

## LETTER

**Threshold Equalization for On-Line Signature Verification**

**Isao NAKANISHI<sup>†a)</sup>, Member, Hiroyuki SAKAMOTO<sup>†\*</sup>, Student Member, Yoshio ITOH<sup>†</sup>, Member,  
and Yutaka FUKUI<sup>†</sup>, Fellow**

**SUMMARY** In on-line signature verification, complexity of signature shape can influence the value of the optimal threshold for individual signatures. Writer-dependent threshold selection has been proposed but it requires forgery data. It is not easy to collect such forgery data in practical applications. Therefore, some threshold equalization method using only genuine data is needed. In this letter, we propose three different threshold equalization methods based on the complexity of signature. Their effectiveness is confirmed in experiments using a multi-matcher DWT on-line signature verification system.

**key words:** *on-line signature verification, score level fusion, threshold equalization, multi-matcher, DWT*

**1. Introduction**

Use of biometrics for person authentication is attracting a lot of attention [1]. One of the well-known behavioral biometrics is on-line signature, which recognizes the signatures of users based on time-varying parameters such as pen-position, pen-pressure, pen-inclination and so on [2]–[5]. Especially, only the pen-position parameter is available when the on-line signature verification is assumed to be carried out in the context of a personal digital assistant (PDA). However, the pen-position data can be forged since the signature shape is visible.

In order to cope with the above problem, we have introduced the notion of sub-band decomposition by the discrete Wavelet transform (DWT) into the on-line signature verification process [6]. Moreover, in order to improve the verification performance, we have proposed a multi-matcher DWT on-line signature verification system [7], [8]. However, we noticed that fusing the best matchers did not always result in the best matching pair. One of the reasons is that the optimal threshold for each signature is different. This effect has been observed in a number of related studies [3], [10].

In this letter, we introduce threshold equalization methods into the decision making stage. Their effectiveness is confirmed based on experimental results using the multi-matcher DWT on-line signature verification system described in [7].

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<sup>†</sup>The authors are with the Graduate School of Engineering, Tottori University, Tottori-shi, 680-8552 Japan.

\*Presently, with the Sumitomo Heavy Industries Ltd.

a) E-mail: nakanishi@ele.tottori-u.ac.jp

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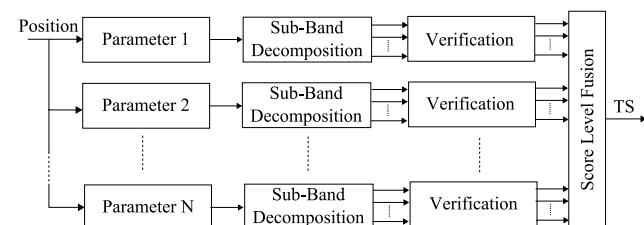
**2. Multi-Matcher DWT On-Line Signature Verification**

First, we briefly explain the multi-matcher DWT approach for on-line signature verification. See [6]–[9] for more details. The block diagram of the multi-matcher DWT on-line signature verification approach is shown in Fig. 1. The pen-position input consists of x and y coordinates. Moreover, we also use the pen-movement angle, the pen-movement acceleration, and the pen-movement vector parameters, all of which can be derived from the pen-position data [7]–[9]. These parameters are, respectively, as position, angle, acceleration, and vector for convenience.

Figure 2 shows the definition of parameters, where  $s$  is a constant that defines the time shift in each parameter. It has an influence on verification performance [7]. Small  $s$  causes excess fluctuation in a parameter and inversely large  $s$  neglects the difference between a genuine signature and its forgery. The vector\*\* is given by multiplying unit pen-movement distance  $d(n)$  by the angle  $\theta(n)$  [8].

The time-varying signal of each parameter is decomposed into sub-band signals using the DWT. A verification score is obtained at each sub-band level for each parameter. At the decision stage, all verification scores are fused based on a weighted-sum rule to obtain the total score (TS). When the total score is larger than a threshold, the user is recognized as genuine.

Based on this scheme, the multi-matcher DWT approach was evaluated in [7]. As a result, the best matched pair was obtained by combining the angle at  $s = 2$  and the acceleration at  $s = 8$ . On the other hand, the results also suggested that if the parameters which individually achieved



**Fig. 1** Multi-matcher DWT on-line signature verification.

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\*\*Strictly speaking, this is not a vector but we call so for convenience.

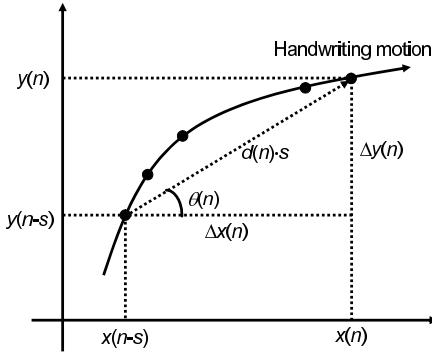


Fig. 2 Definition of pen-movement parameters.

the best performance were fused, it was not always guaranteed to obtain the best performance. In fact, the best performance (matcher) was individually obtained using the angle with  $s = 4$  and the acceleration with  $s = 16$ .

One of the reasons is that the optimal threshold for each signature is different. Figure 3 shows examples of error rate curves of four users, where 2(a) shows the results of fusing individual best matchers, that is, the angle at  $s = 4$  and the acceleration at  $s = 16$  and 2(b) shows the results of the best matcher pair, that is, the angle at  $s = 2$  and the acceleration at  $s = 16$ . While it is easier to set an optimal threshold which is common to all users in 2(b), it is clear that such a common threshold achieves lower performance as shown in 2(a).

This result is related to the idea of writer-dependent threshold selection, which sets an optimal threshold depending on the writer (user) [3], [10]. Furthermore, it is generalized as the user-specific matching threshold in [11]. The writer-dependent threshold values are found empirically based on error rates: Equal Error Rate (EER), False Acceptance Rate (FAR) and False Rejection Rate (FRR), which require the availability of forgeries. In general, biometric data of other users are used as impostor data. In the case of signatures, to use of other users' signatures as forgeries is called the random forgery scenario. However, as mentioned earlier the signature has the problem that it can be easily forged; therefore, it is reasonable to assume more severe or adverse conditions, that is, simulated forgery in which genuine signatures are traced by forgers and/or the skilled forgery in which genuine writing styles are learned by forgers. However, it is difficult to collect such forgery data in practical applications since others must imitate genuine signatures whenever a new user is registered to a system.

In this letter, we propose to equalize optimal thresholds for users based on the complexity of their own signatures. This scheme does not require any forgery data and so is called threshold equalization to distinguish it from conventional user-specific threshold matching.

### 3. Threshold Equalization Methods

As a result of investigating the relationship between opti-

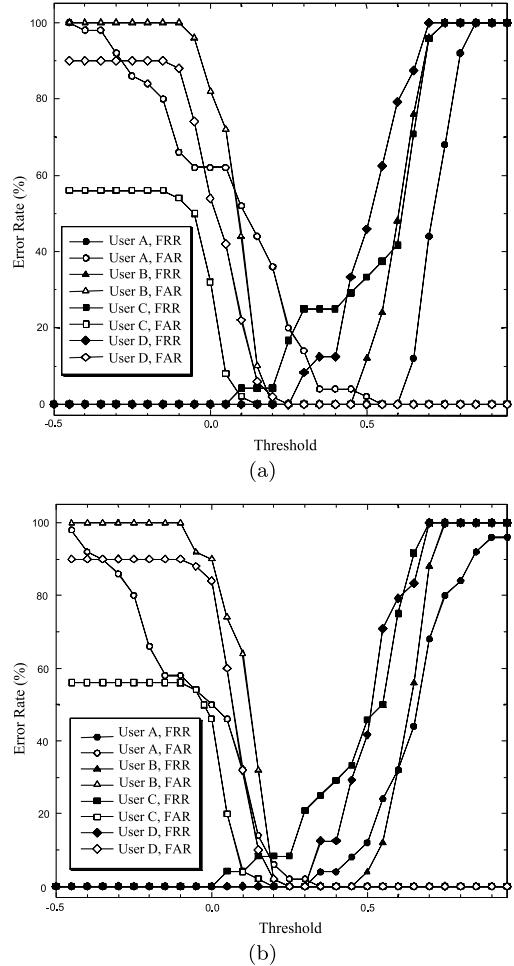


Fig. 3 Error rate curves of individual users in (a) individual best matchers and (b) the best matcher pair.

mal thresholds and individual user signatures, we observed that the higher the complexity of a signature, the smaller the optimal threshold. This fact is explained as follows. In general, a complex signature requires long writing time, resulting in large variation in the signature of one individual (large intra-class variation). Such large variation makes the total match score smaller even when the signature is genuine. On the other hand, when the signature shape is simple, the writing time is short and the signature is relatively stable. This makes the intra-class variation small and then the total match score becomes larger for genuine trials.

Thus, the different complexities of signature shape might cause different optimal thresholds for individual signatures. In the following sections, we propose three threshold equalization methods based on the complexity of a signature.

#### 3.1 Method Based on Average Number of Samples

The first method is based on the simple idea that a complex signature requires more time for its writing, so that it increases the number of sampled data of the signature. The

complexity of a signature is measured by counting the number of samples in the template. The equalized threshold is achieved by normalizing a total score as follows:

$$TS_{eq}^i = k_{ave} N^i TS^i \quad (1)$$

where  $i$  is the user (signature) number,  $TS^i$  is the total score of the  $i^{th}$  user,  $TS_{eq}^i$  is the equalized total score,  $N^i$  is the average number of samples in the templates of the  $i^{th}$  user and  $k_{ave}$  is a coefficient for adjusting the equalized total score between 0 and 1.

If the complexity of a signature is high, then we have hypothesized that the total score  $TS^i$  will be small. In such a case, the small  $TS^i$  is multiplied by the large  $N^i$ , so that we obtain an equalized  $TS_{eq}^i$ .

### 3.2 Method Based on Number of Negative Samples

In this method we examine the complexity of a signature using the number of samples with a negative value in the pen-movement angle parameter. Assuming that the average direction of pen-movement is from left to right on a tablet, the negative value of the angle parameter means that the pen moves in the opposite direction. Therefore, a large number of samples with a negative value in the angle suggests that the signature includes frequent back and forth movements and is complicated.

The equalized threshold is achieved by

$$TS_{eq}^i = k_{min} M_i TS^i \quad (2)$$

where  $k_{min}$  is the coefficient for adjustment,  $M_i$  is the average number of samples with a negative value of the angle in the user  $i$  template. The scheme for equalization the threshold is similar to that of the averaging method.

### 3.3 Method Based on Number of Sign-Inversions

Lastly, we propose to examine the complexity of a signature using the number of sign-inversions in the angle parameter. The sign-inversion in the angle corresponds to the direction

change in pen-movement. In general, a complex signature has many direction changes. Thus, we define the threshold equalization method as

$$TS_{eq}^i = k_{sin} S_i TS^i \quad (3)$$

where  $k_{sin}$  is the coefficient for adjustment and  $S_i$  is the average number of sign-inversions in the angle in the user  $i$  template. The scheme for equalization is similar to the other two methods.

## 4. Verification Experiments

In order to evaluate the effectiveness of the three proposed threshold equalization methods which are called Average method, Negative method, and Sign method for convenience sake, we carried out the following verification experiments.

Four subjects were requested to sign their own signatures and then we obtained 118 genuine signatures. Five genuine signatures for each subject were used to make a template and the remaining 98 genuine signatures were used for verification. Five subjects were required to counterfeit the genuine signature 10 times each, so that 200 forgeries were prepared in total. In order to obtain the number of samples with a negative value or of sign-inversions in the angle parameter, we utilized the data with the time shift  $s = 2$ . The coefficient for adjustment for each method was selected empirically. When pen-position parameters ( $x, y$ ) are fused with another parameter, the weighting factors for fusing of  $x, y$  and another parameter were  $2/5, 1/5$  and  $2/5$ , respectively. When two parameters except the pen-position were fused, each weighting factor was  $1/2$ . For lack of space, we omit other conditions. Please refer to [6]–[8] for details.

Tables 1–4 show the variations in EER by applying the three proposed threshold equalization methods in several fusion cases. In each column, the EERs with no equalization, by the Average method, by the Negative method and by the Sign method are presented from the left to the right.

Among total 153 cases, the verification performance was improved or unchanged in 122 cases by using the proposed methods. It corresponds to the improvement rate of

**Table 1** Variations of EER in fusing the pen-position and other parameters.

	$s = 2$	$s = 4$	$s = 8$	$s = 16$
Angle				
	2.8, 4.1, 3.9, 4.5	3.6, 3.0, 2.9, 3.0	3.6, 2.0, 2.2, 2.0	4.7, 2.8, 2.9, 2.6
Acceleration				
Position	6.8, 2.8, 2.9, 3.1	7.5, 2.7, 2.0, 2.6	6.3, 2.0, 2.8, 2.8	4.8, 2.0, 2.2, 2.0
Vector				
	2.6, 3.9, 2.9, 3.7	3.0, 3.4, 2.0, 3.1	4.4, 2.8, 2.0, 2.8	

**Table 2** Variations of EER in fusing the angle and the acceleration.

	Acceleration			
	$s = 2$	$s = 4$	$s = 8$	$s = 16$
Angle	5.3, 4.6, 3.9, 3.9	5.0, 4.9, 3.7, 4.6	2.0, 2.9, 2.0, 2.3	2.0, 3.2, 2.7, 2.8
	8.4, 1.9, 3.0, 1.8	7.3, 3.0, 2.5, 2.2	6.0, 1.6, 1.4, 1.3	4.6, 1.6, 1.0, 1.0
	8.8, 3.0, 2.0, 2.1	8.4, 2.8, 2.9, 2.7	6.2, 1.5, 1.0, 1.0	3.5, 2.0, 1.9, 1.9
	8.8, 3.3, 3.1, 2.7	8.0, 4.6, 3.8, 3.0	7.0, 1.7, 1.7, 1.2	4.6, 3.9, 2.9, 2.3

**Table 3** Variations of EER in fusing the acceleration and the vector.

		Vector		
		$s = 4$	$s = 8$	$s = 16$
Acceleration	$s = 2$	6.3, 4.7, 4.1, 4.9	6.6, 3.8, 3.2, 3.8	9.7, 4.6, 4.0, 2.8
	$s = 4$	5.9, 5.1, 4.2, 3.7	6.1, 4.8, 3.8, 3.6	9.9, 5.1, 4.0, 3.7
	$s = 8$	5.0, 3.1, 2.0, 3.0	5.0, 3.0, 2.1, 2.7	7.5, 4.0, 3.0, 2.1
	$s = 16$	3.8, 2.3, 2.0, 2.0	3.8, 2.8, 2.0, 2.3	4.7, 3.2, 2.3, 2.0

**Table 4** Variations of EER in fusing the vector and the angle.

		Angle			
		$s = 2$	$s = 4$	$s = 8$	$s = 16$
Vector	$s = 4$	4.6, 7.0, 6.3, 6.5	3.0, 4.7, 4.0, 4.6	4.0, 4.0, 4.0, 4.0	4.0, 4.4, 3.8, 4.3
	$s = 8$	3.3, 6.6, 6.4, 6.7	3.0, 4.2, 3.8, 4.6	4.0, 3.9, 3.3, 4.0	4.2, 4.6, 3.8, 4.2
	$s = 16$	4.1, 6.8, 5.3, 5.7	4.0, 3.8, 3.2, 3.0	4.9, 4.1, 3.0, 3.3	5.3, 5.0, 4.6, 4.3

about 80%. The Averaging method was suitable in fusing the pen-position parameter. On the other hand, the Negative and the Sign methods achieved comparable improvement of the EER when other parameters were fused. In addition, although the purpose of this letter is not to evaluate the absolute verification performance, it is interesting that the best verification rate of 99.0% in our system was achieved when the angle ( $s = 4, 8$ ) was fused with the acceleration ( $s = 8, 16$ ).

We can also observe in some cases that the EER was degraded by the proposed threshold equalization methods, for example when combining the vector and the angle (see Table 4), which requires further examination. In addition, more work needs to be done in order to estimate the adjustment parameters, based either on experimental knowledge or statistical formulation.

## 5. Conclusions

In order to equalize optimal thresholds of individual signatures, we have proposed to adjust the total score according to the complexity of a signature and evaluated the scheme in verification experiments. The results confirmed that the proposed threshold equalization methods are effective for improving the verification performance. On the other hand, it is not always guaranteed that the performance will be improved using the proposed methods. In addition, there may be some more sources which cause different optimum thresholds. The effect of the proposed methods is not evaluated yet when other combinations of parameters are attempted. These are future problems to be studied.

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