The Measurement of Reynolds Stresses with a Hot-Wire Anemometer

by

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This paper deals with the measurement of the mean velocity and the Reynolds stresses in a turbulent flow by using a linearized constant temperature hot-wire anemometer. A system of analytical response equations was derived, in this study, to determine all the components of mean velocity and the Reynolds stresses in a three-dimensional flow field by using a single slanted wire in combination with a straight wire. Here the direction of the flow itself is initially unknown. This method of measurement with two types of hot-wires is tested experimentally in the initial region of a round free jet. The results obtained by this method are compared with experimental results in the noise-producing region of a circular jet by P. Bradshaw and those in the flow-establishment regions of a round jet by S. Sami, and found satisfactory except the radial and tangential components of the mean velocity.

1 Introduction

Despite the limitation, the hot-wire anemometer is still an indispensable instrument for investigating instantaneously unsteady flow such as turbulence. The measurements of mean velocity and the Reynolds stresses have been carried out by many workers, e. g., Hinze,[1] Champagne and Sleicher,[2] Kovasznay and Fujita[3] .

Although the hot-wire technique may now appear to have been well established, it is noticeable that the majority of published works on this subject is concerned only with one or two-dimensional flow field and has some restrictive assumptions.

Hence this paper describes the method of the measurement of the three components of mean velocity and the six components of Reynolds stresses in a three-dimensional turbulent flow field of unknown direction with the least assumptions.

For this purpose, the hot-wire response equations were derived for evaluating the three components of mean velocity and the six components of Reynolds stresses.

These measurements are carried out with two types of hot-wire probes, i. e., the

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slanted wire and the straight wire probes, at a given position in a known flow field, i. e., the initial region of a round free jet. The results obtained by this method are compared with the experimental results in the noise-producing region of a circular jet measured with the x-probe by P. Bradshaw[4] and those in the flow establishment region of a round jet measured with four types of probes by S. Sami[5] .

2 Nomenclatures

Symbols

D	= Nozzle diameter;
h	= Pitch factor;
k	= Yaw factor;
u, v, w	= Velocity components in a fixed reference system;
u_{eff}	= Effective cooling velocity;
x, y, z	= Coordinate axes;

Superscripts

—	= Indicates time average;
′	= Indicates the fluctuating component;

Subscripts

0. 1, $\frac{1}{2}$, 0.9	= Indicate the points at which velocity deficit is 0.1, half, 0.9 of the maximum velocity $\bar{u}_o$, respectively.

3 Derivation of the hot-wire equations

Fig. 1 shows the velocity vector components, u_n , u_b and u_t at the hot-wire sensor which are the components in the directions of the axis x_n perpendicular to the sensor prong plane, of the probe axis x_b and of the sensor x_t, respectively.

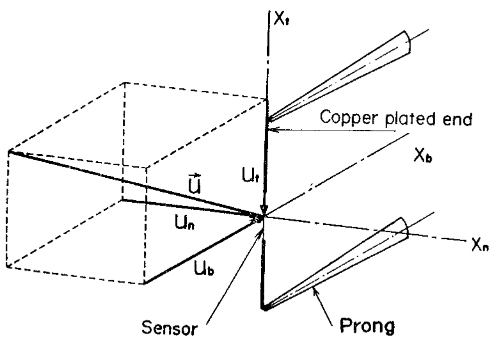


Fig. 1 Attached reference system of the hot-wire.

If the wire were infinitely long, only the normal components u_n and u_b would contribute to the cooling. In reality, the wire is about 200 diameters long and must be supported on the prong for the measurement. In consequence u_t contributes to the cooling of the wire and u_b acts differently from u_n because of the blockage effect of the prong. The effective cooling velocity u_{eff} acting on the hot-wire sensor can be expressed as :

$$u_{eff}^2 = u_n^2 + h^2 u_b^2 + k^2 u_t^2 \qquad (1),$$

where h is the pitch factor and k the yaw factor. The estimate of the pitch factor and the yaw factor is based on the calibration curve of the probe obtained from the experiment. The electrical signal through a linearizer is proportional to the effective cooling velocity.

As Eq. (1) is based on the reference system (called the attached system from now on) which is fixed to the hot-wire sensor as shown in Fig. 1, it is necessary to express Eq. (1) by using the other reference system (called the fixed reference system from now on) which is fixed to the nozzle because the flow field is represented in this system. The components of the mean velocity and Reynolds stresses in the fixed reference can be determined through the matrix of transformation from the attached reference system. If the angles, for a given position of the hot-wire, between the attached system (x'', y'', z'') and the fixed system (x, y, z) are defined as in Fig. 2, the matrix of transformation can be obtained by the following procedure.

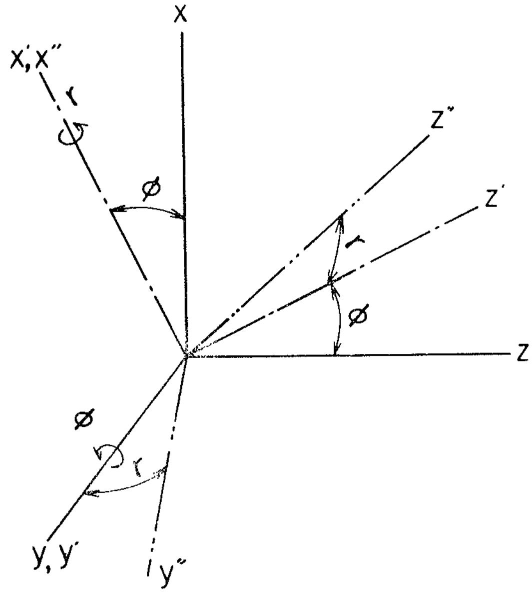


Fig. 2 Definition of angles ϕ and γ.

First, the fixed reference system is rotated in the counter-clockwise direction by an angle ϕ around the axis of y to yield

$$\begin{aligned} x' &= x\cos\phi - z\sin\phi \\ y' &= y \\ z' &= x\sin\phi + z\cos\phi \end{aligned} \qquad (2).$$

Second, (x', y', z') system is rotated in the counter-clockwise direction by an angle γ around the axis of x', then (x'', y'', z'') system, i. e., the hot-wire reference system (the attached system), is obtained as

$$\begin{aligned} x_n &= x'' = x' \\ x_b &= y'' = y'\cos\gamma + z'\sin\gamma \\ x_t &= z'' = -y'\sin\gamma + z'\cos\gamma \end{aligned} \qquad (3).$$

Combining Eq's. (2) with (3),

$$\begin{bmatrix} x_n \\ x_b \\ x_t \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ \sin\gamma\sin\phi & \cos\gamma & \sin\gamma\cos\phi \\ \cos\gamma\sin\phi & -\sin\gamma & \cos\gamma\cos\phi \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = M_{(j)} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (4).$$

The $M_{(j)}$ is the matrix of transformation. u_{eff} in Eq. (1), can be represented in terms of u, v and w, which are the components of instantaneous velocity in the fixed reference system, by the help of the matrix of transformation $M_{(j)}$

$$u^2_{eff} = [u\ v\ w] \cdot {}^T H_{(j)} \cdot H_{(j)} \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad (5),$$

where

$$H_{(j)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & k \end{bmatrix} \cdot M_{(j)} \ .$$

${}^T H_{(j)}$ is the transpose of $H_{(j)}$.

Taking time-average of Eq. (5) and dividing the velocities into the mean velocities and the Reynolds stresses, we have

$$\bar{u}^2_{eff\,(j)} = A_{(j)}\,\bar{u}^2 + B_{(j)}\,\bar{v}^2 + C_{(j)}\bar{w}^2 + D_{(j)}\,\bar{u}\bar{v} + E_{(j)}\,\bar{v}\bar{w} + F_{(j)}\,\bar{w}\bar{u}$$

$$\overline{u'^2_{eff\,(j)}} = A_{(j)}\overline{u'^2} + B_{(j)}\,\overline{v'^2} + C_{(j)}\,\overline{w'^2} + D_{(j)}\,\overline{u'v'} + E_{(j)}\,\overline{v'w'} + F_{(j)}\,\overline{w'u'} \qquad (6).$$

where

$$A_{(j)} = cos^2\phi + h^2 sin^2\phi\, sin^2\gamma + k^2 sin^2\phi\, cos^2\gamma, \quad B_{(j)} = h^2 cos^2\gamma + k^2 sin^2\gamma,$$

$$C_{(j)} = sin^2\phi + h^2 cos^2\phi\, sin^2\gamma + k^2 cos^2\phi\, cos^2\gamma, \quad D_{(j)} = (h^2 - k^2)\, sin\phi\, sin2\gamma,$$

$$E_{(j)} = (h^2 - k^2)\, cos\phi\, sin2\gamma, \quad F_{(j)} = -(1 - h^2 sin^2\gamma - k^2 cos^2\gamma)\, sin2\phi,$$

where ϕ represents the angle of rotation of the probe around its axis and γ represents the angle of the inclination of the sensor to the probe axis.

To solve the resulting six simultaneous equations, it is necessary to take a set of six readings measured at the different orientations. Furthermore to make the determinant of the coefficients of Eq. (6) nonvanishing, it is also necessary to use the two types of probes which have different γ. Therefore three readings out of six are obtained by rotating a straight wire probe ($\gamma = 0°$), and the others by rotating a slanted wire probe ($\gamma \neq 0°$).

4 Experimental equipment and procedure

The idea introduced in the preceding section is tested experimentally in the initial region of a round free jet. The experimental equipment is shown in Fig. 3.

The flow rate of air supplied with the air compressor is adjusted by the pressure regulator and the air flow is rectified in the settling chamber and issued into stag-

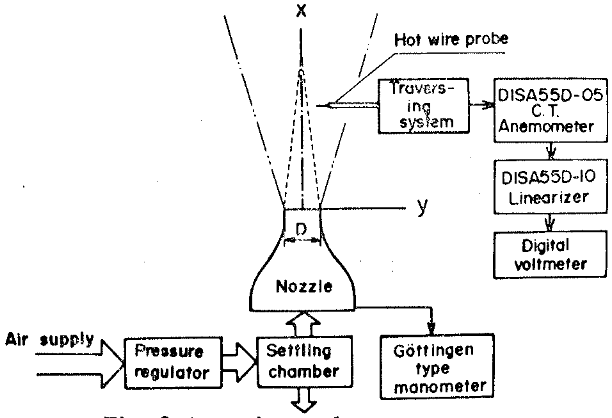
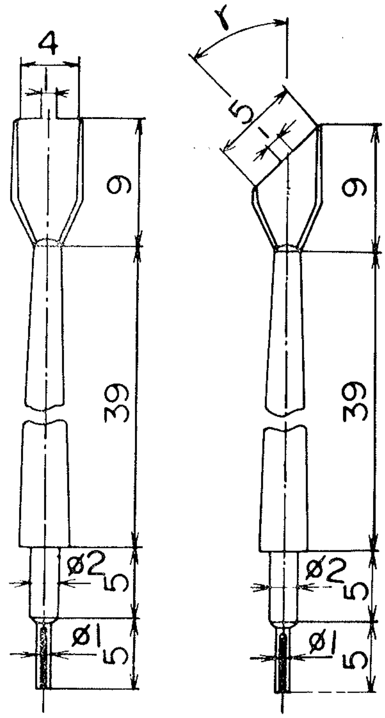


Fig. 3 Experimental set-up.

nant air from the nozzle. The exit velocity is maintained constant within 1% of itself and Reynolds number is 7.5×10^4. The jet and the room temperatures are equal with each other during any test run. The nozzle is 30*mm* in throat diameter, and has the contraction ratio of 9 to 100 and three static holes on the side wall of its inlet.



(a) Straight wire probe (b) Slanted wire probe

Fig. 4 Configurations of the hot-wire probes.

The configurations of hot-wire probes are shown in Fig. 4. (a) is the straight wire probe and (b) the slanted wire probe. The sensors of these are made of tungsten wire with diameter of 0.005*mm* and length of 1*mm* (aspect ratio 200) with copper plated ends. The probe dimension was measured with a profile projector. The experiment consists of two parts. First, the straight wire traverse was made in the half of the jet along the axis of y, and rotated about its axis at each points. On this traverse, the mean and the root-mean-square values of the linearized signal are recorded against the angle $\phi = 0°$, 40°, −30°. Second, the same values are measured with the slanted wire probe in the same manner.

Substituting in Eq's. (6) and (7) a set of six signals, the wire orientation ϕ and γ, and the pitch and the yaw factors of each wire could determine three components of mean velocity and the six components of Reynolds stresses.

5 Results and Discussion

In this section, the results obtained by our method are presented and compared with those by Bradshaw and Sami.

In the following presentations, the velocities are made dimensionless by the center-line velocity $\bar{u}_0$. The radial distance y, however, is made dimensionless in three ways, i. e., by $y_{1/2}$ (the distance at which the velocity has half its maximum value), flow width $(y_{0.1}-y_{0.9})$ or the nozzle diameter D.

The distribution of components of mean velocity is shown in Fig's. 5 and 6 together with the profile of $\bar{u}_p/\bar{u}_{p0}$ obtained by means of the total pressure pitot tube. The agreement of $\bar{u}/\bar{u}_0$ with $\bar{u}_p/\bar{u}_{p0}$ is satisfactory except for the velocity near the edge.

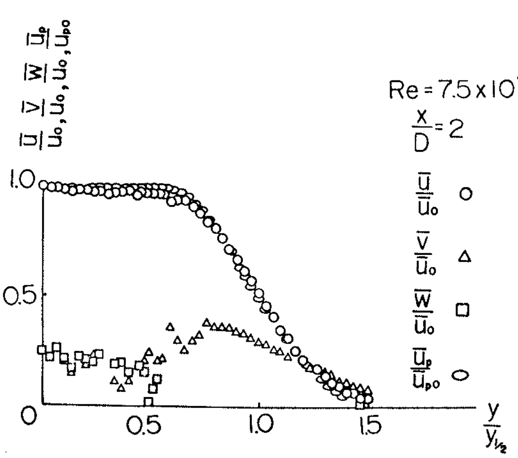


Fig. 5 Distributions of $\bar{u}/\bar{u}_0$, $\bar{v}/\bar{u}_0$ and $\bar{w}/\bar{u}_0$ at x/D=2.

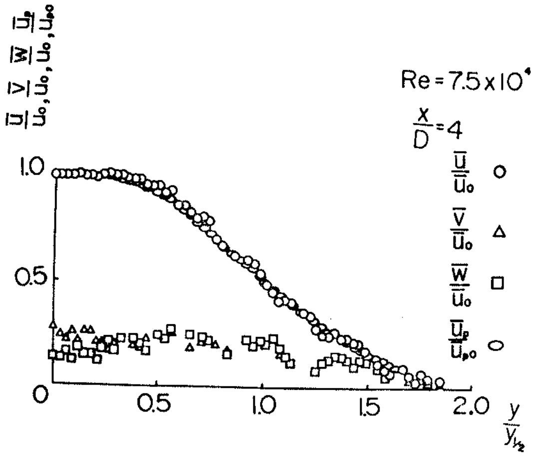


Fig. 6 Distributions of $\bar{u}/\bar{u}_0$, $\bar{v}/\bar{u}_0$ and $\bar{w}/\bar{u}_0$ at x/D=4.

The velocity profiles $\bar{v}/\bar{u}_0$ and $\bar{w}/\bar{u}_0$ are widely scattered, and seem to include inaccuracies in the values obtained by experiments. In order to show the comparison of our results with those of Bradshaw, the distributions of normal stresses obtained at two axial stations x/D=2 and 4 are plotted in Fig's. 7 and 8. The agreement of magnitude of the streamwise fluctuations $\sqrt{\overline{u'^2}}/\bar{u}_0$ is good but the values of the radial fluctuations $\sqrt{\overline{v'^2}}/\bar{u}_0$ and the tangential fluctuations $\sqrt{\overline{w'^2}}/\bar{u}_0$ are small compared with the results of Bradshaw. The distributions of shear stress $\overline{u'v'}/\bar{u}_0^2$ are plotted to compare with the results by Bradshaw and Sami at the axial stations x/D =1, 2, 3, 4, as shown in Fig. 9. Although the maximum values of $\overline{u'v'}/\bar{u}_0^2$ are as large as those of Bradshaw and Sami, their profiles are different from ours.

This method produces considerable scatter in the profiles. This is consequence of the magnification of measurement-errors through the computation of simultaneous equations. If a higher accuracy of the results is required, it could be obtained with more adequate measurement conditions and more accurate recording of the output signal corresponding to the fluctuating velocity.

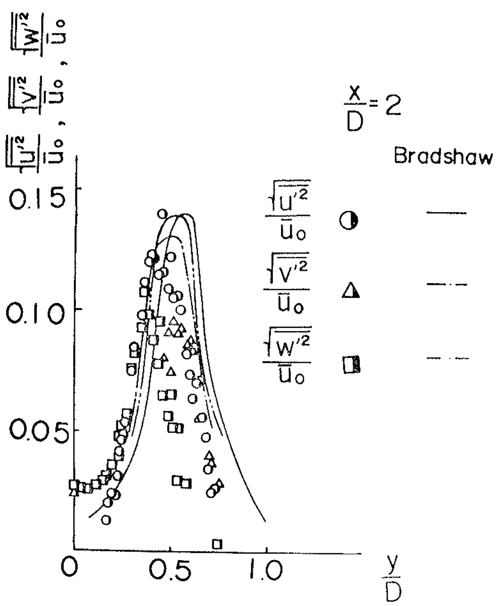


Fig. 7 Comparison of our normal stresses with other worker's at x/D=2.

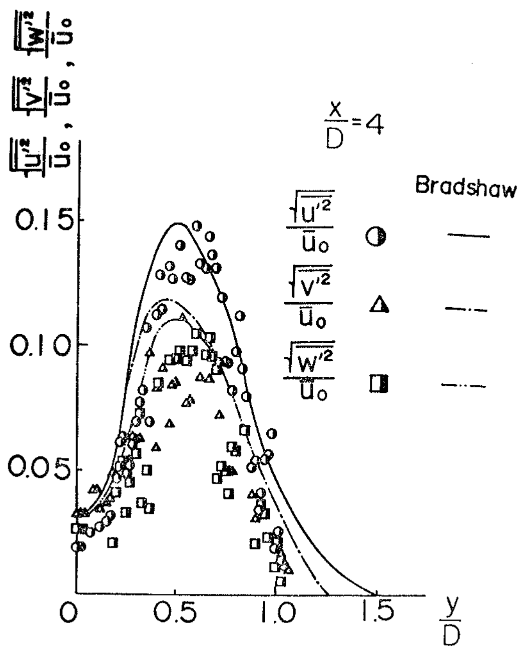


Fig. 8 Comparison of our normal stresses with other worker's at x/D=4.

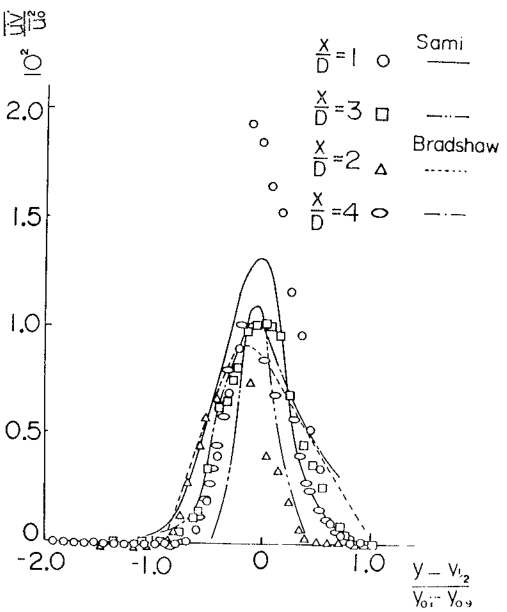


Fig. 9 Comparison of our shear stresses with other workers' x/D=1,2,3,4.

6 Conclusion

The method by the combined use of a straight wire probe and a slanted wire probe has been successfully applied to the measurement of the mean velocity and the Reynolds stresses in three-dimensional turbulent flow. The derived hot-wire equations in terms of velocity components with least assumptions can be used to investigate a three-dimensional turbulent flow. In order to establish more accurate measurement, some modification of the recording of the electric signals from the hot-wire system and the accurate measurements of the orientation of the hot-wire are necessary.

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