# A New Integrator Using Operational Amplifier Pole

by

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A new integrator is realized by using an operational amplifier pole. Its gain constant can be varied easily by changing the resistance ratio or operational amplifiers. This integrator is dc stable, and there is no effect of the leakage current. Both the noninverting and inverting outputs can be obtained simultaneously. The analytical and experimental results show clealy that this new integrator works successfully over the wide frequency range.

## 1. Introduction

The conventional RC integrator finds many applications to the analog computations, signal processing, signal generating and control. In such an integrator, however, the main difficulties lie in making the output to reach one or the other of its saturation levels due to the drift of the operational amplifier ( op amp ) and leakage current of a feedback capacitor. Furthermore, we should note that capacitors of very small value cannot be easily obtained accurately because of the stray capacity of wiring, and that capacitors of very large value are rather expensive. Electrolytic capacitors do not permit bipolar potentials to be applied and have a large amount of dissipation, that is, they present an appreciable leakage current. Therefore, the selection of a capacitor with desired characteristics is very difficult.

From the above mentioned point of view, we have been trying to eliminate a feedback capacitor of the integrator by using op amp pole. In this paper, we propose a new integrator which does not make use of the feedback capacitor. Although many papers on active filters using op amp pole[1)-3)] have been published, the integrator as described in this paper has not been given yet. It is composed of two op amps and five resistors. The gain constant of the integrator depends not only on the gain bandwidth product of op amp but also on the resistance ratio. Therefore, we can realize an integrator with arbitrary gain. The integrator is dc stable since it is not in need of a capacitor, and both the noninverting and inverting outputs can be obtained at the same time. This integrator works successfully over the wide

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frequency range from 100 Hz to 100 KHz, and the experimental results agree well with the theoretical ones. Sensitivity analysis and consideration on transient response are also given.

## 2. Single pole rolloff model

Figs. 1 and 2 show the open-loop inverting op amp and the frequency response of amplitude, respectively. If the op amp has a -6 dB octave rolloff, its open-loop gain can be represented by a single pole model. The mathematical expression for the gain A ( $s$ ) can be written as[4)-7),9)]

$$A(s)=\frac{V_o}{V_i}=-\frac{A_o\omega_p}{s+\omega_p}=-\frac{GB}{s+\omega_p} \qquad (1)$$

where $A_0$ is its open-loop d. c. gain, $\omega_P$ is the open-loop 3 dB bandwidth in radians per second ( *rad* $s^{-1}$ ), and the product $A_0$ $\omega_p$ ( $=GB$ ) is the gain bandwidth product in *rad* $s^{-1}$. Typical values for $\mu$A 741 are $A_0=10^5$ and $\omega_p=20$ *rad* $s^{-1}$. In Eq. ( 1 ), letting $s=j\omega$, and if $\omega \gg \omega_p$, the open-loop gain $A$ ( $j\omega$ ) is given by $A$ ( $j\omega$ ) $=-GB/j\omega$.

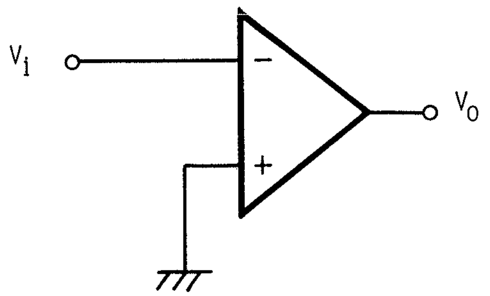


**Fig. 1 Open-loop inverting op amp**

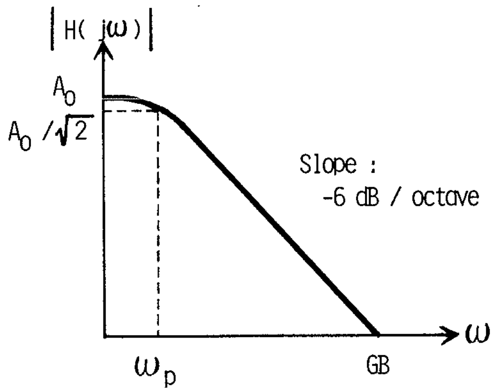


**Fig. 2 Frequency response of amplitude in open-loop mode**

In practice, we cannot use an op amp in the open-loop mode as an integrator because of the very large gain, paticularly for low frequency signals. Further, $A_0$ may vary considerably from one unit to another, and it may change with temperature in a given unit.

## 3. Theoretical analysis

An inverting voltage amplifier is shown in Fig. 3. For the typical case of $\{R_b A_0/(R_a+R_b)\}\gg 1$, the closed loop gain $A_{ab}$ ( $s$ ) of inverting voltage amplifier can be expressed as[4)-7),9)]

$$A_{ab}(s)=\frac{V_o}{V_i}=-\frac{K_{ab}}{1+K_{ab}}\ \frac{GB}{s+\dfrac{GB}{1+K_{ab}}} \qquad (2)$$

where $K_{ab}=R_b/R_a$. If op amp is considered ideal, that is, $GB = \infty$, it is clear that $A_{ab}(s) = -K_{ab} = -R_b/R_a$.

Fig. 4 shows the experimental circuit of the integrator. In this figure, if the closed loop gain $(V'/V_i)$, $(V_o/V')$ and $(V'/V_o)$ are written as $A_{12}(s)$, $A_{34}(s)$ and $A_{f2}(s)$, respectively, loop gain $H(s)$ is obtained as

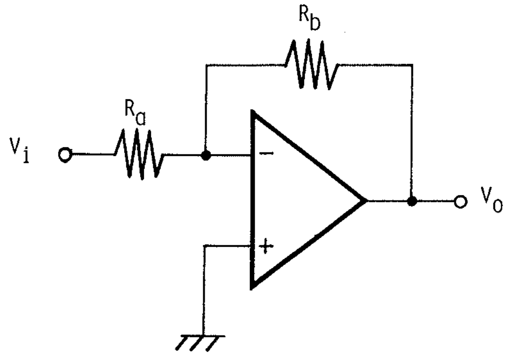


**Fig. 3 Inverting voltage amplifier**

$$H(s)=\frac{V_o}{V_i}$$

$$=\frac{A_{12}(s)A_{34}(s)}{1-A_{34}(s)A_{f2}(s)} \quad \cdots\cdots (3)$$

Where it is assumed that gain bandwidth product $GB$ of each op amp is epual, and

$$A_{12}(s)=-\frac{K_{12}}{1+K_{12}}\,\frac{GB}{s+\frac{GB}{1+K_{12}}} \quad \cdots\cdots (4)$$

$$A_{34}(s)=-\frac{K_{34}}{1+K_{34}}\,\frac{GB}{s+\frac{GB}{1+K_{34}}} \quad \cdots\cdots (5)$$

$$A_{f2}(s)=-\frac{K_{f2}}{1+K_{f2}}\,\frac{GB}{s+\frac{GB}{1+K_{f2}}} \quad \cdots\cdots (6)$$

In Eqs. (4), (5)and (6), $K'$ s are given as $K_{12}=R_2/R_1$, $K_{34}=R_4/R_3$ and $K_{f2}=R_f/R_2$. Letting $s=j\omega$, and substituting Eqs. (4), (5) and (6)to Eq. (3), we can obtain

$$H(j\omega)=\frac{K_{12}K_{34}U_{f2}GB^2s+K_{12}K_{34}GB^3}{[U_{12}U_{34}U_{f2}s^3+\{U_{12}(U_{34}+U_{f2})+U_{34}U_{f2}\}GBs^2 +(U_{12}U'+U_{34}+U_{f2})GB^2s+U'GB^3]} \quad \cdots\cdots (7)$$

Where $U_{12}=1+K_{12}$, $U_{34}=1+K_{34}$, $U_{f2}=1+K_{f2}$ and $U'=1-K_{34}K_{f2}$.

For frequencies such that

$$GB \gg U_{f2}\omega,$$

$$U'GB^2 \gg \{U_{12}(U_{34}+U_{f2})+U_{34}U_{f2}\}\omega^2$$

and $(U_{12}U'+U_{34}+U_{f2})GB^2 \gg U_{12}U_{34}U_{f2}\omega^2$, we can rewrite Eq. (7) as follow.

$$H(j\omega)=\frac{K_{12}K_{34}GB}{U'GB+j\omega(U_{12}U'+U_{34}+U_{f2})} \quad \cdots\cdots (8)$$

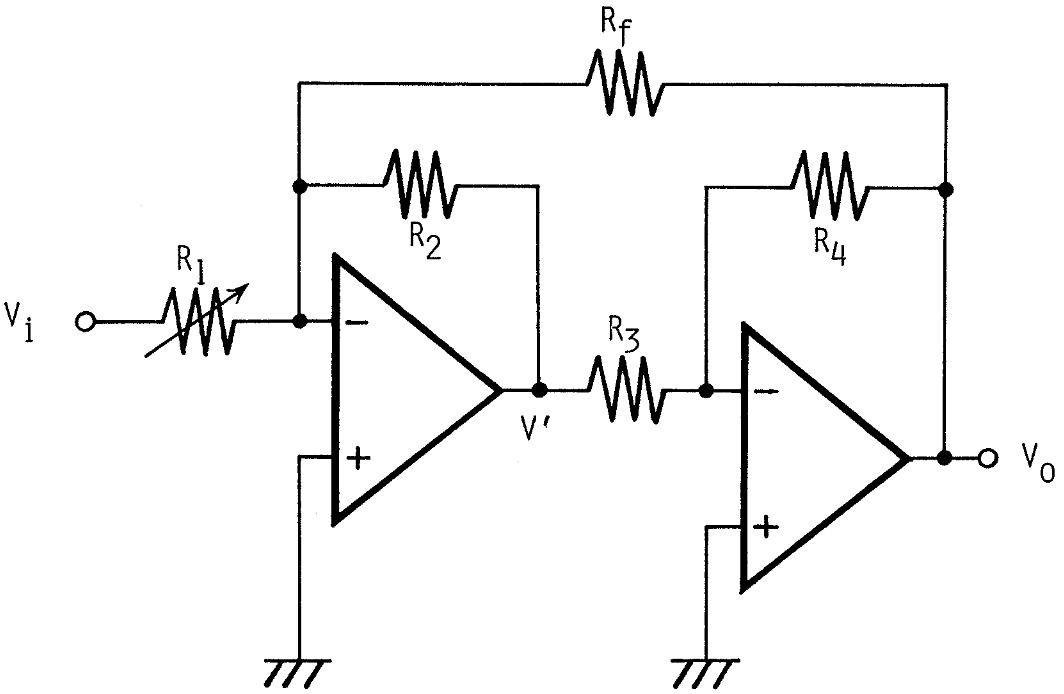


**Fig. 4 Experimental circuit**

As an example, if the circuit is adjusted so that $K_{34}=K_{f2}=1$ by the suitable choice of resistors $R_2$, $R_3$, $R_4$ and $R_f$, the loop gain $H$ ( $j\omega$ ) can be written in a simple form as

$$H(j\omega)=0.25K_{12}GB/j\omega \qquad (9)$$

It is clear that Eq. (9) has the property of an ideal integrator, and that the gain constant 0.25 $K_{12}$ $GB$ which corresponds to ( $1/RC$ ) in the RC integrator can be varied by changing the resistance ratio $K_{12}$ or op amps. It should be taken notice that exact value of resistor is immaterial but their ratio is important. In order to realize an integrator the resistance ratio must be trimmed so that $K_{34}=K_{f2}=1$.

## 4. Sensitivity function

By sensitivity, in general, we consider a measure of the displacement in some performance characteristic of the network or the network function resulting from some displacement in one or more element values. It is desirable to choose realization having low sensitivity and to minimize the sensitivity of realization. In this chapter, we deal with the sensitivity of the new integrator.

If we let y be the performance characteristic and $x$ be the element, we generally define sensitivity as follow[8)–9)]

$$S_x^y=\frac{\partial y}{\partial x}\frac{x}{y}=\frac{\partial y/y}{\partial x/x} \qquad (10)$$

If the quantity $y$ is chosen to be the network function $H(s)$, and the element $x$ is usually chosen as some passive or active element in the circuit realization of the function, the sensitivity function is defined as[8)-9)]

$$S_x^{H(s)}=\frac{\partial H(s)}{\partial x}\ \frac{x}{H(s)} \qquad \cdots\cdots (11)$$

Thus, the sensitivity function of the integrator with respect to the passive and active components can be determined. Let us derive the sensitivity function of the integrator for the case of resistance ratio being $K_{12}=K_{34}=K_{f2}=1$. The sensitivity function with respect to $K_{12}$ is

$$S_{K_{12}}^{H(s)}=\frac{U_{34}U_{f2}s^3+(U_{34}+U_{34}U_{f2}+U_{f2})GBs^3+(U_{34}+}{U_{12}U_{34}U_{f2}s^3+(U_{12}U_{34}+U_{12}U_{f2}+U_{34}U_{f2})GBs^2}\ *$$

$$*\ \frac{U_{f2}+U')GB^2s+U'GB^3}{+(U_{12}U'+U_{34}+U_{f2})GB^2s+U'GB^3} \qquad \cdots\cdots (12)$$

Evaluating the sensitivity function along the j$\omega$ axis, we can obtain

$$S_{K_{12}}^{H(j\omega)}=\frac{4^2\omega^4+24GB^2\omega^2+7GB^4}{8(4\omega^4+5GB^2\omega^2+GB^4)}-j\frac{(\omega^2+GB^2)GB\omega}{4\omega^4+5GB^2\omega^2+GB^4} \qquad \cdots\cdots (13)$$

Similarly, the sensitivity function with respect to $K_{34}$, $K_{f2}$ and $GB$ are given by

$$S_{K_{34}}^{H(s)}=\frac{U_{12}U_{f2}s^3+(U_{12}+K_{12}U_{f2}+2U_{f2})GBs^2+(1+U_{12}+U_{f2})GB^2s+GB^3}{\{U_{12}U_{34}U_{f2}s^3+(U_{12}U_{34}+U_{12}U_{f2}+U_{34}U_{f2})GBs^2+(U_{12}U'+U_{34}+U_{f2})GB^2s+U'GB^3\}} \qquad (14)$$

$$S_{K_{f2}}^{H(s)}=\frac{K_{34}K_{f2}GB^2\{U_{12}s^2+(1+U_{12})GBs+GB^2\}}{(U_{f2}s+GB)\{U_{12}U_{34}U_{f2}s^3+(U_{12}U_{34}+U_{12}U_{f2}+U_{34}U_{f2})GBs^2+(U_{12}U'+U_{34}+U_{f2})GB^2s+U'GB^3\}} \qquad \cdots\cdots (15)$$

$$S_{GB}^{H(s)}=\frac{2U_{12}U_{34}U_{f2}^2s^4+U_{f2}(4U_{12}U_{34}+U_{12}U_{f2}+U_{34}U_{f2})GBs^3}{(U_{f2}s+GB)\{U_{12}U_{34}U_{f2}s^3+(U_{12}U_{34}+U_{12}U_{f2}+U_{34}U_{f2})GBs^2}\ *$$

$$*\ \frac{+2(U_{12}U_{34}+U_{12}U_{f2}+U_{34}U_{f2})GB^2s^2+(U_{12}U'+U_{34}+U_{f2}-U_{f2}U')GB^3s}{+(U_{12}U'+U_{34}+U_{f2})GB^2s+U'GB^3\}} \qquad (16)$$

For the $K's$ given above, Eqs. (14), (15) and (16)are simplified to

$$S_{K_{34}}^{H(j\omega)}=0.5-j\ \frac{GB}{4\omega} \qquad \cdots\cdots (17)$$

$$S_{K_{f2}}^{H(j\omega)}+\frac{-(8\omega^4+5GB^2\omega^2+2GB^4)}{4(2\cdot 6^2\omega^4-4^2GB^2\omega^2+5^2GB^2+GB^4)\omega^2}$$

$$-j\frac{GB(8\omega^4+3GB^2\omega^2+GB^4)}{4(2\cdot 6^2\omega^4-4^2GB^2\omega^2+5^2GB^2+GB^4)\omega^3} \qquad \cdots\cdots (18)$$

$$S_{GB}^{H(j\omega)}=\frac{8^2\omega^6+2^2\cdot 19GB^2\omega^4+15GB^4\omega^2+2GB^6}{2(2\cdot 6^2\omega^4-4^2GB^2\omega^2+5^2GB^2+GB^4)GB^2\omega^2}$$

$$-j\frac{3\cdot 4^2\omega^4-4\cdot 7^2GB^2\omega^2+5GB^4}{(2\cdot 6^2\omega^4-4^2GB^2\omega^2+5^2GB^2+GB^4)GB} \qquad \cdots\cdots (19)$$

Assuming that $GB \gg \omega$, we obtain

$$\mathrm{Re}\ S_{K_{12}}^{H(j\omega)}=\frac{7}{8} \quad , \mathrm{Im}\ S_{K_{12}}^{H(j\omega)}=-\frac{\omega}{GB} \qquad (20)$$

$$\mathrm{Re}\ S_{K_{34}}^{H(j\omega)}=0.5 \quad , \mathrm{Im}\ S_{K_{34}}^{H(j\omega)}=-\frac{GB}{4\omega} \qquad (21)$$

$$\mathrm{Re}\ S_{K_{f2}}^{H(j\omega)}=-\frac{0.5}{\omega^2} \quad , \mathrm{Im}\ S_{K_{f2}}^{H(j\omega)}=-\frac{GB}{4\omega^3} \qquad (22)$$

$$\mathrm{Re}\ S_{GB}^{H(j\omega)}=\frac{1}{\omega^2} \quad , \mathrm{Im}\ S_{GB}^{H(j\omega)}=-\frac{5}{\omega GB} \qquad (23)$$

where Re and Im stand for the real and imaginary part, respectively. It is clear that most of the sensitivity functions except Im $S_{K_{34}}^{H(j\omega)}$ and Im $S_{K_{f2}}^{H(j\omega)}$ are less than or equal to nearly zero.

## 5. Transient response

The transient response[10] of the integrator is studied by calculating the response to a step function. Letting $K_{34}=K_{f2}=1$ in Eq. (7), the closed loop gain $H(s)$ is shown as

$$H(s)=\frac{K_{12}GB^2(2s+GB)}{4s(s+GB)\{(1+K_{12})s+GB\}} \qquad (24)$$

If the input to the integrator is positive step function $Vi(s)$

$$V_i(s)=\frac{E}{s} \qquad (25)$$

then the response $V_o(s)$ is given as

$$V_o(s)=H(s)\ V_i(s)$$

$$=\frac{K_{12}GB^2(2s+GB)}{4s^2(s+GB)(1+K_{12})s+GB}\cdot E \qquad (26)$$

The partial-fraction expansion of Eq. (26) consists of four terms as given by

$$V_o(s)=\frac{E}{4}\{-K_{12}^2\frac{1}{s}+K_{12}GB\frac{1}{s^2}+\frac{1}{s+GB}+(K_{12}^2-1)\frac{1}{s+\frac{GB}{K_{12}+1}}\} \qquad (27)$$

Taking the inverse Laplace transformation of Eq. (27), we obtain

$$V_o(t)=\frac{E}{4}\{-K_{12}^2+K_{12}GB\,t+e^{-GBt}+(K_{12}^2-1)e^{-\frac{GB}{K_{12}+1}t}\} \qquad (28)$$

Fig. 5 represent the transient response of this integrator and an ideal one. From this figure, we Know that there is principal error in actual response due to the finite bandwidth of op amp which causes a time lag.

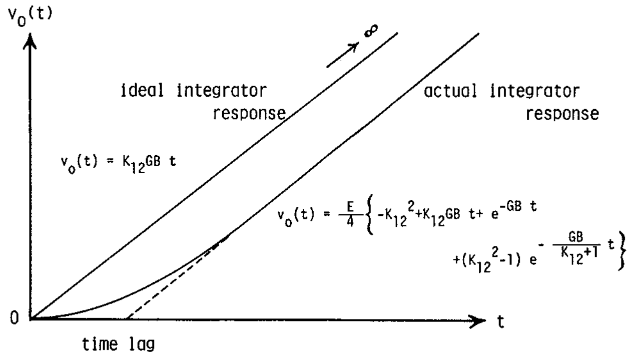


**Fig. 5 Transient response of the integrator**

## 6. Experimental results

The experimental circuit shown in Fig. 4 consists of two op amps, four resistors with 10 percent tolerance and a variable resistor. The op amps are $\mu A$ 741 types, and the resistor values are $R_2 = R_3 = R_4 = R_f = 10\,\mathrm{k\Omega}$. The power supply voltage is 15.0 V. Fig. 6 shows the frequency response of the integrator for various $K_{12}$ along with the Bode plot of Eq, (9) and the op amp's open loop frequency response. Experimental results agree well with the theoretical results. It is obvious that this circuit operates successfully over the wide frequency range from 100 Hz to 100 KHz which depends on the resistance ratio $K_{12}$. Exchanging the $\mu A$741 op amps for the other op amps with various gain bandwidth product value, we can obtain the desired output value. Similar results as above are obtained in case of changing the resistance ratio $K_{12}$.

Figs. 7 and 8 show the output waveform of the integrator for sinusoidal input of 1 KHz and the square wave input of 1 KHz, respectively. The output waveform swing is 12.5 $V$ peak to peak. This novel scheme also produce an output V′ which is 180 degrees out of phase with the output $V_o$.

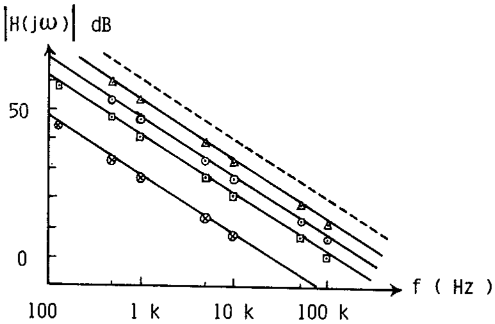


**Fig. 6 Bode plot of Eq. (9) and experimental results**
**--- : op amp open-loop gain**
**— : theoretical**
**△ : measured for $K_{12} = 2$**
**⊙ : = 1**
**⊡ : = 0.5**
**⊗ : = 0.1**

## 7. Conclusion

A new integrator using op amp pole has been proposed, and closed loop gain, transient response and sensitivity function have been analyzed. As a result of the analysis and experiment, it has been shown that the new integrator works successfully over the wide frequency range from 100 Hz to 100 kHz. The gain constant can be varied easily by changing resistance ratio $K_{12}$ or op amps. Both the

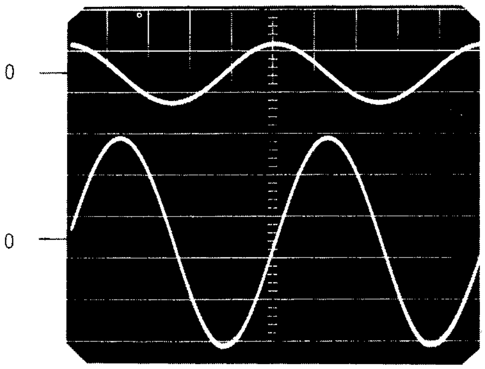


**Fig. 7 Output waveform of the integrator for the sinusoidal input**
**Upper trace : input waveform, 1KHz**
**($V$ : 0.1 V/div)**
**Lower trace : output waveform**
**($V$ : 5.0 V/div)**

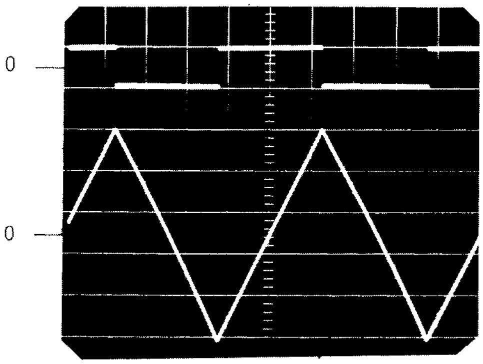


**Fig. 8 Output waveform of the integrator for the square wave input**
**Upper trace : input waveform, 1KHz**
**($V$ : 0.1 V/div)**
**Lower trace : output waveform**
**(V : 5.0 V/div)**

noninverting and inverting output can be obtained simultaneously. Since the new integrator has no capacitor, it presents some distinct features as follows

1. dc stable since it has dc feedback path
2. Suitable for IC implementation
3. No effect of the leakage current
4. Being able to adjust the dc offset with ease

We expect that this novel integrator will find many applications.

## References

1 ) Soliman, A. M. and Fawzy, M. : "A bandpass filter using operational amplifier pole", IEEE J. Solid-State Circuits, **SC-12**, pp. 429-430, 1977.

2 ) Ananda Mohan, P. V. : "Low-pass and notch filters using the operational amplifier pole", Proc. IEEE, vol. 67, pp. 1160-1162, 1979.

3 ) Ananda Mohan, P. V. : "Novel active filters using operational amplifier pole", Electron. Lett., vol. 16, pp. 378-380, 1980.

4 ) Budak, A. and Petrela, D. M. : "Frequency limitations of active filters using operational amplifiers", IEEE Trans. Circuit Theory, vol. **CT-19**, pp. 322-328, 1972.

5 ) Wait, J. V., Huelsman, L. P. and Korn, G. A. :"Intorduction to operational amplifier theory and applications ", McGraw-Hill, New York, pp. 13-54, 1975.

6 ) Wong, Y. J. and Ott, W, E. : "Function circuits design and applications", McGraw-Hill, New York, pp. 37-42, 1976.

7 ) Budak, A. : Passive and active network analysis and synthesis",Houghton Mifflin, Chap. 7, pp. 203-240, 1974.

8 ) Huelsman, L. P. : "Theory and design of active RC circuits", McGraw-Hill, pp. 14-19, 1968.
9 ) Huelsman, L. P. and Allen, P. E. : "Introduction to the theory and design of active filters", McGraw-Hill, New York, Chap. 3, pp. 102-114 and Chap. 5, pp. 232-241, 1980.
10 ) Graeme, J. G., Tobey, G. E. and Huelsman, L. P. : "Operational amplifiers design and applications", McGraw–Hill, New York, Chap. 2, pp. 213-218, 1971.