Influence of field penetration ratios and filamentation on end-effect related hysteretic loss reductions for superconducting strips

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Abstract. There are a few key conductor-specific factors which influence the power loss of superconductors; these include critical current, geometry, and normal metal resistivity. This paper focuses on the influence of sample geometry on the power loss of superconducting strips and the effect of filamentation and sample length as a function of the field penetration state of the superconductor. We start with the analytical equations for infinite slabs and strips and then consider the influence of end effects for both unstriated and striated conductor. The loss is then calculated and compared as a function of applied field for striated and unstriated conductors. These results are much more general than they might seem at first glance, since they will be important building blocks for analytic loss calculations for twisted geometries for coated conductors, including helical (Conductor on Round Core, CORC), and twisted (e.g., twist stack cables) geometries. We show that for relatively low field penetration, end effects and reduced field penetration both reduce loss. In addition, for filamentary samples the relevant ratio of length scales becomes the filament width to sample length, thus modifying the loss ratios.

1. Introduction

Understanding and reducing the AC loss of coated conductor and the cables wound from superconducting strips is important for enabling superconducting AC machines. Superconducting machines of interest include motors, generators, fault current limiters, fast ramping magnets, and a number of other devices. YBCO coated conductors of increasing interest for these applications. Based on the critical state model [1-3], loss expressions for semi-infinite superconducting slabs are findable in most textbooks on superconductivity, e.g., [4-6], while the expressions for loss in simple YBCO tapes are well known from the work of Brandt [7, 8] and later Muller [9].

While the losses for monofilaments or even multifilamentary coated conductor tapes is understood for the simple case of a flat tape in a perpendicular applied field, the loss for coated conductors in helical or twisted geometries is not well known. An approach which considers the applied field to vary as a trigonometric function along the sample length suggests a simple rule that the loss of a twisted or helical sample should be reduced by a factor of $2/\pi$. This treatment is correct for long pitches, but a consideration of end effects – i.e. the reduction in magnetization and loss for samples where the sample length is larger but not very much greater than the sample width – should be important to consider for tighter pitches.

Below we first remind the reader of the loss expressions for slabs and strips. We then describe the basic geometry of helical and twisted coated conductor samples, and describe the simple approach of averaging a sinusoidal spatial for larger pitches. We then consider a helical conductor as a series of segments where an end effect or termination effect is present, but one where an additional spatial field

variation also exists. The difficulty of this simple problem leads us back to a description of the simpler case where the field is uniform, and a termination is present, but both coated conductor geometries and filament striation modify the termination effect.

1.1. Superconducting semi-infinite slab with $t \gg w$

Consider the critical state of a slab ($t \gg w$, and infinitely long) of superconductor of width $w = 2a$ in a field H, applied along $t = 2t_h$ (the y direction) as shown in Figure 1. For low applied field the sample is only partially penetrated, and the loss per cycle, per volume (Q) is [4]

$$Q = \frac{2\mu_0 H_0^3}{3H_p} \tag{1}$$

where the penetration field, H_p is given by $H_p = J_c\, w/2$.
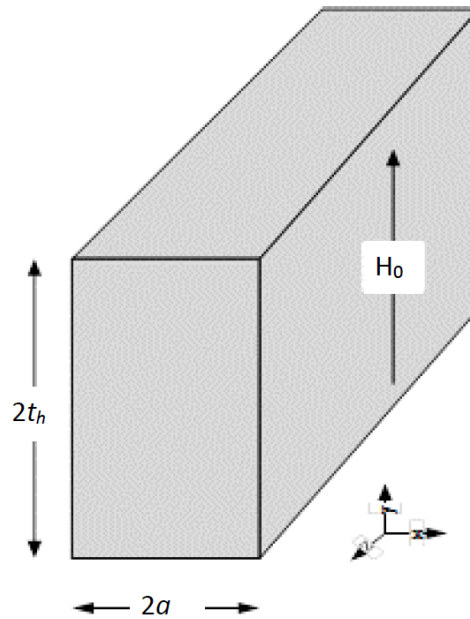
Figure 1. A superconducting sample whose cross section is rectangular.

When $H_0 \geq H_P$, the slab becomes fully-penetrated and the per cycle, per volume loss is given by [4]

$$Q = 2\,\mu_0 J_c a H_0 \left(1 - \frac{2}{3}\frac{J_c a}{H_0}\right) = 2\mu_0 H_0^2 \left(\frac{H_p}{H_0} - \frac{2}{3}\left(\frac{H_p}{H_0}\right)^2\right) \tag{2}$$

Comparing equations (1) and (2), the loss initially increases rapidly as H_0^3 until $H_0 > H_p$, after which

$$Q = 2\mu_0 H_0^2 \tau(\beta) \tag{3}$$

where $\beta = H_0/H_P$ and $\tau(\beta)$ given by

$$\tau(\beta) = \begin{cases} \dfrac{\beta}{3} & \beta \leq 1 \\[2mm] \dfrac{1}{\beta} - \dfrac{2}{3\beta^2} & \beta \geq 1 \end{cases} \tag{4}$$

Thus when the applied field is just above the penetration field both linear and quadratic loss term are seen. But as $H_0 \gg H_p$, $Q \approx 2\mu_0 H_0 H_P = \mu_0 H_0 J_c w$

1.2. An infinitely long superconducting strip

The result for semi-infinite slabs which are thick along the direction of the applied field but have zero demagnetization are well described by Eq (1)-(4). However, if the sample thickness, t, becomes much thinner that the sample width ($w = 2a$), the situation will be different. For $H_0 >> H_p$, the loss equations are identical to those of the semi-infinite slab (Eq(1)-(4)). However, as H_0 drops below the penetration field, which is itself modified, the loss expressions are significantly modified by a kind of demagnetization effect. Brandt [7, 8] and Muller [9] showed that

$$Q = N\mu_0 H_0 J_c w \tag{5}$$

Where

$$N = \left(\frac{H_0}{H_d}\right) g\left\{\frac{H_0}{H_d}\right\}$$

$$g\left\{\frac{H_0}{H_d}\right\} = \frac{H_d}{H_0}\left[\frac{2H_d}{H_0}\ln\left(\cosh(\frac{H_0}{H_d})\right) - \tanh(\frac{H_0}{H_d})\right] \text{ and } H_d = 0.4 J_c t \tag{6}$$

The penetration field is given by

$$H_p = \frac{J_c t}{\pi}\left[Ln\left(\frac{w}{t} + 1\right)\right] = \frac{5}{2\pi}H_d\left[Ln\left(\frac{w}{t} + 1\right)\right] \tag{7}$$

We note that, from Ref [10], that

$$N \approx 1 - 2\left(\frac{H_d}{H_0}\right)Ln(2) \tag{8}$$

Therefore, when $H_0 >> H_d$, the loss strip is the same as that of a slab with the same width and J_c. At lower fields, the loss is modified by N.

1.3. Twisted and Helical Geometries

Figure 2 shows the geometry of a twisted sample (left) and a helical wrap coated conductor (right). We might imagine a simple loss treatment which is an extension of either the slab or the strip model, but modified by the space varying field. Our task is simplified if we take fields well above the penetration field, where the slab and strip model are equivalent.
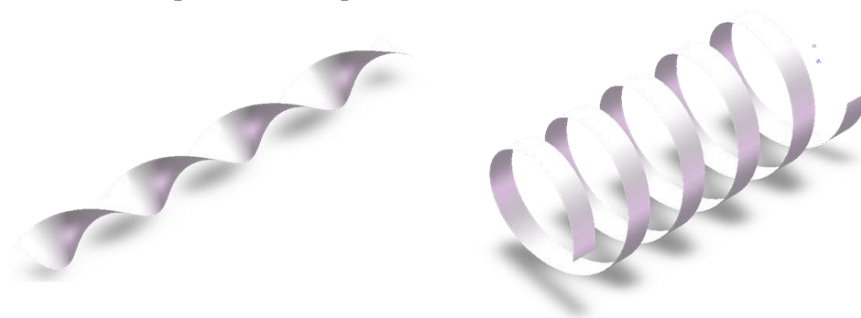
Figure 2. Coated Conductor in a twisted geometry (left) and a helical geometry (right).

Let us consider Eq 5 with $N = 1$, but take $H = H_0 sin(2\pi z/L_p)sin(\omega t)$ as the field applied perpendicular to the twisted strand (left), or to the helical wrap strand (right). Here z is the position along the length of the axis of symmetry, and L_p is the twist pitch in the case of the twisted strand, and for the case of the helix an effective twist pitch

$$L_{peff} = \sqrt{L_h^2 + \pi^2 D_h^2} \tag{9}$$

where L_h is the length of one helix period as taken along the length (z-axis) of the helix. Following Carr [11], we can integrating the average of Eq (5) over a spatial field cycle, such that

$$Q = \frac{N\mu_0 J_c w H_0}{L_p/2} \int_0^\pi sin\left(\frac{2\pi z}{L_p}\right) dz = \frac{N\mu_0 J_c w H_0}{L_p/2} \frac{L_p}{2\pi}(2) = \left(\frac{2}{\pi}\right) N\mu_0 J_c w H_0 = \left(\frac{2}{\pi}\right) Q_0 \tag{10}$$

Where Q_0 is the loss for a slab or strip where the field is a field that is time varying and spatially uniform of maximum amplitude H_0. For $L_p \gg w$, this leads to a proper result. However, we have neglected to account for the details of the current paths, which will be important as pitch is reduced. The currents generally flow along the length of the tape, but as the polarity of H is changing over the twist pitch, the sense of the shielding (or trapping) currents do too, requiring them to flow across the tape at periods like $L_p/2$ (or $L_{peff}/2$ for a helix). This can be seen more clearly if we consider a finite sample and its end effects.

1.4. Influence of Sample End Effects

As shown by a number of authors [12-14], finite length superconductors in applied fields have flux gradients which penetrate into the sample in multiple dimensions. If we consider, paralleling the case of the infinite slab, a sample which is infinitely thick (y-direction), and has a length L (along z) and a half width $a=w/2$ (along x), and field is applied along y, as shown in Figure 3, flux will penetrate in the x and z directions. Because the sample is longer than it is wide, the flux penetrates most effectively in the width (x) direction, and over most of the length of the sample the shielding (or trapping) currents flow along the length of the sample. However, Kirchhoff's law requires that they do not diverge and thus must flow across the width of the sample at the ends of the sample, leading to the current flow regions shown in Figure 3. The end effect regions are $2a$ wide, and penetrate a maximum depth of $L_t = a$ assuming that J_c is isotropic.
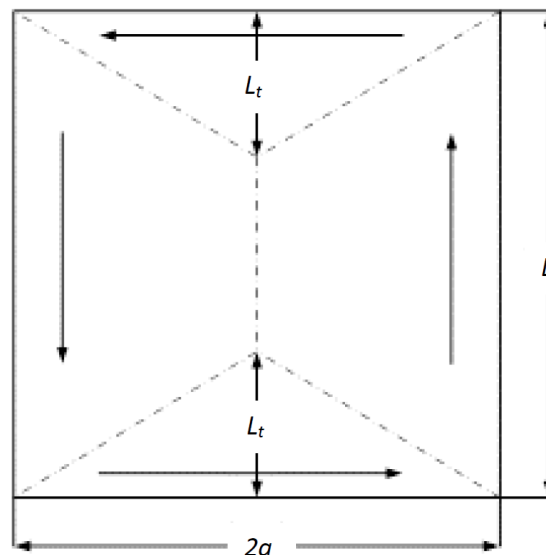
Figure 3. The transferring path of induced current with the consideration of terminal effect

More generally

$$L_t = a\frac{J_{cz}}{J_{cy}} \tag{11}$$

where J_{cz} and J_{cy} are the critical currents in z and y directions respectively. Here we assume the superconducting sample is isotropic ($J_{cz}/J_{cy} = 1$) and the loss for a slab becomes

$$Q = \mu_0 J_c H_0 w \left(1 - \frac{w}{3L}\right) \tag{12}$$

A straightforward extension of this to a strip should be possible by the addition of the prefactor N, such that

$$Q = N\mu_0 J_c H_0 w \left(1 - \frac{w}{3L}\right) \tag{13}$$

Let us explore the behaviour of expression (13) by inserting some specific and relevant values for a coated conductor and then plotting Q/H_0 as a function of applied field. Using $w = 1.2$ cm, $J_c = 10^{10}$ A/m^2, and $L = (4w, 5w, 10w, 20w, 50w, 100w)$ we plot Q/H_0 vs H_0/H_d in Figure 4 (a). Using Eq (6) we find that $B_d = \mu_0 H_d = 5$ mT, and using Eq (7) we find that $B_p = \mu_0 H_p = 37.6$ mT. In Figure 4(a) we see that Q/H_0 tends to 150 J/(m^{2}A) for samples with $L \gg w$ and $H_0 \gg H_p$ agreeing with the value for the infinite slab (or infinite strip for fields well above penetration). As L/w is reduced, the saturation value for a given L/w ratio is reduced. If $H_0 \leq H_p \approx 8H_d$, loss is further reduced by field penetration effects related to the coated conductor's highly aspected geometry.

2. Effect of Filamentarization

The loss of coated conductors can be reduced by striation (cutting filaments into the superconducting strip). The per cycle loss per unit volume loss is then given by

$$Q = N\mu_0 H_0 \frac{J_c w}{N_f} \left(1 - \frac{w}{3LN_f}\right) \tag{14}$$

where N_f is the number of filaments. Using the same parameters as above, and setting $L/w = 4$, QN_f/H_0 can be plotted vs H_0/H_d for tapes with various numbers of filaments, as shown in Fig. 4 (b).
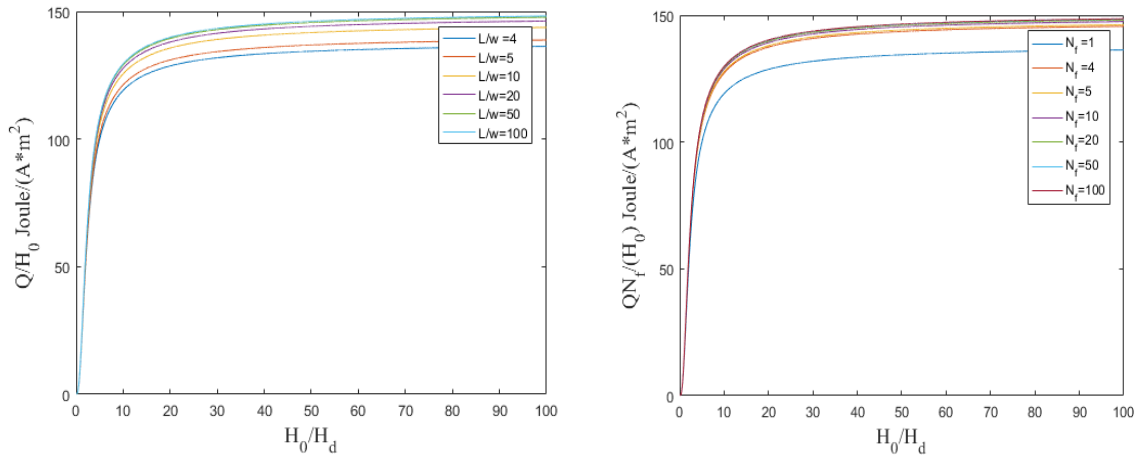
Figure. 4. (a) Q/H_0 vs H_0/H_d for an unstriated coated conductor strip for $w = 1.2$ cm, $J_c = 10^{10}$ A/m^2, for various L/w ratios, (b) Q/H_0 vs H_0/H_d for a striated coated conductor strip for $w = 1.2$ cm, $J_c = 10^{10}$ A/m^2, for various $L/w = 4$.

As we can see in Figure 4 (b), for a one filament (unstriated) strand with $L/w = 4$, Q/H_0 is about 135 J/Am^2, reduced from its value for infinite length (150 J/Am^2) by about 10% due to end effects. As we introduce more filaments, the loss is reduced proportional to the number of filaments, but by plotting Q/H_0N_f, we can see instead the effect of filamentation on sample length. We see that as the filament number reaches 100 the sample end effect based loss reduction is removed.

Conclusions

This paper investigates the influence of sample geometry on the loss of superconducting strips and the effect of filamentation and sample length as a function of the field penetration state of the superconductor. The loss for finite segments is calculated as a function of applied field for striated and unstriated conductors. These results are much more general than they might seem at first glance, since they will be important building blocks for analytic loss calculations for twisted geometries for coated conductors, including helical (Conductor on Round Core, CORC), and twisted (e.g., twist stack cables) geometries. For high levels of flux penetration, the end effects are those for superconducting slabs, while for relatively low field penetration, end effects and reduced field penetration both reduce loss. However, for filamentary samples the ratio of length scales becomes filament width to sample length, thus modifying the loss ratios. This leads to an apparent reduction in the end effects, since the relevant ratio which controls them is the filament width to sample length, rather than the whole conductor width to sample length.

Acknowledgments

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