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FUNCTIONAL GRAPH AS A POWERFUL TOOL IN MATHEMATICAL PROBLEM SOLVING: ITS CURRICULUM AND INSTRUCTION[1), 2)]

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In our research and practice, to solve an equation and to represent a graph are placed as tools for problem solving in the curriculum. We set the new integrated unit called "*Function and Equation*" which is reorganized conventional units in Japan, intended to cultivate students' ability and attitude to apply those in effectively in each school grade. To realize this, we edited the experimental textbooks along the program that we reconstructed. These are verified through lesson studies.

Key words: functional graph, problem solving, lesson study, experimental textbook, curriculum

PROBLEMATIC OF THE RESEARCH

The present situation of Japanese students

It is pointed out as the present situation of the Japanese junior high school students from various kinds of domestic and international surveys that they cannot make an equation and represent it into a graph necessary for the given problem situation while they can solve the given equation and represent the graph of the given equation solely. In other words, expressions or equations, graphs, and tables cannot be effective tools for their problem solving.

The conventional condition in Japanese mathematics curriculum

In the conventional curriculum condition of the Japanese junior high school mathematics, students usually learn the function after the equation in all school grades. At the 7th grade, they learn linear function limited to proportionality ($y=ax$) and inverse proportionality ($y=\frac{a}{x}$) after linear equation ($ax+b=0$). At the 8th, they learn linear function in general ($y=ax+b$) after simultaneous equations. At the 9th, they learn quadratic function limited to in proportion to the square ($y=ax^2$) after quadratic equation ($ax^2+bx+c=0$, *limited to real solutions*).

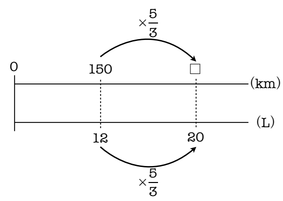
However, the arrangement of the curriculum manifests students' obstacles in their problem solving, *i.e.*, the graph cannot function as an effective tool when a student expresses the quantities relation of the problem situation algebraically -in an expression-. In addition, from the interaction of the equation and the graph, the student cannot have a standard to judge the validity of the solution of the problem. In the current curriculum, these activitis are placed in the last of the function unit. That is irrational and uneconomical.

Basic way of thinking about the integration of function and equation

It is necessary for comprehending a phenomenon mathematically to identify variables inherent in a phenomenon, and to represent the relation into a table, a graph, and a literal expression. This is because it can grasp the characteristic clearly by expressing a phenomenon in a table, a graph, an expression. We think that a table, a graph, and a expression are not skill but tools for mathematical problem solving or decision making. If the relations between variables inherent in a phenomenon are expressed in tables, graphs, and expressions, it is able to solve many problems with utilizing them. We want to think about the curriculum that students can realize this(Sugiyama, et al., 2003).

A figure of tape

A figure of segment



A set of number line

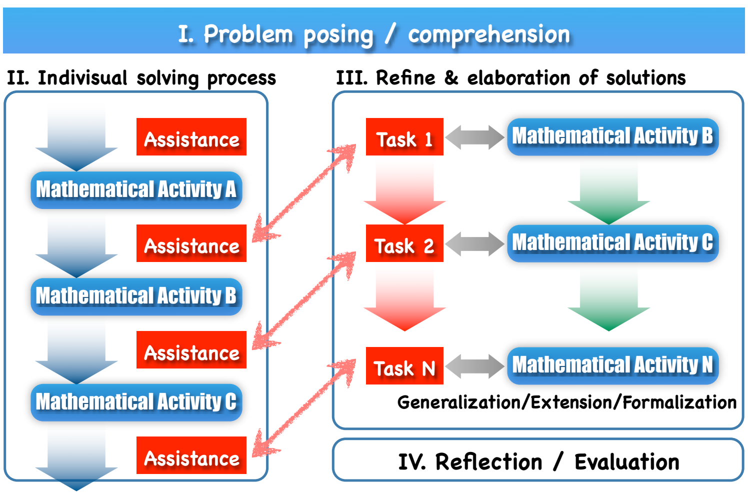
Particularly, in Japanese elementary school, *a figure of tape* or *a figure of segment* (*addition* and *subtraction*), and *a set of number line* (*multiplication* and *division*) are used as tools for deciding operation.

Therefore, in the junior high school, we expect our students to utilize a graph as an effective tool for making an equation.

PURPOSE AND METHODOLOGY OF THE RESEARCH

The purpose of our project is to develop the local curriculum (*Function* and *Equation*) and the pedagogical mathematical contents for teaching intended that students use equation and graph as effective tools of their problem solving.

We edit the experimental textbooks – ***Function and Equation*** **I** (for 7th), **II** (for 8th)**, III** (for 9th, III *is not included in this report, under practice now*) – for the junior high school mathematics, and verify the effectiveness and the problems of the textbooks that we developed through ***lesson studies***. Our lesson style is called ***Problem Solving Lesson*** as shown in the figure below(*cf*. Mizoguchi, 2008).



PROCESS OF THE RESEARCH

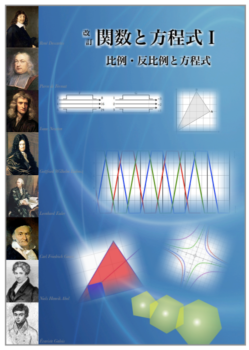
We think that to solve an equation and to represent a graph should be as tools for problem solving in the curriculum. We set the new integrated unit called “*Function and Equation*” that reorganized conventional units, intended to cultivate students’ ability and attitude to apply these in effectively in each school grade. For this, we edited the experimental textbooks along the program that we reconstructed.

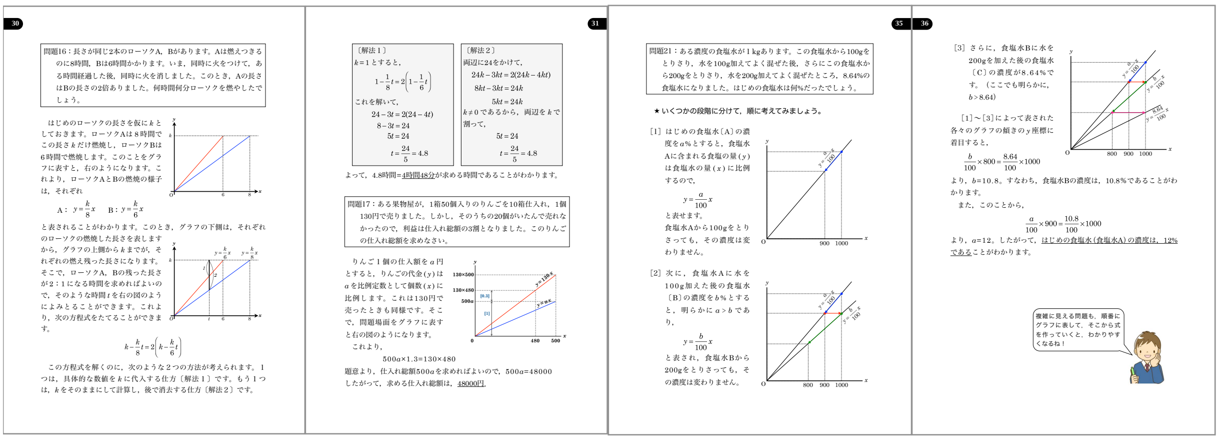
In 2010 and 2011 school year, the researcher from Tottori University and the teachers from Junior High School Attached Tottori University carried out above mentioned research collaboratively about the 7th grade and the 8th grade program. In 2012, we work on the 9th grade program likewise, and improve the textbooks of the 7th and the 8th grade.

EXPERIMENTAL TEXTBOOKS – *Function and Equation* I, II –

***Function and Equation* I** contains *proportion*, *inverse proportion* and *linear equation*.

I. Functional way of thinking
  §1 Problem of crane and tortoise
  §2 Two quantities relation which vary correspondingly
II. Proportion
  §1 Coordinates
  §2 Graph of the proportion
  §3 The use of the proportion
III. Inverse proportion
  §1 Graph of the inverse proportion
  §2 The use of the inverse proportion
IV. How to solve equations
  §1 Equation
  §2 Property of the equation
V. The graph of the function and equation
  §1 The use of the expression of the graph of the proportion
  §2 The use of the slope of the graph of the proportion
  §3 Graph of the proportion and inequality





***Function and Equation* II** contains *linear function* and *simultaneous equations*.

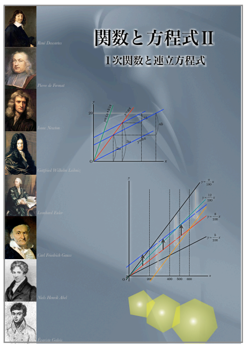
I. Linear function and equation
  §1 Problem of crane and tortoise(Part 2): What do we learn at this unit?
  §2 Proportion and linear function

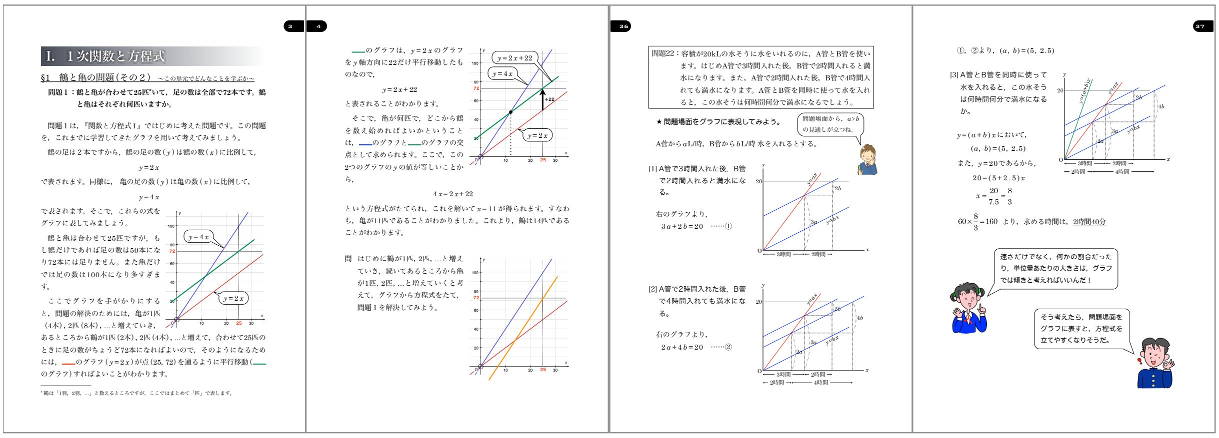
(*continued*)

II. Graph of linear function
- §1 Rate of change
- §2 Graph of linear function
- §3 Linear function and linear equation in two variables

III. Linear function and inequality & simultaneous equations
- §1 Graph of the linear function and inequality & simultaneous equation
- §2 Various how to solve simultaneous equations
- §3 Complicated simultaneous equation

IV. Various inequalities and simultaneous equations
- §1 Problem solving(1)
- §2 Problem solving(2)



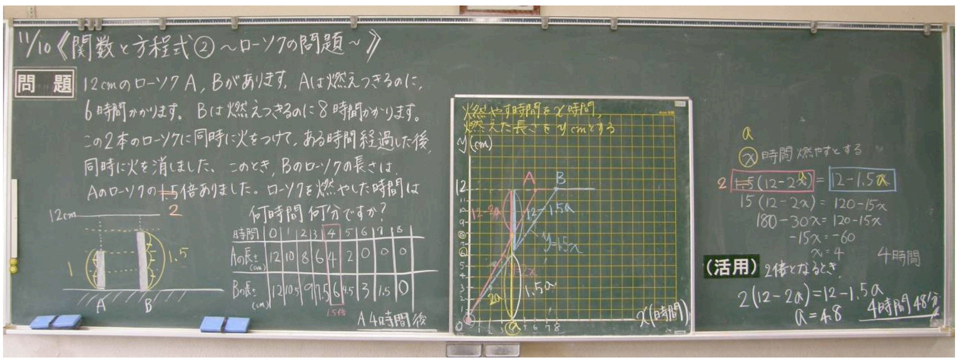


LESSON STUDIES – *partially* –

A.Problem of Candle (7th grade)

◉ *Purpose of the lesson:* To understand that to represent a phenomenon by a graph is useful to make an equation.

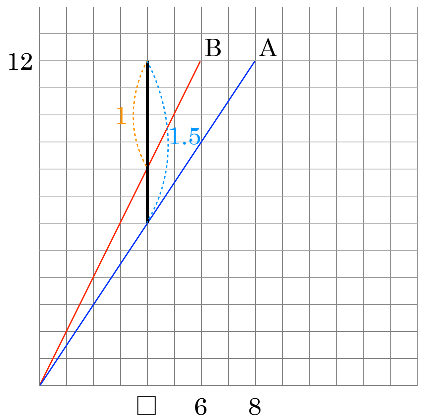
Problem: There are candle A and B of 12cm both. Candle A takes eight hours to burn out, and candle B takes six hours. Setting fire at the same time, having passed at a certain time, and putting out fire at the same time. Then the length of A was 1.5 times of B. How long did two candles burn?



◉*Students' activities* are:

- to investigate by a table;
- to represent graphs because the length that candles burnt is proportional to time, and to read it from coordinate plane directly;

$$A : y = \frac{12}{8}x \quad B : y = \frac{12}{6}x$$



- to make an equation from the graphs.

$$1.5 \times \left(12 - \frac{12}{6}t\right) = 12 - \frac{12}{8}t$$

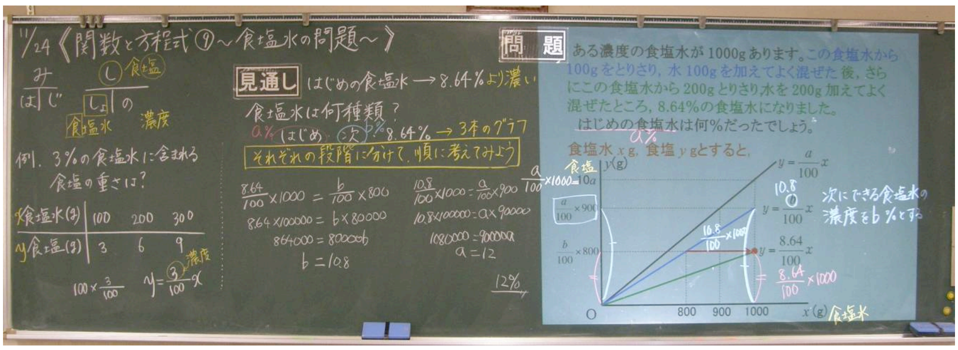
◉*Discussion (excerpted)*

It was difficult for the students to see two graphs at the same time, especially, to compare the length that each candle burns with a change of the time.

B. Problem of Salt Solutions (7th grade)

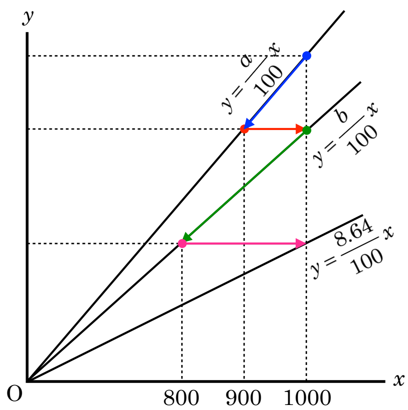
◉ *Purpose of the lesson:* To represent a problem situation to graph, and make equations from it.

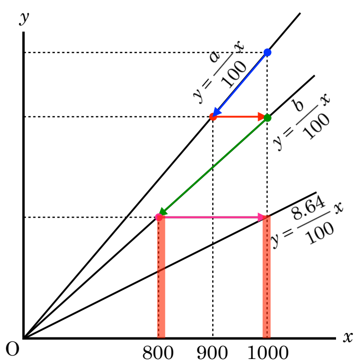
> *Problem: There are 1000g of salt solutions of a certain density. At first, removing 100g from this salt solution, and having added 100g of water, second, removing 200g from this salt solution, and having added 200g of water, then it becomes to the salt solution of 8.64%. What percentage the salt solution of the beginning would be?*



◉*Students' activities* are:

- to reproduce the problem situation by the movement of the point on graphs;
- to make equations from the graphs with focusing to quantity of the salt.

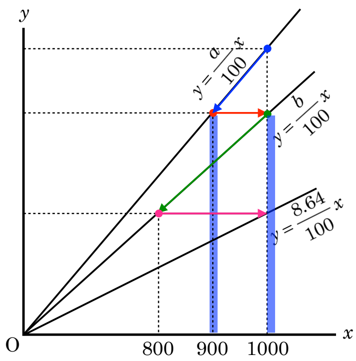




$$\frac{b}{100} \times 800 = \frac{8.64}{100} \times 1000$$

$$8b = 86.4$$

$$b = 10.8(\%)$$



$$\frac{a}{100}\times 900=\frac{10.8}{100}\times 1000$$
$$9a=108$$
$$a=12(\%)$$

◉*Discussion (excerpted)*

Many students were able to solve by oneself with observing the movement of the point.

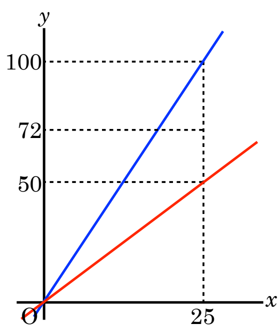
C. Problem of Crane and Tortoise (8th grade)

◉ *Purpose of the lesson:* To understand that to represent a phenomenon by graphs is useful to make equations.

> *Problem: There are cranes and a tortoises in total 25, and the number of feet is 72 in total. How many is the number of crane and tortoise respectively?*

◉*Students' activities* are as follows:

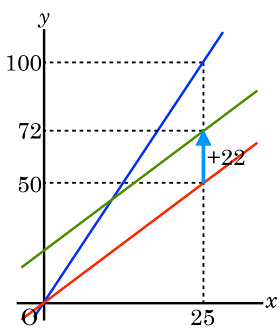
- If the number of crane is 25, the number of feet is 50, then it is fewer than 72. If the number of tortoise is 25, the number of feet is 100, then it is more than 72;



Crane: $y=2x$

Tortoise: $y=4x$

- If several tortoises are there at the beginning and several cranes come afterwards... (*Translation of the graph of tortoise, also similar of the case of crane*)



$y=4x$

$y=2x+22$ ➡ $4x=2x+22$

◉*Discussion (excerpted)*

Students were able to find the graph of the linear function from the graph of the proportion.

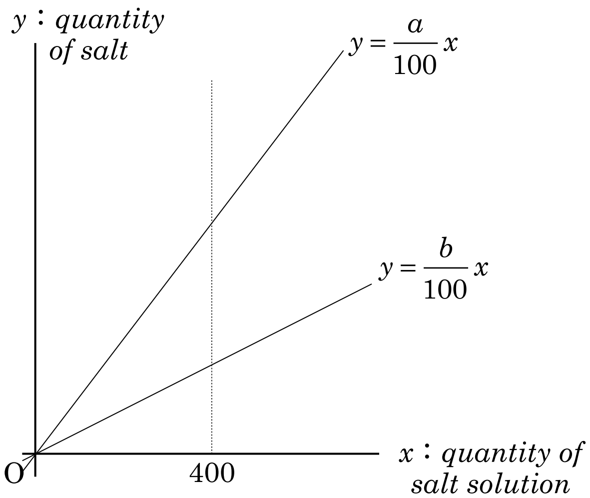
D.Problem of Salt Solution -another- (8th grade)

◉ *Purpose of the lesson:* To represent a phenomenon in simultaneous equations and to solve the problem by using the translation of the graphs of the linear function.

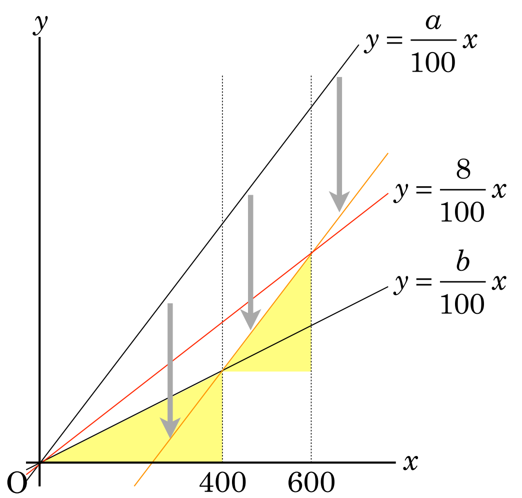
> *Problem: There are two container A and B. The salt solution of a% is in A, and the salt solution of b% is in B and both salt solutions are 400g. The following operations are carried out for A and B.*
>
> [*Operation* 1] *Adding half quantity of the salt solution in A to B, and having mixed them.*
>
> [*Operation* 2] *Adding half quantity of the salt solution in B which was made with the former operation to A.*
>
> *After the operations, the density of salt solution of A is 12%, the density of salt solution of B is 8%. Find the each density of salt solution of A and B before the operations.*

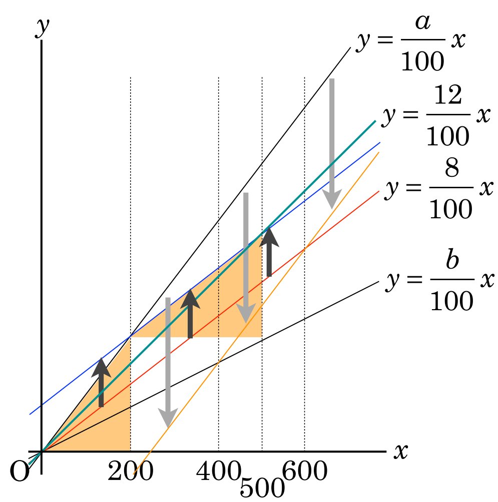
◉*Students' activities* are almost as follows:

‣ y is proportional to x.



‣ Representing the operations by the movement of the graphs.





$$\begin{cases} \dfrac{b}{100} \times 400 + \dfrac{a}{100} \times 200 = \dfrac{8}{100} \times 600 \\ \dfrac{a}{100} \times 200 + \dfrac{8}{100} \times 300 = \dfrac{12}{100} \times 500 \end{cases}$$

◉*Discussion (excerpted)*

Students were almost able to use graphs and to translate them for comprehending and solving the problem with making simultaneous equations.

Notes

1) This report is a revised version of our handout for the 12th International Congress on Mathematical Education(8 July – 15 July, 2012, COEX, Seoul, Korea).
2) This research project is funded by Japan Association of Universities of Education.

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