

# A Grounded Inductance Using an Operational Amplifier Pole and Its Application to Filters

by

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This paper proposes a new simulation circuit of a grounded inductance using an operational amplifier pole. The value of simulated inductance can be varied by a resistance ratio and a gain bandwidth product of the operational amplifier. The proposed structure works well over a wide frequency range and has low sensitivity to active and passive elements. Theoretical and experimental results indicate that the realized circuit is stable over the wide frequency range and can be used effectively for a low-Q active network. The practical examples in a highpass ladder filters are also given together with experimental verifications.

## 1. Introduction

Recently, operational amplifiers (op amps) are widely used as inductorless filters, oscillators, inductor and capacitor simulations in addition to use as active elements. However, a practical op amp does not generally ideal because of characteristics of frequency-dependent gain and finite input and output impedances. As the result, the use of op amp is limited by their finite gain bandwidth product. Therefore, many authors have exploited op amp pole in designing active R filter,<sup>1)-3)</sup> inductance and capacitance simulations<sup>4)-6)</sup> etc. In the previous paper,<sup>7)</sup> we also proposed a new integrator using this technique. The utilization of the op amp pole gives the reliable high frequency performance, and eliminates the external capacitances from the circuit. Therefore, it is suitable for monolithic IC implementation.

In this paper, we proposed a new grounded inductance simulation circuit which is composed of two op amps and only resistances without requiring any external capacitances. Since the value of simulated inductance depends primarily on resistance ratios, the proposed circuit is suitable for IC implementation. From the sensitivity analysis, we proved that the circuit has low sensitivities to all the network parameters. It is shown that the configuration can be used

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for active network realizing low-Q and the experimental results agree well with the analytic results.

The network analysis based on the single-pole rolloff model of op amp, and stability analysis are also given. Then, the highpass filters are designed using the proposed circuit. The experimental results show that these filters operate successfully as the conventional ones.

### 2. Theoretical analysis

The new circuit for inductance simulation is shown in Fig. 1. Assuming the single-pole rolloff characteristic for the two op amps, the gain is

$$A_i(s) = -\frac{A_{oi} \omega_{pi}}{s + \omega_{pi}} = -\frac{GB_i}{s + \omega_{pi}}, \quad i = 1, 2 \quad \dots\dots\dots (1)$$

where  $A_{oi}$ ,  $\omega_{pi}$  and  $GB_i$  are the open loop gain, the 3 dB frequencies and the gain bandwidth product of op amps, respectively, and subscripts  $i$  are related to op amp 1 and op amp 2. Assuming a typical case  $|s| \gg \omega_{pi}$ , the open loop gain  $A_i(s)$  is given by

$$A_i(s) = -\frac{GB_i}{s} \quad \dots\dots\dots (2)$$

In Fig. 1, the input impedance  $Z_{in}(s)$  at the input terminals 1-1' is obtained as

$$Z_{in}(s) = \frac{A^2 a_1 R_1 R' - A(a_2 R_3 + a_5 R_1 R_f)}{A^2(a_1 a_3 + R_2 R_3 R_f) - A[a_3(a_4 R_3 + a_5 R_f)]} * \frac{+ a_2 a_5}{R_2 R_f(a_5 + R_3) + a_5 R'[(1 + R_1)a_4 + R_2 R_f]} \quad \dots\dots\dots (3)$$

where

$$\left. \begin{aligned} a_1 &= R_3 R_f - R_2 R_4 \\ a_2 &= R_1 R_2 + R_2 R_f + R_1 R_f \\ a_3 &= R_1 + R' \\ a_4 &= R_2 + R_f \\ a_5 &= R_3 + R_4 \end{aligned} \right\} \quad \dots\dots\dots (4)$$

and it is assumed that  $A_1(s) = A_2(s) = A(s)$ . Substituting Eq. (2) to Eq. (3), we have

$$Z_{in}(s) = \frac{s^2 R' (U_1 R_1 + U_2) + s GB R' U_3}{s^2[(R' + R_1) U_1 + U_2] + s GB(U_3 + U_1 R' U_2)} * \frac{+ GB^2(R_3 R_f - R_2 R_4) R_1 R'}{+ GB^2[(R_3 R_f - R_2 R_4)(R_1 + R') + R_2 R_3 R_f]} \quad \dots\dots\dots (5)$$

where

$$\left. \begin{aligned} U_1 &= R_3 R_f + R_2 R_3 + R_4 R_f + R_2 R_4 \\ U_2 &= (R_3 + R_4) R_2 R_f \\ U_3 &= 2R_1 R_3 R_f + R_2 R_3 R_f + R_1 R_2 R_3 + R_1 R_4 R_f \end{aligned} \right\} \quad \dots\dots\dots (6)$$

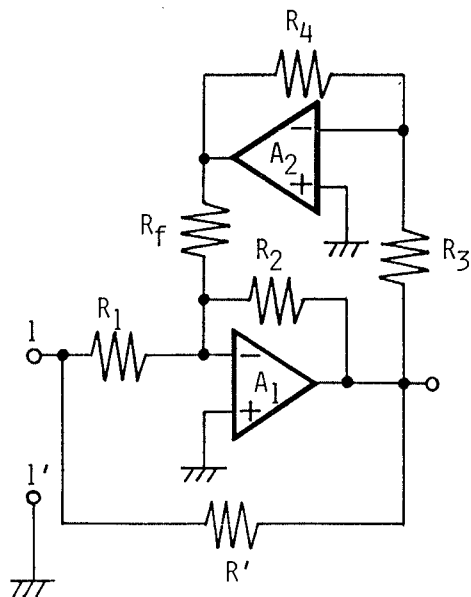


Fig. 1 Circuit for inductance simulation

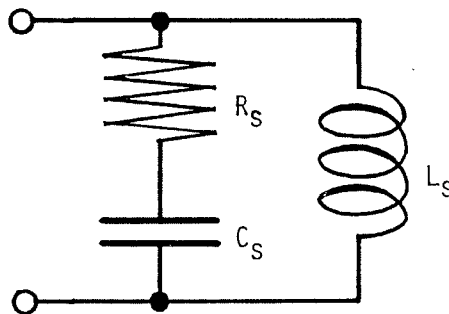


Fig. 2 Equivalent circuit of Fig. 1

The circuit is adjusted so that  $(R_2R_4/R_3R_f) = 1$  by the suitable choice of resistances  $R_2$ ,  $R_3$ ,  $R_4$  and  $R_f$ ,  $Z_{in}(s)$  reduces to

$$Z_{in}(s) = \frac{s^2 R' (U_1 R_1 + U_2)}{s^2 [(R' + R_1) U_1 + U_2] + s GB (U_3 + U_1 R' + R_2)} * \frac{+s GB R' U_3}{+GB^2 R_2 R_3 R_f} \dots\dots\dots(7)$$

In Eq. (7), it is seen that the circuit of Fig. 1 behaves as an equivalent inductance  $L_s$  and a resistance  $R_s$  in series with a capacitance  $C_s$  as shown in Fig. 2, where the simulated components are

$$L_s = \frac{R' U_3}{GB R_2 R_3 R_f} \dots\dots\dots(8)$$

$$R_s = \frac{(R' + R_1) U_1 + U_2}{R' U_3 GB} \dots\dots\dots(9)$$

$$C_s = \frac{U_3 + U_1 R' + R_2}{GB R_2 R_3 R_f} \dots\dots\dots(10)$$

The quality factor  $Q(\omega)$  of a simulated circuit is given by

$$Q(\omega) = \frac{\omega^2 GB (T_1 T_4 - T_2 T_3) + GB^3 T_2 T_5}{\omega^3 T_1 T_3 + \omega GB^2 (T_2 T_4 - T_1 T_5)} \dots\dots\dots(11)$$

where

Table 1 Calculated values of simulated components and Q-factor

Type	$K_{34}$	$K_{f2}$	$R_1, R_2$ ( $\Omega$ )	$R_3$ ( $\Omega$ )	$R_4$ ( $\Omega$ )	$R_f$ ( $\Omega$ )	Q (1 KHz)	$L_s$ (H)	$R_s$ ( $\Omega$ )	$C_s$ (pF)
1	.0001	10000	1 M	1 M	100.	100	.148	10.6	$2.12 \cdot 10^{-3}$	10.7
2	.001	1000	100 K	1 M	1 K	100	1.359	1.06	$2.12 \cdot 10^{-4}$	11.6
3			1 M	100 K	100	1 K	1.480	1.06	$2.12 \cdot 10^{-4}$	10.7
4	.01	100	10 K	1 M	10 K	100	7.316	$1.09 \cdot 10^{-1}$	$2.18 \cdot 10^{-5}$	21.1
5			100 K	100 K	1 K	1 K	13.248	$1.09 \cdot 10^{-1}$	$2.18 \cdot 10^{-5}$	11.7
6			1 M	10 K	100	10 K	14.418	$1.09 \cdot 10^{-1}$	$2.18 \cdot 10^{-5}$	10.7
7	.1	10	10 K	100 K	10 K	1 K	59.306	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	20.5
8			100 K	10 K	1 K	10 K	104.150	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	11.7
9			1 K	1 M	100 K	100	111.780	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	108.
10			1 M	1 K	100	100 K	112.670	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	10.7
11	1.0	1.0	10 K	10 K	10 K	10 K	153.060	$5.31 \cdot 10^{-3}$	$9.55 \cdot 10^{-7}$	21.2
12			100 K	1 K	1 K	100 K	241.940	$5.31 \cdot 10^{-3}$	$9.55 \cdot 10^{-7}$	13.6
13			1 K	100 K	100 K	1 K	32.751	$5.31 \cdot 10^{-3}$	$9.55 \cdot 10^{-7}$	97.6
14			1 M	100	100	1 M	256.650	$5.31 \cdot 10^{-3}$	$9.55 \cdot 10^{-7}$	12.8
15			100	1 M	1 M	100	3.696	$5.31 \cdot 10^{-3}$	$9.55 \cdot 10^{-7}$	862.
16	10.0	.1	10 K	1 K	10 K	100 K	43.558	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	28.5
17			1 K	10 K	100 K	10 K	10.465	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	116.
18			100 K	100	1 K	1 M	63.703	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	19.7
19			100	100 K	1 M	1 K	1.217	$1.39 \cdot 10^{-2}$	$2.67 \cdot 10^{-6}$	999.
20	100	.01	10 K	100	10 K	1 M	4.933	$1.09 \cdot 10^{-1}$	$2.18 \cdot 10^{-5}$	31.4
21			1 K	1 K	100 K	100 K	1.227	$1.09 \cdot 10^{-1}$	$2.18 \cdot 10^{-5}$	125.
22			100	10 K	1 M	10 K	.144	$1.09 \cdot 10^{-1}$	$2.18 \cdot 10^{-5}$	1071.
23	1000	.001	1 K	100	100 K	1 M	.124	1.06	$2.13 \cdot 10^{-4}$	127.
24			100	1 K	1 M	100 K	.014	1.06	$2.13 \cdot 10^{-4}$	1081.
25	10000	.0001	100	100	1 M	1 M	.0014	10.6	$2.12 \cdot 10^{-3}$	1082.

$$\left. \begin{aligned}
 T_1 &= (U_1 R_1 + U_2) R' \\
 T_2 &= U_3 R' \\
 T_3 &= (R' + R_1) U_1 + U_2 \\
 T_4 &= U_2 + U_3 + (U_1 + R_3 R_f - R_2 R_4) R' \\
 T_5 &= R_2 R_3 R_f
 \end{aligned} \right\} \dots\dots\dots (12)$$

Thus we note that in order to obtain a fixed value of inductance it is necessary to keep the condition  $R_3 R_f = R_2 R_4 (= K_{34} K_{f2} = 1)$ . Table 1 shows the calculated values of  $L_s$ ,  $R_s$ ,  $C_s$  and  $Q$  for the various cases. The quality factor  $Q(\omega)$  indicates a large value in the region of resistance ratio  $K_{12} = K_{34} = K_{f2} = 1$ .

In order to obtain a pure inductance, the following conditions must be satisfied.

$$\left. \begin{aligned} |sR'(U_1 R_1 + U_2)| &\ll GB U_3 \\ GB^2 R_2 R_3 R_f &\gg |s^2[(R' + R_1) U_1 + U_2]| \\ GB R_2 R_3 R_f &\gg |s(U_3 + U_1 R' + R_2)| \end{aligned} \right\} \dots\dots\dots (13)$$

Then we can be rewritten Eq. (7) in a simple form

$$Z_{in}(s) = \frac{2 + K_{12} + K_{34} + K_{f2}}{K_{12}} \cdot \frac{R'}{GB} s \dots\dots\dots (14)$$

where  $K_{12} = R_2/R_1$ ,  $K_{34} = R_4/R_3$  and  $K_{f2} = R_2/R_f$ . It is clear that Eq. (14) has the property of an ideal inductance, and the inductance value can be varied by changing the resistance ratios  $K_{12}$ ,  $K_{34}$  and  $K_{f2}$ , the gain bandwidth product  $GB$  of op amp and element value  $R'$ . The value of the ideal inductance  $L_s$  is expressed as

$$L_s = \frac{2 + K_{12} + K_{34} + K_{f2}}{K_{12}} \cdot \frac{R'}{GB} \dots\dots\dots (15)$$

It should be noted that exact value of resistance is immaterial but their ratio is important. Calculated values of  $L_s$  are shown in Table 1, where the values of  $R_s$  and  $C_s$  become zero under the conditions Eq. (13).

**3. Sensitivity function and stability**

The sensitivity is a measure of the displacement in some performance characteristics of a network resulting from a displacement in value of one or more the element of the network. In this chapter, the sensitivities of the magnitude of the driving point impedance  $Z_{in}(s)$ , the coefficient for the numerator and denominator polynomial of the  $Z_{in}(s)$  and the quality factor  $Q(\omega)$  with respect to displacement in  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$ ,  $R_f$ ,  $R'$  and  $GB$  are derived.

The sensitivity of a network function  $N(\omega)$  with respect to the element  $x$  is defined by<sup>8)</sup>

$$S_x^{N(\omega)} = \frac{dN(\omega)}{N(\omega)} \cdot \frac{x}{dx} \dots\dots\dots (16)$$

We can rewrite Eq. (7) as shown below :

$$Z_{in}(s) = \frac{T_1 s^2 + T_2 GBs}{T_3 s^2 + T_4 GBs + T_5 GB^2} \dots\dots\dots (17)$$

where  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$  and  $T_5$  are the same as Eq. (12). The sensitivity function of the driving point impedance  $Z_{in}(s)$  with respect to a element  $x$  are given by

$$\text{Re } S_x^{Z_{in}(s)} = \frac{R(X_0 - X_1) + I(Y_0 - Y_1)}{R^2 + I^2} \cdot x \dots\dots\dots (18)$$

$$\text{Im } S_x^{Z_{in}(s)} = \frac{R(Y_0 - Y_1) - I(X_0 - X_1)}{R^2 + I^2} \cdot x \dots\dots\dots (19)$$

where  $\text{Re } S_x^{Z_{in}(s)}$  and  $\text{Im } S_x^{Z_{in}(s)}$  stand for the real and imaginary part of  $S_x^{Z_{in}(s)}$ , respectively, and

$$\left. \begin{aligned}
 X_0 &= A_0 T_3 \omega^4 - GB^2(A_0 T_5 + B_0 T_4) \omega^2 \\
 Y_0 &= GB[B_0 T_5 GB^2 - (A_0 T_4 + B_0 T_3) \omega^2] \omega \\
 X_1 &= A_0 T_3 \omega^4 - GB^2(T_1 G_0 + T_2 F_0) \omega^2 \\
 Y_1 &= GB[T_2 G_0 GB^2 - (T_1 F_0 + T_2 E_0) \omega^2] \omega \\
 R &= T_1 T_3 \omega^4 - GB^2(T_1 T_5 + T_2 T_4) \omega^2 \\
 I &= GB[T_2 T_5 GB^2 - (T_1 T_4 + T_2 T_3) \omega^2] \omega
 \end{aligned} \right\} \dots\dots\dots (20)$$

In the sensitivity functions for each element, we substitute each of element for  $x$ , and the equations as shown below insert in Eq. (20). For example, we deduce the sensitivity function  $Re S_x^{Z_{in}(s)}$  for  $R_1$ . Substituting Eq. (21) into Eq. (20), and further inserting the resulting Eq. (20) in Eq. (18), then we have the  $Re S_x^{Z_{in}(s)}$

$R_1$ ;

$$\left. \begin{aligned}
 A_0 &= R'(R_3 + R_4)(R_2 + R_f) \\
 B_0 &= R'[R_f(2R_3 + R_4) + R_2 R_3] \\
 E_0 &= (R_2 + R_f)(R_3 + R_4) \\
 F_0 &= R_f(R_3 + R_4) + R_3(R_2 + R_f) \\
 G_0 &= 0
 \end{aligned} \right\} \dots\dots\dots (21)$$

$R_2$ ;

$$\left. \begin{aligned}
 A_0 &= R'(R_3 + R_4)(R_1 + R_f) \\
 B_0 &= R' R_3(R_1 + R_f) \\
 E_0 &= (R_3 + R_4)(R' + R_1 + R_f) \\
 F_0 &= R_f(R_3 + R_4) + R_3(R' + R_1 + R_f) \\
 G_0 &= 0
 \end{aligned} \right\} \dots\dots\dots (22)$$

$R_3$ ;

$$\left. \begin{aligned}
 A_0 &= R'(R_1 R_f + R_2 R_f + R_1 R_2) \\
 B_0 &= R'(2R_1 R_f + R_2 R_f + R_1 R_2) \\
 E_0 &= (R' + R_1)(R_2 + R_f) + R_2 R_f \\
 F_0 &= (R' + R_1)(R_2 + R_f) + R_2 R_f + R_f(R' + R_1 + R_2) \\
 G_0 &= R_2 R_f
 \end{aligned} \right\} \dots\dots\dots (23)$$

$R_4$ ;

$$\left. \begin{aligned} A_0 &= R'(R_1 R_f + R_2 R_f + R_1 R_2) \\ B_0 &= R' R_1 R_f \\ E_0 &= (R' + R_1)(R_2 + R_f) + R_2 R_f \\ F_0 &= R_f(R' + R_1 + R_2) \\ G_0 &= 0 \end{aligned} \right\} \dots\dots\dots (24)$$

$R_f$ ;

$$\left. \begin{aligned} A_0 &= R'(R_1 + R_2)(R_3 + R_4) \\ B_0 &= R'(2R_1 R_3 + R_1 R_4 + R_2 R_3) \\ E_0 &= (R_3 + R_4)(R' + R_1 + R_2) \\ F_0 &= (R' + R_1 + R_2)(2R_3 + R_4) \\ G_0 &= R_2 R_3 \end{aligned} \right\} \dots\dots\dots (25)$$

$R'$ ;

$$\left. \begin{aligned} A_0 &= (R_1 R_f + R_2 R_f + R_1 R_2)(R_3 + R_4) \\ B_0 &= R_1 R_f(2R_3 + R_4) + R_2 R_3(R_1 + R_f) \\ E_0 &= (R_f + R_2)(R_3 + R_4) \\ F_0 &= R_f(R_3 + R_4) + R_3(R_2 + R_f) \\ G_0 &= 0 \end{aligned} \right\} \dots\dots\dots (26)$$

In the same manner, the sensitivity function with respect to  $GB$  is as follows.

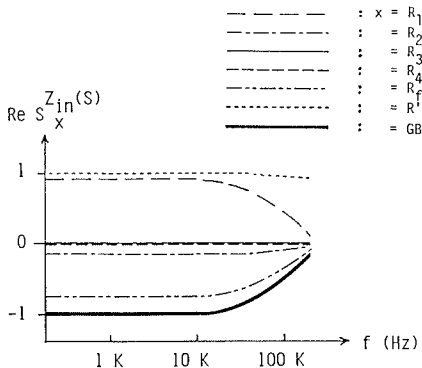


Fig. 3 Calculated values of  $Re S_x^{Z_{in}(s)}$  in type 8

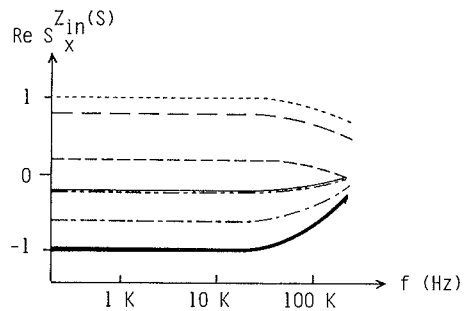


Fig. 4 Calculated values of  $Re S_x^{Z_{in}(s)}$  in type 11

$$\text{Re } S_{CB}^{Z_{in}(s)} = \frac{(2T_1 T_5 GB \omega^2)R - [(T_2 T_3 - T_1 T_4) \omega^3 + T_2 T_5 GB^2 \omega]I}{R^2 + I^2} \dots\dots\dots (27)$$

$$\text{Im } S_{CB}^{Z_{in}(s)} = \frac{[(T_2 T_3 - T_1 T_4) \omega^3 + T_2 T_5 GB^2 \omega]R + 2T_1 T_5 GB \omega^2 I}{R^2 + I^2} \dots\dots\dots (28)$$

When we take typical value listed up in Table 1, each calculated sensitivity factor are shown in Figs. 3, 4 and 5 to real part and Figs. 6, 7 and 8 to imaginary part, respectively.

Next, we deal with the coefficient sensitivity of the driving point impedance  $Z_{in}(s)$ . We let  $N(s)$  be network function as

$$N(s) = \frac{a_0 + a_1 s + a_2 s^2 + \dots\dots\dots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \dots\dots\dots + b_n s^n} \dots\dots\dots (29)$$

Then, we define the coefficient sensitivity as follows :

$$S_x^{a_i} = \frac{da_i}{a_i} \cdot \frac{x}{x} \dots\dots\dots (30)$$

$$S_x^{b_i} = \frac{db_i}{b_i} \cdot \frac{x}{x} \dots\dots\dots (31)$$

where  $x$  is any arbitrary network element. From Eqs. (17), (30) and (31), the coefficient sensitivity of  $Z_{in}(s)$  for each element can be determined as follows :

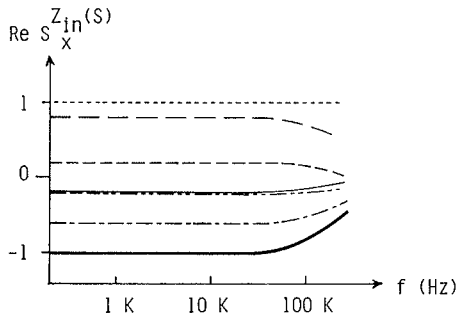


Fig. 5 Calculated values of  $\text{Re } S_x^{Z_{in}(s)}$  in type 14

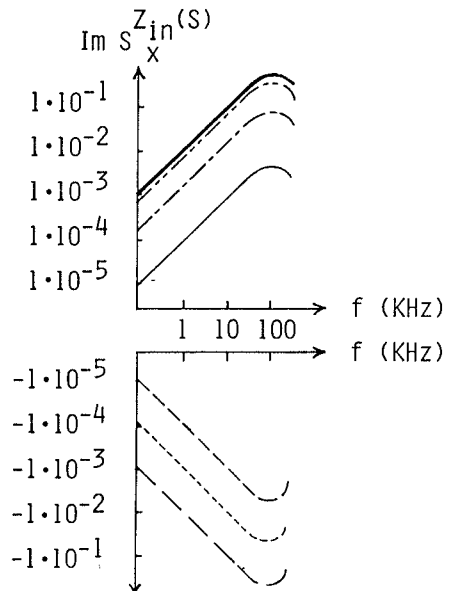


Fig. 6 Calculated values of  $\text{Im } S_x^{Z_{in}(s)}$  in type 8



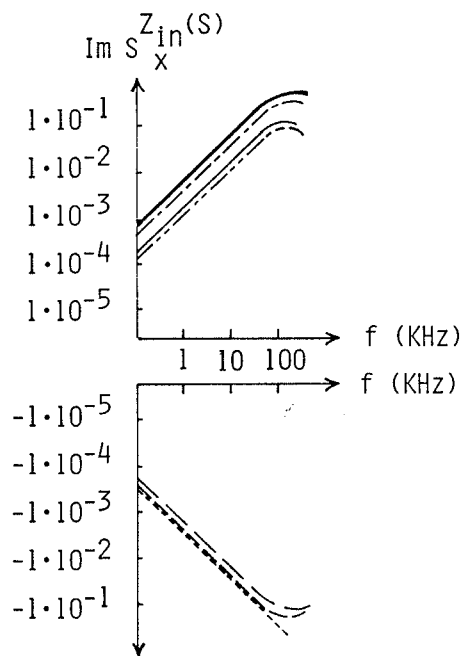


Fig. 7 Calculated values of  $\text{Im } S_X^{Z_{in}(s)}$  in type 11

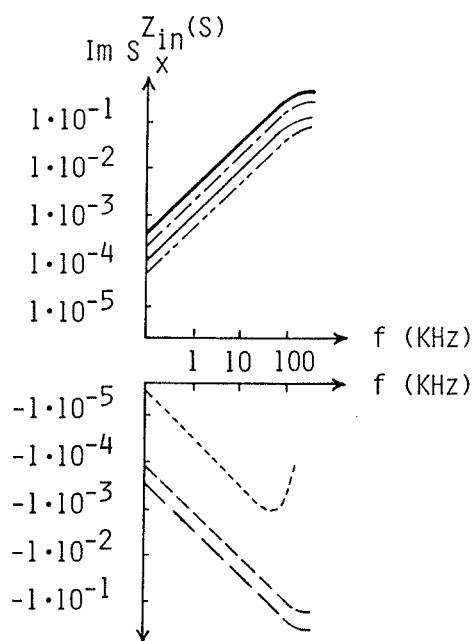


Fig. 8 Calculated values of  $\text{Im } S_X^{Z_{in}(s)}$  in type 14

$$\left. \begin{aligned}
 S_{R_1}^{T_1} &= (R_2 + R_f) R_1 / [(R_2 + R_f) R_1 + R_2 R_f] \\
 S_{R_2}^{T_1} &= (R_1 + R_f) R_2 / [(R_1 + R_f) R_2 + R_1 R_f] \\
 S_{R_3}^{T_1} &= R_3 / (R_3 + R_4) \\
 S_{R_4}^{T_1} &= R_4 / (R_3 + R_4) \\
 S_{R_f}^{T_1} &= (R_1 + R_2) R_f / [(R_1 + R_f) R_2 + R_1 R_2] \\
 S_{R'}^{T_1} &= 1 \quad , \quad S_{GB}^{T_1} = 0
 \end{aligned} \right\} \dots \dots \dots (32)$$

$$\left. \begin{aligned}
 S_{R_1}^{T_2^{GB}} &= 1 - R_2 R_3 R_f R' / T_2 \\
 S_{R_2}^{T_2^{GB}} &= 1 - R_1 R_f R' (2R_3 + R_4) / T_2 \\
 S_{R_3}^{T_2^{GB}} &= 1 - R_1 R_4 R_f R' / T_2 \\
 S_{R_4}^{T_2^{GB}} &= 1 - R_3 R' (2R_1 R_f + R_2 R_f + R_1 R_2) / T_2 \\
 S_{R_f}^{T_2^{GB}} &= 1 - R_1 R_2 R_3 R' / T_2 \\
 S_{R'}^{T_2^{GB}} &= 1 \quad , \quad S_{GB}^{T_2} = 1
 \end{aligned} \right\} \dots \dots \dots (33)$$

$$\left. \begin{aligned}
 S_{R_2}^{T_3} &= R_1(R_2 + R_f)/J_3 \\
 S_{R_2}^{T_3} &= R_2(R' + R_1 + R_f)/J_3 \\
 S_{R_3}^{T_3} &= R_3/(R_3 + R_4) \\
 S_{R_4}^{T_3} &= R_4/(R_3 + R_4) \\
 S_{R_f}^{T_3} &= R_f(R' + R_1 + R_2)/J_3 \\
 S_{R'}^{T_3} &= R'(R_2 + R_f)/J_3 \quad , \quad S_{GB}^{T_3} = 0
 \end{aligned} \right\} \dots\dots\dots (34)$$

$$\left. \begin{aligned}
 S_{R_1}^{T_4 \cdot GB} &= R_1(2R_3 R_f + R_2 R_3 + R_4 R_f)/J_4 \\
 S_{R_2}^{T_4 \cdot GB} &= R_2(2R_3 R_f + R_2 R_3 + R_4 R_f)/J_4 \\
 S_{R_3}^{T_4 \cdot GB} &= R_3(R_1 + R_f + R')(2R_2 + R_f)/J_4 \\
 S_{R_4}^{T_4 \cdot GB} &= R_4 R_f(R_1 + R_f + R')/J_4 \\
 S_{R_f}^{T_4 \cdot GB} &= R_f(R_1 + R_f + R')(2R_3 + R_4)/J_4 \\
 S_{R_1}^{T_4 \cdot GB} &= R'(2R_3 R_f + R_2 R_3 + R_4 R_f)/J_4 \\
 S_{GB}^{T_4 \cdot GB} &= 1
 \end{aligned} \right\} \dots\dots\dots (35)$$

$$\left. \begin{aligned}
 S_{R_1}^{T_5 \cdot GB^2} &= 0 \quad , \quad S_{R_2}^{T_5 \cdot GB^2} = 1 \\
 S_{R_3}^{T_5 \cdot GB^2} &= 1 \quad , \quad S_{R_4}^{T_5 \cdot GB^2} = 0 \\
 S_{R_f}^{T_5 \cdot GB^2} &= 1 \quad , \quad S_{R'}^{T_5 \cdot GB^2} = 0 \\
 S_{GB}^{T_5 \cdot GB^2} &= 2
 \end{aligned} \right\} \dots\dots\dots (36)$$

where

$$\left. \begin{aligned}
 J_3 &= (R' + R_1)(R_2 + R_f) + R_2 R_f \\
 J_4 &= (2R_3 R_f + R_2 R_3 + R_4 R_f)(R_1 + R') + R_2 R_f(2R_3 + R_4)
 \end{aligned} \right\} \dots\dots\dots (37)$$

Calculated coefficient sensitivity factors of the driving point impedance  $Z_{in}(s)$  by using the nominal element values in Table 1 are given in Table 2—5. Here,  $S_x^{T_5}$  is not included in the table, as it is obvious from Eq. (36). The variation in any elements are found to be within 1 and  $10^{-4}$  order as is evident from these tables. It is clear that the proposed circuit presents the low-Q sensitivity to all types and to all elements.

In the same manner as before, the values of sensitivity of  $Q(\omega)$ -factor<sup>8)</sup> evaluated using Eq. (11) and Eq. (16) are shown in Table 6.

Finally, we deal with the stability problem. In general, for an network to be stable, a numerator and denominator polynomials of the driving point impedance  $Z_{in}(s)$  must be Hurwitz polynomial. From Eq. (7), the driving point impedance  $Z_{in}(s)$  is written as a ratio of polynomials  $N_0(s)$  and  $D_0(s)$ . Thus, it is clear that the polynomials  $N_0(s)$  and  $D_0(s)$  are given

as follows :

$$N_0(s) = s^2 R' (U_1 R_1 + U_2) + s G B R' U_3 \dots\dots\dots (38)$$

$$D_0(s) = s^2 [(R' + R_1) U_1 + U_2] + s G B (U_3 + U_1 R' + R_2) + G B^2 R_2 R_3 R_f \dots\dots\dots (39)$$

Table 2 Calculated values of  $S_x^{T_1}$

Type	$S_{R'}$	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_f}$	$S_{GB}$
1	1.0	1.0	1.0	1.0	$1.00 \cdot 10^{-4}$	$2.00 \cdot 10^{-4}$	0
2	1.0	.999	.999	.999	$9.90 \cdot 10^{-4}$	$2.00 \cdot 10^{-4}$	0
3	1.0	.999	.999	.999	$9.90 \cdot 10^{-4}$	$2.00 \cdot 10^{-4}$	0
4	1.0	.990	.990	.990	$9.90 \cdot 10^{-3}$	$1.96 \cdot 10^{-2}$	0
5	1.0	.990	.990	.990	$9.90 \cdot 10^{-3}$	$1.96 \cdot 10^{-2}$	0
6	1.0	.990	.990	.990	$9.90 \cdot 10^{-3}$	$1.96 \cdot 10^{-2}$	0
7	1.0	.917	.917	.909	$9.02 \cdot 10^{-2}$	.167	0
8	1.0	.917	.917	.909	$9.02 \cdot 10^{-2}$	.167	0
9	1.0	.917	.917	.909	$9.02 \cdot 10^{-2}$	.167	0
10	1.0	.917	.917	.909	$9.02 \cdot 10^{-2}$	.167	0
11	1.0	.667	.667	.500	.500	.667	0
12	1.0	.667	.667	.500	.500	.667	0
13	1.0	.667	.667	.500	.500	.667	0
14	1.0	.667	.667	.500	.500	.667	0
15	1.0	.667	.667	.500	.500	.667	0
16	1.0	.524	.524	$9.09 \cdot 10^{-2}$	.909	.952	0
17	1.0	.524	.524	$9.09 \cdot 10^{-2}$	.909	.952	0
18	1.0	.524	.524	$9.09 \cdot 10^{-2}$	.909	.952	0
19	1.0	.524	.524	$9.09 \cdot 10^{-2}$	.909	.952	0
20	1.0	.502	.502	$9.90 \cdot 10^{-2}$	.990	.995	0
21	1.0	.502	.502	$9.90 \cdot 10^{-2}$	.990	.995	0
22	1.0	.502	.502	$9.90 \cdot 10^{-2}$	.990	.995	0
23	1.0	.500	.500	$9.99 \cdot 10^{-4}$	.999	1.0	0
24	1.0	.500	.500	$9.99 \cdot 10^{-4}$	.999	1.0	0
25	1.0	.500	.500	$1.00 \cdot 10^{-4}$	1.0	1.0	0

Table 4 Calculated values of  $S_x^{T_3}$

Type	$S_{R'}$	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_f}$	$S_{GB}$
1	$9.90 \cdot 10^{-3}$	.990	1.0	1.0	$1.00 \cdot 10^{-4}$	$1.99 \cdot 10^{-4}$	0
2	$9.08 \cdot 10^{-2}$	.908	.999	.999	$9.99 \cdot 10^{-4}$	$1.91 \cdot 10^{-3}$	0
3	$9.89 \cdot 10^{-3}$	.989	.999	.999	$9.99 \cdot 10^{-4}$	$1.99 \cdot 10^{-3}$	0
4	.498	.498	.990	.990	$9.90 \cdot 10^{-3}$	$1.48 \cdot 10^{-2}$	0
5	$9.01 \cdot 10^{-2}$	.901	.990	.990	$9.90 \cdot 10^{-3}$	$1.87 \cdot 10^{-2}$	0
6	$9.80 \cdot 10^{-3}$	.980	.990	.990	$9.90 \cdot 10^{-3}$	$1.95 \cdot 10^{-2}$	0
7	.478	.478	.913	.909	$9.09 \cdot 10^{-3}$	.130	0
8	$8.40 \cdot 10^{-2}$	.840	.916	.909	$9.09 \cdot 10^{-2}$	.160	0
9	.902	$9.02 \cdot 10^{-2}$	.910	.909	$9.09 \cdot 10^{-2}$	$9.84 \cdot 10^{-2}$	0
10	$9.08 \cdot 10^{-3}$	.908	.917	.909	$9.09 \cdot 10^{-2}$	.166	0
11	.400	.400	.600	.500	.500	.600	0
12	$6.25 \cdot 10^{-2}$	.625	.656	.500	.500	.656	0
13	.870	$8.70 \cdot 10^{-2}$	.522	.500	.500	.522	0
14	$6.62 \cdot 10^{-3}$	.662	.666	.500	.500	.666	0
15	.985	$9.85 \cdot 10^{-3}$	.502	.500	.500	.502	0
16	.344	.344	.375	$9.09 \cdot 10^{-2}$	.909	.938	0
17	.840	$8.40 \cdot 10^{-2}$	.165	$9.09 \cdot 10^{-2}$	.909	.916	0
18	$4.98 \cdot 10^{-2}$	.498	.502	.909	.909	.950	0
19	.981	$9.81 \cdot 10^{-3}$	$9.90 \cdot 10^{-2}$	$9.09 \cdot 10^{-2}$	.909	.910	0
20	.334	.334	.338	$9.09 \cdot 10^{-3}$	.990	.993	0
21	.834	$8.42 \cdot 10^{-2}$	$9.17 \cdot 10^{-2}$	$9.90 \cdot 10^{-3}$	.990	.991	0
22	.980	$9.80 \cdot 10^{-3}$	$1.95 \cdot 10^{-2}$	$9.90 \cdot 10^{-3}$	.990	.990	0
23	.833	$8.33 \cdot 10^{-2}$	$8.42 \cdot 10^{-2}$	$9.99 \cdot 10^{-4}$	.999	.999	0
24	.980	$9.80 \cdot 10^{-3}$	$1.08 \cdot 10^{-2}$	$9.99 \cdot 10^{-4}$	.999	.999	0
25	.980	$9.80 \cdot 10^{-3}$	$9.90 \cdot 10^{-3}$	$1.00 \cdot 10^{-4}$	1.0	1.0	0

Table 3 Calculated values of  $S_x^{T_2GB}$

Type	$S_{R'}$	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_f}$	$S_{GB}$
1	1.0	1.0	1.0	1.0	$1.00 \cdot 10^{-8}$	$3.00 \cdot 10^{-4}$	1.0
2	1.0	.990	.990	1.0	$9.90 \cdot 10^{-7}$	$2.90 \cdot 10^{-3}$	1.0
3	1.0	.990	.990	1.0	$9.90 \cdot 10^{-7}$	$2.90 \cdot 10^{-3}$	1.0
4	1.0	.990	.990	1.0	$9.71 \cdot 10^{-7}$	$2.92 \cdot 10^{-2}$	1.0
5	1.0	.990	.990	1.0	$9.71 \cdot 10^{-7}$	$2.92 \cdot 10^{-2}$	1.0
6	1.0	.990	.990	1.0	$9.71 \cdot 10^{-7}$	$2.92 \cdot 10^{-2}$	1.0
7	1.0	.924	.840	.992	$7.63 \cdot 10^{-3}$	.237	1.0
8	1.0	.924	.840	.992	$7.63 \cdot 10^{-3}$	.237	1.0
9	1.0	.924	.840	.992	$7.63 \cdot 10^{-3}$	.237	1.0
10	1.0	.924	.840	.992	$7.63 \cdot 10^{-3}$	.237	1.0
11	1.0	.800	.400	.800	.200	.800	1.0
12	1.0	.800	.400	.800	.200	.800	1.0
13	1.0	.800	.400	.800	.200	.800	1.0
14	1.0	.800	.400	.800	.200	.800	1.0
15	1.0	.800	.400	.800	.200	.800	1.0
16	1.0	.924	$8.40 \cdot 10^{-2}$	.237	.763	.992	1.0
17	1.0	.924	$8.40 \cdot 10^{-2}$	.237	.763	.992	1.0
18	1.0	.924	$8.40 \cdot 10^{-2}$	.237	.763	.992	1.0
19	1.0	.924	$8.40 \cdot 10^{-2}$	.237	.763	.992	1.0
20	1.0	.990	$9.80 \cdot 10^{-3}$	$2.92 \cdot 10^{-2}$	.971	1.0	1.0
21	1.0	.990	$9.80 \cdot 10^{-3}$	$2.92 \cdot 10^{-2}$	.971	1.0	1.0
22	1.0	.990	$9.80 \cdot 10^{-3}$	$2.92 \cdot 10^{-2}$	.971	1.0	1.0
23	1.0	.999	$9.98 \cdot 10^{-4}$	$2.99 \cdot 10^{-3}$	.997	1.0	1.0
24	1.0	.999	$9.98 \cdot 10^{-4}$	$2.99 \cdot 10^{-3}$	.997	1.0	1.0
25	1.0	1.0	$1.00 \cdot 10^{-4}$	$3.00 \cdot 10^{-3}$	1.0	1.0	1.0

Table 5 Calculated values of  $S_x^{T_4GB}$

Type	$S_{R'}$	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_f}$	$S_{GB}$
1	$9.90 \cdot 10^{-3}$	.990	1.0	1.0	$1.99 \cdot 10^{-8}$	$3.98 \cdot 10^{-4}$	1.0
2	$9.07 \cdot 10^{-2}$	.907	.998	.999	$1.90 \cdot 10^{-6}$	$3.81 \cdot 10^{-3}$	1.0
3	$9.88 \cdot 10^{-3}$	.988	.998	.999	$1.98 \cdot 10^{-6}$	$3.97 \cdot 10^{-3}$	1.0
4	.495	.495	.980	.995	$1.46 \cdot 10^{-4}$	$2.93 \cdot 10^{-2}$	1.0
5	$8.93 \cdot 10^{-2}$	.893	.981	.991	$1.84 \cdot 10^{-4}$	$3.70 \cdot 10^{-2}$	1.0
6	$9.71 \cdot 10^{-3}$	.971	.981	.990	$1.91 \cdot 10^{-4}$	$3.85 \cdot 10^{-2}$	1.0
7	.460	.460	.840	.958	$1.14 \cdot 10^{-2}$	.240	1.0
8	$7.85 \cdot 10^{-2}$	.785	.850	.934	$1.36 \cdot 10^{-2}$	.286	1.0
9	.895	$8.95 \cdot 10^{-2}$	.829	.985	$8.89 \cdot 10^{-3}$	.186	1.0
10	$8.45 \cdot 10^{-3}$	.845	.852	.930	$1.40 \cdot 10^{-2}$	.295	1.0
11	.364	.364	.455	.818	.273	.818	1.0
12	$5.41 \cdot 10^{-2}$	.541	.554	.851	.284	.851	1.0
13	.851	$8.51 \cdot 10^{-2}$	.298	.766	.255	.766	1.0
14	$5.68 \cdot 10^{-3}$	.568	.570	.857	.286	.857	1.0
15	.983	$9.85 \cdot 10^{-3}$	.256	.752	.251	.752	1.0
16	.334	.334	.337	.696	.829	.994	1.0
17	.834	$8.34 \cdot 10^{-2}$	$9.03 \cdot 10^{-2}$	.304	.827	.992	1.0
18	$4.78 \cdot 10^{-2}$	.478	.478	.921	.830	.996	1.0
19	.980	$9.80 \cdot 10^{-3}$	$1.79 \cdot 10^{-2}$	.189	.827	.992	1.0
20	.333	.333	.333	.670	.980	1.0	1.0
21	.833	$8.33 \cdot 10^{-2}$	$8.84 \cdot 10^{-2}$	.182	.980	1.0	1.0
22	.980	$9.80 \cdot 10^{-3}$	$9.90 \cdot 10^{-3}$	$3.88 \cdot 10^{-2}$	.980	1.0	1.0
23	.833	$8.33 \cdot 10^{-2}$	$8.33 \cdot 10^{-2}$	.168	.998	1.0	1.0
24	.980	$9.80 \cdot 10^{-3}$	$9.80 \cdot 10^{-3}$	$2.16 \cdot 10^{-2}$	.998	1.0	1.0
25	.980	$9.80 \cdot 10^{-3}$	$9.80 \cdot 10^{-3}$	$1.98 \cdot 10^{-2}$	1.0	1.0	1.0

In Eqs. (38) and (39), since the resistances and the parameters  $GB$  of op amp are positive, zeros of  $N_0(s)$  and  $D_0(s)$  are in the left-half  $S$ -plane and origin. It is obvious that this proposed circuit is stable.

**Table 6** Calculated values of  $S_z^{Q(\omega)}$

Type	$s_{z1}$	$s_{z2}$	$s_{z3}$	$s_{z4}$	$s_{z5}$	$s_{z6}$	$s_{z7}$
1	-9.90·10 <sup>-3</sup>	-9.90·10 <sup>-1</sup>	2.00·10 <sup>-4</sup>	0	-2.00·10 <sup>-6</sup>	1.0	1.0
2	-9.08·10 <sup>-2</sup>	-9.08·10 <sup>-1</sup>	2.00·10 <sup>-3</sup>	4.90·10 <sup>-6</sup>	-2.00·10 <sup>-6</sup>	1.0	1.0
3	-9.89·10 <sup>-1</sup>	-9.89·10 <sup>-1</sup>	2.00·10 <sup>-3</sup>	5.00·10 <sup>-6</sup>	-2.00·10 <sup>-6</sup>	1.0	1.0
4	-4.98·10 <sup>-1</sup>	-4.98·10 <sup>-1</sup>	1.97·10 <sup>-2</sup>	4.82·10 <sup>-4</sup>	-1.96·10 <sup>-4</sup>	1.03	1.0
5	-9.01·10 <sup>-2</sup>	-9.01·10 <sup>-1</sup>	1.96·10 <sup>-2</sup>	4.83·10 <sup>-4</sup>	-1.96·10 <sup>-4</sup>	1.03	1.0
6	-9.80·10 <sup>-3</sup>	-9.80·10 <sup>-1</sup>	1.95·10 <sup>-2</sup>	4.84·10 <sup>-4</sup>	-1.96·10 <sup>-4</sup>	1.01	1.0
7	-4.78·10 <sup>-1</sup>	-4.79·10 <sup>-1</sup>	1.69·10 <sup>-1</sup>	3.60·10 <sup>-2</sup>	-1.62·10 <sup>-2</sup>	1.24	1.0
8	-8.40·10 <sup>-2</sup>	-8.40·10 <sup>-1</sup>	1.66·10 <sup>-1</sup>	3.65·10 <sup>-2</sup>	-1.64·10 <sup>-2</sup>	1.07	1.0
9	-9.02·10 <sup>-1</sup>	-9.02·10 <sup>-2</sup>	1.73·10 <sup>-1</sup>	3.55·10 <sup>-2</sup>	-1.60·10 <sup>-2</sup>	1.24	1.0
10	-9.09·10 <sup>-3</sup>	-9.09·10 <sup>-1</sup>	1.65·10 <sup>-1</sup>	3.66·10 <sup>-2</sup>	-1.64·10 <sup>-2</sup>	-1.52·10 <sup>-1</sup>	1.0
11	-4.08·10 <sup>-1</sup>	-4.24·10 <sup>-1</sup>	6.45·10 <sup>-1</sup>	6.57·10 <sup>-1</sup>	-4.69·10 <sup>-1</sup>	1.33	1.0
12	-6.45·10 <sup>-2</sup>	-6.71·10 <sup>-1</sup>	5.84·10 <sup>-1</sup>	6.90·10 <sup>-1</sup>	-4.81·10 <sup>-1</sup>	-4.63·10 <sup>1</sup>	1.0
13	-8.73·10 <sup>-1</sup>	-9.08·10 <sup>-2</sup>	7.28·10 <sup>-1</sup>	6.12·10 <sup>-1</sup>	-4.54·10 <sup>-1</sup>	1.08	1.0
14	-6.85·10 <sup>-3</sup>	-7.12·10 <sup>-1</sup>	5.74·10 <sup>-1</sup>	6.96·10 <sup>-1</sup>	-4.83·10 <sup>-1</sup>	-4.82·10 <sup>3</sup>	1.0
15	-9.86·10 <sup>-1</sup>	-1.03·10 <sup>-2</sup>	7.47·10 <sup>-1</sup>	6.03·10 <sup>-1</sup>	-4.50·10 <sup>-1</sup>	1.08	1.0
16	-3.51·10 <sup>-1</sup>	-3.72·10 <sup>-1</sup>	7.19·10 <sup>-1</sup>	1.13	-1.63	-1.43·10 <sup>1</sup>	1.0
17	-8.44·10 <sup>-1</sup>	-8.94·10 <sup>-2</sup>	9.26·10 <sup>-1</sup>	1.06	-1.60	-1.83	1.0
18	-5.14·10 <sup>-2</sup>	-5.44·10 <sup>-1</sup>	5.94·10 <sup>-1</sup>	1.16	-1.64	-1.63·10 <sup>3</sup>	1.0
19	-9.81·10 <sup>-1</sup>	-1.08·10 <sup>-2</sup>	9.84·10 <sup>-1</sup>	1.04	-1.59	1.99	9.99·10 <sup>-1</sup>
20	-3.35·10 <sup>-1</sup>	-3.39·10 <sup>-1</sup>	6.74·10 <sup>-1</sup>	1.02	-1.96	-1.94·10 <sup>1</sup>	1.0
21	-8.34·10 <sup>-1</sup>	-8.47·10 <sup>-2</sup>	9.19·10 <sup>-1</sup>	1.01	-1.95	4.73·10 <sup>-2</sup>	9.99·10 <sup>-1</sup>
22	-9.76·10 <sup>-1</sup>	-1.43·10 <sup>-2</sup>	9.80·10 <sup>-1</sup>	1.01	-1.95	1.98	9.91·10 <sup>-1</sup>
23	-8.29·10 <sup>-1</sup>	-8.83·10 <sup>-2</sup>	9.17·10 <sup>-1</sup>	9.96·10 <sup>-1</sup>	2.00	-1.80·10 <sup>1</sup>	9.89·10 <sup>-1</sup>
24	-9.38·10 <sup>-1</sup>	-5.27·10 <sup>-2</sup>	9.91·10 <sup>-1</sup>	9.58·10 <sup>-1</sup>	-2.00	1.80	9.13·10 <sup>-1</sup>
25	-6.75·10 <sup>-1</sup>	-3.19·10 <sup>-1</sup>	9.93·10 <sup>-1</sup>	6.88·10 <sup>-1</sup>	-2.00	4.98·10 <sup>-4</sup>	3.76·10 <sup>-2</sup>

**4. Experimental results and discussion**

The circuit is consisted of six resistances with 1 percent tolerance, one potentiometer and two  $\mu A$  747 PC-type op amps satisfying the condition of the gain bandwidth products  $GB_1 = GB_2 = GB$ . The gain bandwidth product of the op amp for a power supply voltage of  $\pm 15 V$  is  $GB = 2\pi \cdot 1.5 \cdot 10^6 \text{ rad/sec}$ . The experiment was carried out mainly using the component values in table 1. The values of simulated inductance  $L$  and  $Q$ -factor were measured by using parallel resonance method and are given by

$$L = 1/2\pi f_0^2 C \dots\dots\dots (40)$$

$$Q = f_0/(f_2 - f_1) \dots\dots\dots (41)$$

where  $f_0$  is a resonance frequency, and  $f_1$  and  $f_2$  ( $f_1 < f_2$ ) are the frequencies at cut-off frequency.

In Fig. 9, the measured values of the simulated inductance are compared with the theoretical curve for four cases, such as  $L = 1.39 \cdot 10^{-2} H$ ,  $8.7 \cdot 10^{-3} H$ ,  $5.31 \cdot 10^{-3} H$  and  $1.48 \cdot 10^{-3} H$ . As is obvious from this figure, the experimental results agree well with the theoretical ones.

Fig. 10 is an example of a  $Q$ -factor against frequency taking the resistance ratio  $K_{12}$  as a parameter. It shows how  $Q$  varies with frequency depending upon the resistance ratio  $K_{12}$ . Four typical resistance ratios of  $K_{12}$  are chosen in Fig. 10, such as  $K_{12} = 0.5$  ( $= 10 \text{ k}\Omega / 20 \text{ k}\Omega$ ),  $0.8$  ( $= 10 \text{ k}\Omega / 12.5 \text{ k}\Omega$ ),  $1.0$  ( $= 10 \text{ k}\Omega / 10 \text{ k}\Omega$ ),  $10.0$  ( $= 100 \text{ k}\Omega / 10 \text{ k}\Omega$ ) and  $100$  ( $= 100 \text{ k}\Omega / 1 \text{ k}\Omega$ ).

For the measurements of  $Q$ -factor versus frequency, the resonance frequencies were changed by adjusting the external standard capacitance. In order to obtain  $Q$ -factor of over 30, the resistance ratio  $K_{12}$  must be chosen within  $1 \leq K_{12}$ . Furthermore, it is difficult for practical implementation to adjust  $K_{12}$  beyond 100.

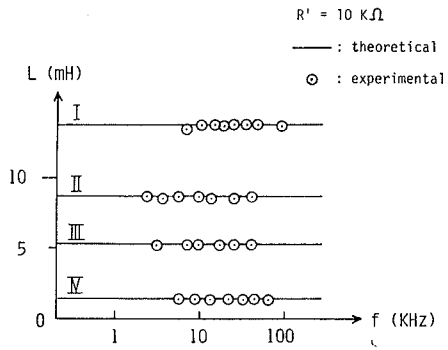


Fig. 9 Theoretical and experimental values of simulated inductance

- I : type 10
- II :  $R_1 = R_2 = 20 \text{ K}\Omega$  ,  $R_3 = 2 \text{ K}\Omega$   
 $R_4 = 10 \text{ K}\Omega$  ,  $R_f = 100 \text{ K}\Omega$
- III : type 11
- IV :  $R_1 = R_3 = R_4 = 10 \text{ K}\Omega$   
 $R_2 = R_f = 100 \text{ K}\Omega$

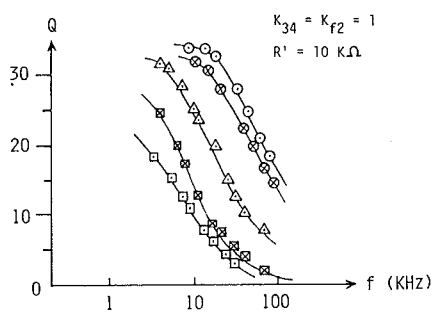


Fig. 10 Experimental Q-factor versus frequency by varying the resistance ratio  $K_{12}$

- : 0.5 (=  $10 \text{ K}\Omega / 20 \text{ K}\Omega$  )
- : 0.8 (=  $10 \text{ K}\Omega / 12.5 \text{ K}\Omega$  )
- △ : 1.0 (=  $100 \text{ K}\Omega / 100 \text{ K}\Omega$  )
- : 10 (=  $100 \text{ K}\Omega / 10 \text{ K}\Omega$  )
- ⊙ : 100 (=  $100 \text{ K}\Omega / 1 \text{ K}\Omega$  )

Next, an example of a Q-factor variation with frequency are plotted in Fig. 11 for the case of varying the resistance value for a fixed  $K_{12}$  at  $R' = R_3 = R_4 = 10 \text{ k}\Omega$ , and  $K_{f2} = 1$ . From this figure, it is obvious that the circuit attains a high Q-factor by making  $R_1, R_2 \geq 10 \text{ k}\Omega$ . For example, the Q-factor varies from 4 to 25 at 10 KHz depending on the resistance values. This means that Q-factor can be adjusted without changing the inductance value. It is better to choose  $R_1$  and  $R_2$  from 10 kΩ to 100 kΩ in order to be the influence of a op amp's loop gain.

Fig. 12 show how the condition  $K_{34} K_{f2} = 1$  for realizing the inductance effects the Q-factor.

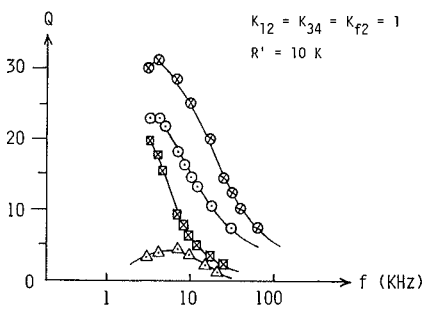


Fig. 11 Experimental Q-factor versus frequency for the various values of  $R_1$  and  $R_2$

- △ :  $R_1 = R_2 = 1 \text{ K}$
- :  $R_1 = R_2 = 2 \text{ K}$
- :  $R_1 = R_2 = 10 \text{ K}$
- ⊙ :  $R_1 = R_2 = 100 \text{ K}$

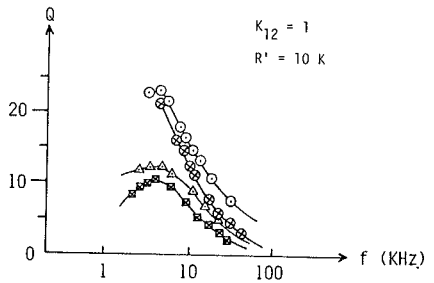
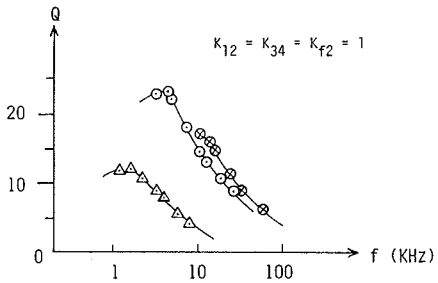


Fig. 12 Experimental Q-factor versus frequency by varying the resistance ratios  $K_{34}$  and  $K_{f2}$

- ⊠ :  $K_{34} = 5.0$  ,  $K_{f2} = 0.2$
- ⊙ :  $K_{34} = 2.0$  ,  $K_{f2} = 0.5$
- :  $K_{34} = 1.0$  ,  $K_{f2} = 1.0$
- △ :  $K_{34} = 0.1$  ,  $K_{f2} = 10.0$



**Fig. 13 Experimental Q-factor versus frequency by varying the resistance value  $R'$**

- :  $R' = 1 \text{ K}$
- :  $R' = 10 \text{ K}$
- △ :  $R' = 100 \text{ K}$

It is clear from the figure that values of  $K_{34}$  and  $K_{f2}$  around unity are recommended to achieve high Q-factor.

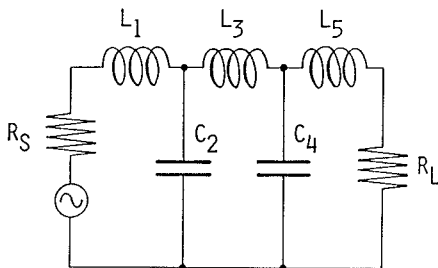
Fig. 13 shows the Q vs frequency characteristics for various  $R'$ 's. In order to obtain a high Q-factor it is necessary to keep the resistance  $R'$  around 10 k $\Omega$ .

As is clear from above, Q depends on many element values. When we design the inductance with moderate Q at a given frequency, we must select a suitable combination of element values to satisfy the specifications.

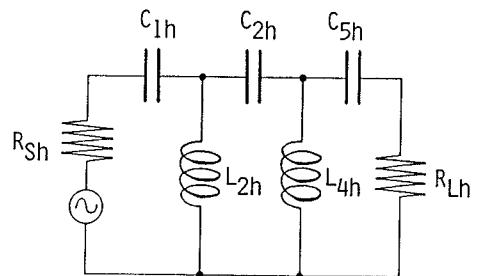
### 5. Highpass ladder filters

In this chapter, we deal with the design<sup>9,10</sup> of 5th-order Butterworth highpass filter and 5th-order 1 dB ripple Chebyshev highpass filter. The inductance in highpass ladder filter is replaced by the simulated inductance.

Fig. 14 is conventional the 5th-order Butterworth lowpass filter.<sup>9</sup> The normalized element values of an equally terminated 5th-order Butterworth lowpass filter are  $R_{s1} = R_{L1} = 1 \Omega$ ,  $L_{1l} = L_{5l} = 0.168\text{H}$ ,  $C_{21} = C_{41} = 1.618\text{F}$  and  $L_{31} = 2\text{H}$ . To transform the normalized lowpass filter circuit to a highpass configuration, replace each inductance with capacitance and vice versa using reciprocal element values as shown in Fig. 15. Then, for realizing the corresponding active highpass filter with a cutoff frequency of 20.0 KHz and an impedance level of 300  $\Omega$ , the following component values are obtained :  $R_{sh} = R_{Lh} = 300 \Omega$ ,  $C_{1h} = C_{5h} = 0.043 \mu\text{F}$ ,  $L_{2h} = L_{4h} = 1.48 \text{ mH}$ ,  $C_{3h} = 0.013 \mu\text{F}$ . For example, the resistance values corresponding to inductance value  $L_{2h} = L_{4h}$ , using  $\mu\text{A} 747 \text{ PC}$ - type op amp with gain bandwidth product  $GB = 2\pi \cdot 1.5 \cdot 10^6 \text{ rad/sec}$  for power supply voltage  $\pm 15 \text{ V}$ ,



**Fig. 14 Normalized lowpass filter**



**Fig. 15 Transformed highpass filter**

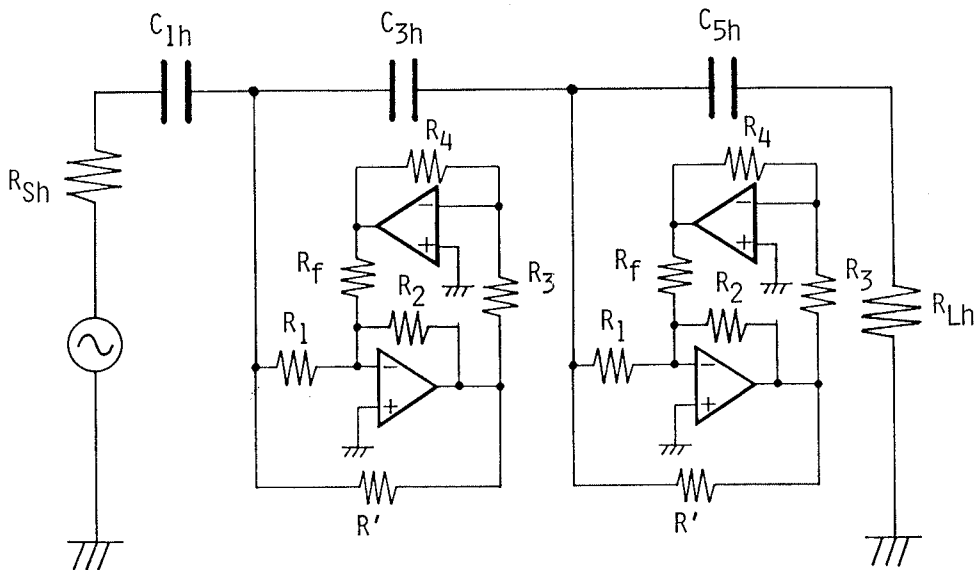


Fig. 16 Experimental circuit of highpass ladder filter

are  $R = R_3 = R_4 = R' = 10 \text{ k}\Omega$  and  $R_2 = R_f = 100 \text{ k}\Omega$ . The final filter circuit is given in fig. 16.

The 5th-order 1 dB ripple Chebyshev filter is realized in the same manner as above. The normalized element values of an equally terminated 5th-order Chebyshev lowpass filter<sup>9)</sup> are  $R_{S1} = R_{L1} = 1 \Omega$ ,  $L_{11} = L_{51} = 2.2072 \text{ H}$ ,  $C_{21} = C_{41} = 1.1279 \text{ F}$  and  $L_{31} = 3.1025 \text{ H}$  in Fig. 14. The corresponding active highpass filter with a cutoff frequency of 28.5 KHz and an impedance level of  $300 \Omega$ , has the following element values :  $R_{S_h} = R_{L_h} = 300 \Omega$ ,  $C_{1_h} = C_{5_h} = 0.0084 \mu\text{F}$ ,  $L_{2_h} = L_{4_h} = 1.48 \text{ mH}$  and  $C_{3_h} = 0.006 \mu\text{F}$ . The resistance values corresponding to inductance values  $L_{2_h}$  and  $L_{4_h}$  are the same as in the example above. Then, the final filter circuit is

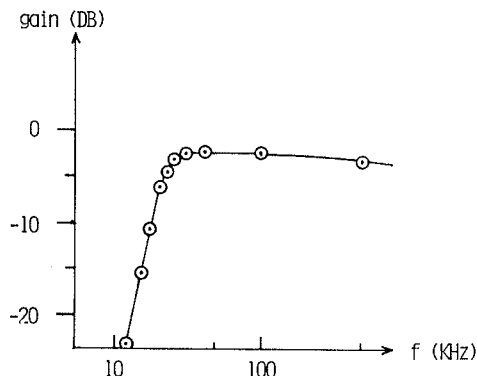


Fig. 17 Experimental frequency response of 5th-order Butterworth highpass filter

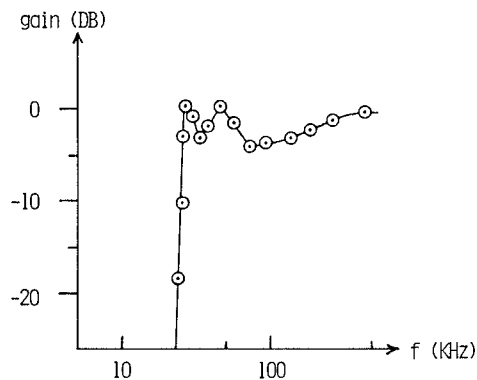


Fig. 18 Experimental frequency response of 5th-order 1 dB ripple Chebyshev highpass filter

shown in Fig. 16.

These highpass filters were composed of  $\mu A 747$  PC-type op amps with gain bandwidth product  $GB = 2\pi \cdot 1.5 \cdot 10^6$  rad/sec and 1 percent tolerance resistors and capacitances. The power supply voltage is  $\pm 15$  V. It is seen that the experimental results agree approximately with theoretical as is evident from the Figs. 17 and 18. We can conclude from the experiment described above that the inductance simulation circuit shown in Fig. 1 operates successfully as an inductance.

## 6. Conclusion

We have described the grounded inductance using an op amp pole. The structure simulates an inductance and a resistance in series with a capacitance and can be operated at much higher frequencies than classical active networks. The circuit is suitable for microcircuit fabrication of the network. Furthermore, it has been obvious that the realized circuit has low sensitivity to active and passive elements from the sensitivity analysis, and the inductance value can be changed with the resistance ratios  $K_{12}$ ,  $K_{34}$  and  $K_{2}$ , which assures better temperature performance. The proposed circuit is suitable for realizing inductance with relatively low  $Q$ -factor. The  $Q$ -factor can be adjusted by either the resistance ratios or values. As examples of application of such simulated inductance, the 5th-order Butterworth and 5th-order 1 dB ripple Chebyshev highpass ladder filter were presented. It has been obvious that these filter configurations work successfully over the wide frequency.

We expect that this proposed scheme can be readily applied to other filter realizations, FDNR and GIC i.e.

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