Open Court.

Mattern, K. D., Shaw, E. J., & Xiong, X. (2009). *The relationship between AP exam performance and college outcomes.* New York, NY: College Board.

Mazzarol, T., Soutar, N. G., & Seng, S. Y. M. (2003). The third wave: Future trends in international education. *International Journal of Educational Management*, *17*(3), 90-99.

Mertens, D. M. (2014). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods*. Thousand Oaks, CA: Sage.

Mestre, J. P., & Lochhead, J. (1990). *Academic preparation in science: Teaching for transition from high school to college*. New York, NY: College Entrance Examination Board.

Moore, T. S., Lapan, S. D., & Quartaroli, M. T. (2012). Case study research. In S. Lapan, M. T. Quartaroli, & F. J. Riemer (Eds.), *Qualitative research: An introduction to methods and designs* (pp. 243-270). New York, NY: Wiley.

Morgan, R., & Crone, C. (1993). *Advanced Placement examinees at the University of California: An examination of the freshman year courses and grades of examinees in biology, calculus, and chemistry* (ETS Statistical Report No. 93-210). Princeton, NJ: Educational Testing Service.

Morgan, R., & Klaric, J. (2007). *AP® Students in college: An analysis of five-year academic careers* (College Board Research Report 2007-4). New York, NY: College Board.

Morgan, R., & Ramist, L. (1998). *Advanced Placement students in college: An investigation of course grades at 21 colleges* (ETS Statistical Report No. 98-13). Princeton, NJ: Educational Testing Service.

Nugent, S. A., & Karnes, F. A. (2002). The advanced placement program and the International Baccalaureate Programme: A history and update. *Gifted Child Today*, *25*(1), 30-39.

Odin, J. K., & Manicas, P. T. (Eds.). (2004). *Globalization and higher education*. Honolulu University of Hawaii Press.

Onwuegbuzie, A. J., & Collins, K. M. (2007). A typology of mixed methods sampling designs in social science research. *The Qualitative Report*, *12*(2), 281-316.

Onwuegbuzie, A. J., & Leech, N. L. (2004). Enhancing the interpretation of significant findings: The role of mixed methods research. *The Qualitative Report*, *9*(4), 770-792.

Onwuegbuzie, A. J., Leech, N. L., & Collins, K. M. (2010). Innovative data collection strategies in qualitative research. *The Qualitative Report*, *15*(3), 696.

Overview. (n.d.). Retrieved October 15, 2018, from https://reports.collegeboard.org/ap-program-results/overview

Peterson, A. D. C. (2003). *Schools across frontiers: The story of the International Baccalaureate and the United World Colleges*. Chicago, IL: Open Court.

Phillips, G., & Pound, T. (Eds.). (2003). *The Baccalaureate: A model for curriculum reform*. London, England: Kogan Page.

Plucker, J. A., Chien, R. W., & Zaman, K. (2006). Enriching the high school curriculum through postsecondary credit-based transition programs. *Education Policy Brief*, *4*(2), 1-11.

Poelzer, G. H., & Feldhusen, J. F. (1997). The International Baccalaureate: A program for gifted secondary students. *Roeper Review*, *19*(3), 168-171.

Professional development. (n.d.). Retrieved October 15, 2018, from https://apcentral. collegeboard.org/professional-development

Program summary report. (n.d.). Retrieved October 15, 2018, from https://secure-media.collegeboard.org/digitalServices/pdf/research/2018/Program-Summary-Report-2018.pdf

Recognition of the IB. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/ university-admission/recognition-of-the-ib-diploma-by-countries-and-universities/

Ren, C. (2007). Commitment to excellence: Comments on the AP courses in US senior high schools: About the improvement of US senior high school's education quality. *Curriculum, Teaching Material and Method*, *27*(12), 81-86.

Ren, C. (2008). SAT and ACT: College Entrance Examination in the United States: Some suggestions to the innovation of college entrance examinations in China [J]. *Theory and Practice of Education*, *28*(3), 27-30.

Renaud, G. (1974). *Experimental period of the International Baccalaureate: Objectives and results*. Paris, France: UNESCO Press.

Rossman, G. B., & Wilson, B. L. (1985). Numbers and words: Combining quantitative and qualitative methods in a single large-scale evaluation study. *Evaluation Review*, *9*(5), 627-643.

Russo, F. (2000). Beyond advanced placement. *Village Voice*, *45*(15), 90-91.

Saavedra, A. R. (2016). Academic civic mindedness and model citizenship in the International Baccalaureate diploma programme. *The Social Studies*, *107*(1), 1-13.

Score setting and scoring. (n.d.). Retrieved October 15, 2018, from https://apcentral. collegeboard.org/about-ap/how-ap-develops-courses-and-exams/score-setting-and-scoring

Sechrest, L., & Sidani, S. (1995). Quantitative and qualitative methods: Is there an alternative? *Evaluation and Program Planning*, *18*(1), 77-87.

Shah, S., Dean, M., & Chen, Y. C. (2010a). *High school student engagement among IB and non-IB students in the United States: A comparison study*. Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/globalassets/publications/ib-research/myp/highschoolstudentengagementamongibandnonibstudentsintheus2010.pdf

Shah, S., Dean, M., & Chen, Y. C. (2010b). *Academic performance of IB students entering the University of California System from 2000-2002*. Geneva, Switzerland: International Baccalaureate Organization.

Simms, D. (1982). *Comparison of academic performance between AP and non-AP students at the University of Michigan*. Unpublished manuscript.

Soares, B. (2001). *Can student aptitude and attitude assist in placing students in appropriate advanced placement mathematics courses?* (Doctoral Dissertation). Retrieved from ProQuest Dissertations Publishing. (No. 3007550)

Song, H. (2008). *An analysis of high school's curricula in international school* (Masters thesis). Retrieved from CNKI. (No. G632)

Song, J. (2012). *IB shuxue kecheng jianjie ji qishi [An introduction of IB Mathematics and its enlightments]* (Masters thesis). Retrieved from CNKI. (No. G633.6)

Song, J., & Wang, L. (2015). Shixi meiguo AP kecheng he IB kecheng de xuefen rending: Yi xiaweiyi zhou de daxue weili [Analysis of credit recognition of American AP and IB courses: Taking colleges and universities in the State of Hawaii]. *Journal of Xinyu University*, *20*(5), 139-141.

Tang, S., Wen, T., & Liu, J. (1999). Zhongxue shuxue jiaoyu de zhongxi bijiao [A Sino-Western comparative study of high school mathematics education]. *Journal of Shanghai Education Research*, 5, 2-9.

Tao, M. (2012). You guojiban shuangyu shuxue jiaoxue yinfa de sikao [Reflection on bilingual mathematics teaching in international schools]. *Gaozhong Shuxue Jiaoyuxue*, *4*, 9-10.

Taylor, M. L., & Porath, M. (2006). Reflections on the International Baccalaureate program: Graduates' perspectives. *Journal of Secondary Gifted Education*, *17*(3), 149-158.

Teddlie, C., & Tashakkori, A. (2003). Major issues and controversies in the use of mixed methods in the social and behavioral sciences. In A. Tashakkori, & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 3-50). Thousand Oaks, CA: Sage.

Teddlie, C., & Tashakkori, A. (2009). *Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioral sciences*. Thousand Oaks, CA: Sage.

Tian, L. (2004). Zai gaozhong shang daxue: Zoujin meiguo daxue yuxiu kecheng [Take college-level courses in high school: Approaching Advanced Placement Program]. *Journal of World Education*, (6), 27-30.

Tookey, M. E. (1999). The International Baccalaureate: A program conducive to the continued growth of the gifted adolescent. *Journal of Secondary Gifted Education*, *11*(2), 52-66.

Understanding DP assessment. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/assessment-and-exams/understanding-ib-assessment/

Wang, L., & Wang, X. (2016). Meiguo zhongdeng jiaoyu yu gaodeng jiaoyu xianjie de daxue xianxiu kecheng: AP kecheng he IB kecheng de tese yu sikao [The bridge between secondary education and higher education in America: Feature and reflection of the AP and IB Courses]. *Examinations Research*, (3), 91-96.

Wang, Y. (2016). *IB shuxue jiaocai zhong gailv zhuanti de yanjiu [A Study of the Probability Content in IB Mathematics Curriculum]* (Masters thesis). Retrieved from CNKI. (No. G633.6)

Wang, Z. (2017). Qiantan zhongxuesheng xuexi AP weijifen de yiyi [Discussion of the meaning of learning AP calculus in high school]. *Shuxue Xuexi yu Yanjiu*, *2*, 15-16.

Weng, Y. (2008). International high school courses in the background of globalization - analyzing and commenting IB and AP courses. *Journal of Ningbo Institute of Education*, *10*(4), 30-33.

What is CAS Project? (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/curriculum/creativity-activity-and-service/cas-projects/

What is EE? (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/curriculum/extended-essay/

What is the DP? (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/what-is-the-dp/

What is TOK? (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/programmes/diploma-programme/curriculum/theory-of-knowledge/what-is-tok/

What you need to know about IB. (n.d.). Retrieved October 15, 2018, from https://www.ibo.org/globalassets/digital-tookit/presentations/1506-presentation-whatyouneedtoknow-en.pdf

Wray, D. (2013). *Student perceptions of the value of the International Baccalaureate extended essay in preparing for university studies*. Coventry, England. University of Warwick. Retrieved from https://www.ibo.org/globalassets/publications/ib-research/dp/dp_eewarwick_summary_may132013_eng.pdf

Wu, C. (2014). IB shuxue biaozhun shuiping he gaoshuiping de pingheng [The balance of IB mathematics standard level and higher level]. *Shanghai Education*, (32), 34-36.

Xiao, H. (2014). *Zhongmei gongli gaozhong guoji kecheng yinjin yu shishi yanjiu [Research of imported international curriculum of Chinese-foreign cooperative course project]* (Masters thesis). Retrieved from CNKI. (No. G632.3)

Xie, Y., & Shen, Y. (2013). Trends of development on international preparatory mathematics curriculum and some inspirations. *Primary and Secondary Schooling Abroad*, *5*, 61-65.

Xu, D. (2017). Ideal and reality gap: AP in the United States and China "College Courses." *Examinations Research*, *1*, 104-110.

Xu, H. (2001). Comment on international school and their curriculum. *Theory and Practice of Education*, *21*(6), 41-44.

Xu, H., & Xiang, Y. (2015). A comparative study on mathematics curriculum between IBDP and high school. *Primary and Secondary Schooling Abroad, 7*, 53-59.

Xu, P., Xia, H., & Chen, F. (2015). IB international curriculum: Philosophy and action. *Primary and Secondary Schooling Abroad, 2*, 54-58.

Xu, S., & Gao, G. (2012). Problem and recommendation on implementing international curriculum to high school students from mainland of China: Case of Shanghai. *Research in Educational Development*, *6*, 11-15.

Yamamoto, B. A., Saito, T., Shibuya, M., Ishikura, Y., Gyenes, A., Kim, V., Mawer, K., & Kitano, C. (2016). *Implementation and impact of the Dual Language IB DP Programme in Japanese secondary schools*. Bethesda, MD: International Baccalaureate Organization. Retrieved from https://www.ibo.org/contentassets/d1c0accb5b804676ae9e782b78c8bc1c/research-dp-in-japan-full-eng.pdf

Yang, B., & Dai, Y. (2012a), Three cores of DP. *Shanghai Education, 14*, 29-33.

Yang, B., & Dai, Y. (2012b), Six subject groups of DP. *Shanghai Education, 14*, 29-33.

Yang, M. (2018). Quanqiuhua beijing xia jiaoshi guoji siwei fazhan de yiyun, kunjing yu lujing: Jiyu IB kecheng jiaoshi de kaocha [The meaning, dilemmas and ways of teachers' international mindedness in the context of globalization: Based on the study of IB teachers]. *Journal of Guizhou Education University*, *34(*02), 78-84.

Yang, X. (2013). Meiguo gaozhong AP kecheng gailun ji duiyu jiaoyu moshi de qishi [An introduction of the AP courses in the US high schools and its inspiration for education mode]. *Modern Education Science*, *5*, 31-32.

Zha, Q. (2003). Internationalization of higher education: Towards a conceptual framework. *Policy Futures in Education*, *1*(2), 248-270.

Zhang, H. (2014). IB kecheng zai zhongguo [International Baccalaureate in China]. *Journal of Tianjin Academy of Educational Science*, *1*, 56-58.

Zhang, Y. (2012). Gaozhong weijifen jiaoyuxue xianzhuang de diaocha yanjiu [Research on the status of teaching and learning of high school calculus]. *Curriculum, Teaching Material and Method*, *8*, 83-89.

Zhao, D. (2010). A study on the practices of Advanced Placement in the pursuit of educational equity. *Journal of Schooling Studies*, *7*(11), 52-55.

Zhao, H., & Zhang, X. (2009a). Meiguo gaozhong AP kecheng pingshu [Comments on the AP Courses in high schools in America]. *Journal of Shanghai Educational Research*, *4*, 39-41.

Zhao, H., & Zhang, X. (2009b). Enlightenments to the high school curriculum reform based on AP course in America. *Modern Education Journal*, *3*, 80-83.

Zhuang, L. (2017). *Research on calculus exam for China Advanced Placement –based on the CAP, AP, IB, A-Level of the calculus exam* (Masters thesis). Retrieved from CNKI. (No. G642.4)

Appendix A

AP School Recruitment Handbook

1 宁外AP中心简介

宁波外国语学校中美合作国际高中课程教育项目是2011年由宁波市教育局同意，浙江省教育厅批准，教育部备案的中外合作办学项目。中美合作高中课程教育项目坚持充分结合中美基础教育的优势，在完成高中课程的基础上，为有志于进入美国、加拿大、英国等一流名校的优秀学生提供更具有挑战性高级课程，学校引进提供广受美国大学认可的AP课程、对名校入学申请有重要影响的SAT和TOEFL预备课程、美国名校申请指导课程，培养提高学生获得全球高等教育所需要的素质，提高学生申请美国名校的综合竞争力，为优秀的高中学生提供优质的国际教育资源，开启通往世界一流大学的快捷通道；为将来成长为具有国际视野、全面发展的精英人才打下良好基础。

中美合作高中国际课程教育项目从2011年9月招生以来，在各方的关心和支持下健康稳步发展，现有高一年级3个班，高二年级3个班，高三年级3个班，合计学生近240人；2013年首届毕业生有近68%的学生进入美国前50名文理学院和大学深造；2014届毕业生有近82%的学生进入美国排名前50名的大学和文理学院深造，有不少学生被加州伯克利、佐治亚理工，明德学院、克莱蒙特麦肯纳学院、多伦多大学，瑞士洛桑酒店管理学院等世界著名高校录取。2015届毕业生在延续前两届辉煌的基础上再创佳绩，有近85%的学生进入美国排名前50大学和文理学院及英国的前20名大学深造，其中不乏有剑桥大学、芝加哥大学、香港大学、圣母大学等全球顶尖学府。

2016届53名毕业生近70%的学生进入美国排名前50的大学和文理学院，近100%的学生进入美国排名前100的大学和文理学院，并获近千万奖学金。

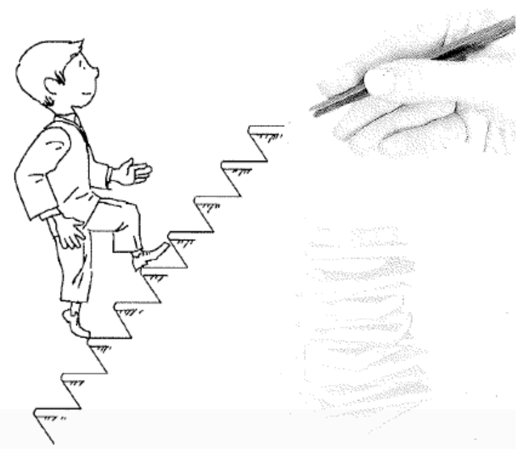
宁波外国语学校中美合作国际高中课程教育项目，继续以“奋发进取、探索创新”的钻研精神和“实事求是、讲求实效”的务实作风，以办成宁波市“教育国际化窗口”为目标而奋发努力！

2 美国AP课程介绍

美国AP课程全称为Advanced Placement Program，即大学先修课程，是为那些优秀的美国高中学生在完成普通高中教学任务后，提前学习美国大学专业课程的一种课程，由美国大学理事会（College Board）开发，已在美国17,000多所高中里普遍开设。通过AP课程的学习和考试，学生更容易被大学，尤其是名校录取。学生修学AP课程获得的学分可在其进入大学后底座大学学分，达到免修课程、缩短学士、折抵学分、跳级、节省学费和获奖学金等待遇。

AP课程具有很高的国际认可度。AP考试在全球80个国家举行，英国、加拿大、澳大利亚等过也将此作为发放奖学金的主要条件之一；同时，80多个国家超过5,000所大学得到认可AP为其入学标准或参考标准，包括哈佛、耶鲁、牛津、剑桥等世界名牌大学。

根据美国大学升学顾问委员会（NACAC）在全美范围内所做的调查，美国大学已经普遍把学生在AP课程学习及考试中的表现作为衡量其能否胜任大学学习的依据，AP课程的学习及考试成绩也已经成为影响美国大多数大学决定是否录取时最为重要的因素之一。



3 AP 中心培养目标

外语见长、全面发展、个性鲜明，兼具中国心、世界眼、民族魂、国际范的国际化人才。

1）中方课程设置及各学科课时分布

年段	学期	语	数	英	物	化	生	政	史	地	计	技	音美	班会	体育	社团	选修	总计
高三	S2	/	/	/	/	/	/	/	/	/	/	/	/	1	2	1	1	5
	S1	/	/	/	/	/	/	/	/	/	/	/	/	1	2	1	1	5
高二	S2	/	/	/	/	/	3	/	/	/	/	2	1	1	2	1	1	11
	S1	4	4	2	/	/	3	3	/	/	/	2	1	1	2	1	1	24
高一	S2	3	4	2	3	3	/	2	2	2	2	/	1	1	2	1	1	29
	S1	3	4	2	3	3	/	2	2	2	2	/	1	1	2	1	1	29

2）中美合作高中国际课程教育项目学生学业水平考试方案 （学考方案）

年级	时间	会考科目	备注Notes
高二	10月	物理、化学、历史、地理	为不影响毕业以及AP课程，学考课程在高二基本结束。
	4月	语文、数学、政治	
	6月	英语	
高三	10月	生物、技术	

注：1.浙江省新课程改革规定：从2015年开始，学业水平考试（除英语外）每年10月和4月全科开考。

2.所有中方课程都坚持形成性评价。根据形成性评价的指标，如课堂参与、作业表现、阶段性测试或项目化作业等，由卷面成绩和形成性评价成绩按一定比例折合成分数后，学生的期中期末成绩以等级形式（A, B, C, D, F）呈现，记入学生学业报告单。

(3) 外方课程设置

概述

宁波外国语学校AP中心致力于为学生创造一个能够参与充满严苛的，具有学术性挑战的课程的机会，这有助于他们进入正规的西方大学，并顺利完成大学的学业。通过AP课程的学习，学生可以在学科上取得进步，而且不但能够帮助他们在该学科上取得成功，更能使他们的思维和情感达到大学的水平。

通过大学同级别的AP课程的学习，学生可以获得在普高所获取不到的知识；通过AP考试，他们能够有机会修得学分或是提前进入多数的学院和大学。通过AP课程的学习可以让学生：

- 拓宽国际视野：用多种视角去探索世界
- 在大学预备中获得优势：强化解决问题的能力和发展学习的能力，并能够应对严格的课程要求
- 在大学录取中脱颖而出：专业学术上的强化，行为举止上的成熟，大学进入的准备

宁波外国语学校开设的AP课程是国内公立学校中最综合的，目前已开设25门AP课程。课程结构旨在为学生提供课程选择的多样性和机会，使其能发掘自身潜能和兴趣。

AP 课程项目旨在鼓励学生认真考虑他们的专业以及（或者）未来长期的学习目标及未来从事领域。每一学年，选课期间都将由中心专业的升学指导团队面向学生和家长，根据学生情况，个性化了解和指导帮助学生完成选课，这样个性化的设计能确保学生在学习期间成为某一领域的“专家”，而这一领域即将成为学生兴趣最大化和潜力最大化的结合点。

AP课程的学习比普高课程的学习难度更高，要求更多。普高课程需要花大量的时间和完成大量的练习，但是AP课程相对比较有深度。学生需要意识到学习AP课程是一件很严肃的事情。学生所选课程的难度和要求严格程度与其未来申请成功是正相关的。因此，我们鼓励学生选修难度大、要求高的课程，AP中心对此有存档记录在案，将有利于学生未来申请进入海外知名大学。

十年级：AP预备课程

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<tr><th colspan="3">组一</th><th colspan="3">组二</th><th colspan="3">组三</th></tr>
<tr><td colspan="3">学术英语I [8]</td><td colspan="3">学术英语I [6]</td><td colspan="3">学术英语I [4]</td></tr>
<tr><td colspan="3">全球视野 [4]</td><td colspan="3">全球视野 [4]</td><td colspan="3">AP人文地理[5]</td></tr>
<tr><td colspan="3">AP微积分预备 [4]</td><td colspan="3">AP微积分预备 [4]</td><td colspan="3">AP微积分预备[4]</td></tr>
<tr><td colspan="3">SAT®科学[4]</td><td colspan="3">AP 物理 I [6]</td><td colspan="3">AP 物理 I [5]</td></tr>
<tr><td>AP美术预备</td><td>AP 电脑科学预备</td><td>批判性思维</td><td>AP美术预备</td><td>AP 电脑科学预备</td><td>批判性思维</td><td colspan="3">德语 I [4]</td></tr>
<tr><td colspan="6">*循环上课</td><td>AP美术预备</td><td>AP 电脑科学预备</td><td>批判性思维</td></tr>
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十一年级:

<table>
<tr><th>英语</th><th>数学</th><th>组块 1</th><th>组块 2</th><th>组块 3</th><th>组块 4</th></tr>
<tr><td>AP英语语言A</td><td>AP 微积分 AB</td><td colspan="4" rowspan="2">每个组块可供选择的科目有：
科学：AP物理I 和II；AP化学
人文学：AP经济学；AP世界史；AP美国史；AP人文地理
语言学：德语II
美术：2D设计；绘画
电脑科学A和应用电脑科学</td></tr>
<tr><td>学术英语 II</td><td>AP 微积分 BC</td></tr>
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十二年级:

<table>
<tr><th>英语</th><th>数学</th><th>组块 1</th><th>组块 2</th><th>组块 3</th><th>组块 4</th></tr>
<tr><td>AP 英语语言 B</td><td>AP 统计学</td><td colspan="4" rowspan="3">每个组块可供选择的科目有：
科学：AP物理C；AP化学；AP生物学；AP环境科学
人文学：AP经济学；AP世界史；AP美国史；AP心理学
语言：德语III
美术：2D设计；绘画；3D设计；AP美术史
应用电脑科学</td></tr>
<tr><td>AP 英语文学</td><td>AP 微积分BC</td></tr>
<tr><td></td><td>AP 电脑科学 A</td></tr>
</table>

11年级和12年级的美术和人文学因为人数或安排的问题会被合并。Ap课程一般一周会上5节。

高一

我们的高一是AP学科的预备年，是学术上趋向严谨并为未来学习打下坚实基础的一年，我们将会帮助学生在英语能力培养、学习技能提升、学习习惯养成三方面得到提升。希望学生们通过高一一年来适应高中生活，并用效率武装自己为高二的全面提高自身语言以及学科能力做好准备。

学术英语 I：

本课程在阅读各种体裁、撰写多种作文、听、说等方面有一定的指导和实践。通过这门课学生的各种技能会有所提升，这将为他们在今后完成阅读和写作任务方面，以及完成AP英语语言和AP英语文学的科目起到极大的帮助。学生在完成项目的时候，要求他们了解和评估各种文本和视觉材料。

全球视野：

学生通过批判性分析和反思全球意义性的问题，可以开阔视野。全球视角课程更倾向于技能而不是具体内容学习。学生通过分析和评估被称为关键路径的观点和看法来发展研究，思考，推理和沟通技巧。同时，通过大量的团队合做还可以提升协作能力。

AP 微积分预科：

本课程侧重于在用符号推理和分析方法来表示数学的情况下，强调概括和研究数学概念和它们之间的关系。学生还使用演说，工具和技术来模拟函数，包括多项式，指数和三角函数，并解决现实生活中的问题。本课程的重点是在准备上AP微积分的学生有高等微积分的概念。

批判性思维：

这一学期的课程介绍了分析思维的基础，包含所有理性的话语和询问的基础。它让学生在参与社会重大问题讨论时，能够在有机会能够深入思考，能够拥有有结构化的说话方式。该课程可以让学生作出基于证据和论据的合理的决定，而不是假设和偏见。

AP 电脑科学预科：

这学期课程教授计算机编程语言。学生将通过学习图形设计以及实际应用，为AP计算机科学课程打下基础。

AP 美术预科：

这一学期的课程为学生提供了创造艺术的机会，反映了他们对元素的先验知识，艺术的主体认知和他们对艺术的应用。学生在这个课程中，通常会担心三个问题：（1）学生作业的质量问题；（2）学生对视觉兴趣或问题的注意力问题；（3）学生对艺术家的形式、技术和表现方式的广泛体验的需要。

*对于招生过程中确实有才华的学生，我们提供了一个AP美术2D设计的特殊路径

SAT科学：

这门课是为了让学生更好的准备SAT生物，化学和物理考试。学生可以学习各种主题，如细胞和病毒的结构和功能；生物演化；原子结构；元素周期表；气体行为；键合；能量和动量守恒；热力学；力；波的特性和行为；量子物理。

AP 人文地理：

这门AP课程全面的介绍了人类的理解、使用模式和过程的研究，地球的表面的变更。学生使通过空间概念和景观分析，研究人类社会组织及其带来的环境后果。他们也会学习地理学家在他们研究和应用

时，所使用的方法和工具。该课程体现了国家地理标准的目标（2012）。

AP 物理I

AP 物理1是一个基于代数，介绍大学水平级别物理的课程。这门课程基于六大理念，包括核心科学原理、理论和过程，跨越传统界限，以及提供对物理世界的广泛思考。学生通过探究性的调查来培养自己对物理学的理解，例如探索诸如牛顿力学（包括旋转运动）、工作、能量和动力、机械波和声音以及简单的电路。

德语I

在当今全球社会中，掌握多种语言的能力是沟通和文化理解的重要组成部分。歌德考试A1：开始德语1成人德语考试。它证明了考生获得基础的语言技能，在欧洲共同语言参考框架（CEFR）六个级别中对应的第一级别（A1）。

十年级组别的确定主要是由英语基本词汇测试决定的，而这个也将带领学生进入学术的一年。它的测试包含书面接受词汇知识的能力，这个能力也是阅读的需要。我们会通过进行阅读，写作和口语测试对现有的十年级学生进行分组。

高二

高二学生可以根据高一的选修情况进行自行选课，学生会被要求选择AP微积分或AP微积分ABC其中之一，需在AP英语和学术英语II之间选修一门英语课程（学生需满足一定要求才能继续学习AP英语，但如若他们未达到要求而决定选修AP英语课程，我们将提供必要的辅导来使其满足要求）。我们鼓励学生修读AP自然科学课程。

学生必须从下述列表中选择3-4们课程学习：

*AP人文地理或AP世界历史； *AP物理 II或AP环境科学； *Ap美术二维设计或绘画；

*AP微观经济学；*AP预科德语II；*AP计算机科学编程或计算机科学原理；AP美国历史

高三

高三学生的学术重心将集中于完成强势的学科，以及选修严谨的学术安排为大学做准备，以及完成一流大学的申请工作。学生必须从下述列表学科中选取5-7门课程学习：

*AP计算机科学编程/*AP微积分BC/ AP统计学(学生必须从这三门课中选取两门进行学习；*AP宏观经济学； *AP英语与写作B/AP英国文学与写作（学生需通过高二AP英语与写作B）/学术英语 III；*AP德语（学生需完成预科德语I和II）；*AP物理C或AP生物；*AP化学或AP环境科学；*AP心理学；*AP世界史/AP 人文地理/AP艺术史；AP欧洲历史

学生成绩将以形成性评价方式呈现，与美国高中评估标准一致。评等第的过程将运用标准的测评方式对于学生在某一门学科上所取得的成绩施以不同的标准。不同于其他评价体系，即一考定成绩的制度，相反，大学前期课程（AP课程）的评估是由几种评估方式共同完成的，包括：单元测试、课堂测试、家庭作业、课后作业、课堂参与、以及项目或实验操作。每一部分的比重都将在学期初通过教学大纲呈现其所占比重。

学分

宁波外国语学校跟很多典型的美国高中一样都采取5分学分制

分数段	得分	所获学分		
		非AP课程	荣誉课程*	AP课程
90-100	A	4	4	5
80-89	B	3	3	4
70-79	C	2	2	3
60-69	D	1	1	2
0-59	F	0	0	0

浙江省中方课程等第标准：

A: 85-100; B: 75-84; C: 65-74; D: 55-64; F: 54以下

*为了能够获得AP课程的学分，学生必须要参加AP考试。如果学生没有参加AP考试，那么他所修的AP课程会变成荣誉课程。

平均学分绩点

平均学分绩点是学分绩点的总和除以学分总和。学分取决于该学期学生所修的课程数量，平均绩点的范围在0.0到4.81。

课程评估

AP 课程评估分为两种方式，即大学理事会组织的AP统考及学期成绩。每学期末（十月下旬、一月上旬、四月中旬）将进行学科期末测试。这些考试均占该门学科总评的25%-45%（与学期初发放的课程大纲一致）。由大学理事会组织的AP统考于每年5月份进行。评价标准由1-5分5个档次相应反应由低到高的学术水平。学校要求选修了该门课程的学生参加将该课的AP考试。

美国大学录取时评价标准主要是来自平时测试成绩。通常来说，3分的等第被视为“通过”，而4分、5分的等第则被认为是“卓越”，将会为该生大学学习挣得相应学分。在某些情况下，对于一些特别的课程，2分的成绩也会被认为是“卓越”。AP考试并非仅是一个抽象的指标来计算GPA。

Engrade 课程体系

宁波外国语学校国际部AP中心采用的engrade评价系统有别于应试型评价系统，是持续的、渐进的且符合美国高中评估标准。作为家校间全面沟通体系的一部分，学生成绩均输入到engrade系统中，家长和学生可上网查看分数，以便家长能及时了解孩子的学习动态，而非等到期末考试后才能收到孩子的成绩单。开学初，所有学生会发到一张课程大纲，从而了解这一学年他们在这门课上的表现是如何被评价的。该大纲还包含关于该门课程的细则，如评分细则。

不同于一学年两学期制，AP课程一学年分三学期，因此一学年有三学期的成绩报告。Engrade系统中，老师的评语为一学年两次，同样可供学生及家长查阅。

7 升学指导

“AP项目学生大学升学指导计划”由狄邦教育创设，是国内首次将美国高中所普遍实施的counseling（大学升学指导）理念引入中国。大学升学指导计划是美国大学升学顾问委员会根据美国一流名校的录取标准，结合本土顶尖高中的大学升学指导体系而专门设计的课程。美国本土的重点高中，尤其是美国著名大学的生源基地学校，广泛采用该课程。基于对美国教育的深刻理解和对中美高中课程体系的对比研究，狄邦教育与美国本土的专业咨询机构、非盈利性组织合作，共同设计了针对中国AP课程学生的美国大学升学指导课程。

“大学升学指导课程”从学生的自我认知开始，注重个人的全方位成长。通过有效地利用高中三年的课余时间，从SAT、TOEFL考试准备、课外活动参与和大学升学指导三个方面，以小型讲座、一对一辅导、小组活动、实地参观与考察等形式，让学生充分认识美国名校的申请要素，进而制定个性化的申请方案，在专家的指导下，完成申请文件；同时，通过采用美国名校广泛认可的申请资料存储传输系统，提高学生申请资料的可信度，从而帮助学生成功申请世界名校。

Appendix B

IB School Recruitment Handbook

课程使命：
宁波效实中学IB中心的使命是让学生成为富有创造力、好奇心、责任感，坚韧不拔的全球公民，让他们能在未来生活和事业中发挥最大的潜力，使之拥有一种学习态度，能够在大学毕业后的未来人生中出类拔萃。

学校的目标是在培养学生全球视野，使之保持深厚爱国主义情怀的同时，开拓国际视野，能与不同文化和平共处。

IB课程使命：
IB课程致力于培养善于探究、知识渊博、具有爱心的青年，使他们通过跨文化理解和相互尊重，携手创造更加美好、和平的世界。为此，IB组织与学校、政府、国际组织一起合作，开发在国际教育和评估严谨度方面有竞争力的项目。这些项目鼓励遍布全球的学生成为积极主动、富有同情心的终身学习者，他们能求同存异，包容理解。

宁波效实中学IB中心理念：
我们的教育理念是为能力各异、背景不同的学生提供高质量的国际教育。我们重视文化多样性，力图提供关怀、创新和具启发性的环境，充分激发学生潜能，为每个学生的成功而欢呼。

我们相信，IB国际中心将把孩子培养成全面发展的人，为他们提供学会学习的技能，并帮助孩子在跨文化环境中拥有成功的生活。IB国际中心鼓励父母积极参与这个过程，希冀其提供更为广泛的学习途径。

我们致力于把学生培养成高层次的思考者，使他们具有创业和自我激励的研究技能以及社会技能，并成为高效的团队成员。在追求我们的理念中，我们的目标是:
●使所有的学生都能以优异的状态进入下一阶段的教育；
●培养学生对其他文化和民族的理解和尊重；
●培养学生热爱学习，谙学术研究之道，成为理想的大学生和终身学习者；
●培养学生的自信心、融入社会的技能和行为能力，使学生在每个发展阶段获得个人满足感和成就感；
●培养学生的主动性，以及投身到广泛的教育、社会和体育活动中的能力；
●促进其行为规范，促使其尊重他人需要和权利；
●为学生提供丰富多样、启发式和引人入胜的学习经验；
●成为宁波和该地区的教育资源，成为卓越教育的灯塔。

宁波效实中学IBDP课程实验班于2012年经宁波市教育局同意，浙江省教育厅批准，获教育部正式备案（项目批准书编号：PDE33US5A20110505N），并于2012年9月正式开课。

秉承“国际化的教学融入本土化的教育”，IBDP国际课程实验班纳入效实中学统一管理。实验班采用小班制授课，外籍学术校长负责教学管理，制定教学计划，实施教学质量管理。外籍授课教师具有丰富的国际课程教学经验，并均接受过IBO的相关培训。学校设立的升学指导办公室由多名从海外学成归来、经验丰富的老师组成，三年全程指导学生的大学升学工作，为学生的海外大学申请和入学保驾护航。

从2012年9月开办以来，IBDP课程实验班在各方的关心和支持下健康稳步向上发展，现有在校人数184人。凭借出色的学术能力和过硬的综合素质，毕业生海外大学录取情况喜人。2015届至2017届毕业生录取美国大学前50排名的比率（其他国家同比）高达80%，并获得超过300万美金奖学金。效实中学IB课程实验班结合中西方基础教育的优势，为优秀学生提供优质的国际教育资源，培养具有全球视野的国际化人才，帮助他们实现进入世界一流名校的梦想。

In April, 2012, having duly registered with the National Ministry of Education, the provincial and municipal education authorities approved the International Baccalaureate Program (IBDP) at Xiaoshi High School, which was launched the same academic year.

To localize international teaching, the International Baccalaureate Diploma Program is managed by Xiaoshi High School. The program is operated with small-size classes, and a center principal takes charge of its academic and daily management. All the foreign teachers are certified with rich international teaching experience, and receive IBO training. The college counseling team is made up of highly experienced college counselors with overseas educational backgrounds. Their work ensures the success of students' college applications and admissions.

Since its establishment in September 2012, IBDP has achieved steady progress with support from all parties. Currently there are 190 students enrolled. Our graduates, with their outstanding academic performance and overall ability, have received offers from many international universities. Up to 80% of students from class of 2015 to 2017 achieving offers from top 50 American universities (same ratio in other countries), winning accumulated scholarship more than USD 3 million. The IB program combines the best of both Chinese and western based education. It provides high quality educational resources and aims to cultivate students with a global vision. It helps students realizetheir dream of studying at leading universities around the world.

学制与课程设置 /Program Duration & Course Setting

预备课程及IB文凭课程

效实中学国际课程实验班学制三年，修满学分并通过IB全球统考后可获IB课程高中毕业文凭。

高一年级学习预备课程

预备课程采用IGCSE课程体系，并进行改良设计，开设英语、数学、物理、化学、生物、地理、商业、经济学、艺术、体育、全球视野与IB技能（学术探究、活动、创意和服务等课程）等课程。学习预备课程将帮助学生在语言、学术能力等下阶段的学习做好全面准备和衔接。

高二、高三年级学习IB文凭课程

包括三门核心课程和六个组别的学科。在IB文凭课程中，学生可选择学科，规则如下：
学生选择6门学科进行学习；
学生选择3门高级(HL)学科和3门标准级(SL)学科；
学生应根据自己的强项选择高级（HL）学科；
学生应选择自己爱好或者感兴趣的学科。

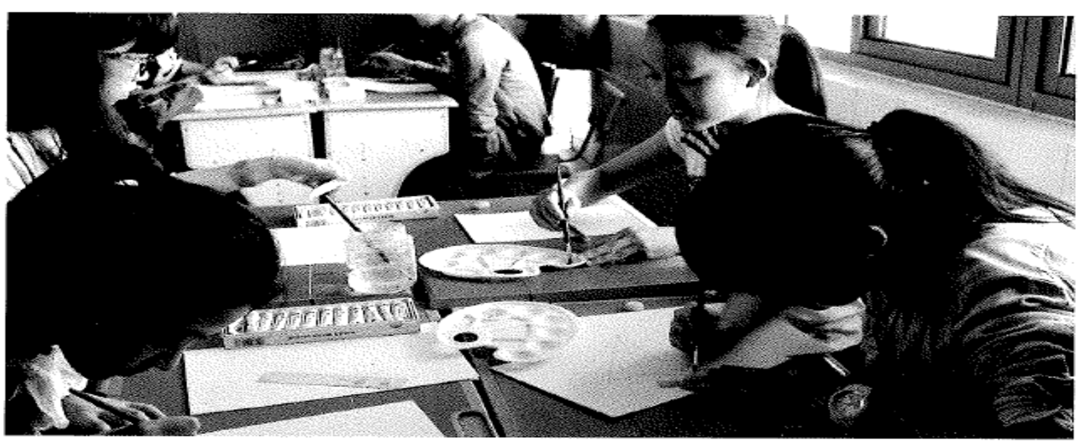
中方学考课程

为了培养兼具家国情怀和国际视野的学生，学校将中方课程纳入浙江省学考管理系统。自2016级新生起效实中学国际课程实验班学生将修学国家规定的高中课程，并参加浙江省普通高中学业水平考试，达标后可获国内高中毕业文凭。

三门核心课程 /IB Diploma Programme Core

知识理论课是一门必修的跨学科课程，目的是培养学生的判断与综合归纳能力，鼓励学生对基础知识进行质疑，防止主观臆断和思想意识上的偏见，增强学生以理性为基础进行分析和表达的能力。

拓展论文是从6门学科中选择某一主题进行有针对性的深入研究。旨在提高学生进行高层次的研究与写作的能力，同时激发学生的知性与创造力。它为学生提供了一个在学科老师指导下，自行选题并展开独立研究的机会。最终成果将是一篇结构清晰、论证合理、思想连贯的4000字的正式论文。

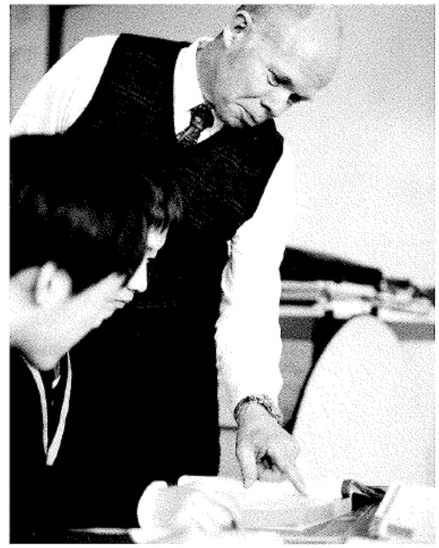


创造、行动与服务课程鼓励学生进行创新，提高艺术修养，进行坚持不懈的锻炼，关心他人，发挥与别人合作的精神。该课程将由外方教师和升学团队共同担任指导教师。

六类学科课程 /Subjects

语言A1(母语)与世界文学

语言A1课程关注的是我们对世界的概念、阐释和经历；文学鼓励我们进行独立、原创、批判以及清晰的思考。文学课程鼓励学生通过广泛的文学研究，在欣赏文学的同时养成批判性阅读的能力。作品研读将依据其文学与文化背景，对个别文字和段落进行仔细研究，并将重要手法纳入考量范畴。



Language A1(Chinese & Literature, HI /SL)
Chinese & Literature, HL/SL

Literature is concerned with our conceptions, interpretations and experiences of the world. It encourages independent, original, critical and clear thinking. Through the study of a wide range of literature, the literature course encourages students to appreciate literature and to develop an ability to reflect critically on their reading. Works are studied in their literary and cultural contexts, through close study of individual texts and passages, and by considering a range of critical approaches.

语言B（母语以外的现代语言）

语言B课程是为具备一定英语基础的学生设计的语言课程。课程着重于英语语言技能的获得和发展。这些语言技能将通过学习和应用一系列书面和会话材料来得到提高。

Second Language (English B, HL/SL)
English B, HL/SL

English B, is an English language - learning course designed for students with some previous learning of English (approximately 2-5 years).The main focus of the course is on English acquisition and development of language skills. These language skills will be developed through the study and use of a range of written and spoken material. Such material will extend from everyday oral exchanges to literary texts and should be related to the cultures concerned. The material will be chosen to enable students to develop mastery of language skills and intercultural understanding.

人文与社会学
（历史、地理、商业与管理、经济学、哲学、心理学、IT与全球化社会、社会与文化人类学、世界宗教）

地理学侧重于时间和空间上的个体，社会和物理过程之间的相互作用。它旨在确定这些互动中的趋势和模式，并调查人们如何适应和应对变化，并评估与此类变化相关的实际和可能的管理策略。地理描述和帮助解释不同地方之间的异同。这些可以在不同的范围内进行定义，也可以从不同角色的角度来定义，这将会对决策过程产生不同效果。

商业与管理课程主要研究商业决策过程、其中发生的个人与组织行为，以及资源向商品和服务的转化。学科培养学生对商业理论的理解和应用。授课内容包括账目与财务、市场营销、运营管理、人力资源、企业战略和日常组织与管理。学科通过这些内容为学生提供了商业运营的整体框架。

经济课程主要研究稀缺问题、资源分配，以及如何为了满足人们的愿望和需求而做出选择。在这门社会科学的研究中将引进定量分析和定性分析。IB国际文凭课程的微观经济学部分着重于个人、企业与市场；其宏观经济学部分则探讨国家、政府与社会。经济理论的学习依托于真实案例分析，强调知识的应用技能。此外还将讨论国际贸易，经济发展与环境的可持续性，以及经济理论和政策的道德尺度。

Individuals & Societies (Geography, HL/SL; Business & Management, HL/SL; Economics, HL/SL)

Geography, HL/SL

Geography focuses on the interactions between individuals, societies and physical processes in both time and space. It seeks to identify trends and patterns in these interactions and investigates the way in which people adapt and respond to change, and evaluates actual and possible management strategies associated with such change. Geography describes and helps to explain the similarities and differences between different places. These may be defined on a variety of scales and from the perspectives of a different range of actors, with varying powers over decision-making processes.

Business & Management, HL/SL

This dynamic and rigorous discipline examines business decision making processes, individuals, organizational behavior and the transformation of resources into goods and services. Students develop an understanding of business theory and learn to apply business principles, practices and skills. Topics in Business & Management included in both higher and standard level course are accounts and finance, marketing, operations management, human resources, business strategy and day-to-day business organization and management. The topics are integrated to present a holistic view of business operations.

Economics, HL/SL

The study of economics deals with scarcity, resource allocation and making choices to satisfy human desires and needs. Scientific methodologies including quantitative and qualitative analysis are employed in this social science. IBDP Economics emphasizes microeconomics: individuals and firms and markets, and macroeconomics: countries, government and societies. Economic theories are studied in a real - world context with case studies to enhance practicality of knowledge and skills. International trade, economic development and environmental sustainability as well as ethical dimensions of economic theories and policies are reviewed in the course.

below is the subject (old version):

人文与社会学
（历史、地理、商业与管理、经济学、哲学、心理学、IT与全球化社会、社会与文化人类学、世界宗教）

商业与管理课程主要研究商业决策过程、其中发生的个人与组织行为，以及资源向商品和服务的转化。学科培养学生对商业理论的理解和应用。授课内容包括账目与财务、市场营销、运营管理、人力资源、企业战略和日常组织与管理。学科通过这些内容为学生提供了商业运营的整体框架。

经济课程主要研究稀缺问题、资源分配，以及如何为了满足人们的愿望和需求而做出选择。在这门社会科学的研究中将引进定量分析和定性分析。IB国际文凭课程的微观经济学部分着重于个人、企业与市场；其宏观经济学部分则探讨国家、政府与社会。经济理论的学习依托于真实案例分析，强调知识的应用技能。此外还将讨论国际贸易，经济发展与环境的可持续性，以及经济理论和政策的道德尺度。

Individuals & Societies (Business & Management, HL/SL; Economics, HL/SL; History, HL/SL)

Business & Management, HL/SL

This dynamic and rigorous discipline examines business decision making processes, individuals, organizational behavior and the transformation of resources into goods and services. Students develop an understanding of business theory and learn to apply business principles, practices and skills. Topics in Business & Management included in both higher and standard level course are accounts and finance, marketing, operations management, human resources, business strategy and day-to-day business organization and management. The topics are integrated to present a holistic view of business operations.

Economics, HL/SL

The study of economics deals with scarcity, resource allocation and making choices to satisfy human desires and needs. Scientific methodologies including quantitative and qualitative analysis are employed in this social science. IBDP Economics emphasizes microeconomics: individuals and firms and markets, and macroeconomics: countries, government and societies. Economic theories are studied in a real - world context with case studies to enhance practicality of knowledge and skills. International trade, economic development and environmental sustainability as well as ethical dimensions of economic theories and policies are reviewed in the course.

实验科学（生物、化学、物理、设计技术）

生物课程主要研究生命体之间的相互作用以及它们的化学、物理环境。生物学是一门实验科学，同其他课程一样，该课程将学术研究与实用调查研究技能的习得相结合，并将实用的方法贯穿始终。

化学课程是一门实验科学，该课程将学术研究与实用调查研究技能的习得相结合。由于其本身的学科性质，化学课程将以实验为主。

物理课程是实验科学中最根本的学科，目的是对宇宙进行阐释。从最小的粒子，到巨大星系之间的距离，都是物理学研究的范畴。同其他科学学科一样，学生的实践能力和技能将在课程中获得提高。

Experimental Science (Biology, HL/SL; Chemistry, HL/SL; Physics, HL/SL)

Biology, HL/SL

The word biology means the study of life. It is the scientific study of the interactions that take place between and within living organisms and their chemical and physical environments. Biology is an experimental science that, as with the other sciences, combines academic study with the acquisition of practical and investigative skills and a practical approach will be used throughout the course.

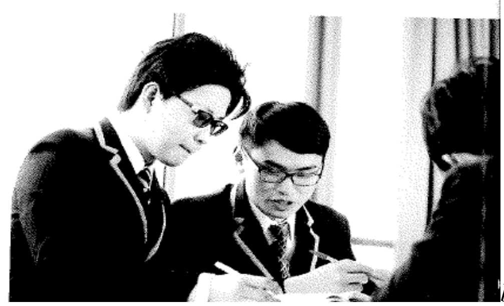
Chemistry, HL/SL

Chemistry is an experimental science that combines academic study with the acquisition of practical and investigative skills. By its very nature, chemistry lends itself to an experimental approach and this will be reflected throughout the course.

Physics, HL/SL

This is the most fundamental of the experimental sciences as it seeks to explain the Universe; from the smallest particle to massive distances between galaxies. As with the other sciences, students will develop practical skills and techniques.

数学（数学包括标准水平数学、高级水平数学）



数学课程针对希望为将来的专业学习夯实数学基础的学生，如果学生将来的目标专业期望申请者具备高等数学知识或与数学有较大关联，则学生应该修读高级数学课程。

Mathematics Mathematics, HL/SL

SL Mathematics is aimed at those who expect to need a sound mathematical background as they prepare for future studies. Students who will be expecting to take tertiary mathematics or a subject with a major mathematical component should take HL Mathematics.

below is the subject (old version):

数学与计算机科学（数学包括标准水平数学研究、标准水平数学、高级水平数学、标准水平进阶数学；计算机科学包括标准水平计算机、科学和高级水平计算机科学）

数学课程针对希望为将来的专业学习夯实数学基础的学生，如果学生将来的目标专业期望申请者具备高等数学知识或与数学有较大关联，则学生应该修读高级数学课程。

Mathematics and Computer Science (Mathematics, HL/SL Further Mathematics, HL)Mathematics, HL/SL

SL Mathematics is aimed at those who expect to need a sound mathematical background as they prepare for future studies. Students who will be expecting to take tertiary mathematics or a subject with a major mathematical component should take HL Mathematics.

艺术（视觉艺术、音乐、戏剧、电影、舞蹈）

音乐课程的教学目标是帮助学生了解音乐家的工作和沟通方式；在个人与集体协作的层面发展学生的音乐知识与潜能。

视觉艺术课程的教学目标是研究过去、现在和正在兴起的视觉艺术，并参与艺术作品的创作、欣赏与评估；培养学生在地方、国家和国际层面上对视觉艺术的理解能力；使学生能够自信地对个人和文化的体验进行创作；培养学生的绘画技巧、对作品的敏感度以及创作能力，作品能反映其积极的个体参与与特色；通过有效的实践帮助引导学生自主学习。

The Arts (Music, HL/ SL; Visual Arts, HL/ SL)

Music, HL/SL

The aim of Music is for students to:

- Become aware of how musicians work and communicate.
- Develop knowledge and potential as musicians both personally and collaboratively.
- Develop musical perception and analysis skills.
- Enjoy lifelong engagement with Music.
- Explore the diversity of Music across time.
- Express ideas with confidence and competance.

Visual Arts, HL/ SL

The specific aims of Visual Arts are to:

- Investigate past, present and emerging forms of visual arts and engage in producing, appreciating and evaluating these.
- Develop an understanding of visual arts from a local, national and international perspective.
- Build confidence in responding visually and creatively to personal and cultural experiences.
- Develop skills in, and sensitivity to, the creation of works that reflect active and individual involvement.
- Help to direct students' own learning through the acquisition of effective working practices.

评估体系 /Assessment

IB国际文凭课程是由内部评估和外部评估两部分组成；学生在校内参加内部考试评估，考核由学科老师按照评价标准完成，提交IBO国际文凭组织审核，内部评估一般占最终成绩的25%-50%。学生在完成IB第二年的课程之后参加IBO组织的全球统考，学生在学校进行考试，最后成绩由IBO发布。

IB每门学科以1-7分进行评分，6门课程共42分；拓展论文和知识理论3分，总分45分。

文凭要求 /Diploma Requirements

- 总分至少得到24分
- 所有6门学科、专题论文和认识论均有得分
- 达到创造、行动、服务 (CAS) 的所有要求
- 专题论文和认识论均至少得到D
- 各HL和SL学科均不得低于2分
- 低于4分的学科不得超过3门

What does a student need in order to get the full IB Diploma Program?

- At least 24 points overall
- Grades in all six subjects and Extended Essay (EE) and Theory of Knowledge (TOK)
- All Creativity, Action, Service (CAS) requirements met
- At least grade D in EE or TOK
- No grade 2 in any HL or SL subject
- No more than three subjects grade 4 or below

总分在28分以上

- HL学科单科得分低于3分不得超过一门
- HL学科平均得分不低于4分
- SL学科单科得分低于3分不得超过两门
- SL学科平均得分不低于3分

If 28+ Points

- No more than one 2 in any HL subject
- An average of 4 points in each HL subject
- No more than two 2s in any SL subject
- An average of 3 points in each SL subject

升学指导

宁波效实中学升学指导团队为学生和家长提供最专业的海外升学指导和服务。根据学生未来不同国家的选择和自身特点，量身定制三年升学指导计划，包括各项考试时间安排，大学专业与就业指导，海外大学申请等内容。升学指导以多种形式定期开展，如为学生和家长进行一对一面谈，邀请国外大学招生官来访，每周开展升学指导课程等。我们的目标是三年后为学生进入全球化的教育环境做好充足的申请准备。

升学指导办公室将为学生提供以下方面指导：

· 升学指导课程
· 课外活动指导
· 标准化考试规划，报名和组织出行
· 根据学生大学阶段专业意向等要求合理特供IB选择科目的建议
· 分析各海外大学特点并提出择校建议
· 定期与学生和家长一对一面谈，量身定制大学申请规划
· 邀请国外大学招生官来访，与学生直接沟通与交流
· 优秀学生将获得向国外名校招生官直接推荐的机会
· 签证申请
· 及其他相关方面的咨询



Appendix C

Interview Questions

1. How did you make the decision to teach mathematics in an international school? What factors contributed to that decision?
2. What do you see as the difference between a general Chinese mathematics education and the mathematics program (AP/IB) where you are teaching right now?
3. How well have you adapted to the curricular design of the imported (AP/IB) program? (Question for Chinese instructors only)
4. Do you think your in-class teaching is well suited to the imported programs (AP/IB)? Are there any specific areas that you feel can be improved? (Question for Chinese instructors only)
5. How do you assess your students' performance while instructing the imported programs (AP/IB)? Is there any difference between the assessment of general Chinese programs and imported program? (Question for Chinese instructors only)
6. Are there any general weaknesses that you notice in Chinese mathematics learning and teaching? Can you offer any advice to help innovation in Chinese mathematics education? Did the experience of teaching the imported programs (AP/IB) inspire the advice?
7. How do you support your students' mathematics learning?
8. For your own secondary education, did you attend an international school or did you attend a standard school?
9. Have you ever taught in any international/standard school before? Is there a difference between teaching in an international school and a standard school?
10. What kinds of in-class activities will you encourage students in your class to participate in? What is your purpose for creating those activities and what is their effect?
11. What kinds of courses or out-of-classroom activities would you recommend to

improve Chinese students' performance as well as their participation in mathematics?

12. How does your relationship with your Chinese students differ (if at all) from the relationship you had with students in your own country? Why is it similar/different? (Question for non-Chinese instructor only)
13. What (if anything) do you see as unusual about teaching mathematics in China? (Question for non-Chinese instructor only)
14. What is your observation about students' performance when learning mathematics in a bilingual environment? (Question for non-Chinese instructor only)
15. How would you inspire students learn mathematics? Do you think your classroom style is more teacher-centered or student-centered?
16. What experiences would you share with your students who show a strong interest in the field of mathematics?
17. How are your experiences in teaching mathematics today different from your experiences, say, three years ago?
18. Do you prefer focusing instruction on the breadth or the depth of a subject, especially for mathematics?
19. What do you think of the likelihood of any future development of foreign curriculums in China under the government policy (such as requiring every student in the program to take the General Examination)?
20. What would you describe as your priorities as a mathematics teacher? [probe: research/teaching/pushing/mentoring]
21. What is your nationality?
22. Select the most appropriate a) Male b) Female c) Undecided

Appendix D

AP (Advanced Placement) Concept Outline

The Concept Outline

Big Idea 1: Limits

Many calculus concepts are developed by first considering a discrete model and then the consequences of a limiting case. Therefore, the idea of limits is essential for discovering and developing important ideas, definitions, formulas, and theorems in calculus. Students must have a solid, intuitive understanding of limits and be able to compute various limits, including one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to work with tables and graphs in order to estimate the limit of a function at a point. Students should know the algebraic properties of limits and techniques for finding limits of indeterminate forms, and they should be able to apply limits to understand the behavior of a function near a point. Students must also understand how limits are used to determine continuity, a fundamental property of functions.

Enduring Understandings (Students will understand that ...)	**Learning Objectives** (Students will be able to ...)	**Essential Knowledge** (Students will know that ...)
EU 1.1: The concept of a limit can be used to understand the behavior of functions.	**LO 1.1A(a):** Express limits symbolically using correct notation. **LO 1.1A(b):** Interpret limits expressed symbolically.	**EK 1.1A1:** Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$. **EXCLUSION STATEMENT (EK 1.1A1):** *The epsilon-delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. However, teachers may include this topic in the course if time permits.*
		EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
		EK 1.1A3: A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right. **EXAMPLES OF LIMITS THAT DO NOT EXIST:** $\lim_{x \to 0} \frac{1}{x^2} = \infty$ $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ *does not exist* $\lim_{x \to 0} \frac{\lvert x \rvert}{x}$ *does not exist* $\lim_{x \to 0} \frac{1}{x}$ *does not exist*
	LO 1.1B: Estimate limits of functions.	**EK 1.1B1:** Numerical and graphical information can be used to estimate limits.

Note: In the Concept Outline, subject matter that is included only in the BC course is indicated with blue shading.

Enduring Understandings (Students will understand that ...)	**Learning Objectives** (Students will be able to ...)	**Essential Knowledge** (Students will know that ...)
EU 1.1: The concept of a limit can be used to understand the behavior of functions. *(continued)*	**LO 1.1C:** Determine limits of functions.	**EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
		EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
		EK 1.1C3: Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.
	LO 1.1D: Deduce and interpret behavior of functions using limits.	**EK 1.1D1:** Asymptotic and unbounded behavior of functions can be explained and described using limits.
		EK 1.1D2: Relative magnitudes of functions and their rates of change can be compared using limits.
EU 1.2: Continuity is a key property of functions that is defined using limits.	**LO 1.2A:** Analyze functions for intervals of continuity or points of discontinuity.	**EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $\lim_{x \to c} f(x) = f(c)$.
		EK 1.2A2: Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
		EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
	LO 1.2B: Determine the applicability of important calculus theorems using continuity.	**EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

Big Idea 2: Derivatives

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. In AP Calculus, students build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), and solving problems involving rectilinear motion. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.	**LO 2.1A**: Identify the derivative of a function as the limit of a difference quotient.	**EK 2.1A1**: The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
		EK 2.1A2: The instantaneous rate of change of a function at a point can be expressed by $\lim\limits_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $\lim\limits_{x\to a}\frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
		EK 2.1A3: The derivative of f is the function whose value at x is $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ provided this limit exists.
		EK 2.1A4: For $y=f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.
		EK 2.1A5: The derivative can be represented graphically, numerically, analytically, and verbally.
	LO 2.1B: Estimate derivatives.	**EK 2.1B1**: The derivative at a point can be estimated from information given in tables or graphs.

Enduring Understandings (Students will understand that ...)	**Learning Objectives** (Students will be able to ...)	**Essential Knowledge** (Students will know that ...)
EU 2.1: The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies. *(continued)*	**LO 2.1C**: Calculate derivatives.	**EK 2.1C1**: Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
		EK 2.1C2: Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
		EK 2.1C3: Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
		EK 2.1C4: The chain rule provides a way to differentiate composite functions.
		EK 2.1C5: The chain rule is the basis for implicit differentiation.
		EK 2.1C6: The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
		EK 2.1C7: (BC) Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.
	LO 2.1D: Determine higher order derivatives.	**EK 2.1D1**: Differentiating f' produces the second derivative f'', provided the derivative of f' exists; repeating this process produces higher order derivatives of f.
		EK 2.1D2: Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.2: A function's derivative, which is itself a function, can be used to understand the behavior of the function.	**LO 2.2A**: Use derivatives to analyze properties of a function.	**EK 2.2A1**: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
		EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
		EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.
		EK 2.2A4: (BC) For a curve given by a polar equation $r = f(\theta)$, derivatives of r, x, and y with respect to θ and first and second derivatives of y with respect to x can provide information about the curve.
	LO 2.2B: Recognize the connection between differentiability and continuity.	**EK 2.2B1**: A continuous function may fail to be differentiable at a point in its domain.
		EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point.
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.	**LO 2.3A**: Interpret the meaning of a derivative within a problem.	**EK 2.3A1**: The unit for $f'(x)$ is the unit for f divided by the unit for x.
		EK 2.3A2: The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
	LO 2.3B: Solve problems involving the slope of a tangent line.	**EK 2.3B1**: The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
		EK 2.3B2: The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
	LO 2.3C: Solve problems involving related rates, optimization, rectilinear motion, (BC) and planar motion.	**EK 2.3C1**: The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
		EK 2.3C2: The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
		EK 2.3C3: The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
		EK 2.3C4: (BC) Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector-valued functions.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 2.3: The derivative has multiple interpretations and applications including those that involve instantaneous rates of change. *(continued)*	**LO 2.3D**: Solve problems involving rates of change in applied contexts.	**EK 2.3D1**: The derivative can be used to express information about rates of change in applied contexts.
	LO 2.3E: Verify solutions to differential equations.	**EK 2.3E1**: Solutions to differential equations are functions or families of functions.
		EK 2.3E2: Derivatives can be used to verify that a function is a solution to a given differential equation.
	LO 2.3F: Estimate solutions to differential equations.	**EK 2.3F1**: Slope fields provide visual clues to the behavior of solutions to first order differential equations.
		EK 2.3F2: (BC) For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.
EU 2.4: The Mean Value Theorem connects the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval.	**LO 2.4A**: Apply the Mean Value Theorem to describe the behavior of a function over an interval.	**EK 2.4A1**: If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a,b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

Big Idea 3: Integrals and the Fundamental Theorem of Calculus

Integrals are used in a wide variety of practical and theoretical applications. AP Calculus students should understand the definition of a definite integral involving a Riemann sum, be able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. The interpretation of a definite integral is an important skill, and students should be familiar with area, volume, and motion applications, as well as with the use of the definite integral as an accumulation function. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus — a central idea in AP Calculus. Students should be able to work with and analyze functions defined by an integral.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 3.1: Antidifferentiation is the inverse process of differentiation.	**LO 3.1A:** Recognize antiderivatives of basic functions.	**EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f.
		EK 3.1A2: Differentiation rules provide the foundation for finding antiderivatives.
EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.	**LO 3.2A(a):** Interpret the definite integral as the limit of a Riemann sum. **LO 3.2A(b):** Express the limit of a Riemann sum in integral notation.	**EK 3.2A1:** A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
		EK 3.2A2: The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is, $\int_a^b f(x)dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where x_i^* is a value in the ith subinterval, Δx_i is the width of the ith subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$, where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the ith subinterval.
		EK 3.2A3: The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.

Enduring Understandings (Students will understand that ...)	**Learning Objectives** (Students will be able to ...)	**Essential Knowledge** (Students will know that ...)
EU 3.2: The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies. *(continued)*	**LO 3.2B**: Approximate a definite integral.	**EK 3.2B1**: Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
		EK 3.2B2: Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
	LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.	**EK 3.2C1**: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
		EK 3.2C2: Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
		EK 3.2C3: The definition of the definite integral may be extended to functions with removable or jump discontinuities.
	LO 3.2D: (BC) Evaluate an improper integral or show that an improper integral diverges.	EK 3.2D1: (BC) An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration.
		EK 3.2D2: (BC) Improper integrals can be determined using limits of definite integrals.
EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.	**LO 3.3A**: Analyze functions defined by an integral.	**EK 3.3A1**: The definite integral can be used to define new functions; for example, $f(x)=\int_0^x e^{-t^2}\,dt$.
		EK 3.3A2: If f is a continuous function on the interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)dt\right)=f(x)$, where x is between a and b.
		EK 3.3A3: Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x)=\int_a^x f(t)dt$.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 3.3: The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration. *(continued)*	LO 3.3B(a): Calculate antiderivatives.	EK 3.3B1: The function defined by $F(x)=\int_a^x f(t)dt$ is an antiderivative of f.
	LO 3.3B(b): Evaluate definite integrals.	EK 3.3B2: If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)dx=F(b)-F(a)$.
		EK 3.3B3: The notation $\int f(x)dx=F(x)+C$ means that $F'(x)=f(x)$, and $\int f(x)dx$ is called an indefinite integral of the function f.
		EK 3.3B4: Many functions do not have closed form antiderivatives.
		EK 3.3B5: Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, (BC) integration by parts, and nonrepeating linear partial fractions.
EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.	LO 3.4A: Interpret the meaning of a definite integral within a problem.	EK 3.4A1: A function defined as an integral represents an accumulation of a rate of change.
		EK 3.4A2: The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
		EK 3.4A3: The limit of an approximating Riemann sum can be interpreted as a definite integral.
	LO 3.4B: Apply definite integrals to problems involving the average value of a function.	EK 3.4B1: The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.
	LO 3.4C: Apply definite integrals to problems involving motion.	EK 3.4C1: For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
		EK 3.4C2: (BC) The definite integral can be used to determine displacement, distance, and position of a particle moving along a curve given by parametric or vector-valued functions.

Enduring Understandings (Students will understand that …)	Learning Objectives (Students will be able to …)	Essential Knowledge (Students will know that …)
EU 3.4: The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation. *(continued)*	LO 3.4D: Apply definite integrals to problems involving area, volume, (BC) and length of a curve.	EK 3.4D1: Areas of certain regions in the plane can be calculated with definite integrals. (BC) Areas bounded by polar curves can be calculated with definite integrals.
		EK 3.4D2: Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
		EK 3.4D3: (BC) The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral.
	LO 3.4E: Use the definite integral to solve problems in various contexts.	EK 3.4E1: The definite integral can be used to express information about accumulation and net change in many applied contexts.
EU 3.5: Antidifferentiation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determining a function or relation given its rate of change.	LO 3.5A: Analyze differential equations to obtain general and specific solutions.	EK 3.5A1: Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, (BC) and logistic growth.
		EK 3.5A2: Some differential equations can be solved by separation of variables.
		EK 3.5A3: Solutions to differential equations may be subject to domain restrictions.
		EK 3.5A4: The function F defined by $F(x)=c+\int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx}=f(x)$, and $F(x)=y_0+\int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx}=f(x)$ satisfying $F(a)=y_0$.
	LO 3.5B: Interpret, create, and solve differential equations from problems in context.	EK 3.5B1: The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt}=ky$.
		EK 3.5B2: (BC) The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt}=ky(a-y)$.

Big Idea 4: Series (BC)

The AP Calculus BC curriculum includes the study of series of numbers, power series, and various methods to determine convergence or divergence of a series. Students should be familiar with Maclaurin series for common functions and general Taylor series representations. Other topics include the radius and interval of convergence and operations on power series. The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection to the tangent-line problem and is a natural extension that helps achieve a better approximation. The concept of approximation is a common theme throughout AP Calculus, and power series provide a unifying, comprehensive conclusion.

Enduring Understandings (Students will understand that ...)	**Learning Objectives** (Students will be able to ...)	**Essential Knowledge** (Students will know that ...)
EU 4.1: The sum of an infinite number of real numbers may converge.	**LO 4.1A:** Determine whether a series converges or diverges.	**EK 4.1A1:** The nth partial sum is defined as the sum of the first n terms of a sequence.
		EK 4.1A2: An infinite series of numbers converges to a real number S (or has sum S), if and only if the limit of its sequence of partial sums exists and equals S.
		EK 4.1A3: Common series of numbers include geometric series, the harmonic series, and p-series.
		EK 4.1A4: A series may be absolutely convergent, conditionally convergent, or divergent.
		EK 4.1A5: If a series converges absolutely, then it converges.
		EK 4.1A6: In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term test, the comparison test, the limit comparison test, the integral test, the ratio test, and the alternating series test.
		EXCLUSION STATEMENT (EK 4.1A6): *Other methods for determining convergence or divergence of a series of numbers are not assessed on the AP Calculus AB or BC Exam. However, teachers may include these topics in the course if time permits.*

Enduring Understandings (Students will understand that ...)	Learning Objectives (Students will be able to ...)	Essential Knowledge (Students will know that ...)
EU 4.1: The sum of an infinite number of real numbers may converge. (continued)	LO 4.1B: Determine or estimate the sum of a series.	**EK 4.1B1**: If a is a real number and r is a real number such that $\|r\|<1$, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.
		EK 4.1B2: If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.
		EK 4.1B3: If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.
EU 4.2: A function can be represented by an associated power series over the interval of convergence for the power series.	**LO 4.2A**: Construct and use Taylor polynomials.	**EK 4.2A1**: The coefficient of the nth-degree term in a Taylor polynomial centered at $x=a$ for the function f is $\frac{f^{(n)}(a)}{n!}$.
		EK 4.2A2: Taylor polynomials for a function f centered at $x=a$ can be used to approximate function values of f near $x=a$.
		EK 4.2A3: In many cases, as the degree of a Taylor polynomial increases, the nth-degree polynomial will converge to the original function over some interval.
		EK 4.2A4: The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function.
		EK 4.2A5: In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function.
	LO 4.2B: Write a power series representing a given function.	**EK 4.2B1**: A power series is a series of the form $\sum_{n=0}^{\infty} a_n(x-r)^n$ where n is a non-negative integer, $\{a_n\}$ is a sequence of real numbers, and r is a real number.
		EK 4.2B2: The Maclaurin series for $\sin(x)$, $\cos(x)$, and e^x provide the foundation for constructing the Maclaurin series for other functions.
		EK 4.2B3: The Maclaurin series for $\frac{1}{1-x}$ is a geometric series.
		EK 4.2B4: A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$.

Enduring Understandings (Students will understand that …)	**Learning Objectives** (Students will be able to …)	**Essential Knowledge** (Students will know that …)
EU 4.2: A function can be represented by an associated power series over the interval of convergence for the power series. *(continued)*	**LO 4.2B:** Write a power series representing a given function. *(continued)*	**EK 4.2B5:** A power series for a given function can be derived by various methods (e.g., algebraic processes, substitutions, using properties of geometric series, and operations on known series such as term-by-term integration or term-by-term differentiation).
	LO 4.2C: Determine the radius and interval of convergence of a power series.	**EK 4.2C1:** If a power series converges, it either converges at a single point or has an interval of convergence.
		EK 4.2C2: The ratio test can be used to determine the radius of convergence of a power series.
		EK 4.2C3: If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval.
		EK 4.2C4: The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.

Appendix E

IB Mathematics SL (Standard Level) Curriculum

Prior learning topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematics SL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics SL. This table lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics SL course.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topic	Content
Number	Routine use of addition, subtraction, multiplication and division, using integers, decimals and fractions, including order of operations. Simple positive exponents. Simplification of expressions involving roots (surds or radicals). Prime numbers and factors, including greatest common divisors and least common multiples. Simple applications of ratio, percentage and proportion, linked to similarity. Definition and elementary treatment of absolute value (modulus), $\|a\|$. Rounding, decimal approximations and significant figures, including appreciation of errors. Expression of numbers in standard form (scientific notation), that is, $a \times 10^k$, $1 \le a < 10$, $k \in \mathbb{Z}$.
Sets and numbers	Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets. Operations on sets: union and intersection. Commutative, associative and distributive properties. Venn diagrams. Number systems: natural numbers; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$. Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation. Mappings of the elements of one set to another. Illustration by means of sets of ordered pairs, tables, diagrams and graphs.
Algebra	Manipulation of simple algebraic expressions involving factorization and expansion, including quadratic expressions. Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included. The linear function and its graph, gradient and *y*-intercept. Addition and subtraction of algebraic fractions. The properties of order relations: $<, \le, >, \ge$. Solution of equations and inequalities in one variable, including cases with rational coefficients. Solution of simultaneous equations in two variables.

Topic	Content
Trigonometry	Angle measurement in degrees. Compass directions and three figure bearings. Right-angle trigonometry. Simple applications for solving triangles. Pythagoras' theorem and its converse.
Geometry	Simple geometric transformations: translation, reflection, rotation, enlargement. Congruence and similarity, including the concept of scale factor of an enlargement. The circle, its centre and radius, area and circumference. The terms "arc", "sector", "chord", "tangent" and "segment". Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes. Volumes of prisms, pyramids, spheres, cylinders and cones.
Coordinate geometry	Elementary geometry of the plane, including the concepts of dimension for point, line, plane and space. The equation of a line in the form $y = mx + c$. Parallel and perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$. Geometry of simple plane figures. The Cartesian plane: ordered pairs (x, y), origin, axes. Mid-point of a line segment and distance between two points in the Cartesian plane and in three dimensions.
Statistics and probability	Descriptive statistics: collection of raw data; display of data in pictorial and diagrammatic forms, including pie charts, pictograms, stem and leaf diagrams, bar graphs and line graphs. Obtaining simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range, interquartile range. Calculating probabilities of simple events.

Syllabus content

Topic 1—Algebra 9 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Technology may be used to generate and display sequences in several ways. Link to 2.6, exponential functions. Examples include compound interest and population growth.	**Int:** The chess legend (Sissa ibn Dahir). **Int:** Aryabhatta is sometimes considered the "father of algebra". Compare with al-Khawarizmi. **TOK:** How did Gauss add up integers from 1 to 100? Discuss the idea of mathematical intuition as the basis for formal proof. **TOK:** Debate over the validity of the notion of "infinity": finitists such as L. Kronecker consider that "a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps". **TOK:** What is Zeno's dichotomy paradox? How far can mathematical facts be from intuition?

	Content	Further guidance	Links
1.2	Elementary treatment of exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	*Examples*: $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$; $\log 32 = 5\log 2$; $(2^3)^{-4} = 2^{-12}$. *Examples*: $\log_4 7 = \frac{\ln 7}{\ln 4}$, $\log_{25} 125 = \frac{\log_5 125}{\log_5 25}\left(= \frac{3}{2}\right)$. Link to 2.6, logarithmic functions.	**Appl:** Chemistry 18.1 (Calculation of pH). **TOK:** Are logarithms an invention or discovery? (This topic is an opportunity for teachers to generate reflection on “the nature of mathematics”.)
1.3	The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal’s triangle and $\binom{n}{r}$. **Not required:** formal treatment of permutations and formula for ${}^nP_r$.	Counting principles may be used in the development of the theorem. $\binom{n}{r}$ should be found using **both** the formula and technology. *Example*: finding $\binom{6}{r}$ from inputting $y = 6^nC_rX$ and then reading coefficients from the table. Link to 5.8, binomial distribution.	**Aim 8:** Pascal’s triangle. Attributing the origin of a mathematical discovery to the wrong mathematician. **Int:** The so-called “Pascal’s triangle” was known in China much earlier than Pascal.

Topic 2—Functions and equations

24 hours

The aims of this topic are to explore the notion of a function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic, rather than elaborate analytical techniques. On examination papers, questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus, and students may need to choose the appropriate viewing window. For those functions explicitly mentioned, questions may also be set on composition of these functions with the linear function $y = ax + b$.

	Content	Further guidance	Links
2.1	Concept of function $f : x \mapsto f(x)$. Domain, range; image (value).	*Example*: for $x \mapsto \sqrt{2-x}$, domain is $x \le 2$, range is $y \ge 0$. A graph is helpful in visualizing the range.	**Int:** The development of functions, Rene Descartes (France), Gottfried Wilhelm Leibniz (Germany) and Leonhard Euler (Switzerland).
	Composite functions.	$(f \circ g)(x) = f(g(x))$.	**TOK:** Is zero the same as "nothing"? **TOK:** Is mathematics a formal language?
	Identity function. Inverse function f^{-1}.	$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.	
	Not required: domain restriction.	On examination papers, students will only be asked to find the inverse of a *one-to-one* function.	
2.2	The graph of a function; its equation $y = f(x)$.		**Appl:** Chemistry 11.3.1 (sketching and interpreting graphs); geographic skills. **TOK:** How accurate is a visual representation of a mathematical concept? (Limits of graphs in delivering information about functions and phenomena in general, relevance of modes of representation.)
	Function graphing skills. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.	**Note** the difference in the command terms "draw" and "sketch".	
	Use of technology to graph a variety of functions, including ones not specifically mentioned.	An analytic approach is also expected for simple functions, including all those listed under topic 2.	
	The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.	Link to 6.3, local maximum and minimum points.	

	Content	Further guidance	Links
2.3	Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p: $y = pf(x)$. Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f(qx)$. Composite transformations.	Technology should be used to investigate these transformations. Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down. *Example*: $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.	**Appl:** Economics 1.1 (shifting of supply and demand curves).
2.4	The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry. The form $x \mapsto a(x-p)(x-q)$, x-intercepts $(p, 0)$ and $(q, 0)$. The form $x \mapsto a(x-h)^2 + k$, vertex (h, k).	Candidates are expected to be able to change from one form to another. Links to 2.3, transformations; 2.7, quadratic equations.	**Appl:** Chemistry 17.2 (equilibrium law). **Appl:** Physics 2.1 (kinematics). **Appl:** Physics 4.2 (simple harmonic motion). **Appl:** Physics 9.1 (HL only) (projectile motion).

	Content	Further guidance	Links
2.5	The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature. The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph. Vertical and horizontal asymptotes.	*Examples*: $h(x) = \frac{4}{3x-2}$, $x \neq \frac{2}{3}$; $y = \frac{x+7}{2x-5}$, $x \neq \frac{5}{2}$. Diagrams should include all asymptotes and intercepts.	
2.6	Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$. Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$. Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.	Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits.	**Int:** The Babylonian method of multiplication: $ab = \frac{(a+b)^2 - a^2 - b^2}{2}$. Sulba Sutras in ancient India and the Bakhshali Manuscript contained an algebraic formula for solving quadratic equations.

	Content	Further guidance	Links
2.7	Solving equations, both graphically and analytically. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Solving $ax^2+bx+c=0$, $a \neq 0$. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. Solving exponential equations.	Solutions may be referred to as roots of equations or zeros of functions. Links to 2.2, function graphing skills; and 2.3–2.6, equations involving specific functions. *Examples*: $e^x = \sin x$, $x^4 + 5x - 6 = 0$. *Example*: Find k given that the equation $3kx^2 + 2x + k = 0$ has two equal real roots. *Examples*: $2^{x-1} = 10$, $\left(\frac{1}{3}\right)^x = 9^{x+1}$. Link to 1.2, exponents and logarithms.	
2.8	Applications of graphing skills and solving equations that relate to real-life situations.	Link to 1.1, geometric series.	**Appl:** Compound interest, growth and decay; projectile motion; braking distance; electrical circuits. **Appl:** Physics 7.2.7–7.2.9, 13.2.5, 13.2.6, 13.2.8 (radioactive decay and half-life)

Topic 3—Circular functions and trigonometry 16 hours

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π, or decimals.	**Int:** Seki Takakazu calculating π to ten decimal places. **Int:** Hipparchus, Menelaus and Ptolemy. **Int:** Why are there 360 degrees in a complete turn? Links to Babylonian mathematics. **TOK:** Which is a better measure of angle: radian or degree? What are the "best" criteria by which to decide? **TOK:** Euclid's axioms as the building blocks of Euclidean geometry. Link to non-Euclidean geometry.
3.2	Definition of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.	The equation of a straight line through the origin is $y = x\tan\theta$. *Examples*: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan 210° = \frac{\sqrt{3}}{3}$.	**Aim 8:** Who really invented "Pythagoras' theorem"? **Int:** The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca. 510). **TOK:** Trigonometry was developed by successive civilizations and cultures. How is mathematical knowledge considered from a sociocultural perspective?

	Content	Further guidance	Links
3.3	The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine.	Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities).	
	Relationship between trigonometric ratios.	*Examples*: Given $\sin\theta$, finding possible values of $\tan\theta$ without finding θ. Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x.	
3.4	The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs.		**Appl:** Physics 4.2 (simple harmonic motion).
	Composite functions of the form $f(x) = a\sin(b(x+c)) + d$.	*Examples*: $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2\cos(3(x-4)) + 1$.	
	Transformations.	*Example*: $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction. Link to 2.3, transformation of graphs.	
	Applications.	Examples include height of tide, motion of a Ferris wheel.	

	Content	Further guidance	Links
3.5	Solving trigonometric equations in a finite interval, both graphically and analytically.	*Examples*: $2\sin x = 1$, $0 \le x \le 2\pi$, $2\sin 2x = 3\cos x$, $0^\circ \le x \le 180^\circ$, $2\tan\left(3(x-4)\right) = 1$, $-\pi \le x \le 3\pi$.	
	Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$. **Not required:** the general solution of trigonometric equations.	*Examples*: $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \le x < 4\pi$, $2\sin x = \cos 2x$, $-\pi \le x \le \pi$.	
3.6	Solution of triangles.	Pythagoras' theorem is a special case of the cosine rule.	**Aim 8:** Attributing the origin of a mathematical discovery to the wrong mathematician.
	The cosine rule. The sine rule, including the ambiguous case.	Link with 4.2, scalar product, noting that: $\boldsymbol{c} = \boldsymbol{a} - \boldsymbol{b} \Rightarrow \lvert \boldsymbol{c} \rvert^2 = \lvert \boldsymbol{a} \rvert^2 + \lvert \boldsymbol{b} \rvert^2 - 2\boldsymbol{a}\cdot\boldsymbol{b}$.	**Int:** Cosine rule: Al-Kashi and Pythagoras.
	Area of a triangle, $\frac{1}{2}ab\sin C$.		
	Applications.	Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	**TOK:** Non-Euclidean geometry: angle sum on a globe greater than 180°.

Topic 4—Vectors

16 hours

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

	Content	Further guidance	Links
4.1	Vectors as displacements in the plane and in three dimensions.	Link to three-dimensional geometry, x, y and z-axes.	**Appl:** Physics 1.3.2 (vector sums and differences) Physics 2.2.2, 2.2.3 (vector resultants). **TOK:** How do we relate a theory to the author? Who developed vector analysis: JW Gibbs or O Heaviside?
	Components of a vector; column representation; $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.	Components are with respect to the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ (standard basis).	
	Algebraic and geometric approaches to the following:	Applications to simple geometric figures are essential.	
	• the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v} - \boldsymbol{w} = \boldsymbol{v} + (-\boldsymbol{w})$. Vector sums and differences can be represented by the diagonals of a parallelogram.	
	• multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors;	Multiplication by a scalar can be illustrated by enlargement.	
	• magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	• unit vectors; base vectors; $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$;		
	• position vectors $\overrightarrow{OA} = \boldsymbol{a}$;		
	• $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \boldsymbol{b} - \boldsymbol{a}$.	Distance between points A and B is the magnitude of $\overrightarrow{AB}$.	

	Content	Further guidance	Links
4.2	The scalar product of two vectors. Perpendicular vectors; parallel vectors. The angle between two vectors.	The scalar product is also known as the "dot product". Link to 3.6, cosine rule. For non-zero vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = 0$ is equivalent to the vectors being perpendicular. For parallel vectors, $\boldsymbol{w} = k\boldsymbol{v}$, $\lvert \boldsymbol{v} \cdot \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert$.	
4.3	Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. The angle between two lines.	Relevance of $\boldsymbol{a}$ (position) and $\boldsymbol{b}$ (direction). Interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert \boldsymbol{b} \rvert$ representing speed.	**Aim 8:** Vector theory is used for tracking displacement of objects, including for peaceful and harmful purposes. **TOK:** Are algebra and geometry two separate domains of knowledge? (Vector algebra is a good opportunity to discuss how geometrical properties are described and generalized by algebraic methods.)
4.4	Distinguishing between coincident and parallel lines. Finding the point of intersection of two lines. Determining whether two lines intersect.		

Topic 5—Statistics and probability 35 hours

The aim of this topic is to introduce basic concepts. It is expected that most of the calculations required will be done using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context. Statistical tables will no longer be allowed in examinations. While many of the calculations required in examinations are estimates, it is likely that the command terms “write down”, “find” and “calculate” will be used.

	Content	Further guidance	Links
5.1	Concepts of population, sample, random sample, discrete and continuous data. Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals; box-and-whisker plots; outliers. Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class. **Not required:** frequency density histograms.	Continuous and discrete data. Outlier is defined as more than $1.5\times \mathrm{IQR}$ from the nearest quartile. Technology may be used to produce histograms and box-and-whisker plots.	**Appl:** Psychology: descriptive statistics, random sample (various places in the guide). **Aim 8:** Misleading statistics. **Int:** The St Petersburg paradox, Chebychev, Pavlovsky.

	Content	Further guidance	Links
5.2	Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles. Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data. Applications.	On examination papers, data will be treated as the population. Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data. Calculation of standard deviation/variance using only technology. Link to 2.3, transformations. *Examples*: If 5 is subtracted from all the data items, then the mean is decreased by 5, but the standard deviation is unchanged. If all the data items are doubled, the median is doubled, but the variance is increased by a factor of 4.	**Appl:** Psychology: descriptive statistics (various places in the guide). **Appl:** Statistical calculations to show patterns and changes; geographic skills; statistical graphs. **Appl:** Biology 1.1.2 (calculating mean and standard deviation); Biology 1.1.4 (comparing means and spreads between two or more samples). **Int:** Discussion of the different formulae for variance. **TOK:** Do different measures of central tendency express different properties of the data? Are these measures invented or discovered? Could mathematics make alternative, equally true, formulae? What does this tell us about mathematical truths? **TOK:** How easy is it to lie with statistics?
5.3	Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.	Values of the median and quartiles produced by technology may be different from those obtained from a cumulative frequency graph.	

	Content	Further guidance	Links
5.4	Linear correlation of bivariate data. Pearson's product–moment correlation coefficient r. Scatter diagrams; lines of best fit. Equation of the regression line of y on x. Use of the equation for prediction purposes. Mathematical and contextual interpretation. **Not required:** the coefficient of determination R^2.	Independent variable x, dependent variable y. Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Positive, zero, negative; strong, weak, no correlation. The line of best fit passes through the mean point. Technology should be used find the equation. Interpolation, extrapolation.	**Appl:** Chemistry 11.3.3 (curves of best fit). **Appl:** Geography (geographic skills). Measures of correlation; geographic skills. **Appl:** Biology 1.1.6 (correlation does not imply causation). **TOK:** Can we predict the value of x from y, using this equation? **TOK:** Can all data be modelled by a (known) mathematical function? Consider the reliability and validity of mathematical models in describing real-life phenomena.
5.5	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A is $\mathrm{P}(A)=\frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams and tables of outcomes.	The sample space can be represented diagrammatically in many ways. Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between (experimental) relative frequency and (theoretical) probability. Simulations may be used to enhance this topic. Links to 5.1, frequency; 5.3, cumulative frequency.	**TOK:** To what extent does mathematics offer models of real life? Is there always a function to model data behaviour?

	Content	Further guidance	Links
5.6	Combined events, $\mathrm{P}(A \cup B)$. Mutually exclusive events: $\mathrm{P}(A \cap B) = 0$. Conditional probability; the definition $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$. Independent events; the definition $\mathrm{P}(A \mid B) = \mathrm{P}(A) = \mathrm{P}(A \mid B')$. Probabilities with and without replacement.	The non-exclusivity of "or". Problems are often best solved with the aid of a Venn diagram or tree diagram, without explicit use of formulae.	**Aim 8:** The gambling issue: use of probability in casinos. Could or should mathematics help increase incomes in gambling? **TOK:** Is mathematics useful to measure risks? **TOK:** Can gambling be considered as an application of mathematics? (This is a good opportunity to generate a debate on the nature, role and ethics of mathematics regarding its applications.)
5.7	Concept of discrete random variables and their probability distributions. Expected value (mean), $\mathrm{E}(X)$ for discrete data. Applications.	Simple examples only, such as: $\mathrm{P}(X = x) = \frac{1}{18}(4 + x)$ for $x \in \{1,2,3\}$; $\mathrm{P}(X = x) = \frac{5}{18}, \frac{6}{18}, \frac{7}{18}$. $\mathrm{E}(X) = 0$ indicates a fair game where X represents the gain of one of the players. Examples include games of chance.	

	Content	Further guidance	Links
5.8	Binomial distribution. Mean and variance of the binomial distribution. **Not required:** formal proof of mean and variance.	Link to 1.3, binomial theorem. Conditions under which random variables have this distribution. Technology is usually the best way of calculating binomial probabilities.	
5.9	Normal distributions and curves. Standardization of normal variables (z-values, z-scores). Properties of the normal distribution.	Probabilities and values of the variable must be found using technology. Link to 2.3, transformations. The standardized value (z) gives the number of standard deviations from the mean.	**Appl:** Biology 1.1.3 (links to normal distribution). **Appl:** Psychology: descriptive statistics (various places in the guide).

Topic 6—Calculus

40 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications.

	Content	Further guidance	Links
6.1	Informal ideas of limit and convergence.	*Example*: 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$. Technology should be used to explore ideas of limits, numerically and graphically.	**Appl:** Economics 1.5 (marginal cost, marginal revenue, marginal profit). **Appl:** Chemistry 11.3.4 (interpreting the gradient of a curve). **Aim 8:** The debate over whether Newton or Leibnitz discovered certain calculus concepts. **TOK:** What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? **TOK:** Opportunities for discussing hypothesis formation and testing, and then the formal proof can be tackled by comparing certain cases, through an investigative approach.
	Limit notation.	*Example*: $\lim\limits_{x\to\infty}\left(\frac{2x+3}{x-1}\right)$ Links to 1.1, infinite geometric series; 2.5–2.7, rational and exponential functions, and asymptotes.	
	Definition of derivative from first principles as $f'(x)=\lim\limits_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for derivatives of simple polynomial functions only. Technology could be used to illustrate other derivatives. Link to 1.3, binomial theorem. Use of both forms of notation, $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $f'(x)$, for the first derivative.	
	Derivative interpreted as gradient function and as rate of change.	Identifying intervals on which functions are increasing or decreasing.	
	Tangents and normals, and their equations. **Not required:** analytic methods of calculating limits.	Use of both analytic approaches and technology. Technology can be used to explore graphs and their derivatives.	

	Content	Further guidance	Links
6.2	Derivative of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		
	Differentiation of a sum and a real multiple of these functions.		
	The chain rule for composite functions.	Link to 2.1, composition of functions.	
	The product and quotient rules.	Technology may be used to investigate the chain rule.	
	The second derivative.	Use of both forms of notation, $\frac{d^2 y}{dx^2}$ and $f''(x)$.	
	Extension to higher derivatives.	$\frac{d^n y}{dx^n}$ and $f^{(n)}(x)$.	

	Content	Further guidance	Links
6.3	Local maximum and minimum points. Testing for maximum or minimum.	Using change of sign of the first derivative and using sign of the second derivative. Use of the terms “concave-up” for $f''(x) > 0$, and “concave-down” for $f''(x) < 0$.	**Appl:** profit, area, volume.
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	
	Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. Optimization.	Both “global” (for large $\|x\|$) and “local” behaviour. Technology can display the graph of a derivative without explicitly finding an expression for the derivative. Use of the first or second derivative test to justify maximum and/or minimum values.	
	Applications. **Not required:** points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.	Examples include profit, area, volume. Link to 2.2, graphing functions.	

	Content	Further guidance	Links
6.4	Indefinite integration as anti-differentiation.		
	Indefinite integral of x^n $(n \in \mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x.	$\int \frac{1}{x} dx = \ln x + C$, $x > 0$.	
	The composites of any of these with the linear function $ax + b$.	*Example*: $f'(x) = \cos(2x+3) \Rightarrow f(x) = \frac{1}{2}\sin(2x+3) + C$.	
	Integration by inspection, or substitution of the form $\int f(g(x))g'(x)dx$.	*Examples*: $\int 2x\left(x^2+1\right)^4 dx$, $\int x \sin x^2 dx$, $\int \frac{\sin x}{\cos x} dx$.	
6.5	Anti-differentiation with a boundary condition to determine the constant term.	*Example*: if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	**Int:** Successful calculation of the volume of the pyramidal frustum by ancient Egyptians (Egyptian Moscow papyrus). Use of infinitesimals by Greek geometers.
	Definite integrals, both analytically and using technology.	$\int_a^b g'(x)dx = g(b) - g(a)$. The value of some definite integrals can only be found using technology.	Accurate calculation of the volume of a cylinder by Chinese mathematician Liu Hui
	Areas under curves (between the curve and the x-axis). Areas between curves. Volumes of revolution about the x-axis.	Students are expected to first write a correct expression before calculating the area. Technology may be used to enhance understanding of area and volume.	**Int:** Ibn Al Haytham: first mathematician to calculate the integral of a function, in order to find the volume of a paraboloid.
6.6	Kinematic problems involving displacement s, velocity v and acceleration a.	$v = \frac{ds}{dt}$; $a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$.	**Appl:** Physics 2.1 (kinematics).
	Total distance travelled.	Total distance travelled $= \int_{t_1}^{t_2} \lvert v \rvert \, dt$.	

Appendix F

IB Mathematics HL (Higher Level) Curriculum

Syllabus

Prior learning topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematics HL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics HL. This table lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics HL course.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topic	Content
Number	Routine use of addition, subtraction, multiplication and division, using integers, decimals and fractions, including order of operations. Rational exponents. Simplification of expressions involving roots (surds or radicals), including rationalizing the denominator. Prime numbers and factors (divisors), including greatest common divisors and least common multiples. Simple applications of ratio, percentage and proportion, linked to similarity. Definition and elementary treatment of absolute value (modulus), $\|a\|$. Rounding, decimal approximations and significant figures, including appreciation of errors. Expression of numbers in standard form (scientific notation), that is, $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$.
Sets and numbers	Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets. Operations on sets: union and intersection. Commutative, associative and distributive properties. Venn diagrams. Number systems: natural numbers; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$. Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation. Mappings of the elements of one set to another; sets of ordered pairs.

Topic	Content
Algebra	Manipulation of linear and quadratic expressions, including factorization, expansion, completing the square and use of the formula. Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included. Linear functions, their graphs, gradients and y-intercepts. Addition and subtraction of simple algebraic fractions. The properties of order relations: $<$, $\leq$, $>$, $\geq$. Solution of linear equations and inequalities in one variable, including cases with rational coefficients. Solution of quadratic equations and inequalities, using factorization and completing the square. Solution of simultaneous linear equations in two variables.
Trigonometry	Angle measurement in degrees. Compass directions. Right-angle trigonometry. Simple applications for solving triangles. Pythagoras' theorem and its converse.
Geometry	Simple geometric transformations: translation, reflection, rotation, enlargement. Congruence and similarity, including the concept of scale factor of an enlargement. The circle, its centre and radius, area and circumference. The terms arc, sector, chord, tangent and segment. Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes. Volumes of cuboids, pyramids, spheres, cylinders and cones. Classification of prisms and pyramids, including tetrahedra.
Coordinate geometry	Elementary geometry of the plane, including the concepts of dimension for point, line, plane and space. The equation of a line in the form $y = mx + c$. Parallel and perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$. The Cartesian plane: ordered pairs (x, y), origin, axes. Mid-point of a line segment and distance between two points in the Cartesian plane.
Statistics and probability	Descriptive statistics: collection of raw data, display of data in pictorial and diagrammatic forms, including frequency histograms, cumulative frequency graphs. Obtaining simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range, interquartile range and percentiles. Calculating probabilities of simple events.

Syllabus content

Topic 1—Core: Algebra 30 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications.	Sequences can be generated and displayed in several ways, including recursive functions. Link infinite geometric series with limits of convergence in 6.1. Examples include compound interest and population growth.	**Int:** The chess legend (Sissa ibn Dahir). **Int:** Aryabhatta is sometimes considered the "father of algebra". Compare with al-Khawarizmi. **Int:** The use of several alphabets in mathematical notation (eg first term and common difference of an arithmetic sequence). **TOK:** Mathematics and the knower. To what extent should mathematical knowledge be consistent with our intuition? **TOK:** Mathematics and the world. Some mathematical constants (π , e, ϕ , Fibonacci numbers) appear consistently in nature. What does this tell us about mathematical knowledge? **TOK:** Mathematics and the knower. How is mathematical intuition used as a basis for formal proof? (Gauss' method for adding up integers from 1 to 100.) *(continued)*

	Content	Further guidance	Links
			(see notes above) **Aim 8:** Short-term loans at high interest rates. How can knowledge of mathematics result in individuals being exploited or protected from extortion? **Appl:** Physics 7.2, 13.2 (radioactive decay and nuclear physics).
1.2	Exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.	Exponents and logarithms are further developed in 2.4.	**Appl:** Chemistry 18.1, 18.2 (calculation of pH and buffer solutions). **TOK:** The nature of mathematics and science. Were logarithms an invention or discovery? (This topic is an opportunity for teachers and students to reflect on "the nature of mathematics".)
1.3	Counting principles, including permutations and combinations. The binomial theorem: expansion of $(a+b)^n$, $n \in \mathbb{N}$. **Not required:** Permutations where some objects are identical. Circular arrangements. Proof of binomial theorem.	The ability to find $\binom{n}{r}$ and ${}^nP_r$ using both the formula and technology is expected. Link to 5.4. Link to 5.6, binomial distribution.	**TOK:** The nature of mathematics. The unforeseen links between Pascal's triangle, counting methods and the coefficients of polynomials. Is there an underlying truth that can be found linking these? **Int:** The properties of Pascal's triangle were known in a number of different cultures long before Pascal (eg the Chinese mathematician Yang Hui). **Aim 8:** How many different tickets are possible in a lottery? What does this tell us about the ethics of selling lottery tickets to those who do not understand the implications of these large numbers?

	Content	Further guidance	Links
1.4	Proof by mathematical induction.	Links to a wide variety of topics, for example, complex numbers, differentiation, sums of series and divisibility.	**TOK:** Nature of mathematics and science. What are the different meanings of induction in mathematics and science? **TOK:** Knowledge claims in mathematics. Do proofs provide us with completely certain knowledge? **TOK:** Knowledge communities. Who judges the validity of a proof?
1.5	Complex numbers: the number $\mathrm{i} = \sqrt{-1}$; the terms real part, imaginary part, conjugate, modulus and argument. Cartesian form $z = a + \mathrm{i}b$. Sums, products and quotients of complex numbers.	When solving problems, students may need to use technology.	**Appl:** Concepts in electrical engineering. Impedance as a combination of resistance and reactance; also apparent power as a combination of real and reactive powers. These combinations take the form $z = a + \mathrm{i}b$. **TOK:** Mathematics and the knower. Do the words imaginary and complex make the concepts more difficult than if they had different names? **TOK:** The nature of mathematics. Has “i” been invented or was it discovered? **TOK:** Mathematics and the world. Why does “i” appear in so many fundamental laws of physics?

	Content	Further guidance	Links
1.6	Modulus–argument (polar) form $z = r(\cos\theta + \mathrm{i}\sin\theta) = r\,\mathrm{cis}\theta = r\mathrm{e}^{\mathrm{i}\theta}$. The complex plane.	$r\mathrm{e}^{\mathrm{i}\theta}$ is also known as Euler's form. The ability to convert between forms is expected. The complex plane is also known as the Argand diagram.	**Appl:** Concepts in electrical engineering. Phase angle/shift, power factor and apparent power as a complex quantity in polar form. **TOK:** The nature of mathematics. Was the complex plane already there before it was used to represent complex numbers geometrically? **TOK:** Mathematics and the knower. Why might it be said that $\mathrm{e}^{\mathrm{i}\pi} + 1 = 0$ is beautiful?
1.7	Powers of complex numbers: de Moivre's theorem. n^{th} roots of a complex number.	Proof by mathematical induction for $n \in \mathbb{Z}^+$.	**TOK:** Reason and mathematics. What is mathematical reasoning and what role does proof play in this form of reasoning? Are there examples of proof that are not mathematical?
1.8	Conjugate roots of polynomial equations with real coefficients.	Link to 2.5 and 2.7.	
1.9	Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinity of solutions or no solution.	These systems should be solved using both algebraic and technological methods, eg row reduction. Systems that have solution(s) may be referred to as consistent. When a system has an infinity of solutions, a general solution may be required. Link to vectors in 4.7.	**TOK:** Mathematics, sense, perception and reason. If we can find solutions in higher dimensions, can we reason that these spaces exist beyond our sense perception?

Topic 2—Core: Functions and equations 22 hours

The aims of this topic are to explore the notion of function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic.

	Content	Further guidance	Links
2.1	Concept of function $f : x \mapsto f(x)$: domain, range; image (value). Odd and even functions. Composite functions $f \circ g$. Identity function. One-to-one and many-to-one functions. Inverse function f^{-1}, including domain restriction. Self-inverse functions.	$(f \circ g)(x) = f(g(x))$. Link with 6.2. Link with 3.4. Link with 6.2.	**Int:** The notation for functions was developed by a number of different mathematicians in the 17th and 18th centuries. How did the notation we use today become internationally accepted? **TOK:** The nature of mathematics. Is mathematics simply the manipulation of symbols under a set of formal rules?

	Content	Further guidance	Links
2.2	The graph of a function; its equation $y = f(x)$. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes and symmetry, and consideration of domain and range. The graphs of the functions $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$. The graph of $y = \dfrac{1}{f(x)}$ given the graph of $y = f(x)$.	Use of technology to graph a variety of functions.	**TOK:** Mathematics and knowledge claims. Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically (analytically)? **Appl:** Sketching and interpreting graphs; Geography SL/HL (geographic skills); Chemistry 11.3.1. **Int:** Bourbaki group analytical approach versus Mandlebrot visual approach.
2.3	Transformations of graphs: translations; stretches; reflections in the axes. The graph of the inverse function as a reflection in $y = x$.	Link to 3.4. Students are expected to be aware of the effect of transformations on both the algebraic expression and the graph of a function.	**Appl:** Economics SL/HL 1.1 (shift in demand and supply curves).
2.4	The rational function $x \mapsto \dfrac{ax+b}{cx+d}$, and its graph. The function $x \mapsto a^x$, $a > 0$, and its graph. The function $x \mapsto \log_a x$, $x > 0$, and its graph.	The reciprocal function is a particular case. Graphs should include both asymptotes and any intercepts with axes. Exponential and logarithmic functions as inverses of each other. Link to 6.2 and the significance of e. Application of concepts in 2.1, 2.2 and 2.3.	**Appl:** Geography SL/HL (geographic skills); Physics SL/HL 7.2 (radioactive decay); Chemistry SL/HL 16.3 (activation energy); Economics SL/HL 3.2 (exchange rates).

	Content	Further guidance	Links
2.5	Polynomial functions and their graphs. The factor and remainder theorems. The fundamental theorem of algebra.	The graphical significance of repeated factors. The relationship between the degree of a polynomial function and the possible numbers of x-intercepts.	
2.6	Solving quadratic equations using the quadratic formula. Use of the discriminant $\Delta = b^2 - 4ac$ to determine the nature of the roots.	May be referred to as roots of equations or zeros of functions.	**Appl:** Chemistry 17.2 (equilibrium law). **Appl:** Physics 2.1 (kinematics). **Appl:** Physics 4.2 (energy changes in simple harmonic motion). **Appl:** Physics (HL only) 9.1 (projectile motion). **Aim 8:** The phrase "exponential growth" is used popularly to describe a number of phenomena. Is this a misleading use of a mathematical term?
	Solving polynomial equations both graphically and algebraically.	Link the solution of polynomial equations to conjugate roots in 1.8.	
	Sum and product of the roots of polynomial equations.	For the polynomial equation $\sum_{r=0}^{n} a_r x^r = 0$, the sum is $\frac{-a_{n-1}}{a_n}$, the product is $\frac{(-1)^n a_0}{a_n}$.	
	Solution of $a^x = b$ using logarithms. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.		

	Content	Further guidance	Links
2.7	Solutions of $g(x) \geq f(x)$. Graphical or algebraic methods, for simple polynomials up to degree 3. Use of technology for these and other functions.		

Topic 3—Core: Circular functions and trigonometry — 22 hours

The aims of this topic are to explore the circular functions, to introduce some important trigonometric identities and to solve triangles using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated, for example, by $x \mapsto \sin x^{\circ}$.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles. Length of an arc; area of a sector.	Radian measure may be expressed as multiples of π, or decimals. Link with 6.2.	**Int:** The origin of degrees in the mathematics of Mesopotamia and why we use minutes and seconds for time.
3.2	Definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ in terms of the unit circle. Exact values of sin, cos and tan of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. Definition of the reciprocal trigonometric ratios $\sec\theta$, $\csc\theta$ and $\cot\theta$. Pythagorean identities: $\cos^2\theta + \sin^2\theta = 1$; $1+\tan^2\theta = \sec^2\theta$; $1+\cot^2\theta = \csc^2\theta$.		**TOK:** Mathematics and the knower. Why do we use radians? (The arbitrary nature of degree measure versus radians as real numbers and the implications of using these two measures on the shape of sinusoidal graphs.) **TOK:** Mathematics and knowledge claims. If trigonometry is based on right triangles, how can we sensibly consider trigonometric ratios of angles greater than a right angle? **Int:** The origin of the word "sine". **Appl:** Physics SL/HL 2.2 (forces and dynamics).
3.3	Compound angle identities. Double angle identities. **Not required:** Proof of compound angle identities.	Derivation of double angle identities from compound angle identities. Finding possible values of trigonometric ratios without finding θ, for example, finding $\sin 2\theta$ given $\sin\theta$.	**Appl:** Triangulation used in the Global Positioning System (GPS). **Int:** Why did Pythagoras link the study of music and mathematics? **Appl:** Concepts in electrical engineering. Generation of sinusoidal voltage.

(continued)

	Content	Further guidance	Links
3.4	Composite functions of the form $f(x) = a\sin(b(x+c)) + d$. Applications.		*(see notes above)* **TOK:** Mathematics and the world. Music can be expressed using mathematics. Does this mean that music is mathematical, that mathematics is musical or that both are reflections of a common "truth"? **Appl:** Physics SL/HL 4.1 (kinematics of simple harmonic motion).
3.5	The inverse functions $x \mapsto \arcsin x$, $x \mapsto \arccos x$, $x \mapsto \arctan x$; their domains and ranges; their graphs.		
3.6	Algebraic and graphical methods of solving trigonometric equations in a finite interval, including the use of trigonometric identities and factorization. **Not required:** The general solution of trigonometric equations.		**TOK:** Mathematics and knowledge claims. How can there be an infinite number of discrete solutions to an equation?
3.7	The cosine rule The sine rule including the ambiguous case. Area of a triangle as $\frac{1}{2}ab\sin C$.		**TOK:** Nature of mathematics. If the angles of a triangle can add up to less than 180°, 180° or more than 180°, what does this tell us about the "fact" of the angle sum of a triangle and about the nature of mathematical knowledge?
	Applications.	Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.	**Appl:** Physics SL/HL 1.3 (vectors and scalars); Physics SL/HL 2.2 (forces and dynamics). **Int:** The use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.

Topic 4—Core: Vectors 24 hours

The aim of this topic is to introduce the use of vectors in two and three dimensions, and to facilitate solving problems involving points, lines and planes.

	Content	Further guidance	Links
4.1	Concept of a vector. Representation of vectors using directed line segments. Unit vectors; base vectors $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$. Components of a vector: $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$.		**Aim 8:** Vectors are used to solve many problems in position location. This can be used to save a lost sailor or destroy a building with a laser-guided bomb. **Appl:** Physics SL/HL 1.3 (vectors and scalars); Physics SL/HL 2.2 (forces and dynamics). **TOK:** Mathematics and knowledge claims. You can perform some proofs using different mathematical concepts. What does this tell us about mathematical knowledge?
	Algebraic and geometric approaches to the following: • the sum and difference of two vectors; • the zero vector $\boldsymbol{0}$, the vector $-\boldsymbol{v}$; • multiplication by a scalar, $k\boldsymbol{v}$; • magnitude of a vector, $\lvert\boldsymbol{v}\rvert$; • position vectors $\vec{\mathrm{OA}} = \boldsymbol{a}$.	Proofs of geometrical properties using vectors.	
	$\vec{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a}$	Distance between points A and B is the magnitude of $\vec{\mathrm{AB}}$.	

	Content	Further guidance	Links
4.2	The definition of the scalar product of two vectors. Properties of the scalar product: $\boldsymbol{v} \cdot \boldsymbol{w} = \boldsymbol{w} \cdot \boldsymbol{v}$; $\boldsymbol{u} \cdot (\boldsymbol{v} + \boldsymbol{w}) = \boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \cdot \boldsymbol{w}$; $(k\boldsymbol{v}) \cdot \boldsymbol{w} = k(\boldsymbol{v} \cdot \boldsymbol{w})$; $\boldsymbol{v} \cdot \boldsymbol{v} = \lvert \boldsymbol{v} \rvert^2$. The angle between two vectors. Perpendicular vectors; parallel vectors.	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$. Link to 3.6. For non-zero vectors, $\boldsymbol{v} \cdot \boldsymbol{w} = 0$ is equivalent to the vectors being perpendicular. For parallel vectors, $\lvert \boldsymbol{v} \cdot \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert$.	**Appl:** Physics SL/HL 2.2 (forces and dynamics). **TOK:** The nature of mathematics. Why this definition of scalar product?
4.3	Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$. Simple applications to kinematics. The angle between two lines.	Knowledge of the following forms for equations of lines. Parametric form: $x = x_0 + \lambda l$, $y = y_0 + \lambda m$, $z = z_0 + \lambda n$. Cartesian form: $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$.	**Appl:** Modelling linear motion in three dimensions. **Appl:** Navigational devices, eg GPS. **TOK:** The nature of mathematics. Why might it be argued that vector representation of lines is superior to Cartesian?
4.4	Coincident, parallel, intersecting and skew lines; distinguishing between these cases. Points of intersection.		

	Content	Further guidance	Links
4.5	The definition of the vector product of two vectors. Properties of the vector product: $\boldsymbol{v} \times \boldsymbol{w} = -\boldsymbol{w} \times \boldsymbol{v}$; $\boldsymbol{u} \times (\boldsymbol{v} + \boldsymbol{w}) = \boldsymbol{u} \times \boldsymbol{v} + \boldsymbol{u} \times \boldsymbol{w}$; $(k\boldsymbol{v}) \times \boldsymbol{w} = k(\boldsymbol{v} \times \boldsymbol{w})$; $\boldsymbol{v} \times \boldsymbol{v} = \boldsymbol{0}$. Geometric interpretation of $\lvert \boldsymbol{v} \times \boldsymbol{w} \rvert$.	$\boldsymbol{v} \times \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \sin\theta\, \boldsymbol{n}$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ and $\boldsymbol{n}$ is the unit normal vector whose direction is given by the right-hand screw rule. Areas of triangles and parallelograms.	**Appl:** Physics SL/HL 6.3 (magnetic force and field).
4.6	Vector equation of a plane $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$. Use of normal vector to obtain the form $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$. Cartesian equation of a plane $ax + by + cz = d$.		
4.7	Intersections of: a line with a plane; two planes; three planes. Angle between: a line and a plane; two planes.	Link to 1.9. Geometrical interpretation of solutions.	**TOK:** Mathematics and the knower. Why are symbolic representations of three-dimensional objects easier to deal with than visual representations? What does this tell us about our knowledge of mathematics in other dimensions?

Topic 5—Core: Statistics and probability 36 hours

The aim of this topic is to introduce basic concepts. It may be considered as three parts: manipulation and presentation of statistical data (5.1), the laws of probability (5.2–5.4), and random variables and their probability distributions (5.5–5.7). It is expected that most of the calculations required will be done on a GDC. The emphasis is on understanding and interpreting the results obtained. Statistical tables will no longer be allowed in examinations.

	Content	Further guidance	Links
5.1	Concepts of population, sample, random sample and frequency distribution of discrete and continuous data. Grouped data: mid-interval values, interval width, upper and lower interval boundaries. Mean, variance, standard deviation. **Not required:** Estimation of mean and variance of a population from a sample.	For examination purposes, in papers 1 and 2 data will be treated as the population. In examinations the following formulae should be used: $\mu = \frac{\sum_{i=1}^{k} f_i x_i}{n}$, $\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$.	**TOK:** The nature of mathematics. Why have mathematics and statistics sometimes been treated as separate subjects? **TOK:** The nature of knowing. Is there a difference between information and data? **Aim 8:** Does the use of statistics lead to an overemphasis on attributes that can easily be measured over those that cannot? **Appl:** Psychology SL/HL (descriptive statistics); Geography SL/HL (geographic skills); Biology SL/HL 1.1.2 (statistical analysis). **Appl:** Methods of collecting data in real life (census versus sampling). **Appl:** Misleading statistics in media reports.

	Content	Further guidance	Links
5.2	Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A as $\mathrm{P}(A)=\frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams, counting principles and tables of outcomes to solve problems.		**Aim 8:** Why has it been argued that theories based on the calculable probabilities found in casinos are pernicious when applied to everyday life (eg economics)? **Int:** The development of the mathematical theory of probability in 17^{th} century France.
5.3	Combined events; the formula for $\mathrm{P}(A\cup B)$. Mutually exclusive events.		
5.4	Conditional probability; the definition $\mathrm{P}(A\mid B)=\frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$. Independent events; the definition $\mathrm{P}(A\mid B)=\mathrm{P}(A)=\mathrm{P}(A\mid B')$. Use of Bayes' theorem for a maximum of three events.	Use of $\mathrm{P}(A\cap B)=\mathrm{P}(A)\mathrm{P}(B)$ to show independence.	**Appl:** Use of probability methods in medical studies to assess risk factors for certain diseases. **TOK:** Mathematics and knowledge claims. Is independence as defined in probabilistic terms the same as that found in normal experience?

	Content	Further guidance	Links
5.5	Concept of discrete and continuous random variables and their probability distributions. Definition and use of probability density functions.		**TOK:** Mathematics and the knower. To what extent can we trust samples of data?
	Expected value (mean), mode, median, variance and standard deviation.	For a continuous random variable, a value at which the probability density function has a maximum value is called a mode.	
	Applications.	Examples include games of chance.	**Appl:** Expected gain to insurance companies.
5.6	Binomial distribution, its mean and variance. Poisson distribution, its mean and variance. **Not required:** Formal proof of means and variances.	Link to binomial theorem in 1.3. Conditions under which random variables have these distributions.	**TOK:** Mathematics and the real world. Is the binomial distribution ever a useful model for an actual real-world situation?
5.7	Normal distribution.	Probabilities and values of the variable must be found using technology. The standardized value (z) gives the number of standard deviations from the mean.	**Appl:** Chemistry SL/HL 6.2 (collision theory); Psychology HL (descriptive statistics); Biology SL/HL 1.1.3 (statistical analysis). **Aim 8:** Why might the misuse of the normal distribution lead to dangerous inferences and conclusions? **TOK:** Mathematics and knowledge claims. To what extent can we trust mathematical models such as the normal distribution? **Int:** De Moivre's derivation of the normal distribution and Quetelet's use of it to describe *l'homme moyen*.
	Properties of the normal distribution. Standardization of normal variables.	Link to 2.3.	

Topic 6—Core: Calculus

48 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.

	Content	Further guidance	Links
6.1	Informal ideas of limit, continuity and convergence. Definition of derivative from first principles $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. The derivative interpreted as a gradient function and as a rate of change. Finding equations of tangents and normals. Identifying increasing and decreasing functions. The second derivative. Higher derivatives.	Include result $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$. Link to 1.1. Use of this definition for polynomials only. Link to binomial theorem in 1.3. Both forms of notation, $\frac{dy}{dx}$ and $f'(x)$, for the first derivative. Use of both algebra and technology. Both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$, for the second derivative. Familiarity with the notation $\frac{d^ny}{dx^n}$ and $f^{(n)}(x)$. Link with induction in 1.4.	**TOK:** The nature of mathematics. Does the fact that Leibniz and Newton came across the calculus at similar times support the argument that mathematics exists prior to its discovery? **Int:** How the Greeks' distrust of zero meant that Archimedes' work did not lead to calculus. **Int:** Investigate attempts by Indian mathematicians (500–1000 CE) to explain division by zero. **TOK:** Mathematics and the knower. What does the dispute between Newton and Leibniz tell us about human emotion and mathematical discovery? **Appl:** Economics HL 1.5 (theory of the firm); Chemistry SL/HL 11.3.4 (graphical techniques); Physics SL/HL 2.1 (kinematics).

	Content	Further guidance	Links
6.2	Derivatives of x^n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$. Differentiation of sums and multiples of functions. The product and quotient rules. The chain rule for composite functions. Related rates of change. Implicit differentiation. Derivatives of $\sec x$, $\csc x$, $\cot x$, a^x, $\log_a x$, $\arcsin x$, $\arccos x$ and $\arctan x$.		**Appl:** Physics HL 2.4 (uniform circular motion); Physics 12.1 (induced electromotive force (emf)). **TOK:** Mathematics and knowledge claims. Euler was able to make important advances in mathematical analysis before calculus had been put on a solid theoretical foundation by Cauchy and others. However, some work was not possible until after Cauchy's work. What does this tell us about the importance of proof and the nature of mathematics? **TOK:** Mathematics and the real world. The seemingly abstract concept of calculus allows us to create mathematical models that permit human feats, such as getting a man on the Moon. What does this tell us about the links between mathematical models and physical reality?
6.3	Local maximum and minimum values. Optimization problems. Points of inflexion with zero and non-zero gradients. Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. **Not required:** Points of inflexion, where $f''(x)$ is not defined, for example, $y = x^{1/3}$ at $(0,0)$.	Testing for the maximum or minimum using the change of sign of the first derivative and using the sign of the second derivative. Use of the terms "concave up" for $f''(x) > 0$, "concave down" for $f''(x) < 0$. At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change).	

	Content	Further guidance	Links
6.4	Indefinite integration as anti-differentiation. Indefinite integral of x^n, $\sin x$, $\cos x$ and e^x. Other indefinite integrals using the results from 6.2. The composites of any of these with a linear function.	Indefinite integral interpreted as a family of curves. $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$. Examples include $\int (2x-1)^5\,dx$, $\int \frac{1}{3x+4}\,dx$ and $\int \frac{1}{x^2+2x+5}\,dx$.	
6.5	Anti-differentiation with a boundary condition to determine the constant of integration. Definite integrals. Area of the region enclosed by a curve and the x-axis or y-axis in a given interval; areas of regions enclosed by curves. Volumes of revolution about the x-axis or y-axis.	The value of some definite integrals can only be found using technology.	**Appl:** Industrial design.

	Content	Further guidance	Links
6.6	Kinematic problems involving displacement s, velocity v and acceleration a. Total distance travelled.	$v=\frac{\mathrm{d}s}{\mathrm{d}t}$, $a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2s}{\mathrm{d}t^2}=v\frac{\mathrm{d}v}{\mathrm{d}s}$. Total distance travelled $=\int_{t_1}^{t_2}\lvert v\rvert\,\mathrm{d}t$.	**Appl:** Physics HL 2.1 (kinematics). **Int:** Does the inclusion of kinematics as core mathematics reflect a particular cultural heritage? Who decides what is mathematics?
6.7	Integration by substitution Integration by parts.	On examination papers, non-standard substitutions will be provided. Link to 6.2. Examples: $\int x\sin x\,\mathrm{d}x$ and $\int \ln x\,\mathrm{d}x$. Repeated integration by parts. Examples: $\int x^2\mathrm{e}^x\,\mathrm{d}x$ and $\int \mathrm{e}^x\sin x\,\mathrm{d}x$.	

Topic 7—Option: Statistics and probability 48 hours

The aims of this option are to allow students the opportunity to approach statistics in a practical way; to demonstrate a good level of statistical understanding; and to understand which situations apply and to interpret the given results. It is expected that GDCs will be used throughout this option, and that the minimum requirement of a GDC will be to find probability distribution function (pdf), cumulative distribution function (cdf), inverse cumulative distribution function, p-values and test statistics, including calculations for the following distributions: binomial, Poisson, normal and t. Students are expected to set up the problem mathematically and then read the answers from the GDC, indicating this within their written answers. Calculator-specific or brand-specific language should not be used within these explanations.

	Content	Further guidance	Links
7.1	Cumulative distribution functions for both discrete and continuous distributions. Geometric distribution. Negative binomial distribution. Probability generating functions for discrete random variables. Using probability generating functions to find mean, variance and the distribution of the sum of n independent random variables.	$G(t) = \mathrm{E}(t^X) = \sum_x P(X = x)t^x$.	**Int:** Also known as Pascal's distribution. **Aim 8:** Statistical compression of data files.
7.2	Linear transformation of a single random variable. Mean of linear combinations of n random variables. Variance of linear combinations of n independent random variables. Expectation of the product of independent random variables.	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$. $\mathrm{E}(XY) = \mathrm{E}(X)\mathrm{E}(Y)$.	

	Content	Further guidance	Links
7.3	Unbiased estimators and estimates. Comparison of unbiased estimators based on variances. $\overline{X}$ as an unbiased estimator for μ. S^2 as an unbiased estimator for σ^2.	T is an unbiased estimator for the parameter θ if $\mathrm{E}(T)=\theta$. T_1 is a more efficient estimator than T_2 if $\mathrm{Var}(T_1)<\mathrm{Var}(T_2)$. $\overline{X}=\sum_{i=1}^{n}\frac{X_i}{n}$. $S^2=\sum_{i=1}^{n}\frac{\left(X_i-\overline{X}\right)^2}{n-1}$.	**TOK**: Mathematics and the world. In the absence of knowing the value of a parameter, will an unbiased estimator always be better than a biased one?
7.4	A linear combination of independent normal random variables is normally distributed. In particular, $X\sim \mathrm{N}(\mu,\sigma^2)\Rightarrow \overline{X}\sim \mathrm{N}\left(\mu,\frac{\sigma^2}{n}\right)$. The central limit theorem.		**Aim 8/TOK:** Mathematics and the world. "Without the central limit theorem, there could be no statistics of any value within the human sciences." **TOK:** Nature of mathematics. The central limit theorem can be proved mathematically (formalism), but its truth can be confirmed by its applications (empiricism).

	Content	Further guidance	Links
7.5	Confidence intervals for the mean of a normal population.	Use of the normal distribution when σ is known and use of the *t*-distribution when σ is unknown, regardless of sample size. The case of matched pairs is to be treated as an example of a single sample technique.	**TOK:** Mathematics and the world. Claiming brand A is "better" on average than brand B can mean very little if there is a large overlap between the confidence intervals of the two means. **Appl:** Geography.
7.6	Null and alternative hypotheses, H_0 and H_1. Significance level. Critical regions, critical values, *p*-values, one-tailed and two-tailed tests. Type I and II errors, including calculations of their probabilities. Testing hypotheses for the mean of a normal population.	Use of the normal distribution when σ is known and use of the t-distribution when σ is unknown, regardless of sample size. The case of matched pairs is to be treated as an example of a single sample technique.	**TOK:** Mathematics and the world. In practical terms, is saying that a result is significant the same as saying that it is true? **TOK:** Mathematics and the world. Does the ability to test only certain parameters in a population affect the way knowledge claims in the human sciences are valued? **Appl:** When is it more important not to make a Type I error and when is it more important not to make a Type II error?

	Content	Further guidance	Links
7.7	Introduction to bivariate distributions.	Informal discussion of commonly occurring situations, eg marks in pure mathematics and statistics exams taken by a class of students, salary and age of teachers in a certain school. The need for a measure of association between the variables and the possibility of predicting the value of one of the variables given the value of the other variable.	**Appl:** Geographic skills. **Aim 8:** The correlation between smoking and lung cancer was "discovered" using mathematics. Science had to justify the cause.
	Covariance and (population) product moment correlation coefficient ρ.	$\mathrm{Cov}(X,Y) = \mathrm{E}[(X-\mu_x)(Y-\mu_y)]$ $= \mathrm{E}(XY) - \mu_x\mu_y$, where $\mu_x = \mathrm{E}(X)$, $\mu_y = \mathrm{E}(Y)$. $\rho = \dfrac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}$.	**Appl:** Using technology to fit a range of curves to a set of data.
	Proof that $\rho = 0$ in the case of independence and ± 1 in the case of a linear relationship between X and Y.	The use of ρ as a measure of association between X and Y, with values near 0 indicating a weak association and values near +1 or near −1 indicating a strong association.	**TOK:** Mathematics and the world. Given that a set of data may be approximately fitted by a range of curves, where would we seek for knowledge of which equation is the "true" model?
	Definition of the (sample) product moment correlation coefficient R in terms of n paired observations on X and Y. Its application to the estimation of ρ.	$R = \dfrac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2\sum_{i=1}^{n}(Y_i-\bar{Y})^2}}$ $= \dfrac{\sum_{i=1}^{n}X_iY_i - n\bar{X}\bar{Y}}{\sqrt{\left(\sum_{i=1}^{n}X_i^2 - n\bar{X}^2\right)\left(\sum Y_i^2 - n\bar{Y}^2\right)}}$.	**Aim 8:** The physicist Frank Oppenheimer wrote: "Prediction is dependent only on the assumption that observed patterns will be repeated." This is the danger of extrapolation. There are many examples of its failure in the past, eg share prices, the spread of disease, climate change. *(continued)*

	Content	Further guidance	Links
	Informal interpretation of r, the observed value of R. Scatter diagrams.	Values of r near 0 indicate a weak association between X and Y, and values near ± 1 indicate a strong association.	*(see notes above)*
	The following topics are based on the assumption of bivariate normality.	It is expected that the GDC will be used wherever possible in the following work.	
	Use of the t-statistic to test the null hypothesis $\rho = 0$.	$R\sqrt{\frac{n-2}{1-R^2}}$ has the student's t-distribution with $(n-2)$ degrees of freedom.	
	Knowledge of the facts that the regression of X on Y $\left(\mathrm{E}(X) \mid Y = y\right)$ and Y on X $\left(\mathrm{E}(Y) \mid X = x\right)$ are linear. Least-squares estimates of these regression lines (proof not required). The use of these regression lines to predict the value of one of the variables given the value of the other.	$x - \bar{x} = \left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2}\right)(y - \bar{y})$ $= \left(\frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\,\bar{y}}{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}\right)(y - \bar{y}),$ $y - \bar{y} = \left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)(x - \bar{x})$ $= \left(\frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\,\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}\right)(x - \bar{x}).$	

Topic 8—Option: Sets, relations and groups — 48 hours

The aims of this option are to provide the opportunity to study some important mathematical concepts, and introduce the principles of proof through abstract algebra.

	Content	Further guidance	Links
8.1	Finite and infinite sets. Subsets. Operations on sets: union; intersection; complement; set difference; symmetric difference.		**TOK:** Cantor theory of transfinite numbers, Russell's paradox, Godel's incompleteness theorems.
	De Morgan's laws: distributive, associative and commutative laws (for union and intersection).	Illustration of these laws using Venn diagrams. Students may be asked to prove that two sets are the same by establishing that $A \subseteq B$ and $B \subseteq A$.	**Appl:** Logic, Boolean algebra, computer circuits.
8.2	Ordered pairs: the Cartesian product of two sets.		
	Relations: equivalence relations; equivalence classes.	An equivalence relation on a set forms a partition of the set.	**Appl, Int:** Scottish clans.
8.3	Functions: injections; surjections; bijections.	The term codomain.	
	Composition of functions and inverse functions.	Knowledge that the function composition is not a commutative operation and that if f is a bijection from set A to set B then f^{-1} exists and is a bijection from set B to set A.	

	Content	Further guidance	Links
8.4	Binary operations. Operation tables (Cayley tables).	A binary operation $*$ on a non-empty set S is a rule for combining any two elements $a, b \in S$ to give a unique element c. That is, in this definition, a binary operation on a set is not necessarily closed.	
8.5	Binary operations: associative, distributive and commutative properties.	The arithmetic operations on $\mathbb{R}$ and $\mathbb{C}$. Examples of distributivity could include the fact that, on $\mathbb{R}$, multiplication is distributive over addition but addition is not distributive over multiplication.	**TOK:** Which are more fundamental, the general models or the familiar examples?
8.6	The identity element e. The inverse a^{-1} of an element a. Proof that left-cancellation and right-cancellation by an element a hold, provided that a has an inverse. Proofs of the uniqueness of the identity and inverse elements.	Both the right-identity $a * e = a$ and left-identity $e * a = a$ must hold if e is an identity element. Both $a * a^{-1} = e$ and $a^{-1} * a = e$ must hold.	

	Content	Further guidance	Links
8.7	The definition of a group $\{G,*\}$. The operation table of a group is a Latin square, but the converse is false.	For the set G under a given operation $*$: • G is closed under $*$; • $*$ is associative; • G contains an identity element; • each element in G has an inverse in G.	**Appl:** Existence of formula for roots of polynomials. **Appl:** Galois theory for the impossibility of such formulae for polynomials of degree 5 or higher.
	Abelian groups.	$a*b=b*a$, for all $a,b \in G$.	
8.8	Examples of groups: • $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$ and $\mathbb{C}$ under addition; • integers under addition modulo n; • non-zero integers under multiplication, modulo p, where p is prime;		**Appl:** Rubik's cube, time measures, crystal structure, symmetries of molecules, strut and cable constructions, Physics H2.2 (special relativity), the 8–fold way, supersymmetry.
	symmetries of plane figures, including equilateral triangles and rectangles; invertible functions under composition of functions.	The composition $T_2 \circ T_1$ denotes T_1 followed by T_2.	
8.9	The order of a group. The order of a group element. Cyclic groups. Generators. Proof that all cyclic groups are Abelian.		**Appl:** Music circle of fifths, prime numbers.

	Content	Further guidance	Links
8.10	Permutations under composition of permutations. Cycle notation for permutations. Result that every permutation can be written as a composition of disjoint cycles. The order of a combination of cycles.	On examination papers: the form $p = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ or in cycle notation (132) will be used to represent the permutation $1 \to 3$, $2 \to 1$, $3 \to 2$.	**Appl:** Cryptography, campanology.
8.11	Subgroups, proper subgroups.	A proper subgroup is neither the group itself nor the subgroup containing only the identity element.	
	Use and proof of subgroup tests.	Suppose that $\{G,*\}$ is a group and H is a non-empty subset of G. Then $\{H,*\}$ is a subgroup of $\{G,*\}$ if $a*b^{-1} \in H$ whenever $a,b \in H$. Suppose that $\{G,*\}$ is a finite group and H is a non-empty subset of G. Then $\{H,*\}$ is a subgroup of $\{G,*\}$ if H is closed under $*$.	
	Definition and examples of left and right cosets of a subgroup of a group.		
	Lagrange's theorem. Use and proof of the result that the order of a finite group is divisible by the order of any element. (Corollary to Lagrange's theorem.)		**Appl:** Prime factorization, symmetry breaking.

	Content	Further guidance	Links
8.12	Definition of a group homomorphism.	Infinite groups as well as finite groups. Let $\{G,*\}$ and $\{H,\circ\}$ be groups, then the function $f:G \to H$ is a homomorphism if $f(a*b) = f(a)\circ f(b)$ for all $a,b \in G$.	
	Definition of the kernel of a homomorphism. Proof that the kernel and range of a homomorphism are subgroups.	If $f:G \to H$ is a group homomorphism, then $\text{Ker}(f)$ is the set of $a \in G$ such that $f(a) = e_H$.	
	Proof of homomorphism properties for identities and inverses.	Identity: let e_G and e_H be the identity elements of $(G,*)$ and $(H,\circ)$, respectively, then $f(e_G) = e_H$. Inverse: $f(a^{-1}) = \left(f(a)\right)^{-1}$ for all $a \in G$.	
	Isomorphism of groups.	Infinite groups as well as finite groups. The homomorphism $f:G \to H$ is an isomorphism if *f* is bijective.	
	The order of an element is unchanged by an isomorphism.		

Topic 9—Option: Calculus 48 hours

The aims of this option are to introduce limit theorems and convergence of series, and to use calculus results to solve differential equations.

	Content	Further guidance	Links
9.1	Infinite sequences of real numbers and their convergence or divergence.	Informal treatment of limit of sum, difference, product, quotient; squeeze theorem. Divergent is taken to mean not convergent.	**TOK:** Zeno's paradox, impact of infinite sequences and limits on our understanding of the physical world.
9.2	Convergence of infinite series. Tests for convergence: comparison test; limit comparison test; ratio test; integral test.	The sum of a series is the limit of the sequence of its partial sums. Students should be aware that if $\lim_{x\to\infty} x_n = 0$ then the series is not necessarily convergent, but if $\lim_{x\to\infty} x_n \neq 0$, the series diverges.	**TOK:** Euler's idea that $1-1+1-1+\ldots=\frac{1}{2}$. Was it a mistake or just an alternative view?
	The *p*-series, $\sum \frac{1}{n^p}$.	$\sum \frac{1}{n^p}$ is convergent for $p>1$ and divergent otherwise. When $p=1$, this is the harmonic series.	
	Series that converge absolutely. Series that converge conditionally.	Conditions for convergence.	
	Alternating series. Power series: radius of convergence and interval of convergence. Determination of the radius of convergence by the ratio test.	The absolute value of the truncation error is less than the next term in the series.	

	Content	Further guidance	Links
9.3	Continuity and differentiability of a function at a point. Continuous functions and differentiable functions.	Test for continuity: $\lim_{x\to a-} f(x) = f(a) = \lim_{x\to a+} f(x)$. Test for differentiability: f is continuous at a and $\lim_{h\to 0-} \frac{f(a+h)-f(a)}{h}$ and $\lim_{h\to 0+} \frac{f(a+h)-f(a)}{h}$ exist and are equal. Students should be aware that a function may be continuous but not differentiable at a point, eg $f(x)=\|x\|$ and simple piecewise functions.	
9.4	The integral as a limit of a sum; lower and upper Riemann sums. Fundamental theorem of calculus. Improper integrals of the type $\int_a^\infty f(x)\,\mathrm{d}x$.	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_a^x f(y)\,\mathrm{d}y\right] = f(x)$.	**Int:** How close was Archimedes to integral calculus? **Int:** Contribution of Arab, Chinese and Indian mathematicians to the development of calculus. **Aim 8:** Leibniz versus Newton versus the "giants" on whose shoulders they stood—who deserves credit for mathematical progress? **TOK:** Consider $f(x)=\frac{1}{x}$, $1 \le x \le \infty$. An infinite area sweeps out a finite volume. Can this be reconciled with our intuition? What does this tell us about mathematical knowledge?

	Content	Further guidance	Links
9.5	First-order differential equations. Geometric interpretation using slope fields, including identification of isoclines. Numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$ using Euler's method. Variables separable. Homogeneous differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = f\left(\frac{y}{x}\right)$ using the substitution $y = vx$. Solution of $y' + P(x)y = Q(x)$, using the integrating factor.	$y_{n+1} = y_n + hf(x_n, y_n)$, $x_{n+1} = x_n + h$, where h is a constant.	**Appl:** Real-life differential equations, eg Newton's law of cooling, population growth, carbon dating.
9.6	Rolle's theorem. Mean value theorem. Taylor polynomials; the Lagrange form of the error term. Maclaurin series for e^x, $\sin x$, $\cos x$, $\ln(1+x)$, $(1+x)^p$, $p \in \mathbb{Q}$. Use of substitution, products, integration and differentiation to obtain other series. Taylor series developed from differential equations.	Applications to the approximation of functions; formula for the error term, in terms of the value of the $(n+1)^{\text{th}}$ derivative at an intermediate point. Students should be aware of the intervals of convergence.	**Int, TOK:** Influence of Bourbaki on understanding and teaching of mathematics. **Int:** Compare with work of the Kerala school.

	Content	Further guidance	Links
9.7	The evaluation of limits of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ and $\lim_{x \to \infty} \frac{f(x)}{g(x)}$. Using l'Hôpital's rule or the Taylor series.	The indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$. Repeated use of l'Hôpital's rule.	

Topic 10—Option: Discrete mathematics 48 hours

The aim of this option is to provide the opportunity for students to engage in logical reasoning, algorithmic thinking and applications.

	Content	Further guidance	Links
10.1	Strong induction. Pigeon-hole principle.	For example, proofs of the fundamental theorem of arithmetic and the fact that a tree with n vertices has $n-1$ edges.	**TOK:** Mathematics and knowledge claims. The difference between proof and conjecture, eg Goldbach's conjecture. Can a mathematical statement be true before it is proven? **TOK:** Proof by contradiction.
10.2	$a \mid b \Rightarrow b = na$ for some $n \in \mathbb{Z}$.		
	The theorem $a \mid b$ and $a \mid c \Rightarrow a \mid (bx \pm cy)$ where $x, y \in \mathbb{Z}$.	The division algorithm $a = bq + r$, $0 \le r < b$.	
	Division and Euclidean algorithms. The greatest common divisor, $\gcd(a,b)$, and the least common multiple, $\operatorname{lcm}(a,b)$, of integers a and b. Prime numbers; relatively prime numbers and the fundamental theorem of arithmetic.	The Euclidean algorithm for determining the greatest common divisor of two integers.	**Int:** Euclidean algorithm contained in Euclid's *Elements*, written in Alexandria about 300 BCE. **Aim 8:** Use of prime numbers in cryptography. The possible impact of the discovery of powerful factorization techniques on internet and bank security.
10.3	Linear Diophantine equations $ax + by = c$.	General solutions required and solutions subject to constraints. For example, all solutions must be positive.	**Int:** Described in Diophantus' *Arithmetica* written in Alexandria in the 3rd century CE. When studying *Arithmetica*, a French mathematician, Pierre de Fermat (1601–1665) wrote in the margin that he had discovered a simple proof regarding higher-order Diophantine equations—Fermat's last theorem.

	Content	Further guidance	Links
10.4	Modular arithmetic. The solution of linear congruences. Solution of simultaneous linear congruences (Chinese remainder theorem).		**Int:** Discussed by Chinese mathematician Sun Tzu in the 3^{rd} century CE.
10.5	Representation of integers in different bases.	On examination papers, questions that go beyond base 16 will not be set.	**Int:** Babylonians developed a base 60 number system and the Mayans a base 20 number system.
10.6	Fermat's little theorem.	$a^p = a \pmod p$, where p is prime.	**TOK:** Nature of mathematics. An interest may be pursued for centuries before becoming "useful".

	Content	Further guidance	Links
10.7	Graphs, vertices, edges, faces. Adjacent vertices, adjacent edges. Degree of a vertex, degree sequence. Handshaking lemma.	Two vertices are adjacent if they are joined by an edge. Two edges are adjacent if they have a common vertex.	**Aim 8:** Symbolic maps, eg Metro and Underground maps, structural formulae in chemistry, electrical circuits. **TOK:** Mathematics and knowledge claims. Proof of the four-colour theorem. If a theorem is proved by computer, how can we claim to know that it is true?
	Simple graphs; connected graphs; complete graphs; bipartite graphs; planar graphs; trees; weighted graphs, including tabular representation. Subgraphs; complements of graphs.	It should be stressed that a graph should not be assumed to be simple unless specifically stated. The term adjacency table may be used.	**Aim 8:** Importance of planar graphs in constructing circuit boards.
	Euler's relation: $v-e+f=2$; theorems for planar graphs including $e \le 3v-6$, $e \le 2v-4$, leading to the results that κ_5 and $\kappa_{3,3}$ are not planar.	If the graph is simple and planar and $v \ge 3$, then $e \le 3v-6$. If the graph is simple, planar, has no cycles of length 3 and $v \ge 3$, then $e \le 2v-4$.	**TOK:** Mathematics and knowledge claims. Applications of the Euler characteristic $(v-e+f)$ to higher dimensions. Its use in understanding properties of shapes that cannot be visualized.
10.8	Walks, trails, paths, circuits, cycles.		
	Eulerian trails and circuits.	A connected graph contains an Eulerian circuit if and only if every vertex of the graph is of even degree.	**Int:** The "Bridges of Königsberg" problem.
	Hamiltonian paths and cycles.	Simple treatment only.	
10.9	Graph algorithms: Kruskal's; Dijkstra's.		

	Content	Further guidance	Links
10.10	Chinese postman problem. **Not required:** Graphs with more than four vertices of odd degree.	To determine the shortest route around a weighted graph going along each edge at least once.	**Int:** Problem posed by the Chinese mathematician Kwan Mei-Ko in 1962.
	Travelling salesman problem. Nearest-neighbour algorithm for determining an upper bound. Deleted vertex algorithm for determining a lower bound.	To determine the Hamiltonian cycle of least weight in a weighted complete graph.	**TOK:** Mathematics and knowledge claims. How long would it take a computer to test all Hamiltonian cycles in a complete, weighted graph with just 30 vertices?
10.11	Recurrence relations. Initial conditions, recursive definition of a sequence.		**TOK:** Mathematics and the world. The connections of sequences such as the Fibonacci sequence with art and biology.
	Solution of first- and second-degree linear homogeneous recurrence relations with constant coefficients.	Includes the cases where auxiliary equation has equal roots or complex roots.	
	The first-degree linear recurrence relation $u_n = au_{n-1} + b$.		
	Modelling with recurrence relations.	Solving problems such as compound interest, debt repayment and counting problems.	

Appendix G

General Chinese Mathematics Curriculum

Standard Curriculum

Requirements 1

1.1 集合

基本要求	1. 了解集合的含义、元素与集合的"属于"关系、集合相等的含义。 2. 理解列举法和描述法，能选择自然语言、图形语言、集合语言来表示集合。 3. 掌握常用数集的记法。 4. 理解空集的含义。 5. 了解集合与集合之间的"包含"关系，理解子集、真子集的概念，会写出给定集合的子集、真子集。 6. 理解两个集合的并集与交集的含义，掌握有关术语和符号，会求两个简单集合的并集与交集。 7. 理解全集、补集的含义，会求给定子集的补集。 8. 会使用 Venn 图表达集合的关系及运算，体会直观图示对理解抽象概念的作用。
发展要求	能利用集合的关系和运算及 Venn 图来求有限集合中元素的个数。
说明	在教学时，要把握好难度，不要求补充集合运算的性质及证明。

1.2 函数及其表示

基本要求	1. 理解函数的概念，理解构成函数的三要素。 2. 掌握区间的表示方法。 3. 能根据给定的函数解析式及自变量计算函数值，会求一些简单函数的定义域、值域。 4. 理解函数的三种表示法：解析法、图象法和列表法，能根据不同的要求选择恰当的方法表示简单的函数。 5. 理解分段函数的本质，能用分段函数解决一些简单的数学问题。 6. 能用描点法画出一些简单函数的图象。 7. 了解映射的概念，了解函数是一种特殊的映射，并能根据映射的概念判别哪些对应关系是映射。
发展要求	1. 会求一些简单复合函数的值域。 2. 可利用计算器或计算机研究函数图象，帮助学生更深刻地理解函数的概念。
说明	函数教学应基于具体的函数，有关抽象函数的内容不宜涉及；函数值域的教学应控制难度；变量代换不宜太难。

1.3 函数的基本性质

基本要求	1. 理解函数的单调性及其几何意义，能根据函数图象判断单调区间。 2. 会利用定义讨论和证明一些简单函数的单调性。 3. 理解函数的最大(小)值及其几何意义，能根据函数图象和单调性求出一些简单函数的最大(小)值。 4. 理解函数奇偶性的含义，会判断简单函数的奇偶性。 5. 了解奇(偶)函数图象的对称性。 6. 会运用函数图象来理解、研究函数性质。
发展要求	能研究某些简单的复合函数及分段函数的奇偶性、单调性、最大(小)值和图象。
说明	研究函数性质的例题和训练不宜太难，应局限于具体的函数；奇(偶)函数的图象对称性在本节教学时不要求证明。

2.1 指数函数

基本要求	1. 了解指数函数模型的实际背景，认识学习指数函数的必要性。 2. 理解 n 次方根与 n 次根式的概念，理解分数指数幂的含义，熟练掌握用根式与分数指数幂表示一个正实数的 n 次方根。 3. 能运用有理指数幂的运算性质进行运算和化简，会进行根式与分数指数幂的相互转化。 4. 通过经历用有理指数幂逼近无理指数幂的过程，了解实数指数幂的含义。 5. 理解指数函数的概念。 6. 能用描点法或借助计算机(计算器)画出指数函数的图象，探索并理解指数函数的性质(定义域、值域、特殊点、单调性)。 7. 在解决简单的实际问题过程中，体会指数函数是一类重要的函数模型。 8. 会求一些与指数函数有关的简单复合函数的定义域、值域、单调性等。
发展要求	1. 了解函数图象的平移与对称变换。 2. 体会数学的逼近、数形结合等思想。
说明	教学时，有关根式的运算、化简应控制难度。

2.2 对数函数

基本要求	1. 经历由指数得到对数的过程，理解对数的概念，会熟练地进行指数式与对数式的互化。 2. 理解对数的运算性质，并能灵活、准确地运用对数的运算性质进行对数式的化简与计算。 3. 了解对数的换底公式，能将一般对数化成自然对数或常用对数。

续表

基本要求	4. 理解对数函数的概念。 5. 能用描点法或借助计算机画出对数函数的图象,探索并掌握对数函数的性质(定义域、值域、特殊点、单调性)。 6. 通过实例,体会对数函数是一类重要的函数模型。 7. 了解指数函数 $y=a^x$ ($a>0$,且 $a\neq1$)与对数函数 $y=\log_a x$ ($a>0$,且 $a\neq1$)互为反函数。 8. 了解 $y=a^x$($a>0$,且 $a\neq1$)与对数函数 $y=\log_a x$($a>0$,且 $a\neq1$)的图象关于直线 $y=x$ 对称。
发展要求	会求一些与对数函数有关的简单的复合函数的定义域、值域、单调性。
说明	不必讨论形式化的反函数定义,也不要求求已知函数的反函数。

2.3 幂函数

基本要求	1. 了解幂函数的概念。 2. 掌握以下五个幂函数的图象和性质: $y=x$, $y=x^2$, $y=x^3$, $y=x^{\frac{1}{2}}$, $y=x^{-1}$。
发展要求	
说明	不必在一般的幂函数上作引申。

3.1 函数与方程

基本要求	1. 了解函数零点的概念，了解函数零点与方程根的联系。 2. 理解并掌握连续函数在某个区间上存在零点的判定方法。 3. 能利用函数的图象和性质判断函数零点的个数。
发展要求	体验函数与方程、数形结合、算法等数学基本思想。
说明	连续函数在某个区间上存在零点的判定方法，只要求学生理解并会运用，教学中不需要给出证明。

3.2 函数模型及其应用

基本要求	1. 理解直线上升、指数爆炸、对数增长的含义。 2. 理解指数函数、对数函数以及幂函数增长速度的差异。 3. 能利用给定的函数模型解决实际问题；能选择适当的函数模型进行拟合，实现问题的解决；了解指数函数、对数函数、幂函数、分段函数等函数模型在社会生活中的广泛应用。 4. 初步掌握建立函数模型解决问题的过程和方法。
发展要求	通过建立和运用函数基本模型，体验数学建模、拟合等数学基本思想，发展学生的创新意识和数学应用意识。
说明	建立及应用函数模型是学习函数的重要目的之一，在函数模型的选择上应以简单函数为主，不追求问题的难度。可指导学生利用计算器或计算机运算。

Requirements 2

1.1 空间几何体的结构

基本要求	1. 理解柱、锥、台、球的结构特征。 2. 了解棱柱、棱锥、棱台的底面、侧棱、侧面、顶点的意义。 3. 了解圆柱、圆锥、圆台的底面、母线、侧面、轴的意义。 4. 了解与正方体、球有关的简单组合体及其结构特征。 5. 能根据条件判断几何体的类型。
发展要求	
说明	1. 柱、锥、台、球的结构特征只需通过实例概括,不必证明。 2. 空间几何体的性质不必深入挖掘。

1.2 空间几何体的三视图和直观图

基本要求	1. 了解中心投影和平行投影的意义。 2. 理解三视图画法的规则,能画简单几何体的三视图。 3. 掌握斜二测画法,能画简单几何体的直观图。 4. 能识别三视图所表示的空间几何体。 5. 理解三视图和直观图的联系,并能进行转化。
发展要求	
说明	对于画三视图和直观图的几何体,只要求前一节介绍的柱、锥、台、球及它们的一些简单组合,不必研究较复杂的几何体。

1.3 空间几何体的表面积与体积

基本要求	1. 了解表面与展开图的关系。 2. 了解柱、锥、台、球表面积的计算公式,并能计算一些简单组合体的表面积。 3. 了解柱、锥、台、球的体积公式,并能计算一些简单组合体的体积。
发展要求	了解柱体、锥体、台体之间的关系。
说明	球的体积公式的推导不要求学生掌握。

2.1 空间点、直线、平面之间的位置关系

基本要求	1. 了解平面的概念,掌握平面的画法及表示方法。 2. 了解平面的基本性质,即公理 1,2,3。 3. 会进行“文字语言”“符号语言”“图形语言”之间的转化。 4. 掌握空间中点与直线、点与平面位置关系的分类与表示。 5. 理解异面直线的定义,并能正确画出两条异面直线。 6. 掌握直线与直线、直线与平面、平面与平面的位置关系的分类与表示。 7. 理解公理 4 和等角定理。
发展要求	1. 会用反证法证明两条直线是异面直线。 2. 初步体验将空间问题转化为平面问题的思想方法。
说明	确定平面的 3 个推论和两条异面直线的公垂线、距离及有关概念不作要求。

2.2 直线、平面平行的判定及其性质

基本要求	1. 通过直观感知、操作确认,归纳出直线与平面、平面与平面平行的判定定理。 2. 证明并掌握直线与平面平行、平面与平面平行的性质定理。 3. 能运用上述定理证明一些空间位置关系的简单命题。
发展要求	
说明	平行关系的判定定理的证明不作要求。

2.3 直线、平面垂直的判定及其性质

基本要求	1. 理解直线与平面垂直的定义。 2. 通过直观感知、操作确认,归纳直线与平面、平面与平面垂直的判定定理。 3. 证明并掌握直线与平面、平面与平面垂直的性质定理。 4. 理解直线和平面所成角的概念。 5. 了解二面角及其平面角的概念。 6. 能运用判定定理、性质定理证明一些空间位置关系的简单命题。
发展要求	
说明	垂直关系的判定定理的证明不作要求。

3.1 直线的倾斜角与斜率

基本要求	1. 理解直线的倾斜角的定义,掌握直线倾斜角的范围。 2. 理解直线的斜率,掌握过两点的直线的斜率公式。 3. 掌握用斜率判定两条直线平行和垂直的方法。 4. 掌握直线斜率和倾斜角之间的关系:$k=\tan\alpha=\frac{y_1-y_2}{x_1-x_2}$。 5. 能利用斜率解决具体问题。
发展要求	能用三角函数描述斜率。
说明	

3.2 直线的方程

基本要求	1. 掌握直线方程的点斜式、斜截式、两点式，能根据条件熟练地求出直线的方程。 2. 了解直线方程的截距式。 3. 能正确理解直线方程一般式的含义。 4. 能将直线方程的点斜式、斜截式、两点式等几种形式转化为一般式，知道这几种形式的直线方程的局限性。 5. 根据所给条件，灵活选取并求出适当形式的直线方程。
发展要求	了解直线和直线方程之间的对应关系。
说明	截距式方程只作为两点式方程的一种应用例子，不必单独提出这种直线的形式。

3.3 直线的交点坐标与距离公式

基本要求	1. 会求两条直线的交点坐标。 2. 理解两条直线的平行、相交与相应的直线方程所组成的二元一次方程组的解的对应关系。 3. 掌握平面上两点间的距离公式。 4. 掌握平面上两点连线的中点坐标公式。 5. 能运用距离公式和中点坐标公式解决一些简单的问题。 6. 掌握点到直线的距离公式，能运用它解决一些简单的问题。 7. 会求两条平行直线间的距离。
发展要求	1. 通过对点到直线距离公式的推导，渗透化归思想，并使学生进一步了解用代数方程研究几何问题的方法。 2. 渗透数形结合思想，对学生进行对立统一观点的教育。 3. 理解两条直线垂直时，它们的斜率之间的关系。
说明	两条平行线的距离公式不必记忆。

4.1 圆的方程

基本要求	1. 探索并掌握圆的标准方程和一般方程。 2. 会根据圆的方程求出圆心坐标和半径。 3. 能用代数方法判定点与圆的位置关系。 4. 会用待定系数法求圆的方程。 5. 体验求曲线方程(点的轨迹)的基本方法,概括其基本步骤。
发展要求	认识圆的方程与 x^2,y^2 项系数相同的二元二次方程之间的联系。
说明	

4.2 直线、圆的位置关系

基本要求	1. 能判断直线与圆、圆与圆的位置关系。 2. 能利用坐标法解决一些简单的位置关系问题。 3. 理解坐标法解决几何问题的一般步骤。 4. 能根据条件求直线或圆的方程。
发展要求	1. 通过研究圆上任意一点与直线上任意一点之间距离的最值问题,体会数形结合、化归的思想方法。 2. 通过两圆关于直线对称问题的研究,进一步体会解析法思想。
说明	

4.3 空间直角坐标系

基本要求	1. 了解空间直角坐标系的概念,理解三维空间的点可以用三个量来表示。 2. 通过所有棱分别与坐标轴平行的特殊长方体的顶点的坐标,探索并得出空间两点间的距离公式。 3. 会用空间两点间的距离公式,求两点间的距离,比较线段的长度。
发展要求	能建立空间直角坐标系表示一些特殊的几何体(如正三棱锥、正三棱柱)。
说明	该内容主要为后续学习打基础;重点应放在空间直角坐标系的理解上。

Requirements 4

1.1 任意角、弧度

基本要求	1. 认识角的扩充的必要性，了解任意角的概念。 2. 能用集合和数学符号表示终边相同的角。 3. 能用集合和数学符号表示象限角。 4. 了解弧度制，能进行弧度与角度的换算。 5. 了解弧长公式，能进行简单应用。
发展要求	能用集合和数学符号表示终边满足一定条件的角。
说明	对于弧长公式，教学中不必在应用方面加深。

1.2 任意角的三角函数

基本要求	1. 理解任意角三角函数(正弦、余弦、正切)的定义。 2. 能判断各象限角的正弦、余弦、正切函数的符号。 3. 理解终边相同的角的同一三角函数的值相等。 4. 认识单位圆中任意角的正弦线、余弦线和正切线。 5. 理解同角三角函数的两个基本关系：$\sin^2x+\cos^2x=1$，$\frac{\sin x}{\cos x}=\tan x$，能进行简单应用。
发展要求	利用单位圆中的三角函数线解决简单的三角函数问题。
说明	用同角三角函数基本关系证明三角恒等式和进行求值计算，教学中不必作太多的拓展和补充。

1.3 三角函数的诱导公式

基本要求	1. 能借助单位圆中的三角函数线推导$\frac{\pi}{2}\pm\alpha$的正弦、余弦，以及$\pi\pm\alpha$的正弦、余弦、正切的诱导公式，能进行简单的应用。 2. 掌握用单位圆中三角函数线研究三角函数问题的方法。
发展要求	
说明	

1.4 三角函数的图象与性质

基本要求	1. 能画出$y=\sin x$，$y=\cos x$，$y=\tan x$的图象。 2. 了解三角函数的周期性。 3. 借助图象理解正弦函数、余弦函数在$[0,2\pi]$，正切函数在$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$上的性质(单调性、最大和最小值、图象与$x$轴的交点等)。 4. 会用“五点法”画正弦函数、余弦函数的图象。 5. 了解$y=\cos x$的图象与$y=\sin x$的图象之间的联系。
发展要求	会求形如$y=A\sin(\omega x+\varphi)$的函数的单调区间、最值、周期。
说明	有条件的学校可选择一种计算机软件绘制函数的图象。

1.5 函数$y=A\sin(\omega x+\varphi)$的图象

基本要求	1. 了解$y=A\sin(\omega x+\varphi)$的实际意义，能借助计算器或计算机画出它的图象。 2. 掌握参数A，ω，φ对函数图象变化的影响规律。 3. 会用“五点法”画函数$y=A\sin(\omega x+\varphi)$的图象。 4. 掌握运用平移变换和伸缩变换把$y=\sin x$的图象变换为$y=A\sin(\omega x+\varphi)$的图象的方法。
发展要求	掌握函数$y=A\cos(\omega x+\varphi)$的图象与函数$y=A\sin(\omega x+\varphi)$的图象的联系。
说明	教学中提倡用计算机辅助研究函数$y=A\sin(\omega x+\varphi)$的图象。

1.6 三角函数模型的简单应用

基本要求	1. 会用三角函数解决一些简单的实际问题。 2. 初步学会由图象求解析式的方法。 3. 体验将实际问题抽象为数学问题的过程。 4. 体会三角函数是描述周期变化现象的重要函数模型。
发展要求	能运用三角函数知识分析和处理实际问题。
说明	教学中应突出三角函数的工具性，重点是引导学生建立三角函数模型。

2.1 平面向量的实际背景及基本概念

基本要求	1. 理解向量、零向量、向量的模、单位向量、相等向量、平行向量、共线向量的概念。 2. 理解平面向量的几何意义。
发展要求	
说明	

2.2 平面向量的线性运算

基本要求	1. 掌握向量加、减法的定义，并理解其几何意义，会用向量加法的三角形法则和平行四边形法则作出两个向量的和与差。 2. 掌握向量加法的交换律与结合律，并会用它们进行向量运算。 3. 掌握实数与向量积的定义及向量数乘的运算，并理解其几何意义。 4. 理解两个向量共线的条件。 5. 理解向量的线性运算性质及其几何意义。
发展要求	
说明	1. 向量的线性表示应控制在基本要求的范围内，不宜作太多的扩展。 2. 对于向量运算的交换律、数乘的结合律和分配律，只要求会用即可。对于基础较好的学生可以介绍证明方法。

2.3 平面向量的基本定理与坐标运算

基本要求	1. 理解平面向量的基本定理及其意义，会用平面向量的基本定理解决简单问题。 2. 理解平面向量的坐标的概念，掌握平面向量的正交分解及其坐标表示。 3. 掌握平面向量的坐标运算，会用坐标表示平面向量的加、减与数乘运算。 4. 理解用坐标表示的平面向量共线的条件，会依据向量的坐标判断向量是否共线。
发展要求	掌握利用向量求分点坐标的方法。
说明	平面向量基本定理不作严格证明。

2.4 平面向量的数量积

基本要求	1. 理解平面向量数量积及其几何意义。 2. 体会平面向量数量积与向量投影的关系。 3. 掌握平面向量数量积的性质、运算律和几何意义。 4. 掌握向量数量积的坐标表达式，会进行平面向量数量积的坐标运算。 5. 能运用数量积表示两个向量的夹角。 6. 会用数量积处理有关长度、角度和垂直等问题。
发展要求	掌握平面向量数量积的应用。
说明	

2.5 平面向量应用举例

基本要求	1. 了解向量知识在实际生活中有着广泛的应用。 2. 能运用向量方法解决某些简单的平面几何问题、力学问题和其他一些实际问题。
发展要求	合理选择基向量解决数学问题。
说明	平面向量的应用主要在平面几何和简单的物理学这两个方面，不必在其他方面拓展。

3.1 两角和与差的正弦、余弦和正切公式

基本要求	1. 了解学习两角和与差的三角函数公式的必要性。 2. 理解用三角函数线、向量推导两角差的余弦公式的思路。 3. 能利用两角差的余弦公式推出两角和与倍角的其他三角函数公式。 4. 能利用这些公式进行和、差、倍角的求值和简单的化简。
发展要求	1. 理解在两角差的余弦公式的推导过程中所体现的向量方法。 2. 理解和、差、倍角的相对性,能对角进行合理、正确的拆分。 3. 能对公式进行简单的逆用。
说明	控制好拆分角度的难度。

3.2 简单的三角恒等变换

基本要求	1. 能利用和、差、倍角的公式进行基本的变形,并证明三角恒等式。 2. 能利用三角恒等变换研究三角函数的性质。 3. 能把一些实际问题转化为三角问题,通过三角变换解决。
发展要求	1. 了解和、差、倍角公式的特点,并进行变形应用。 2. 理解三角变换的基本特点和基本功能。 3. 了解三角变换中蕴含的数学思想和方法。
说明	积化和差、和差化积、半角公式等只作为练习,不要求记忆。

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1.1 正弦定理和余弦定理

基本要求	1. 会证明正弦定理、余弦定理。 2. 理解正弦定理、余弦定理在讨论三角形边角关系时的作用。 3. 能用正弦定理、余弦定理解斜三角形。 4. 会用正弦定理、余弦定理讨论三角形解的情形。
发展要求	了解正弦定理与三角形外接圆半径的关系。
说明	可以利用计算器进行解三角形计算，但不要求太复杂或繁琐的运算。

1.2 应用举例

基本要求	1. 掌握利用正弦定理、余弦定理解任意三角形的方法。 2. 通过解三角形在实际中的一些应用，培养学生分析问题、解决问题的能力。 3. 理解三角形的面积公式 $S=\frac{1}{2}ab\sin C$，并能应用。
发展要求	利用正弦定理、余弦定理讨论三角形中的边角关系。
说明	1. 在本章学习时不必增加在空间情况下求解三角形的问题，这类问题可在学习立体几何时适当拓展。 2. 应用问题应限制在正弦定理、余弦定理的简单应用上。

2.1 数列的概念与简单表示法

基本要求	1. 理解数列的定义，了解数列是一种特殊函数。 2. 了解数列的几种简单的表示方法(列表、图象、通项公式)。 3. 认识数列是反映自然规律的基本模型。
发展要求	1. 能根据数列的前几项写出一个通项公式。 2. 能根据给出的递推公式写出数列的前几项。
说明	1. 复杂的递推关系式不作要求。 2. 已知数列前几项写出一个通项公式，习题不必太难。

2.2 等差数列

基本要求	1. 理解等差数列的概念。 2. 掌握等差数列的通项公式。 3. 了解等差数列与一次函数的关系。 4. 能在具体的问题情境中识别数列的等差关系，进而用等差数列的有关知识解决相应的问题。
发展要求	掌握等差数列的典型性质及应用。
说明	

2.3 等差数列的前 n 项和

基本要求	1. 掌握等差数列的前 n 项和公式，并能用公式解决简单的问题。 2. 理解等差数列前 n 项和公式的推导方法。 3. 能利用等差数列前 n 项和公式及其性质求一些特殊数列的和。 4. 理解 S_n 与 a_n 的关系。
发展要求	能灵活运用等差数列的求和公式。
说明	

2.4 等比数列

基本要求	1. 理解等比数列的概念。 2. 掌握等比数列的通项公式。 3. 了解等比数列与指数函数的关系。 4. 能在具体的问题情境中识别数列的等比关系，进而用等比数列的有关知识解决相应的问题。
发展要求	1. 掌握等比数列的典型性质及应用。 2. 能用类比观点推导等比数列的性质。
说明	

2.5 等比数列的前 n 项和

基本要求	1. 掌握等比数列的前 n 项和公式，并能用公式解决简单的问题。 2. 理解等比数列前 n 项和公式的推导方法。 3. 能利用等比数列前 n 项和公式及其性质求一些特殊数列的和。
发展要求	1. 等比数列的求和公式应达到灵活运用。 2. 理解等差数列与等比数列简单组合的数列的前 n 项和。
说明	注意等比数列求和公式中 $q\neq 1$ 的条件。

3.1 不等关系与不等式

基本要求	1. 了解现实世界和日常生活中的不等关系,了解不等式(组)的实际背景。 2. 理解不等式(组)对于刻画不等关系的意义和价值。 3. 会用不等式(组)表示实际问题中的不等关系,能用不等式(组)研究含有不等关系的实际问题。 4. 理解并掌握不等式的基本性质。
发展要求	体会不等式的基本性质在不等式证明中所起的作用。
说明	不等式的进一步介绍将在选修 4-5 中给出。

3.2 一元二次不等式及其解法

基本要求	1. 了解从实际情境中抽象出一元二次不等式模型的过程。 2. 理解一元二次不等式的概念。 3. 通过图象,理解并掌握一元二次不等式、二次函数及一元二次方程之间的关系。 4. 理解并掌握解一元二次不等式的过程。 5. 会求一元二次不等式的解集。 6. 掌握求解一元二次不等式的程序框图及隐含的算法思想,会设计求解的程序框图。
发展要求	
说明	淡化解不等式的技巧,突出不等式的实际背景及其应用。

3.3 二元一次不等式(组)与简单的线性规划问题

基本要求	1. 了解从实际情境中抽象出二元一次不等式(组)模型的过程。 2. 理解二元一次不等式(组)、二元一次不等式(组)的解集的概念。 3. 了解二元一次不等式的几何意义,理解(区域)边界的概念及实线、虚线边界的含义。 4. 会用二元一次不等式(组)表示平面区域,能画出给定的不等式(组)表示的平面区域。 5. 了解线性约束条件、目标函数、线性目标函数、线性规划、可行解、可行域、最优解的概念。 6. 掌握简单的二元线性规划问题的解法。
发展要求	会从实际情境中抽象出一些简单的二元线性规划问题并加以解决。
说明	

3.4 基本不等式：$\sqrt{ab} \leqslant \frac{a+b}{2}$

基本要求	1. 了解基本不等式的代数背景、几何背景以及它的证明过程。 2. 理解算术平均数、几何平均数的概念。 3. 会用基本不等式解决简单的最大(小)值的问题。 4. 通过基本不等式的实际应用，感受数学的应用价值。
发展要求	
说明	突出用基本不等式解决问题的基本方法，不必推广到三个变量以上的情形。

3.5 绝对值不等式

基本要求	1. 理解绝对值三角不等式的代数证明和几何意义。 2. 能利用绝对值三角不等式证明一些简单的绝对值不等式。 3. 掌握最简单的绝对值不等式$\|x\|<a$和$\|x\|>a$的解法和几何意义。 4. 掌握$\|ax+b\|\leqslant c$和$\|ax+b\|\geqslant c$型不等式的解法。 5. 掌握$\|x-a\|+\|x-b\|\leqslant c$和$\|x-a\|+\|x-b\|\geqslant c$型不等式的解法。
发展要求	理解绝对值不等式在解决简单的最大(小)值问题中的应用。
说明	控制绝对值不等式的教学难度。

Option 2-1

1.1 命题及其关系

基本要求	1. 理解命题的概念。 2. 了解命题的逆命题、否命题和逆否命题。 3. 会分析四种命题间的相互关系。 4. 会利用互为逆否命题的两个命题之间的关系判断命题的真假。
发展要求	会列举四种命题的相互转化。
说明	四种命题的转化不必要求太高。

1.2 充分条件与必要条件

基本要求	1. 理解必要条件、充分条件与充要条件的含义。 2. 通过具体命题，掌握判断充分条件、必要条件与充要条件的方法。
发展要求	会证明具体问题中的必要性和充分性。
说明	

2.1 曲线与方程

基本要求	1. 了解曲线的方程、方程的曲线等概念。 2. 掌握求曲线方程的基本方法。
发展要求	了解曲线方程的完备性与纯粹性。
说明	

2.2 椭圆

基本要求	1. 了解椭圆的实际背景,体会椭圆在刻画现实世界和解决实际问题中的作用。 2. 掌握椭圆的定义,会建立椭圆的标准方程。 3. 能利用椭圆的标准方程研究椭圆的简单几何性质(范围、对称性、顶点、离心率等)。 4. 能根据椭圆的几何性质,写出椭圆的方程。 5. 会利用椭圆的标准方程和几何性质处理一些简单的实际问题。 6. 掌握求曲线方程的一些基本方法。
发展要求	掌握利用曲线的方程研究曲线几何性质的基本方法。
说明	

2.3 双曲线

基本要求	1. 了解双曲线的实际背景,体会双曲线在刻画现实世界和解决实际问题中的作用。 2. 了解双曲线的定义,会建立双曲线的标准方程。 3. 了解双曲线的几何性质(范围、对称性、顶点、渐近线和离心率等)。 ~~4. 会利用双曲线的几何性质(特别是渐近线)~~求双曲线的标准方程。 5. 会利用双曲线的标准方程和几何性质处理一些简单的实际问题。
发展要求	了解双曲线与椭圆的区别与联系。
说明	

2.4 抛物线

基本要求	1. 了解抛物线的实际背景,体会抛物线在刻画现实世界和解决实际问题中的作用。 2. 掌握抛物线的定义,会建立并掌握抛物线的标准方程。
基本要求	3. 能根据条件求出抛物线的标准方程。 4. 掌握抛物线的几何性质(范围、对称性、顶点、离心率等)。 5. 会利用抛物线方程解决简单的实际问题。
发展要求	了解椭圆、双曲线、抛物线的一些共同性质。
说明	

2.5 直线与圆锥曲线的位置关系

基本要求	1. 了解直线与圆锥曲线的交点个数与相应方程组的解的对应关系。 2. 能用判别式法研究直线与圆锥曲线之间的位置关系。 3. 掌握直线与椭圆、双曲线、抛物线位置关系的简单问题的基本解法。
发展要求	1. 掌握直线与圆锥曲线有关的综合问题的解决方法。 2. 了解圆锥曲线的有关光学性质。
说明	

3.1 空间向量及其运算

基本要求	1. 了解空间向量的意义及相关概念。 2. 掌握空间向量的加减运算及其运算律。 3. 理解空间向量的数乘运算的意义和运算律。 4. 理解共线(平行)向量、共面向量的意义,能利用它们证明简单的空间向量共线和共面的问题。 5. 了解直线的方向向量的意义,理解空间向量的长度和夹角的意义。 6. 理解空间向量的数量积的意义及其运算律。 7. 能利用空间向量的运算,解决直线和直线垂直、直线和平面垂直、两点间距离或线段长度等相关问题。 8. 了解空间向量基本定理及其意义。 9. 了解空间向量的正交分解,掌握其坐标表示;会在简单的问题中选用合适的基底表示其他向量。 10. 掌握向量的长度公式、两向量夹角公式、空间两点间的距离公式,并会解决简单的立体几何问题。
发展要求	
说明	三垂线定理作为用向量解决立体几何问题的一个例题,对三垂线定理及其逆定理本身不需作深人探究。

3.2 立体几何中的向量方法

基本要求	1. 能利用空间向量表示空间的点、直线、平面等元素,建立立体图形与空间向量之间的联系。 2. 理解平面的法向量的意义。 3. 通过具体的实例,明确用空间向量解决立体几何问题的“三步曲”。 4. 利用直线的方向向量解决两直线平行、垂直及夹角的问题,利用法向量解决两平面平行、垂直及二面角的问题,能通过选择适当的坐标系,解决简单的立体几何问题。
发展要求	
说明	

Extended Curriculum

OPTION 2-2

Chapter 1 Derivative

Section	Topic
Rate of Change and Derivative	1. Rate of Change 1.1 Background of derivative and necessity of learning derivative 1.2 Average rate of change and instantaneous rate of change; and their connection
	2. Derivative of a Function 2.1 Derivative at a point 2.2 Tangents to a curve
	3. Geometric Meaning of Derivative 3.1 Geometric meaning of derivative, understand the concept of derived function 3.2 Physical significance of derivative 3.3 Solve simple geometric and physical problems by using derivative
Calculation of Derivative	1. Derivatives of some common functions 1.1 derivatives of y=c, y=x, y=x^2 , y=$\frac{1}{x}$, y=$\sqrt{x}$ 1.2 Geometric meaning of the derivatives of y=c, y=x, y=x^2
	2. Derivative Formula of Basic Elementary Functions and the Algorithm of Derivative 2.1 Use derivative formula of basic elementary functions and the algorithm of derivative to get basic elementary functions' derivatives 2.2 A Composite Function and its derivative 2.3 Derivative formula table of basic elementary functions 2.4 Solve simple questions by using derivatives
Application of Derivative	1. Monotonicity of a Function and Derivative 1.1 the relationship between monotonicity of a function and derivative 1.2 Monotone interval of polynomial function (within cube)
	2. Extreme Value of a Function and Derivative 2.1 Definition of Extreme Value 2.2 Use monotonicity of a function to explore the relationship between extreme value and derivative 2.3 Method of getting extreme value of a function 2.4 Extreme value of polynomial function (within cube)
	3. Maximum and Minimum Value of a Function and Derivative 3.1 Maximum and Minimum Value of polynomial function (within cube) in a closed interval

Chapter 2 Reasoning and Proof

Section	Topic
Direct and Indirect Proof	1. Synthesis Method and Analytical Method
	2. Proof by Contradiction
Mathematical Induction	1. Mathematical Induction

Chapter 3 Expansion of a Number System and Introduction of Complex Number

Section	Topic
Expansion of a Number System and Definition of Complex Number	1. Expansion of a Number System and Definition of Complex Number 1.1 Necessity of introducing complex number 1.2 Method of expansion of a number system 1.3 Definition of complex number 1.4 Definition of complex algebraic form 1.5 Classification of complex number 1.6 Geometric meaning of complex number 1.7 Corresponding relationship between Complex set and set of points, and Complex set and plane vector 1.8 Definition of Complex Plane 1.9 Modulus of complex number
Calculation of Complex Algebraic Form	1. Four Fundamental Rules of Complex Algebraic Form 1.1 Addition and subtraction of complex algebraic form 1.2 Equality of two complex numbers 1.3 Geometric meaning of addition and subtraction of complex algebraic form 1.4 Multiplication and division of complex algebraic form 1.5 Conjugate complex number 1.6 Fundamental theorem of algebra

OPTION 2-3

Chapter 1 Counting Principle

Section	Topic
Classification of Addition Counting Principle and Fractional Step Method of Multiplication Counting Principle	1. Classification of Addition Counting Principle and Fractional Step Method of Multiplication Counting Principle 1.1 Definition of these two counting principles 1.2 Connection and difference of these two counting principles 1.3 Use two counting principles to solve problems 1.4 Use two counting principles to analyze and solve some simple counting problems
Permutation and Combination	1. Permutation 1.1 Meaning of permutation and number of permutation 1.2 Formula of number of permutation 1.3 Properties of number of permutation
	2. Combination 2.1 Meaning of combination and number of combination 2.2 Formula of number of combination 2.3 Connection and Difference between Permutation and Combination
Binomial Theorem	1. Binomial Theorem 1.1 Multiplication Formula and Binomial Theorem 1.2 Use Counting Principle to Prove Binomial Theorem 1.3 Application of Binomial Theorem 1.4 General Formula of Binomial Expansion and its Application
	2. Pascal's Triangle and Binomial Coefficient 2.1 Concept of Pascal's Triangle 2.2 Properties of Binomial Coefficient 2.3 Difference and Connection of Binomial Coefficient and the Coefficient of the terms 2.4 Application of Binomial Coefficient

Chapter 2 Probability

Section	Topic
Probability of Random Event	1. Probability of Random Event 1.1 Meanings of Certain Event, Impossible Event and Random Event 1.2 Indeterminacy of Random Event and Frequency Stability
	2. Meaning of Probability 2.1 Meaning of Probability 2.2 Connection and difference between Probability and Frequency 2.3 Meanings and Formulas of Mutually Exclusive Events and Complementary Events
	3. Properties of Probability
Classical Probability	1. Classical Probability 1.1 Meaning of Elementary Event 1.2 Classical Probability and its formula 1.3 Enumeration Method 1.4 Application of Probability Calculation Formulas
Discrete Random Variable and Distribution Sequence	1. Discrete Random Variable and Distribution Sequence 1.1 Meanings of Random Variable and Discrete Random Variable 1.2 Definition of Distribution Sequence 1.3 Calculation of Simple Discrete Radom Variable and its Distribution Sequence 1.4 Two-Point Distribution
Binomial Distribution and its Application	1. Conditional Probability 1.1 Meaning of Conditional Probability and its Application
	2. Mutual Independence of Events 2.1 Meaning of Mutual Independence of Events and its Application
	3. Independent Repeated Trials and Binomial Distribution 3.1 Meaning of Independent Repeated Trials 3.2 Meaning of Binomial Distribution
Mean Value and Variance of Discrete Random Variables	1. Mean Value of Discrete Random Variables 1.1 Meaning of Mean Value of Discrete Random Variables 1.2 Connection and Difference between Mean Value of Random Variables and the Sample Mean 1.3 Use Discrete Random Variable and its Distribution Sequence to Calculate Mean Value 1.4 Mean Value of Two-Point Distribution and Binomial Distribution
	2. Variance of Discrete Random Variables 2.1 Meaning of Variance of Discrete Random Variables 2.2 Use Discrete Random Variable and its Distribution Sequence to Calculate Variance 2.3 Application of Mean Value and Variance

Appendix H

General Chinese Mathematics Curriculum vs AP Calculus Curriculum

REQUIREMENTS 1

Chapter 1 Sets and Function

Section	Topic
Set	1. Meanings and Notations of Sets 1.1 Meaning of Sets 1.2 Element Characteristics 1.3 Equality of Sets 1.4 Relations between Set and Element 1.5 Memorization of Common Sets 1.6 Set Notations
	2. Set Relations 2.1 Introduction of Subset and Proper Subset 2.2 Introduction of Empty Set
	3. Set Operation 3.1 Union 3.2 Intersection 3.3 Universal Set and Complement
Functions and Notations	*1. Concepts of Functions* *1.1 Definition of a Function* *1.2 Notation y=f(x)* *1.3 Domain of a Function* *1.4 Range of a Function* *1.5 Definition of Interval and Its Notation*
	2. Notations of Functions *2.1 Algebraic expression of a Function* *2.2 Graph Notation and Tracing Point Method* *2.3 Table Notation of a Function* *2.4 Definition and Application of a Piecewise Function* *2.5 Definition of Mapping*
Properties of Functions	*1. Monotonicity and Max(Min) Value* *1.1 Definitions of Increasing and Decreasing Functions* 1.2 Monotonicity and Monotone Interval *1.3 Maximum and Minimum Value of a Function*
	2. Parity of a Function *2.1 Definitions of odd and even functions* *2.2 Properties of odd and even functions*

Chapter 2 Elementary Functions

Section	Topic
Exponential Functions	*1. Exponents and Exponential Operation* *1.1 Definition of Root* *1.2 Definition of a Rational Exponent* *1.3 Definition of an Irrational Exponent* *1.4 Laws of Rational Exponents*
	2. Exponential Function and Properties *2.1 Definition of an Exponential Function* *2.2 Graph of an Exponential Function* *2.3 Properties of an Exponential Function*
Logarithmic Functions	*1. Logarithm and Logarithmic Operation* *1.1 Definition of a Logarithm* *1.2 Common Logarithm and Natural Logarithm* *1.3 Laws and Properties of Logarithms* *1.4 Change of Base*
	2. Logarithmic Function and Its Properties *2.1 Definition of a Logarithmic Function* *2.2 Graph of a Logarithmic Function* *2.3 Properties of a Logarithmic Function* *2.4 Relation between Exponential and Logarithmic Functions*
Power Functions	*1. Power Function* ($y = x$, $y = x^2$, $y = x^{\frac{1}{2}}$, $y = x^{-1}$) *1.1 Definition of a Power Function* *1.2 Graph of a Power Function* *1.3 Properties of a Power Function*

Chapter 3 Application of Functions

Section	Topic
Functions And Equations	*1. Root of an Equation and Zero of a Function* *1.1 Definition of Zero* *1.2 Relation between "f(x) = 0 has defined roots" & "y = f(x) has zeros"* *1.3 Determine Continuous Function Has Zero(s) in Domain (a, b)*
Models of Functions and their Applications	*1. Increasing Models of Different Functions* *1.1 Speed of Growth of Exponential Function* $y = a^x$ *(a>1) in Domain (0, +∞)* *1.2 Speed of Growth of Logarithmic Function* $y = \log_a x$ *(a>1) in (0, +∞)* *1.3 Speed of Growth of Power Function* $y = x^n$ *(n>0) in Domain (0, +∞)* *1.4 Comparison of the Speeds of Growth of* $y = a^x$ *(a>1),* $y = \log_a x$ *(a>1),* $y = x^n$ *(n>0) in Domain (0, +∞)*
	2. Examples of Application of Models *2.1 Application of Models in Practical Cases* *2.2 Develop Models according to Practical Cases*
	3. Comprehensive Application

REQUIREMENTS 2

Chapter 1 Geometry in 3D-Space

Section	Topic
Structure of 3D Geometry	1. Structures of Cylinder, Cone and Frustum 1.1 Definitions of a Prism, a Pyramid and Frustum of a Pyramid 1.2 Bases, Side Edges, Lateral Faces and Vertices of a Prism, a Pyramid and Frustum of a Pyramid 1.3 Definitions of a Cylinder, Cone, Frustum of a Cone and Sphere 1.4 Bases, Generatrices, Lateral Faces, Axis of a Cylinder, Cone, Frustum of a Cone and Sphere 1.5 Center, Radius, Diameter of a Sphere
	2. Features of Simple Geometry 2.1 Cube and Sphere Related Geometries and their features 2.2 Determine Geometries with Givens
Three-view Drawing and Trimetric Drawing of 3D Geometry	1. Perspective Projection and Parallel Projection 1.1 Definitions of Projection, Projection Line and Projection Plane 1.2 Definitions of Perspective Projection and Parallel Projection
	2. Three-view Drawing of 3D Geometry 2.1 Definitions of Top View, Side View, and Front View 2.2 Rules of Three-view Drawing 2.3 Three-view Drawing of Simple Geometry
	3. Trimetric View of 3D Geometry 3.1 Definition of Oblique Bimetric projection 3.2 Steps of Oblique Bimetric projection 3.3 Draw Trimetric View of Simple Geometry 3.4 Determine the Geometry Given Three-view Drawing 3.5 Draw Three-view Drawing Given Trimetric View and Vice Versa
Surface Area and Volume of 3D Geometry	1. Surface Area and Volume of a Prism, a Pyramid, and Frustum of a Pyramid 1.1 Relation between Surface Area and Expansion View 1.2 Volume Formula of a Prism, a Pyramid, and Frustum of a Pyramid 1.3 Formula of Surface Area of a Prism, a Pyramid, and Frustum of a Pyramid 1.4 Relation between a Prism, a Pyramid, and Frustum of a Pyramid 1.5 Relation of Graphical Alternation between Triangular Prism and Triangular Pyramid
	2. Surface Area and Volume of a Sphere
	3. Surface Area and Volume of Composition of Simple 3D Geometry

Chapter 2 Position Relations between Point, Line and Plane

Section	Topic
Position Relation between Point, Line and Plane in 3D Space	1. Plane 1.1 Definition of Plane 1.2 Drawing and Notation of Plane 1.3 Properties of Plane i.e. Theorem1, 2, and 3 1.4 Alternation between “Word”, “Symbol”, “Graph”
	2. Position Relation between Lines in 3D Space 2.1 Definition and Notation of Lines in Different Planes 2.2 Theorem 4 2.3 Isogonal Theorem 2.4 Angle Formed by Lines in Different Planes 2.5 Definition of Orthogonal Lines
	3. 3 Positional Relations between Lines and Planes
	4. Position Relation between Plane and Plane
Test and Properties of Parallel	1. Methods to Determine Parallel Lines and Planes 2. Methods to Determine Two Parallel Planes 3. Properties of Parallel between a Line and a Plane 4. Properties of Parallel between Two Planes
Test whether Lines and Planes are Orthogonal	1. Determine Orthogonal Lines and Planes 1.1 Definition of a Line and a Plane that are Orthogonal 1.2 Methods to Determine an Orthogonal Line and a Plane 1.3 Angle Formed by a Line and a Plane
	2. Determine Two Orthogonal Planes 2.1 Definition of Dihedral Angle and Its Planar Angle 2.2 Calculation of Planar Angle 2.3 Definition of Two Planes that are Orthogonal 2.4 Methods of Two Planes that are Orthogonal
	3. Properties of a Line and a Plane that are Orthogonal
	4. Properties of Two Planes that are Orthogonal

Chapter 3 Line and Function

Section	Topic
Angle of Inclination and Slope	*1. Angle of Inclination and Slope* *1.1 Angle of Inclination and Its Range* *1.2 Definition of Slope* *1.3 Formula of Slope of a Line Passing Through Points (a, b) and (c, d)*
	2. Parallel/Perpendicular Determination of Two Lines *2.1 Parallel Determination of Two Lines* *2.2 Perpendicular Determination of Two Lines*
Linear Function	*1. Point-Slope Form of a Line* *1.1 Point-Slope Form of a Line* *1.2 Slope-Intercept Form of a Line*
	2. Two-Point Form of a Line *2.1 Two-Point Form of a Line* *2.2 Intercept Form of a Line* *2.3 Midpoint Coordinates Formula of a Line Segment by Two Points*
	3. General Form of a Line *3.1 General Form of a Line* *3.2 Transform Point-Slope Form, Slope-Intercept Form and Two-Point to General Form*
Intercept Coordinates and Distance Formula	*1. Intercept Coordinates of Intersection of Two Lines* *1.1 Intercept Coordinates of Intersection of Two Lines* *1.2 Determine Position Relation of Two Lines by Their Linear Equations*
	2. Distance *2.1 Distance between Two Points* *2.2 Distance from a Point to a Line* *2.3 Distance between Two Parallel Lines*

Chapter 4 Function of Circle

Section	Topic
Function of Circle	*1. Standard Form of a Function of a Circle* *1.1 Standard Form of a Function of a Circle* *1.2 Determine Position Relation between a Point and a Circle*
	2. General Form of a Function of a Circle 2.1 General Form of a Function of a Circle 2.2 Transform General Form to Standard Form 2.3 Determine Function of a Curve
Position Relations between Lines and Circles	1. Position Relations between Lines and Circles 1.1 Determining Position Relations between Lines and Circles 1.2 Determining Function of a Line or Circle Given Its Position Relation
	2. Position Relations between Circles
	3. Application of Functions of Lines and Circles 3.1 Solve Functions of a Line and Circle by Coordinates 3.2 Comprehensive Application in Complicated Problems
Coordinates in 3D Space	1. Coordinates in 3D Space 1.1 Definition of Coordinates in 3D Space 1.2 Notation of Coordinates in 3D Space
	2. Distance between Two Points in 3D Space

REQUIREMENTS 4

Chapter 1 Trigonometric Functions

Section	Topic
Angles and Radian Measure	*1. Angle* *1.1 Definition of an Angle* *1.2 Angles with Same Terminal Side* *1.3 Definition of a Quadrant Angle*
	2. Radian Measure *2.1 Definition of Radian Measure* *2.2 Conversion between Degrees and Radians* *2.3 Arc Length Formula*
Trigonometric Functions of Any Angles	*1. Trigonometric Functions of Any Angles* *1.1 Definitions of Sine, Cosine and Tangent* *1.2 ASTC i.e. Sign in Each Quadrant* *1.3 Relation between Values of Same Trigonometric Function of Angles with Same Terminal Side* *1.4 Unit Circle and Trigonometric Functions*
	2. Relations between Sin, Cos and Tan of an Angle
Induced formula of Trig Functions	*1. Induced formula of Trig Functions* *1.1 Relation between Sin, Cos, Tan of "π+α" and those of "α"* *1.2 Relation between Sin, Cos, Tan of "-α" and those of "α"* *1.3 Relation between Sin, Cos, Tan of "π-α" and those of "α"* *1.4 Relation between Sin, Cos, Tan of "π/2±α" and those of "α"*
Graphs and Properties of Trig Functions	*1. Graph of Sin and Cos*
	2. Properties of Sin and Cos *2.1 Definition of a Periodic Function* *2.2 Periodicity and Parity of Sin and Cos* *2.3 Intervals of Increasing and Decreasing of Sin and Cos* *2.4 Max and Min Value of Sin and Cos*
	3. Properties and Graph of Tan *3.1 Periodicity and Parity of Tan* *3.2 Monotone Interval of Tan* *3.3 Graph of Tan*
Graph of y=Asin(ωx+φ)	*1. Graph of y=Asin (ωx+φ)* *1.1 Drawing Graph of y=Asin (ωx+φ) by Five-point Sketch* *1.2 Relation of graphs of y=Asin (ωx+φ) and y=sinx* *1.3 Period and Amplitude of Function y=Asin(ωx+φ)* *1.4 Frequency, Phase and Initial Phase of Function y=Asin(ωx+φ)*
Basic Application of Trig Functions	*1. Application of Trig Functions in Practical Situation*

Chapter 2 Vector in 2D Plane

Definition of Vector	*1. Background of Vector in Physics Field and Its Definition*
	2. Definitions of Zero Vector, Unit Vector, and Vector
	3. Definitions of Equal Vector, Parallel Vector, and Collinear Vectors
Vector Operation	*1. Vector Addition and Its Meaning* *1.1 Definition of Vector Addition and its Geometric Meaning* *1.2 Basic Addition Operation of Vector*
	2. Vector Subtraction and Its Meaning *2.1 Definition of Negative Vector* *2.2 Definition of Vector Subtraction and Its Geometric Meaning*
	3. Scalar Multiplication and Its Meaning *3.1 Scalar Multiplication* *3.2 Geometric Meaning of Vector Multiplication*
Theorems and Coordinates of Vector	*1. Theorems of Vector* *1.1 Theorems of Vector* *1.2 Base Vectors in the Coordinate Plane* *1.3 Definition of Angle between Two Vectors*
	2. Vector Decomposition and Coordinates *2.1 Definition of Vector Decomposition* *2.2 Coordinate Form of Vector*
	3. Coordinates Operations of Vector *i.e. Addition, Subtraction, Multiplication*
	4. Coordinates of Collinear Vectors
Dot Product	*1. Physical Background and Meaning of Dot Product* *1.1 Dot Product and Its Geometric Representation* *1.2 Dot Product and Vector Projection* *1.3 Properties and Laws of Dot Product*
	2. Coordinate Forms of Dot Product, Modulus, and Angle between Vectors *2.1 Coordinate Form of Dot Product* *2.2 Calculation of Angle between Two Vectors by Coordinates* *2.3 Calculation of Modulus by Coordinates*
Applications of Vector	*1. Vector Application in Plane Geometry*
	2. Vector Application in Physics

Chapter 3 Trigonometric Identities

Compound Angle Identities	*1. Derivation of Identity for the cosine of the difference of two Numbers*
	2. Sum/Difference Identities of Sin, Cos and Tan
	3. Double Angle Identities of Sin, Cos and Tan
Basic Identities	*1. Basic Identities* *1.1 Triangular Properties from Basic Identities* *1.2 Transform Practical Problem to Triangle Problem and Solve by Trig Identities*

REQUIREMENTS 5

Chapter 1 Solve Triangles

Section	Topic
Sine Rule and Cosine Rule	*1. Sine Rule* *1.1 Sine Rule* *1.2 Solve Triangles by Sine Rule*
	2. Cosine Rule *2.1 Cosine Rule* *2.2 Solve Triangles by Cosine Rule*
Case Study	*1. Case Study* *1.1 Application of Solving Triangles* *1.2 Application of Area of a Triangle*

Chapter 2 Sequences and Series

Section	Topic
Definitions and Notations of Sequence	1. Definitions and Notations of Sequence 1.1 Definition of Sequence 1.2 Notation of Sequence 1.3 Recursive Definition of Sequence
Arithmetic Sequences	1. Arithmetic Sequences 1.1 Definition of Arithmetic Sequences 1.2 n^{th} Term of an Arithmetic Sequence 1.3 Common Difference 1.4 Arithmetic Sequences and First Order Function
Arithmetic Series	1. Arithmetic Series 1.1 Formula of Sum 1.2 Arithmetic Series Operations 1.3 S_n and a_n 1.4 Application of Arithmetic Series
Geometric Sequences	1. Geometric Sequences 1.1 Definition of Geometric Sequences 1.2 n^{th} Term of Geometric Sequences 1.3 Common Ratio 1.4 Geometric Sequences and Exponential Functions
Geometric Series	1. Geometric Series 1.1 Formula of Sum 1.2 Geometric Series Operations 1.3 Application of Geometric Series
Application of Sequences	1. Applications of Sequences 1.1 Sum of Special Sequences 1.2 Applications of Sequences

Chapter 3 Inequalities

Section	Topic
Unequal Relation and Inequalities	*1. Unequal Relation and Inequalities* *1.1 Actual Background of Unequal Relation and Inequalities* *1.2 Describe Unequal Relation by Inequalities* *1.3 Express and Solve Real-Life Problem by Inequalities* *1.4 Properties of Inequalities*
One-Variable Quadratic Inequalities and their solutions	*1. One-Variable Quadratic Inequalities and Their Solutions* *1.1 Extracting One-Variable Quadratic Inequalities Models from Real-Life Situations* *1.2 Definition of One-Variable Quadratic Inequalities* *1.3 3 Quadratic Relations* *1.4 Solve One-Variable Quadratic Inequalities* *1.5 Application of One-Variable Quadratic Inequalities*
Linear Inequalities in Two Unknown Variables and Simple Linear Programming Questions	1. Two-Variable Linear Inequalities and Planar Area 1.1 Extracting Two-variable Linear Inequalities Models from Real-Life Situations 1.2 Definition of Solution Sets of Two-Variable Linear Inequalities 1.3 Geometric Representation of Two-Variable Linear Inequalities 1.4 Definitions of Planar Area, Boundary, Solid Line and Dotted Line 1.5 Denote Planar Area by Two-Variable Linear Inequalities
	2. Simple Linear Programming Questions 2.1 Definitions of Linear Constraint, Objective Function, Linear Programming, Feasible Solution, Feasible Region and Optimum Solution 2.2 Solve Simple Linear Programming Questions
Fundamental Inequalities	**1.** Fundamental Inequalities: $\sqrt{ab} \leq \frac{\mathbf{a+b}}{\mathbf{2}}$ 1.1 Background of $a^2 + b^2 \geq 2ab$ and $\sqrt{ab} \leq \frac{\mathbf{a+b}}{\mathbf{2}}$ 1.2 Definitions of Arithmetic Mean and Geometric Mean 1.3 Max/Min Value Question that Sum/Product of Two Positive Variables is Constant 1.4 Application of Fundamental Inequalities
Inequalities with Absolute Values	1. Inequalities with Absolute Values 1.1 Deriving Triangular Inequalities with Absolute Values and Their Geometric Meaning 1.2 Applications of $\|a\|-\|b\| \leq \|a+b\| \leq \|a\|+\|b\|$ 1.3 Solving Inequalities in forms of $\|ax+b\| \leq c$ and $\|ax+b\| \geq c$ 1.4 Solving Inequalities in forms of $\|x-a\|+\|x-b\| \leq c$ and $\|x - a\|+\|x - b\| \leq c$

OPTION 2-1

Chapter 1 Common Logic Terms

Section	Topic
Propositions and their Relations	1. Concept of Proposition
	2. Inverse, Negative and Inverse Negative Propositions
	3. Relationship between Four Types of Propositions 3.1 Relationship between Four Types of Propositions 3.2 Judge Whether the Proposition is True or False Based on Relationship between Two Mutually Inverse Negative Propositions
Sufficient and Necessary Condition	1. Definitions of Sufficient Condition and Necessary Condition
	2. Definition of Necessary and Sufficient Condition

Chapter 2 Conic Sections and Equations

Section	Topic
Curve and Equation	1. Concepts of Equation of a Curve and Curve of an Equation
	2. Basic Methods to Deduct Equation of Curve
Ellipse	1. Ellipse and Its Standard Equation 1.1 Definition of Ellipse 1.2 Stand Equation of Ellipse 1.3 Concepts of Focus and Focal Length of Ellipse
	2. Simple Geometric Properties of Ellipse 2.1 Simple Geometric Properties of Ellipse 2.2 Calculations and Proofs about Ellipse 2.3 Spatial Relation between Line and Ellipse
Hyperbola	1. Hyperbola and Its Standard Equation 1.1 Definition of Hyperbola 1.2 Standard Equation of Hyperbola 1.3 Concepts of Focus and Focal Length of Hyperbola
	2. Simple Geometric Properties of Hyperbola 2.1 Simple Geometric Properties of Hyperbola 2.2 Calculations and Proofs about Hyperbola
Parabola	1. Parabola and Its Standard Equation 1.1 Definition of Parabola 1.2 Standard Equation of Parabola 1.3 Concepts of Focus and Directrix of Parabola
	2. Simple Geometric Properties of Parabola 2.1 Simple Geometric Properties of Parabola 2.2 Calculations and Proofs about Parabola 2.3 Spatial Relations between Lines and Parabolas

Chapter 3 Space Vector and Solid Geometry

Section	Topic
Space Vector and Its Operations	1. Space Vector and Its Addition and Subtraction 1.1 Meaning and Related Concepts of Space Vector 1.2 Addition and Subtraction of Space Vector with its Operation Laws
	2. Scalar Multiplication of Space Vector 2.1 Scalar Multiplication of Space Vector with its Operational Laws 2.2 Meaning of Collinear (Parallel) Vector and Coplanar Vector 2.3 Direction Vector of Lines
	3. Dot Product of Space Vector 3.1 Angle between Two Space Vectors 3.2 Definition and Laws of Dot Product
	4. Decomposition and Coordinate System of Space Vector 4.1 Theorem and Definition of Vector in 3D Space 4.2 Decomposition of Space Vector 4.3 Coordinate System of Space Vector 4.4 Denote other Vectors using Base Vectors in Simple Questions
	5. Coordinate Forms of Space Vector Operations 5.1 Length Formula, Distance between Two Points in 3D Space Formula 5.2 Angle Calculation between Two Space Vectors
Vectors in 3D Geometry	1. Vectors in 3D Geometry 1.1 Represent Point, Line, and Plane by Space Vectors 1.2 Definition of Normal Vector 1.3 3 Steps of Solving 3D Geometric Problems by Space Vectors 1.4 Using Space Vectors, Determine Positional Relations between Lines and Planes, and Calculating Angle in 3D Space 1.5 Solving Simple 3D Geometric Problems by Setting Appropriate Coordinate Systems

Appendix I

Pre-AP Calculus Textbook Content

Chapter P. Prerequisites
P.1 Real Numbers
- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation
P.2 Cartesian Coordinate System
- Cartesian Plane
- Absolute Value of a Real Number
- Distance Formulas
- Midpoint Formulas
- Equations of Circles
- Applications
P.3 Linear Equations and Inequalities
- Equations
- Solving Equations
- Linear Equations in One Variable
- Linear Inequalities in One Variable
P.4 Lines in the Plane
- Slope of a Line
- Point-Slope Form Equation of a Line
- Slope-Intercept Form Equation of a Line
- Graphing Linear Equations in Two Variables
- Parallel and Perpendicular Lines
- Applying Linear Equations in Two Variables
P.5 Solving Equations Graphically, Numerically, and Algebraically
- Solving Equations Graphically
- Solving Quadratic Equations
- Approximating Solutions of Equations Graphically
- Approximating Solutions of Equations Numerically with Tables
- Solving Equations by Finding Intersections
P.6 Complex Numbers
- Complex Numbers
- Operations with Complex Numbers
- Complex Conjugates and Division
- Complex Solutions of Quadratic Equations
P.7 Solving Inequalities Algebraically and Graphically
- Solving Absolute Value Inequalities
- Solving Quadratic Inequalities
- Approximating Solutions to Inequalities
- Projectile Motion

Chapter 1. Functions and Graphs
1.1 Modeling and Equation Solving
- Numerical Models
- Algebraic Models
- Graphical Models
- The Zero Factor Property
- Problem Solving
- Grapher Failure and Hidden Behavior
- A Word About Proof
1.2 Functions and Their Properties
- Function Definition and Notation
- Domain and Range
- Continuity
- Increasing and Decreasing Functions
- Boundedness
- Local and Absolute Extrema
- Symmetry
- Asymptotes
- End Behavior
1.3 Twelve Basic Functions
- What Graphs Can Tell Us
- Twelve Basic Functions
- Analyzing Functions Graphically
1.4 Building Functions from Functions
- Combining Functions Algebraically
- Composition of Functions
- Relations and Implicitly Defined Functions
1.5 Parametric Relations and Inverses
- Relations Defined Parametrically
- Inverse Relations and Inverse Functions
1.6 Graphical Transformations
- Transformations
- Vertical and Horizontal Translations
- Reflections Across Axes
- Vertical and Horizontal Stretches and Shrinks
- Combining Transformations
1.7 Modeling with Functions
- Functions from Formulas
- Functions from Graphs
- Functions from Verbal Descriptions
- Functions from Data

Chapter 2. Polynomial, Power, and Rational Functions
2.1 Linear and Quadratic Functions and Modeling
- Polynomial Functions
- Linear Functions and Their Graphs

- Average Rate of Change
- Association, Correlation, and Linear Modeling
- Quadratic Functions and Their Graphs
- Applications of Quadratic Functions

2.2 Power Functions with Modeling
- Power Functions and Variation
- Monomial Functions and Their Graphs
- Graphs of Power Functions
- Modeling with Power Functions

2.3 Polynomial Functions of Higher Degree with Modeling
- Graphs of Polynomial Functions
- End Behavior of Polynomial Functions
- Zeros of Polynomial Functions
- Intermediate Value Theorem
- Modeling

2.4 Real Zeros of Polynomial Functions
- Long Division and the Division Algorithm
- Remainder and Factor Theorems
- Synthetic Division
- Rational Zeros Theorem
- Upper and Lower Bounds

2.5 Complex Zeros and the Fundamental Theorem of Algebra
- Two Major Theorems
- Complex Conjugate Zeros
- Factoring with Real Number Coefficients

2.6 Graphs of Rational Functions
- Rational Functions
- Transformations of the Reciprocal Function
- Limits and Asymptotes
- Analyzing Graphs of Rational Functions
- Exploring Relative Humidity

2.7 Solving Equations in One Variable
- Solving Rational Equations
- Extraneous Solutions
- Applications

2.8 Solving Inequalities in One Variable
- Polynomial Inequalities
- Rational Inequalities
- Other Inequalities
- Applications

Chapter 3. Exponential, Logistic, and Logarithmic Functions

3.1 Exponential and Logistic Functions
- Exponential Functions and Their Graphs
- The Natural Base e
- Logistic Functions and Their Graphs

- Population Models
3.2 Exponential and Logistic Modeling
- Constant Percentage Rate and Exponential Functions
- Exponential Growth and Decay Models
- Using Regression to Model Population
- Other Logistic Models
3.3 Logarithmic Functions and Their Graphs
- Inverses of Exponential Functions
- Common Logarithms—Base 10
- Natural Logarithms—Base e
- Graphs of Logarithmic Functions
- Measuring Sound Using Decibels
3.4 Properties of Logarithmic Functions
- Properties of Logarithms
- Change of Base
- Graphs of Logarithmic Functions with Base b
- Re-expressing Data
3.5 Equation Solving and Modeling
- Solving Exponential Equations
- Solving Logarithmic Equations
- Orders of Magnitude and Logarithmic Models
- Newton's Law of Cooling
- Logarithmic Re-expression
3.6 Mathematics of Finance
- Simple and Compound Interest
- Interest Compounded k Times per Year
- Interest Compounded Continuously
- Annual Percentage Yield
- Annuities—Future Value
- Loans and Mortgages -Present Value

Chapter 4. Trigonometric Functions
4.1 Angles and Their Measures
- The Problem of Angular Measure
- Degrees and Radians
- Circular Arc Length
- Angular and Linear Motion
4.2 Trigonometric Functions of Acute Angles
- Right Triangle Trigonometry
- Two Famous Triangles
- Evaluating Trigonometric Functions with a Calculator
- Common Calculator Errors when Evaluating Trig Functions
- Applications of Right Triangle Trigonometry
4.3 Trigonometry Extended: The Circular Functions
- Trigonometric Functions of Any Angle
- Trigonometric Functions of Real Numbers

Appendix J

AP Calculus AB & BC Textbook Content[1]

Chapter 1. Pre-requisites for AP Calculus
1 Prerequisites for Calculus
1.1 Linear Functions
1.2 Functions and Graphs
1.3 Exponential Functions
1.4 Parametric Equations
1.5 Inverse Functions and Logarithms
1.6 Trigonometric Functions

Chapter 2. Limits and Continuity
2.1 Rates of Change and Limits
- Average and Instantaneous Speed
- Definition of Limit
- Properties of Limits
- One-Sided and Two-Sided Limits
- Squeeze Theorem
2.2 Limits involving Infinity
- Finite Limits as $x \to \pm\infty$
- Squeeze Theorem Revisited
- Infinite Limits as $x \to \alpha$
- End Behavior Models
- "Seeing" Limits as $x \to \pm\infty$
2.3 Continuity
- Continuity at a Point
- Continuous Functions
- Algebraic Combinations
- Composites
- Intermediate Value Theorem for Continuous Functions
2.4 Rates of Change, Tangent Lines, and Sensitivity
- Average Rates of Change
- Tangent to a Curve
- Slope of a Curve
- Normal to a Curve
- Speed Revisited

[1]Contents of AP Calculus BC are highlighted in italics.

Chapter 3. Derivatives
3.1 Derivative of a Function
- Definition of Derivative
- Notation
- Relationships Between the Graphs of f and f'
- Graphing the Derivative from Data
- One-Sided derivatives
3.2 Differentiability
- How f'(a) Might Fail to Exist
- Differentiability Implies Local Linearity
- Numerical Derivatives on a Calculator
- Differentiability Implies Continuity
- Intermediate Value Theorem for Derivatives
3.3 Rules for Differentiation
- Positive Integer Powers, Multiples, Sums, and Differences
- Products and Quotients
- Negative Integer Powers of x
- Second and Higher Order Derivatives
3.4 Velocity and Other Rates of Change
- Instantaneous Rate of Change
- Motion Along a Line
3.5 Derivatives of Trigonometric Functions
- Derivative of the Sine Function
- Derivative of the Cosine Function
- Derivatives of the Other Basic Trigonometric Function

Chapter 4. More Derivatives
4.1 Chain Rule
- Derivative of a Composite Function
- "Outside-Inside" Rule
- Repeated Use of the Chain Rule
- Slopes of Parametrized Curves
- Power Chain Rule
4.2 Implicit Differentiation
- Implicitly Defined Functions
- Lenses, Tangents, and Normal Lines
- Derivatives of Higher Order
- Rational Powers of Differentiable Functions
4.3 Derivatives of Inverse Trigonometric Functions
- Derivatives of Inverse Functions
- Derivative of the Arcsine
- Derivative of the Arctangent
- Derivative of the Arcsecant
- Derivatives of the Other Three
4.4 Derivatives of Exponential and Logarithmic Functions
- Derivative of e^x

Chapter 8. Applications of Definite Integrals
8.1 Accumulation and Net Change
- Linear Motion Revisited
- General Strategy
- Consumption over Time
- Coming and Going
- Net Change from Data
8.2 Areas in the Plane
- Area Between Curves
- Area Enclosed by Intersecting Curves
- Boundaries with Changing Functions
- Integrating with Respect to y
- Saving Time with Geometry Formulas
8.3 Volumes
- Volume as an Integral
- Square Cross Sections
- Circular Cross Sections
- Other Cross Sections
8.4 Lengths of Curves
- A Sine Wave
- Length of a Smooth Curve
- Vertical Tangents, Corners, and Cusps
8.5 Applications from Science and Statistics
- Work Revisited
- Fluid Force and Fluid Pressure
- Normal Probabilities

Chapter 9. Sequences, L'Hospital's Rule, and Improper Integrals
9.1 Sequences
- Defining a Sequence
- Arithmetic and Geometric Sequences
- Graphing a Sequence
- Limit of a Sequence
9.2 L'Hospital's Rule
- Indeterminate Form 0/0
- Indeterminate Forms ∞/∞, ∞·0, ∞-∞
- Indeterminate Forms
9.3 Relative Rates of Growth
- Comparing Rates of Growth
- *Using L' Hospital's Rule to Compare Growth Rates*
9.4 Improper Integrals
- Infinite Limits of Integration
- Integrands with Infinite Discontinuities
- Test for Convergence and Divergence
- Applications

Chapter 10. Infinite Series
10.1 Power Series
- Geometric Series
- Representing Functions by Series
- Differentiation and Integration
- Identifying a Series
10.2 Taylor Series
- Constructing a Series
- Series for sin x and cos x
- Beauty Bare
- Maclaurin and Taylor Series
- Combining Taylor Series
- Table of Maclaurin Series
10.3 Taylor's Theorem
- Taylor Polynomials
- The Remainder
- Bounding the Remainder
- Analyzing Truncation Error
- Euler's Formula
10.4 Radius of Convergence
- Convergence
- nth-Term Test
- Comparing Nonnegative Series
- Ratio Test
- Endpoint Convergence
10.5 Testing Convergence at Endpoints
- Integral Test
- Harmonic Series and p-series
- Comparison Tests
- Alternating Series
- Absolute and Conditional Convergence
- Intervals of Convergence
- A word of Caution

Chapter 11. Parametric, Vector, and Polar Functions
11.1 Parametric Functions
- Parametric Curves in the Plane
- Slope and Concavity
- Arc Length
- Cycloids
11.2 Vectors in the Plane
- Modeling Planar Motion
- Velocity, Acceleration, and Speed
- Displacement and Distance Traveled

11.3 Polar Functions
- Polar Coordinates
- Polar Curves
- Slopes of Polar Curves
- Areas Enclosed by Polar Curves
- A Small Polar Gallery

Appendix K

AP Data

Year 2016

Student	General Examination Mathematics	SAT Mathematics	AP Calculus AB	AP Calculus BC
1	C	740	4	
2	C	760	4	
3	C	730	3	
4	C	730	3	
5	B	710	5	5
6	C	710	3	
7	D	770	4	4
8	B	710	2	
9	B	680	3	
10	C	750	5	3
11	D	690	1	
12	C	780	3	
13	C	800	4	
14	C	710	4	
15	D	750	1	
16	C	730	3	3
17	C	770	5	5
18	C	770	5	
19	D	670	4	
20	D	770	3	
21	C	700	4	
22	C	770	5	5
23	D	760	4	
24	B	770	5	5
25	D	720	1	
26	D	760	5	
27	D	580	1	
28	D	680	1	
29	C	710	2	
30	C	730	5	
31	D	750	1	
32	D	680	3	
33	D	680	1	
34	B	770	5	
35	B	770	2	3
36	D	790	3	
37	C	730	1	
38	B	760	3	
39	C	750	4	
40	B	760	5	4

Student	General Examination Mathematics	SAT Mathematics	AP Calculus AB	AP Calculus BC
41	D	680	1	
42	C	760	3	
43	C	690	1	
44	B	760	5	5
45	C	730	5	
46	B	760	4	4
47	B	770	5	5
48	C	770	5	5
49	D	760	3	
50	C	720	5	
51	C	770	3	3
AVERAGE		735	3.333333333	4.214

Year 2017

Student	General Examination Mathematics	SAT Mathematics	AP Calculus AB	AP Calculus BC
1	D	680	2	
2	D	770	5	
3	C	770	5	
4	D	750	3	3
5	D	730	3	3
6	D	700	2	
7	D	650	1	
8	D	730	1	
9	C	790	5	5
10	C	740	5	
11	C	790	5	5
12	D	760	4	4
13	D	750	4	
14	D	730	4	
15	D	710	5	
16	C	700	5	
17	D	650	2	
18	D	720	3	
19	C	740	3	3
20	D	720	1	
21	D	770	4	3
22	C	680	3	
23	D	700	3	2
24	D	690	3	
25	C	800	5	
26	D	740	3	
27	D	690	3	
AVERAGE		727.7777778	3.407407407	3.5

Appendix L

General Chinese Mathematics Curriculum vs IB Mathematics Curriculum

REQUIREMENTS 1

Chapter 1 Sets and Function

Section	Topic
Set	*1. Meanings and Notations of Sets* *1.1 Meaning of Sets* *1.2 Element Characteristics* *1.3 Equality of Sets* *1.4 Relations between Set and Element* *1.5 Memorization of Common Sets* *1.6 Set Notations*
	2. Set Relations *2.1 Introduction of Subset and Proper Subset* *2.2 Introduction of Empty Set*
	3. Set Operation *3.1 Union* *3.2 Intersection* *3.3 Universal Set and Complement*
Functions and Notations	*1. Concepts of Functions* *1.1 Definition of a Function* *1.2 Notation y=f(x)* *1.3 Domain of a Function* *1.4 Range of a Function* **1.5 Definition of Interval and Its Notation**
	2. Notations of Functions **2.1 Algebraic expression of a Function** **2.2 Graph Notation and Tracing Point Method** 2.3 Table Notation of a Function **2.4 Definition and Application of a Piecewise Function** **2.5 Definition of Mapping**
Properties of Functions	**1. Monotonicity and Max(Min) Value** **1.1 Definitions of Increasing and Decreasing Functions** **1.2 Monotonicity and Monotone Interval** *1.3 Maximum and Minimum Value of a Function*
	2. Parity of a Function **2.1 Definitions of odd and even functions** **2.2 Properties of odd and even functions**

Chapter 2 Elementary Functions

Section	Topic
Exponential Functions	1. Exponents and Exponential Operation 1.1 Definition of Root 1.2 Definition of a Rational Exponent 1.3 Definition of an Irrational Exponent 1.4 Laws of Rational Exponents
	2. Exponential Function and Properties 2.1 Definition of an Exponential Function 2.2 Graph of an Exponential Function 2.3 Properties of an Exponential Function
Logarithmic Functions	1. Logarithm and Logarithmic Operation 1.1 Definition of a Logarithm 1.2 Common Logarithm and Natural Logarithm 1.3 Laws and Properties of Logarithms 1.4 Change of Base
	2. Logarithmic Function and Its Properties 2.1 Definition of a Logarithmic Function 2.2 Graph of a Logarithmic Function 2.3 Properties of a Logarithmic Function 2.4 Relation between Exponential and Logarithmic Functions
Power Functions	1. Power Function ($y = x$, $y = x^2$, $y = x^{\frac{1}{2}}$, $y = x^{-1}$) 1.1 Definition of a Power Function 1.2 Graph of a Power Function 1.3 Properties of a Power Function

Chapter 3 Application of Functions

Section	Topic
Functions And Equations	1. Root of an Equation and Zero of a Function 1.1 Definition of Zero 1.2 Relation between "f(x) = 0 has defined roots" & "y = f(x) has zeros" 1.3 Determine Continuous Function Has Zero(s) in Domain (a, b)
Models of Functions and their Applications	**1. Increasing Models of Different Functions** **1.1 Speed of Growth of Exponential Function** $y = a^x$ **(a>1) in Domain (0, +∞)** **1.2 Speed of Growth of Logarithmic Function** $y = \log_a x$ **(a>1) in (0, +∞)** **1.3 Speed of Growth of Power Function** $y = x^n$ **(n>0) in Domain (0, +∞)** **1.4 Comparison of the Speeds of Growth of** $y = a^x$ (a>1), $y = \log_a x$ **(a>1),** $y = x^n$ **(n>0) in Domain (0, +∞)**

	2. Examples of Application of Models 2.1 Application of Models in Practical Cases 2.2 Develop Models according to Practical Cases
	3. Comprehensive Application

REQUIREMENTS 2

Chapter 1 Geometry in 3D-Space

Section	Topic
Structure of 3D Geometry	1. Structures of Cylinder, Cone and Frustum 1.1 Definitions of a Prism, a Pyramid and Frustum of a Pyramid 1.2 Bases, Side Edges, Lateral Faces and Vertices of a Prism, a Pyramid and Frustum of a Pyramid 1.3 Definitions of a Cylinder, Cone, Frustum of a Cone and Sphere 1.4 Bases, Generatrices, Lateral Faces, Axis of a Cylinder, Cone, Frustum of a Cone and Sphere 1.5 Center, Radius, Diameter of a Sphere
	2. Features of Simple Geometry 2.1 Cube and Sphere Related Geometries and their features 2.2 Determine Geometries with Givens
Three-view Drawing and Trimetric Drawing of 3D Geometry	**1. Perspective Projection and Parallel Projection** **1.1 Definitions of Projection, Projection Line and Projection Plane** **1.2 Definitions of Perspective Projection and Parallel Projection**
	2. Three-view Drawing of 3D Geometry 2.1 Definitions of Top View, Side View, and Front View 2.2 Rules of Three-view Drawing 2.3 Three-view Drawing of Simple Geometry
	3. Trimetric View of 3D Geometry 3.1 Definition of Oblique Bimetric projection 3.2 Steps of Oblique Bimetric projection 3.3 Draw Trimetric View of Simple Geometry 3.4 Determine the Geometry Given Three-view Drawing 3.5 Draw Three-view Drawing Given Trimetric View and Vice Versa
Surface Area and Volume of 3D Geometry	1. Surface Area and Volume of a Prism, a Pyramid, and Frustum of a Pyramid 1.1 Relation between Surface Area and Expansion View 1.2 Volume Formula of a Prism, a Pyramid, and Frustum of a Pyramid 1.3 Formula of Surface Area of a Prism, a Pyramid, and Frustum of a Pyramid 1.4 Relation between a Prism, a Pyramid, and Frustum of a Pyramid 1.5 Relation of Graphical Alternation between Triangular Prism and Triangular Pyramid
	2. Surface Area and Volume of a Sphere
	3. Surface Area and Volume of Composition of Simple 3D Geometry

Chapter 2 Vector in 2D Plane

Section	Topic
Position Relation between Point, Line and Plane in 3D Space	**1. Plane** **1.1 Definition of Plane** **1.2 Drawing and Notation of Plane** **1.3 Properties of Plane i.e. Theorem1, 2, and 3** **1.4 Alternation between "Word", "Symbol", "Graph"**
	2. Position Relation between Lines in 3D Space **2.1 Definition and Notation of Lines in Different Planes** **2.2 Theorem 4** **2.3 Isogonal Theorem** **2.4 Angle Formed by Lines in Different Planes** **2.5 Definition of Orthogonal Lines**
	3. 3 Positional Relations between Lines and Planes
	4. Position Relation between Plane and Plane
Test and Properties of Parallel	**1. Methods to Determine Parallel Lines and Planes** **2. Methods to Determine Two Parallel Planes** **3. Properties of Parallel between a Line and a Plane** **4. Properties of Parallel between Two Planes**
Test whether Lines and Planes are Orthogonal	**1. Determine Orthogonal Lines and Planes** **1.1 Definition of a Line and a Plane that are Orthogonal** **1.2 Methods to Determine an Orthogonal Line and a Plane** **1.3 Angle Formed by a Line and a Plane**
	2. Determine Two Orthogonal Planes **2.1 Definition of Dihedral Angle and Its Planar Angle** **2.2 Calculation of Planar Angle** **2.3 Definition of Two Planes that are Orthogonal** **2.4 Methods of Two Planes that are Orthogonal**
	3. Properties of a Line and a Plane that are Orthogonal
	4. Properties of Two Planes that are Orthogonal

Chapter 3 Trigonometric Identities

Section	Topic
Angle of Inclination and Slope	*1. Angle of Inclination and Slope* *1.1 Angle of Inclination and Its Range* *1.2 Definition of Slope* *1.3 Formula of Slope of a Line Passing Through Points (a, b) and (c, d)*
	2. Parallel/Perpendicular Determination of Two Lines *2.1 Parallel Determination of Two Lines* *2.2 Perpendicular Determination of Two Lines*
Linear Function	*1. Point-Slope Form of a Line* *1.1 Point-Slope Form of a Line* *1.2 Slope-Intercept Form of a Line*
	2. Two-Point Form of a Line *2.1 Two-Point Form of a Line* *2.2 Intercept Form of a Line*

Section	Topic
	2.3 Midpoint Coordinates Formula of a Line Segment by Two Points
	3. General Form of a Line *3.1 General Form of a Line* *3.2 Transform Point-Slope Form, Slope-Intercept Form and Two-Point to General Form*
Intercept Coordinates and Distance Formula	*1. Intercept Coordinates of Intersection of Two Lines* *1.1 Intercept Coordinates of Intersection of Two Lines* *1.2 Determine Position Relation of Two Lines by Their Linear Equations*
	2. Distance *2.1 Distance between Two Points* *2.2 Distance from a Point to a Line* *2.3 Distance between Two Parallel Lines*

Chapter 4 Function of Circle

Section	Topic
Function of Circle	1. Standard Form of a Function of a Circle 1.1 Standard Form of a Function of a Circle 1.2 Determine Position Relation between a Point and a Circle
	2. General Form of a Function of a Circle 2.1 General Form of a Function of a Circle 2.2 Transform General Form to Standard Form 2.3 Determine Function of a Curve
Position Relations between Lines and Circles	1. Position Relations between Lines and Circles 1.1 Determining Position Relations between Lines and Circles 1.2 Determining Function of a Line or Circle Given Its Position Relation
	2. Position Relations between Circles
	3. Application of Functions of Lines and Circles 3.1 Solve Functions of a Line and Circle by Coordinates 3.2 Comprehensive Application in Complicated Problems
Coordinates in 3D Space	1. Coordinates in 3D Space 1.1 Definition of Coordinates in 3D Space 1.2 Notation of Coordinates in 3D Space
	2. Distance between Two Points in 3D Space

REQUIREMENTS 4

Chapter 1 Trigonometric Functions

Section	Topic
Angles and Radian Measure	1. Angle 1.1 Definition of an Angle 1.2 Angles with Same Terminal Side 1.3 Definition of a Quadrant Angle
	2. Radian Measure 2.1 Definition of Radian Measure 2.2 Conversion between Degrees and Radians 2.3 Arc Length Formula
Trigonometric Functions of Any Angles	1. Trigonometric Functions of Any Angles 1.1 Definitions of Sine, Cosine and Tangent 1.2 ASTC i.e. Sign in Each Quadrant 1.3 Relation between Values of Same Trigonometric Function of Angles with Same Terminal Side 1.4 Unit Circle and Trigonometric Functions
	2. Relations between Sin, Cos and Tan of an Angle
Induced formula of Trig Functions	1. Induced formula of Trig Functions 1.1 Relation between Sin, Cos, Tan of "$\pi+\alpha$" and those of "α" 1.2 Relation between Sin, Cos, Tan of "$-\alpha$" and those of "α" 1.3 Relation between Sin, Cos, Tan of "$\pi-\alpha$" and those of "α" 1.4 Relation between Sin, Cos, Tan of "$\pi/2\pm\alpha$" and those of "α"
Graphs and Properties of Trig Functions	1. Graph of Sin and Cos
	2. Properties of Sin and Cos 2.1 Definition of a Periodic Function 2.2 Periodicity and Parity of Sin and Cos 2.3 Intervals of Increasing and Decreasing of Sin and Cos 2.4 Max and Min Value of Sin and Cos
	3. Properties and Graph of Tan 3.1 Periodicity and Parity of Tan 3.2 Monotone Interval of Tan 3.3 Graph of Tan
Graph of $y=A\sin(\omega x+\varphi)$	1. Graph of $y=A\sin(\omega x+\varphi)$ 1.1 Drawing Graph of $y=A\sin(\omega x+\varphi)$ by Five-point Sketch 1.2 Relation of graphs of $y=A\sin(\omega x+\varphi)$ and $y=\sin x$ 1.3 Period and Amplitude of Function $y=A\sin(\omega x+\varphi)$ 1.4 Frequency, Phase and Initial Phase of Function $y=A\sin(\omega x+\varphi)$
Basic Application of Trig Functions	1. Application of Trig Functions in Practical Situation

Chapter 2 Vector in 2D Plane

Section	Topic
Definition of Vector	1. Background of Vector in Physics Field and Its Definition
	2. Definitions of Zero Vector, Unit Vector, and Vector
	3. Definitions of Equal Vector, Parallel Vector, and Collinear Vectors
Vector Operation	1. Vector Addition and Its Meaning 1.1 Definition of Vector Addition and its Geometric Meaning 1.2 Basic Addition Operation of Vector
	2. Vector Subtraction and Its Meaning 2.1 Definition of Negative Vector 2.2 Definition of Vector Subtraction and Its Geometric Meaning
	3. Scalar Multiplication and Its Meaning **3.1 Scalar Multiplication** **3.2 Geometric Meaning of Vector Multiplication**
Theorems and Coordinates of Vector	**1. Theorems of Vector** **1.1 Theorems of Vector** **1.2 Base Vectors in the Coordinate Plane** **1.3 Definition of Angle between Two Vectors**
	2. Vector Decomposition and Coordinates **2.1 Definition of Vector Decomposition** **2.2 Coordinate Form of Vector**
	3. Coordinates Operations of Vector **i.e. Addition, Subtraction, Multiplication**
	4. Coordinates of Collinear Vectors
Dot Product	**1. Physical Background and Meaning of Dot Product** **1.1 Dot Product and Its Geometric Representation** **1.2 Dot Product and Vector Projection** **1.3 Properties and Laws of Dot Product**
	2. Coordinate Forms of Dot Product, Modulus, and Angle between Vectors **2.1 Coordinate Form of Dot Product** **2.2 Calculation of Angle between Two Vectors by Coordinates** **2.3 Calculation of Modulus by Coordinates**
Applications of Vector	**1. Vector Application in Plane Geometry**
	2. Vector Application in Physics

Chapter 3 Trigonometric Identities

Section	Topic
Compound Angle Identities	**1. Derivation of Identity for the cosine of the difference of two Numbers**
	2. Sum/Difference Identities of Sin, Cos and Tan
	3. Double Angle Identities of Sin, Cos and Tan
Basic Identities	**1. Basic Identities** **1.1 Triangular Properties from Basic Identities** **1.2 Transform Practical Problem to Triangle Problem and Solve by Trig Identities**

REQUIREMENTS 5

Chapter 1 Solve Triangles

Section	Topic
Sine Rule and Cosine Rule	**1. Sine Rule** **1.1 Sine Rule** **1.2 Solve Triangles by Sine Rule**
	2. Cosine Rule **2.1 Cosine Rule** **2.2 Solve Triangles by Cosine Rule**
Case Study	**1. Case Study** **1.1 Application of Solving Triangles** **1.2 Application of Area of a Triangle**

Chapter 2 Sequences and Series

Section	Topic
Definitions and Notations of Sequence	**1. Definitions and Notations of Sequence** **1.1 Definition of Sequence** **1.2 Notation of Sequence** **1.3 Recursive Definition of Sequence**
Arithmetic Sequences	**1. Arithmetic Sequences** **1.1 Definition of Arithmetic Sequences** **1.2 n^{th} Term of an Arithmetic Sequence** **1.3 Common Difference** **1.4 Arithmetic Sequences and First Order Function**
Arithmetic Series	**1. Arithmetic Series** **1.1 Formula of Sum** **1.2 Arithmetic Series Operations** **1.3 S_n and a_n** **1.4 Application of Arithmetic Series**
Geometric Sequences	**1. Geometric Sequences** **1.1 Definition of Geometric Sequences** **1.2 n^{th} Term of Geometric Sequences**

Section	Topic
	1.3 Common Ratio **1.4 Geometric Sequences and Exponential Functions**
Geometric Series	**1. Geometric Series** **1.1 Formula of Sum** **1.2 Geometric Series Operations** **1.3 Application of Geometric Series**
Application of Sequences	**1. Applications of Sequences** **1.1 Sum of Special Sequences** **1.2 Applications of Sequences**

Chapter 3 Inequalities

Section	Topic
Unequal Relation and Inequalities	**1. Unequal Relation and Inequalities** **1.1 Actual Background of Unequal Relation and Inequalities** **1.2 Describe Unequal Relation by Inequalities** **1.3 Express and Solve Real-Life Problem by Inequalities** **1.4 Properties of Inequalities**
One-Variable Quadratic Inequalities and their solutions	**1. One-Variable Quadratic Inequalities and Their Solutions** **1.1 Extracting One-Variable Quadratic Inequalities Models from Real-Life Situations** **1.2 Definition of One-Variable Quadratic Inequalities** **1.3 3 Quadratic Relations** **1.4 Solve One-Variable Quadratic Inequalities** **1.5 Application of One-Variable Quadratic Inequalities**
Linear Inequalities in Two Unknown Variables and Simple Linear Programming Questions	1. Two-Variable Linear Inequalities and Planar Area 1.1 Extracting Two-variable Linear Inequalities Models from Real-Life Situations 1.2 Definition of Solution Sets of Two-Variable Linear Inequalities 1.3 Geometric Representation of Two-Variable Linear Inequalities 1.4 Definitions of Planar Area, Boundary, Solid Line and Dotted Line 1.5 Denote Planar Area by Two-Variable Linear Inequalities
	2. Simple Linear Programming Questions 2.1 Definitions of Linear Constraint, Objective Function, Linear Programming, Feasible Solution, Feasible Region and Optimum Solution 2.2 Solve Simple Linear Programming Questions
Fundamental Inequalities	**1. Fundamental Inequalities:** $\sqrt{ab} \leq \frac{a+b}{2}$ **1.1 Background of** $a^2 + b^2 \geq 2ab$ **and** $\sqrt{ab} \leq \frac{a+b}{2}$ **1.2 Definitions of Arithmetic Mean and Geometric Mean** **1.3 Max/Min Value Question that Sum/Product of Two Positive Variables is Constant** **1.4 Application of Fundamental Inequalities**
Inequalities with Absolute	**1. Inequalities with Absolute Values** **1.1 Deriving Triangular Inequalities with Absolute Values and Their Geometric Meaning**

Section	Topic
Values	**1.2 Applications of |a|−|b|≤|a+b|≤|a|+|b|** **1.3 Solving Inequalities in forms of |ax+b|≤c and |ax+b|≥c** **1.4 Solving Inequalities in forms of |x−a|+|x−b|≤c and $\|x-a\|+\|x-b\| \le c$**

OPTION 2-1

Chapter 1 Common Logic Terms

Section	Topic
Propositions and their Relations	1. Concept of Proposition
	2. Inverse, Negative and Inverse Negative Propositions
	3. Relationship between Four Types of Propositions 3.1 Relationship between Four Types of Propositions 3.2 Judge Whether the Proposition is True or False Based on Relationship between Two Mutually Inverse Negative Propositions
Sufficient and Necessary Condition	1. Definitions of Sufficient Condition and Necessary Condition
	2. Definition of Necessary and Sufficient Condition

Chapter 2 Conic Sections and Equations

Section	Topic
Curve and Equation	1. Concepts of Equation of a Curve and Curve of an Equation
	2. Basic Methods to Deduct Equation of Curve
Ellipse	1. Ellipse and Its Standard Equation 1.1 Definition of Ellipse 1.2 Stand Equation of Ellipse 1.3 Concepts of Focus and Focal Length of Ellipse
	2. Simple Geometric Properties of Ellipse 2.1 Simple Geometric Properties of Ellipse 2.2 Calculations and Proofs about Ellipse 2.3 Spatial Relation between Line and Ellipse
Hyperbola	1. Hyperbola and Its Standard Equation 1.1 Definition of Hyperbola 1.2 Standard Equation of Hyperbola 1.3 Concepts of Focus and Focal Length of Hyperbola
	2. Simple Geometric Properties of Hyperbola 2.1 Simple Geometric Properties of Hyperbola 2.2 Calculations and Proofs about Hyperbola
Parabola	1. Parabola and Its Standard Equation **1.1 Definition of Parabola** **1.2 Standard Equation of Parabola** 1.3 Concepts of Focus and Directrix of Parabola

Section	Topic
	2. Simple Geometric Properties of Parabola 2.1 Simple Geometric Properties of Parabola 2.2 Calculations and Proofs about Parabola 2.3 Spatial Relations between Lines and Parabolas

Chapter 3 Space Vector and Solid Geometry

Section	Topic
Space Vector and Its Operations	**1. Space Vector and Its Addition and Subtraction** **1.1 Meaning and Related Concepts of Space Vector** **1.2 Addition and Subtraction of Space Vector with its Operation Laws**
	2. Scalar Multiplication of Space Vector **2.1 Scalar Multiplication of Space Vector with its Operational Laws** **2.2 Meaning of Collinear (Parallel) Vector and Coplanar Vector** **2.3 Direction Vector of Lines**
	3. Dot Product of Space Vector **3.1 Angle between Two Space Vectors** **3.2 Definition and Laws of Dot Product**
	4. Decomposition and Coordinate System of Space Vector **4.1 Theorem and Definition of Vector in 3D Space** **4.2 Decomposition of Space Vector** **4.3 Coordinate System of Space Vector** **4.4 Denote other Vectors using Base Vectors in Simple Questions**
	5. Coordinate Forms of Space Vector Operations **5.1 Length Formula, Distance between Two Points in 3D Space Formula** **5.2 Angle Calculation between Two Space Vectors**
Vectors in 3D Geometry	**1. Vectors in 3D Geometry** **1.1 Represent Point, Line, and Plane by Space Vectors** **1.2 Definition of Normal Vector** **1.3 3 Steps of Solving 3D Geometric Problems by Space Vectors** **1.4 Using Space Vectors, Determine Positional Relations between Lines and Planes, and Calculating Angle in 3D Space** **1.5 Solving Simple 3D Geometric Problems by Setting Appropriate Coordinate Systems**

Appendix M

Pre-IB (Grade 10) Mathematics Textbook Content

Chapter 1. Sets
1.1 Introduction to Sets
1.2 Intersection and Union of Sets
1.3 Applications

Chapter 2. Simultaneous Equations
2.1 Simultaneous Linear Equations in Two Unknowns
2.2 Simultaneous Linear and Non-Linear Equations in Two Unknowns

Chapter 3. Indices, Surds and Logarithms
3.1 Indices (Exponents) and Surds
3.2 Exponential Equations
3.3 Logarithms
3.4 Common and Natural Logarithms
3.5 Laws of Logarithms
3.6 Logarithmic Equations

Chapter 4. Quadratic Expressions and Equations
4.1 Maximum/Minimum Value of a Quadratic Expression
4.2 Roots of a Quadratic Equation
4.3 Solving Quadratic Inequalities

Chapter 5. Remainder and Factor Theorems
5.1 Polynomial Identities
5.2 Remainder Theorem
5.3 Factor Theorem
5.4 Solving Cubic Equations

Chapter 6. Matrices
6.1 Represent Information as a Matrix
6.2 Addition, Subtraction and Scalar Multiplication of Matrices
6.3 Multiplication of Matrices
6.4. Determinant and Inverse of a 2 x 2 Matrix
6.5 Solving Simultaneous Equations by a Matrix Method

Chapter 7. Coordinate Geometry
7.1 Distance between Two Points
7.2 Midpoint of the Line Joining Two Points
7.3 Gradient of a Line Passing through Two Points
7.4 Equations of Straight Lines
7.5 Equations of Parallel and Non-Parallel Lines

Appendix N

IB Mathematics SL (Standard Level) Textbook Content

Chapter 1. Fundamentals
1.1 The real numbers
1.2 Roots and radicals (surds)
1.3 Exponents (indices)
1.3 Scientific notation (standard form)
1.5 Algebraic expressions
1.6 Equations and formulae

Chapter 2. Functions and Equations
2.1 Relations and functions
2.2 Composition of functions
2.3 Inverse functions
2.4 Transformations of functions
2.5 Quadratic functions
2.6 Rational functions

Chapter 3. Sequences and Series
3.1 Sequences
3.2 Arithmetic sequences
3.3 Geometric sequences
3.4 Series
3.5 The binomial theorem

Chapter 4. Exponential and Logarithmic Functions
4.1 Exponential functions
4.2 Exponential growth and decay
4.3 The number e
4.4 Logarithmic functions
4.5 Exponential and logarithmic equations

Chapter 5. Matrix Algebra
5.1 Basic definitions
5.2 Matrix operations
5.3 Applications to systems

Chapter 6. Trigonometric Functions and Equations
6.1 Angles, circles, arcs and sectors
6.2 The unit circle and trigonometric functions
6.3 Graphs of trigonometric functions
6.4 Solving trigonometric equations and trigonometric identities

Appendix O

IB Mathematics HL (Higher Level) Textbook Content

Chapter 1. Fundamentals
1.1 Sets, inequalities, absolute value and properties of real numbers
1.2 Roots and radicals (surds)
1.3 Exponents (indices)
1.4 Scientific notation (standard form)
1.5 Algebraic expressions
1.6 Equations and formulae

Chapter 2. Functions
2.1 Definition of a functions
2.2 Composite functions
2.3 Inverse functions
2.4 Transformations of functions

Chapter 3. Algebraic Functions, Equations and Inequalities
3.1 Polynomial functions
3.2 Quadratic functions
3.3 Zeros, factors and remainders
3.4 Rational functions
3.5 Other equations and inequalities
3.6 Partial fractions (optional)

Chapter 4. Sequences and Series
4.1 Sequences
4.2 Arithmetic sequences
4.3 Geometric sequences
4.4 Series
4.5 Counting principles
4.6 The binomial theorem
4.7 Mathematical induction

Chapter 5. Exponential and Logarithmic Functions
5.1 Exponential functions
5.2 Exponential growth and decay
5.3 The number e
5.4 Logarithmic functions
5.5 Exponential and logarithmic equations

Chapter 6. Matrix Algebra (optional)
6.1 Basic definitions
6.2 Matrix operations

Appendix P

IB Mathematics Option (Topic 9 – Calculus) Textbook Content

Chapter 1. Limits of Sequences and Functions
1.1 The limit of a sequence
1.2 The Squeeze Theorem
1.3 The limit of a function
1.4 L'Hôpital's Rule
1.5 Continuous functions
1.6 Differentiable functions
1.7 Rolle's Theorem and the Mean Value Theorem

Chapter 2. Improper Integrals
2.1 The Fundamental Theorem of Calculus
2.2 Convergent and divergent improper integrals
2.3 Approximation of improper integrals

Chapter 3. Infinite Series
3.1 Convergence of series
3.2 Tests for convergence and divergence
3.3 Power series

Chapter 4. Maclaurin aria Taylor series
4.1 Maclaurin series
4.2 Approximations to the Maclaurin series
4.3 Maclaurin series of composite functions
4.4 Taylor series
4.5 Applications

Chapter 5. Differential equations
5.1 Setting up differential equations
5.2 Separation of variables
5.3 Homogeneous differential equations
5.4 Linear differential equations
5.5 Approximations to solutions

Appendix Q

IB Data

Year 2016

Student	SAT Mathematics	IB Mathematics all Higher Level (HL)
1	720	4
2	800	4
3	800	6
4	790	4
5	800	5
6	770	5
7	800	3
8	740	3
9	790	7
10	740	4
11	750	5
12	780	3
13	800	3
14	730	6
15	800	2
16	720	4
17	680	4
18	690	3
19	750	4
20	780	5
21	740	3
22	710	4
23	790	5
24	740	4
25	790	4
26	800	5
27	760	4
28	800	4
29	790	5
30	760	5
31	800	7
32	720	4
33	720	5
34	780	4
35	720	3
36	740	2
37	680	3
38	740	5
AVERAGE	758.1578947	4.210526316

Year 2017

Student	SAT Mathematics	IB Mathematics Higher Level (HL)	IB Mathematics Standard Level (SL)
1	690		2
2	790	4	
3	770	6	
4	780	3	
5	790	5	
6	770	5	
7	770	4	
8	760		7
9	800	6	
10	790	4	
11	760	3	
12	770	4	
13	790	6	
14	760	3	
15	790		6
16	740		5
17	800	4	
18	760	4	
19	790	7	
20	800	6	
21	800	5	
22	790	3	
23	770		6
24	780		6
25	760	4	
26	800	7	
27	760	5	
28	790	3	
29	770		4
30	760	2	
31	790	5	
32	700		5
33	780	4	
AVERAGE	773.3333333	4.48	5.125