A Theory of Circulation Control by Tangential Blowing, Applied to a Circular Cylinder in Uniform Shear Flow

by

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The flow around the two-dimensional circular cylinder with tangential blowing immersed in the uniform shear flow was examined analytically by aid of the perturbation method and the inviscid potential theory. The effect of introduction of shear in the upstream flow was compared with the experimental result. The theoretical curves representative of the pressure distribution on the circular cylinder was enough to reproduce the experimental result. The correlation between upstream shear, characterized by a nondimensional shear parameter K, and the pressure distribution was shown satisfactory by even the simplified theory.

1. Introduction

There has been considerable study on the effect of boundary control for lift generation on various configuration of bodies such as an aerofoil or a circular cylinder. Various studies have considered boundary-layer control by blowing air through narrow tangential slot on the circular cylinder. Two-dimensional aerodynamic characteristics on the circular cylinder with tangential blowing have been illuminated by detail measurements of authors,[(1)~(3)] but these studies have been concerned with cylinders immersed in uniform flow in the wind tunnel. The theoretical expression for the flow field has been proposed by J. Dumham[(4)] that distribution of normal pressure on the surface of circular cylinders with tangential blowing can be discribed by inviscid, incompressible, unseparated, potential flow theory. In various practical applications, however, the existence of the velocity gradient in the upstream complicates the aerodynamic characteristics on the circular cylinder with tangential blowing. Therefore knowledge of the effect of shear on the distribution of normal pressure on the cylinder is desirable for the practical application.

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The purpose of the present study is to further illuminate theoretically the effect of introduction of shear in the upstream flow on aerodynamic characteristics for circular cylinder. Specifically, the primary goal is to calculate the pressure distribution included the effect of shear flow and to see how well the theory reproduces the experimental result.

2. Nomenclature

C_l	:	lift coefficient
C_p	:	pressure coefficient
C_{pb}	:	base pressure coefficient
C_{pm}	:	minimum pressure coefficient
C_q	:	coefficient of source $(=q/RU_c)$
C_Γ	:	coefficient of circulation $(=\Gamma/RU_c)$
$C\mu$	:	momentum of the blowing air per unit span$/(\rho U_c^2 R)$
K	:	shear parameter of the uniform shear flow $(=2R/U_c \times \partial U/\partial Z)$
R	:	radius of the model cylinder
R_e	:	Reynolds number $(=2U_cR/\nu)$
U	:	velocity of the uniform shear flow
U_c	:	velocity at the centre (see **Fig. 1**)
$r,\ \theta$	:	polar coordinates (see **Fig. 1**)
α	:	angular location at source measured unclockwise from the rear stagnation point which would be predicted with potential theory in uniform flow (see **Fig. 1**)
Γ	:	strength of circulation
θ_{st}	:	angular location of the forward stagnation point
Ψ	:	stream function in the whole field
Ψ_o	:	stream function in the uniform flow field
Ψ_1	:	stream function in the pertubed shear flow field

3. The Flow Model

A circular cylinder with its axis normal to a uniform incident flow has symmetrical separation points and the pressure distribution on upper and lower surfaces. When the circular cylinder is submerged in uniform shear flow, small lift due to unsymmetrical pressure distribution on the cylinder is produced for a function of the upstream velocity gradient. While a method for delaying separation by jets through the slots is used to increase the momentum of the retarded fluid in the boundary layer on the surface. When the slot on the upper surface only is in use, the flow follows that surface and increases the lift coefficient acting on the cylinder.

To reproduce the flow field theoretically, including such separated flow or wake around the cylinder, is generally difficult. Dumham however has proposed that the distribution of normal pressure on the surface of the circular cylinder with tangential blowing could be discribed by inviscid, incompressible, unseparated, potential flow theory. Since this theory however has been developed for the cylinder submerged in uniform flow without velocity gradient, the

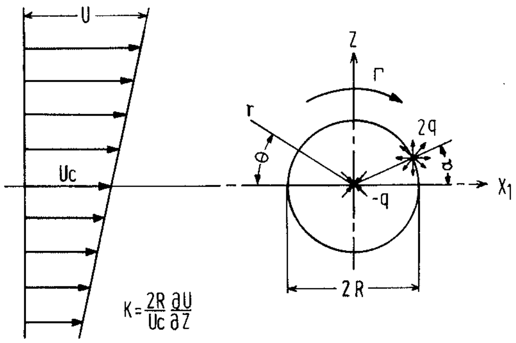

Fig. 1 The flow model and coordinate system

present study will be enlarged to include the flow around circular cylinder submerged in uniform shear flow.

Figure 1 shows the model considered and the coordinate system. The flow around the circular cylinder is generated by adding a circulation at the centre of the circle, a doublet, a sink, and a source to the upstream shear flow. The circulation Γ at the centre of the circle represents the circulating flow which in actuality in generated by blowing air through a narrow tangential slot on the surface of the circular cylinder. It will be seen that the separated flow or wake region is so wide that the theory gives a very poor approximation to the pressure distribution. The source $2q$ is therefore added to represent the displacement effect of the wake. And also the sink q of half the strength of the source is placed at the centre of the circle to ensure that no streamline crosses the surface.

4. Theory

The perturbation method is used to represent the field of the uniform shear flow far upstream. Velocity distribution of the uniform shear flow is represented in according with the principle of superposition of a constant velocity U_c and a slight pertubed flow with lineary varied small velocity $(U_c/2R)\,Kz$ as in the following equation :

$$U = U_c\left(1+\frac{K}{2R}z\right) = U_c\left(1+\frac{K}{2R}r\sin\theta\right) \qquad (1)$$

It is convenient to work with the stream function to solve the problem. Therefore its stream function is

$$\begin{aligned}\Psi_\infty &= U_c\left(z+\frac{K}{4R}z^2\right)\\ &= U_c\left\{r\sin\theta+\frac{Kr^2}{8R}(1-\cos 2\theta)\right\} \qquad (2)\end{aligned}$$

with small constant vorticity $\omega_\infty = KU_c/2R$.
In the whole flow field the stream function should be satisfies the differential equation

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2} = \frac{U_c K}{2R} \quad \cdots\cdots(3)$$

which expresses the physical fact that vorticity is constant along streamlines in absence of viscosity. The boundary conditions are

$$\left.\begin{array}{lll} \text{upstream :} & \Psi = U_c\left\{ r\sin\theta + \frac{Kr^2}{8R}(1-\cos 2\theta)\right\} & \text{as } r\to\infty \\ \text{surface :} & \Psi = 0 & \text{as } r = R \end{array}\right\} \quad \cdots\cdots(4)$$

If the dimensionless shear parameter K is small, it seems likely that the flow will depart only slightly from the solution for irrotational motion. On that assumption the stream function Ψ is tentatively stated

$$\Psi(r, \theta : K) = \Psi_0(r, \theta) + \frac{K}{2}\Psi_1(r, \theta) \quad \cdots\cdots(5)$$

where Ψ_o is the basic solution which indicate the stream function around the circular cylinder with tangential blowing immersed in the uniform flow without vorticity. For an application with the inviscid and irrotational motion, far upstream the vorticity vanishes, leaving Laplace's equation $\nabla^2 \Psi_0 = 0$ so that its solution is [4]

$$\Psi_0 = U_c\left(r - \frac{R^2}{r}\right)\sin\theta + \frac{\Gamma}{2\pi}\log\frac{r}{R} + \frac{q}{2\pi}\left\{2\cos^{-1}\frac{r - R\cos(\pi-\alpha-\theta)}{\sqrt{\{r^2+R^2-2rR\cos(\pi-\alpha-\theta)}} - \alpha - \theta\right\} \quad \cdots(6)$$

Substituting into the full equation (3) the relation (5) and (6) and equalling terms include the shear parameter K yields for the first-order perturbation Ψ_1, it should be satisfied that

$$\frac{\partial^2 \Psi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi_1}{\partial \theta^2} = \frac{U_c}{R} \quad \cdots\cdots(7)$$

The boundary conditions are

$$\left.\begin{array}{lll} \text{upstream:} & \Psi_1 = \frac{U_c r^2}{4R}(1-\cos 2\theta) & \text{as } r\to\infty \\ \text{surface :} & \Psi_1 = 0 & \text{as } r = R \end{array}\right\} \quad \cdots\cdots(8)$$

This is as readily solved by separation of variables, so that its stream function is

$$\Psi_1 = \frac{U_c r^2}{4R}(1-\cos 2\theta) + \frac{U_c R^3}{4r^2}\cos 2\theta - \frac{U_c R}{4} \quad \cdots\cdots(9)$$

Consequently, collecting result of equations (5), (6) and (9) gives the complete solution.

Because the circumferential velocity on the surface of the circular cylinder is $(V_\theta)_{r=R} = \left(\frac{\partial \Psi}{\partial r}\right)_{r=R}$, the coefficient of normal pressure at any angular location θ is given by

$$C_p = 1-\{(V_\theta)_{r=R}/U_c\}^2 = 1-\left\{2\sin\theta+\frac{K}{4}(1-2\cos 2\theta)+\frac{C_\Gamma}{2\pi}-\frac{C_q}{2\pi}\cot\left(\frac{\pi-\alpha-\theta}{2}\right)\right\}^2 \quad \cdots\cdots(10)$$

where $C_\Gamma = \Gamma/RU_c$ and $C_q = q/RU_c$. The results calculated numerically for the pressure coefficient C_p is plotted against θ in **Fig. 2** which also give the variation of curves against the parameters K, C_Γ and C_q.

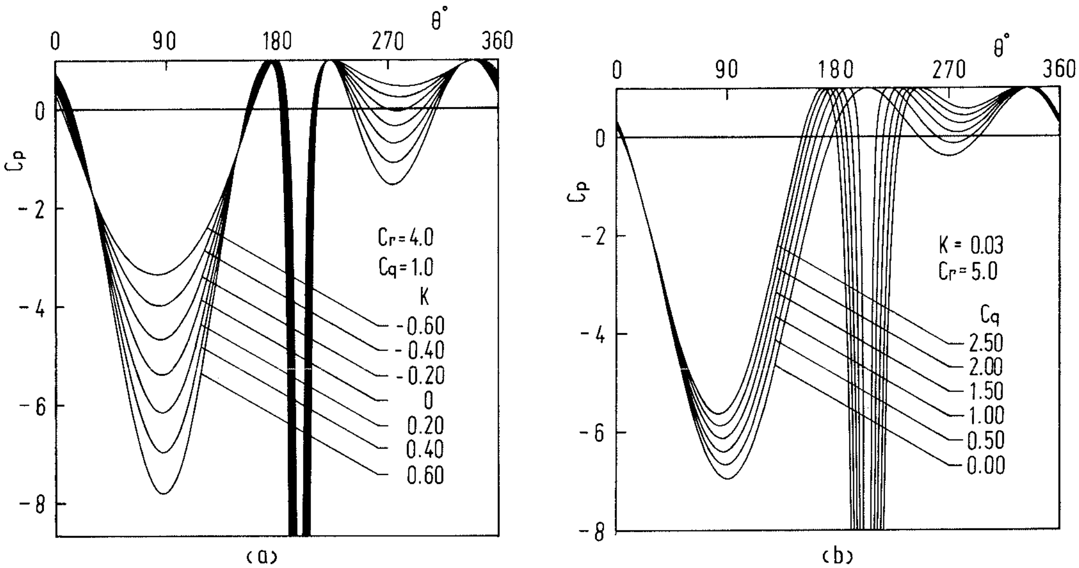

Fig. 2 Theoretical curves representative of the variation of pressure distribution against the parameters K, C_Γ, and C_q: (a) $C_\Gamma=4.0$, $C_q=1.0$, and K is a variable, (b) $K=0.03$, $C_\Gamma=5.0$, C_q is a variable

However, algebraical expressions for the characteristic values such as an angle of forward stagnation point θ_{st} or an angle at the minimum pressure coefficient θ_m can not obtain easily from the above formula for C_p. Then in the special case for no source at rear portion of the circle, *i. e.* $C_q=0$, it is possible to make fomulations for θ_{st} and θ_m using a condition of $\partial C_p/\partial\theta = 0$.

$$\left.\begin{aligned} \theta_{st} &= \sin^{-1}[\{\sqrt{1-(C_\Gamma/2\pi)K+(1/4)K^2}-1\}/K] \\ \theta_m &= \pi/2 \end{aligned}\right\} \quad \cdots\cdots(11)$$

Substituting the values of $\theta_m=\pi/2$ and $C_q=0$ into equation (10), the coefficient of the minimum pressure is

$$C_{pm} = 1-\{2+(3/4)K+C_\Gamma/2\pi\}^2 \quad \cdots\cdots(12)$$

The lift coefficient due to normal pressure is obtained by integration which is taken right round the counter of the cylinder.

$$C_l=-\frac{1}{2}\int_0^{2\pi} C_p \sin\theta \, d\theta = C_\Gamma + \pi K \qquad (13)$$

For purposes of convenience, in the comparison between the theoretical evaluations and the measured results, the lift coefficient C_l is used instead of the circulation coefficient C_Γ. Substituting the relation (13) into equation (11), the angular location of the forward stagnation point is

$$\theta_{st}=\sin^{-1}\frac{\sqrt{1-(C_l/2\pi)K+(3/4)K^2}-1}{K} \qquad (14)$$

It have been point out in the uniform flow ($K=0$) that as the lift increases the wake contracts and the need for a source diminishes ; above a lift coefficient C_l of about five, a simple potential flow as such a special case $C_q=0$ is good enough.[4]

5. Comparison with Experiments

Although numerous studies have been made of pressure distributions around circular cylinders, most studies have been not concerned with cylinders immersed in the uniform shear flow. There is not much available information on the effect of shear on distribution of normal pressure on the cylinder, especially in the study of the cylinder with tangential blowing. Therefore the theory is compared with measured results for two distinct cases ; (1) a simple circular cylinder without slot immersed in the uniform shear flow, (2) a circular cylinder with the tangential blowing immersed in the uniform shear flow, to see how well the theory reproduces the experimental results.

A theoretical curve which gives the variation of the pressure coefficient for no source and no circulation ($C_q=0$, $C_\Gamma=0$) is shown against $|\theta|$ in **Fig. 3** which also contains plots of the measured pressure distribution according to Adachi and Kato[5] for a simple circular cylinder at 8×10^3 of the Reynolds number. It will be seen that there is fair agreement with the experimental result in the neighbourhood of the forward stagnation point, but the results differ enormously around the rear of the cylinder.

In the case of the tangential blowing, a typical example for the comparison between theoretical curves and experimental results is shown in **Fig. 4**. In these experiments, a measured condition for two given shear parameters $K=+0.045$ and -0.045, is $C\mu=0.30$ and $\theta_j=100°$ at the Reynolds number $R_e=6.0\times10^4$. Although the Reynolds number also has an important influence on the flow, at the near region of this Reynolds number the laminar separation is confirmed at about $\theta=\pm80°$ on both surfaces of an unblown cylinder. Operating the tangential blowing on the upper surface only, the laminar separation is followed quickly by re-attachment as a turbulent boundary layer which only separates much later, i. e. it is shown in **Fig. 4** that the retarted separation reach at about 175° near the rear stagnation

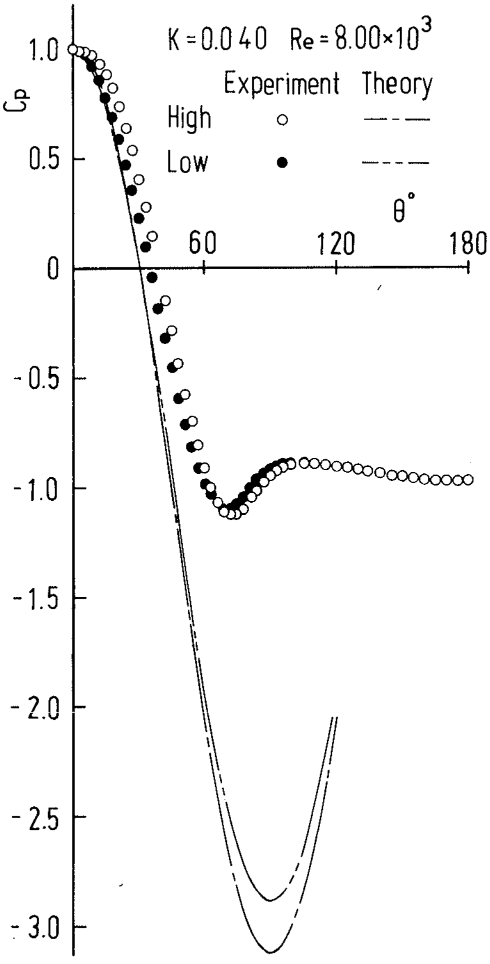

Fig. 3 Comparison between simplified theory and experiment

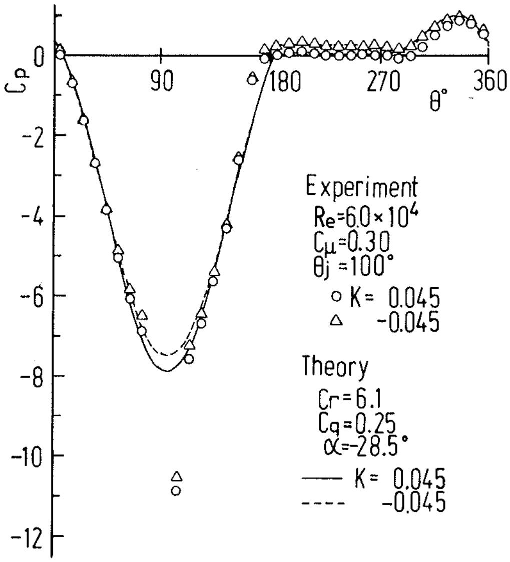

Fig. 4 A typical example for the comparison between theory and experiment

point. The shear parameter bring to effect on the region of minimum pressure and back pressure. Selecting the theoretical parameter C_r, C_q and α so as to obtain good agreement between the theoretical curve and the experimental curve under arbitrary blowing condition ($C\mu$, θ_j), the experimental result can be reproduced by the theory.

Any unseparated-flow theory predicts a rear stagnation point which is a singularity in this theory, whereas in practice the boundary layers separate from both upper and lower surfaces, leaving a region of constant pressuer around the back. Therefore the calculated distribution should be truncated at the observed base pressure, ignoring the theoretical singularity and its immediate neighbourhood. The calculated distributions for given parameters $K=\pm 0.045$, $C_r=6.1$, $C_q=0.25$ and $\alpha=-28.5°$ are also shown in **Fig. 4**, and it agrees fairly well with experimental plots except for the region of the back pressure, and also reproduces the effect of the difference of the shear parameter.

Because an angular distance of the stagnation point from the horizontal axis is a characteristics value which indicate an effect of the difference of the shear parameter K only, the relation of the forward stagnation point θ_{st} as a function of the lift coefficient C_l is illustrated in **Fig.5**, and it shows the good agreement between theory and experiment, especially in the range of small lift coefficient. However, in the theory the graphs of θ_{st} against C_l are calculated from equation (14), whereas in the experimental results the resultant lift coefficient due to measured normal pressure distribution is obtained by graphical evaluation of the integral.

Because the minimum pressure coefficient C_{pm} is also a characteristic value which is under the influence of the shear parameter, the relation of C_{pm} against the lift coefficient is shown in **Fig. 6**. Solid curves are calculated from the equation (12) in which the relations of $C_\Gamma = C_l - \pi K$ and $K = \pm 0.045$ are substituted. The comparison with experiment show the excellent agreement in the range of larger lift coefficient than about five. This excellent agreement means that as the lift increases the wake contracts and the need for a source dimenishes ; simplified potential theory ($C_q = 0$) is good enough to indicate the effect of the shear.

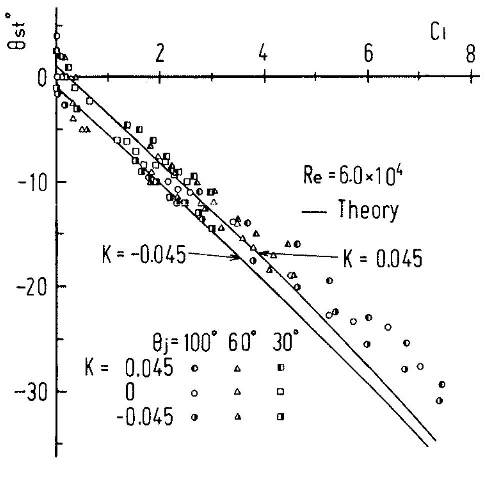

Fig. 5 The relation of the forward stagnation point against the lift coefficient

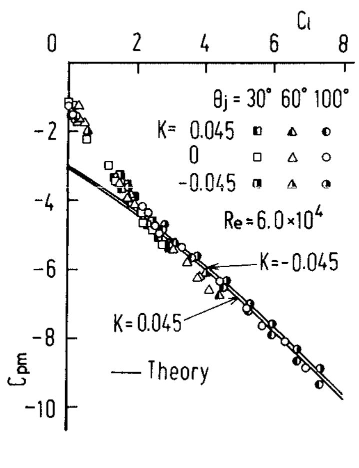

Fig. 6 The relation of the minimum pressure coefficient against the lift coefficient

6. Conclusions

The flow around the two-dimensional circular cylinder with tangential blowing immersed in the uniform shear flow was examined analytically by aid of the perturbation method and the inviscid, incompressible, unseparated, potential theory. The theoretical curves representative of the distribution of the pressure coefficient on the surface of the circular cylinder or the aerodynamic characteristics such as the angular distance of the forward stagnation point or the minimum pressure coefficient were compared with the experimental results, to see how well the theory reproduces the experimental results, especially in the introduction of the shear in the upstream flow. And we obtained the following conclusions.

(1) The theoretical curves representative of the distribution of the pressure coefficient are enough to reproduce the experimental results on the cylinder immersed in the uniform shear flow with the shear parameter K.

(2) Even the simplified theory ($C_q=0$) is also enough to show the effect of the shear parameter on the angular distance of the forward stagnation point and the minimum pressure coefficient.

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