Optimal configuration of a power grid system with a dynamic performance sharing mechanism[1]

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Abstract: Performance sharing is an effective policy for a power grid system to satisfy the power demand of different districts to greatest extent. Through transmission lines, the districts with sufficient power can share the redundant power with the districts with power deficit. The existing research has incorporated the performance sharing mechanism into systems with simple structures such as parallel systems and series-parallel systems. However, little concentration has been spent on more complex structures. This necessitates the need of this paper that models a power distribution with a more complex reliability structure. We assume that the system is composed of generators and nodes. Both the performance of each generator and the demand of each node in the network are assumed to be random variables. This paper first proposes a dynamic performance sharing policy to minimize the unsupplied demand for a given system with fixed capacity and demand. The optimal allocation of generators, which minimizes the expected system unsupplied demand, is then studied. Numerical examples are proposed to illustrate the applications of the proposed procedures.

Keywords: power grid system; performance sharing mechanism; optimal generator allocation policy; Hybridized Particle Swarm Optimization (HPSO) algorithm

1. Introduction

Power grid systems are essential for people's daily life and industrial production processes (Li and Zio, 2012; Rodgers et al., 2018; Figueroa-Candia et al., 2018). Therefore, it is important to ensure that the power demand is met (Zimmerman et al., 2011; Zhai et al., 2018). One of the solutions is to optimally allocate electricity in real time according to the supply and demand within a power grid system (Wang et al., 2018). Frank et al. (2016) and Lavaei et al. (2012) studied the optimization problems in the electric power systems, known collectively as Optimal Power Flow (OPF). For the future smart grids, another solution to meet power demand is Demand Response (DR), which is achieved by active customer participation in real time to maintain balance between generation and demand with two-way communication (Pourmousavi and Nehrir, 2014). DR solutions can be deployed to encourage electricity consumers in

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scheduling their controllable demands to off-peak hours (Bahrami et al., 2017). A power grid system may contain many generators, which are used to satisfy the power demand in different districts (Xiang et al., 2017). As power production may be influenced by the state of generators and the demand may also vary, the power produced in a district may not be able to satisfy its power demand (Faza, 2017). In light of this, the performance sharing mechanism is essential such that the districts with sufficient power supply can share the redundant power with the districts with deficient power supply (Tolani & Sensarma, 2017). The power grid system can be regarded reliable as long as the demands of all the districts can be satisfied after proper power redistribution.

Some works studied the reliability of systems with performance sharing mechanism. Levitin (2011) studied a system consisting of some unrepairable units, each of which has a random performance and a random demand to be satisfied. The units can share performance with each other but the total amount of shared performance is limited by the bandwidth of the performance sharing mechanism. The system works if the demand of all the units can be satisfied with proper performance sharing. A Universal Generating Function (UGF) technique is proposed to evaluate the reliability of the system. Xiao and Peng (2014) analyzed the case where the system has several subsystems to share performance with each other and each subsystem contains several units structured in parallel. The optimal allocation of units and the optimal preventive maintenance interval of the units are investigated. Xiao and Peng (2014) studied the optimal maintenance and protection of the system units, which are subject to both internal failures and external impacts. Yu et al. (2014) extended Levitin (2011) to the case of repairable systems. The availability of each system unit is evaluated first by considering the state transitions, which are due to degradation or maintenance, and then the UGF technique is used to calculate the system availability. Peng et al. (2017) extended Levitin (2011) to the case where the performance sharing group has limited size and studied the optimal choice of units to include in the performance sharing group. Yu et al. (2017) proposed a model of a system with multiple phases, where the units can share performance with each other in each phase. Zhai et al. (2017) studied the optimal defense and attack of a system with a performance sharing mechanism. Zhao et al. (2018) studied a k-out-of-n system with performance sharing mechanism. Wu et al. (2019) studied the reliability of capacitated systems with a performance sharing mechanism.

However, all the above works are restricted to systems with single performance sharing group. Peng et al. (2016) considered a different scenario, where a system has two performance sharing groups. Later, Peng (2019) studied a system with hierarchical performance sharing groups. However, these models are still not able to accommodate more distinct system structures and more arbitrary performance sharing among the system units. In practical systems, what is constrained may not be the total amount of performance that can be shared in the whole system or a subsystem. Instead, the performance sharing between any two nodes may be constrained by the performance sharing mechanism between them. For example, a power grid system may be of a networked structure with many nodes representing different districts, and the power sharing between each two nodes may be constrained by the bandwidth of the transmission lines between them. In this paper, a general network model is proposed to model a power grid system with the performance sharing mechanism. Each node in the network has a random performance and a random demand to be satisfied. The link between any two nodes serves a channel for the performance sharing between the two nodes, and has a bandwidth that restricts the amount of performance that can be shared. For the case where the allocation of generators, the realization of the generators' capacity and nodes' demand is given, a dynamic performance sharing mechanism is proposed to minimize the unsupplied demand. The expected system unsupplied demand can be obtained by taking into account all the possible realizations of the nodes' performance and demand. The allocation of generators, which minimizes the expected system unsupplied demand, can be solved.

The remainder of this paper is as follows. Section 2 describes the system and assumptions. Section 3 focuses on the optimal power sharing problem for given realization of the nodes' demand and the generators' capacity and given generators' allocation. The case where the power transmission loss can be neglected and the case where the power transmission loss cannot be neglected are considered. Section 4 formulates the optimal allocation of the generators that minimizes the expected unsupplied demand of the systems. Section 5 presents the optimization techniques adopted. Section 6 presents numerical examples to illustrate the application of the proposed model. Section 7 presents a case study based on practical background. Section 8 concludes the paper.

2. System description

Consider a system consisting of N nodes, from node 1 to node N, representing N districts in need of power supply. The number of available generators is M. M generators are assigned to N nodes, and the system structure is shown in the Fig 1.

Fig 1. A typical power grid system

Where, a box represents a node, a line represents a power transmission line, and M_{ij_r} refers to the j_r th generator in the i th node. $i = 1, 2, \ldots$ $j_1 + j_2 + \ldots$ M .

At each node, several power generators can be deployed. It is assumed that both the power to be generated by each generator and the power demand of each district are random variables. It is also assumed that the power generated by different generators is statistically independent, so are the power demands on different nodes. In fact, the demand and generating capacity are changing from time to time (Lisnianski & Ding, 2016). However, if one focuses on the demand and the generated power for a specific period of concern, then both the demand and capacity are random variables. In practical applications, the probability distribution of a generator's capacity can be estimated from historical data. Similarly, the power demand at each node may also change from time to time under the influence of various factors and its probability distribution can be estimated from historical data of power consumption. To facilitate discussion, the distribution for the generators' capacity is also called a capacity distribution, and the distribution for the demand at each node is also called a demand distribution.

Suppose that the capacity of generator k $(k = 1, 2, ..., M)$ has realizations $a_k(1) < a_k(2) < ... < a_k(L_k)$, where L_k is the number of possible realizations for the capacity of generator k. The corresponding probabilities for $a_k(1), a_k(2), ..., a_k(L_k)$ are $p_k(1), p_k(2), ..., p_k(L_k)$, respectively, where $p_k(1) + p_k(2) + ... + p_k(L_k) = 1$. The demand of node $i(i = 1, 2, ..., N)$ has realizations $d_i(1) < d_i(2) < ... < d_i(W_i)$, where W_i is the number of possible realizations for the demand of node i. The corresponding probabilities for $d_i(1), d_i(2), ..., d_i(W_i)$ are $q_i(1), q_i(2), ..., q_i(W_i)$, where $q_i(1) + q_i(2) + ... + q_i(W_i) = 1$. Between each pair of districts, there may be a power distribution line facilitating the power transmission between them. The amount of power transmission is constrained by the bandwidth of the transmission line between them. Typically, the maximum amount of the power transmission between nodes i and j is denoted as c_{ij}. The case of no transmission line between nodes i and j corresponds to $c_{ij} = 0$. If the allocation of generators, the realization of the generators' capacity and the nodes' demand are given, the power is dynamically transmitted between the system nodes so that the unsupplied demand of the system is minimized. The optimal allocation of the generators that minimizes the expected unsupplied demand of the system needs to be solved.

The notations used in this paper are defined as follows.

Notations	Definitions
N	The number of nodes
M	The number of generators
c_{ij}	The transmission bandwidth between nodes i and j
d_i	The power demand at node i
g_i	The amount of power produced by generators from node i
X_{ij}	The performance transmitted from any node i to any other node j

$K_{ij},\left(0\le K_{ij}\le 1\right)$	The loss rate for power transmission from node i to node j
f_{before}	The unsupplied power demand before performance sharing
f_{after}	The unsupplied power demand after performance sharing
L_k	The number of possible realizations for the capacity of generator k
$u_k\left(z\right)=\sum_{l=1}^{L_k}p_k\left(l\right)z^{a_k(l)},$	The UGF used to represent the capacity distribution of generator k
$a_k\left(l\right)$	The capacity realization of generator
$p_k\left(l\right)$	The corresponding probability of the capacity realization of the generator
$d_i\left(l\right)$	The demand realization of the node i
$q_i\left(l\right)$	The corresponding probability of the demand realization of the node i.
$n\left(i\right)$	The number of generators allocated into node i
$g_i\left(l\right)$	The capacity realization of the node i
$p_i\left(l\right)$	The corresponding probability of the capacity realization of the node i
$H=\left\{h\left(k\right),1\le k\le M\right\}$	The generator allocation strategy where each generators k is assigned to the $h(k)$ th node
W_i	The number of demand realizations for the node i
$V=\prod_{i=1}^{N}L_{ig}W_i$	The number of different realizations of the generators' capacity and the nodes' demand
f_t	The minimized unsupplied demand for each realization t

$P_{f_t}(t=1,2,\ldots$	The probability for the realization

3. Optimal power sharing policy

For a given generator allocation policy and given realizations of the generators' performances and the nodes' demand, the power sharing policy that minimizes the unsupplied demand of the power grid needs to be solved. To facilitate discussion, the following assumptions are made:

(1) Suppose the total amount of power produced by generators from node i is g_i, the power demand at node i is d_i. The excess supply at node i is therefore given by $\max(g_i - d_i, 0)$, the deficit at node is $\max(d_i - g_i, 0)$.

(2) Between each pair of districts, there may be a power distribution line facilitating the power transmission. The amount of power that can be transmitted between each pair of nodes is constrained by the bandwidth of the transmission line between them. We assume that the maximum amount of power that can be transmitted from node i to node j is constrained by the bandwidth c_{ij}.

(3) Let X_{ij} be the power exchange from any node i to any other node j. In practice, the power value is usually rounded to a finite number of digits. After choosing a proper unit, the power value can be integer. So, it is assumed that both X_{ij} and X_{ji} are integer variables. In addition, it is assumed $X_{ij}=0$ for $i=j$, and $\min(X_{ij}, X_{ji}) = 0$ for $i \neq j$.

3.1. The case where power transmission loss is neglected

Based on the above assumptions, if the power transmission loss between the system nodes can be neglected, the optimal power sharing problem can be represented by the following integer programming model

$$Min\ f = \sum_{i=1}^{N} \left\{ Max\left(d_i - \left(g_i + \sum_{j=1}^{N} X_{ji} - \sum_{j=1}^{N} X_{ij} \right),\ 0 \right) \right\}; \quad (1)$$

Subject to:

$$0 \le X_{ij} \le c_{ij},\ \ i, j = 1, 2, \ldots \quad (2)$$

$$X_{ij} = 0, \quad i = j, \quad (3)$$

$$\min(X_{ij}, X_{ji}) = 0, \quad i \neq j, \tag{4}$$

$$g_i + \sum_{j=1}^{N} X_{ji} - \sum_{j=1}^{N} X_{ij} \geq 0, \ (i = 1, 2, \ldots \tag{5}$$

$$X_{ij} \in Z, \ (\ i, j = 1, 2, \ldots \tag{6}$$

where (1) is the objective function to be minimized, which represents the minimum total amount of unsupplied demand for all the system nodes; $g_i + \sum_{j=1}^{N} X_{ji} - \sum_{j=1}^{N} X_{ij}$ is regarded as the amount of power in node i after performance sharing. $d_i - \left(g_i + \sum_{j=1}^{N} X_{ji} - \sum_{j=1}^{N} X_{ij} \right)$ is regarded as the difference between node i 's demand and capacity. $Max\left(d_i - \left(g_i + \sum_{j=1}^{N} X_{ji} - \sum_{j=1}^{N} X_{ij} \right), \ 0 \right)$ represents the deficiency of node i after performance sharing. (2) ensures that the power sharing amount between any two nodes does not exceed the bandwidth of the transmission line between them. (3) ensures that there no power transmitted from a node to itself. (4) ensures that the power transmission between any two nodes happens for at most one direction. (5) ensures that the power supply of each node after power redistribution is not negative, and (6) is assumed for simplicity. The model was solved by the integer programming toolbox of MATLAB software.

3.2. The case where power transmission loss is incorporated

In the actual transmission process, power loss may occur due to the line resistance. Taking into account the transmission loss, the objective function of the optimal power sharing problem can be formulated as:

$$Min \ f = \sum_{i=1}^{N} \left\{ Max\left(d_i - \left(g_i + \sum_{j=1}^{N} \left(1 - K_{ji}\right) X_{ji} - \sum_{j=1}^{N} X_{ij} \right), \ 0 \right) \right\} ; \tag{7}$$

where K_{ij} $\left(0 \leq K_{ij} \leq 1\right)$ is the loss rate for power transmission from node i to node j and $\sum_{j=1}^{N} \left(1 - K_{ji}\right) X_{ji}$ is the actual amount of power that can be transmitted to node i from all other nodes after deducting the power transmission loss. For simplicity, it is assumed that $K_{ij} = K_{ji}$ $\left(i, j = 1, 2, \ldots\right.$ which implies that the power transmission loss rate does not depend on the direction of the transmission. $g_i + \sum_{j=1}^{N}\left(1-K_{ji}\right)X_{ji} - \sum_{j=1}^{N}X_{ij}$ is regarded as the amount of power in node i after performance sharing. Eq. (7) is the objective function that needs minimizing and it represents the total amount of unsupplied demand for all the system nodes after performance sharing. The constraints are still as the same as Section 3.1, as formulated by Eqs (2)-(6). The model is solved by the integer programming toolbox of MATLAB software.

3.3. The power sharing rate

In order to know the effect of performance sharing in reducing the unsupplied demand, we introduce an index called Power Sharing Rate (PSR). In particular, when the allocation of generators is given and both the generators' capacity and the nodes' demand are also given, the unsupplied power demand f can be calculated. We assume that the unsupplied power demand before power sharing is f_{before}, and the minimum achievable unsupplied power demand after power sharing is f_{after}. The power sharing rate corresponding to a fixed power sharing policy is given by:

$$PSR = \frac{f_{before} - f_{after}}{f_{before}}.$$

where $f_{\text{before}} > 0$ .When $f_{\text{before}} = 0,\ PSR = 0$.

4. The optimal allocation of generators

As the capacity of each generator and the demand of each node in the network are random, it is essential to allocate the generators in an optimal way to minimize the expected unsupplied demand of the power grid system, by taking into account all the possible realizations of generators' capacity and the nodes' demand. Section 4.1 presents a UGF based approach to represent the unsupplied demand of the power grid systems, and Section 4.2 models the optimal generators allocation problem.

4.1. Generator capacity distribution and node demand distribution

Based on the system description in Section 2, this paper constructs a generator capacity distribution function and a node demand distribution function using UGF (Lisnianski, 2007; Lisnianski & Ding, 2009; Meena & Vasanthi, 2016; Khorshidi et al., 2016; Liu et al., 2017). The UGF approach is based on the definition of a u-function of multistate elements, which are of discrete random variables and composition operators over u-functions. It is a polynomial used to represent the distribution of discrete random variables, where the exponent represents the realization of the random variables and the coefficient represents the corresponding probability of the realization. In particular, the UGF used to represent the capacity distribution of generator k is defined as

$$u_k(z) = \sum_{l=1}^{L_k} p_k(l) z^{a_k(l)}, \tag{8}$$

where $a_k(l)$ is the *l*-th capacity realization of generator, and $p_k(l)$ is the corresponding probability of the capacity realization of the generator.

The demand distribution of the node i is defined as below

$$u_{id}(z) = \sum_{l=1}^{W_i} q_i(l) z^{d_i(l)}, \tag{9}$$

where $d_i(l)$ is the demand realization of the node i, and $q_i(l)$ is the corresponding probability of the demand realization of the node i.

For each node i, the total amount of capacity equals to the summation of the capacity for all the generators allocated into node i. Therefore, its capacity distribution can be obtained from the capacity distribution of all the generators belonging to the node i as

$$u_{ig}(z) = \prod_{k=1}^{n(i)} u_{i(k)}(z) = \prod_{k=1}^{n(i)} \left(\sum_{l=1}^{L_{ig}} p_i(l) z^{g_i(l)} \right), \tag{10}$$

where $n(i)$ is the number of generators allocated into node i, $i(k)$ is the index for the k -th generator allocated to node i, $g_i(l)$ is the capacity realization of the node i, and $p_i(l)$ is the corresponding probability of the capacity realization of the node i. Note that, in case where node i does not contain any generators, the capacity distribution of node i can be expressed by the UGF $u_{ig}(z) = \mathrm{z}^0$.

4.2. Representation of the generators' allocation

The generator allocation problem can be considered as a problem of partitioning M generators into N mutually disjoint subsets $F_i(1 \le i \le N)$ such that

$$\bigcup_{\mathrm{i=1}}^{\mathrm{N}} \tag{11}$$

and

$$F_i \cap F_j = \phi, \ \ i \neq j. \tag{12}$$

The partition of the set F can be represented by the vector $H = \{h(k), 1 \leq k \leq M\}$, which denotes that any generator k is assigned to the $h(k)$ th node. The cardinality of each subset $F_i\left(1 \leq i \leq N\right)$ can be easily obtained as

$$n(i) = \left|F_i\right| = \sum_{j=1}^{M} 1\left(h\left(k\right) = i\right), \ \left(i = 1,2,...\mathrm{N}\right). \tag{13}$$

Where, “1”is an indicator function, satisfying 1(TRUE)=1 and 1(FALSE)=0.

4.3. Optimal allocation of generators

For fixed allocation of generators, the expected unsupplied demand of the power grid system can be obtained by considering all the possible realizations of the generators’ capacity and the nodes’ demand. It can be seen from Section 3, Subsections 4.1 and 4.2 that the numbers of states for the *N* nodes are $L_{1g}, L_{2g}, \cdots$, respectively. The number of demand realizations for the node i is W_i. Therefore, there are $V = \prod_{i=1}^{N} L_{ig} W_i$ different realizations of the generators’ capacity and the nodes’ demand, where the unsupplied demand needs to be minimized under each realization using the procedures proposed in Section 3. Denote the minimized unsupplied demand for each realization t as f_t and the corresponding probability for the realization as $P_{f_t} (t = 1, 2, \cdots$. Finally, the expected value of the minimum unsupplied power demand of the system is obtained by

$$\mathrm{E}\left(f_t\right) = \sum_{t=1}^{V} P_{f_t} f_t. \tag{14}$$

In addition, the expected power sharing rate can be obtained by taking into account all the possible realizations of the generators’ capacity and the nodes’ demand as:

$$E(PSR) = \sum_{t=1}^{V} P_{f_t} PSR_t,$$

where PSR_t is the power sharing rate under the optimal sharing policy for the realization t .

Based on the above description, the optimization problem is to find vector $H=\{h(k),1\le k\le M\}$ that minimizes the expected unsupplied demand of the system. That is

$$H_{optimal}=\arg minE\left[f_t\left(H\right)\right]; \tag{15}$$

Subject to

$$H=\{h(k),1\le k\le M\}, \tag{16}$$

$$h(k)=1,\ldots \qquad \ldots \tag{17}$$

5. Optimization technique

To solve the optimal generator allocation policy in order to minimize the objective function (15) is a difficult combinatorial optimization problem. Though the exact solution is possible to find with enumeration technique, it may be too time consuming. In practice, the quality of the solution is of concern, thus a typical way for such combinatorial optimization problem is to employ a heuristic to find the near optimal solution. In particular, this paper adopts a Hybridized Particle Swarm optimization (HPSO) algorithm that combines Particle Swarm optimization algorithm (PSO) with other algorithms (Chen et al., 2014; Jamrus et al., 2018; Zhang et al., 2016).

PSO is inspired from the behaviour characteristics of the biological population and is used to solve optimization problems in some research fields. In PSO, each particle represents a potential solution of the problem and each particle corresponds to a fitness value determined by fitness function (Wang ＆ Tang, 2011). The velocity of the particle determines the direction and distance of the particle moving. The velocity is dynamically adjusted with the movement experience of the particle itself and other particles, so as to realize the optimization of the individual in the solvable space. To apply the described swarm optimization technique to a particular problem, the key actually lies in linking the solution to the fitness function of the solution. In our case, the fitness function of each solution *H* is defined in Eq. (14) in Section 4, which again needs to use the result from Eq. (7) in Section 3.

PSO initializes a group of particles in the feasible solution space. Each particle represents a potential optimal solution of the extremum optimization problem. The characteristics of the particle are represented by the position, velocity and fitness values. The fitness function represents the objective function, which indicates the advantages and disadvantages of the particles. Particles move in the solution space and update individual positions by tracking individual extremum and group extremum.

Although PSO is simple and can converge quickly, with the increasing number of iterations, the convergence of the population is concentrated, and the examples are more and more similar, which may not jump out around the local optimal solution. To overcome the shortcoming, the HPSO introduces crossover and mutation operations from genetic algorithms, and searches for the optimal solution by crossing the individual extreme value with the population extreme value and mutation of the particle itself (Gong et al., 2016; Shi et al., 2003).

6. Illustrative examples

Consider the allocation of 5 generators into the 4 nodes in the following illustrative example. The topology of the system is as shown in Fig 2. The capacity distribution for each generator is as listed in Table 1, where the capacity distribution of each generator has three states.

Fig 2. The topology diagram of power grid with four nodes

Table 1. The capacity distribution for each generator

The demand distribution for each node is as listed in Table 2. There are three states for each node's demand distribution.

Table 2. The demand distribution for each node

6.1. Fixed allocation

Suppose the allocation vector $H=\{h(k),1\le k\le 5\}=\{2,\ 3,\ 4,\ 1,\ 1\}$.Through the definition of H, it is easy to know that the sets of generators allocated to the 4 nodes are {4,5}, {1}, {2}, and {3}, respectively. That is, it is a generator allocation policy.

According to the given generator allocation policy, the capacity distribution of the four nodes based on the formulas (8) and (10) are given respectively by

$$\begin{aligned} u_{1g}(z) &= \left(0.7z^4+0.2z^3+0.1z^0\right)\left(0.6z^4+0.2z^3+0.2z^0\right) \\ &= 0.42z^8+0.26z^7+0.04z^6+0.2z^4+0.06z^3+0.02z^0, \end{aligned}$$

$$u_{2g}(z)=0.8z^4+0.1z^3+0.1z^0,$$

$$u_{3g}(z)=0.8z^4+0.1z^3+0.1z^0,$$

and

$$u_{4g}(z)=0.7z^4+0.2z^3+0.1z^0.$$

It is known from the above discussion that there are six states for the capacity distribution of the first node, and there are three states for the capacity distribution of the other three nodes. In addition, the demand for each node has three possible realizations. Thus, there are $3^7\times 6=13122$ different realizations for the generators' capacity and the nodes' demand.

Take one of these cases as an example, as shown in Table 3. The probability of this case is $0.26\times0.1\times0.1\times0.7\times0.1\times0.1\times0.6\times0.15=1.638\times10^{-6}$. The optimal power sharing policy is discussed for the case where the power transmission loss can be neglected and the case where the power transmission loss cannot be neglected.

Table 3. A specific case for illustration

6.1.1. The case where power transmission loss is neglected

(1) The case where the transmission bandwidth between nodes is the same

Assuming that the transmission bandwidth between nodes i and j is $c_{ij}=0$. The minimum value of the unsupplied power demand that can be achieved for the case presented in Table 3 is solved to be 4. The contribution to the expected unsupplied power demand is $1.638\times10^{-6}\times4=6.552\times10^{-6}$. The PSR for this case is 0.

If the transmission bandwidth between nodes i and j is $c_{ij}=1$, the optimal solution, the minimum value of the unsupplied power demand that can be achieved for the case presented in Table 3, the contribution to the expected unsupplied power demand, and the PSR are respectively $x_{12}=x_{13}=x_{14}=x_{23}=x_{34}=x_{42}=1$, $x_{21}=x_{24}=x_{31}=x_{32}=x_{41}=x_{43}=0$, 1, $1.638\times10^{-6}\times1=1.638\times10^{-6}$, and 3/4=0.75. If the transmission bandwidth between nodes i and j is $c_{ij}=2$, they become $x_{12}=2$,

$x_{21}=x_{23}=x_{31}=x_{32}=x_{34}=x_{43}=x_{41}=x_{24}=x_{42}=0$, $x_{13}=x_{14}=1$, 0, $1.638\times10^{-6}\times0=0$ and 4/4=1, respectively.

It can be seen that with the increase of the transmission bandwidth between nodes i and j, the minimum value of the unsupplied power demand becomes smaller and the PSR increases.

Considering all the realizations of the generators' capacity and the nodes' power demand, the expected values of the minimum unsupplied power demand of the system $E(f_t)$ and the PSR are obtained as below.

When the transmission bandwidth between nodes i and j is $c_{ij}=0$, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ is 2.1050 and the expected PSR is 0. When the transmission bandwidth between nodes i and j is $c_{ij}=1$, these values become 1.0156 and 0.5175, respectively. When the transmission bandwidth between nodes i and j is $c_{ij}=2$, these values become 0.8955 and 0.5746, respectively .

It can be seen that with the increase of the transmission bandwidth between nodes i and j, the expected value of the minimum unsupplied power demand of the system $E(f_t)$

becomes smaller and the expected PSR increases.

(2) The case where the transmission bandwidth between nodes is different

In order to examine the importance of different transmission lines, we investigate the case where the bandwidths of the transmission lines differ. Specifically, it is assumed that the bandwidths of the transmission lines are as shown in Table 4. The corresponding expected value of the minimum unsupplied demand together with the expected PSR are also shown for each setting of the bandwidths. To be consistent with some other papers in the reliability field (Song & Schmeiser, 2009), the expected value of minimum unsupplied demand is accurate to the last four decimal points. For guaranteeing the quality of our obtained optimal solution, for each setting, the optimization code is run 5 times and consistent results are observed.

Table 4. Results for conditions of different transmission bandwidths

From Table 4, we can see that the transmission line between nodes 1 and 4 and the transmission line between nodes 1 and 3 are more important than other lines. Actually, two generators instead of one are allocated to node 1 which makes it more likely to have excess. On the other hand, the demand of node 3 and node 4 are relatively higher. Therefore, the power transmission from node 1 to node 3 and node 4 is more important than other lines. To further confirm the finding, the case presented in Table 5 is examined, which also implies the importance of power transmission from node 1 to node 3 and node 4.

Table 5. Results for conditions of different transmission bandwidths

6.1.2. The case where power transmission loss is incorporated

(1) The case where the transmission bandwidth between nodes is the same

Assuming loss rate $K_{ij} = 0.05$, where the capacity and the demand of the nodes are still as shown in Table 3. When the transmission bandwidth between nodes i and j is $c_{ij} = 0$, it is not allowed to transmit power, so there is no power loss. The minimum value of the unsupplied power demand is 4, and the contribution to the expected unsupplied power demand is $1.638 \times 10^{-6} \times 4 = 6.552 \times 10^{-6}$. The PSR for this case is 0. When the transmission bandwidth between nodes i and j is $c_{ij} = 1$, the optimal solution is obtained as $x_{12} = x_{13} = x_{14} = 1$ and $x_{21} = x_{23} = x_{24} = x_{31} = x_{32} = x_{34} = x_{41} = x_{42} = x_{43} = 0$, respectively. The minimum value of the unsupplied power demand is 1.15, and the contribution to the expected unsupplied power demand is $1.6380 \times 10^{-6} \times 1.15 = 1.8837 \times 10^{-6}$.

The PSR for this case is 2.85/4=0.7125. When the transmission bandwidth between nodes i and j is $c_{ij} = 2$, the optimal solution is obtained as $x_{12} = 2$, $x_{13} = x_{14} = 1$, and $x_{41} = x_{24} = x_{42} = x_{21} = x_{23} = x_{31} = x_{32} = x_{34} = x_{43} = 0$. The minimum value of the unsupplied power demand is 0.2, and the contribution to the expected unsupplied power demand is

$1.6380 \times 10^{-6} \times 0.2 = 3.276 \times 10^{-7}$. The PSR for this case is 3.8/4=0.95. It can be seen that with the increase of the transmission bandwidth between nodes i and j, the minimum value of the unsupplied power demand becomes smaller and the PSR increases.

Combined with all cases, the expected value of minimum unsupplied power demand of the system $E(f_t)$ and the PSR can be obtained. When the transmission bandwidth between nodes i and j is $c_{ij} = 0$, the expected value of minimum unsupplied power demand of the system $E(f_t)$ is 2.1050 and the expected PSR is 0. When the transmission bandwidth between nodes i and j is $c_{ij} = 1$, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ is 1.0774 and the expected PSR is 0.4882. When the transmission bandwidth between nodes i and j is $c_{ij} = 2$, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ is 0.9736 and the expected PSR is 0.5375.

The results of the optimal solution with or without power loss are listed in Table 6. It is obvious that the transmission bandwidth and the power transmission loss are positively proportional. Actually, more power can be transmitted when the bandwidth is bigger, which also causes more power to be lost through transmission, which indeed results in more unsupplied demand.

Table 6. Comparison of solution with or without power loss

(2) The case where the transmission bandwidth between nodes is different

Similarly, the results for the case where transmission bandwidth between different nodes are obtained, as shown in Table 7, which also shows the importance of power transmission from node 1 to node 3 and node 4.

Table 7. Results for conditions of different transmission bandwidths

6.2. Optimal allocation

In order to minimize the power demand of the system, we analyze the optimal allocation policy of generators below. The optimal allocation policy of generator is obtained by the hybridized particle swarm optimization (HPSO) algorithm. The hybridized particle swarm optimization algorithm in this paper combines particle swarm optimization algorithm and genetic algorithm. The algorithm flow chart is as shown in Fig 3.

Fig 3. Flow chart of hybridized particle swarm optimization algorithm

The algorithm is realized by programming in MATLAB. The size of the selected population is 200, the number of times of evolution is 100. The computer is a Windows 10 operating system, i7-8550u CPU, 4.0GHZ, Intel Quad-core processor, 16GB memory.

6.2.1. The optimal allocation policy of generators without considering power transmission loss

(1) The case where the transmission bandwidth between nodes is the same

When the transmission bandwidth between nodes i and j is $c_{ij} = 0$, the optimal allocation policy of generator is {2}, {1}, {4,5}, {3}. The minimum expected unsupplied demand value of the system is 2.0450 and the expected PSR is 0. When the transmission bandwidth between nodes i and j is $c_{ij} = 1$, the optimal allocation policy of generator is {1}, {2}, {3,5}, {4}. The minimum expected unsupplied demand value of the system is 1.0141 and the expected PSR is 0.5041. When the transmission bandwidth between nodes i and j is $c_{ij} = 2$, the optimal allocation policy of generator is {1}, {2}, {4,5}, {3}. The minimum expected unsupplied demand value of the system is 0.8954 and the expected PSR is 0.5622. It can be seen that with the increase of the transmission bandwidth between nodes i and j, the minimum expected unsupplied demand value of the system becomes smaller and the expected PSR increases.

(2) The case where the transmission bandwidth between nodes is different

Table 8 presents the results for the case where the transmission bandwidths of different transmission lines are different. It can be seen that the transmission line between nodes 3 and node 4 is more important in this case. Actually, the demand of node 3 and node 4 are relatively higher, which makes either of them more likely to experience deficit given the optimal allocation. Therefore, the power transmission between them is essential.

Table 8. Results for conditions of different transmission bandwidths

6.2.2. The optimal allocation policy of generators considering power transmission loss

(1) The case where the transmission bandwidth between nodes is the same

Still consider the power transmission loss rate $K_{ij} = 0.05$. When the transmission bandwidth between nodes i and j is $c_{ij} = 0$, the optimal allocation policy of generators, the minimum expected unsupplied demand value of the system, and the expected PSR are respectively {2}, {1}, {4,5}, {3}, 2.0450 and 0. When the transmission bandwidth between nodes i and j is $c_{ij} = 1$, they become {2},{1},{3,5},{4}, 1.0774, and 0.4732. When the transmission bandwidth between nodes i and j is $c_{ij} = 2$, they become {1}, {2}, {4,5}, {3}, 0.9724 , and 0.5245. It can be seen that with the increase of the transmission bandwidth between nodes i and j, the minimum expected unsupplied demand value of the system becomes smaller and the expected PSR increases. Under the same bandwidth, the minimum expected unsupplied demand value of the system for the case with power loss is larger than that of the system for the case without power loss, and the expected PSR for the case with power loss is smaller than that of the system for the case without power loss.

(2) The case where the transmission bandwidth between nodes is different

The results for the cases where the transmission widths are different are presented in Table 9. Similar as Table 8, it also shows that the transmission line between nodes 3 and 4 is more important.

Table 9. Results for conditions of different transmission bandwidths

7. Case study

Decentralized wind power generation projects are being undertaking in different areas of China, in particular, in western China. Imagine that there is an area with six cities, each of which is connected with a wind power station. Suppose that the power stations for different cities are connected with transmission lines, as shown in Fig. 4. The power transmission bandwidth between nodes is the same as the transmission lines of the same material and specification are used. Due to increasing power consumption of this area, it is intended to allocate eight wind power generators. The capacity distribution of the wind power generators and the distribution of demand for wind power for the six cities are as shown in Tables 10 and 11, respectively:

Fig 4. The topology diagram of power grid with six cities

Table 10. The capacity distribution for each generator

Table 11. The demand distribution for each node

7.1. Fixed allocation

Suppose the allocation is fixed and the allocation vector is $H=\{h(k),1\le k\le 8\}=\{3,\ 3,\ 6,\ 2,\ 1,4,5,5\}$. Through the definition of H, it is easy to know that the sets of generators allocated to the 6 nodes are {5}, {4}, {1, 2}, {6}, {7, 8} and {3}, respectively.

According to the given generator allocation policy, the capacity distribution of the six nodes based on the formulas (8) and (10) are given respectively by

$$u_{1g}(z)=0.7z^4+0.2z^3+0.1z^0,$$

$$u_{2g}(z)=0.8z^4+0.1z^3+0.1z^0,$$

$$\begin{aligned}u_{3g}(z)&=\left(0.8z^4+0.1z^3+0.1z^0\right)\left(0.8z^4+0.1z^3+0.1z^0\right)\\&=0.64z^8+0.16z^7+0.01z^6+0.16z^4+0.02z^3+0.01z^0,\end{aligned}$$

$$u_{4g}(z)=0.7z^4+0.2z^3+0.1z^0,$$

$$u_{5g}(z)=\left(0.7z^4+0.2z^3+0.1z^0\right)\left(0.6z^4+0.2z^3+0.2z^0\right)$$
$$=0.42z^8+0.26z^7+0.04z^6+0.2z^4+0.06z^3+0.02z^0,$$

and

$$u_{6g}(z)=0.8z^4+0.1z^3+0.1z^0.$$

It is known from the above discussion that there are 2125764 (=$3^{10}\times6^2$) different realizations for the generators' capacity and the nodes' demand.

Considering all the realizations of the generators' capacity and the nodes' power demand, we discuss the optimal power sharing policy for the case where the power transmission loss can be neglected and the case where the power transmission loss cannot be neglected.

7.1.1. The case where power transmission loss is neglected

When power transmission loss is neglected, we consider all the realizations of the generators' capacity and the nodes' power demand, respectively. The expected values of the minimum unsupplied power demand of the system $E(f_t)$ and the PSR are obtained as below.

When the transmission bandwidth between nodes i and j is $c_{ij}=0$, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ is 2.5360 and the expected PSR is 0. When the transmission bandwidth between nodes i and j is $c_{ij}=1$, these values become 1.1963 and 0.5283, respectively. When the transmission bandwidth between nodes i and j is $c_{ij}=2$, these values become 0.6964 and 0.7254, respectively .

It can be seen that with the increase of the transmission bandwidth between nodes i and j, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ becomes smaller and the expected PSR increases.

7.1.2. The case where power transmission loss is incorporated

Assume that loss rate is $K_{ij}=0.05$. When the transmission bandwidth between nodes i and j is $c_{ij}=0$, the expected value of minimum unsupplied power demand of the system $E(f_t)$ is 2.1657 and the expected PSR is 0. When the transmission bandwidth between nodes i and j is $c_{ij}=1$, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ is 1.2740 and the expected PSR is 0.4117. When the transmission bandwidth between nodes i and j is $c_{ij}=2$, the expected value of the minimum unsupplied power demand of the system $E(f_t)$ is 0.8506 and the expected PSR is 0.6072.

It can be seen that with the increase of the transmission bandwidth between nodes i and j, the minimum expected unsupplied demand value of the system becomes smaller and the expected PSR increases. Under the same bandwidth, the minimum expected unsupplied demand value of the system for the case with power loss is larger than that of the system for the case without power loss, and the expected PSR for the case with power loss is smaller than that of the system for the case without power loss.

7.2. Optimal allocation

In order to minimize the power demand of the system, we analyze the optimal allocation policy of generators below.

7.2.1. The optimal allocation policy of generators without considering power transmission loss

If power transmission loss is neglected, the optimal allocation vector $H=\{h(k),1\le k\le 8\}=\{5,\ 5,\ 6,\ 1,\ 2, 4, 3, 3\}$. That is, the optimal allocation policy of generator is {4}, {5}, {7, 8}, {6}, {1, 2}, and {3}. If the transmission bandwidth between nodes i and j is $c_{ij}=0$, the minimum expected unsupplied demand value of the system is 2.4270 and the expected PSR is 0. If the transmission bandwidth between nodes i and j is $c_{ij}=1$, the minimum expected unsupplied demand value of the system is 1.0094 and the expected PSR is 0.5841. If the transmission bandwidth between nodes i and j is $c_{ij}=2$, the minimum expected unsupplied demand value of the system is 0.5940 and the expected PSR is 0.7553. It can be seen that with the increase of the transmission bandwidth between nodes i and j, the minimum expected unsupplied demand value of the system decreases and the expected PSR increases.

7.2.2. The optimal allocation policy of generators considering power transmission loss

Suppose that the power transmission loss rate $K_{ij}=0.05$. The optimal allocation policy of generators is {2}, {6}, {5, 8}, {7}, {3, 4}, {1}. If the transmission bandwidth between nodes i and j is $c_{ij}=0$, the minimum expected unsupplied demand value of the system is 2.4270 and the expected PSR is 0. If the transmission bandwidth between nodes i and j is $c_{ij}=1$, the minimum expected unsupplied demand value of the system, and the expected PSR are respectively 1.0753 and 0.5569. If the transmission bandwidth between nodes i and j is $c_{ij}=2$, they become 0.7284, and 0.6999.

Similarly, the same conclusion can be obtained that with the increase of the transmission bandwidth between nodes i and j, the minimum expected unsupplied demand value of the system becomes smaller and the expected PSR increases. Under the same bandwidth, the minimum expected unsupplied demand value of the system for the case with power loss is larger than that of the system for the case without power loss, and the expected PSR for the case with power loss is smaller than that of the system for the case without power loss.

8. Conclusions

Performance sharing is a method of improving the reliability of systems. However, the existing works are restricted to systems of simple structures. Differently, this paper considers a complex power grid system with performance sharing mechanisms, where each node of the power network can be distributed with power generators. For fixed generators' allocation, procedures are proposed to evaluate the expected system unsupplied demand. After that, a hybridized particle swarm optimization (HPSO) algorithm is used to find out the optimal allocation policy of generators that minimizes the expected unsupplied demand of the system. Both the case where power transmission loss can and cannot be neglected and the case where the power transmission loss cannot be neglected are considered. Examples are presented to illustrate show the applications of the proposed procedures.

There are some limitations in this paper. First, this paper studies a static optimization problem where the demand and the capacity are random variables. In practice, the demand and the capacity are changing from time to time, and thus the dynamic system configuration and dynamic performance sharing may be of interest. In addition, the demands of different nodes are assumed to be independent, so as to the capacities of different capacities. The possible dependence between demands, between capacities, and even between demand and capacities can be investigated in the future.

Besides the above mentioned limitations, this work can also be extended in the following ways. In this paper, only the generators' allocation is studied. In the future, policies of maintenance and protection of the generators can be incorporated as well. For example, each generator may be subject to internal failures and external attacks. The availability of each generator depends on the preventive maintenance interval and the protection effort on each generator. Besides generators' allocation, one may also be concerned with the optimal maintenance and protection strategy which minimizes the expected system cost per unit time (Peng et al., 2012; Peng et al., 2014). In addition, this paper only considers the distribution of electricity. In the future, the integrated electricity and gas distribution system can be studied. Another possible direction is to incorporate the framework the effects of cascading failures by physical and cyber attacks.

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Table 1. The capacity distribution for each generator

Generator/ capacity	4	3	0
a_1	0.8	0.1	0.1
a_2	0.8	0.1	0.1
a_3	0.7	0.2	0.1
a_4	0.7	0.2	0.1
a_5	0.6	0.2	0.2

Table 2. The demand distribution for each node

Node/demand	3	4	5
d_1	0.1	0.8	0.1
d_2	0.1	0.8	0.1

d_3	0.2	0.6	0.2
d_4	0.15	0.7	0.15

Table 3. A specific case for illustration

Node	capacity (g_i)	probability	demand (d_i)	probability
n_1	7	0.26	3	0.1
n_2	3	0.1	5	0.1
n_3	3	0.1	4	0.6
n_4	4	0.7	5	0.15

Table 4. Results for conditions of different transmission bandwidths

Bandwidths of transmission lines	The expected value of minimum unsupplied demand	The expected PSR
$c_{12}=c_{21}=1,\ \text{other } c_{ij}=0$	1.8977	0.0985
$c_{13}=c_{31}=1,\ \text{other } c_{ij}=0$	1.8384	0.1267
$c_{14}=c_{41}=1,\ \text{other } c_{ij}=0$	1.8176	0.1365
$c_{23}=c_{32}=1,\ \text{other } c_{ij}=0$	2.0346	0.0334
$c_{24}=c_{42}=1,\ \text{other } c_{ij}=0$	2.0466	0.0277
$c_{34}=c_{43}=1,\ \text{other } c_{ij}=0$	2.0093	0.0455

Table 5. Results for conditions of different transmission bandwidths

Bandwidths of transmission lines	The expected value of minimum unsupplied demand	The expected PSR
$c_{12}=c_{21}=2, \text{other } c_{ij}=0$	1.8189	0.1359
$c_{13}=c_{31}=2, \text{ other } c_{ij}=0$	1.7524	0.1675
$c_{14}=c_{41}=2, \text{ other } c_{ij}=0$	1.7245	0.1808
$c_{23}=c_{32}=2, \text{ other } c_{ij}=0$	2.0346	0.0334
$c_{24}=c_{42}=2, \text{ other } c_{ij}=0$	2.0466	0.0277
$c_{34}=c_{43}=2, \text{ other } c_{ij}=0$	2.0093	0.0455

Table 6. Comparison of solution with or without power loss

	Bandwidth （c_{ij}）	No loss	Loss
The minimum value of the unsupplied power demand for the case in Table 3	0	4	4
	1	1	1.15
	2	0	0.2
The expected value of minimum unsupplied power demand of the system	0	2.1050	2.1050
	1	1.0156	1.0774
	2	0.8955	0.9736

Table 7. Results for conditions of different transmission bandwidths

Bandwidths of transmission lines	The expected value of minimum unsupplied demand	The expected PSR
$c_{12}=c_{21}=1, \text{ other } c_{ij}=0$	1.9080	0.0936
$c_{13}=c_{31}=1, \text{ other } c_{ij}=0$	1.8517	0.1203

$c_{14} = c_{41} = 1,\ \text{other } c_{ij} = 0$	1.8320	0.1297
$c_{23} = c_{32} = 1,\ \text{other } c_{ij} = 0$	2.0381	0.0318
$c_{24} = c_{42} = 1,\ \text{other } c_{ij} = 0$	2.0496	0.0263
$c_{34} = c_{43} = 1,\ \text{other } c_{ij} = 0$	2.0141	0.0432
$c_{12} = c_{21} = 2, \text{other } c_{ij} = 0$	1.8275	0.1318
$c_{13} = c_{31} = 2,\ \text{other } c_{ij} = 0$	1.7622	0.1629
$c_{14} = c_{41} = 2,\ \text{other } c_{ij} = 0$	1.7348	0.1759
$c_{23} = c_{32} = 2,\ \text{other } c_{ij} = 0$	2.0381	0.0318
$c_{24} = c_{42} = 2,\ \text{other } c_{ij} = 0$	2.0496	0.0263
$c_{34} = c_{43} = 2,\ \text{other } c_{ij} = 0$	2.0141	0.0432

Table 8. Results for conditions of different transmission bandwidths

Bandwidths of transmission lines	The expected value of minimum unsupplied demand	The expected PSR
$c_{12} = c_{21} = 1,\ \text{other } c_{ij} = 0$	2.0018	0.0211
$c_{13} = c_{31} = 1,\ \text{other } c_{ij} = 0$	1.8312	0.1046
$c_{14} = c_{41} = 1,\ \text{other } c_{ij} = 0$	1.9866	0.0285
$c_{23} = c_{32} = 1,\ \text{other } c_{ij} = 0$	1.8312	0.1046
$c_{24} = c_{42} = 1,\ \text{other } c_{ij} = 0$	1.9866	0.0285

$c_{34}=c_{43}=1,\ \text{other } c_{ij}=0$	1.7487	0.1499
$c_{12}=c_{21}=2,\text{other } c_{ij}=0$	2.0018	0.0211
$c_{13}=c_{31}=2,\ \text{other } c_{ij}=0$	1.7528	0.1429
$c_{14}=c_{41}=2,\ \text{other } c_{ij}=0$	1.9866	0.0285
$c_{23}=c_{32}=2,\ \text{other } c_{ij}=0$	1.7528	0.1429
$c_{24}=c_{42}=2,\ \text{other } c_{ij}=0$	1.9866	0.0285
$c_{34}=c_{43}=2,\ \text{other } c_{ij}=0$	1.6561	0.1902

Table 9. Results for conditions of different transmission bandwidths

Bandwidths of transmission lines	The expected value of minimum unsupplied demand	The expected PSR
$c_{12}=c_{21}=1,\ \text{other } c_{ij}=0$	2.0040	0.0201
$c_{13}=c_{31}=1,\ \text{other } c_{ij}=0$	1.8419	0.0993
$c_{14}=c_{41}=1,\ \text{other } c_{ij}=0$	1.9896	0.0272
$c_{23}=c_{32}=1,\ \text{other } c_{ij}=0$	1.8419	0.0993
$c_{24}=c_{42}=1,\ \text{other } c_{ij}=0$	1.9896	0.0272
$c_{34}=c_{43}=1,\ \text{other } c_{ij}=0$	1.7635	0.1377
$c_{12}=c_{21}=2,\text{other } c_{ij}=0$	2.0040	0.0201
$c_{13}=c_{31}=2,\ \text{other } c_{ij}=0$	1.7618	0.1385

$c_{14} = c_{41} = 2,\ \text{other } c_{ij} = 0$	1.9896	0.0271
$c_{23} = c_{32} = 2,\ \text{other } c_{ij} = 0$	1.7618	0.1385
$c_{24} = c_{42} = 2,\ \text{other } c_{ij} = 0$	1.9896	0.0272
$c_{34} = c_{43} = 2,\ \text{other } c_{ij} = 0$	1.6668	0.1849

Table 10. The capacity distribution for each generator

Generator/ capacity	4	3	0
a_1	0.8	0.1	0.1
a_2	0.8	0.1	0.1
a_3	0.8	0.1	0.1
a_4	0.8	0.1	0.1
a_5	0.7	0.2	0.1
a_6	0.7	0.2	0.1
a_7	0.7	0.2	0.1
a_8	0.6	0.2	0.2

Table 11. The demand distribution for each node

Node	Demand 1	Demand 2	Demand 3
	2	3	4
d_1	0.1	0.8	0.1

	2	3	4
d_2	0.15	0.7	0.15
	4	5	6
d_3	0.1	0.8	0.1
	2	3	4
d_4	0.1	0.8	0.1
	4	6	8
d_5	0.15	0.7	0.15
	3	4	5
d_6	0.1	0.8	0.1

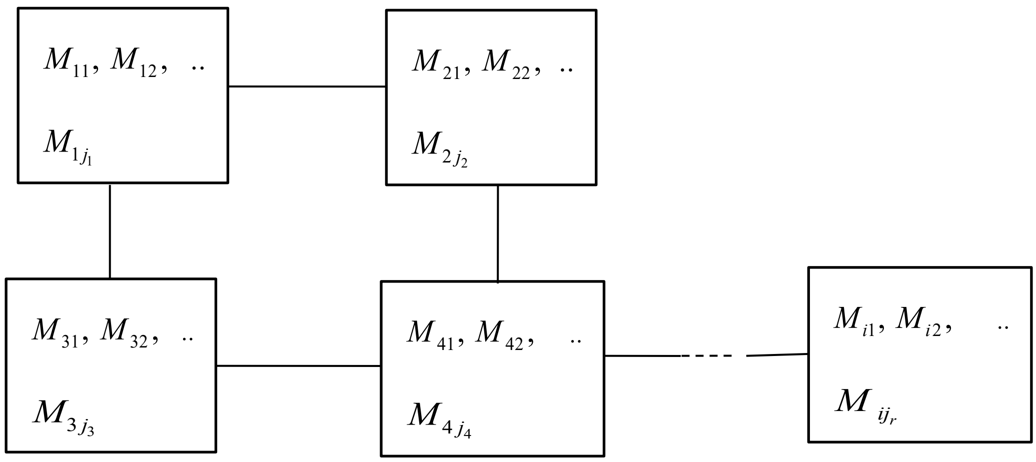

Fig 1. The power grid system structure diagram

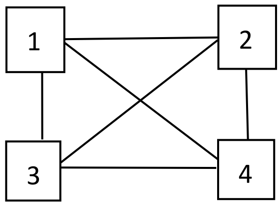

Fig 2. The topology diagram of power grid with four nodes

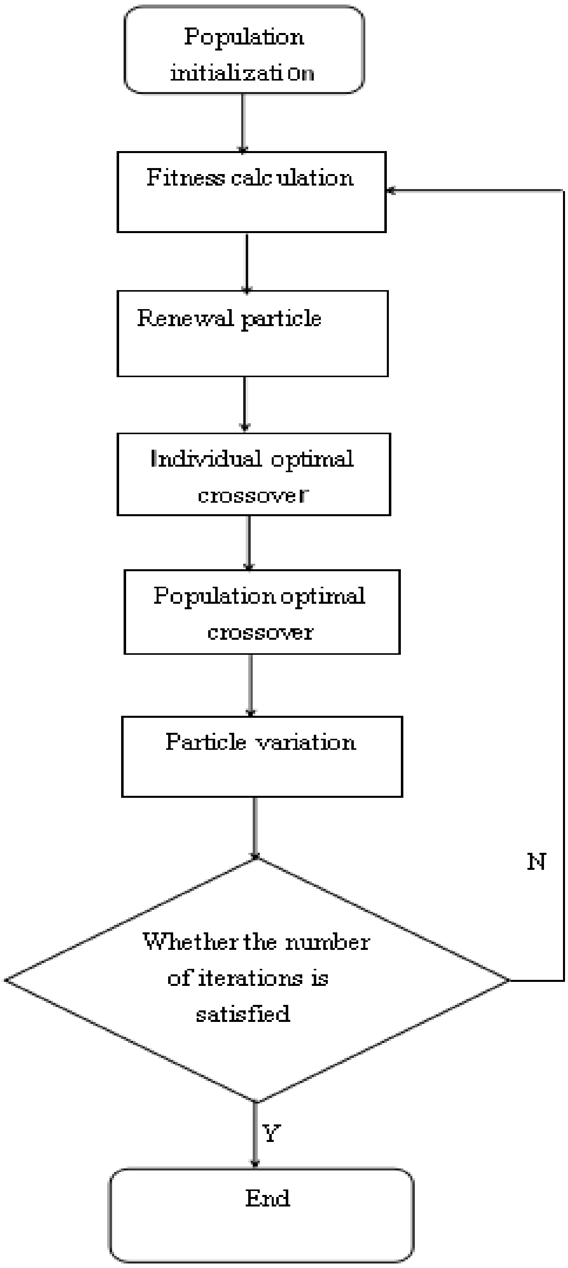

Fig 3. Flow chart of hybridized particle swarm optimization algorithm

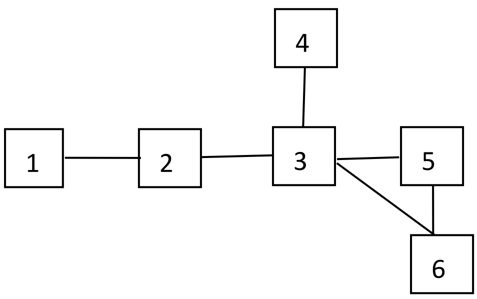

Fig 4.The topology diagram of power grid with six cities