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Use of inerter devices for weight reduction of tuned mass-dampers for seismic protection of multi-storey buildings: the tuned mass-damper-inerter (TMDI)

Agathoklis Giaralis*[a], Laurentiu Marian[a]
[a]Dept of Civil Engineering, City University London, Northampton Square, EC1V 0HB, London, UK

ABSTRACT

This paper explores the practical benefits of the recently proposed by the authors tuned mass-damper-inerter (TMDI) vis-à-vis the classical tuned mass-damper (TMD) for the passive vibration control of seismically excited linearly building structures assumed to respond linearly. Special attention is focused on showcasing that the TMDI requires considerably reduced attached mass/weight to achieve the same vibration suppression level as the classical TMD by exploiting the mass amplification effect of the ideal inerter device. The latter allows for increasing the inertial property of the TMDI without a significant increase to its physical weight. To this end, novel numerical results pertaining to a seismically excited 3-storey frame building equipped with optimally designed TMDIs for various values of attached mass and inertance (i.e., constant of proportionality of the inerter resisting force in mass units) are furnished. The seismic action is modelled by a non-stationary stochastic process compatible with the elastic acceleration response spectrum of the European seismic code (Eurocode 8), while the TMDIs are tuned to minimize the mean square top floor displacement. It is shown that the TMDI achieves the same level of performance as an unconventional "large mass" TMD for seismic protection (i.e., more than 10% of attached mass of the total building mass), by incorporating attached masses similar to the ones used for controlling wind-induced vibrations via TMDs (i.e., 1%-5% of the total building mass). Moreover, numerical data from response history analyses for a suite of Eurocode 8 compatible recorded ground motions further demonstrate that optimally tuned TMDIs for top floor displacement minimization achieve considerable reductions in terms of top floor acceleration and attached mass displacement (stroke) compared to the classical TMD with the same attached mass.

Keywords: passive vibration control, tuned mass damper, inerter, optimum design, earthquake resistant buildings, non-stationary stochastic process, Eurocode 8

1. INTRODUCTION

For several decades, the concept of the tuned mass-damper (TMD) has been considered as a viable solution for the protection of building structures exposed to the earthquake hazard in the context of passive vibration control[1,2]. In its simplest form, the linear TMD comprises a mass attached towards the top of the building whose oscillatory motion is to be controlled (primary structure) via optimally designed/"tuned" linear stiffeners, or hangers in case of pendulum-like TMD implementations, in conjunction with linear energy dissipation devices (i.e., viscous dampers). The effectiveness of the TMD relies on "tuning" its stiffness and damping properties for a given primary structure and attached mass, such that significant kinetic energy is transferred from the vibrating primary structure to the TMD mass and eventually dissipated through the damping devices. No matter what performance criteria are adopted in this design, it is widely recognized that the performance of TMDs for the seismic protection of civil engineering structures depends heavily on its inertia properties[2-4]. Practically speaking, the larger the attached TMD mass that can be accommodated, subject to structural design and architectural constraints, the more effective and robust the TMD becomes for passive vibration control of earthquake induced oscillations.

*agathoklis@city.ac.uk; phone +44 (0)207040 8104

In this context, various researchers proposed the implementation of unconventionally large mass TMDs by connecting the top floor, or the last few top floors, to the rest of the building via isolators and, therefore, by treating the top floor(s) of buildings as the “attached” TMD mass[3-6]. In this manner, the TMD mass may reach up to 50% of the total mass of the building or more[3,4]. Such “exotic” solutions are not only demanding (and costly) from the structural design and construction viewpoint, but they also add uncertainty and complexity to the optimum TMD design/tuning since under severe ground motions the isolators exhibit non-linear behavior.

In this regard, this paper explores the potential of the recently proposed by the authors tuned mass-damper-inerter (TMDI) configuration[7,8] to control earthquake induced vibrations in multi-storey buildings using *significantly reduced attached mass* compared to the previously discussed large mass TMDs, while achieving similar performance levels. Specifically, the TMDI benefits from the mass amplification property of an inerter: a two-terminal device developing a resisting force proportional to the relative acceleration of its terminals having a constant of proportionality, termed inertance, in mass units[9]. Remarkably, the inertance can be orders of magnitude larger than the physical mass in typical inerter device prototypes[10]. In this respect, it was shown analytically and numerically that optimally designed TMDIs outperform the classical TMD *for the same attached mass* in terms of relative displacement variance of linear primary structures under broad-band (white noise) and narrow-band stationary stochastic base excitations[8]. Furthermore, the TMDI is linear and constitutes a generalization of the classical TMD. Therefore, all standard optimum design techniques for the classical TMD are also applicable to the TMDI, with certain modifications, as shown by Marian and Giaralis for both stochastically[8] and harmonically[11] base-excited primary structures. Moreover, the manufacturing cost of inerter devices, which are not yet commercially available for earthquake engineering applications despite the recent research interest[12,13], is expected to be similar to the cost of the various passive energy dissipation devices widely used in such applications. This is because inerter devices are relatively easy and cost-efficient to build exploiting either simple mechanical gearing arrangements[10] or fluid mechanics principles[14,15]. In this regard, the TMDI may circumvent the previously identified practical shortcomings of large mass TMDs once inerter devices for large-scale civil engineering applications become available.

The remainder of the paper is organized as follows. First, a brief review of the TMDI system for ground excited multi-storey building structures is provided, followed by the description of the herein adopted numerical approach for optimum design of TMDI equipped buildings under earthquake excitation modeled as a non-stationary stochastic process. Next, novel numerical results involving a 3-storey frame building structure excited by a non-stationary process compatible with the elastic response spectrum of European seismic code provisions[16] (Eurocode 8) are provided to gauge the level of mass/weight reduction achieved in the optimum design of the TMDI vis-à-vis the classical TMD. Finally, the performance of the optimally designed TMDI vis-à-vis the classical TMD is assessed in terms of top floor displacement and acceleration and attached mass displacement by means of response history analyses for a suite of Eurocode 8 spectrum compatible accelerograms.

2. THE TMDI FOR BASE EXCITED MULTI-STOREY SHEAR FRAME BUILDINGS

Conceptually introduced by Smith[9], the ideal inerter is a linear two terminal device of negligible mass/weight developing an internal (resisting) force F proportional to the relative acceleration of its terminals which are free to move independently. Its resisting force is expressed as

$$F = b(\ddot{u}_1 - \ddot{u}_2), \tag{1}$$

where u_1 and u_2 are the displacement coordinates of the inerter terminals as shown in the inlet of Figure 1 and, hereafter, a dot over a symbol signifies differentiation with respect to time. In the above equation, the constant of proportionality b is the so-called inertance and has mass units; it fully characterizes the behavior of the ideal inerter. Importantly, the physical mass of actual inerter devices can be two or more orders of magnitude lower than b. This has been experimentally validated by testing several flywheel-based prototyped inerter devices incorporating rack-and-pinion or ball-screw mechanisms to transform the translational kinetic energy into rotational kinetic energy “stored” in a relatively light rotating disk[10]. More recently, fluid inerters achieving inertance values b that are almost independent of the physical device mass were also built and experimentally verified[14,15]. In this regard, the ideal inerter can be construed as an inertial amplification device, since by “grounding” any one of its terminals, the device acts as a “weightless” mass b[9].

This property of the inerter is exploited by the herein considered TMDI (tuned mass-damper-inerter) to enhance the vibration suppression capabilities of the classical TMD[7,8].

The topology of the TMDI system proposed by the authors[7,8] for planar base-excited n-storey shear frame building primary structures modeled as lumped-mass multi-degree-of-freedom (MDOF) "chain-like" linear damped systems is shown in Figure 1. It involves a classical TMD (tuned mass-damper) located at the top floor of the primary structure comprising a mass m_d attached to the structure via a linear spring of stiffness k_d and a linear dashpot of damping coefficient c_d. The TMD mass is linked to the penultimate frame floor by an inerter device with inertance b. The $n+1$ equations of motion of the resulting MDOF system subject to a lateral ground motion represented by an acceleration stochastic process $\alpha_g(t)$ are written in matrix form as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=-\mathbf{M}_o\boldsymbol{\delta}a_g(t), \tag{2}$$

where $\boldsymbol{\delta}$ is the (n+1)-length unit column vector, and $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\mathbf{x}$ are the mass matrix, the damping matrix, the stiffness matrix, and the response displacements vector expressed as

$$\mathbf{M}=\begin{bmatrix} m_d+b & 0 & -b & 0 & \cdots & 0 \\ 0 & m_1 & 0 & 0 & \cdots & \vdots \\ -b & 0 & m_2+b & 0 & \cdots & \vdots \\ 0 & 0 & 0 & m_3 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & m_n \end{bmatrix}, \mathbf{C}=\begin{bmatrix} c_d & -c_d & 0 & \cdots & 0 \\ -c_d & c_1+c_d & -c_1 & 0 & \vdots \\ 0 & -c_1 & c_1+c_2 & -c_2 & \vdots \\ 0 & 0 & -c_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & -c_{n-1} \\ 0 & \cdots & 0 & -c_{n-1} & c_{n-1}+c_n \end{bmatrix},$$

$$\mathbf{K}=\begin{bmatrix} k_d & -k_d & 0 & \cdots & 0 \\ -k_d & k_1+k_d & -k_1 & 0 & \vdots \\ 0 & -k_1 & k_1+k_2 & -k_2 & \vdots \\ 0 & 0 & -k_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & -k_{n-1} \\ 0 & \cdots & 0 & -k_{n-1} & k_{n-1}+k_n \end{bmatrix}, \text{ and } \mathbf{x}(t)=\begin{bmatrix} x_d(t) \\ x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix}, \tag{3}$$

respectively, while $\mathbf{M_o}$ is coincides with $\mathbf{M}$ for b=0. In the above expressions, m_i, k_i, and c_i are the mass, lateral stiffness, and damping of floor i (i=1,2,…,n) and m_d, k_d, c_d, and b are the TMDI mass, stiffness, damping, and inertance coefficients (see also Figure 1). Further, x_i (i=1,2,…,n) are the lateral floor deflections and x_d is the displacement of the attached TMDI mass relative to the ground displacement.

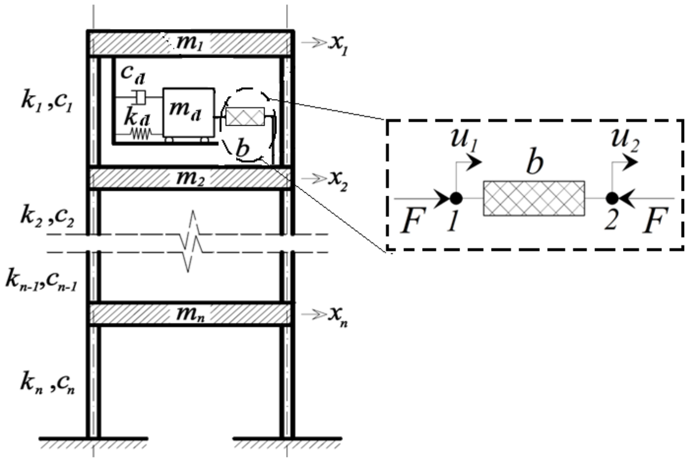

Figure 1. The tuned mass-damper-inerter (TMDI) system for multi-storey frame buildings[7,8]

Note that for $b=0$, the Eqs. (2) and (3) govern the response of a seismically excited multi-storey shear frame building equipped with the classical TMD attached to its top floor. The latter TMD topology is widely considered to control the first mode of vibration of multi-storey buildings which, for regular in elevation structures, dominates their dynamic response to broadband earthquake excitations[1]. The inclusion of the inerter device alters the mass matrix which is no longer diagonal. However, the overall structural system remains linear and, from a practical viewpoint, the same well-established computational methods for optimal design of the classical TMD to control the seismic response of multi-storey buildings according to their first mode[1,17] can be readily applied for the optimal design of the TMDI system.

Moreover, Eq. (3) suggests that the total inertia of the TMDI is equal to (m_d+b). This observation motivates the definition of the following dimensionless frequency ratio υ_{TMDI} and damping ratio ζ_{TMDI}

$$\upsilon_{TMDI} = \frac{\sqrt{\dfrac{k_d}{(m_d + b)}}}{\omega_1} \quad , \; \zeta_{TMDI} = \frac{c_d}{2(m_d + b)\omega_d} , \tag{4}$$

to characterize the design of the TMDI, where ω_1 is the first (fundamental) natural frequency of the primary structure. The following section presents the adopted optimum TMDI design strategy which seeks to determine the above parameters given the properties of the primary structure (assumed to behave linearly), the attached mass m_d and the inertance b such that the displacement variance of the top floor is minimized under a given stochastic seismic excitation.

3. OPTIMAL DESIGN OF THE TMDI FOR MULTI-STOREY SEISMICALLY EXCITED SHEAR FRAME BUILDING STRUCTURES

Since the input seismic excitation is represented by a stochastic process and the TMDI equipped system is linear, it is convenient to cast the optimum design problem in the frequency domain to benefit from random vibrations input-output relationships of linear systems. To this end, the following transfer function can be defined relating the (input) support excitation in terms of acceleration to the (output) relative displacement of the top floor mass m_1 of the primary structure

$$G_1(s) = \mathbf{C_o} \cdot (s\mathbf{I}_{(2n+2)} - \mathbf{A})^{-1}\mathbf{B} , \tag{5}$$

where $\mathbf{C_o}$ is the (2n+2)-length measurement row vector given by [0 1 0 0 … 0], $\mathbf{I}_{(p)}$ is the square identity matrix of dimension p, the superscript (-1) denotes matrix inversion, and **A** and **B** are the system matrix and the input matrix given by

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{(n+1)} & \mathbf{I}_{(n+1)} \\ \mathbf{-M^{-1}K} & \mathbf{-M^{-1}C} \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{(n+1)} \\ \mathbf{I}_{(n+1)} \end{bmatrix}, \tag{6}$$

respectively. Further, in the previous expression $\mathbf{0_{(p)}}$ is the zero matrix of dimension p and s is the standard Laplacian coordinate on the complex plane. Evaluation of $G_1(s)$ along the imaginary axis $s= j\omega$ where $j=(-1)^{1/2}$ yields the frequency response function denoted henceforth as $G_1(\omega)$.

In this study, the input seismic excitation is modeled by an evolutionary power spectral density function (EPSD) $S(\omega,t)$ representing a non-stationary seismic stochastic process[18,19]. Therefore, optimal design values for the frequency ratio υ_{TMDI}, the damping ratio ζ_{TMDI} are sought to minimize the mean square displacement of the top floor of the primary structure, at the time when the input EPSD is maximized for a given primary structure, inerter coefficient b, and modal mass ratio μ. The latter quantity is defined as

$$\mu = \frac{m_d}{\boldsymbol{\varphi}_1^{\mathrm{T}} \mathbf{M_p} \boldsymbol{\varphi}_1} , \tag{7}$$

where $\mathbf{M_p}$ is the mass matrix of the primary structure (uncontrolled), the superscript "T" denotes matrix transposition and $\boldsymbol{\varphi}_1$ is the fundamental mode shape vector (eigenvector) of the primary structure normalized by the modal coordinate corresponding to the top floor mass (see also Rana and Soong[1]). Under the above assumptions, the following non-dimensional performance index (PI) is considered in the optimization problem (cost function)

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$$\mathrm{PI} = J^{TMDI} / J^{0} \quad ; \quad J^{TMDI} = \int_0^\infty \left|G_1(\omega)\right|^2 \max_t \left\{S(\omega,t)\right\} d\omega \ . \tag{8}$$

In the above equation, J^0 denotes the variance of the top floor displacement of the uncontrolled primary structure. Note that, notation-wise, for b=0: $J^{TMDI}= J^{TMD}$ and for $b=\mu=0$: $J^{TMDI}=J^0$.

In all ensuing numerical work a MATLAB® built-in "min-max" constraint optimization algorithm employing a sequential programming method is used to minimize the PI in Eq. (8) for the design parameters υ_{TMDI} and ζ_{TMDI}[17]. The algorithm is initialized using the following "seed" values for the design

$$\upsilon_{TMDI} = \frac{1}{1+\beta+\mu} \frac{\sqrt{[\beta(\mu-1)+(2-\mu)(1+\mu)]}}{\sqrt{2(1+\mu)}} , \tag{9}$$

and

$$\zeta_{TMDI} = \frac{\sqrt{(\beta+\mu)}\sqrt{\beta(3-\mu)+(4-\mu)(1+\mu)}}{2\sqrt{2(1+\beta+\mu)[\beta(1-\mu)+(2-\mu)(1+\mu)]}} , \tag{10}$$

where $\beta= b/m_d$. The latter two expressions yield optimally designed TMDI parameters minimizing J^{TMDI} for an undamped linear single degree of freedom primary structure under white noise base excitation as derived by Marian and Giaralis[8]. As a final note, the following constraints are imposed to the sought design parameters relying on physical considerations: $0.5\leq \upsilon_{TMDI}\leq 1.10$ and $0\leq \zeta_{TMDI}\leq 1.0$.

4. LIGHTWEIGHT TMDI OPTIMUM DESIGN FOR SHEAR FRAME BUILDING STRUCTURES

This section presents novel numerical results to demonstrate the potential of the TMDI to achieve a significantly more lightweight passive vibration control solution for the seismic protection of building structures compared to the classical TMD. A 3-storey building frame is used as the test-bed primary structure. The properties of the structure are given in Table 1 with the note that the damping matrix is taken as proportional to the stiffness matrix with coefficient of proportionality equal to $2\zeta_1/\omega_1$, where ζ_1= 2% is the critical damping ratio of the fundamental mode shape and ω_1= 4π (rad/s) is the fundamental natural frequency corresponding to a natural period of $T=0.5$s. Also reported in Table 1 is the fundamental mode shape normalized to the top floor displacement as required in Eq. (7).

Table 1. Inertial and elastic properties of the considered primary structure

Floor	Mass (kg)	Stiffness (N/m)	Damping (Ns/m)	Fundamental mode shape $\boldsymbol{\varphi}_1$ (T_1= 0.5s)
1 (top)	30 x 10^3	10 x 10^5	3183	1
2	30 x 10^3	30 x 10^5	9549	0.527
3	30 x 10^3	30 x 10^5	9549	0.286

The Eurocode 8 pseudo-acceleration response spectrum for peak ground acceleration 0.36g (g=981cm/s^2), ground type "B" (gray thick curve in Figure 2(a)), and damping ratio ζ=5% is assumed to represent the seismic action. Note that the damping ratio of the input response spectrum corresponds to a higher damping ratio than the considered (presumably bare steel) primary structure. This choice is made to account for the fact that the TMDI equipped structure will have an overall higher level of viscous/linear damping due to the presence of the viscous damper with (*a priori* unknown) damping coefficient c_d. A uniformly modulated non-stationary stochastic process compatible, in the mean sense, with the above EC8 spectrum is used as the ground excitation $a_g(t)$ in Eq. (2). The considered process has been derived as detailed in Giaralis and Spanos[19] and is defined by means of the EPSD

$$S(t,\omega) = C^2 t^2 \exp(-bt) \times \frac{\omega_g^4 + 4\zeta_g^2\omega^2\omega_g^2}{\left(\omega_g^2-\omega^2\right)^2 + 4\zeta_g^2\omega_g^2\omega^2} \times \frac{\omega^4}{\left(\omega_f^2-\omega^2\right)^2 + 4\zeta_f^2\omega_f^2\omega^2} , \tag{11}$$

with ω_g=10.73 rad/s, ζ_g=0.78, ω_f= 2.33 rad/s, ζ_f=0.90, C= 17.76 cm/s$^{2.5}$, and b=0.58 s^{-1}. The above EPSD is plotted in Figure 2(b) on the time-frequency plane and the time instant 3.44s at which the above function attains its maximum value is indicated. Further, the median response spectrum of an ensemble of 100 realizations (i.e., non-stationary artificial accelerograms) compatible with the EPSD in Eq.(11) is plotted in Figure 2(a) to illustrate the good level of mean sense compatibility achieved by the underlying stochastic process with the considered Eurocode 8 spectrum. These realizations have been generated using an auto-regressive-moving-average (ARMA) filtering technique as outlined in Giaralis and Spanos[18].

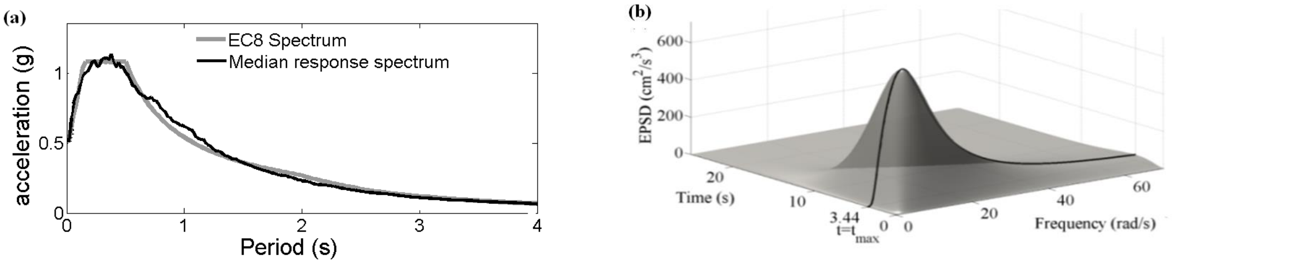

Figure 2: (a) Considered Eurocode 8 response spectrum and median response spectrum of 100 realizations compatible with the EPSD of Eq.(11) plotted in (b).

The optimization procedure described in the previous section is used to derive optimum TMDI parameters for both $b=0$ (i.e., classical TMD) and $b>0$ while considering pre-specified attached m_d mass values within a wide range: 1% to 50% of the total mass of the primary structure. Numerical results from the optimization algorithm in terms of optimum frequency ratio, optimum damping ratio, and achieved performance index are plotted in Figures 3(a), 3(b), and 3(c), respectively, as functions of the attached m_d mass and for various values of inertance b. These numerical data show that for attached mass m_d up to 10% of the mass of the primary structure, an increase of the attached masses m_d for fixed inertance b has similar effects to an increase of the inertance b for a fixed attached mass. That is, the optimum frequency ratio decreases, the optimum damping ratio increases and a better performance index (reduction of top floor displacement variance compared to the uncontrolled primary structure) is achieved. These observations are in accordance to what has been previously reported by the authors[7,8].

However, not the same trends are observed for m_d mass higher than 10% of the mass of the primary structure (i.e., large mass TMD(I)s). Specifically, in this range of attached mass, the optimum damping ratio tends to saturate. The level of saturation depends on the values of the inertance. In fact, for inertance values above 50% of the mass of the primary structure, the optimum damping ratio remains constant or even slightly decreases with an increase as a function of the attached mass m_d. Accordingly, the decrease of the frequency ratios tend to saturate as well with increasing attached mass and asymptotically converge to a value of about 0.30. More importantly, the convexity of the curves of performance index vs attached mass m_d change for inertance values above 50%. As a result, it appears that the classical TMD becomes more effective than the TMDI to suppress the top floor displacement (fundamental mode shape) for m_d greater than about 12% of the mass of the primary structure. This result verifies engineering intuition that the physical mass may only be replaced by the inertance up to a certain level, above which, a larger mass TMD will always outperform the TMDI. Indeed, the inclusion of the inerter appears to be more beneficial for relatively small attached m_d mass. Nevertheless, it is herein pointed out that perhaps the most remarkable practical benefit of the TMDI for earthquake engineering applications is that it can achieve the same level of performance for significantly reduced attached m_d mass (and thus reduced overall additional weight to the structure). This point can be appreciated by taking horizontal stripes in Figure 3(c) corresponding to a fixed performance and read the value of the attached mass required to achieve this performance for different values of the inertance b. To facilitate this interpretation, Figure 4(d) reports the value of the attached mass required to achieve different levels of performance for the classical TMD and for a TMDI with inertance equal to about 55% the mass of the primary structure. The reductions noted are dramatic and indicate that TMDIs with very high inertance values perform equally well with unconventional large mass TMDs for attached masses considered for wind vibrations mitigation (about 1%-5%)[20].

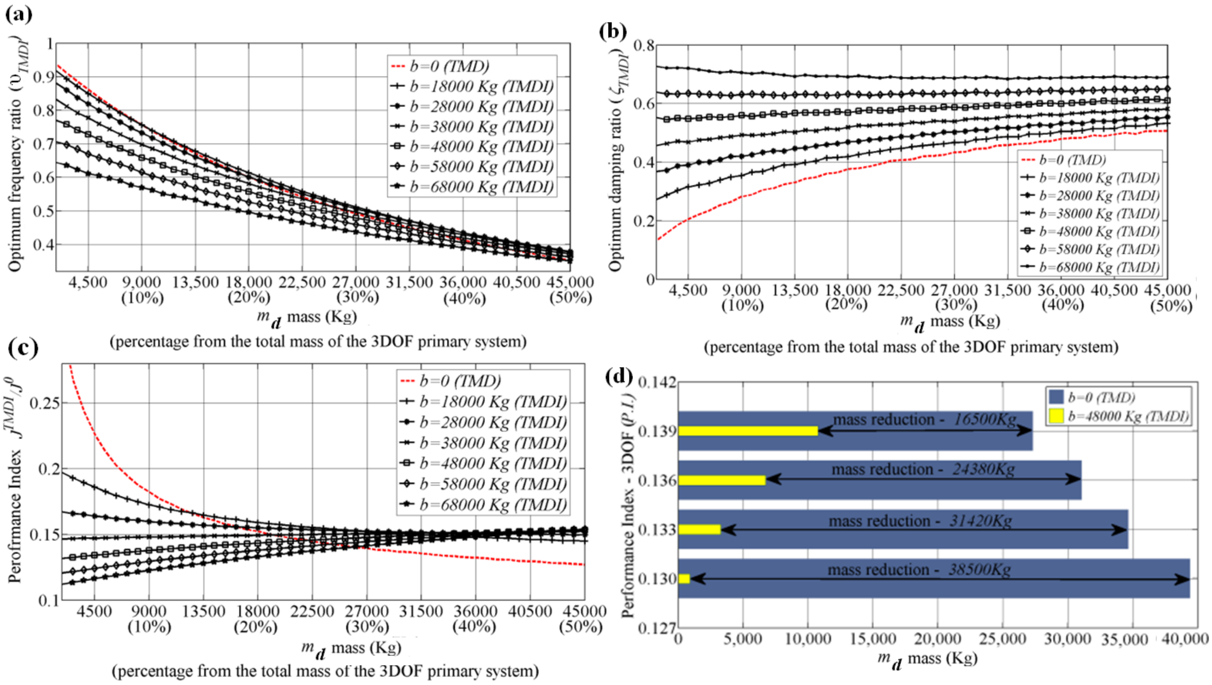

Figure 3: (a) Optimum frequency ratio; (b) Optimum damping ratio; (c) Performance index; (d) mass reduction for a given performance index, for various values of inertance *b* and attached mass m_d.

5. PERFORMANCE ASSESSMENT FOR RECORDED GROUND MOTIONS

This section furnishes further numerical results to assess the effectiveness of the TMDI vis-à-vis the classical TMD for passive vibration control of seismically excited building structures. To this aim, the peak top floor deflection and acceleration, as well as the peak relative displacement of the attached mass for the previously considered 3-storey primary structure equipped with a TMD with $m_d=900$ *kg* (10% of the total mass of the structure) and with a TMDI with the same attached mass and inertance $b=68000$ *kg* is obtained for a suite of 7 field recorded ground motions reported in Table 2. These ground motions have been chosen out of a larger ground motion data-bank constructed for the design and assessment of passively controlled building structures[21]. The two controlled structures considered are optimally designed as discussed in the previous section for the Eurocode 8 spectrum of Figure 2(a). To this end, the field recorded accelerograms have been non-uniformly scaled by means of the harmonic wavelet based approach proposed by Giaralis and Spanos[18], such that they comply to the Eurocode 8 compatibility criteria. Specifically, their average response spectral ordinates are greater than 90% of the target spectrum within a [$0.2T_1$ $2T_1$] period interval where T_1=0.5s is the fundamental natural period of the considered primary structure (Figure 4(a)). This numerical study is motivated by the fact that Eurocode 8 prescribes the use of the *average* of pertinent peak response quantities for design purposes when at least 7 response history analyses are performed for spectrum compatible accelerograms. Pertinent results for the uncontrolled primary structure are also included in Table 2.

On average (Table 2), the TMD achieves more than 50% peak response displacement and acceleration reduction compared to the uncontrolled primary structure. The TMDI achieves a further 30% average reduction in terms of peak top floor deflection. Less expected is the fact that the TMDI achieves about 3 times reduced peak top floor acceleration and attached mass displacement compared to the classical TMD system. The former response quantity is associated with the vulnerability of secondary systems, while the latter quantity is important in the practical design of the TMDI and is related to the required "stroke" of the damping device. The above observations in terms of peak response are confirmed from the time-histories of response quantities as those shown in Figures 4(b) and 4(c) for illustration.

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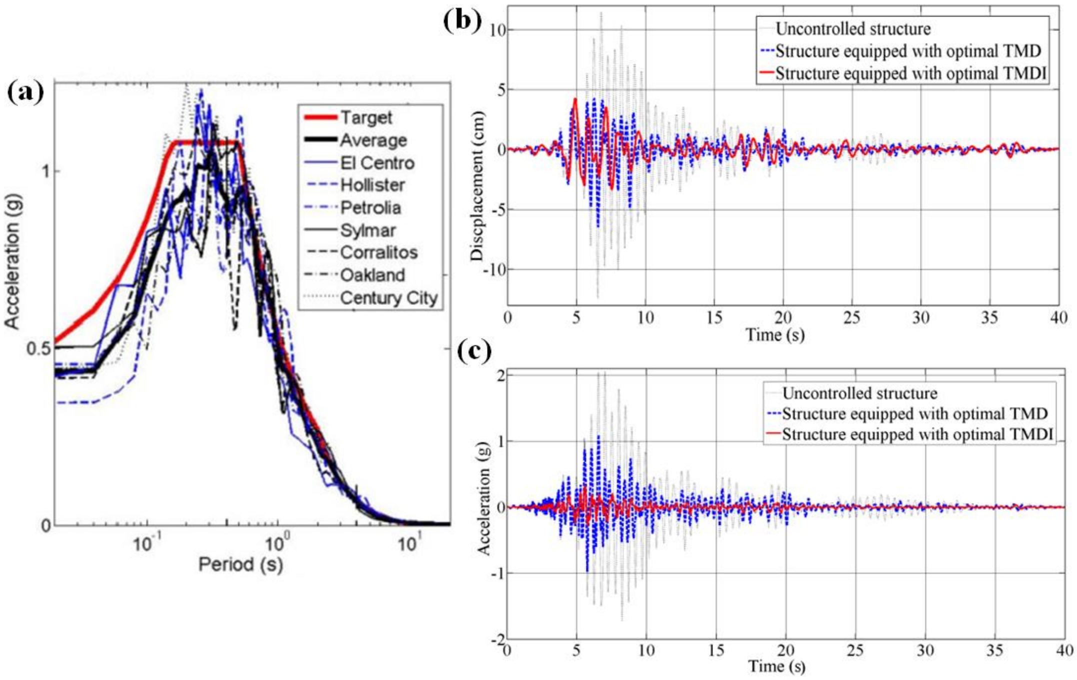

Figure 4: (a) Response spectra of the considered Eurocode 8 compatible ground motions listed in Table 2; (b) and (c) Top floor deflection and acceleration for the Century City ground motion component.

Table 2: Peak response quantities for a suite of 7 Eurocode 8 compatible ground motions

Earthquake Component (Seismic event)	**Peak top floor displacement (cm)**			**Peak top floor acceleration (g)**			**Peak attached mass displacement (cm)**	
	Primary structure(1)	TMD(2)	TMDI(3)	Primary structure(1)	TMD(2)	TMDI(3)	TMD(2)	TMDI(3)
Petrolia- 90° (Petrolia,1992)	14.92	5.58	3.98	2.25	1.15	0.28	11.11	4.09
Corralitos- 90° Eureka Canyon (Loma Prieta, 1989)	10.36	5.90	3.73	1.67	1.03	0.32	11.43	3.80
El Centro #6-230° Huston Rd. (Imperial Val., 1979)	11.28	6.79	4.86	2.04	0.99	0.47	13.92	4.20
Hollister-90° South St & Pine Dr (Loma Prieta,1989)	10.02	6.93	4.09	1.79	1.14	0.37	13.87	4.43
Oakland-35° Outer harbor wharf (Loma Prieta1989)	11.54	5.30	4.15	1.82	1.22	0.33	12.56	4.47
Century City-90° LACC North (Northridge, 1994)	12.43	6.46	4.28	2.06	1.10	0.34	11.51	4.58
Sylmar- 90° County Hospital (Northridge, 1994)	14.45	5.54	5.05	2.57	1.23	0.3	11.01	5.60
Average	12.05	6.07	4.31	2.03	1.12	0.35	12.20	4.45

(1) Uncontrolled structure; (2) TMD with m_d= 9000Kg; (3) TMDI with m_d= 9000Kg and b=68000Kg

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6. CONCLUDING REMARKS

Novel numerical results have been reported shedding new light to the potential practical benefits of the recently proposed by the authors tuned mass-damper-inerter (TMDI) vis-à-vis the classical tuned mass-damper (TMD) for the seismic protection of building structures. It is noted that the TMDI constitutes a generalization of the TMD incorporating a mass amplification inerter device in addition to the spring and viscous damper elements of the TMD to link an attached mass to the primary building structure. The considered results pertain to optimally designed TMDIs for various attached masses and values of inertance minimizing the mean square top floor displacement of a specific 3-storey frame building structure base-excited by a non-stationary acceleration stochastic process compatible with the elastic design spectrum of Eurocode 8. It is concluded that although the classical TMD outperforms the TMDI for large values of attached mass, similar to those proposed by some researchers for the seismic protection of building structures, a TMDI can achieve the same level of performance as the classical TMD for significantly reduced attached mass/weight of the same order to those used for wind-induced vibrations mitigation. This advantage of the TMDI may arguably be the most significant one in practical terms and warrants further numerical investigation, currently undertaken by the authors. Furthermore, numerical data from response history analyses for a suite of Eurocode 8 compatible recorded ground motions have been also reported to demonstrate that optimally tuned TMDIs achieve considerable reduction compared to the classical TMD with the same attached mass not only to the top floor displacement, but also to the top floor acceleration and to the displacement of the attached oscillating mass (stroke).

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