# Theoretical Studies on the Behavior of Concrete under Bearing Pressure

by

Shinzo NISHIBAYASHI, Hideo KIYAMA, Kenji SAKATA.

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**Synopsis**

In this paper, the authors describe theoretical studies on the bearing strength of concrete. Main results which have been obtained by them are as follows.

(1) In the bearing pressure test of concrete, the dimension ratio, the Poison's ratio, the compressive: tensile strength ratio and the loading ratio have an effect on the stress distributions and severity in the specimen, and that is on the bearing strength of it. The effects of such factors have been estimated in detail.

(2) The theoretical formulas of bearing strength of concrete have been suggested using the above estimated coefficient, and examined in connection with the experimental formulas have been pointed out so far.

## 1 Introduction

Bearing capacity which is a kind of compressive strength of concrete shall be taken in consideration for the concrete under the actions of local stresses, in such as end anchorage zones of PC wire or bar, as supports with shoe in bridge pier and as doweled joints of combined girder.

Bearing strength or capacity for a cylindrical concrete specimen which is set on the flat plate having diameter ($d$) and height ($h$) can be evaluated by means of its ultimate compressive load due to circular pressure at the center of the top, as shown in Fig. 1. In this respect, bearing strength ($\sigma_u$) is defined as follows,

$$\sigma_u = -\frac{P}{A_b} = -\frac{4P}{\pi d_0^2} \tag{1}$$

where $P$ is ultimate compressive load when the failure of specimen is occured, $A_b$ and $d_0$ are the area and the diameter of a circular bearing plate respectively.

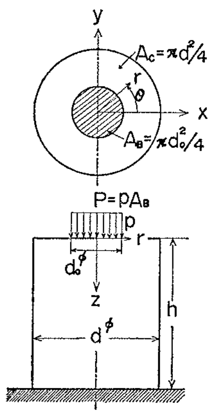


**Fig. 1 Diagrams illustrating the bearing pressure on the specimen accompanied by the cylindrical coordinates adopted.**

Many laboratory tests have been carried out to make certain about the effects of the dimension ratio in the specimen $(h/d)$ , the ratio of the area in specimen to that in circular bearing plate $(A_c/A_b)$, the concrete mix proportion, the types and kinds of aggregate, and the strength of concrete, on the bearing capacity and the failure conditions, and the various experimental formulas on the relations between bearing strength $(\sigma_u)$ and compressive strength $(\sigma_c)$ have been suggested.

The authors describe the results of theoretical investigations on the relations between bearing capacity and above-mentioned factors, under considering the internal stress distributions in the specimen and the failure criterion of concrete in this paper.

## 2 Stress Distributions

At the analysis of the distribution of stresses in the concrete specimen by means of the elastic theory, following assumptions are required.

A footing plate is rigid, and any frictions between the plate and bottom of the specimen may be negligible (Assumption 1) . However, if the height of specimen is too short (such as $h/d < 1$) , there are some problems in this assumption. The distribution of the load due to the circular bearing plate at the top of the specimen is uniform (Assumption 2) .

Then, the distributions of stresses in the cylindrical specimen subjected to bearing pressure will be analysed by using the Michell's stress function[1] based on the above-mentioned assumptions. In this case, stress states at any points in the specimen depend on the dimensions of the specimen that are expressed in $h/d$, the ratio of loading area $(A_c/A_b$ or $d_o/d=\sqrt{A_b/A_c})$ and the Poisson's number of used material $(m)$. Vertical stress $(\sigma_z)$ and horizontal stresses ($\sigma_r$ and $\sigma_\theta$), each of them being perpendicular to the other, producing in each point on the central axis of specimen, are calculated as to combine various dimensional ratios $(h/d)$; 0.5, 1.0, 1.5, 2.0, and various loading ratios $(d_o/d)$; 0.1, 0.2, 0.3, ……, 0.9 (when its ratios are expressed in $A_c/A_b$, those values become 100, 25, ……… , 1.2), and Poisson's number $(m)$; 4, 6 and 8. Along the central axis, that is, $z$-axis, those stresses express the three principal stresses and $\sigma_r$ is equal as $\sigma_\theta$ . An example of calculated stresses is shown in Fig. 2. In Fig. 2, the ordinate shows the ratio of height and distance from top of specimen $(z/h)$. and the abscissa shows the divided value of stress by $-P/A_c$ .

Some charactristics for stress distribution may be observed from Fig. 2, as follows.

(i) The principal stress on vertical direction presents always compressive stress, but those on horizontal show tensile at lower parts and compressive at upper parts. In the near of bearing pressure both principal stresses become large value, and produce the state of tri-axial confining stresses.

(ii) At any point, tensile stress $\sigma_r$ is smaller by one order than compressive stress $\sigma_z$ . On the other hand, $\sigma_z$ is hardly effected by Poisson's number but $\sigma_r$ may

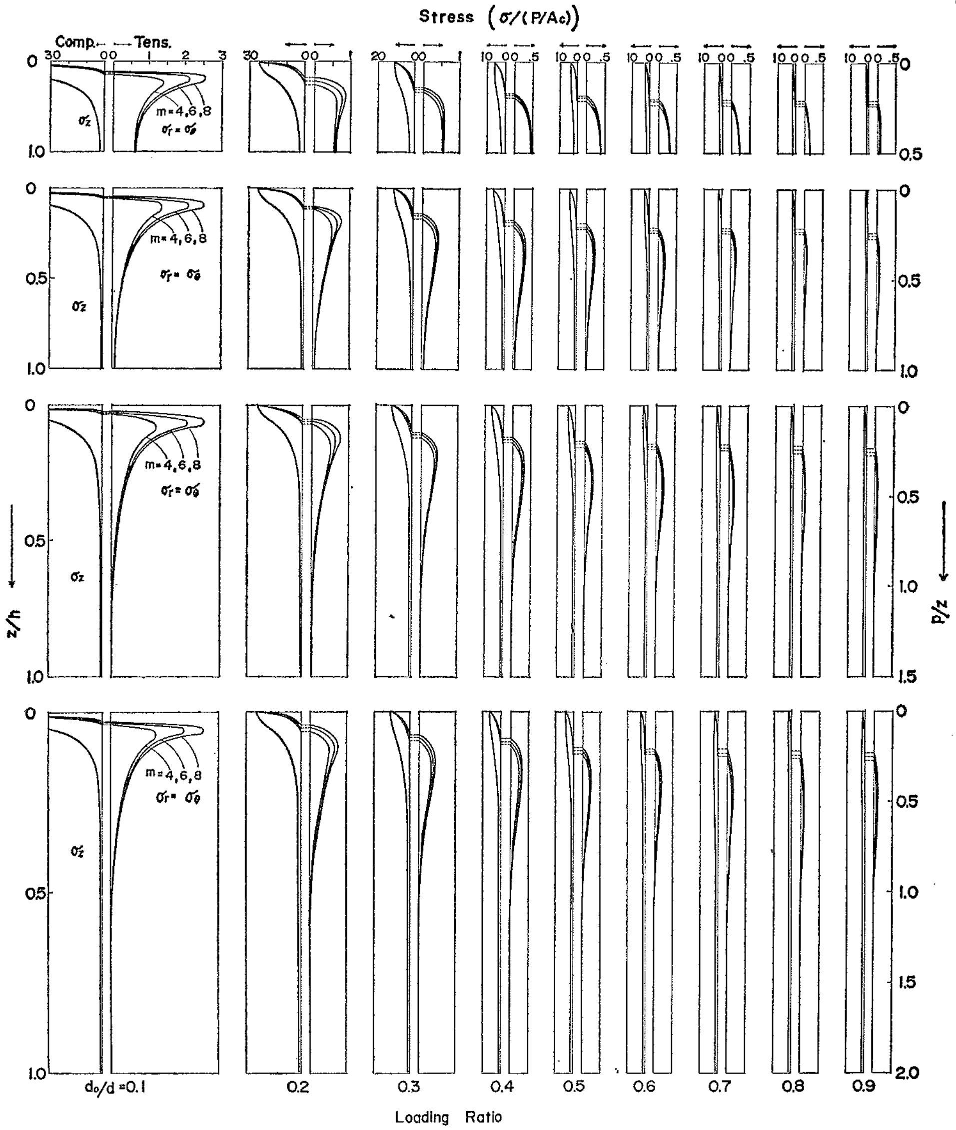


**Fig. 2 Stresses along z-axis of cylindrical specimen subjected to the bearing pressure.**

be achieved relative large effect by $m$. As the loading ratio is small (that is, $do/d < 0.3$), maximum of tensile stress that is produced at the near of bearing pressure (such as $z/h=0.3\sim0.1$) is particularly effected by Poisson's number. Thus, if the states of stresses at that local position dominate the failure of concrete specimen, the effects of Poisson's number may not be negligible.

(iii) As the loading condition spread over whole surface of specimen, $\sigma_z$ is equal to $-1\times$ $(-P/A_c)$ at any cross section. If the load is applied partially, as principle of Saint-Venant have been shown, $\sigma_z$ develops progressively at whole cross section according as increase distance from loading point, and put up in $\sigma_z = -1 \times (-P/A_c)$ lastly. Therefore, it can be recognized that the effect of dimension in bearing capacity is negligible when $h/d$ is more than 1.5, because the stress states become uniform, that is, $\sigma_z = -1$ and $\sigma_r = \sigma_\theta = 0$.

## 3 Criterion of Failure for Concrete

It has been assumed in the above analysis that the distribution of the load under the bearing plate is uniform. But generally considering the pressure between two elastic bodies in contact, it may be thought that the distribution of the load is not uniform but varying such as its smallest value is at the center ($r=0$) and its largest value is at the boundary of the circular area of contact ($r=d_0$). Moreover it may be that the diametrical stress ($\sigma_r$) is the largest tensile stress at the same boundary ($r=d_0$) although only near the surface. Due to these stress states a circular tensile crack along that boundary is produced when the propotion of the loading ratio $d_0/d$ is relatively small.

It has been reported at the previous papers[2] that the crack however is of local character which does not substantially affect ultimate failure of the specimen, and that if the bearing plate with rounded corner is used the distribution of the load becomes such uniform that the crack is almost hard to be produced.

Then the failure of the specimen, apart from that local fracture, must be depended on the distribution of stresses along the $z$-axis as shown in Fig. 2. Because stress states along the axis differ from points to points, it, however, is complicated to predict where the ultimate failure of the specimen begins and to estimate the ultimate load at the failure.

Now the authors adopt such a procedure as follows. At first we will determine the equation of the parabolic Mohr envelope which enclosed a circle of the uniaxial tensile strength of concrete ($\sigma_t$), touching it at the vertex ($\sigma_t$,0), and a circle of the uniaxial compressive strength of concrete ($\sigma_c$), that is

$$\tau^2 = \left\{\sqrt{n+1} - 1\right\}^2 \sigma_t \ (\sigma_t - \sigma) , \qquad (2)$$

where $n = -\sigma_c/\sigma_t$. Changing the value of $n$ adequately, the corresponding envelopes can include most actually generalized criterions of failure of concrete, such as reducing to the Griffith Criterion when $n = 8$.

Next let us assume that when a Mohr stress circle at a point, theoretically calculated, may touch the envelope the failure occurs at that point (Assumption 3).

Then we can obtain the measure of the stress severity[3], in other words the measure of the likelihood of failure at each point along the $z$-axis. That is, the stress severity is defined as the ratio of the (theoretical) load at failure of the specimen to the load which would (theoretically) have been required to cause failure at the point, and its calculation is as follows. The relative failure load ($\overline{P}$) of each point which would theoretically have been required for the Mohr stress circle of the each point to touch the envelope of Eq.(2) is calculated from the stress states as shown in Fig. 2. Letting $\bar{P}_{min}$ be the minimum value of $\bar{P}$, $\bar{P}_{min}$ would indicate the load at the first failure of the specimen, and the stress severity $s$ at each point is given by the ratio of $\bar{P}_{min}$ to $\bar{P}$ at the point,

$$s = \bar{P}_{min}/\bar{P}. \qquad (3)$$

Thus, $s$ at the point where being the most likely to fail is equal to 1, and the other points where $s \neq 1$ $(1 > s > 0)$ have not yet failed untill the load would increased to $1/s$ $( > 1)$ times of the load that has caussed failure of the point where $s = 1$. For an example to clarify the above discription, the Mohr critical envelope of Eq. (2) and the Mohr stress circles accompanied by the values of $s$ of points along the $z$-axis when the circle of the point where $s=1$ touches the envelope are graphically shown in Fig. 3. It is clearly seen from Fig. 3 that the points where $s \fallingdotseq 1$ $(z/h = 0.2$ and $0.3)$ may be the likelyhood of failure while the other points where $s << 1$ are not yet so when the point where $s = 1$ $(z/h = 0.25)$ is failed.

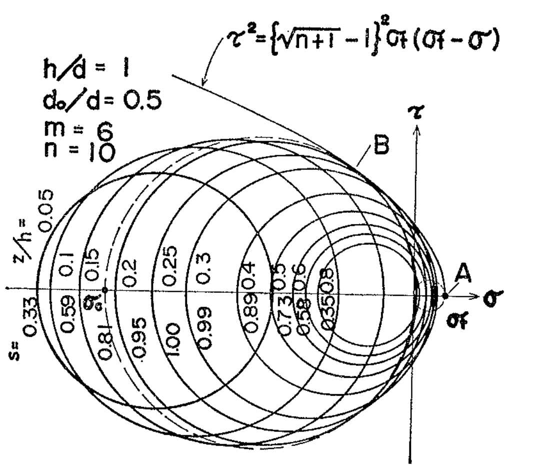


**Fig. 3 Diagram explaining the stress severity (s) by Mohr stress circles of points along z-axis at failure of the pont where s=1.0, accmpanied by the Mohr critical envelope. On this example, h/d=1, do/d=0.5, m=6 and n=10.**

It is to be noted that the value of $s$ may vary naturally corresponding with $n$ $(= -\sigma_c / \sigma_t)$ although the stress states are the same. Therefor in each of such three cases as $n=8$, 10 and 15, the values of $s$ are calculated on every stress states as shown in Fig. 2. Some of the results, i. e. when $m=6$ are shown in Fig. 4. In Fig. 4 the left, middle and right figures are when $n=8$, 10 and 15 respectively, and the upper, second, third and lower figures are when $h/d = 0.5$, 1.0, 1.5 and 2.0 respectively. At each figure the contour lines of $s$ are drawn on the coordinates whose horizontal and vertical lines are the propotion of the loading ratio $(d_o/d)$ and the position of points on the $z$-axis of each test piece $(z/h)$ respectively,and the zones Ⅰ, Ⅱ and Ⅲ enclosed by the dotted lines correspond to those characterizing the stress states of each point by where Mohr stress circle of the point would touch the envelope as shown in Fig.3. That is to say, failure at any points in the zone Ⅰ would occur under the condition that three principal stresses are compressive, while failure in the zones Ⅱ and Ⅲ would do under the condition that at least maximum stress $\sigma_r$ $(= \sigma_\theta)$ must be tensile, and failure in the zones Ⅱ and Ⅲ, especially in the zone Ⅲ, is therefor affected by these tensile stresses. Thus an aspect

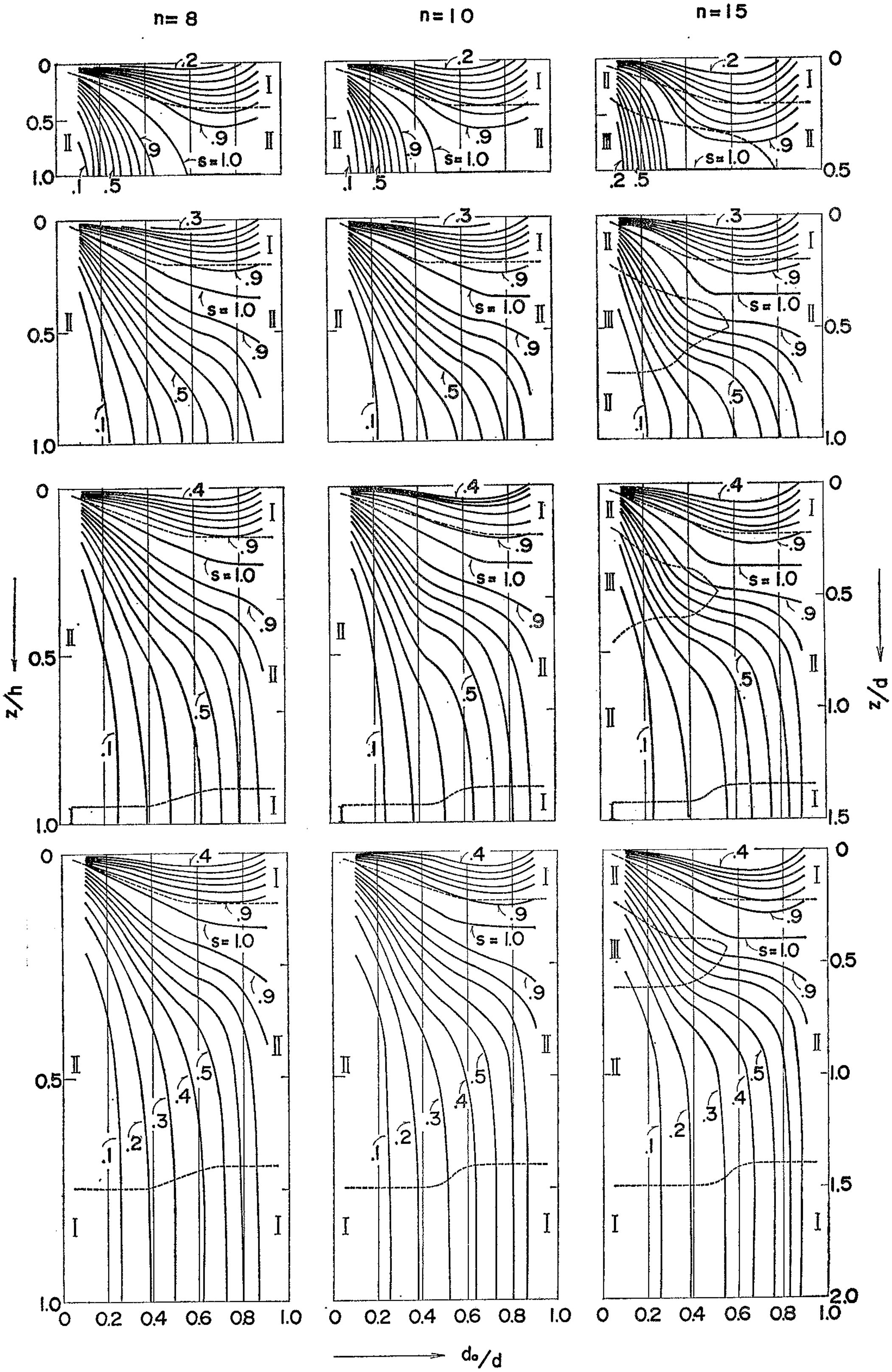


**Fig. 4 Diagrams showing the stress severity (s) at points along z-axis, where m=6.**

of failure of a specimen would vary according as in which zone Ⅰ, Ⅱ or Ⅲ substantial failure of the specimen may be caused.

It is clear from the results,

(i) that the values of $s$ of the points in the near of the bearing pressure is relatively little, i.e. showing little likely to failure, no matter how high compressive the three principal stresses would be,

(ii) that the greater zone is critically stressed, i.e. $s \cong 1$, as the larger loading ratio $d_0/d$ is approached, so that failure throughout the test piece is easy to occur, while the smaller zone is critical as the smaller $d_0/d$ is,

(iii) that the point where $s = 1$ positions near the upper end of the specimen when the smaller $d_0/d$ is, while the inner or lower of the specimen as the larger $d_0/d$ is, and that this position is dependent on $n$,

(iv) that the point where $s = 1$, almost agreeing with the point where the maximum tensile stress arises, belongs always to the zone Ⅱ, and that the failure is therefor affected also by the tensile stress $\sigma_r$ $(=\sigma_\theta)$ and, then, the values of $m$ and $n$.

## 4 Theoretical Formula of Bearing Strength

Under the above assumptions, the ultimate compressive load when the specimen is failed can be calculated from $\sigma_c$ and $\sigma_t$.

At first let us assume that the ultimate load is equal to the load $(P_1)$ which would cause failure at the point where $s = 1$, the relations between $P_1$ and $\sigma_c$ (or $\sigma_t$) are shown, for an example, by the dotted lines in Fig. 5 where $h/d = 1$ and $n = 10$. Hereupon $k_c$ is given by the equation,

$$k_c = (-P_1/A_c)/\sigma_c = (P_1/A_c)/n\sigma_t \quad (4)$$

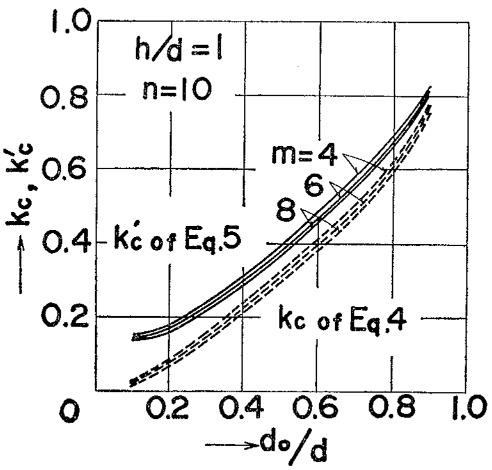


Fig. 5 Relations between the coefficients, Kc of Eq. (4) and Kc′ of Eq. (5), and the values of do /d and m, where h/d=1, and n=10.

As has been mentioned, the extension of the zone where $s \cong 1$ is varied with the loading ratio $(d_0/d)$; the smaller the ratio $d_0/d$ is, the smaller that zone is. In the case of the small ratio $d_0/d$, when the load is increased untill the stress circle of the point where $s = 1$ would theoretically touch the envelope of faifure, the small zone is critically stressed but most part of specimen is not at all. In such a case the specimen, as is experimentally known, does not entirely fail at that load as far as new stress states of equilibrium in the specimen may be achieved by the redistribution of the load. Consequently when the bearing strength ($\sigma_u$) defined by Eq. (1) is regarded as a ultimate strength of the specimen subjected to the bearing pressure, the value of $k_c$ determined from the load ($P_1$) which would cause failure at the point where $s = 1$ must be suitably modified as follows.

Considering the characteristics of the stress states and the actual conditions of failure of the specimen, it is assumed that the ultimate load is equal to the load ($P$) which would cause failure at the points in the some range on the $z$-axis, such as the range equal to a half of radius of crosssection of the specimen($d/4$), including the point where $s = 1$ (Assumption 4) .

Then for the above mentioned example, the relations between $P$ and $\sigma_c$ (or $\sigma_t$) are shown by the solid line in Fig. 5 where $k_c'$ is given by the equation,

$$k_c' = (-P_2/A_c)/\sigma_c = (P_2/A_c)/n\sigma_t \,. \tag{5}$$

Substituting Eq. (5) in Eq. (1), we obtain the relations between $\sigma_u$ and $\sigma_c$ (or $\sigma_t$) as follows.

$$\sigma_u / \sigma_c = k_c' \ A_c / \ A_b \,. \tag{6-a}$$

$$\sigma_u / \sigma_t = -nk_c' \ A_c / \ A_b \,. \tag{6-b}$$

Whenever considering $n = -\sigma_c / \sigma_t$ both the above expressions are the same and then the formula of bearing strength can generally be expressed in Eq. (6-a) inspite of conditions of failure of the specimen. One should however note that the coefficient $k_c'$ in itself is dependent on $n$ as well as the values of $h/d$, $d_0/d$ and $m$.

Converting the value of $k_c'$ theoretically, the various formulas of bearing strength which have been proposed experimentally can be deduced. But the coefficients of those formulas should also be functions of the factors, such as $h/d$, $d_0/d$ and $m$, which affect the stress states of the specimen and the other factor, such as $n$, which affects the criterion of failure. Consequently it is difficult to obtain such a formula with simple form

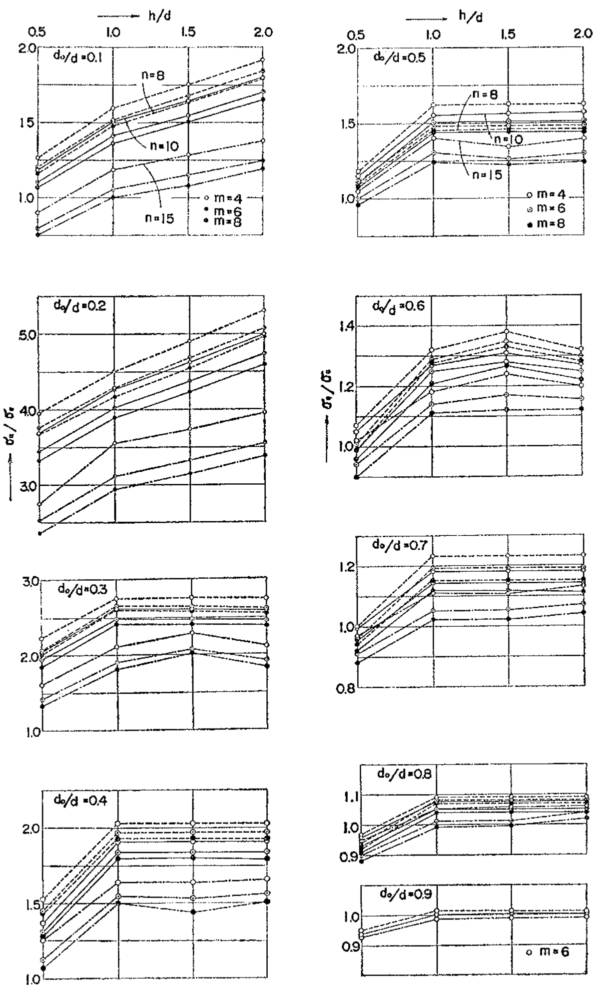


**Fig. 6 Theoretical relations between $\sigma_u/\sigma_c$ and the values of h/h, $d_0/d$, m and n.**

and a constant coefficient which would be available under very different conditions at bearing pressure test.

At the end, the values of $\sigma_u/\sigma_c$ theoretically calculated from Eq. (6-a) under various conditions are shown in Fig. 6. When the ratio $(h/d)$ is less than 1.0 or the ratio $(d_0/d)$ is more than 0.5 effects of friction should not be ignored in actual test. This is the subject for a future study.

## 5 Conclusions

The effects of the various factors on bearing strength $(\sigma_u)$, such as the dimension ratio of the specimen $(h/d)$ , the loading ratio $(d_0/d)$, the Poisson's number$(m)$ and the strength ratio $(n)$ have been made clear theoretically. It has also clarified from the stress severity diagrams that the failure conditions of the specimen vary from tensile failure to entire compressive failure according as the conditions provided by the above mentioned factors.

Using the coefficient $(k_c')$ estimating the effects of the same factors, the theoretical formula of bearing strength has been shown in Eq. (6-a). These results may be useful in discussion of the experimental results and formulas of bearing strength quantitatively as well as qualitatively. It should be noted that the formulas so far have not merely been sufficient because of the disregard for the relationship between $(\sigma_u)$ and $(\sigma_t)$, but also overlooking the effect of Poisson's number $(m)$ on $(\sigma_u)$.

### References

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