Forecasting Realized Volatility of Agricultural Commodities☆

Stavros Degiannakis[a,b], George Filis[b,*], Tony Klein[c], Thomas Walther[d,e]

[a] *Department of Economics and Regional Development, Panteion University of Social and Political Sciences, Greece*
[b] *Department of Accounting, Finance and Economics, Bournemouth University, UK*
[c] *Queen's Management School, Queen's University Belfast, UK*
[d] *Utrecht School of Economics, Utrecht University, The Netherlands*
[e] *Faculty of Business and Economics, Technische Universität Dresden, Germany*

Abstract

We forecast the realized and median realized volatility of agricultural commodities using variants of the Heterogeneous AutoRegressive (HAR) model. We obtain tick-by-tick data for five widely traded agricultural commodities (Corn, Rough Rice, Soybeans, Sugar, and Wheat) from the CME/ICE. Real out-of-sample forecasts are produced for 1- up to 66-days ahead. Our in-sample analysis shows that the variants of the HAR model which decompose volatility measures into their continuous path and jump components and incorporate leverage effects offer better fitting in the predictive regressions. However, we convincingly demonstrate that such HAR extensions do not offer any superior predictive ability in the out-of-sample results, since none of these extensions produce significantly better forecasts compared to the simple HAR model. Our results remain robust even when we evaluate them in a Value-at-Risk framework. Thus, there is no benefit by adding more complexity, related to volatility decomposition or relative transformations of volatility, in the forecasting models.

Keywords: Agricultural Commodities, Realized Volatility, Median Realized Volatility, Heterogeneous Autoregressive model, Forecast.

JEL classification: C22; C53; Q02; Q17

1. Introduction & Brief Review of the Literature

Examining the behaviour of agricultural commodity prices and volatilities is of significant importance since they represent a major component of household consumption. They also have a pronounced impact on food security, which primarily affects the poorer parts of the population (Ordu et al., 2018).

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*Corresponding Author, Mail: gfilis@bournemouth.ac.uk

The Food and Agricultural Organisation (FAO) of the United Nations (2010) have claimed that food prices had rarely experienced any significant volatility prior to 2008 (FAO, 2010); however, over the last decade (2008-2018) agricultural commodities have experienced enormous price swings resulting in both high and low volatility regimes (Greb & Prakash, 2015). This new normal suggests that the food system is becoming progressively more vulnerable to price volatility (FAO, 2010) and this led the G20 to request a report from several international bodies (including the World Bank, IMF, UNCTAD, OECD, and FAO, among others) in order "to develop options for G20 consideration on how to better mitigate and manage the risks associated with the price volatility of food and other agriculture commodities, without distorting market behaviour, ultimately to protect the most vulnerable." (FAO, 2011, p.3).

The Council on Foreign Relations (CFR, 2011) promotes that such increased volatility is the result of extreme weather events, biofuels production, market speculation but also rising demand coupled with declines in food stocks. von Braun & Tadesse (2012) also show that agricultural commodities price volatility is impacted by the increasing linkages among agricultural prices, energy commodities, and financial markets. Ordu et al. (2018) further suggest that the agricultural market is becoming financialized since institutional investors are increasing their holdings in the commodity markets, which further suggests the rise in the speculative activity in this market. It is rather easy to understand that such cross-market linkages and financialization processes could have destabilizing effects on agricultural food prices.

Proper modeling and detection of long-memory dynamics in the volatility of commodity futures improves risk-management techniques, such as volatility forecasting and hedging performance, and better characterizes equilibrium relationships. Over the last few years there is an increased effort either to model agricultural price volatility (focusing primarily on GARCH-type and wavelet-based modelling approaches) or to provide evidence of potential predictors of such volatility, within an in-sample setting (Egelkraut & Garcia, 2006, Elder & Jin, 2007, Anderluh & Borovkova, 2008, Triantafyllou et al., 2015, Li et al., 2017).

Given the aforementioned market conditions and previous research effort to model agricultural volatility, it becomes central to develop the necessary frameworks that would allow successful forecasts for agricultural commodity price volatility so that policy institutions can get prepared for high price volatility periods or design preventing policies, as also implied by Greb & Prakash (2017).

Despite the recent evidence provided by policy institutions for the need of successful agricultural price volatility forecasts, the fact that modelling approaches for agricultural price volatility have developed for over 15 years now, as well as, the fact that the first effort to produce real out-of-sample forecasts was by Giot & Laurent (2003), we observe the paradox that there are only four other recent studies in this strand of the literature, those by Tian et al. (2017a,b), Yang et al. (2017), and Luo et al. (2019).

Starting with the former study, Giot & Laurent (2003) focus on Cocoa, Coffee, and Sugar futures price volatility and use GARCH-type models to generate the forecasts. By contrast, Tian et al. (2017a), Tian et al. (2017b) and Yang et al. (2017) utilize the increased availability of ultra-high frequency data and extend Corsi (2009) Heterogeneous AutoRegressive (HAR) model to produce short-run volatility forecasts (up to 20-days ahead).

More specifically, Tian et al. (2017a) use a two regime-switching Markov models to forecast realized volatility for five agricultural commodities traded in the Chinese market, namely, Soybean, Soybean oil, White Sugar, Gluten Wheat and Cotton. They find evidence that regime switching dynamics offer predictive gains compared to both a simple AR(1) and a Markov-Switching AR(1) model. Yang et al. (2017) also use intra-day data from the Chinese commodity futures markets (Zhenzhou Commodity Exchange and Dalian Commodity Exchange) of Soybean, Cotton, Gluten Wheat and Corn futures prices and employ a similar strategy with Tian et al. (2017b), where the HAR model is extended with potential predictors (such as day-of-the-week dummies, past cumulative returns and the jump component) and forecasts are generated based on bagging and combination methods. Their conclusions suggest the forecasts based on the HAR models with bagging and principal component combination methods are able to outperform the AR model. Finally, Tian et al. (2017b) use Soybean, Cotton, Gluten Wheat, Corn, Early Indica Rice and Palm futures prices, traded in the Chinese market, to construct and forecast their realized volatility measure. Furthermore, the authors use several other realized volatility measures (such as daily log-range volatility, realized threshold multi-power variation and the realized threshold bi-power variation) and the jump component, as potential predictors of the realized volatility. Their predictive models allow both predictors and coefficients to vary over time. Their findings show that the Dynamic Model Average and Bayesian Model Average models are able to exhibit superior predictive ability, relatively to the simple HAR model. More importantly, they show that the HAR model with time-varying sparsity produces the most accurate forecasts for all the chosen commodities.

Given the limited research efforts on agricultural price volatility forecasting as well as the importance of such forecasts, it is imperative to further extend this line of research. Currently, the limited number of studies have not considered three rather important issues when it comes to agricultural commodities volatility forecasting. First, all previous papers use data from the Chinese futures markets, whereas there are no efforts to forecast volatility of agricultural commodities traded in the U.S., which is the most established market as well as the market with the highest penetration to both speculators and hedgers[1]. Second, the main focus has been on realized volatility forecasting, whereas other intra-day volatility measures have been ignored. Finally, the current literature focuses on the aggregation of the information of agricultural commodities volatility (through bagging, combination

[1]See for example Bloomberg (2019).

techniques, or time-varying approaches); nevertheless, they do not provide an answer as to whether specific volatility components, such as the jump component, the continuous component, the signed jumps, and the volatility or return leverage can provide better forecasts than simple HAR models. Thus, this study fills these voids and provides clear evidence as to whether the aforementioned components can provide predictive gains. This is rather important, given that complexity to forecasting models should only be added if this provide material predictive gains.

Succinctly, we add to this extremely scarce strand in the literature by applying several HAR-type models that accommodate the jump and continuous component, the signed jumps, and the volatility or return leverage (namely the HAR-J, HAR-CJ, HAR-PS and LHAR-CJ) to forecast different realized volatility measures (such as the realized volatility RV and the median realized volatility $MedRV$). For this study we focus on five important agricultural commodities traded in the Chicago Mercantile Exchange (CME) and the Intercontinental Exchange (ICE), namely, Corn, Rough Rice, Soybeans, Sugar, and Wheat and we produce forecasts for 1-day to 66-days ahead.

The choice of the RV and $MedRV$ volatility measures stems from the fact that the former is the most well-known volatility measure within past research but also among practitioners, whereas the latter is a more robust measure, compared to multipower variations, as large absolute returns associated with jumps tend to be eliminated from the calculation by the median operators. In addition, the $MedRV$ offers a number of advantages over alternative measures of integrated variance in the presence of infrequent jumps and it is less sensitive to the presence of occasional zero intra-day returns (Theodosiou & Zikes, 2011).

Our in-sample analysis shows that variants of the HAR model which decompose the volatility measure in its continuous path and jump component and take the volatility or return leverage effects into consideration (and in particular the LHAR-CJ model) are capable of offering better fit of the predictive equation for both the RV and $MedRV$ volatility measures. Turning to the out-of-sample results, these strongly suggest that the simple HAR model significantly outperforms the Random Walk and AR models. However, contrary to the in-sample findings, none of the HAR extensions is able to generate forecasts that are statistically significantly better compared to the simple HAR model. Hence, we cannot support the view that the decomposition of the volatility measure into its continuous path and jump component or even by taking into consideration the volatility or return leverage effect in a HAR-type model adds any incremental predictive accuracy. These results hold for both the RV and $MedRV$, hence the results are not volatility measure specific. Finally, we show that all HAR models have a marginally better directional accuracy compared to the random walk and AR models for the shorter forecasting horizons.

The remainder of this study is structured as follows. Section 2 describes the construction of the volatility measures, the predictive models and the loss functions for the forecast evaluations. Section 3 presents the data and their descriptive statistics. Section 4 presents the results followed by a thorough discussion of the in-sample and real out-of-sample evaluation. Section 5 discusses the results from a risk management application. Finally, Section 6 concludes the study and provides avenues for further research.

2. Methodology

2.1. Realized Variance measures and jump detection

Let the number of intraday observations be m and the total number of observation days be M. Intraday returns are then defined as log-difference of two consecutive prices

$$r_{t,i} = (\log\, P_{t,i} - \log\, P_{t,i-1}) * 100, \tag{1}$$

at day $t = 1, \ldots, M$ for $i = 2, \ldots, m$. The realized volatility of a given day t is then defined as

$$RV_t = \sum_{i=1}^{m} r_{t,i}^2. \tag{2}$$

Following Andersen & Bollerslev (1998) and under the assumption of no serial correlation and other noise[2] in this discrete return data sampling, it holds that

$$\underset{m\to\infty}{\text{p-lim}} \left(\int_0^1 \sigma_{t+\tau}^2 \mathrm{d}\tau - \sum_{i=1}^{m} r_{t,i}^2 \right) = 0, \tag{3}$$

where the integral describes the daily, continuous time volatility and the sum is the estimator of the daily realized volatility.

Discretizing data by equidistant sampling, where Eq. (3) does not hold any longer, might introduce intra-day price jumps which translate to higher realized variances. In order to have a more robust measure of realized volatility, Barndorff-Nielsen & Sheppard (2004) introduce the concept of the bi-power variation (BPV_t) which is defined as

$$BPV_t = \frac{\pi}{2} \left(\frac{m}{m-1} \right) \sum_{j=1}^{m-1} |r_{t,j}||r_{t,j+1}|. \tag{4}$$

This bi-power variation is being used to separate the realized variance in a continuous and discontinuous (jump) part. We use the approach of Huang (2004) to identify the jump component

$$J_t = I_{\{Z_t > \Phi_\alpha\}} \left(RV_t - BPV_t \right), \tag{5}$$

[2] We use tick-data of 5-minute price intervals to circumvent some of the microstructure issues.

where $\Phi(\cdot)$ refers to the density of a Standard Normal distribution with excess value

$$Z_t = \sqrt{m}\frac{1 - BPV_t \cdot RV_t^{-1}}{\sqrt{\left(\mu_1^{-4} + 2\mu_1^{-2} - 5\right) \max\left(1, TQ_t \cdot BPV_t^{-2}\right)}}, \tag{6}$$

and $\mu_1 = \mathbb{E}\left(Z\right) = \sqrt{2/\pi}$. The tri-power quarticity TQ_t is defined as

$$TQ_t = m\mu_{4/3}^{-3} \sum_{j=1}^{m-2} |r_{t,j}|^{4/3} |r_{t,j+1}|^{4/3} |r_{t,j+2}|^{4/3}, \tag{7}$$

where $\mu_p = 2^{p/2} \cdot \Gamma\left(1/2 \cdot (p+1)\right) \cdot \Gamma\left(1/2\right)$. We set $\alpha = 0.99$. The continuous component C_t is then calculated as

$$C_t = I_{\{Z_t > \Phi_\alpha\}} BPV_t + I_{\{Z_t \leq \Phi_\alpha\}} RV_t. \tag{8}$$

As the BPV_t is not free of flaws, e.g. a downward-bias if there are zero-return ticks, an alternative is introduced by Andersen et al. (2012). This median realized volatility $MedRV_t$ is defined as

$$MedRV_t = \frac{\pi}{6 - 4\sqrt{3} + \pi} \frac{m}{m-2} \sum_{j=2}^{m-1} \text{median}\left(|r_{t,j-1}|, |r_{t,j}|, |r_{t,j+1}|\right)^2, \tag{9}$$

which yields alternative continuous and jump components

$$J_{t,\alpha}^{MedRV} = I_{\left\{Z_t^{MedRV_t} > \Phi_\alpha\right\}} \left(RV_t - MedRV_t\right), \text{ and} \tag{10}$$

$$C_t^{MedRV} = I_{\{Z_t > \Phi_\alpha\}} MedRV_t + I_{\{Z_t \leq \Phi_\alpha\}} RV_t \tag{11}$$

with

$$Z_t^{MedRV} = \sqrt{m}\frac{1 - MedRV_t \cdot RV_t^{-1}}{\sqrt{0.96 \max\left(1, MedRQ_t \cdot MedRV_t^{-2}\right)}}, \tag{12}$$

$$MedRQ_t = \frac{3\pi}{9\pi + 72 - 52\sqrt{3}} \frac{m}{m-2} \sum_{j=2}^{m-1} \text{median}\left(|r_{t,j-1}|, |r_{t,j}|, |r_{t,j+1}|\right)^4. \tag{13}$$

In order to further disaggregate realized volatilities to account for asymmetries, we also apply realized semi-variances which are based on Barndorff-Nielsen et al. (2010) and Patton & Sheppard (2015)

$$RS_t^+ = \sum_{j=1}^{m} I_{\{r_{t,j} > 0\}} r_{t,j}^2, \tag{14}$$

$$RS_t^- = \sum_{j=1}^{m} I_{\{r_{t,j} < 0\}} r_{t,j}^2, \tag{15}$$

and it naturally holds that $RV_t = RS_t^+ + RS_t^-$.

2.2. RV Models

In this section, we present the forecasting models for the realized volatility (RV), whereas by replacing the RV with the $MedRV$, we can obtain the equivalent predictive models for the latter volatility measure. We follow the formulations of Corsi & Renò (2012). Thus, for each forecasting horizon h and each forecasting model we generate a different regression estimation. Doing so allows us to circumvent the use of recursive long-term forecasts based on the relative weights for 1-day ahead predictions for $h > 1$.[3] In particular, we define

$$\log(RV_{t+h}^{(h)}) = \frac{1}{h}\sum_{j=1}^{h}\log(RV_{t+h-j+1}) \quad \text{and} \tag{16}$$

$$\log(RV_t^{(h)}) = \frac{1}{h}\sum_{j=1}^{h}\log(RV_{t-j+1}), \tag{17}$$

where $h \in \{1, ..., 66\}$ denotes the days ahead forecasting horizons. Note that, $\log(RV_{t+h}^{(h)})$ is the average realized volatility for time $t+1$ to $t+h$, $\log(RV_t^{(h)})$ is the average realized volatility for time $t-h+1$ to t, while $\log(RV_t)$ is the realized volatility at time t and equivalent to $\log(RV_t^{(1)})$.

As a baseline estimation, we use a simple Random Walk (RW), defined as:

$$\log(RV_{t+h}^{(h)}) = \log(RV_t^{(h)}) + \varepsilon_{t+h}^{(h)}, \tag{18}$$

a simple autoregressive model of order one (AR(1)), defined as:

$$\log(RV_{t+h}^{(h)}) = \beta_0^{(t)} + \beta_1^{(t)}\log(RV_t^{(h)}) + \varepsilon_{t+h}^{(h)}, \tag{19}$$

as well as an autoregressive moving average model of order one (ARMA(1,1)), defined as:

$$\log(RV_{t+h}^{(h)}) = \beta_0^{(t)} + \beta_1^{(t)}\log(RV_t^{(h)}) + \beta_2^{(t)}\varepsilon_t^{(h)} + \varepsilon_{t+h}^{(h)}. \tag{20}$$

Subsequent to the three aforementioned naive and simple models, we focus on the standard HAR model of Corsi (2009) and a number of extensions. The standard HAR model reads as follows:

$$\log(RV_{t+h}^{(h)}) = \beta_0^{(t)} + \beta_1^{(t)}\log(RV_t) + \beta_2^{(t)}\log(RV_t^{(5)}) + \beta_3^{(t)}\log(RV_t^{(22)}) + \varepsilon_{t+h}^{(h)}, \tag{21}$$

where the RV_t denotes the previous day's volatility, $RV_t^{(5)}$ denotes the averaged volatility during the previous week and finally, $RV_t^{(22)}$ denotes the averaged volatility over the

[3] As pointed out by Ederington & Guan (2010), the recursive forecast procedure introduces a bias to longer-term forecasts.

previous month.

Next, in order to account for possible jumps, we augment the standard HAR with the simple jump measure J_t to define the HAR-J model:

$$\log(RV_{t+h}^{(h)}) = \beta_0^{(t)} + \beta_1^{(t)} \log(RV_t) + \beta_2^{(t)} \log(RV_t^{(5)}) + \beta_3^{(t)} \log(RV_t^{(22)}) + \beta_4^{(t)} \log(J_t + 1) + \varepsilon_{t+h}^{(h)}. \tag{22}$$

Andersen et al. (2007) further propose to make use of bi-power variations to separate realized volatilites in a continuous and jump components, which we subsequently labeled HAR-CJ:

$$\begin{aligned} \log(RV_{t+h}^{(h)}) = \beta_0^{(t)} &+ \beta_1^{(t)} \log(J_t + 1) + \beta_2^{(t)} \log(J_t^{(5)} + 1) + \beta_3^{(t)} \log(J_t^{(22)} + 1) \\ &+ \beta_4^{(t)} \log(C_t) + \beta_5^{(t)} \log(C_t^{(5)}) + \beta_6^{(t)} \log(C_t^{(22)}) + \varepsilon_{t+h}^{(h)}. \end{aligned} \tag{23}$$

In analogy to the definition of $RV_t^{(h)}$ above, we define $\log\left(C_t^{(h)}\right) = \frac{1}{h}\sum_{j=1}^{h} \log\left(C_{t-j+1}\right)$ and $J_t^{(h)} = \sum_{j=1}^{h} J_{t-j+1}$. Note that jumps are aggregated, not averaged.

The next model is one of the HAR specifications outlined in Patton & Sheppard (2015) who separate realized volatilites in semi-variances to include measures for positive and negative daily log-returns (r_t) as well as possible leverage effects. This model is labeled HAR-PS:

$$\begin{aligned} \log(RV_{t+h}^{(h)}) = \beta_0^{(t)} &+ \beta_1^{(t)} \log(RS_t^{+}) + \beta_2^{(t)} \log(RS_t^{-}) + \beta_3^{(t)} I_{\{r_t<0\}} \log(RV_t) \\ &+ \beta_4^{(t)} \log(RV_t^{(5)}) + \beta_5^{(t)} \log(RV_t^{(22)}) + \varepsilon_{t+h}^{(h)}. \end{aligned} \tag{24}$$

Finally, we use a leverage variant of the HAR-CJ, which is proposed by Corsi & Renò (2012). This model separates the aggregated negative daily log-returns over the corresponding periods to account for leverage effects. The LHAR-CJ reads:

$$\begin{aligned} \log(RV_{t+h}^{(h)}) = \beta_0^{(t)} &+ \beta_1^{(t)} \log(J_t + 1) + \beta_2^{(t)} \log(J_t^{(5)} + 1) + \beta_3^{(t)} \log(J_t^{(22)} + 1) \\ &+ \beta_4^{(t)} \log(C_t) + \beta_5^{(t)} \log(C_t^{(5)}) + \beta_6^{(t)} \log(C_t^{(22)}) \\ &+ \beta_7^{(t)} r_t^{-} + \beta_8^{(t)} r_t^{(5)-} + \beta_9^{(t)} r_t^{(22)-} + \varepsilon_{t+h}^{(h)}, \end{aligned} \tag{25}$$

with

$$r_t^{(h)-} = \frac{1}{h} I_{\{(r_t+\ldots+r_{t-h+1})<0\}} \left(r_t + \ldots + r_{t-h+1}\right). \tag{26}$$

The choice of the HAR model and its extensions is motivated by the fact that the existing literature has convincingly shown that this model class is the most appropriate framework to model and forecast intra-day volatility (such as the realized volatility and the median realized volatility). This has been shown not only for agricultural commodities (Tian et al., 2017a,b, Yang et al., 2017), but also for other commodities, such as crude oil, copper, and aluminum as well as stock market indices (Corsi & Renò, 2012, Sévi, 2014, Zhang, 2017, Degiannakis & Filis, 2017).

2.3. Forecasting & Evaluation

As outlined in Section 2.2, we use different regression models for each forecasting horizon h. For example, when estimating the HAR model for $h = 66$, i.e. $\log(RV^{(66)}_{t+66})$, we obtain a prediction for the average RV for the next 66 days and we use it as an estimate for the realized volatility in 66 days. Doing so allows us firstly, to circumvent any iterative forecasting procedure and secondly, to use the one-day ahead prediction for each model regardless of the forecasting horizon h. The idea is directly taken from Corsi & Renò (2012).

To evaluate our forecasting results from the presented models over the h-days ahead horizons, for $h = 1, 5, \ldots, 66$, we employ three widely used loss functions, namely the Mean Squared Prediction Error (MSPE), the Mean Absolute Percentage Error (MAPE), and the QLIKE (Patton, 2011):

$$MSPE = \sqrt{N^{-1} \sum_{t=1}^{N} \left(RV_t - \widehat{RV_t} \right)^2}, \tag{27}$$

$$MAPE = N^{-1} \sum_{t=1}^{N} \frac{\left| RV_t - \widehat{RV_t} \right|}{RV_t}, \tag{28}$$

$$QLIKE = N^{-1} \sum_{t=1}^{N} \left(\log(\widehat{RV_t}) + \frac{\widehat{RV_t}}{RV_t} \right), \tag{29}$$

where RV_t and $\widehat{RV_t}$ are the actual realized volatility and the forecasted RV, respectively, at the different forecasting horizons and N is the number of real out-of-sample forecasts. The forecasting errors are then compared using the Model Confidence Set (MCS, Hansen et al., 2011). The MCS is built by iteratively comparing all forecasts under consideration, the set $\mathcal{M}^0$, and by creating a subset of models with statistically indistinguishable performance from the best model, $\mathcal{M}^*$. Here, the best model refers to the one with the lowest loss function (MSPE, MAPE, and QLIKE). Thus, all models belonging to the set $\overline{\mathcal{M}^*}$, which are not part of the MCS, are performing statistically worse than all models included in the MCS. Following Hansen et al. (2011), we calculate two MCS sets, the $\mathcal{M}^*_{90\%}$ for $\alpha = 10\%$ and $\mathcal{M}^*_{75\%}$ for $\alpha = 25\%$, i.e. we construct a larger set of models with a confidence level of 90% and a more restrictive subset of the best models at the cost of a lower confidence of 75%. We use the T_R statistic and 10 000 bootstraps with a block length of 3 to calculate the MCS.[4]

Moreover, we evaluate the directional accuracy of the predicted RV. To this end, we

[4] Our code on the estimation and forecasting is based on the code provided by Andrew Patton (`http://public.econ.duke.edu/~ap172/`). The calculations of the MCS are performed using the MFE MatLab toolbox of Kevin Sheppard available from his personal webpage `https://www.kevinsheppard.com/MFE_Toolbox`. All estimations, forecasts, and calculations are carried out in Matlab 2018a using an Intel i7-7700 and 32GB RAM.

calculate the Success Ratio (SR) by

$$SR = N^{-1} \sum_{t=1}^{N} \mathbf{I}_{RV_t \cdot \widehat{RV_t} > 0}, \tag{30}$$

where $\mathbf{I}_{RV_t \cdot \widehat{RV_t} > 0}$ is an indicator function which is one if $RV_t \cdot \widehat{RV_t} > 0$ and zero otherwise.

Thus, the SR displays the ratio of a model's success to correctly predict the directional movement of the actual time series. In order to obtain directions from non-negative volatility forecasts, we de-mean the actual realized volatility RV_t and its forecast ($\widehat{RV_t}$) by their corresponding overall mean beforehand. The SR is then tested using the test statistic presented by Pesaran & Timmermann (1992).[5]

We should note here that the same loss functions and evaluation methods are utilized for the $MedRV$ forecasts as well.

3. Data

Our data set consists of tick-by-tick prices of the most liquid front month futures contracts of Corn, Rough Rice, Soybean, Sugar, and Wheat, traded at the CME and ICE, sampled from January 4, 2010 to June 30, 2017. The period of time is dictated by the data availability of these futures contracts. In order to circumvent microstructure noise, we aggregate our data to 5-min prices; see also Andersen & Bollerslev (1998), Degiannakis (2008), and Liu et al. (2015). Subsequently, we obtain data on $M = 1\,898$ trading days with a total number of intra-day prices ranging from $m_{\text{total}} = 234\,798$ (Sugar) to $m_{\text{total}} = 399\,190$ (Rice). For our in-sample analysis we use the full number of daily observations, whereas for the real out-of-sample forecasts we use the period January 4, 2010 to December 31, 2012 for our estimation period and the period January 2, 2013 to June 30, 2017 for the out-of-sample forecasts, based on a rolling window approach with fixed window length of 3 years (roughly 750 observations). We opt for a rolling window approach given its superior ability to capture changes in the market conditions, as suggested by Engle et al. (1990), Degiannakis & Filis (2017), and Degiannakis et al. (2018).

Table 1 provides an overview of the sampling times and data sources.

Descriptive statistics and test statistics of the Ljung-Box test for five, ten, and 22 lags (trading days), corresponding to the aggregation in the HAR-type models, are presented in Tables 2-6. We report statistics for the realized volatility (RV_t), the discontinuous jump component (J_t), and the continuous component (C_t) according to definitions given in Eq. (2) and Eq. (5)-(8), respectively. Statistics for the alternative measure of realized

[5]Pesaran & Timmermann (1992) provide the test statistic $\frac{SR - SR^*}{\sqrt{\text{var}(SR) - \text{var}(SR^*)}} \overset{a}{\sim} N(0,1)$, where $SR^* = P \cdot \widehat{P} + (1-P) \cdot (1-\widehat{P})$, $\text{var}(SR) = SR^* \cdot (1 - SR^*)/N$, $\text{var}(SR^*) = \left(2\widehat{P} - 1\right)^2 \cdot P \cdot (1-P)/N + (2P-1)^2 \cdot \widehat{P} \cdot (1-\widehat{P})/N + 4P \cdot \widehat{P} \cdot (1-P) \cdot (1-\widehat{P})/N^2$, $P = N^{-1} \sum_{t=1}^{N} \mathbf{I}_{RV_t > 0}$, and $\widehat{P} = N^{-1} \sum_{t=1}^{N} \mathbf{I}_{\widehat{RV}_t > 0}$.

Commodity	Exchange	Ticker	Sampling times (GMT)	Trading pauses (GMT)
Corn	CBOT/CME	CN	Monday (01:00:05) - Friday (23:59:59)	20:01-22:00
Rough Rice	CBOT/CME	RR	Monday (08:15:05) - Friday (23:59:59)	20:01-22:00
Soybeans	CBOT/CME	SY	Monday (01:00:05) - Friday (23:59:59)	20:01-22:00
Wheat	CBOT/CME	WC	Monday (11:00:05) - Friday (23:59:59)	14:01-17:00
Sugar	ICE Futures U.S.	SB	Monday (05:31:00) - Friday (19:00:00)	–

Table 1: Overview of the acquired data, its source for each agricultural commodity futures, and sampling times.

volatility with the median RV measure, $MedRV$, defined in Eq. (9), are given in the rightmost columns of those tables.

Sugar (Table 5) futures present the highest mean of realized volatilities at 3.8498 as well as the highest maximum daily volatility of 44.1071 which is almost twofold the second-highest value of the maximum of RV_t (Wheat). Soybean (Table 4) shows the lowest values of mean and maximum of RV_t as well as the lowest standard deviation. The statistics for Corn, Rough Rice, and Wheat are quite similar and less extreme than Sugar or Soybean. The results for the alternative measure of realized volatility, $MedRV_t$, are qualitatively the same.

We find that for all five commodities, the measures for realized volatilities show significant autocorrelation on all lags, tested with the Ljung-Box test. This further motivates the application of autoregressive models such as the HAR and its extensions. Surprisingly, even the jump components J_t for all commodities show autoregressive behaviour, indicating that agricultural commodity futures are indeed a special case if compared to high-frequency prices of crude oil or metal futures. Albeit with lower test statistics compared to its measures for realized volatilities, autocorrelated jump measures suggest that jumps in realized volatilities are a very common occurrence. We follow that high intraday price movements are the rule instead of an exception for agricultural prices in our sample period. This is supported by the relatively high Kurtosis of the realized volatility measures for all commodities. As the continuous component C_t refers to the remaining realized volatility after removing jumps, the Ljung-Box test statistics are naturally much higher and take dimensions similar to RV_t. The findings for $MedRV_t$ and its jump and continuous part decomposition are qualitatively the same. Since $MedRV_t$ is more robust against small and high jumps compared to RV_t, we can expect a better forecasting performance given this highly volatile data set.

	RV_t	J_t	C_t	$MedRV_t$	J_t^{MedRV}	C_t^{MedRV}
Mean	2.4523	0.2565	2.1958	2.1194	0.1880	2.2644
Minimum	0.2171	0.0000	0.0750	0.0905	0.0000	0.2171
Maximum	18.4265	5.9690	16.4022	13.2742	6.4750	16.4022
StD	1.7878	0.5188	1.6563	1.4810	0.5805	1.6324
Skewness	2.7020	4.5461	2.6810	2.7628	5.3108	2.8505
Kurtosis	14.2612	34.5191	14.0081	14.4542	39.8801	15.4021
Q(5)	1987.95***	18.83***	1883.92***	1955.56***	24.07***	1818.43***
Q(10)	3119.27***	23.68***	3009.55***	3047.09***	45.80***	2854.76***
Q(22)	4696.17***	47.82***	4686.03***	4564.83***	96.88***	4318.17***

Table 2: Descriptive statistics for Corn, sampled from January 4, 2010 to June 30, 2017 with $M = 1\,898$ trading days and a total number of $m_{\text{total}} = 399\,114$ prices at the 5 minutes interval.

Fig. 1 visualizes two measures of realized volatility (RV_t and $MedRV_t$) and the jump measure ($J_{t,\alpha}$) for Corn, Rough Rice, Soybean, Sugar, and Wheat in our sample period

	RV_t	J_t	C_t	$MedRV_t$	J_t^{MedRV}	C_t^{MedRV}
Mean	3.1809	1.3842	1.7967	1.8069	0.9439	2.2370
Minimum	0.1245	0.0000	0.0158	0.0040	0.0000	0.0040
Maximum	19.7251	10.1585	18.6148	17.2740	11.1076	18.6148
StD	2.6145	1.4413	1.9326	1.7684	1.4327	2.2589
Skewness	2.0028	2.1103	2.8934	2.6134	2.4035	2.4388
Kurtosis	8.5347	9.0724	16.1702	13.9032	10.5472	11.5642
Q(5)	1395.49***	349.95***	737.04***	940.26***	69.50***	799.13***
Q(10)	2486.75***	632.27***	1332.10***	1687.71***	101.03***	1474.60***
Q(22)	4434.15***	1137.62***	2352.03***	3050.09***	169.53***	2693.43***

Table 3: Descriptive statistics for Rough Rice, sampled from January 4, 2010 to June 30, 2017 with $M = 1\,898$ trading days and a total number of $m_{\text{total}} = 399\,190$ prices at the 5 minutes interval.

	RV_t	J_t	C_t	$MedRV_t$	J_t^{MedRV}	C_t^{MedRV}
Mean	1.5387	0.1475	1.3912	1.2902	0.1471	1.3916
Minimum	0.0201	0.0000	0.0043	0.0019	0.0000	0.0019
Maximum	8.4025	5.4881	8.2409	10.7046	5.2553	8.2409
StD	1.0307	0.3581	0.9419	0.8652	0.4306	0.9196
Skewness	2.4411	6.0922	2.4766	2.9417	5.8652	2.5342
Kurtosis	11.4026	61.5027	12.1233	18.7540	50.2153	12.9593
Q(5)	1680.15***	22.01***	1976.96***	2052.24***	16.96***	2172.94***
Q(10)	2479.40***	29.62***	2916.09***	3037.15***	19.63***	3234.40***
Q(22)	3581.27***	43.12***	4120.91***	4289.34***	33.76***	4518.27***

Table 4: Descriptive statistics for Soybean, sampled from January 4, 2010 to June 30, 2017 with $M = 1\,898$ trading days and a total number of $m_{\text{total}} = 399\,126$ prices at the 5 minutes interval.

	RV_t	J_t	C_t	$MedRV_t$	J_t^{MedRV}	C_t^{MedRV}
Mean	3.8498	0.2875	3.5623	3.2655	0.3095	3.5403
Minimum	0.2853	0.0000	0.2665	0.3312	0.0000	0.2853
Maximum	44.1071	5.3501	44.1071	46.5043	6.5751	44.1071
StD	3.1993	0.6453	3.0801	2.8760	0.8004	3.0541
Skewness	3.2517	3.1877	3.5814	4.3797	3.5174	3.6207
Kurtosis	24.5723	15.5835	28.9848	43.1246	17.7357	29.5813
Q(5)	3260.05***	22.47***	3081.98***	2775.52***	34.32***	2987.32***
Q(10)	5641.97***	42.31***	5320.00***	4792.44***	56.22***	5193.60***
Q(22)	10486.96***	107.35***	9812.76***	8760.67***	119.38***	9588.06***

Table 5: Descriptive statistics for Sugar, sampled from January 4, 2010 to June 30, 2017 with $M = 1\,898$ trading days and a total number of $m_{\text{total}} = 234\,798$ prices at the 5 minutes interval.

	RV_t	J_t	C_t	$MedRV_t$	J_t^{MedRV}	C_t^{MedRV}
Mean	3.2421	0.3139	2.9282	2.7216	0.2813	2.9608
Minimum	0.1040	0.0000	0.1040	0.0601	0.0000	0.1040
Maximum	26.0959	6.4052	26.0959	24.4544	6.7609	26.0959
StD	2.2858	0.6015	2.1958	1.9768	0.6948	2.1531
Skewness	2.8804	3.3899	3.1174	3.2825	3.6660	3.1371
Kurtosis	17.5920	21.0279	20.2421	22.2619	20.6146	20.9127
Q(5)	2343.74***	48.49***	2104.90***	2076.85***	71.31***	2124.42***
Q(10)	3413.07***	66.45***	3022.29***	3007.03***	103.52***	3007.07***
Q(22)	5421.44***	101.57***	4755.90***	4833.95***	162.93***	4705.53***

Table 6: Descriptive statistics for Wheat, sampled from January 4, 2010 to June 30, 2017 with $M = 1\,898$ trading days and a total number of $m_{\text{total}} = 399\,114$ prices at the 5 minutes interval.

January 4, 2010 to June 30, 2017. Interestingly enough we show that the two volatility measures are closely related yet there are certain peaks, especially in the case of Rice, that are not observed for both measures. This is due to the fact that the $MedRV$ measure is more robust against jumps. Similarly, the jump component behaves rather differently for the different commodities, with a common feature that fewer jumps are apparent during 2013-2014.[6]

[6]We note that the daily data for the RV, $MedRV$, and their jump components are available upon request by the authors.

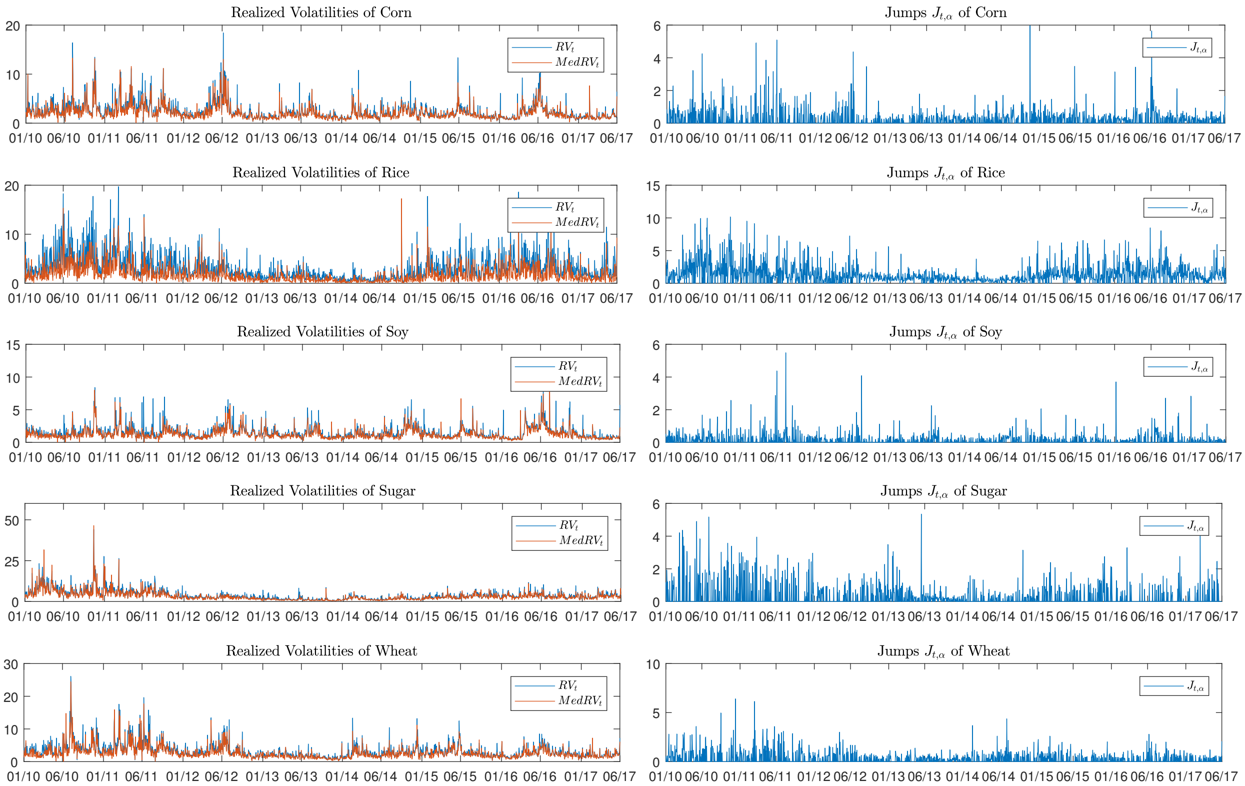

Figure 1: Realized volatility measures (RV_t and $MedRV_t$) and jump measure ($J_{t,\alpha}$) for Corn, Rough Rice, Soybean, Sugar, and Wheat for the sample period January 4, 2010 to June 30, 2017.

4. Results & Discussion

4.1. In-Sample Results

Our in-sample results are presented in Tables 7-11 for RV and in Tables A.17-A.21 (in Appendix A) for $MedRV$, given that the results are qualitatively similar for both volatility measures. Each table shows the parameter estimates as well as loss function for all seven models over all five forecasting horizons.

The best model over all commodities and horizons appears to be the LHAR-CJ, which consistently has the highest R^2 and, with a few exemptions, the lowest loss functions, i.e. it always belongs to the $\mathcal{M}^*_{75\%}$. Comparing the class of HAR models with the naive Random Walk and the AR(1), we conclude that, except for a few instances, the HAR models are superior with regards to model fit. This reveals the fact that a long-term component in the volatility helps to explain the variance of the volatility. The high t-statistics for the RV and C parameters with 5 and 22 days support this assessment. For the Random Walk with forecasting horizons greater than one day, we notice that it performs even worse than the sample mean which is depicted by negative R^2.

Another interesting observation is that the leverage effect appears weak. Thus, in the in-sample series of Corn, Rice, Soybean, Sugar, and Wheat the interaction term between dummy variable of a negative return and the RV is merely of statistical importance. For the negative return parameters in the LHAR-CJ model, we find that the lag of r_t^- somewhat corresponds to the forecasting horizon h, i.e. for small forecasting horizons we observe higher t-statistics and decreasing t-statistics for higher horizons for the first lag. For the fifth and twenty-second lag, however, we find reverse behaviour. We presume that this association roots in the way the regression models for the different forecasting horizons are constructed, e.g. for $h = 22$ the model forecasts the average volatility over 22 days and the leverage component for 22-days of average negative returns contains more information for this regression than the last leverage component for the preceding day.

A similar pattern is noticeable from the jump components in the HAR-CJ and LHAR-CJ models. Again, we see a correlated behavior of the components' lag with the forecasting horizon for most commodities. However, for the HAR-J, i.e. for the model with only one lagged jump, the statistical significance varies over the five commodities. While we observe slightly statistically significant parameters for Corn, Rice, Sugar, and Wheat over all horizons from 1-day to 66-days ahead, the parameter is not distinguishable from zero for Soybean.[7]

From the in-sample analysis, we conclude that the best performing model is the most complex one: the LHAR-CJ depicting long memory, leverage effect, and a differentiation between continuous and jump components. Moreover, the importance of the lags of the leverage and the jump parameters appear to be positively associated with the forecasting horizon. Thus, we conclude that stylized facts are important to describe the in-sample volatility of agricultural commodities. The fact that LHAR-CJ includes all those components at different time horizons makes it consistently superior to its peer over all horizons.

4.2. Real out-of-sample forecasting results

From the in-sample evaluation we show that the LHAR-CJ is the best performing model for all agricultural commodities, across all horizons, and for both volatility measures. Nevertheless, to be able to generate solid conclusions we need to assess the performance of our models in real out-of-sample forecasts.

Thus, we turn our attention to the real out-of-sample forecasting evaluation based on the MSPE, MAPE, and QLIKE. Furthermore, we use the MCS test to identify the set of the best models with equal predictive accuracy. The results are depicted in Tables 12-16 for RV.[8] At a first glance, we notice that none of the competing models can consistently improve the forecast accuracy that we obtain from the simple HAR model.

More specifically, the Random Walk, AR, and ARMA models largely underperform compared to the HAR-type models under any loss function and for all commodities; although they are among the best performing models in the longer forecasting horizons for the RV measure under specific loss function per commodity.

Turning to the HAR-type models, we observe that they significantly outperform the Random Walk and AR models based on the MCS test, except from the cases outlined earlier. The most important finding, though, is that the simple HAR model is not con-

[7]Note that this finding is only robust for higher horizons if we look at RV measure.
[8]Results for $MedRV$ are presented in Tables A.22-A.26 in Appendix A.

h	1	5	10	22	44	66
			Random Walk			
adj. R^2	0.2291	0.4768	0.4601	0.3986	0.1035	-0.2900
MSPE	1.6701	1.6512	1.6991**	1.7574*	1.9139	1.9736
MAPE	0.4281	0.4417	0.4640	0.4954	0.5877	0.6530
QLIKE	1.8564	1.8607**	1.8720**	1.9001*	1.9708	2.0336
			AR(1)			
c	0.2715	0.1828	0.1888	0.2056	0.3070	0.4549
	(13.7709)	(14.1551)	(15.1709)	(14.9843)	(19.4514)	(24.9167)
$RV^{(h)}$	0.6144	0.7377	0.7296	0.7006	0.5538	0.3460
	(26.6586)	(44.3740)	(44.8092)	(36.9599)	(28.4631)	(15.8790)
adj. R^2	0.3770	0.5452	0.5329	0.4874	0.2972	0.1109
MSPE	1.4961	1.6007	1.6591**	1.7216*	1.8356	1.8628
MAPE	0.3763	0.4121*	0.4324*	0.4630**	0.5186*	0.5340**
QLIKE	1.8214**	1.8384**	1.8489**	1.8636**	1.8969**	1.9134**
			ARMA			
c	0.0361	0.0886	0.1202	0.2298	0.5748	0.7806
	(4.4106)	(7.4290)	(8.6521)	(11.4874)	(22.7837)	(16.3591)
$RV^{(h)}$	0.9485	0.8733	0.8279	0.6692	0.1860	-0.1078
	(98.4321)	(60.7486)	(51.8560)	(28.6283)	(5.8400)	(1.7522)
$\varepsilon^{(h)}$	-0.6590	-0.3186	-0.2148	0.0671	0.6195	0.5319
	(31.1119)	(11.5396)	(7.4564)	(1.9298)	(21.7651)	(10.2136)
adj. R^2	0.4574	0.5645	0.5441	0.4908	0.3950	0.1507
MSPE	1.4134**	1.5866*	1.6557**	1.7181**	1.8065**	1.8751
MAPE	0.3493**	0.4043**	0.4300**	0.4614**	0.5113*	0.5362**
QLIKE	1.8098**	1.8351**	1.8483**	1.8632**	1.8987**	1.9177**
			HAR			
c	0.0769	0.1143	0.1485	0.1974	0.2830	0.3627
	(3.9033)	(8.2035)	(11.2609)	(15.1118)	(21.7301)	(25.6239)
$RV^{(1)}$	0.2710	0.1642	0.1352	0.0931	0.0723	0.0559
	(8.4593)	(7.4805)	(6.4997)	(4.6027)	(3.7074)	(2.7557)
$RV^{(5)}$	0.3677	0.3480	0.2780	0.2294	0.2333	0.2096
	(7.1703)	(9.9696)	(8.5140)	(7.3187)	(7.5987)	(6.5342)
$RV^{(22)}$	0.2507	0.3220	0.3723	0.3915	0.2867	0.2130
	(5.2222)	(9.5482)	(11.5066)	(12.6777)	(9.8721)	(6.8432)
adj. R^2	0.4616	0.5829	0.5719	0.5378	0.4416	0.3271
MSPE	1.4138**	1.5823	1.6542**	1.7199**	1.8160	1.8440
MAPE	0.3474**	0.4036**	0.4299*	0.4611**	0.5101*	0.5271**
QLIKE	1.8094**	1.8347**	1.8475**	1.8635**	1.8910**	1.9091**
			HAR-J			
c	0.0803	0.1228	0.1562	0.2069	0.2860	0.3600
	(3.8114)	(8.0783)	(10.9821)	(14.8998)	(20.6688)	(24.1734)
$RV^{(1)}$	0.2767	0.1788	0.1485	0.1095	0.0776	0.0512
	(7.7882)	(7.2318)	(6.3356)	(4.7209)	(3.5889)	(2.3196)
$RV^{(5)}$	0.3672	0.3466	0.2768	0.2280	0.2329	0.2100
	(7.1720)	(9.9612)	(8.4759)	(7.2561)	(7.5777)	(6.5449)
$RV^{(22)}$	0.2496	0.3190	0.3695	0.3880	0.2856	0.2139
	(5.1870)	(9.4131)	(11.3890)	(12.5545)	(9.8227)	(6.8864)
$J^{(1)}$	-0.0204	-0.0522	-0.0473	-0.0583	-0.0187	0.0169
	(-0.3689)	(-1.3986)	(-1.4217)	(-1.8507)	(-0.5884)	(0.5274)
adj. R^2	0.4614	0.5832	0.5721	0.5385	0.4414	0.3268
MSPE	1.4137**	1.5804*	1.6531**	1.7181**	1.8163	1.8437
MAPE	0.3475**	0.4035**	0.4296*	0.4613**	0.5103*	0.5271**
QLIKE	1.8095**	1.8345**	1.8473**	1.8636**	1.8911**	1.9091**
			HAR-CJ			
c	0.1979	0.2560	0.2925	0.3037	0.2720	0.2804
	(4.4140)	(8.1272)	(10.0820)	(11.4628)	(10.7127)	(10.4344)
$J^{(1)}$	0.1488	0.0859	0.0595	0.0366	0.0223	0.0142
	(3.2641)	(2.8191)	(2.1244)	(1.4198)	(0.8341)	(0.5198)
$J^{(5)}$	0.0730	0.0575	0.0326	-0.0206	-0.0183	-0.0133
	(2.3276)	(2.6926)	(1.6555)	(-1.0230)	(-0.9383)	(-0.6589)
$J^{(22)}$	-0.0291	-0.0381	-0.0331	0.0076	0.0769	0.1126
	(-0.9481)	(-1.6990)	(-1.6055)	(0.3973)	(4.1672)	(5.9815)
$C^{(1)}$	0.2383	0.1439	0.1178	0.0816	0.0654	0.0516
	(7.8100)	(6.9921)	(6.1939)	(4.3552)	(3.6248)	(2.7543)
$C^{(5)}$	0.3260	0.3144	0.2675	0.2477	0.2479	0.2211
	(6.8119)	(9.6263)	(8.8319)	(8.5036)	(8.4374)	(7.1935)
$C^{(22)}$	0.2685	0.3373	0.3702	0.3501	0.2074	0.1179
	(5.6877)	(9.5194)	(11.0729)	(11.4272)	(6.9175)	(3.6910)
adj. R^2	0.4609	0.5863	0.5783	0.5435	0.4410	0.3314
MSPE	1.4159**	1.5747**	1.6495**	1.7179**	1.8133	1.8432
MAPE	0.3480**	0.4036**	0.4268**	0.4612**	0.5086*	0.5245**
QLIKE	1.8089**	1.8337**	1.8458**	1.8637**	1.8907**	1.9079**
			HAR-PS			
c	0.2720	0.2315	0.2404	0.2625	0.3322	0.4021
	(8.8286)	(10.3589)	(10.9672)	(12.3914)	(16.5556)	(18.8381)
RS^{+}	0.0985	0.0574	0.0376	0.0207	0.0117	0.0009
	(3.6287)	(3.1694)	(2.1823)	(1.2009)	(0.6690)	(0.0485)
RS^{-}	0.1751	0.1069	0.0912	0.0708	0.0574	0.0545
	(5.0766)	(4.4604)	(3.8958)	(3.1795)	(2.6153)	(2.4125)
$\mathbf{I}_{r_t<0}RV^{(1)}$	-0.0152	-0.0076	0.0010	-0.0114	-0.0171	-0.0264
	(-0.5522)	(-0.3611)	(0.0521)	(-0.5884)	(-0.9656)	(-1.5331)
$RV^{(5)}$	0.3762	0.3537	0.2856	0.2385	0.2469	0.2256
	(7.3238)	(10.0583)	(8.6690)	(7.5252)	(7.9694)	(6.9823)
$RV^{(22)}$	0.2516	0.3226	0.3726	0.3914	0.2860	0.2117
	(5.2345)	(9.5383)	(11.4684)	(12.6516)	(9.8359)	(6.8047)
adj. R^2	0.4593	0.5815	0.5709	0.5366	0.4399	0.3258
MSPE	1.4157**	1.5840	1.6548**	1.7197**	1.8165	1.8439
MAPE	0.3477**	0.4040**	0.4305*	0.4611**	0.5099*	0.5270**
QLIKE	1.8096**	1.8350**	1.8479**	1.8633**	1.8909**	1.9092**
			LHAR-CJ			
c	0.1869	0.2310	0.2741	0.2979	0.2578	0.2612
	(3.7655)	(6.8504)	(8.8028)	(10.1694)	(9.1067)	(8.7459)
$J^{(1)}$	0.1429	0.0806	0.0548	0.0343	0.0208	0.0139
	(3.1675)	(2.6713)	(1.9646)	(1.3321)	(0.7855)	(0.5176)
$J^{(5)}$	0.0732	0.0574	0.0324	-0.0207	-0.0181	-0.0129
	(2.3617)	(2.7158)	(1.6637)	(-1.0333)	(-0.9329)	(-0.6382)
$J^{(22)}$	-0.0382	-0.0337	-0.0323	0.0051	0.0848	0.1292
	(-1.1599)	(-1.4448)	(-1.4995)	(0.2468)	(4.2261)	(6.2994)
$C^{(1)}$	0.2224	0.1352	0.1085	0.0745	0.0593	0.0478
	(7.2915)	(6.5716)	(5.7081)	(3.9539)	(3.2836)	(2.5466)
$C^{(5)}$	0.3280	0.3064	0.2632	0.2499	0.2524	0.2274
	(6.8660)	(9.3713)	(8.6812)	(8.4827)	(8.5176)	(7.2767)
$C^{(22)}$	0.2793	0.3413	0.3749	0.3531	0.2008	0.1029
	(5.9710)	(9.6185)	(11.2086)	(11.4265)	(6.6458)	(3.1870)
$r_t^{-,(1)}$	-0.0417	-0.0225	-0.0240	-0.0188	-0.0171	-0.0116
	(-3.1915)	(-2.5805)	(-3.0373)	(-2.7349)	(-2.5435)	(-1.7253)
$r_t^{-,(5)}$	-0.0030	0.0009	-0.0001	-0.0009	0.0020	0.0044
	(-1.0386)	(0.4832)	(-0.0528)	(-0.5000)	(1.0786)	(2.2776)
$r_t^{-,(22)}$	-0.0601	-0.0789	-0.0632	-0.0123	0.0553	0.1119
	(-1.1275)	(-2.4668)	(-2.2226)	(-0.4630)	(2.1065)	(4.4947)
adj. R^2	0.4653	0.5896	0.5816	0.5447	0.4431	0.3379
MSPE	1.4050**	1.5757**	1.6520**	1.7181**	1.8090**	1.8375**
MAPE	0.3453**	0.4029**	0.4258**	0.4610**	0.5065**	0.5231**
QLIKE	1.8079**	1.8336**	1.8456**	1.8635**	1.8899**	1.9073**

Table 7: In-Sample regression results for Corn with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
Random Walk						
adj. R^2	0.1138	0.5612	0.6559	0.6563	0.7404	0.7227
MSPE	2.7428	2.3759	2.3925**	2.4649	2.3974**	2.4587
MAPE	0.7216	0.6141	0.6208	0.6423	0.6306	0.6758
QLIKE	2.1942	2.1099**	2.1159**	2.1302**	2.1310**	2.1768
AR(1)						
c	0.3812	0.1890	0.1482	0.1475	0.1063	0.1066
	(15.2813)	(11.7710)	(9.6476)	(9.8653)	(8.2372)	(7.9070)
$RV^{(h)}$	0.5567	0.7807	0.8281	0.8281	0.8715	0.8640
	(26.5044)	(53.6456)	(57.8886)	(55.8951)	(71.8209)	(74.0386)
adj. R^2	0.3093	0.6088	0.6851	0.6854	0.7567	0.7413
MSPE	2.3864	2.3566	2.3908	2.4593	2.4157*	2.4768
MAPE	0.6134	0.5945	0.6074*	0.6284	0.6221	0.6592
QLIKE	2.1165**	2.0991**	2.1084**	2.1206**	2.1173**	2.1445**
ARMA						
c	0.0122	0.0364	0.0360	0.0458	0.0805	0.1260
	(2.7418)	(5.0655)	(5.0083)	(5.5321)	(7.7323)	(9.2143)
$RV^{(h)}$	0.9861	0.9584	0.9581	0.9451	0.8964	0.8344
	(217.6556)	(130.3651)	(134.2256)	(112.2272)	(81.1036)	(62.4972)
$\varepsilon^{(h)}$	-0.8385	-0.5577	-0.5131	-0.4122	-0.0849	0.3436
	(59.7307)	(23.9696)	(21.1455)	(16.0591)	(3.0020)	(11.7358)
adj. R^2	0.4393	0.6644	0.7069	0.7269	0.7705	0.8030
MSPE	2.2252	2.3104**	2.3725**	2.4179**	2.3997**	2.4170**
MAPE	0.5399	0.5713**	0.5949**	0.6118**	0.6186**	0.6470**
QLIKE	2.0656**	2.0848**	2.0996**	2.1092**	2.1195**	2.1481**
HAR						
c	0.0730	0.0962	0.1138	0.1427	0.1596	0.1599
	(2.8141)	(5.8371)	(7.4777)	(9.8667)	(12.1206)	(12.3609)
$RV^{(1)}$	0.1669	0.0724	0.0611	0.0513	0.0366	0.0316
	(5.7515)	(4.1456)	(3.9159)	(3.6719)	(2.7543)	(2.4538)
$RV^{(5)}$	0.2778	0.3124	0.3048	0.1901	0.1135	0.0986
	(4.9183)	(9.5792)	(10.5476)	(7.0905)	(4.3935)	(3.8385)
$RV^{(22)}$	0.4712	0.5036	0.5019	0.5920	0.6630	0.6755
	(8.6123)	(15.5121)	(17.5319)	(22.1786)	(27.9778)	(26.8721)
adj. R^2	0.4396	0.6576	0.7028	0.7034	0.7237	0.7347
MSPE	2.2208	2.3163*	2.3666**	2.4625	2.4473	2.4477
MAPE	0.5375	0.5739**	0.5939**	0.6301	0.6284	0.6483**
QLIKE	2.0651**	2.0867**	2.0979**	2.1221**	2.1235**	2.1378**
HAR-J						
c	0.1168	0.1137	0.1337	0.1700	0.1780	0.1739
	(3.2765)	(5.3059)	(6.9838)	(8.9676)	(10.1453)	(10.2858)
$RV^{(1)}$	0.2117	0.0904	0.0815	0.0794	0.0555	0.0459
	(5.2178)	(3.7525)	(3.9482)	(4.1602)	(3.0467)	(2.6313)
$RV^{(5)}$	0.2793	0.3130	0.3055	0.1911	0.1143	0.0992
	(4.9471)	(9.5988)	(10.5656)	(7.1195)	(4.4124)	(3.8529)
$RV^{(22)}$	0.4668	0.5019	0.4999	0.5892	0.6611	0.6740
	(8.5367)	(15.4413)	(17.4639)	(22.0114)	(27.7839)	(26.7343)
$J^{(1)}$	-0.0974	-0.0390	-0.0444	-0.0610	-0.0411	-0.0310
	(-1.6883)	(-1.2108)	(-1.5683)	(-2.1543)	(-1.5836)	(-1.2662)
adj. R^2	0.4402	0.6577	0.7030	0.7040	0.7239	0.7347
MSPE	2.2182	2.3174*	2.3657**	2.4615	2.4467*	2.4494
MAPE	0.5372	0.5743**	0.5942**	0.6303	0.6278	0.6486**
QLIKE	2.0652**	2.0869**	2.0980**	2.1221**	2.1232**	2.1381**
HAR-CJ						
c	-0.1526	-0.1556	-0.1641	-0.3002	-0.3468	-0.5027
	(-0.8698)	(-1.5025)	(-1.8874)	(-3.4272)	(-4.3784)	(-7.1070)
$J^{(1)}$	0.0606	0.0208	0.0223	0.0225	0.0138	0.0106
	(1.8315)	(1.0752)	(1.2972)	(1.3461)	(0.9314)	(0.7545)
$J^{(5)}$	0.1464	0.1650	0.1411	0.0490	0.0370	0.0162
	(2.9161)	(5.5419)	(5.4709)	(2.0006)	(1.6581)	(0.7645)
$J^{(22)}$	0.1885	0.1877	0.2042	0.2994	0.3231	0.3818
	(2.9139)	(4.9581)	(6.3064)	(9.3110)	(10.9483)	(14.5076)
$C^{(1)}$	0.1160	0.0503	0.0410	0.0340	0.0248	0.0212
	(5.4489)	(3.9183)	(3.7223)	(3.3817)	(2.6925)	(2.3856)
$C^{(5)}$	0.1461	0.1623	0.1675	0.1256	0.0698	0.0707
	(3.4427)	(6.8289)	(7.7370)	(5.9317)	(3.6895)	(3.8224)
$C^{(22)}$	0.2726	0.3013	0.2914	0.3022	0.3438	0.3145
	(5.2465)	(10.1374)	(11.1958)	(12.0409)	(15.0756)	(13.8898)
adj. R^2	0.4395	0.6592	0.7060	0.7104	0.7310	0.7435
MSPE	2.2207	2.3076**	2.3585**	2.4486	2.4377*	2.4330**
MAPE	0.5350	0.5753**	0.5947**	0.6279	0.6283	0.6452**
QLIKE	2.0642**	2.0884**	2.0980**	2.1209**	2.1238**	2.1352**
HAR-PS						
c	0.1780	0.1408	0.1541	0.1753	0.1798	0.1773
	(5.1370)	(6.7532)	(7.8460)	(9.4277)	(10.2804)	(10.4298)
RS^{+}	0.0559	0.0168	0.0151	0.0086	-0.0034	-0.0028
	(2.0366)	(0.9410)	(0.9657)	(0.6119)	(-0.2688)	(-0.2322)
RS^{-}	0.0851	0.0433	0.0394	0.0354	0.0308	0.0262
	(2.5813)	(2.1253)	(2.1929)	(2.1244)	(1.8406)	(1.7066)
$\mathbf{I}_{r_t<0}RV^{(1)}$	0.0384	0.0115	0.0034	0.0040	0.0090	0.0077
	(1.4279)	(0.6939)	(0.2324)	(0.2852)	(0.6788)	(0.6313)
$RV^{(5)}$	0.2861	0.3202	0.3107	0.1966	0.1203	0.1047
	(5.0307)	(9.7252)	(10.6517)	(7.2618)	(4.6319)	(4.0509)
$RV^{(22)}$	0.4685	0.5019	0.5008	0.5907	0.6609	0.6737
	(8.5650)	(15.4629)	(17.5027)	(22.1376)	(27.9160)	(26.7743)
adj. R^2	0.4394	0.6570	0.7023	0.7029	0.7237	0.7346
MSPE	2.2149	2.3176*	2.3669**	2.4628	2.4464*	2.4463
MAPE	0.5377	0.5742**	0.5943**	0.6303	0.6276	0.6480**
QLIKE	2.0649**	2.0870**	2.0980**	2.1221**	2.1232**	2.1378**
LHAR-CJ						
c	-0.1110	-0.0666	-0.0642	-0.1946	-0.2393	-0.3827
	(-0.6211)	(-0.6377)	(-0.7360)	(-2.2751)	(-3.1032)	(-5.4788)
$J^{(1)}$	0.0500	0.0156	0.0182	0.0181	0.0092	0.0065
	(1.5437)	(0.8158)	(1.0629)	(1.0906)	(0.6268)	(0.4656)
$J^{(5)}$	0.1410	0.1579	0.1333	0.0409	0.0292	0.0112
	(2.8643)	(5.4661)	(5.3211)	(1.7391)	(1.3563)	(0.5457)
$J^{(22)}$	0.1591	0.1491	0.1632	0.2543	0.2785	0.3336
	(2.4184)	(3.9173)	(5.0533)	(8.1537)	(9.7224)	(12.8497)
$C^{(1)}$	0.0984	0.0420	0.0351	0.0280	0.0189	0.0159
	(4.5401)	(3.2324)	(3.1438)	(2.7927)	(2.0501)	(1.7946)
$C^{(5)}$	0.1548	0.1647	0.1676	0.1237	0.0682	0.0693
	(3.6572)	(6.8945)	(7.7898)	(5.9659)	(3.7063)	(3.8737)
$C^{(22)}$	0.2811	0.3173	0.3099	0.3233	0.3653	0.3367
	(5.4272)	(10.7287)	(12.0586)	(13.2907)	(16.5240)	(15.2689)
$r_t^{-,(1)}$	-0.0790	-0.0228	-0.0086	-0.0072	-0.0069	-0.0058
	(-4.6098)	(-2.1229)	(-0.9357)	(-0.8101)	(-0.8565)	(-0.7414)
$r_t^{-,(5)}$	-0.0090	-0.0145	-0.0154	-0.0145	-0.0140	-0.0145
	(-1.8354)	(-5.0678)	(-6.0187)	(-5.7797)	(-6.2562)	(-6.5579)
$r_t^{-,(22)}$	-0.1337	-0.1607	-0.1905	-0.2825	-0.2508	-0.2010
	(-1.5981)	(-3.0681)	(-4.1927)	(-6.3198)	(-6.3700)	(-4.7859)
adj. R^2	0.4489	0.6691	0.7177	0.7279	0.7470	0.7572
MSPE	2.1821**	2.3030**	2.3561**	2.4436*	2.4423*	2.4380**
MAPE	0.5278**	0.5716**	0.5915**	0.6203*	0.6265	0.6430**
QLIKE	2.0595**	2.0852**	2.0953**	2.1156**	2.1227**	2.1346**

Table 8: In-Sample regression results for Rough Rice with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
			Random Walk			
adj. R^2	0.0878	0.4370	0.4079	0.2215	-0.2846	-0.8494
MSPE	1.0153	0.9843	1.0273	1.0670	1.1603	1.1986
MAPE	0.4792	0.4786	0.5046	0.5586	0.6631	0.7130
QLIKE	1.4582**	1.4631**	1.4777**	1.5210*	1.6081	1.6563
			AR(1)			
c	0.1179	0.0706	0.0728	0.0942	0.1642	0.2599
	(7.8375)	(8.5864)	(9.2025)	(10.1223)	(13.6241)	(20.4670)
$RV^{(h)}$	0.5438	0.7186	0.7049	0.6131	0.3473	0.0198
	(17.3488)	(42.8571)	(43.0140)	(30.8879)	(13.4734)	(0.7105)
adj. R^2	0.2944	0.5156	0.4940	0.3681	0.1109	-0.0002
MSPE	0.9021	0.9486*	0.9933**	1.0341*	1.0843	1.0782**
MAPE	0.4357	0.4528**	0.4774**	0.5180**	0.5677**	0.5655**
QLIKE	1.4293**	1.4433**	1.4540**	1.4791**	1.5117**	1.5155**
			ARMA			
c	0.0117	0.0388	0.0639	0.1324	0.2731	0.4155
	(2.9799)	(5.4589)	(6.8698)	(11.0727)	(20.3049)	(19.8041)
$RV^{(h)}$	0.9534	0.8434	0.7430	0.4764	-0.0653	-0.5738
	(92.6966)	(46.3489)	(33.2360)	(16.6055)	(1.4788)	(7.0828)
$\varepsilon^{(h)}$	-0.7161	-0.2685	-0.0735	0.2270	0.5765	0.7303
	(35.3722)	(9.4306)	(2.2693)	(6.6787)	(15.7752)	(10.3926)
adj. R^2	0.3857	0.5314	0.4957	0.3842	0.2013	0.0323
MSPE	0.8554*	0.9462*	0.9927**	1.0289**	1.0702**	1.0871
MAPE	0.4124**	0.4469**	0.4771**	0.5152**	0.5620**	0.5705*
QLIKE	1.4263**	1.4398**	1.4542**	1.4789**	1.5081**	1.5163**
			HAR			
c	0.0340	0.0482	0.0633	0.0911	0.1399	0.1858
	(2.6427)	(5.6979)	(7.7652)	(10.4939)	(14.5342)	(18.7151)
$RV^{(1)}$	0.2191	0.1150	0.1053	0.0803	0.0586	0.0449
	(5.8359)	(4.2713)	(4.4042)	(3.5550)	(2.7998)	(2.2614)
$RV^{(5)}$	0.3918	0.4328	0.3734	0.3072	0.2986	0.2323
	(7.2888)	(11.8756)	(11.2228)	(9.0607)	(9.2428)	(7.2685)
$RV^{(22)}$	0.2530	0.2542	0.2623	0.2434	0.0897	0.0080
	(5.1857)	(7.6517)	(8.4722)	(7.7145)	(2.8859)	(0.2511)
adj. R^2	0.3866	0.5395	0.5253	0.4474	0.3028	0.1651
MSPE	0.8546*	0.9408**	0.9916**	1.0314**	1.0741**	1.0890
MAPE	0.4063**	0.4486**	0.4693**	0.5142**	0.5577**	0.5710*
QLIKE	1.4208**	1.4413**	1.4487**	1.4764**	1.5052**	1.5172**
			HAR-J			
c	0.0563	0.0702	0.0824	0.1082	0.1502	0.1875
	(3.6376)	(6.1308)	(7.7444)	(10.0596)	(13.3946)	(16.4568)
$RV^{(1)}$	0.2540	0.1493	0.1351	0.1070	0.0747	0.0476
	(5.4865)	(4.5899)	(4.6437)	(3.9090)	(2.9985)	(2.0698)
$RV^{(5)}$	0.3817	0.4228	0.3647	0.2994	0.2939	0.2316
	(6.9829)	(11.4658)	(10.8152)	(8.7342)	(9.0113)	(7.2123)
$RV^{(22)}$	0.2488	0.2501	0.2587	0.2401	0.0878	0.0077
	(5.0893)	(7.5435)	(8.3703)	(7.6444)	(2.8324)	(0.2416)
$J^{(1)}$	-0.1484	-0.1460	-0.1265	-0.1135	-0.0682	-0.0113
	(-2.1328)	(-2.5815)	(-2.5356)	(-2.4237)	(-1.6642)	(-0.2858)
adj. R^2	0.3881	0.5422	0.5276	0.4497	0.3036	0.1647
MSPE	0.8521*	0.9398**	0.9903**	1.0312**	1.0744**	1.0891
MAPE	0.4048**	0.4467**	0.4687**	0.5128**	0.5579**	0.5711*
QLIKE	1.4199**	1.4399**	1.4485**	1.4749**	1.5057**	1.5173**
			HAR-CJ			
c	0.0629	0.0640	0.0708	0.0657	0.0738	0.0732
	(1.6547)	(2.7843)	(3.4646)	(3.3161)	(3.5716)	(3.7030)
$J^{(1)}$	0.1111	-0.0006	0.0097	0.0063	0.0007	0.0033
	(1.7517)	(-0.0120)	(0.2279)	(0.1592)	(0.0205)	(0.0924)
$J^{(5)}$	-0.0011	0.0538	0.0536	0.0209	0.0348	0.0218
	(-0.0250)	(1.9818)	(2.1268)	(0.8225)	(1.4447)	(0.9746)
$J^{(22)}$	0.0438	0.0386	0.0392	0.0673	0.0799	0.1076
	(1.3187)	(1.8196)	(2.0924)	(3.5805)	(4.2185)	(6.4181)
$C^{(1)}$	0.2091	0.1211	0.1077	0.0806	0.0594	0.0454
	(5.2687)	(4.4631)	(4.4224)	(3.7131)	(3.0685)	(2.5548)
$C^{(5)}$	0.3870	0.4004	0.3429	0.2965	0.2870	0.2227
	(7.7566)	(11.2057)	(10.5988)	(9.3501)	(9.6499)	(7.5763)
$C^{(22)}$	0.2188	0.2306	0.2408	0.1992	0.0351	-0.0594
	(4.6569)	(7.1121)	(7.9743)	(6.6021)	(1.1755)	(-1.9651)
adj. R^2	0.3877	0.5396	0.5231	0.4467	0.3002	0.1737
MSPE	0.8522*	0.9403**	0.9912**	1.0317**	1.0747**	1.0846**
MAPE	0.4041**	0.4485**	0.4692**	0.5120**	0.5595**	0.5668*
QLIKE	1.4193**	1.4417**	1.4484**	1.4743**	1.5070**	1.5151**
			HAR-PS			
c	0.2008	0.1454	0.1480	0.1496	0.1775	0.2161
	(6.2083)	(6.1117)	(6.7029)	(7.4499)	(9.0152)	(11.0303)
RS^{+}	0.1338	0.0621	0.0457	0.0490	0.0397	0.0325
	(4.4723)	(2.9348)	(2.2572)	(2.5062)	(2.1243)	(1.8151)
RS^{-}	0.0967	0.0726	0.0714	0.0317	0.0118	0.0091
	(2.6737)	(2.9442)	(3.1797)	(1.5702)	(0.6005)	(0.4607)
$\mathbf{I}_{r_t<0}RV^{(1)}$	-0.0119	-0.0271	-0.0173	0.0035	0.0172	0.0092
	(-0.2846)	(-0.9108)	(-0.6354)	(0.1329)	(0.6772)	(0.3761)
$RV^{(5)}$	0.3892	0.4281	0.3707	0.3057	0.2973	0.2316
	(7.1398)	(11.5419)	(10.9227)	(8.8975)	(9.0959)	(7.1841)
$RV^{(22)}$	0.2516	0.2542	0.2626	0.2428	0.0890	0.0074
	(5.1662)	(7.6552)	(8.4726)	(7.6823)	(2.8572)	(0.2313)
adj. R^2	0.3856	0.5394	0.5250	0.4468	0.3025	0.1645
MSPE	0.8553*	0.9403**	0.9915**	1.0313**	1.0741**	1.0893
MAPE	0.4064**	0.4481**	0.4693**	0.5139**	0.5570**	0.5713*
QLIKE	1.4212**	1.4411**	1.4487**	1.4762**	1.5046**	1.5174**
			LHAR-CJ			
c	0.0337	0.0402	0.0429	0.0447	0.0508	0.0521
	(0.8559)	(1.6606)	(2.0116)	(2.1050)	(2.3320)	(2.4780)
$J^{(1)}$	0.0938	-0.0108	-0.0026	0.0005	-0.0025	0.0023
	(1.4934)	(-0.2260)	(-0.0609)	(0.0133)	(-0.0686)	(0.0644)
$J^{(5)}$	-0.0094	0.0456	0.0460	0.0143	0.0290	0.0165
	(-0.2084)	(1.7001)	(1.8452)	(0.5663)	(1.2010)	(0.7366)
$J^{(22)}$	0.0483	0.0446	0.0488	0.0771	0.0957	0.1242
	(1.4245)	(2.0762)	(2.6105)	(4.0645)	(5.0468)	(7.3908)
$C^{(1)}$	0.1988	0.1152	0.1007	0.0776	0.0583	0.0457
	(5.0947)	(4.3677)	(4.2943)	(3.6549)	(3.0807)	(2.6161)
$C^{(5)}$	0.3734	0.3869	0.3278	0.2829	0.2712	0.2080
	(7.5246)	(10.8508)	(10.2099)	(8.9297)	(9.1370)	(7.0235)
$C^{(22)}$	0.2215	0.2323	0.2424	0.2003	0.0367	-0.0573
	(4.7071)	(7.1369)	(7.9990)	(6.6144)	(1.2218)	(-1.8823)
$r_t^{-,(1)}$	-0.0300	-0.0158	-0.0198	-0.0069	-0.0015	0.0026
	(-2.2361)	(-1.6274)	(-2.3730)	(-0.8902)	(-0.2026)	(0.3687)
$r_t^{-,(5)}$	0.0076	0.0084	0.0106	0.0103	0.0140	0.0142
	(1.8421)	(3.0841)	(4.1333)	(4.0754)	(5.9767)	(6.4146)
$r_t^{-,(22)}$	-0.1018	-0.0816	-0.0459	-0.0222	0.0393	0.0605
	(-1.8492)	(-2.2507)	(-1.3506)	(-0.6758)	(1.2646)	(2.0464)
adj. R^2	0.3915	0.5440	0.5298	0.4517	0.3115	0.1886
MSPE	0.8476**	0.9403**	0.9921**	1.0328**	1.0721**	1.0829**
MAPE	0.4032**	0.4486**	0.4687**	0.5119**	0.5579**	0.5631**
QLIKE	1.4188**	1.4420**	1.4486**	1.4742**	1.5081**	1.5131**

Table 9: In-Sample regression results for Soybean with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
Random Walk						
adj. R^2	0.5142	0.7405	0.7846	0.7966	0.7220	0.6699
MSPE	2.5965	2.5676	2.5859	2.4934**	2.5379	2.4903
MAPE	0.4287	0.4181	0.4196	0.4367	0.4922	0.5174
QLIKE	2.2141	2.2109**	2.2142**	2.2231**	2.2666	2.2889
AR(1)						
c	0.2624	0.1397	0.1160	0.1102	0.1514	0.1771
	(12.6261)	(9.3988)	(8.6776)	(7.5416)	(7.7898)	(9.1080)
$RV^{(h)}$	0.7571	0.8688	0.8897	0.8940	0.8503	0.8176
	(48.1471)	(73.1783)	(80.8939)	(79.3076)	(58.1268)	(55.4838)
adj. R^2	0.5728	0.7576	0.7966	0.8078	0.7451	0.7058
MSPE	2.3773	2.5252	2.5678	2.5120**	2.5158	2.4661
MAPE	0.4004	0.4062	0.4117	0.4317*	0.4800	0.4924**
QLIKE	2.1973**	2.2009**	2.2031**	2.2114**	2.2388**	2.2382**
ARMA						
c	0.0161	0.0310	0.0457	0.0839	0.0855	0.1003
	(3.0711)	(4.6030)	(6.0132)	(9.4439)	(10.0194)	(9.6682)
$RV^{(h)}$	0.9848	0.9703	0.9555	0.9159	0.9034	0.8844
	(232.3917)	(177.3654)	(153.7074)	(114.8710)	(116.1206)	(99.8488)
$\varepsilon^{(h)}$	-0.7445	-0.5094	-0.3604	-0.1249	-0.2625	-0.2042
	(44.5474)	(24.2486)	(16.9904)	(5.2419)	(10.8704)	(8.3140)
adj. R^2	0.6629	0.7903	0.8131	0.8188	0.7779	0.7517
MSPE	2.2555	2.4541*	2.4968**	2.4430**	2.4190**	2.3960**
MAPE	0.3419**	0.3858	0.4029*	0.4284**	0.4644**	0.4888**
QLIKE	2.1701**	2.1899**	2.1981**	2.2129**	2.2334**	2.2441**
HAR						
c	0.0445	0.0646	0.0799	0.1057	0.1501	0.1917
	(2.0806)	(4.3328)	(5.7612)	(7.6275)	(9.4248)	(10.9989)
$RV^{(1)}$	0.2458	0.1556	0.1084	0.0810	0.0679	0.0613
	(8.3251)	(7.4649)	(5.6673)	(4.3799)	(3.5851)	(3.0761)
$RV^{(5)}$	0.3288	0.2619	0.2388	0.1975	0.2058	0.1721
	(6.5406)	(7.4861)	(7.4609)	(6.0544)	(6.1533)	(4.9690)
$RV^{(22)}$	0.3834	0.5202	0.5760	0.6200	0.5780	0.5734
	(8.8657)	(17.0808)	(20.1447)	(21.8090)	(19.0863)	(18.0935)
adj. R^2	0.6686	0.7974	0.8185	0.8224	0.7950	0.7566
MSPE	2.2218	2.4251**	2.4923**	2.5080**	2.4745*	2.5002
MAPE	0.3401**	0.3855	0.4035	0.4298*	0.4717	0.5057
QLIKE	2.1685**	2.1895**	2.1975**	2.2106**	2.2346**	2.2479**
HAR-J						
c	0.0487	0.0743	0.0893	0.1136	0.1593	0.2012
	(2.1927)	(4.8667)	(6.2730)	(8.0528)	(9.9093)	(11.4917)
$RV^{(1)}$	0.2567	0.1812	0.1333	0.1021	0.0923	0.0862
	(8.1589)	(8.1053)	(6.4539)	(5.0589)	(4.4407)	(3.9588)
$RV^{(5)}$	0.3260	0.2552	0.2323	0.1920	0.1993	0.1655
	(6.4741)	(7.3201)	(7.2783)	(5.8774)	(5.9462)	(4.7642)
$RV^{(22)}$	0.3809	0.5147	0.5705	0.6154	0.5728	0.5681
	(8.7875)	(16.8263)	(19.8649)	(21.6509)	(18.9554)	(17.9948)
$J^{(1)}$	-0.0293	-0.0679	-0.0659	-0.0559	-0.0646	-0.0663
	(-0.7976)	(-2.7444)	(-2.8693)	(-2.6457)	(-3.0618)	(-2.9832)
adj. R^2	0.6685	0.7981	0.8193	0.8229	0.7957	0.7575
MSPE	2.2249	2.4224**	2.4930**	2.5085*	2.4734*	2.4957
MAPE	0.3399**	0.3853	0.4026	0.4301*	0.4713	0.5051
QLIKE	2.1684**	2.1893**	2.1972**	2.2106**	2.2344**	2.2474**
HAR-CJ						
c	0.1111	0.1307	0.1522	0.2102	0.2895	0.3364
	(3.4100)	(5.8602)	(7.4624)	(10.4620)	(13.6178)	(15.1940)
$J^{(1)}$	0.1089	0.0643	0.0431	0.0343	0.0309	0.0285
	(3.1018)	(2.8444)	(2.0590)	(1.8425)	(1.6940)	(1.4476)
$J^{(5)}$	0.0212	0.0123	0.0220	0.0344	0.0340	0.0258
	(0.9170)	(0.7570)	(1.4717)	(2.4911)	(2.4638)	(1.7723)
$J^{(22)}$	0.0304	0.0353	0.0272	-0.0027	-0.0325	-0.0383
	(1.3763)	(2.3602)	(2.0520)	(-0.2215)	(-2.6316)	(-2.9126)
$C^{(1)}$	0.2381	0.1501	0.1052	0.0778	0.0660	0.0601
	(8.2883)	(7.3570)	(5.6387)	(4.2993)	(3.5544)	(3.0947)
$C^{(5)}$	0.3082	0.2488	0.2214	0.1803	0.1973	0.1668
	(6.2990)	(7.3663)	(7.1820)	(5.6773)	(6.0812)	(4.9865)
$C^{(22)}$	0.3443	0.4757	0.5372	0.5941	0.5584	0.5574
	(7.9824)	(15.7123)	(19.3092)	(21.5313)	(19.1130)	(18.2215)
adj. R^2	0.6693	0.7989	0.8210	0.8274	0.8031	0.7670
MSPE	2.2189	2.4303**	2.4938**	2.5134*	2.4712*	2.5054
MAPE	0.3392**	0.3832	0.4004*	0.4265**	0.4675	0.4991**
QLIKE	2.1681**	2.1888**	2.1962**	2.2087**	2.2323**	2.2450**
HAR-PS						
c	0.2053	0.1710	0.1532	0.1621	0.1994	0.2396
	(6.8309)	(7.9936)	(7.6708)	(8.1335)	(9.0974)	(10.1543)
RS^{+}	0.1479	0.1249	0.1002	0.0773	0.0581	0.0516
	(5.5121)	(6.9601)	(5.9390)	(4.7212)	(3.3930)	(2.8382)
RS^{-}	0.0691	0.0159	-0.0045	-0.0035	0.0073	0.0125
	(2.2775)	(0.7312)	(-0.2302)	(-0.1888)	(0.3675)	(0.5846)
$\mathbf{I}_{r_t<0}RV^{(1)}$	0.0479	0.0313	0.0304	0.0244	0.0143	0.0067
	(2.5867)	(2.3671)	(2.5605)	(2.1888)	(1.2851)	(0.5667)
$RV^{(5)}$	0.3336	0.2624	0.2376	0.1932	0.2017	0.1666
	(6.5851)	(7.5075)	(7.4136)	(5.8944)	(5.9728)	(4.7523)
$RV^{(22)}$	0.3812	0.5166	0.5726	0.6177	0.5764	0.5721
	(8.8306)	(17.0820)	(20.1798)	(21.8625)	(19.1224)	(18.0996)
adj. R^2	0.6684	0.7981	0.8196	0.8231	0.7952	0.7567
MSPE	2.2156	2.4238**	2.4911**	2.5052**	2.4744*	2.5000
MAPE	0.3402**	0.3852	0.4034	0.4294*	0.4714	0.5059
QLIKE	2.1685**	2.1894**	2.1975**	2.2103**	2.2345**	2.2480**
LHAR-CJ						
c	0.1247	0.1399	0.1477	0.1861	0.2467	0.2915
	(3.5722)	(5.9190)	(7.0830)	(9.3628)	(11.5448)	(12.8981)
$J^{(1)}$	0.0989	0.0623	0.0423	0.0337	0.0299	0.0269
	(2.8223)	(2.7713)	(2.0152)	(1.7848)	(1.6133)	(1.3371)
$J^{(5)}$	0.0214	0.0099	0.0184	0.0311	0.0313	0.0232
	(0.9338)	(0.6192)	(1.2644)	(2.3204)	(2.3030)	(1.6024)
$J^{(22)}$	0.0296	0.0366	0.0307	0.0025	-0.0262	-0.0316
	(1.3549)	(2.5127)	(2.4201)	(0.2135)	(-2.1867)	(-2.4473)
$C^{(1)}$	0.2124	0.1398	0.0979	0.0724	0.0625	0.0557
	(7.3584)	(6.9047)	(5.2432)	(4.0049)	(3.3902)	(2.8811)
$C^{(5)}$	0.3074	0.2384	0.2067	0.1648	0.1837	0.1542
	(6.2662)	(7.1383)	(6.7955)	(5.3172)	(5.8124)	(4.7191)
$C^{(22)}$	0.3423	0.4734	0.5375	0.5977	0.5654	0.5647
	(7.9402)	(15.7642)	(19.5300)	(22.1264)	(19.9294)	(19.0181)
$r_t^{-,(1)}$	-0.0332	-0.0073	-0.0016	0.0010	0.0018	0.0001
	(-3.3033)	(-1.0424)	(-0.2670)	(0.1981)	(0.3312)	(0.0236)
$r_t^{-,(5)}$	0.0058	0.0049	0.0035	0.0011	-0.0019	-0.0021
	(2.8862)	(3.5960)	(2.9363)	(0.9473)	(-1.5736)	(-1.6513)
$r_t^{-,(22)}$	-0.0844	-0.1434	-0.1821	-0.1992	-0.1847	-0.1832
	(-2.2615)	(-5.7516)	(-8.2723)	(-9.0427)	(-8.3511)	(-7.7072)
adj. R^2	0.6730	0.8024	0.8264	0.8349	0.8118	0.7763
MSPE	2.1443**	2.4123**	2.4760**	2.5191*	2.4946*	2.5341
MAPE	0.3377**	0.3805**	0.3974**	0.4239**	0.4621**	0.4964**
QLIKE	2.1674**	2.1874**	2.1945**	2.2066**	2.2296**	2.2429**

Table 10: In-Sample regression results for Sugar with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
Random Walk						
adj. R^2	0.1756	0.4970	0.5112	0.5163	0.4174	0.2276
MSPE	2.0653	2.1759	2.1633**	2.1861	2.2311**	2.3436
MAPE	0.4737	0.4704	0.4849	0.4974	0.5393	0.5902
QLIKE	2.1772	2.1701**	2.1787**	2.1912**	2.2285	2.2750
AR(1)						
c	0.4086	0.2484	0.2413	0.2344	0.2770	0.3685
	(12.9311)	(14.4719)	(14.4099)	(14.3526)	(12.8627)	(15.0692)
$RV^{(h)}$	0.5877	0.7482	0.7551	0.7588	0.7128	0.6184
	(20.7364)	(45.0175)	(44.6687)	(47.0778)	(37.5835)	(28.5946)
adj. R^2	0.3446	0.5600	0.5708	0.5740	0.4984	0.3692
MSPE	1.9101	2.0802	2.1081**	2.1652**	2.2279	2.3096
MAPE	0.4188	0.4432	0.4569	0.4708**	0.5078**	0.5378**
QLIKE	2.1400**	2.1492**	2.1576**	2.1650**	2.1876**	2.2108**
ARMA						
c	0.0386	0.0857	0.1074	0.1901	0.4718	0.4090
	(4.4988)	(7.0134)	(7.5451)	(10.8720)	(19.1174)	(11.9579)
$RV^{(h)}$	0.9612	0.9126	0.8899	0.8060	0.5232	0.5799
	(123.1420)	(83.6537)	(69.8266)	(49.5912)	(21.8110)	(17.2683)
$\varepsilon^{(h)}$	-0.7114	-0.4264	-0.3405	-0.1030	0.3905	0.0640
	(39.6273)	(16.9608)	(11.9844)	(3.4798)	(12.6367)	(1.3506)
adj. R^2	0.4496	0.5878	0.5937	0.5820	0.5260	0.3708
MSPE	1.8055**	2.0460*	2.0984**	2.1601**	2.2052**	2.3110
MAPE	0.3884**	0.4337	0.4515	0.4704**	0.5076**	0.5392**
QLIKE	2.1248**	2.1451**	2.1556**	2.1650**	2.1948**	2.2150
HAR						
c	0.0978	0.1410	0.1791	0.2264	0.2970	0.3562
	(3.5555)	(7.6910)	(10.5290)	(14.3942)	(17.4647)	(19.2097)
$RV^{(1)}$	0.1949	0.1535	0.1083	0.0804	0.0613	0.0503
	(5.3468)	(5.9368)	(4.4552)	(3.5647)	(2.7974)	(2.3048)
$RV^{(5)}$	0.4178	0.3043	0.2502	0.1886	0.1718	0.1484
	(7.9222)	(8.3324)	(7.4342)	(5.8225)	(5.4687)	(4.6906)
$RV^{(22)}$	0.2885	0.3987	0.4589	0.4985	0.4627	0.4351
	(5.5246)	(11.7133)	(14.9318)	(18.3043)	(16.8009)	(15.2920)
adj. R^2	0.4549	0.6061	0.6090	0.6061	0.5612	0.4975
MSPE	1.7985**	2.0307**	2.1063**	2.1733	2.2318	2.2516
MAPE	0.3848**	0.4272*	0.4496	0.4702**	0.5022**	0.5300**
QLIKE	2.1241**	2.1412**	2.1534**	2.1635**	2.1852**	2.2047**
HAR-J						
c	0.0917	0.1397	0.1797	0.2268	0.2909	0.3451
	(3.3578)	(7.4358)	(10.3125)	(14.0392)	(16.9141)	(18.5417)
$RV^{(1)}$	0.1803	0.1504	0.1097	0.0814	0.0467	0.0240
	(4.4874)	(5.2974)	(4.1279)	(3.2821)	(1.9422)	(1.0154)
$RV^{(5)}$	0.4181	0.3044	0.2502	0.1885	0.1720	0.1490
	(7.9558)	(8.3380)	(7.4288)	(5.8194)	(5.5041)	(4.7586)
$RV^{(22)}$	0.2899	0.3990	0.4587	0.4984	0.4643	0.4381
	(5.5635)	(11.7119)	(14.8971)	(18.2686)	(16.9000)	(15.4672)
$J^{(1)}$	0.0510	0.0110	-0.0048	-0.0035	0.0507	0.0906
	(1.1025)	(0.3367)	(-0.1597)	(-0.1268)	(1.9526)	(3.5606)
adj. R^2	0.4551	0.6059	0.6088	0.6058	0.5619	0.5006
MSPE	1.7973**	2.0314*	2.1064**	2.1733	2.2299	2.2505
MAPE	0.3849**	0.4275*	0.4497	0.4701**	0.5013**	0.5304**
QLIKE	2.1241**	2.1414**	2.1534**	2.1635**	2.1844**	2.2057**
HAR-CJ						
c	0.1324	0.1557	0.1654	0.1499	0.0865	0.1017
	(2.9277)	(5.0306)	(5.4939)	(5.6053)	(3.4936)	(4.1700)
$J^{(1)}$	0.1112	0.0860	0.0511	0.0376	0.0234	0.0227
	(2.8383)	(3.0474)	(1.9466)	(1.5871)	(1.1023)	(1.1280)
$J^{(5)}$	0.0234	0.0007	0.0123	-0.0136	-0.0045	0.0057
	(0.8042)	(0.0325)	(0.6412)	(-0.8228)	(-0.3143)	(0.4109)
$J^{(22)}$	0.0522	0.0718	0.0868	0.1371	0.2224	0.2427
	(1.7119)	(3.3663)	(4.2404)	(7.7988)	(14.1275)	(15.9920)
$C^{(1)}$	0.1886	0.1444	0.1034	0.0767	0.0583	0.0466
	(5.6975)	(6.2422)	(4.7888)	(3.8145)	(3.1028)	(2.5009)
$C^{(5)}$	0.3912	0.2976	0.2427	0.1944	0.1700	0.1428
	(8.1138)	(8.8225)	(7.8325)	(6.5556)	(6.2051)	(5.1981)
$C^{(22)}$	0.2413	0.3340	0.3821	0.3869	0.3052	0.2685
	(4.8565)	(9.9894)	(12.5673)	(14.5925)	(11.9671)	(10.2734)
adj. R^2	0.4566	0.6095	0.6126	0.6126	0.5882	0.5374
MSPE	1.7950**	2.0249**	2.1039**	2.1589**	2.1943**	2.2210**
MAPE	0.3835**	0.4258**	0.4486	0.4697**	0.4968**	0.5250**
QLIKE	2.1231**	2.1403**	2.1529**	2.1645**	2.1833**	2.2043**
HAR-PS						
c	0.2400	0.2513	0.2554	0.2809	0.3341	0.3846
	(6.4359)	(9.1127)	(9.8467)	(11.9562)	(13.9554)	(15.4019)
RS^{+}	0.1301	0.0930	0.0711	0.0551	0.0344	0.0206
	(4.7287)	(4.9477)	(4.2080)	(3.2455)	(2.0982)	(1.2620)
RS^{-}	0.0649	0.0587	0.0330	0.0185	0.0156	0.0181
	(1.5032)	(1.7615)	(1.0594)	(0.6888)	(0.5884)	(0.7022)
$\mathbf{I}_{r_t<0}RV^{(1)}$	-0.0265	-0.0218	-0.0099	-0.0008	0.0058	0.0037
	(-1.0440)	(-1.1457)	(-0.5667)	(-0.0497)	(0.3853)	(0.2558)
$RV^{(5)}$	0.4304	0.3168	0.2590	0.1954	0.1801	0.1583
	(8.0612)	(8.4425)	(7.5256)	(5.9467)	(5.6262)	(4.9189)
$RV^{(22)}$	0.2901	0.3997	0.4595	0.4987	0.4625	0.4347
	(5.5336)	(11.7027)	(14.9139)	(18.2757)	(16.7910)	(15.2812)
adj. R^2	0.4552	0.6053	0.6085	0.6054	0.5599	0.4960
MSPE	1.7932**	2.0286**	2.1064**	2.1733	2.2328	2.2522
MAPE	0.3852**	0.4260**	0.4504	0.4701**	0.5024**	0.5301**
QLIKE	2.1244**	2.1403**	2.1538**	2.1636**	2.1852**	2.2047**
LHAR-CJ						
c	0.1186	0.1330	0.1413	0.1257	0.0569	0.0736
	(2.5508)	(4.2067)	(4.6576)	(4.6868)	(2.2814)	(2.9611)
$J^{(1)}$	0.1122	0.0870	0.0522	0.0375	0.0215	0.0203
	(2.8609)	(3.1014)	(1.9930)	(1.5880)	(1.0196)	(1.0065)
$J^{(5)}$	0.0185	-0.0073	0.0042	-0.0209	-0.0127	-0.0019
	(0.6362)	(-0.3484)	(0.2179)	(-1.2763)	(-0.8887)	(-0.1353)
$J^{(22)}$	0.0518	0.0721	0.0863	0.1364	0.2226	0.2427
	(1.7068)	(3.3859)	(4.2261)	(7.8382)	(14.3470)	(16.2348)
$C^{(1)}$	0.1885	0.1439	0.1029	0.0755	0.0560	0.0442
	(5.7192)	(6.2717)	(4.8431)	(3.8026)	(2.9911)	(2.4042)
$C^{(5)}$	0.3851	0.2878	0.2328	0.1853	0.1600	0.1333
	(8.0231)	(8.5897)	(7.6340)	(6.3103)	(5.8724)	(4.8952)
$C^{(22)}$	0.2527	0.3504	0.4009	0.4051	0.3242	0.2865
	(5.1051)	(10.5295)	(13.2915)	(15.3738)	(12.8772)	(11.0804)
$r_t^{-,(1)}$	0.0026	0.0017	0.0027	-0.0004	-0.0061	-0.0072
	(0.2422)	(0.2386)	(0.4048)	(-0.0588)	(-1.0668)	(-1.2123)
$r_t^{-,(5)}$	-0.0007	-0.0003	-0.0016	-0.0024	-0.0022	-0.0022
	(-0.2198)	(-0.1225)	(-0.8084)	(-1.2225)	(-1.1573)	(-1.1311)
$r_t^{-,(22)}$	-0.1093	-0.1662	-0.1799	-0.1700	-0.1813	-0.1696
	(-1.9617)	(-4.8605)	(-6.0103)	(-6.3813)	(-8.1063)	(-7.6402)
adj. R^2	0.4572	0.6143	0.6194	0.6200	0.5982	0.5471
MSPE	1.7909**	2.0198**	2.1030**	2.1556**	2.1955**	2.2211**
MAPE	0.3837**	0.4237**	0.4447**	0.4665**	0.4971**	0.5234**
QLIKE	2.1234**	2.1399**	2.1512**	2.1633**	2.1837**	2.2048**

Table 11: In-Sample regression results for Wheat with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

sistently outperformed by any of its extended versions, namely the HAR-J, HAR-CJ, HAR-PS and LHAR-CJ models; a finding that holds for both volatility measures. Hence, building forecasting models based on the jump component, the continuous component, the signed jumps, and the volatility or return leverage does not improve the forecasting accuracy. Thus, there is no scope to add complexity in the predictive models without obtaining a significant predictive gain. This finding is in some contrast to Tian et al. (2017a), Tian et al. (2017b), Yang et al. (2017), and Luo et al. (2019) who maintain that the jump component as well as the introduction of time-varying HAR coefficients help improving forecast accuracy of agricultural commodities price volatility. These results are not contradicting, however. The substantial findings of Tian et al. (2017a), Tian et al. (2017b) and Yang et al. (2017) are derived from data of Chinese futures markets, while our price data comes from North-American markets. Comparing realized volatility measures of these two markets as well as the results of the aforementioned studies shows that Chinese and U.S. futures markets behave differently and are possibly driven by dissimilar factors. One of the most important reasons for this apparent difference of market behavior is the investor's structure in the two markets, where the Chinese market is much more driven by speculators than the U.S. market (Bloomberg, 2019, Fan & Zhang, 2018, Klein & Todorova, 2018).

We maintain, though, that the expected outcome for the jump components is not to provide any forecasting gains given that they are built in order to capture the surprised (sudden) changes in volatility. Hence, the jump component is not encompassing any element of either long or short memory; they are simply non-autocorrelated zero mean stochastic processes. This is the reason behind the fact that in the in-sample analysis the jump component is able to provide better fitting of the predictive regressions, whereas this ability no longer exists in a real out-of-sample exercise. Even more, the inability of the leverage effects, of either returns or volatility, to generate significant out-of-sample predictive gains, relatively to the simple HAR model, could be explained by the fact that they are not statistically powerful enough to provide incremental predictive information relative to the heterogeneous beliefs of investors. Another possible explanation might be that previous studies which provide evidence in favour of the predictive ability of the jump components in an out-of-sample setting is due to the incorporation of jump values that are not available to the forecasters at the time that the forecasts are generated, i.e. such studies do not produce real out-of-sample forecasts.

Next, we look at the success ratios of our competing models, as shown at the bottom of Tables 12-16. It is rather interesting that we cannot argue that there is a single model which is able to provide a superior directional accuracy. On the contrary, we reach the conclusion that even though the HAR-type models correctly predict the direction of the volatility at a high level (up to 75% depending on the volatility measure and commodity), these are marginally higher (lower) compared to the Random Walk and AR models for the shorter (longer) forecasting horizons. In any case, however, any differences noticed in predicting the future direction of volatility are not statistically significant.

Overall, our findings are in line with Sévi (2014) who, despite the fact that he focuses on an energy commodity volatility (i.e. oil price volatility), reaches the same conclusion as our study; namely that even though sophisticated HAR-type models outperform the simple HAR in an in-sample setting, they are not capable of outperforming it in an out-of-sample exercise over different forecasting horizons.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.3038	1.1508	1.2400	1.0898**	1.0901**	1.0890**	1.0992	1.0993**
	5	1.2454**	1.2166**	1.2963	1.2045**	1.2036**	1.2031**	1.2079**	1.2035**
	10	1.2983	1.2651**	1.3346	1.2742	1.2726	1.2626**	1.2806	1.2636**
	22	1.3949	1.3423	1.3875	1.3248**	1.3241**	1.3197**	1.3240**	1.3282
	44	1.4981	1.3859	1.3376**	1.3747*	1.3746*	1.3667*	1.3739*	1.3768*
	66	1.5565	1.3870	1.4430	1.3753	1.3759	1.3665**	1.3753	1.3760
MAPE	1	0.6093	0.5899	0.5988	0.5516**	0.5515**	0.5509**	0.5550	0.5570*
	5	0.6232	0.6181	0.6389	0.6056**	0.6057**	0.6043**	0.6070**	0.6038**
	10	0.6325	0.6268**	0.6456	0.6282*	0.6288	0.6232**	0.6305	0.6240**
	22	0.6644	0.6593	0.6875	0.6505**	0.6506**	0.6483**	0.6503**	0.6541
	44	0.7217	0.7002**	0.7020**	0.6913**	0.6917**	0.6939**	0.6912**	0.7016
	66	0.7647	0.7313	0.7701	0.7031**	0.7039*	0.7094*	0.7032**	0.7155
QLIKE	1	1.6887	1.6600	1.6798	1.6517**	1.6519**	1.6520**	1.6535	1.6529**
	5	1.6925	1.6801**	1.7075	1.6770**	1.6768**	1.6771**	1.6770**	1.6775**
	10	1.7014	1.6906**	1.7123	1.6959	1.6955*	1.6933**	1.6968	1.6940**
	22	1.7340	1.7165**	1.7321	1.7113**	1.7113**	1.7111**	1.7114**	1.7139**
	44	1.7852	1.7320	1.7076**	1.7280	1.7278	1.7248	1.7278	1.7287
	66	1.8200	1.7205**	1.7371	1.7267	1.7267	1.7223**	1.7267	1.7265
SR	1	0.7606***	0.7389***	0.7597***	0.7597***	0.7578***	0.7635***	0.7540***	0.7588***
	5	0.6821***	0.6689***	0.6717***	0.6821***	0.6802***	0.6783***	0.6850***	0.6840***
	10	0.6774***	0.6689***	0.6471***	0.6481***	0.6414***	0.6462***	0.6518***	0.6462***
	22	0.6083***	0.5885***	0.5260	0.6140***	0.6140***	0.6140***	0.6121***	0.6159***
	44	0.4787	0.4437	0.5809***	0.5383**	0.5383**	0.5393**	0.5459**	0.5061
	66	0.4172	0.5676	0.5118	0.5307	0.5260	0.5506**	0.5203	0.5336

Table 12: Forecasting Evaluation for Corn Futures with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	2.4100	2.1404	2.3635	1.9934**	1.9968**	1.9967**	1.9943**	1.9838**
	5	2.1062**	2.1123*	2.1350*	2.0759**	2.0767**	2.0843**	2.0789**	2.0708**
	10	2.1401**	2.1503	2.1769	2.1142**	2.1144**	2.1258**	2.1165**	2.1047**
	22	2.1149**	2.1435*	2.1211**	2.1460*	2.1472*	2.1589	2.1460*	2.1264**
	44	2.1017**	2.1329	2.2183	2.1556	2.1575	2.1749	2.1570	2.1632
	66	2.1603**	2.2203	2.3428	2.2066	2.2043	2.2287	2.2063	2.2106
MAPE	1	0.8550	0.7907	0.8478	0.7343**	0.7327**	0.7354**	0.7359*	0.7355**
	5	0.7980	0.7789	0.7986	0.7624*	0.7619**	0.7656*	0.7610**	0.7707
	10	0.7905	0.7788**	0.8049	0.7740**	0.7734**	0.7815	0.7746**	0.7920
	22	0.7904**	0.7902**	0.7856**	0.7918**	0.7927**	0.7990*	0.7911**	0.8078
	44	0.7840**	0.7822**	0.8042*	0.7945**	0.7949**	0.7991*	0.7945**	0.8134
	66	0.8042**	0.8036**	0.8405	0.8082**	0.8074**	0.8127**	0.8086**	0.8305
QLIKE	1	2.0261	1.9471	2.0171	1.8804**	1.8811**	1.8829**	1.8807**	1.8781**
	5	1.9278	1.9234	1.9488	1.9089**	1.9096**	1.9130**	1.9096**	1.9076**
	10	1.9465	1.9422	1.9660	1.9264**	1.9275**	1.9299*	1.9268**	1.9185**
	22	1.9366**	1.9386**	1.9488*	1.9402**	1.9416*	1.9463*	1.9402**	1.9310**
	44	1.9414**	1.9537	2.0278	1.9522**	1.9540	1.9576	1.9526	1.9440**
	66	1.9851*	2.0320	2.1097	1.9874*	1.9869*	1.9947	1.9872*	1.9643**
SR	1	0.7011***	0.6556***	0.7030***	0.7353***	0.7268***	0.7353***	0.7315***	0.7334***
	5	0.7078***	0.6954***	0.7011***	0.7097***	0.7087***	0.7144***	0.7106***	0.7087***
	10	0.7144***	0.6917***	0.6869***	0.7011***	0.7049***	0.6973***	0.7097***	0.7030***
	22	0.7049***	0.6860***	0.6926***	0.6765***	0.6727***	0.6717***	0.6755***	0.6992***
	44	0.7021***	0.6954***	0.6945***	0.6917***	0.6879***	0.6831***	0.6926***	0.6907***
	66	0.6879***	0.6898***	0.6651***	0.6888***	0.6907***	0.6708***	0.6850***	0.6850***

Table 13: Forecasting Evaluation for Rice Futures with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	0.9453	0.8394	0.9104	0.7848**	0.7833**	0.7841**	0.7857**	0.7843**
	5	0.8677*	0.8529*	0.8989	0.8493*	0.8452**	0.8475*	0.8495*	0.8525*
	10	0.9314	0.9085*	0.9421	0.9041*	0.9015**	0.9076	0.9041*	0.9144
	22	1.0220	0.9864	1.0355	0.9603**	0.9603**	0.9647*	0.9609**	0.9762
	44	1.1335	1.0377*	1.0193**	1.0207**	1.0216**	1.0256*	1.0208**	1.0336
	66	1.1765	1.0235**	1.0325**	1.0361**	1.0366**	1.0329**	1.0362**	1.0444
MAPE	1	0.6157	0.6015	0.6076	0.5549*	0.5531**	0.5554*	0.5550*	0.5581*
	5	0.6005	0.6005	0.6202	0.5958*	0.5942**	0.5968*	0.5968	0.6013
	10	0.6296**	0.6282**	0.6620	0.6261**	0.6246**	0.6284*	0.6265**	0.6295*
	22	0.6953	0.6875	0.7305	0.6735**	0.6733**	0.6766	0.6750	0.6826
	44	0.7691	0.7408	0.7513	0.7234**	0.7236**	0.7279	0.7237**	0.7345
	66	0.8075	0.7539	0.7613	0.7401**	0.7405*	0.7449*	0.7403**	0.7517
QLIKE	1	1.3287	1.2983	1.3209	1.2841**	1.2837**	1.2834**	1.2839**	1.2834**
	5	1.3113	1.3026*	1.3277	1.3021*	1.3010**	1.3015**	1.3020*	1.3029*
	10	1.3405	1.3250**	1.3462	1.3274**	1.3272**	1.3288*	1.3275**	1.3315
	22	1.3985	1.3642	1.3864	1.3500**	1.3501**	1.3506**	1.3501**	1.3562
	44	1.4695	1.3865	1.3714**	1.3786**	1.3790**	1.3800**	1.3786**	1.3853
	66	1.5064	1.3769**	1.3833**	1.3850**	1.3852**	1.3805**	1.3851**	1.3885
SR	1	0.7787***	0.7495***	0.7768***	0.7910***	0.7900***	0.7834***	0.7863***	0.7900***
	5	0.7589***	0.7580***	0.7354***	0.7561***	0.7571***	0.7589***	0.7524***	0.7571***
	10	0.7288***	0.7147***	0.6610***	0.7175***	0.7194***	0.7109***	0.7175***	0.7015***
	22	0.6252***	0.5979***	0.5490*	0.6412***	0.6450***	0.6525***	0.6431***	0.6478***
	44	0.5160	0.4595	0.5414**	0.5301	0.5264	0.5339	0.5273	0.5188
	66	0.4030	0.5075	0.4812	0.4699	0.4614	0.4859	0.4557	0.4492

Table 14: Forecasting Evaluation for Soy Futures with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

4.3. Real out-of-sample forecasting results: Further tests

In this section we proceed with the evaluation of the average forecasts over the different horizons, against the actual average volatility over the same horizon. This comparison is motivated by the fact that a number of different stakeholders who are interested in agricultural commodity volatility forecasting (agricultural firms, policy makers, international institutions, etc.) do not require point forecasts at a particular h-day ahead horizon, but rather with the expected average volatility over an h-period ahead. For brevity, we do not include the results in the main part of the study, but make them available in Appendix B.

Overall, the results based on the MCS test suggest that our main conclusions still hold, providing further evidence that the HAR extensions are not capable of generating any incremental predictive gains, relatively to the HAR model U.S. agricultural commodity markets. This holds for both RV and $MedRV$ volatility measures. Even more, we report that as we move further out the forecasting horizons, the RW is also included in the set of the best forecasting models, particularly for the $MedRV$. This is suggestive of the fact that the ability of the HAR model to generate superior average volatility forecasts is economically valuable primarily in shorter horizons.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.6238	1.4626	1.5919	1.3174**	1.3168**	1.3167**	1.3188**	1.3190**
	5	1.4880	1.4721	1.4925	1.4265**	1.4223**	1.4225**	1.4258**	1.4279**
	10	1.4526**	1.4590**	1.5383	1.4582**	1.4564**	1.4519**	1.4571**	1.4528**
	22	1.5126**	1.5350**	1.5995	1.5378**	1.5349**	1.5329**	1.5373**	1.5356**
	44	1.5704**	1.6529	1.6063*	1.6275	1.6225	1.6265	1.6281	1.6441
	66	1.5623**	1.6836	1.6753	1.7081	1.7020	1.6944	1.7084	1.6723
MAPE	1	0.6537	0.6265	0.6480	0.5741**	0.5748**	0.5763*	0.5744**	0.5784*
	5	0.6427	0.6292	0.6403	0.6119**	0.6122**	0.6145**	0.6114**	0.6133**
	10	0.6358*	0.6304**	0.6515	0.6224**	0.6225**	0.6246**	0.6224**	0.6241**
	22	0.6603	0.6468**	0.6723*	0.6431**	0.6442**	0.6507*	0.6433**	0.6503**
	44	0.6878	0.6651**	0.6898*	0.6721**	0.6728**	0.6843	0.6724**	0.6846*
	66	0.6836	0.6659**	0.7222	0.6871	0.6872	0.6949	0.6876	0.6891
QLIKE	1	1.9622	1.9459	1.9591	1.9182**	1.9179**	1.9177**	1.9189**	1.9180**
	5	1.9610	1.9542	1.9604	1.9458**	1.9452**	1.9439**	1.9452**	1.9446**
	10	1.9529**	1.9508**	1.9833	1.9541**	1.9537**	1.9510**	1.9542**	1.9516**
	22	1.9824**	1.9879*	2.0484	1.9854*	1.9843*	1.9791**	1.9851*	1.9781**
	44	2.0311**	2.0622	2.0307**	2.0484	2.0461	2.0352**	2.0490	2.0366**
	66	2.0122**	2.0659	2.0453	2.0752	2.0732	2.0587	2.0762	2.0551
SR	1	0.7333***	0.7352***	0.7362***	0.7662***	0.7624***	0.7681***	0.7624***	0.7718***
	5	0.7352***	0.7277***	0.7211***	0.7343***	0.7352***	0.7390***	0.7399***	0.7390***
	10	0.7230***	0.7362***	0.7052***	0.7239***	0.7324***	0.7174***	0.7192***	0.7230***
	22	0.7108***	0.7146***	0.6854***	0.7221***	0.7080***	0.6977***	0.7239***	0.7136***
	44	0.7174***	0.7042***	0.7202***	0.6920***	0.6854***	0.6845***	0.6901***	0.6845***
	66	0.7484***	0.6723***	0.6704***	0.6563***	0.6516***	0.6638***	0.6516***	0.6714***

Table 15: Forecasting Evaluation for Sugar Futures with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.4871	1.3285	1.4444	1.2476**	1.2497**	1.2575*	1.2418**	1.2579*
	5	1.4991	1.4300	1.5210	1.3900**	1.3899**	1.3962**	1.3908**	1.3894**
	10	1.4746**	1.4403**	1.4991	1.4297**	1.4299**	1.4362**	1.4335**	1.4220**
	22	1.4940**	1.4886**	1.5511	1.4867**	1.4876**	1.4864**	1.4886**	1.4839**
	44	1.5822**	1.5847**	1.6332	1.5589**	1.5591**	1.5622**	1.5597**	1.5555**
	66	1.6453*	1.6520	1.8056	1.6201**	1.6211**	1.6010**	1.6206**	1.5970**
MAPE	1	0.6271	0.6143	0.6204	0.5698**	0.5721	0.5689**	0.5682**	0.5744
	5	0.6329	0.6286	0.6371	0.6072*	0.6081	0.6047**	0.6078*	0.6090
	10	0.6202*	0.6225	0.6364	0.6185*	0.6192	0.6153**	0.6204	0.6214
	22	0.6355**	0.6423**	0.6613	0.6405**	0.6406**	0.6393**	0.6409**	0.6497
	44	0.6733**	0.6745*	0.7199	0.6739**	0.6745*	0.6762*	0.6744*	0.6887
	66	0.7032**	0.7190**	0.7816	0.7081**	0.7088**	0.7121**	0.7082**	0.7286
QLIKE	1	1.9467	1.9167	1.9393	1.9018**	1.9018**	1.9032*	1.9017**	1.9029**
	5	1.9429	1.9324	1.9494	1.9239**	1.9239**	1.9258*	1.9244**	1.9234**
	10	1.9389*	1.9328**	1.9499	1.9341**	1.9340**	1.9366*	1.9347**	1.9324**
	22	1.9574*	1.9544*	1.9915	1.9516*	1.9518*	1.9524*	1.9521*	1.9472**
	44	2.0036	2.0034	2.0022	1.9884	1.9880	1.9875	1.9884	1.9776**
	66	2.0194	2.0083	2.0441	2.0038*	2.0032*	1.9952*	2.0036*	1.9897**
SR	1	0.7124***	0.6805***	0.7143***	0.7105***	0.7077***	0.7049***	0.7180***	0.7068***
	5	0.6513***	0.6344***	0.6560***	0.6786***	0.6739***	0.6842***	0.6795***	0.6880***
	10	0.6692***	0.6363***	0.6541***	0.6673***	0.6692***	0.6729***	0.6664***	0.6758***
	22	0.6654***	0.6297***	0.6062***	0.6288***	0.6269***	0.6457***	0.6269***	0.6598***
	44	0.6071***	0.5470***	0.4643	0.5695***	0.5714***	0.5874***	0.5705***	0.6006***
	66	0.5536***	0.4464	0.4502	0.4812	0.4887	0.4887	0.4868	0.5122

Table 16: Forecasting Evaluation for Wheat Futures with RV. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

5. Value-at-Risk backtesting results

To further demonstrate the economic usefulness of the HAR model and its extensions in risk management applications, we calculate the Value-at-Risk (VaR) for the level α and the forecasting horizon h-days ahead by

$$\mathrm{VaR}_{t+h,\alpha}^{(h)} = \mathrm{RV}_{t+h}^{(h)} z_{\alpha}, \tag{31}$$

where z_α is the α-quantile of the Standard Normal distribution. Based on this VaR forecast, we backtest the performance of the individual models using the unconditional coverage test by Kupiec (1995) and the conditional coverage test by Christoffersen (1998). Coverage means that for example a 99% VaR should have 1% violations. A violation occurs if the actual returns exceeds the VaR prediction (long) or turns out to be below the VaR forecast (short). Both tests compare the actual coverage with the theoretical coverage. While the Kupiec (1995) test assumes the violations to be independent from each other, the Christoffersen (1998) test has the alternative hypothesis of a first-order Markov chain.

The VaR results are available in Appendix C. Evidently, they further confirm that the HAR extensions do not offer any material benefits in a risk management exercise relatively to the simple HAR. Hence, our initial conclusion that the jump component, the continuous component, the signed jumps, and the volatility or return leverage do not offer any significant and economically useful forecasting gains, remains robust.

6. Conclusion

The aim of this paper is to add to the extreme scarce literature on agricultural commodities volatility forecasting. Existing studies concentrate their attention to the Chinese futures markets and the benefit of developing model averaging framework. By contrast, they have not focused on the U.S. market, which is the most established market and they do not provide a clear answer as to whether specific volatility components, e.g. the jump component, the continuous component, the signed jumps and the volatility or return leverage can provide incremental predictive gains. Finally, the current literature provides evidence solely based on the realized volatility measure. This study fills these voids by utilizing naive models (Random Walk, AR, ARMA) and several extensions of the simple HAR model (the simple HAR, HAR-J, HAR-CJ, HAR-PS and LHAR-CJ) to forecast two different realized volatility measures, namely, the realized volatility (RV) and median realized volatility ($MedRV$). For our study we obtain tick-by-tick data from five important agricultural commodities, i.e. Corn, Rough Rice, Soybeans, Sugar and Wheat and we produce forecasts for 1-day to 66-days ahead. The period of study spans from January 4, 2010 to June 30, 2017 and our out-of-sample period is January 2, 2013 to June 30, 2017.

In our in-sample analysis, we show that the variants of the HAR model, which decompose the volatility measures into their continuous path and jump components, provide better fitting in the predictive regressions. However, the real out-of-sample forecasts strongly suggest that such decomposition does not offer any superior predictive ability, since none of the variants of the HAR model produce significantly better forecasts compared to the simple HAR model. Thus, there is no benefit to add more complexity in the forecasting models that relates to the volatility decomposition or its relative transformations. Such finding holds for both the RV and $MedRV$, hence they are not specific to the volatility measure. We note that our findings hold for U.S. markets while other studies, e.g. on Chinese futures markets (Tian et al., 2017a,b, Yang et al., 2017), find increased predictive power of jumps, structural breaks, and time variation of HAR coefficients. We conclude that differing driving factors, such as motive and structure of market participants, could potentially affect differently the behaviour of intra-day volatility and subsequently, its forecastability.

Hence, we maintain that the search for improving the forecasting accuracy of the U.S. agricultural commodities volatility should not be located at the development of extended HAR models that take into account properties such as jump component, the continuous component, the signed jumps and the volatility or return leverage, but rather on other direction, such as the inclusion of exogenous predictors. Degiannakis & Filis (2018), Degiannakis & Filis (2017), and Nguyen & Walther (2019) have already shown that the incorporation of different asset classes volatilities helps improving commodities prices and volatilities (oil prices and volatility in particular) and hence, further study should assess whether such asset classes could also help improve forecasts for agricultural commodities. Even more, future research should consider how extreme weather events, food stocks, biofuels production or even market speculative activity could improve further the agricultural commodity volatility forecasts. Finally, an interesting avenue of further research would be the forecasting accuracy evaluation of alternative forecasting methods, such as machine learning.

References

Anderluh, J., & Borovkova, S. (2008). Commodity volatility modelling and option pricing with a potential function approach. *The European Journal of Finance*, *14*, 91–113. doi:10.1080/13518470701773593.

Andersen, T. G., & Bollerslev, T. (1998). Answering the Skeptics: Yes , Standard Volatility Models do Provide Accurate Forecasts. *International Economic Review*, *39*, 885–905.

Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2007). Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility. *Review of Economics and Statistics*, *89*, 701–720. doi:10.1162/rest.89.4.701.

Andersen, T. G., Dobrev, D., & Schaumburg, E. (2012). Jump-robust volatility estimation using nearest neighbor truncation. *Journal of Econometrics*, *169*, 75–93. doi:10.1016/j.jeconom.2012.01.011.

Barndorff-Nielsen, O. E., Kinnebrock, S., & Shephard, N. (2010). Measuring Downside Risk Realized Semivariance. In *Volatility and Time Series Econometrics* (pp. 117–136). Oxford University Press. doi:10.1093/acprof:oso/9780199549498.003.0007.

Barndorff-Nielsen, O. E., & Sheppard, N. (2004). Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics*, *2*, 1–37. doi:10.1093/jjfinec/nbh001.

Bloomberg (2019). There Are Three New Ways to Bet on Chinese Commodity Markets. *28 January 2019*, . URL: https://www.bloomberg.com/news/articles/2019-01-28/there-are-three-new-ways-to-bet-on-chinese-commodity-markets.

von Braun, J., & Tadesse, G. (2012). Food security, commodity price volatility and the poor. In M. Aoki, T. Kuran, & G. Roland (Eds.), *Institutions and Comparative Economic Development*. Palgrave Macmillan Publications, IAE Conference Volume 2012.

CFR (2011). *Food Price Volatility and Insecurity*. Technical Report. URL: https://www.cfr.org/backgrounder/food-price-volatility-and-insecurity.

Christoffersen, P. F. (1998). Evaluating Interval Forecasts. *International Economic Review*, *39*, 841–862. doi:10.2307/2527341.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, *7*, 174–196. doi:10.1093/jjfinec/nbp001.

Corsi, F., & Renò, R. (2012). Discrete-Time Volatility Forecasting With Persistent Leverage Effect and the Link With Continuous-Time Volatility Modeling. *Journal of Business & Economic Statistics*, *30*, 368–380. doi:10.1080/07350015.2012.663261.

Degiannakis, S. (2008). ARFIMAX and ARFIMAX-TARCH realized volatility modeling. *Journal of Applied Statistics*, *35*, 1169–1180. doi:10.1080/02664760802271017.

Degiannakis, S., & Filis, G. (2017). Forecasting oil price realized volatility using information channels from other asset classes. *Journal of International Money and Finance*, *76*, 28–49. doi:10.1016/j.jimonfin.2017.05.006.

Degiannakis, S., & Filis, G. (2018). Forecasting oil prices: High-frequency financial data are indeed useful. *Energy Economics*, *76*, 388–402. doi:10.1016/j.eneco.2018.10.026.

Degiannakis, S., Filis, G., & Hassani, H. (2018). Forecasting global stock market implied volatility indices. *Journal of Empirical Finance*, *46*, 111–129. doi:10.1016/j.jempfin.2017.12.008.

Ederington, L. H., & Guan, W. (2010). Longer-term time-series volatility forecasts. *Journal of Financial and Quantitative Analysis*, *45*, 1055–1076. doi:10.1017/S0022109010000372.

Egelkraut, T. M., & Garcia, P. (2006). Intermediate Volatility Forecasts Using Implied Forward Volatility: The Performance of Selected Agricultural Commodity Options. *Journal of Agricultural and Resource Economics Journal of Agricultural and Resource Economics*, *31*, 508–528.

Elder, J., & Jin, H. J. (2007). Long memory in commodity futures volatility: A wavelet perspective. *Journal of Futures Markets*, *27*, 411–437. doi:10.1002/fut.20260.

Engle, R., Hong, C.-H., & Kane, A. (1990). *Valuation of Variance Forecast with Simulated Option Markets*. Technical Report National Bureau of Economic Research Cambridge, MA. doi:10.3386/w3350.

Fan, J. H., & Zhang, T. (2018). Demystifying Commodity Futures in China. *SSRN Electronic Journal*, . URL: https://www.ssrn.com/abstract=3124223. doi:10.2139/ssrn.3124223.

FAO (2010). *Price Volatility in Agricultural Markets Evidence, impact on food security and policy responses*. Technical Report. URL: http://www.fao.org/docrep/013/am053e/am053e00.pdf.

FAO (2011). *Price Volatility in Food and Agricultural Markets: Policy Responses*. Technical Report. URL: http://www.fao.org/fileadmin/templates/est/Volatility/Interagency_Report_to_the_G20_on_Food_Price_Volatility.pdf.

Giot, P., & Laurent, S. (2003). Market risk in commodity markets: a VaR approach. *Energy Economics*, *25*, 435–457. doi:10.1016/S0140-9883(03)00052-5.

Greb, F., & Prakash, A. (2015). Has price volatility changed? *Food Outlook. Food and Agriculture Organization of the United Nations, Rome, Italy*, .

Greb, F., & Prakash, A. (2017). Assessing volatility patterns in food crops. URL: http://www.fao.org/3/a-i7066e.pdf.

Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, *79*, 453–497. doi:10.3982/ECTA5771.

Huang, H.-C. R. (2004). A flexible nonlinear inference to the Kuznets hypothesis. *Economics Letters*, *84*, 289–296. doi:10.1016/j.econlet.2004.01.014.

Klein, T., & Todorova, N. (2018). The Impact of SHFE's Night Trading Session on Volume and Realized Volatility of Aluminum and Copper Futures Markets. *SSRN Electronic Journal - Working Paper*, (pp. 1–29). URL: https://www.ssrn.com/abstract=3249598. doi:10.2139/ssrn.3249598.

Kupiec, P. H. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *The Journal of Derivatives*, *3*, 73–84. doi:10.3905/jod.1995.407942.

Li, N., Ker, A., Sam, A. G., & Aradhyula, S. (2017). Modeling regime-dependent agricultural commodity price volatilities. *Agricultural Economics*, *48*, 683–691. doi:10.1111/agec.12366.

Liu, L. Y., Patton, A. J., & Sheppard, K. (2015). Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes. *Journal of Econometrics*, *187*, 293–311. doi:10.1016/j.jeconom.2015.02.008.

Luo, J., Klein, T., Ji, Q., & Hou, C. (2019). Forecasting Realized Volatility of Agricultural Commodity Futures with Infinite Hidden Markov HAR Models. *International Journal of Forecasting*, *forth*.

Nguyen, D. K., & Walther, T. (2019). Modeling and Forecasting Commodity Market Volatility with Long-term Economic and Financial Variables. *Journal of Forecasting*, *forth*. doi:10.1002/for.2617.

Ordu, B. M., Oran, A., & Soytas, U. (2018). Is food financialized? Yes, but only when liquidity is abundant. *Journal of Banking & Finance*, *95*, 82–96. doi:10.1016/j.jbankfin.2017.06.001.

Patton, A. J. (2011). Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics*, *160*, 246–256. doi:10.1016/j.jeconom.2010.03.034.

Patton, A. J., & Sheppard, K. (2015). Good Volatility, Bad Volatility: Signed Jumps and The Persistence of Volatility. *Review of Economics and Statistics*, *97*, 683–697. doi:10.1162/REST{_}a{_}00503.

Pesaran, M. H., & Timmermann, A. (1992). A Simple Nonparametric Test of Predictive Performance. *Journal of Business & Economic Statistics1*, *10*, 461–465. doi:10.1080/07350015.1992.10509922.

Sévi, B. (2014). Forecasting the volatility of crude oil futures using intraday data. *European Journal of Operational Research*, *235*, 643–659. doi:10.1016/j.ejor.2014.01.019.

Theodosiou, M., & Zikes, P. (2011). A comprehensive comparison of alternative tests for jumps in asset prices. *Working Paper, Central Bank of Cyprus*, .

Tian, F., Yang, K., & Chen, L. (2017a). Realized Volatility Forecasting of Agricultural Commodity Futures Using Long Memory and Regime Switching. *Journal of Forecasting*, *36*, 421–430. doi:10.1002/for.2443.

Tian, F., Yang, K., & Chen, L. (2017b). Realized volatility forecasting of agricultural commodity futures using the HAR model with time-varying sparsity. *International Journal of Forecasting*, *33*, 132–152. doi:10.1016/j.ijforecast.2016.08.002.

Triantafyllou, A., Dotsis, G., & Sarris, A. H. (2015). Volatility Forecasting and Time-varying Variance Risk Premiums in Grains Commodity Markets. *Journal of Agricultural Economics*, *66*, 329–357. doi:10.1111/1477-9552.12101.

Yang, K., Tian, F., Chen, L., & Li, S. (2017). Realized volatility forecast of agricultural futures using the HAR models with bagging and combination approaches. *International Review of Economics & Finance*, *49*, 276–291. doi:10.1016/j.iref.2017.01.030.

Zhang, D. (2017). Oil shocks and stock markets revisited: Measuring connectedness from a global perspective. *Energy Economics*, *62*, 323–333. doi:10.1016/j.eneco.2017.01.009.

Appendix A. Results MedRV

Appendix A.1. In-Sample

h	1	5	10	22	44	66
Random Walk						
adj. R^2	0.1393	0.3561	0.2977	0.1964	-0.0923	-0.4312
MSPE	1.3701	1.3857	1.4405	1.4957	1.6236	1.6831
MAPE	0.4198	0.4713**	0.5074**	0.5431**	0.6535	0.7305
QLIKE	1.7198**	1.8561**	1.9108**	1.9650**	2.0696**	2.1551**
AR(1)						
c	0.9108	0.6785	0.7409	0.8426	1.1508	1.5319
	(12.6454)	(12.8731)	(15.5199)	(16.0548)	(23.9994)	(29.8909)
$RV^{(h)}$	0.5696	0.6779	0.6487	0.5987	0.4528	0.2751
	(14.6496)	(23.5169)	(24.9195)	(21.2109)	(19.2723)	(11.9992)
adj. R^2	0.3232	0.4592	0.4203	0.3557	0.1998	0.0719
MSPE	1.2143	1.3108**	1.3648**	1.4170	1.4935	1.5079**
MAPE	0.4378	0.4805*	0.5134**	0.5449**	0.6121	0.6380*
QLIKE	1.9448	1.9707	2.0128	2.0504	2.1212**	2.1637**
ARMA						
c	0.1551	0.4081	0.5141	0.8943	1.7898	2.4003
	(5.4775)	(8.0043)	(9.0119)	(10.6561)	(16.5183)	(12.4831)
$RV^{(h)}$	0.9265	0.8066	0.7563	0.5746	0.1533	-0.1320
	(102.6925)	(44.4168)	(37.3471)	(17.5697)	(3.3155)	(1.5514)
$\varepsilon^{(h)}$	-0.6266	-0.2488	-0.1897	0.0399	0.4086	0.4472
	(38.7335)	(9.7088)	(7.0618)	(0.9944)	(8.6762)	(5.8787)
adj. R^2	0.3899	0.4711	0.4285	0.3580	0.2410	0.0906
MSPE	1.1536**	1.3019**	1.3615**	1.4156*	1.4830	1.5142**
MAPE	0.4002	0.4732**	0.5084**	0.5435**	0.6021	0.6372*
QLIKE	1.8943	1.9625	2.0070	2.0487	2.1111**	2.1611**
HAR						
c	0.3098	0.4695	0.6129	0.8049	1.0913	1.3199
	(3.8434)	(9.1136)	(12.8294)	(16.3233)	(25.4387)	(31.2783)
$RV^{(1)}$	0.2878	0.1601	0.1381	0.0910	0.0637	0.0472
	(5.9968)	(5.8801)	(5.2162)	(3.6860)	(2.9905)	(2.4546)
$RV^{(5)}$	0.3435	0.3542	0.2710	0.2089	0.1930	0.1467
	(4.7716)	(7.5856)	(6.3246)	(5.0017)	(5.4903)	(4.7762)
$RV^{(22)}$	0.2222	0.2620	0.2996	0.3173	0.2263	0.1820
	(3.5670)	(6.4302)	(7.4439)	(7.7915)	(6.8787)	(5.9418)
adj. R^2	0.3933	0.4930	0.4660	0.4165	0.3234	0.2281
MSPE	1.1491**	1.2997**	1.3663**	1.4146	1.4871	1.5003**
MAPE	0.3955	0.4695**	0.5025**	0.5411**	0.5977	0.6222**
QLIKE	1.8889	1.9556	1.9957	2.0413	2.0986**	2.1375**
HAR-J						
c	0.2833	0.4567	0.6010	0.7983	1.0821	1.3080
	(3.4808)	(8.9205)	(12.7789)	(16.4661)	(25.5415)	(31.3998)
$RV^{(1)}$	0.2649	0.1491	0.1280	0.0855	0.0559	0.0370
	(5.3369)	(5.3565)	(4.7597)	(3.3782)	(2.5703)	(1.8984)
$RV^{(5)}$	0.3266	0.3461	0.2635	0.2048	0.1872	0.1391
	(4.7099)	(7.3960)	(6.1448)	(4.9089)	(5.3058)	(4.4873)
$RV^{(22)}$	0.2240	0.2629	0.3005	0.3178	0.2270	0.1829
	(3.6098)	(6.4700)	(7.4972)	(7.8372)	(6.9554)	(6.0443)
$J^{(1)}$	0.2621	0.1263	0.1156	0.0638	0.0899	0.1170
	(2.3993)	(2.3183)	(2.5983)	(1.6828)	(2.2638)	(2.9546)
adj. R^2	0.4000	0.4954	0.4684	0.4172	0.3255	0.2328
MSPE	1.1425**	1.3001**	1.3677**	1.4152	1.4857	1.4976**
MAPE	0.3912**	0.4689**	0.5026**	0.5404**	0.5966	0.6221**
QLIKE	1.8838	1.9545	1.9952	2.0406	2.0973**	2.1371**
HAR-CJ						
c	0.3686	0.5305	0.6808	0.9030	1.2085	1.4232
	(4.7122)	(10.6153)	(14.6331)	(18.6737)	(28.9429)	(34.0533)
$J^{(1)}$	0.1626	0.0887	0.0941	0.0551	0.0361	0.0284
	(1.6227)	(1.6277)	(2.2697)	(1.6383)	(0.9978)	(0.8164)
$J^{(5)}$	0.0777	0.0875	0.0422	-0.0148	0.0001	0.0180
	(2.0670)	(3.6659)	(2.0600)	(-0.7519)	(0.0036)	(1.0146)
$J^{(22)}$	-0.0025	0.0006	0.0179	0.0522	0.0722	0.0678
	(-0.2020)	(0.0622)	(2.3165)	(7.2854)	(10.6087)	(11.3767)
$C^{(1)}$	0.2649	0.1387	0.1199	0.0820	0.0578	0.0426
	(6.5784)	(5.7244)	(5.1117)	(3.7994)	(3.2770)	(2.6690)
$C^{(5)}$	0.2624	0.2845	0.2297	0.1871	0.1613	0.1123
	(4.2929)	(6.8340)	(6.1022)	(5.2061)	(5.6649)	(4.3668)
$C^{(22)}$	0.2042	0.2316	0.2251	0.1707	0.0445	0.0156
	(3.6547)	(5.8049)	(5.8277)	(4.7512)	(1.6043)	(0.5539)
adj. R^2	0.4010	0.5003	0.4761	0.4378	0.3867	0.3075
MSPE	1.1408**	1.2947**	1.3632**	1.3967**	1.4567	1.4993**
MAPE	0.3901**	0.4684**	0.5015**	0.5332**	0.5800	0.6115**
QLIKE	1.8833	1.9538	1.9935	2.0332	2.0829**	2.1249**
HAR-PS						
c	0.3165	0.4761	0.6162	0.8040	1.0879	1.3142
	(3.9645)	(9.3418)	(13.0527)	(16.7068)	(26.0791)	(32.0350)
RS^{+}	0.1467	0.1049	0.0755	0.0333	0.0237	0.0120
	(2.3957)	(3.1172)	(2.6586)	(1.2713)	(0.8861)	(0.4677)
RS^{-}	0.4074	0.1890	0.1874	0.1653	0.1504	0.1546
	(4.7053)	(3.7896)	(3.9352)	(3.8112)	(4.0479)	(4.3229)
$\mathbf{I}_{r_t<0}RV^{(1)}$	-0.0389	0.0003	0.0002	-0.0301	-0.0383	-0.0510
	(-0.9204)	(0.0106)	(0.0082)	(-1.3133)	(-1.9616)	(-2.9747)
$RV^{(5)}$	0.3354	0.3451	0.2584	0.2032	0.1781	0.1265
	(5.0532)	(7.4672)	(6.2539)	(5.0904)	(5.2445)	(4.2486)
$RV^{(22)}$	0.2145	0.2582	0.2967	0.3150	0.2252	0.1815
	(3.4793)	(6.3739)	(7.4231)	(7.8520)	(6.9884)	(6.0727)
adj. R^2	0.4054	0.4986	0.4736	0.4217	0.3309	0.2393
MSPE	1.1369**	1.3007**	1.3663**	1.4139	1.4837	1.4985**
MAPE	0.3907**	0.4691**	0.5025**	0.5388**	0.5949	0.6229**
QLIKE	1.8816	1.9535	1.9942	2.0383	2.0958**	2.1370**
LHAR-CJ						
c	0.3109	0.4923	0.6532	0.9030	1.2261	1.4558
	(3.9367)	(9.5685)	(13.5949)	(18.4390)	(28.8160)	(34.5373)
$J^{(1)}$	0.1570	0.0835	0.0880	0.0499	0.0300	0.0223
	(1.5935)	(1.5699)	(2.1747)	(1.5103)	(0.8602)	(0.6621)
$J^{(5)}$	0.0770	0.0863	0.0412	-0.0151	0.0000	0.0181
	(2.0719)	(3.6433)	(2.0238)	(-0.7721)	(0.0011)	(1.0281)
$J^{(22)}$	-0.0041	-0.0014	0.0171	0.0533	0.0747	0.0710
	(-0.3215)	(-0.1498)	(2.1515)	(7.2455)	(10.9309)	(11.7303)
$C^{(1)}$	0.2486	0.1285	0.1093	0.0750	0.0510	0.0380
	(6.2204)	(5.4253)	(4.7001)	(3.4563)	(2.8884)	(2.3610)
$C^{(5)}$	0.2658	0.2832	0.2342	0.1981	0.1793	0.1326
	(4.3407)	(6.8237)	(6.1774)	(5.4349)	(6.2724)	(5.1090)
$C^{(22)}$	0.2127	0.2397	0.2275	0.1636	0.0295	-0.0038
	(3.8489)	(5.9377)	(5.8137)	(4.5265)	(1.0783)	(-0.1380)
$r_t^{-,(1)}$	-0.1069	-0.0654	-0.0697	-0.0482	-0.0482	-0.0351
	(-2.5449)	(-2.6240)	(-3.1192)	(-2.5132)	(-2.8689)	(-2.2727)
$r_t^{-,(5)}$	-0.0029	0.0068	0.0055	0.0028	0.0013	0.0022
	(-0.4766)	(1.4119)	(1.1699)	(0.6467)	(0.3393)	(0.5479)
$r_t^{-,(22)}$	-0.1099	-0.0626	0.0456	0.2018	0.3454	0.4237
	(-0.7278)	(-0.6529)	(0.5430)	(2.8277)	(5.6021)	(7.8465)
adj. R^2	0.4052	0.5037	0.4798	0.4410	0.3954	0.3215
MSPE	1.1359**	1.2949**	1.3617**	1.3955**	1.4501**	1.4950**
MAPE	0.3871**	0.4679**	0.5009**	0.5343**	0.5761**	0.6107**
QLIKE	1.8779	1.9512	1.9915	2.0327	2.0799**	2.1233**

Table A.17: In-Sample regression results for Corn with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
Random Walk						
adj. R^2	-0.2356	0.2574	0.4694	0.4513	0.6116	0.6230
MSPE	1.9701	1.6796	1.6502	1.6767	1.6303**	1.6859
MAPE	1.3172**	1.4493**	1.4377**	1.5851**	1.5033**	1.7326**
QLIKE	2.0395**	2.4818**	2.5299**	2.7136**	2.6723**	2.9143**
AR(1)						
c	1.1199	0.6711	0.4794	0.4954	0.3452	0.3164
	(16.6458)	(16.0793)	(13.1386)	(14.3639)	(13.8107)	(13.1081)
$RV^{(h)}$	0.3809	0.6286	0.7347	0.7258	0.8071	0.8170
	(9.6683)	(24.1001)	(31.8458)	(32.7801)	(50.3674)	(57.7316)
adj. R^2	0.1435	0.3946	0.5392	0.5258	0.6482	0.6559
MSPE	1.6395	1.6114	1.6131*	1.6446	1.6184**	1.6679
MAPE	1.7584	1.6599	1.5873	1.7126	1.6057	1.7788
QLIKE	3.0462	2.9217	2.8334	2.9712	2.8590	3.0245
ARMA						
c	0.0335	0.0873	0.0984	0.1195	0.2111	0.2736
	(2.1996)	(4.1667)	(4.1662)	(4.6593)	(6.3972)	(6.5555)
$RV^{(h)}$	0.9819	0.9523	0.9455	0.9322	0.8772	0.8311
	(166.1620)	(113.0345)	(101.4047)	(89.4343)	(64.8657)	(48.2801)
$\varepsilon^{(h)}$	-0.8719	-0.6562	-0.5740	-0.4871	-0.1843	0.0839
	(68.6678)	(32.5373)	(30.0047)	(22.4845)	(7.4582)	(2.4454)
adj. R^2	0.2539	0.4888	0.5652	0.5985	0.6687	0.7134
MSPE	1.5309*	1.5699**	1.5938**	1.6148**	1.6164**	1.6371**
MAPE	1.4052	1.4980**	1.5334	1.5913**	1.6076	1.7391**
QLIKE	2.6326	2.7277	2.7655	2.8315*	2.8571	2.9794**
HAR						
c	0.2782	0.3437	0.3932	0.4859	0.5213	0.5318
	(4.0585)	(8.0225)	(10.3285)	(14.2169)	(19.6265)	(19.2724)
$RV^{(1)}$	0.1265	0.0606	0.0376	0.0361	0.0231	0.0210
	(2.8898)	(2.7805)	(1.9825)	(2.1175)	(1.5997)	(1.6164)
$RV^{(5)}$	0.2089	0.1937	0.2377	0.1517	0.0741	0.0661
	(2.7624)	(4.8514)	(6.4200)	(4.5514)	(2.5813)	(2.2948)
$RV^{(22)}$	0.5117	0.5550	0.5064	0.5430	0.6151	0.6158
	(7.2832)	(12.9809)	(13.8234)	(16.4133)	(22.1678)	(21.0952)
adj. R^2	0.2516	0.4696	0.5419	0.5445	0.5883	0.6050
MSPE	1.5316	1.5814	1.6000**	1.6502	1.6362*	1.6544
MAPE	1.4147	1.5303	1.5106	1.6855	1.5928	1.7770
QLIKE	2.6455	2.7678	2.7536	2.9388	2.8619	3.0440
HAR-J						
c	0.2458	0.3114	0.3658	0.4657	0.5036	0.5122
	(3.5846)	(7.2091)	(9.6992)	(13.5702)	(18.4851)	(18.6322)
$RV^{(1)}$	0.1062	0.0403	0.0202	0.0231	0.0118	0.0087
	(2.2696)	(1.7622)	(1.0698)	(1.3486)	(0.8121)	(0.6772)
$RV^{(5)}$	0.1982	0.1830	0.2285	0.1449	0.0684	0.0598
	(2.6145)	(4.6573)	(6.2278)	(4.3556)	(2.3947)	(2.0946)
$RV^{(22)}$	0.4926	0.5359	0.4899	0.5308	0.6039	0.6036
	(7.0574)	(12.5871)	(13.3692)	(16.0809)	(21.8293)	(20.6180)
$J^{(1)}$	0.0804	0.0804	0.0692	0.0512	0.0452	0.0494
	(2.0262)	(3.2810)	(3.5470)	(2.8269)	(2.9412)	(3.5899)
adj. R^2	0.2540	0.4752	0.5470	0.5477	0.5911	0.6087
MSPE	1.5288	1.5777	1.5994**	1.6495	1.6346*	1.6501
MAPE	1.4015	1.5120**	1.4977**	1.6759	1.5917	1.7681*
QLIKE	2.6302	2.7465	2.7390	2.9281	2.8597	3.0343*
HAR-CJ						
c	0.1717	0.2672	0.3316	0.4351	0.4492	0.4154
	(2.1899)	(5.7213)	(8.2535)	(12.1330)	(15.9177)	(14.9847)
$J^{(1)}$	0.1013	0.0374	0.0216	0.0254	0.0148	0.0120
	(2.7255)	(1.7383)	(1.1974)	(1.4477)	(1.0189)	(0.9061)
$J^{(5)}$	0.0240	0.0308	0.0353	0.0124	0.0039	0.0011
	(1.4242)	(3.3838)	(4.5441)	(1.7335)	(0.6638)	(0.2017)
$J^{(22)}$	0.0021	-0.0009	-0.0031	-0.0003	0.0055	0.0117
	(0.3505)	(-0.2993)	(-1.1824)	(-0.1317)	(2.6625)	(5.9781)
$C^{(1)}$	0.1028	0.0463	0.0295	0.0286	0.0184	0.0175
	(3.4187)	(3.0232)	(2.2899)	(2.4622)	(1.8072)	(1.8975)
$C^{(5)}$	0.1612	0.1604	0.1811	0.1170	0.0622	0.0581
	(2.7046)	(5.3464)	(6.7761)	(4.9356)	(2.9895)	(2.8302)
$C^{(22)}$	0.3546	0.4093	0.3940	0.4339	0.4621	0.4292
	(5.4912)	(11.3122)	(13.6094)	(16.3700)	(19.5743)	(18.5807)
adj. R^2	0.2641	0.4926	0.5672	0.5691	0.6117	0.6374
MSPE	1.5175*	1.5702**	1.5901**	1.6452	1.6272**	1.6447
MAPE	1.3717*	1.4845**	1.4932**	1.6600	1.5649*	1.7518**
QLIKE	2.5906	2.7140	2.7298	2.9091	2.8304*	3.0129**
HAR-PS						
c	0.2336	0.3177	0.3738	0.4706	0.5104	0.5207
	(3.3576)	(7.3902)	(9.9496)	(13.8324)	(19.0776)	(19.0210)
RS^{+}	0.0898	0.0623	0.0491	0.0297	0.0168	0.0152
	(1.5669)	(2.2056)	(2.2874)	(1.4608)	(1.0316)	(1.0656)
RS^{-}	0.1287	0.0631	0.0480	0.0534	0.0418	0.0465
	(1.9305)	(1.8422)	(1.8885)	(2.1125)	(1.9240)	(2.4737)
$\mathbf{I}_{r_t<0} RV^{(1)}$	0.0362	0.0122	0.0002	-0.0015	0.0015	-0.0026
	(0.6458)	(0.4433)	(0.0100)	(-0.0781)	(0.0879)	(-0.1777)
$RV^{(5)}$	0.1788	0.1682	0.2131	0.1357	0.0600	0.0500
	(2.4396)	(4.3831)	(5.9570)	(4.1738)	(2.1558)	(1.7901)
$RV^{(22)}$	0.4817	0.5388	0.4939	0.5313	0.6062	0.6062
	(6.9452)	(12.6535)	(13.5256)	(16.1800)	(21.9699)	(20.7381)
adj. R^2	0.2610	0.4763	0.5469	0.5483	0.5908	0.6081
MSPE	1.5212*	1.5793	1.5991**	1.6500	1.6351*	1.6514
MAPE	1.3831*	1.5090**	1.4984**	1.6728	1.5900	1.7844
QLIKE	2.6089	2.7432	2.7401	2.9251	2.8584	3.0507
LHAR-CJ						
c	0.0922	0.2135	0.2878	0.3858	0.4081	0.3793
	(1.1098)	(4.3292)	(6.9719)	(10.6837)	(14.1850)	(13.2847)
$J^{(1)}$	0.0908	0.0327	0.0185	0.0216	0.0116	0.0089
	(2.4694)	(1.5243)	(1.0139)	(1.2177)	(0.8031)	(0.6800)
$J^{(5)}$	0.0242	0.0315	0.0360	0.0133	0.0047	0.0020
	(1.4742)	(3.4917)	(4.6425)	(1.8798)	(0.8164)	(0.3759)
$J^{(22)}$	0.0005	-0.0023	-0.0044	-0.0017	0.0042	0.0104
	(0.0809)	(-0.7530)	(-1.6666)	(-0.7447)	(2.0435)	(5.3214)
$C^{(1)}$	0.0826	0.0370	0.0232	0.0213	0.0126	0.0119
	(2.7496)	(2.4220)	(1.7639)	(1.7825)	(1.2022)	(1.2556)
$C^{(5)}$	0.1664	0.1562	0.1754	0.1121	0.0569	0.0543
	(2.8318)	(5.1939)	(6.4804)	(4.6530)	(2.6960)	(2.6355)
$C^{(22)}$	0.3564	0.4119	0.3968	0.4353	0.4641	0.4291
	(5.5311)	(11.3531)	(13.7210)	(16.5593)	(19.5414)	(18.6349)
$r_t^{-,(1)}$	-0.2147	-0.0723	-0.0381	-0.0438	-0.0278	-0.0276
	(-3.1659)	(-2.2244)	(-1.4738)	(-1.7440)	(-1.3972)	(-1.4934)
$r_t^{-,(5)}$	0.0055	-0.0075	-0.0093	-0.0168	-0.0164	-0.0206
	(0.3858)	(-1.0147)	(-1.5003)	(-2.9359)	(-3.4052)	(-4.1524)
$r_t^{-,(22)}$	-0.3586	-0.4680	-0.4461	-0.4166	-0.3615	-0.2633
	(-1.2494)	(-2.5468)	(-3.2046)	(-3.6660)	(-3.9317)	(-2.8123)
adj. R^2	0.2737	0.5009	0.5748	0.5807	0.6218	0.6472
MSPE	1.5064**	1.5674**	1.5892**	1.6425	1.6269**	1.6376**
MAPE	1.3323**	1.4642**	1.4731**	1.6357**	1.5411**	1.7411**
QLIKE	2.5445	2.6889	2.7076	2.8809*	2.8038*	3.0022**

Table A.18: In-Sample regression results for Rough Rice with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
Random Walk						
adj. R^2	0.1534	0.3170	0.2034	0.0685	-0.3412	-0.8690
MSPE	0.7996	0.8315	0.8775	0.9116	0.9905	1.0368
MAPE	0.7928	0.8616**	0.8009**	0.9920**	1.2138	1.2399*
QLIKE	1.6033**	1.7714**	1.7169**	1.9251**	2.1483	2.1875**
AR(1)						
c	0.5462	0.4386	0.5107	0.5961	0.8745	1.2606
	(9.9397)	(13.4973)	(15.6302)	(20.2573)	(27.1081)	(36.2906)
$RV^{(h)}$	0.5765	0.6584	0.6019	0.5346	0.3209	0.0308
	(12.4866)	(23.3093)	(21.7637)	(22.3262)	(13.8309)	(1.3330)
adj. R^2	0.3318	0.4329	0.3606	0.2811	0.0975	0.0003
MSPE	0.7100	0.7811*	0.8208**	0.8518**	0.8883	0.8830**
MAPE	0.8160	0.8676**	0.8425	0.9653**	1.0591	1.0330**
QLIKE	1.8267	1.8805	1.8593	1.9875**	2.0863	2.0763**
ARMA						
c	0.1173	0.3285	0.3740	0.6977	1.3823	1.9509
	(6.2369)	(9.3806)	(9.8117)	(14.9384)	(17.5942)	(18.0698)
$RV^{(h)}$	0.9086	0.7442	0.7082	0.4561	-0.0727	-0.5001
	(85.9890)	(35.0881)	(27.5323)	(13.6605)	(1.1946)	(5.9322)
$\varepsilon^{(h)}$	-0.5725	-0.1543	-0.1676	0.1134	0.5183	0.6607
	(33.7617)	(5.3514)	(4.8507)	(2.7723)	(10.1166)	(8.8989)
adj. R^2	0.3956	0.4375	0.3676	0.2869	0.1636	0.0330
MSPE	0.6756*	0.7787**	0.8188**	0.8499**	0.8778**	0.8914
MAPE	0.7888**	0.8518**	0.8390	0.9641**	1.0827	1.0253**
QLIKE	1.7927	1.8645	1.8542	1.9854**	2.1056	2.0631**
HAR						
c	0.2019	0.3187	0.4214	0.5679	0.7824	0.9623
	(4.0585)	(9.4380)	(13.6804)	(19.9711)	(28.3770)	(37.0756)
$RV^{(1)}$	0.3014	0.1870	0.1522	0.1004	0.0667	0.0494
	(5.1651)	(4.3318)	(4.2260)	(3.5482)	(3.0192)	(2.7525)
$RV^{(5)}$	0.3670	0.3461	0.2621	0.2071	0.1849	0.1385
	(5.6254)	(6.7383)	(5.7378)	(5.1043)	(5.4105)	(4.8892)
$RV^{(22)}$	0.1745	0.2179	0.2566	0.2497	0.1416	0.0701
	(2.7301)	(5.8351)	(7.6973)	(7.9022)	(4.9059)	(2.7433)
adj. R^2	0.3964	0.4686	0.4219	0.3550	0.2401	0.1352
MSPE	0.6744*	0.7730**	0.8208**	0.8519**	0.8857*	0.8962
MAPE	0.7611**	0.8377**	0.8216**	0.9098**	1.0464**	1.0289**
QLIKE	1.7624	1.8452	1.8317	1.9246**	2.0638**	2.0597**
HAR-J						
c	0.1984	0.3195	0.4217	0.5683	0.7819	0.9611
	(3.9871)	(9.5302)	(13.8126)	(20.1325)	(28.4923)	(37.0932)
$RV^{(1)}$	0.2979	0.1878	0.1525	0.1008	0.0663	0.0482
	(5.0584)	(4.2956)	(4.1831)	(3.5096)	(2.9682)	(2.6671)
$RV^{(5)}$	0.3655	0.3465	0.2623	0.2072	0.1847	0.1379
	(5.6162)	(6.7394)	(5.7385)	(5.1068)	(5.4092)	(4.8729)
$RV^{(22)}$	0.1739	0.2181	0.2567	0.2498	0.1416	0.0699
	(2.7207)	(5.8461)	(7.7001)	(7.9030)	(4.8995)	(2.7333)
$J^{(1)}$	0.0464	-0.0113	-0.0044	-0.0046	0.0055	0.0160
	(0.9395)	(-0.3270)	(-0.1316)	(-0.1434)	(0.2138)	(0.6763)
adj. R^2	0.3964	0.4684	0.4216	0.3546	0.2397	0.1349
MSPE	0.6743*	0.7729**	0.8208**	0.8519**	0.8856*	0.8960
MAPE	0.7606**	0.8369**	0.8217**	0.9089**	1.0458**	1.0280**
QLIKE	1.7616	1.8443	1.8318	1.9237**	2.0633**	2.0589**
HAR-CJ						
c	0.1818	0.3021	0.4000	0.5481	0.7611	0.9411
	(3.4649)	(9.1351)	(13.4957)	(19.8466)	(28.0162)	(35.1316)
$J^{(1)}$	0.0740	0.0203	-0.0059	0.0073	0.0016	0.0020
	(1.7967)	(0.7236)	(-0.2147)	(0.2583)	(0.0680)	(0.0952)
$J^{(5)}$	-0.0321	-0.0354	-0.0124	-0.0027	0.0069	0.0037
	(-1.4070)	(-2.5137)	(-0.8846)	(-0.2001)	(0.5975)	(0.3680)
$J^{(22)}$	0.0068	0.0081	0.0082	0.0063	0.0057	0.0067
	(0.7238)	(1.5101)	(1.6865)	(1.2881)	(1.1906)	(1.6122)
$C^{(1)}$	0.2968	0.1785	0.1500	0.0987	0.0658	0.0486
	(6.8981)	(5.3611)	(5.4101)	(4.5361)	(3.6617)	(3.0182)
$C^{(5)}$	0.3323	0.3319	0.2475	0.1932	0.1709	0.1288
	(6.3988)	(7.7826)	(6.4511)	(5.5533)	(5.6766)	(4.9104)
$C^{(22)}$	0.1606	0.1957	0.2282	0.2250	0.1260	0.0593
	(2.8900)	(5.8374)	(7.3471)	(7.3246)	(4.5121)	(2.4315)
adj. R^2	0.4053	0.4810	0.4328	0.3616	0.2456	0.1395
MSPE	0.6689*	0.7703**	0.8208**	0.8506**	0.8846**	0.8955
MAPE	0.7129**	0.7904**	0.8207**	0.9124**	1.0461**	1.0247**
QLIKE	1.7109	1.7950**	1.8290	1.9269**	2.0643**	2.0555**
HAR-PS						
c	0.1889	0.3121	0.4163	0.5629	0.7782	0.9584
	(3.8946)	(9.5190)	(13.8909)	(20.1616)	(28.4741)	(37.0297)
RS^{+}	0.2189	0.1221	0.1028	0.0878	0.0627	0.0419
	(3.4840)	(2.7344)	(2.4812)	(2.3312)	(2.1504)	(1.7079)
RS^{-}	0.2315	0.1384	0.1076	0.0632	0.0454	0.0459
	(3.6797)	(2.8611)	(2.6585)	(2.0677)	(1.8389)	(2.0617)
$\mathbf{I}_{r_t<0}RV^{(1)}$	-0.0075	-0.0082	0.0016	0.0047	0.0007	-0.0087
	(-0.1936)	(-0.2950)	(0.0654)	(0.2198)	(0.0417)	(-0.5930)
$RV^{(5)}$	0.4245	0.3941	0.2974	0.2222	0.1921	0.1446
	(7.5615)	(8.8086)	(7.3030)	(5.9434)	(6.0406)	(5.3403)
$RV^{(22)}$	0.1640	0.2108	0.2513	0.2468	0.1398	0.0684
	(2.5335)	(5.5277)	(7.4759)	(7.8005)	(4.8465)	(2.6756)
adj. R^2	0.3910	0.4619	0.4170	0.3534	0.2395	0.1347
MSPE	0.6771	0.7759*	0.8206**	0.8507**	0.8848**	0.8956
MAPE	0.7511**	0.8566**	0.8174**	0.9284**	1.0382**	1.0283**
QLIKE	1.7516	1.8638	1.8279	1.9437**	2.0556**	2.0593**
LHAR-CJ						
c	0.1676	0.2952	0.3951	0.5485	0.7648	0.9446
	(3.1795)	(8.8862)	(13.2608)	(20.0308)	(28.6279)	(35.3476)
$J^{(1)}$	0.0578	0.0126	-0.0153	0.0036	-0.0010	0.0016
	(1.4033)	(0.4468)	(-0.5483)	(0.1227)	(-0.0407)	(0.0743)
$J^{(5)}$	-0.0347	-0.0380	-0.0131	-0.0021	0.0090	0.0058
	(-1.4638)	(-2.6431)	(-0.9226)	(-0.1509)	(0.7622)	(0.5689)
$J^{(22)}$	0.0066	0.0078	0.0086	0.0069	0.0069	0.0078
	(0.7379)	(1.4926)	(1.8147)	(1.4555)	(1.4608)	(1.9545)
$C^{(1)}$	0.2821	0.1715	0.1426	0.0967	0.0660	0.0505
	(6.5677)	(5.2542)	(5.1901)	(4.3965)	(3.6131)	(3.0776)
$C^{(5)}$	0.3293	0.3262	0.2426	0.1894	0.1667	0.1260
	(6.3106)	(7.7029)	(6.3373)	(5.4031)	(5.4831)	(4.7569)
$C^{(22)}$	0.1626	0.1980	0.2306	0.2271	0.1287	0.0618
	(2.9323)	(5.9270)	(7.4204)	(7.3555)	(4.6080)	(2.5334)
$r_t^{-,(1)}$	-0.0535	-0.0232	-0.0275	-0.0085	-0.0025	0.0039
	(-1.9576)	(-1.1718)	(-1.6038)	(-0.6377)	(-0.2057)	(0.3506)
$r_t^{-,(5)}$	0.0073	0.0097	0.0100	0.0076	0.0091	0.0065
	(1.2940)	(2.4735)	(2.7562)	(2.1952)	(2.8868)	(2.1385)
$r_t^{-,(22)}$	-0.0798	-0.0577	0.0144	0.0619	0.1309	0.1246
	(-0.7749)	(-0.8824)	(0.2533)	(1.1775)	(2.6252)	(2.7591)
adj. R^2	0.4081	0.4830	0.4354	0.3626	0.2496	0.1427
MSPE	0.6668**	0.7699**	0.8208**	0.8504**	0.8825**	0.8958
MAPE	0.7111**	0.7940**	0.8273**	0.9113**	1.0539**	1.0276**
QLIKE	1.7073	1.7971**	1.8356	1.9256**	2.0731**	2.0582**

Table A.19: In-Sample regression results for Soybean with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
			Random Walk			
adj. R^2	0.2262	0.5608	0.5727	0.6979	0.6498	0.5690
MSPE	2.5390	2.4008	2.4418	2.3006*	2.3514	2.3144
MAPE	0.4220	0.4333**	0.4469**	0.4767**	0.5376**	0.5696**
QLIKE	2.0430**	2.1573**	2.1945**	2.2329**	2.2975**	2.3392**
			AR(1)			
c	1.2607	0.7117	0.6886	0.4861	0.5702	0.6864
	(6.0962)	(8.3957)	(8.3943)	(9.3212)	(14.5089)	(17.4513)
$RV^{(h)}$	0.6129	0.7787	0.7842	0.8438	0.8073	0.7582
	(8.5681)	(25.0790)	(26.0877)	(41.1325)	(51.8637)	(49.1634)
adj. R^2	0.3753	0.6096	0.6192	0.7225	0.6896	0.6363
MSPE	2.2801	2.3163	2.3714	2.2712**	2.2781	2.2191
MAPE	0.5358	0.4879	0.5015	0.5054	0.5591	0.5822
QLIKE	2.4323	2.3518	2.3693	2.3447	2.3947	2.4235
			ARMA			
c	0.0923	0.1286	0.1068	0.2257	0.2564	0.2692
	(2.7729)	(2.8549)	(2.6357)	(5.5043)	(7.9982)	(7.2925)
$RV^{(h)}$	0.9714	0.9588	0.9622	0.9144	0.8861	0.8758
	(193.8690)	(134.4966)	(134.5328)	(124.1920)	(162.0116)	(134.1260)
$\varepsilon^{(h)}$	-0.7478	-0.5879	-0.6050	-0.3614	-0.4168	-0.3466
	(74.8161)	(55.2983)	(42.6080)	(20.4874)	(16.3799)	(14.4741)
adj. R^2	0.4726	0.6437	0.6884	0.7783	0.7841	0.7626
MSPE	2.0959*	2.2448**	2.2484**	2.2126**	2.1831**	2.1689**
MAPE	0.3989	0.4407**	0.4604*	0.4888*	0.5333**	0.5636**
QLIKE	2.2463	2.2795	2.2897	2.3138	2.3499	2.3711*
			HAR			
c	0.2650	0.3589	0.4208	0.4701	0.5936	0.7304
	(2.5424)	(5.4349)	(6.7487)	(9.1831)	(14.1271)	(18.1498)
$RV^{(1)}$	0.2170	0.1279	0.1041	0.0626	0.0502	0.0451
	(2.4736)	(2.9423)	(3.2554)	(2.1464)	(1.9847)	(1.8687)
$RV^{(5)}$	0.3694	0.3086	0.1902	0.0785	0.1287	0.1139
	(3.6601)	(4.5789)	(3.4039)	(1.5283)	(2.8823)	(2.7996)
$RV^{(22)}$	0.3318	0.4500	0.5722	0.7079	0.6213	0.5886
	(4.1014)	(7.5194)	(9.9820)	(17.1428)	(17.9911)	(18.3121)
adj. R^2	0.4843	0.6582	0.6820	0.7317	0.7374	0.7087
MSPE	2.0703*	2.2205**	2.2709**	2.2664**	2.2452**	2.2724
MAPE	0.3877**	0.4406**	0.4695*	0.5029	0.5482**	0.5893
QLIKE	2.2259	2.2808	2.3123	2.3404	2.3800	2.4179
			HAR-J			
c	0.1920	0.3270	0.3954	0.4492	0.5787	0.7210
	(1.6696)	(4.8360)	(6.1801)	(8.5621)	(13.5530)	(17.6302)
$RV^{(1)}$	0.2009	0.1208	0.0984	0.0578	0.0467	0.0429
	(2.4855)	(2.8893)	(3.1829)	(2.0508)	(1.8962)	(1.8030)
$RV^{(5)}$	0.3523	0.3013	0.1845	0.0739	0.1255	0.1118
	(3.5286)	(4.5172)	(3.3462)	(1.4659)	(2.8564)	(2.7809)
$RV^{(22)}$	0.3099	0.4405	0.5646	0.7015	0.6168	0.5857
	(3.8850)	(7.4916)	(9.9932)	(17.3590)	(18.2416)	(18.4361)
$J^{(1)}$	0.4887	0.2124	0.1692	0.1390	0.0995	0.0634
	(2.7537)	(2.4386)	(2.1543)	(2.1475)	(1.8314)	(1.2253)
adj. R^2	0.4942	0.6610	0.6839	0.7332	0.7382	0.7089
MSPE	2.0498*	2.2194**	2.2698**	2.2667**	2.2459**	2.2736
MAPE	0.3871**	0.4409**	0.4713	0.5031	0.5489**	0.5905
QLIKE	2.2162	2.2789	2.3122	2.3399	2.3801	2.4184
			HAR-CJ			
c	0.2060	0.3134	0.3722	0.4365	0.5824	0.7317
	(1.7492)	(4.3983)	(5.9008)	(8.8139)	(14.4660)	(18.5219)
$J^{(1)}$	0.3086	0.1766	0.1026	0.0795	0.0616	0.0521
	(2.8482)	(2.3912)	(1.5252)	(1.5458)	(1.4140)	(1.2420)
$J^{(5)}$	0.0063	-0.0128	0.0028	0.0159	0.0361	0.0179
	(0.1157)	(-0.3339)	(0.0928)	(0.7604)	(1.9433)	(0.9722)
$J^{(22)}$	0.0144	0.0179	0.0199	0.0124	-0.0029	-0.0074
	(0.7554)	(1.4288)	(1.9662)	(1.7812)	(-0.4486)	(-1.0706)
$C^{(1)}$	0.2396	0.1383	0.1097	0.0640	0.0505	0.0457
	(2.9244)	(3.4756)	(3.7786)	(2.4819)	(2.3088)	(2.1805)
$C^{(5)}$	0.3184	0.2718	0.1628	0.0708	0.1140	0.1003
	(3.4719)	(4.4527)	(3.2010)	(1.5124)	(2.9030)	(2.7695)
$C^{(22)}$	0.2484	0.3761	0.4921	0.6201	0.5611	0.5449
	(3.3097)	(6.9532)	(9.4404)	(16.8814)	(18.4632)	(18.7967)
adj. R^2	0.4962	0.6635	0.6860	0.7324	0.7350	0.7063
MSPE	2.0447*	2.2166**	2.2693**	2.2745**	2.2469**	2.2705
MAPE	0.3844**	0.4422**	0.4731	0.5082	0.5522	0.5906
QLIKE	2.2124	2.2763	2.3104	2.3409	2.3805	2.4167
			HAR-PS			
c	0.2433	0.3455	0.4101	0.4629	0.5884	0.7264
	(2.4254)	(5.3605)	(6.6700)	(9.1656)	(14.1364)	(18.1457)
RS^+	-0.1064	0.0529	0.0845	0.0591	0.0415	0.0423
	(-0.6988)	(0.7131)	(1.3450)	(1.2013)	(1.0483)	(1.1104)
RS^-	0.5064	0.1758	0.0883	0.0432	0.0421	0.0247
	(2.4143)	(1.9648)	(1.3526)	(0.9049)	(1.0493)	(0.6266)
$\mathbf{I}_{r_t<0}RV^{(1)}$	0.0048	0.0283	0.0390	0.0331	0.0197	0.0199
	(0.0820)	(0.8322)	(1.3918)	(1.3539)	(0.9317)	(0.9664)
$RV^{(5)}$	0.3602	0.2989	0.1835	0.0707	0.1249	0.1140
	(3.9271)	(4.8684)	(3.4983)	(1.4662)	(2.9657)	(2.9531)
$RV^{(22)}$	0.3180	0.4403	0.5640	0.7030	0.6174	0.5852
	(4.1339)	(7.8192)	(10.1556)	(17.5194)	(18.3943)	(18.6029)
adj. R^2	0.5133	0.6652	0.6857	0.7340	0.7384	0.7090
MSPE	2.0102*	2.2189**	2.2692**	2.2658**	2.2476**	2.2730
MAPE	0.3943	0.4414**	0.4693*	0.5027	0.5481**	0.5897
QLIKE	2.2139	2.2781	2.3104	2.3392	2.3795	2.4179
			LHAR-CJ			
c	0.1856	0.3467	0.4008	0.4576	0.5697	0.7263
	(1.3073)	(4.0550)	(5.5830)	(8.3306)	(12.1648)	(15.3816)
$J^{(1)}$	0.2166	0.1508	0.0872	0.0651	0.0519	0.0438
	(1.9988)	(2.0669)	(1.3315)	(1.2759)	(1.1907)	(1.0308)
$J^{(5)}$	0.0099	-0.0176	-0.0053	0.0082	0.0307	0.0139
	(0.1992)	(-0.4842)	(-0.1793)	(0.3995)	(1.6690)	(0.7467)
$J^{(22)}$	0.0169	0.0203	0.0226	0.0150	-0.0012	-0.0063
	(0.9346)	(1.6300)	(2.2346)	(2.1548)	(-0.1930)	(-0.8983)
$C^{(1)}$	0.1594	0.1159	0.0964	0.0519	0.0420	0.0383
	(2.3619)	(3.1688)	(3.3968)	(2.0253)	(1.9377)	(1.8212)
$C^{(5)}$	0.3479	0.2693	0.1514	0.0597	0.1069	0.0957
	(3.9310)	(4.5367)	(3.0911)	(1.3661)	(2.8613)	(2.7355)
$C^{(22)}$	0.2320	0.3633	0.4804	0.6096	0.5564	0.5412
	(3.1989)	(6.8181)	(9.3522)	(17.3820)	(19.0250)	(19.2053)
$r_t^{-,(1)}$	-0.4428	-0.1075	-0.0513	-0.0447	-0.0337	-0.0310
	(-2.8705)	(-1.7519)	(-1.2284)	(-1.4493)	(-1.1808)	(-1.1072)
$r_t^{-,(5)}$	0.0299	0.0207	0.0191	0.0170	0.0073	0.0062
	(2.9978)	(3.4430)	(3.8154)	(3.8587)	(1.7128)	(1.4511)
$r_t^{-,(22)}$	-0.2537	-0.5299	-0.7131	-0.6846	-0.4538	-0.3312
	(-0.8880)	(-3.1750)	(-4.7871)	(-4.8375)	(-3.7690)	(-2.8484)
adj. R^2	0.5257	0.6697	0.6934	0.7400	0.7388	0.7084
MSPE	1.9823**	2.2120**	2.2609**	2.2732**	2.2471**	2.2725
MAPE	0.3964	0.4407**	0.4691*	0.5035	0.5463**	0.5874*
QLIKE	2.1898	2.2732	2.3082	2.3388	2.3784	2.4154

Table A.20: In-Sample regression results for Sugar with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

h	1	5	10	22	44	66
Random Walk						
adj. R^2	0.1458	0.3011	0.2981	0.2871	0.2930	0.1565
MSPE	1.8343	1.9222	1.8944*	1.9511	1.9702	2.0873
MAPE	0.4884	0.5141*	0.5406**	0.5817**	0.6413	0.7083
QLIKE	2.0051**	2.1349**	2.1839**	2.2425**	2.3071**	2.3766**
AR(1)						
c	1.1644	0.9508	0.9558	0.9655	0.9522	1.1382
	(13.3339)	(13.2848)	(13.9387)	(17.3609)	(21.6686)	(24.0115)
$RV^{(h)}$	0.5727	0.6504	0.6488	0.6440	0.6482	0.5799
	(15.6066)	(21.3878)	(22.4941)	(27.3941)	(39.2914)	(34.5072)
adj. R^2	0.3272	0.4226	0.4207	0.4126	0.4147	0.3288
MSPE	1.6270	1.7989	1.8064**	1.8703**	1.9040	1.9834
MAPE	0.5126	0.5368	0.5633**	0.5917**	0.6356**	0.6827
QLIKE	2.2563	2.2837	2.3136	2.3469	2.3827	2.4369*
ARMA						
c	0.2380	0.3704	0.3991	0.5318	1.2074	0.7315
	(6.2091)	(6.1434)	(6.0785)	(7.4050)	(11.7525)	(8.2796)
$RV^{(h)}$	0.9130	0.8636	0.8529	0.8043	0.5526	0.7273
	(103.5431)	(51.9923)	(44.9880)	(37.3284)	(16.8485)	(25.0687)
$\varepsilon^{(h)}$	-0.5828	-0.4114	-0.3850	-0.2821	0.1677	-0.2205
	(39.4877)	(16.9878)	(14.3651)	(9.8830)	(4.8005)	(4.7750)
adj. R^2	0.3961	0.4459	0.4454	0.4346	0.4220	0.3417
MSPE	1.5423*	1.7628**	1.7958**	1.8598**	1.8959*	1.9491
MAPE	0.4690	0.5207	0.5521**	0.5887**	0.6317**	0.6779**
QLIKE	2.2026	2.2608	2.2967	2.3376	2.3782	2.4364*
HAR						
c	0.3800	0.5884	0.7444	0.9310	1.1321	1.2394
	(3.8705)	(9.5004)	(13.3812)	(17.7823)	(22.7647)	(27.8147)
$RV^{(1)}$	0.2827	0.2087	0.1382	0.0973	0.0637	0.0497
	(4.8478)	(6.1627)	(5.2184)	(4.0073)	(3.1120)	(2.7163)
$RV^{(5)}$	0.3480	0.2211	0.1731	0.1387	0.1039	0.0727
	(4.0167)	(4.8673)	(4.1675)	(3.1657)	(2.7953)	(2.2494)
$RV^{(22)}$	0.2301	0.3540	0.4148	0.4209	0.4167	0.4226
	(3.6690)	(8.8217)	(10.4405)	(11.2239)	(12.2207)	(14.6293)
adj. R^2	0.4003	0.4813	0.4632	0.4541	0.4251	0.4059
MSPE	1.5352*	1.7540**	1.8192**	1.8769	1.9145	1.9221**
MAPE	0.4544**	0.5115*	0.5523**	0.5895**	0.6348	0.6602**
QLIKE	2.1829	2.2471	2.2936	2.3382	2.3878	2.4152**
HAR-J						
c	0.3401	0.5628	0.7277	0.9170	1.1097	1.2120
	(3.5109)	(9.1421)	(13.0778)	(17.3859)	(22.3840)	(27.2433)
$RV^{(1)}$	0.2662	0.1981	0.1314	0.0916	0.0547	0.0388
	(4.6358)	(5.9174)	(4.9592)	(3.7057)	(2.5840)	(2.0477)
$RV^{(5)}$	0.3297	0.2094	0.1655	0.1324	0.0939	0.0609
	(3.8855)	(4.6771)	(4.0158)	(3.0560)	(2.5738)	(1.9175)
$RV^{(22)}$	0.2150	0.3443	0.4085	0.4159	0.4088	0.4133
	(3.4608)	(8.7278)	(10.4084)	(11.1602)	(12.1494)	(14.4943)
$J^{(1)}$	0.3383	0.2160	0.1403	0.1155	0.1834	0.2181
	(3.1893)	(2.9778)	(2.3342)	(2.1151)	(3.7526)	(5.1305)
adj. R^2	0.4083	0.4866	0.4658	0.4562	0.4321	0.4172
MSPE	1.5246**	1.7556**	1.8167**	1.8767	1.9047	1.9184**
MAPE	0.4537**	0.5100*	0.5506**	0.5921**	0.6288**	0.6620**
QLIKE	2.1809	2.2453	2.2918	2.3408	2.3826	2.4170**
HAR-CJ						
c	0.3703	0.5862	0.7489	0.9228	1.0914	1.1864
	(3.9144)	(9.9246)	(13.9807)	(18.8147)	(25.5116)	(29.2157)
$J^{(1)}$	0.2363	0.1947	0.1099	0.0808	0.0526	0.0474
	(2.7220)	(3.2154)	(2.0534)	(1.5856)	(1.2669)	(1.3030)
$J^{(5)}$	0.0328	-0.0057	0.0169	-0.0193	-0.0103	0.0025
	(0.8874)	(-0.2003)	(0.6493)	(-0.8138)	(-0.5818)	(0.1516)
$J^{(22)}$	0.0012	0.0150	0.0183	0.0440	0.0749	0.0729
	(0.1116)	(1.8163)	(2.3217)	(7.0275)	(13.9339)	(15.2208)
$C^{(1)}$	0.2633	0.2094	0.1368	0.0967	0.0615	0.0469
	(5.0116)	(6.4378)	(5.3414)	(4.3533)	(3.5283)	(2.9995)
$C^{(5)}$	0.3275	0.1963	0.1510	0.1138	0.0902	0.0623
	(4.1869)	(4.7345)	(4.1075)	(3.1349)	(3.1671)	(2.5274)
$C^{(22)}$	0.1638	0.2683	0.3213	0.3048	0.2404	0.2489
	(2.8680)	(7.9676)	(9.9496)	(10.1228)	(9.5104)	(11.1179)
adj. R^2	0.4117	0.4898	0.4628	0.4609	0.4879	0.4817
MSPE	1.5193**	1.7563**	1.8200**	1.8551**	1.8671**	1.9015**
MAPE	0.4509**	0.5128*	0.5554**	0.5907**	0.6240**	0.6638**
QLIKE	2.1782	2.2474	2.2965	2.3394	2.3740	2.4146**
HAR-PS						
c	0.3596	0.5721	0.7338	0.9243	1.1240	1.2306
	(3.8122)	(9.4457)	(13.3219)	(17.7089)	(22.8680)	(27.8617)
RS^{+}	0.3555	0.2207	0.1416	0.0889	0.0613	0.0429
	(4.2014)	(5.8919)	(4.4089)	(2.5179)	(1.9288)	(1.5568)
RS^{-}	0.2155	0.2199	0.1473	0.1067	0.0976	0.1012
	(2.4236)	(3.1108)	(2.5105)	(2.1896)	(2.6261)	(2.9482)
$\mathbf{I}_{r_t<0}RV^{(1)}$	-0.0546	-0.0404	-0.0275	-0.0234	-0.0173	-0.0189
	(-1.6081)	(-1.3697)	(-1.1050)	(-1.0840)	(-0.9558)	(-1.1488)
$RV^{(5)}$	0.3468	0.2107	0.1683	0.1420	0.0903	0.0541
	(4.1250)	(4.7452)	(4.1135)	(3.3463)	(2.5548)	(1.7314)
$RV^{(22)}$	0.2108	0.3373	0.4036	0.4126	0.4107	0.4171
	(3.4318)	(8.6355)	(10.3634)	(11.1486)	(12.2474)	(14.6525)
adj. R^2	0.4162	0.4947	0.4694	0.4564	0.4293	0.4109
MSPE	1.5139**	1.7490**	1.8166**	1.8770	1.9096	1.9217**
MAPE	0.4530**	0.5072**	0.5520**	0.5917**	0.6316	0.6605**
QLIKE	2.1791	2.2417	2.2920	2.3400	2.3851	2.4152**
LHAR-CJ						
c	0.3242	0.5197	0.6885	0.8768	1.0349	1.1253
	(3.0446)	(7.8073)	(12.0528)	(17.1058)	(23.4343)	(26.7325)
$J^{(1)}$	0.2410	0.1973	0.1134	0.0817	0.0516	0.0454
	(2.7987)	(3.3254)	(2.1684)	(1.6286)	(1.2596)	(1.2577)
$J^{(5)}$	0.0294	-0.0113	0.0121	-0.0224	-0.0129	0.0004
	(0.8059)	(-0.4090)	(0.4706)	(-0.9464)	(-0.7289)	(0.0242)
$J^{(22)}$	0.0011	0.0151	0.0183	0.0441	0.0751	0.0731
	(0.1063)	(1.8292)	(2.3343)	(7.0944)	(14.0932)	(15.3497)
$C^{(1)}$	0.2641	0.2091	0.1370	0.0962	0.0603	0.0454
	(5.0194)	(6.3344)	(5.3122)	(4.3382)	(3.4585)	(2.9260)
$C^{(5)}$	0.3211	0.1867	0.1425	0.1084	0.0860	0.0591
	(4.1113)	(4.4831)	(3.9379)	(3.0132)	(3.0313)	(2.4089)
$C^{(22)}$	0.1740	0.2795	0.3326	0.3127	0.2501	0.2598
	(2.9965)	(8.1749)	(10.3343)	(10.4124)	(9.9649)	(11.6146)
$r_t^{-,(1)}$	0.0244	0.0084	0.0156	0.0015	-0.0081	-0.0119
	(0.5562)	(0.2809)	(0.5765)	(0.0604)	(-0.4025)	(-0.5987)
$r_t^{-,(5)}$	0.0003	0.0081	0.0046	0.0009	-0.0075	-0.0138
	(0.0317)	(1.0493)	(0.6990)	(0.1450)	(-1.2867)	(-2.3160)
$r_t^{-,(22)}$	-0.3590	-0.4872	-0.4466	-0.2985	-0.2966	-0.2938
	(-1.4935)	(-3.1034)	(-3.4979)	(-2.7605)	(-3.5029)	(-3.7638)
adj. R^2	0.4124	0.4937	0.4666	0.4626	0.4907	0.4861
MSPE	1.5173**	1.7518**	1.8189**	1.8549**	1.8640**	1.8995**
MAPE	0.4516**	0.5112*	0.5530**	0.5892**	0.6224**	0.6624**
QLIKE	2.1782	2.2457	2.2943	2.3387	2.3720	2.4129**

Table A.21: In-Sample regression results for Wheat with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. The t-statistics for the parameter estimates are given in parentheses.

Appendix A.2. Out-of-Sample

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	0.9890	0.8956	0.9527	0.8414**	0.8420**	0.8437**	0.8479**	0.8627*
	5	1.0020*	0.9695**	1.0371	0.9588**	0.9580**	0.9630**	0.9601**	0.9686**
	10	1.0453*	1.0148**	1.0752	1.0207**	1.0223**	1.0294**	1.0280*	1.0405
	22	1.1325	1.0820	1.1263	1.0669**	1.0684**	1.0607**	1.0678**	1.0748
	44	1.2243	1.1221**	1.1184**	1.1124**	1.1122**	1.0982**	1.1089**	1.1077**
	66	1.2886	1.1573	1.2960	1.1263	1.1283	1.1126**	1.1260	1.1240
MAPE	1	0.5958	0.6332	0.5920**	0.5810**	0.5808**	0.5790**	0.5799**	0.5920
	5	0.6396**	0.6650	0.6647*	0.6471**	0.6468**	0.6456**	0.6458**	0.6464**
	10	0.6451**	0.6740	0.6829	0.6744	0.6763	0.6775	0.6756	0.6797
	22	0.6834**	0.7147	0.7430	0.7043*	0.7051	0.6973*	0.7026*	0.7055
	44	0.7443**	0.7644	0.7811	0.7506	0.7514	0.7366**	0.7476	0.7455
	66	0.7860	0.7945	0.8687	0.7696	0.7711	0.7563**	0.7682	0.7666
QLIKE	1	1.5628	1.5390	1.5492	1.5277**	1.5277**	1.5280**	1.5288**	1.5300*
	5	1.5664	1.5559	1.5702	1.5519**	1.5516**	1.5523**	1.5515**	1.5521**
	10	1.5709*	1.5636**	1.5748*	1.5660**	1.5662**	1.5691*	1.5675*	1.5704*
	22	1.5979	1.5828*	1.5954	1.5790**	1.5792**	1.5782**	1.5792**	1.5817*
	44	1.6363	1.5976*	1.5865**	1.5937**	1.5935**	1.5910**	1.5928**	1.5935**
	66	1.6662	1.5976*	1.6295	1.5970	1.5973	1.5935**	1.5965*	1.5970
SR	1	0.7436***	0.7237***	0.7465***	0.7446***	0.7417***	0.7446***	0.7427***	0.7370***
	5	0.6660***	0.6518***	0.6566***	0.6689***	0.6689***	0.6736***	0.6717***	0.6698***
	10	0.6679***	0.6537***	0.6462***	0.6462***	0.6462***	0.6339***	0.6471***	0.6272***
	22	0.6121***	0.5960***	0.5525**	0.6216***	0.6235***	0.6301***	0.6244***	0.6083***
	44	0.4768	0.4910	0.6026***	0.5412	0.5412	0.5572***	0.5468*	0.5241
	66	0.4049	0.5449	0.4617	0.5203	0.5270	0.5374	0.5222	0.5071

Table A.22: Forecasting Evaluation for Corn Futures with *MedRV*. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.7872	1.5022	1.7734	1.4042**	1.4008**	1.3951**	1.3971**	1.3987**
	5	1.5206	1.4697	1.5164	1.4366**	1.4371**	1.4319**	1.4357**	1.4320**
	10	1.4838*	1.4601**	1.5317	1.4497**	1.4521**	1.4510**	1.4505**	1.4508**
	22	1.4654**	1.4655**	1.4404**	1.4700**	1.4675**	1.4693**	1.4677**	1.4674**
	44	1.4340**	1.4448**	1.4916	1.4584**	1.4570**	1.4578**	1.4560**	1.4597**
	66	1.4682**	1.4938	1.5501	1.5012	1.4988	1.5020	1.4989	1.4917*
MAPE	1	1.1032**	1.2800	1.1056**	1.1484	1.1368**	1.1403**	1.1340**	1.1476**
	5	1.2344	1.2811	1.1979**	1.2091	1.2045**	1.2046	1.1893**	1.2155
	10	1.1429**	1.2041	1.2064	1.1988	1.1977	1.2031	1.1979	1.2203
	22	1.1650*	1.2438	1.1372**	1.2448	1.2435	1.2528	1.2388	1.2836
	44	1.1428**	1.1838	1.1342**	1.2380	1.2379	1.2366	1.2368	1.2709
	66	1.1558**	1.1803	1.1804*	1.2648	1.2641	1.2485	1.2632	1.3121
QLIKE	1	1.8923	1.3430	1.6712	1.2643*	1.2611**	1.2567**	1.2609**	1.2571**
	5	1.3534	1.3250	1.3432	1.2931	1.2921	1.2851**	1.2931	1.2847**
	10	1.3306*	1.3096*	1.3594	1.3001**	1.3002**	1.2959**	1.3000**	1.2932**
	22	1.3189**	1.3169**	1.3068**	1.3200**	1.3184**	1.3140**	1.3194**	1.3074**
	44	1.2950**	1.3037*	1.3722	1.3208	1.3209	1.3149*	1.3202	1.3086*
	66	1.3301**	1.3534	1.4147	1.3454*	1.3446*	1.3504	1.3441*	1.3262**
SR	1	0.6907***	0.6129***	0.6879***	0.7068***	0.7125***	0.7059***	0.7125***	0.7135***
	5	0.6898***	0.6433***	0.6803***	0.6784***	0.6765***	0.6736***	0.6755***	0.6784***
	10	0.6717***	0.6565***	0.6499***	0.6537***	0.6556***	0.6556***	0.6594***	0.6670***
	22	0.7040***	0.6347***	0.6879***	0.6376***	0.6433***	0.6357***	0.6490***	0.6546***
	44	0.6869***	0.6784***	0.6822***	0.6319***	0.6395***	0.6518***	0.6461***	0.6698***
	66	0.6793***	0.6879***	0.6452***	0.6129***	0.6186***	0.6471***	0.6243***	0.6641***

Table A.23: Forecasting Evaluation for Rice Futures with *MedRV*. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	0.8109	0.7237	0.7851*	0.6720**	0.6725**	0.6668**	0.6715**	0.6705**
	5	0.7649	0.7400	0.7888	0.7331**	0.7333**	0.7316**	0.7376	0.7377
	10	0.8182	0.7865**	0.8119	0.7816**	0.7821**	0.7823**	0.7822**	0.7869
	22	0.8929	0.8516	0.9107	0.8301**	0.8311*	0.8265**	0.8299**	0.8354
	44	0.9914	0.8973	0.8888**	0.8840**	0.8843**	0.8899	0.8842**	0.9009
	66	1.0476	0.8969**	0.9002**	0.8998**	0.9002**	0.9049**	0.9003**	0.9212
MAPE	1	0.6117	0.6559	0.6081	0.5977	0.5982	0.5922**	0.5961*	0.5984
	5	0.6288**	0.6624	0.6684	0.6511	0.6517	0.6494	0.6530	0.6583
	10	0.6621**	0.6968	0.7215	0.6891	0.6897	0.6868	0.6900	0.6912
	22	0.7293**	0.7618	0.8061	0.7441*	0.7446*	0.7396**	0.7438*	0.7501
	44	0.8140**	0.8207	0.8422	0.8012**	0.8014**	0.8000**	0.8011**	0.8135
	66	0.8676	0.8360	0.8194**	0.8214**	0.8213**	0.8226**	0.8209**	0.8370
QLIKE	1	1.1557	1.1304	1.1413	1.1140**	1.1142**	1.1134**	1.1139**	1.1153*
	5	1.1417	1.1362	1.1545	1.1330**	1.1332**	1.1327**	1.1346	1.1350
	10	1.1651	1.1545**	1.1723	1.1548**	1.1550**	1.1562**	1.1552**	1.1584
	22	1.2243	1.1897	1.2182	1.1791**	1.1794**	1.1791**	1.1789**	1.1828
	44	1.2845	1.2156	1.2068**	1.2077**	1.2078**	1.2118	1.2078**	1.2173
	66	1.3268	1.2234**	1.2157**	1.2179**	1.2180**	1.2195**	1.2181**	1.2297
SR	1	0.7881***	0.7806***	0.7872***	0.7957***	0.7947***	0.7853***	0.7947***	0.7947***
	5	0.7542***	0.7495***	0.7307***	0.7533***	0.7552***	0.7486***	0.7514***	0.7552***
	10	0.7298***	0.7203***	0.6883***	0.7401***	0.7354***	0.7147***	0.7269***	0.7119***
	22	0.6422***	0.6403***	0.5753***	0.6723***	0.6685***	0.6450***	0.6676***	0.6412***
	44	0.5009	0.4849	0.5348*	0.5480**	0.5480**	0.5414*	0.5471**	0.5311
	66	0.4162	0.3889	0.4595	0.4576	0.4595	0.4595	0.4557	0.4407

Table A.24: Forecasting Evaluation for Soy Futures with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.3511	1.2559	1.3330	1.0899**	1.0989**	1.0898**	1.0916**	1.1039
	5	1.2470	1.2311	1.2525	1.1789**	1.1812*	1.1810**	1.1842*	1.1862*
	10	1.2169**	1.2308**	1.2933	1.2125**	1.2131**	1.2173**	1.2134**	1.2210**
	22	1.2859**	1.2813**	1.3286**	1.2850**	1.2859**	1.2949**	1.2869**	1.2974**
	44	1.3138**	1.3412**	1.3569**	1.3317**	1.3318**	1.3415**	1.3321**	1.3413**
	66	1.3051**	1.3783	1.3771	1.4049	1.4046	1.3998	1.4049	1.3830
MAPE	1	0.6343	0.7299	0.6368	0.6109	0.6001**	0.5985**	0.6033**	0.6103*
	5	0.6452**	0.6971	0.6473**	0.6539*	0.6515**	0.6489**	0.6507**	0.6465**
	10	0.6442**	0.7095	0.6666*	0.6748	0.6733	0.6691	0.6722	0.6698
	22	0.6793**	0.7000*	0.6962*	0.6965*	0.6956*	0.6953*	0.6950*	0.7029*
	44	0.7029**	0.7003**	0.7047**	0.7218	0.7217	0.7297	0.7197	0.7338
	66	0.6915**	0.7091	0.7082*	0.7471	0.7480	0.7517	0.7464	0.7473
QLIKE	1	1.8012	1.8068	1.7944	1.7603**	1.7602**	1.7597**	1.7608**	1.9152
	5	1.7956*	1.7961	1.7904*	1.7820**	1.7825**	1.7810**	1.7826**	1.7912**
	10	1.7877**	1.7973**	1.8131	1.7916**	1.7917**	1.7913**	1.7917**	1.8025*
	22	1.8198**	1.8198**	1.8623	1.8182**	1.8186**	1.8191**	1.8185**	1.8246**
	44	1.8480**	1.8518**	1.8567**	1.8526**	1.8524**	1.8474**	1.8525**	1.8495**
	66	1.8327**	1.8595	1.8713	1.8714	1.8709	1.8596	1.8715	1.8601
SR	1	0.7502***	0.7352***	0.7474***	0.7728***	0.7690***	0.7596***	0.7634***	0.7559***
	5	0.7324***	0.7192***	0.7362***	0.7352***	0.7333***	0.7333***	0.7352***	0.7305***
	10	0.7315***	0.7296***	0.7023***	0.7324***	0.7324***	0.7268***	0.7408***	0.7305***
	22	0.7023***	0.7155***	0.6977***	0.7183***	0.7155***	0.7014***	0.7183***	0.7042***
	44	0.7080***	0.6967***	0.7258***	0.7005***	0.6948***	0.6751***	0.6958***	0.6779***
	66	0.7502***	0.6770***	0.7042***	0.6545***	0.6563***	0.6460***	0.6563***	0.6657***

Table A.25: Forecasting Evaluation for Sugar Futures with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.2218	1.1210	1.1825	1.0307**	1.0374**	1.0375**	1.0266**	1.0458*
	5	1.2676	1.2228	1.2854	1.1765**	1.1779**	1.1837**	1.1734**	1.1867**
	10	1.2522**	1.2286**	1.2633**	1.2147**	1.2149**	1.2175**	1.2165**	1.2125**
	22	1.2550**	1.2642**	1.3113*	1.2626**	1.2657**	1.2682**	1.2676**	1.2699**
	44	1.3406**	1.3420**	1.4302	1.3330**	1.3293**	1.3231**	1.3308**	1.3239**
	66	1.3954**	1.4170*	1.6375	1.3967**	1.3997**	1.3800**	1.3989**	1.3910**
MAPE	1	0.6369	0.6978	0.6358	0.6224*	0.6277	0.6223*	0.6179**	0.6402
	5	0.6634**	0.7119	0.6851**	0.6774**	0.6796	0.6759**	0.6747**	0.6930
	10	0.6591**	0.7088	0.6910	0.7007	0.7022	0.6981	0.7007	0.7150
	22	0.6783**	0.7316	0.7292	0.7284	0.7294	0.7250	0.7288	0.7458
	44	0.7234**	0.7634	0.8111	0.7713	0.7703	0.7669	0.7701	0.7905
	66	0.7519**	0.8104	0.8905	0.8074	0.8085	0.8050	0.8079	0.8320
QLIKE	1	1.7837	1.7570	1.7640	1.7348**	1.7354**	1.7351**	1.7342**	1.7384
	5	1.7724	1.7693	1.7754	1.7584**	1.7589**	1.7590**	1.7574**	1.7608**
	10	1.7701**	1.7710**	1.7771**	1.7689**	1.7690**	1.7687**	1.7691**	1.7683**
	22	1.7832**	1.7846**	1.8111	1.7828**	1.7833**	1.7834**	1.7838**	1.7821**
	44	1.8265**	1.8164*	1.8283	1.8069**	1.8060**	1.8040**	1.8064**	1.7988**
	66	1.8418**	1.8247**	1.8784	1.8214**	1.8220**	1.8178**	1.8220**	1.8153**
SR	1	0.7030***	0.6635***	0.7002***	0.7049***	0.7039***	0.7068***	0.7115***	0.7105***
	5	0.6776***	0.6278***	0.6645***	0.6795***	0.6701***	0.6776***	0.6776***	0.6776***
	10	0.6692***	0.6118***	0.6382***	0.6410***	0.6466***	0.6400***	0.6429***	0.6598***
	22	0.6560***	0.5977***	0.5686***	0.6071***	0.6081***	0.6071***	0.6081***	0.6241***
	44	0.5987***	0.5028	0.4737	0.5244**	0.5291**	0.5320**	0.5226**	0.5479***
	66	0.5451***	0.4314	0.4173	0.4549	0.4549	0.4436	0.4455	0.4746

Table A.26: Forecasting Evaluation for Wheat Futures with $MedRV$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

Appendix B. Additional Out-of-Sample Results

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.3038	1.1508	1.2400	1.0898**	1.0901**	1.0890**	1.0992*	1.0993**
	5	0.7970**	0.7839*	0.9020	0.7629**	0.7638**	0.7606**	0.7680**	0.7580**
	10	0.7694*	0.7501	0.8402	0.7252*	0.7246*	0.7164**	0.7325*	0.7122**
	22	0.7619*	0.7307	0.8581	0.6849**	0.6841**	0.6824**	0.6883**	0.6849**
	44	0.8185	0.7428	0.7154	0.6786**	0.6785**	0.6762**	0.6796**	0.6943
	66	0.8722	0.7726	0.7752	0.6547**	0.6546**	0.6471**	0.6550**	0.6642
MAPE	1	0.6093	0.5899	0.5988	0.5516**	0.5515**	0.5509**	0.5550	0.5570*
	5	0.5090	0.5008	0.5307	0.4809**	0.4808**	0.4778**	0.4816**	0.4782**
	10	0.4858	0.4695	0.4877	0.4592*	0.4593*	0.4544**	0.4604*	0.4555**
	22	0.4697	0.4567	0.5277	0.4445*	0.4451*	0.4410**	0.4449*	0.4454*
	44	0.5347	0.5066	0.5215	0.4714**	0.4723	0.4753	0.4722	0.4876
	66	0.5778	0.5411	0.5547	0.4816**	0.4821**	0.4783**	0.4822**	0.4926
QLIKE	1	1.6887	1.6600	1.6798	1.6517**	1.6519**	1.6520**	1.6535	1.6529**
	5	1.6777	1.6686**	1.6962	1.6653**	1.6654**	1.6656**	1.6656**	1.6662**
	10	1.6894	1.6812*	1.6967	1.6778**	1.6777**	1.6769**	1.6783**	1.6774**
	22	1.7045	1.6967	1.7232	1.6902**	1.6901**	1.6902**	1.6907**	1.6914**
	44	1.7444	1.7214	1.7121**	1.7094**	1.7093**	1.7084**	1.7096**	1.7119
	66	1.7787	1.7339	1.7300	1.7197**	1.7195**	1.7177**	1.7197**	1.7208
SR	1	0.7606***	0.7389***	0.7597***	0.7597***	0.7578***	0.7635***	0.7540***	0.7588***
	5	0.7313***	0.7275***	0.6850***	0.7408***	0.7389***	0.7351***	0.7398***	0.7294***
	10	0.7275***	0.7417***	0.7502***	0.7588***	0.7597***	0.7550***	0.7625***	0.7550***
	22	0.7351***	0.7342***	0.5695***	0.7597***	0.7559***	0.7521***	0.7578***	0.7446***
	44	0.5601***	0.5440***	0.5412***	0.6878***	0.6840***	0.6698***	0.6916***	0.6291***
	66	0.4295	0.5629***	0.5317**	0.6339***	0.6216***	0.6254***	0.6310***	0.5781***

Table B.27: Forecasting Evaluation for Corn Futures with average $RV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	2.4100	2.1404	2.3635	1.9934**	1.9968**	1.9967**	1.9943**	1.9838**
	5	1.3149**	1.3366	1.3595	1.2690**	1.2714**	1.2791**	1.2710**	1.2486**
	10	1.0925**	1.1439*	1.2533	1.1146**	1.1180**	1.1284**	1.1168**	1.0867**
	22	0.9978**	1.0516	1.0265**	1.0368*	1.0392	1.0564	1.0379*	0.9993**
	44	0.8255**	0.8944	1.0277	0.9593	0.9616	0.9922	0.9598	0.9241
	66	0.8404**	0.9563	1.1987	0.9673	0.9706	1.0023	0.9676	0.9292
MAPE	1	0.8550	0.7907	0.8478	0.7343**	0.7327**	0.7354**	0.7359*	0.7355**
	5	0.5819	0.5692	0.5996	0.5521**	0.5518**	0.5577*	0.5519**	0.5549**
	10	0.5350**	0.5331**	0.5662	0.5245**	0.5256**	0.5356	0.5249**	0.5279**
	22	0.5086**	0.5233*	0.5598	0.5185**	0.5193**	0.5285	0.5189**	0.5220**
	44	0.4980**	0.5183	0.5574	0.5251	0.5259	0.5379	0.5254	0.5190
	66	0.5050**	0.5507	0.6087	0.5306	0.5307	0.5398	0.5308	0.5179*
QLIKE	1	2.0261	1.9471	2.0171	1.8804**	1.8811**	1.8829**	1.8807**	1.8781**
	5	1.9173	1.9147	1.9352	1.8983**	1.8988**	1.9014**	1.8990**	1.8953**
	10	1.9156*	1.9163	1.9574	1.9087*	1.9095*	1.9118*	1.9093*	1.9017**
	22	1.9237*	1.9260	1.9456	1.9230*	1.9239	1.9269	1.9234*	1.9121**
	44	1.9223**	1.9329	1.9851	1.9319	1.9330	1.9382	1.9321	1.9203**
	66	1.9383*	1.9743	2.0416	1.9459	1.9473	1.9525	1.9460	1.9281**
SR	1	0.7011***	0.6556***	0.7030***	0.7353***	0.7268***	0.7353***	0.7315***	0.7334***
	5	0.7875***	0.7562***	0.7581***	0.7932***	0.7922***	0.7960***	0.7922***	0.7941***
	10	0.8121***	0.7913***	0.7770***	0.8008***	0.8027***	0.8083***	0.8036***	0.8065***
	22	0.8387***	0.7913***	0.7998***	0.7913***	0.7913***	0.7998***	0.7884***	0.8121***
	44	0.8681***	0.8406***	0.8454***	0.8008***	0.8027***	0.8093***	0.8036***	0.8207***
	66	0.8548***	0.7884***	0.7751***	0.7875***	0.7875***	0.7979***	0.7894***	0.7913***

Table B.28: Forecasting Evaluation for Rice Futures with average $RV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	0.9453	0.8394	0.9104	0.7848**	0.7833**	0.7841**	0.7857**	0.7843**
	5	0.5499*	0.5378*	0.5982	0.5281**	0.5262**	0.5280**	0.5280**	0.5319**
	10	0.5268	0.5202	0.5959	0.4986*	0.4962**	0.5039*	0.4989*	0.5116
	22	0.5610	0.5616	0.6847	0.5125**	0.5122**	0.5214	0.5129**	0.5344
	44	0.6527	0.6265	0.6115	0.5528**	0.5529**	0.5618	0.5527**	0.5752
	66	0.6976	0.6006	0.5689**	0.5492**	0.5496**	0.5548**	0.5494**	0.5740
MAPE	1	0.6157	0.6015	0.6076	0.5549*	0.5531**	0.5554*	0.5550*	0.5581*
	5	0.4914	0.4803	0.5087	0.4725**	0.4713**	0.4742*	0.4726**	0.4771
	10	0.4801**	0.4651**	0.5007	0.4639**	0.4633**	0.4668**	0.4642**	0.4684**
	22	0.4911	0.4903	0.5765	0.4650**	0.4655**	0.4701	0.4664	0.4768
	44	0.5549	0.5490	0.5650	0.5069**	0.5073**	0.5097**	0.5071**	0.5177
	66	0.5905	0.5476	0.5400	0.5073**	0.5076**	0.5041**	0.5076**	0.5176
QLIKE	1	1.3287	1.2983	1.3209	1.2841**	1.2837**	1.2834**	1.2839**	1.2834**
	5	1.3014	1.2939**	1.3119	1.2924**	1.2921**	1.2925**	1.2923**	1.2932**
	10	1.3165	1.3082*	1.3302	1.3045**	1.3042**	1.3055*	1.3045**	1.3074
	22	1.3531	1.3401	1.3793	1.3271**	1.3271**	1.3287	1.3271**	1.3323
	44	1.4162	1.3846	1.3778	1.3603**	1.3604**	1.3623**	1.3603**	1.3669
	66	1.4569	1.4004	1.3901**	1.3811**	1.3812**	1.3818**	1.3811**	1.3889
SR	1	0.7787***	0.7495***	0.7768***	0.7910***	0.7900***	0.7834***	0.7863***	0.7900***
	5	0.8117***	0.8145***	0.7768***	0.8145***	0.8136***	0.8136***	0.8107***	0.8098***
	10	0.8117***	0.7994***	0.7250***	0.8173***	0.8154***	0.8126***	0.8136***	0.7957***
	22	0.7363***	0.6770***	0.5753***	0.7316***	0.7279***	0.7373***	0.7316***	0.7326***
	44	0.6186***	0.4849	0.5104	0.6215***	0.6177***	0.6384***	0.6168***	0.6347***
	66	0.4718	0.3484	0.4746	0.5254*	0.5226	0.5678***	0.5226*	0.5104

Table B.29: Forecasting Evaluation for Soy Futures with average $RV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.6238	1.4626	1.5919	1.3174**	1.3168**	1.3167**	1.3188**	1.3190**
	5	0.9428	0.9402	1.0376	0.8756**	0.8733**	0.8722**	0.8734**	0.8797**
	10	0.8091**	0.8237**	0.9256	0.8039**	0.8016**	0.7969**	0.8016**	0.8048**
	22	0.7633**	0.8125	0.9445	0.7857**	0.7818**	0.7768**	0.7841**	0.7877**
	44	0.7468**	0.9151	0.9183	0.8476	0.8427	0.8356	0.8478	0.8646
	66	0.7331**	0.9530	0.8933	0.9321	0.9267	0.9092	0.9321	0.9240
MAPE	1	0.6537	0.6265	0.6480	0.5741**	0.5748**	0.5763*	0.5744**	0.5784*
	5	0.5125	0.5055	0.5425	0.4838**	0.4839**	0.4857**	0.4840**	0.4882**
	10	0.4810**	0.4767**	0.5185	0.4682**	0.4688**	0.4719**	0.4675**	0.4687**
	22	0.4867**	0.4893	0.5345	0.4776**	0.4772**	0.4836**	0.4764**	0.4819**
	44	0.4757**	0.5121	0.5278	0.5010	0.5000	0.5093	0.5003	0.5107
	66	0.4635**	0.5060	0.5241	0.5134	0.5122	0.5180	0.5131	0.5181
QLIKE	1	1.9622	1.9459	1.9591	1.9182**	1.9179**	1.9177**	1.9189**	1.9180**
	5	1.9441	1.9398	1.9603	1.9325**	1.9321**	1.9314**	1.9321**	1.9322**
	10	1.9439**	1.9418**	1.9672	1.9424**	1.9420**	1.9403**	1.9421**	1.9407**
	22	1.9586**	1.9638	2.0159	1.9588*	1.9581*	1.9550**	1.9587*	1.9543**
	44	1.9884**	2.0186	2.0308	1.9908	1.9896	1.9826**	1.9909	1.9830**
	66	1.9919**	2.0413	2.0135	2.0190	2.0176	2.0062*	2.0193	2.0068*
SR	1	0.7333***	0.7352***	0.7362***	0.7662***	0.7624***	0.7681***	0.7624***	0.7718***
	5	0.7822***	0.7784***	0.7793***	0.8038***	0.8103***	0.8047***	0.8150***	0.8103***
	10	0.8282***	0.8319***	0.7953***	0.8291***	0.8300***	0.8169***	0.8300***	0.8169***
	22	0.8188***	0.8244***	0.8028***	0.8357***	0.8254***	0.7962***	0.8376***	0.8254***
	44	0.9296***	0.8451***	0.8648***	0.8761***	0.8601***	0.8441***	0.8667***	0.8648***
	66	0.9408***	0.8141***	0.8197***	0.8225***	0.8216***	0.8225***	0.8310***	0.8319***

Table B.30: Forecasting Evaluation for Sugar Futures with average $RV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.4871	1.3285	1.4444	1.2476**	1.2497**	1.2575*	1.2418**	1.2579*
	5	0.9886*	0.9328*	1.1257	0.8978**	0.8980**	0.9099**	0.8939**	0.9055**
	10	0.8988*	0.8550**	0.9496	0.8192**	0.8198**	0.8288**	0.8182**	0.8156**
	22	0.7592**	0.7657*	0.9012	0.7411**	0.7422**	0.7546**	0.7422**	0.7393**
	44	0.7845**	0.8259	0.9596	0.7539**	0.7557**	0.7676**	0.7547**	0.7531**
	66	0.8367**	0.9112	1.1841	0.7922**	0.7928**	0.8024**	0.7929**	0.7813**
MAPE	1	0.6271	0.6143	0.6204	0.5698**	0.5721	0.5689**	0.5682**	0.5744
	5	0.5057	0.4923	0.5262	0.4726**	0.4729**	0.4742**	0.4735**	0.4779**
	10	0.4655**	0.4551**	0.4824	0.4498**	0.4503**	0.4515**	0.4504**	0.4537**
	22	0.4521*	0.4504	0.5030	0.4409**	0.4415*	0.4458*	0.4411**	0.4459*
	44	0.4675**	0.4843	0.5609	0.4672**	0.4676**	0.4710**	0.4676**	0.4687**
	66	0.4927**	0.5402	0.6319	0.4986**	0.4983**	0.5004*	0.4990*	0.5004**
QLIKE	1	1.9467	1.9167	1.9393	1.9018**	1.9018**	1.9032*	1.9017**	1.9029**
	5	1.9262	1.9188	1.9443	1.9134**	1.9133**	1.9153	1.9134**	1.9139**
	10	1.9304	1.9252**	1.9385	1.9222**	1.9221**	1.9244*	1.9222**	1.9217**
	22	1.9358	1.9350	1.9697	1.9316**	1.9317**	1.9336	1.9318**	1.9297**
	44	1.9672	1.9719	1.9942	1.9537*	1.9538*	1.9554	1.9538*	1.9493**
	66	1.9905	1.9934	2.0376	1.9705*	1.9703*	1.9708	1.9706	1.9648**
SR	1	0.7124***	0.6805***	0.7143***	0.7105***	0.7077***	0.7049***	0.7180***	0.7068***
	5	0.7011***	0.6842***	0.6814***	0.7359***	0.7312***	0.7246***	0.7368***	0.7284***
	10	0.8083***	0.7735***	0.7556***	0.8045***	0.8083***	0.7989***	0.8036***	0.8017***
	22	0.8336***	0.7679***	0.7068***	0.7876***	0.7857***	0.8139***	0.7838***	0.8299***
	44	0.7368***	0.6241***	0.4154	0.6842***	0.6917***	0.7058***	0.6852***	0.7378***
	66	0.6457***	0.4352	0.4126	0.5677***	0.5714***	0.5846***	0.5714***	0.6156***

Table B.31: Forecasting Evaluation for Wheat Futures with average $RV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	0.9890	0.8956	0.9527	0.8414**	0.8420**	0.8437**	0.8479**	0.8627**
	5	0.6689	0.6382	0.7405	0.6127**	0.6114**	0.6196**	0.6127**	0.6292*
	10	0.6323*	0.6115*	0.7012	0.5904**	0.5906**	0.6008**	0.5948**	0.6139
	22	0.6381	0.6140	0.7300	0.5721**	0.5737**	0.5767**	0.5757**	0.5924
	44	0.6936	0.6521	0.7100	0.5903**	0.5916**	0.5790**	0.5909**	0.6061
	66	0.7470	0.7475	0.8598	0.6111	0.6117	0.5895**	0.6100	0.6062
MAPE	1	0.5958	0.6332	0.5920**	0.5810**	0.5808**	0.5790**	0.5799**	0.5920
	5	0.5204**	0.5363	0.5498	0.5115**	0.5112**	0.5119**	0.5101**	0.5125**
	10	0.4905**	0.5096*	0.5213*	0.4977**	0.4980**	0.5015**	0.4970**	0.5045**
	22	0.4827**	0.5140	0.5766	0.4984**	0.4992**	0.4963**	0.4983**	0.5053
	44	0.5486**	0.5674	0.5955	0.5324**	0.5327**	0.5250**	0.5316**	0.5421
	66	0.5882	0.6118	0.6610	0.5504	0.5509	0.5408**	0.5490	0.5509
QLIKE	1	1.5628	1.5390	1.5492	1.5277**	1.5277**	1.5280**	1.5288**	1.5300*
	5	1.5534	1.5466	1.5624	1.5415**	1.5413**	1.5423**	1.5414**	1.5426**
	10	1.5590*	1.5551*	1.5637	1.5519**	1.5518**	1.5534**	1.5520**	1.5541*
	22	1.5712	1.5683	1.5881	1.5633**	1.5634**	1.5641**	1.5636**	1.5664
	44	1.6037	1.5935	1.5954	1.5825**	1.5825**	1.5814**	1.5825**	1.5855
	66	1.6304	1.6141	1.6252	1.5963	1.5963	1.5941**	1.5961	1.5966
SR	1	0.7436***	0.7237***	0.7465***	0.7446***	0.7417***	0.7446***	0.7427***	0.7370***
	5	0.7171***	0.7086***	0.6944***	0.7408***	0.7389***	0.7455***	0.7360***	0.7455***
	10	0.7465***	0.7398***	0.7247***	0.7531***	0.7531***	0.7408***	0.7578***	0.7379***
	22	0.7171***	0.7483***	0.6140***	0.7512***	0.7531***	0.7616***	0.7483***	0.7342***
	44	0.5828***	0.5648***	0.5932***	0.6717***	0.6660***	0.6689***	0.6679***	0.6131***
	66	0.4021	0.5553***	0.4286	0.5837***	0.5847***	0.5629***	0.5818***	0.5118

Table B.32: Forecasting Evaluation for Corn Futures with average $MedRV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.7872	1.5022	1.7734	1.4042**	1.4008**	1.3951**	1.3971**	1.3987**
	5	0.9371	0.8589	0.9408	0.7933**	0.7923**	0.7850**	0.7943**	0.7868**
	10	0.6975**	0.6660**	0.8068	0.6641**	0.6627**	0.6574**	0.6634**	0.6574**
	22	0.6035**	0.5986**	0.5750**	0.5976**	0.5970**	0.5998**	0.5963**	0.5970**
	44	0.4266**	0.4451	0.5269	0.5139	0.5140	0.5133	0.5127	0.5107
	66	0.4194**	0.4707	0.6167	0.5193	0.5203	0.5089	0.5174	0.5115
MAPE	1	1.1032**	1.2800	1.1056**	1.1484	1.1368**	1.1403**	1.1340**	1.1476**
	5	0.7461	0.7943	0.7539	0.7343	0.7304**	0.7230**	0.7301**	0.7280**
	10	0.6414**	0.6791*	0.6962	0.6677*	0.6655**	0.6578**	0.6651**	0.6649**
	22	0.5871**	0.6458	0.5982**	0.6388	0.6379	0.6354	0.6361	0.6459
	44	0.5208**	0.5444	0.5647	0.6033	0.6030	0.5943	0.6015	0.5987
	66	0.5096**	0.5483	0.6212	0.6063	0.6054	0.5913	0.6036	0.5841
QLIKE	1	1.8923	1.3430	1.6712	1.2643*	1.2611**	1.2567**	1.2609**	1.2571**
	5	1.3328	1.3152	1.3344	1.2826	1.2813	1.2748**	1.2828	1.2746**
	10	1.3024	1.2952	1.3368	1.2905	1.2893	1.2835**	1.2901	1.2804**
	22	1.3035**	1.3063*	1.3125*	1.3048*	1.3041*	1.2993**	1.3043*	1.2947**
	44	1.2919**	1.2925**	1.3237	1.3092	1.3090	1.3036	1.3086	1.2989**
	66	1.2975**	1.3093	1.3593	1.3188	1.3189	1.3118	1.3181	1.3038**
SR	1	0.6907***	0.6129***	0.6879***	0.7068***	0.7125***	0.7059***	0.7125***	0.7135***
	5	0.7448***	0.6907***	0.7163***	0.7562***	0.7543***	0.7495***	0.7495***	0.7543***
	10	0.7704***	0.7419***	0.7125***	0.7581***	0.7543***	0.7619***	0.7543***	0.7524***
	22	0.8017***	0.7362***	0.8008***	0.7486***	0.7372***	0.7429***	0.7410***	0.7751***
	44	0.8776***	0.8178***	0.8159***	0.7201***	0.7239***	0.7704***	0.7268***	0.8017***
	66	0.8605***	0.7685***	0.7657***	0.6860***	0.6898***	0.7713***	0.6973***	0.8150***

Table B.33: Forecasting Evaluation for Rice Futures with average $MedRV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	0.8109	0.7237	0.7851*	0.6720**	0.6725**	0.6668**	0.6715**	0.6705**
	5	0.5000	0.4720	0.5388	0.4594**	0.4604*	0.4554**	0.4611*	0.4624
	10	0.4798	0.4562	0.5248	0.4348**	0.4359*	0.4344**	0.4371*	0.4412
	22	0.4929	0.4757	0.6137	0.4395**	0.4408*	0.4384**	0.4405**	0.4499
	44	0.5774	0.5443	0.5439	0.4742**	0.4751**	0.4805	0.4745**	0.4973
	66	0.6207	0.5460	0.5333	0.4774**	0.4780**	0.4895	0.4772**	0.5122
MAPE	1	0.6117	0.6559	0.6081	0.5977	0.5982	0.5922**	0.5961*	0.5984
	5	0.5058**	0.5353	0.5477	0.5183**	0.5193**	0.5167**	0.5185**	0.5254
	10	0.5000**	0.5323	0.5654	0.5165**	0.5171**	0.5144**	0.5175**	0.5221
	22	0.5283**	0.5733	0.6557	0.5432**	0.5438**	0.5410**	0.5428**	0.5520
	44	0.5923**	0.6186	0.6397	0.5725**	0.5727**	0.5704**	0.5720**	0.5854
	66	0.6354	0.6232	0.6027	0.5750**	0.5748**	0.5754**	0.5742**	0.5943
QLIKE	1	1.1557	1.1304	1.1413	1.1140**	1.1142**	1.1134**	1.1139**	1.1153*
	5	1.1344	1.1303	1.1438	1.1264**	1.1267**	1.1262**	1.1268**	1.1279
	10	1.1462	1.1434	1.1610	1.1391**	1.1393**	1.1393**	1.1394**	1.1411
	22	1.1805	1.1720	1.2140	1.1623**	1.1625**	1.1628**	1.1623**	1.1656
	44	1.2379	1.2166	1.2151	1.1953**	1.1955**	1.1976	1.1953**	1.2025
	66	1.2743	1.2488	1.2371	1.2175**	1.2175**	1.2205	1.2172**	1.2283
SR	1	0.7881***	0.7806***	0.7872***	0.7957***	0.7947***	0.7853***	0.7947***	0.7947***
	5	0.8117***	0.8070***	0.7674***	0.8145***	0.8145***	0.8079***	0.8145***	0.8070***
	10	0.8183***	0.7994***	0.7429***	0.8249***	0.8202***	0.8032***	0.8173***	0.7910***
	22	0.7514***	0.7137***	0.6111***	0.7533***	0.7476***	0.7109***	0.7524***	0.7128***
	44	0.6177***	0.5301	0.5631***	0.6290***	0.6271***	0.6243***	0.6318***	0.6215***
	66	0.4934	0.2213	0.4652	0.5122	0.5122	0.5122	0.5104	0.4802

Table B.34: Forecasting Evaluation for Soy Futures with average $MedRV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.3511	1.2559	1.3330	1.0899**	1.0989**	1.0898**	1.0916**	1.1039
	5	0.7882	0.7796	0.8667	0.7097**	0.7089**	0.7059**	0.7089**	0.7124**
	10	0.6749**	0.6971	0.7766	0.6497**	0.6496**	0.6494**	0.6472**	0.6525**
	22	0.6594**	0.6529	0.7906	0.6262**	0.6264**	0.6371**	0.6254**	0.6362**
	44	0.6311**	0.6727*	0.7896	0.6395**	0.6401**	0.6515**	0.6409**	0.6446**
	66	0.6040**	0.7083	0.7029	0.6853	0.6858	0.6865	0.6865	0.6704
MAPE	1	0.6343	0.7299	0.6368	0.6109	0.6001**	0.5985**	0.6033**	0.6103*
	5	0.5163**	0.5705	0.5433	0.5154	0.5120**	0.5084**	0.5121**	0.5124**
	10	0.4844**	0.5457	0.5283	0.5034*	0.5014**	0.4953**	0.5009**	0.4954**
	22	0.5054**	0.5134	0.5429	0.4981**	0.4973**	0.4997**	0.4963**	0.5037**
	44	0.4912**	0.4902**	0.5472	0.5028**	0.5024**	0.5081**	0.5014**	0.5139**
	66	0.4736**	0.4962	0.4878**	0.5143	0.5145	0.5175	0.5134	0.5125
QLIKE	1	1.8012	1.8068	1.7944	1.7603**	1.7602**	1.7597**	1.7608**	1.9152
	5	1.7809	1.7855	1.7894	1.7721**	1.7717**	1.7703**	1.7718**	1.7783*
	10	1.7796**	1.7876	1.7971	1.7799**	1.7797**	1.7782**	1.7795**	1.7848**
	22	1.7956**	1.7952	1.8384	1.7912**	1.7912**	1.7917**	1.7911**	1.7966**
	44	1.8165**	1.8198*	1.8474	1.8090**	1.8091**	1.8076**	1.8091**	1.8074**
	66	1.8145**	1.8315	1.8276*	1.8246*	1.8245*	1.8192**	1.8245*	1.8179**
SR	1	0.7502***	0.7352***	0.7474***	0.7728***	0.7690***	0.7596***	0.7634***	0.7559***
	5	0.7972***	0.7897***	0.8028***	0.8113***	0.8094***	0.8113***	0.8038***	0.8085***
	10	0.8282***	0.8225***	0.7822***	0.8254***	0.8254***	0.8254***	0.8282***	0.8178***
	22	0.8000***	0.8188***	0.8028***	0.8272***	0.8263***	0.8141***	0.8272***	0.8131***
	44	0.9277***	0.8394***	0.8685***	0.8695***	0.8657***	0.8385***	0.8648***	0.8620***
	66	0.9418***	0.8272***	0.8601***	0.8085***	0.8085***	0.7925***	0.8085***	0.8479***

Table B.35: Forecasting Evaluation for Sugar Futures with average $MedRV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

	h	RW	AR	ARMA	HAR	HAR-J	HAR-CJ	HAR-PS	LHAR-CJ
MSPE	1	1.2218	1.1210	1.1825	1.0307**	1.0374**	1.0375**	1.0266**	1.0458*
	5	0.8464	0.8188	0.9583	0.7602*	0.7629*	0.7723*	0.7522**	0.7835
	10	0.7746**	0.7530*	0.8184*	0.7146**	0.7172**	0.7246**	0.7117**	0.7305**
	22	0.6433**	0.6860**	0.7689	0.6682**	0.6692**	0.6742**	0.6683**	0.6810**
	44	0.6649**	0.7111**	0.9099	0.6943**	0.6947**	0.7009**	0.6934**	0.7125**
	66	0.7146**	0.8147	1.2100	0.7438**	0.7415**	0.7333**	0.7416**	0.7469**
MAPE	1	0.6369	0.6978	0.6358	0.6224*	0.6277	0.6223*	0.6179**	0.6402
	5	0.5194**	0.5659	0.5526	0.5284*	0.5312	0.5302*	0.5253**	0.5473
	10	0.4911**	0.5327	0.5246*	0.5173*	0.5189	0.5157*	0.5160*	0.5332
	22	0.4704**	0.5298	0.5557	0.5180	0.5189	0.5155	0.5177	0.5384
	44	0.4968**	0.5586	0.6352	0.5510	0.5509	0.5489	0.5502	0.5665
	66	0.5213**	0.6044	0.7160	0.5800	0.5788	0.5740	0.5788	0.5916
QLIKE	1	1.7837	1.7570	1.7640	1.7348**	1.7354**	1.7351**	1.7342**	1.7384
	5	1.7570	1.7588	1.7702	1.7487*	1.7487*	1.7491*	1.7478**	1.7522
	10	1.7606**	1.7631	1.7676	1.7585**	1.7587**	1.7587**	1.7581**	1.7606**
	22	1.7643**	1.7714	1.7917	1.7688**	1.7688**	1.7689**	1.7687**	1.7701**
	44	1.7898**	1.7926	1.8220	1.7855**	1.7853**	1.7876**	1.7853**	1.7867**
	66	1.8105**	1.8116	1.8903	1.7997**	1.7994**	1.8002**	1.7995**	1.7988**
SR	1	0.7030***	0.6635***	0.7002***	0.7049***	0.7039***	0.7068***	0.7115***	0.7105***
	5	0.7491***	0.6729***	0.7115***	0.7434***	0.7265***	0.7246***	0.7378***	0.7397***
	10	0.7716***	0.6880***	0.7237***	0.7415***	0.7453***	0.7293***	0.7434***	0.7509***
	22	0.8205***	0.6945***	0.6353***	0.7171***	0.7180***	0.7115***	0.7143***	0.7472***
	44	0.7491***	0.5836***	0.3703	0.6297***	0.6344***	0.6335***	0.6297***	0.6720***
	66	0.6551***	0.3853	0.3449	0.5122	0.5179	0.5254**	0.5122	0.5677***

Table B.36: Forecasting Evaluation for Wheat Futures with average $MedRV_{t+h}^{(h)}$. Note that * and ** indicate the inclusion in the $\mathcal{M}^*_{90\%}$ and $\mathcal{M}^*_{75\%}$, respectively. For the Success Ratio (SR), the asterisk *, **, and *** indicate a statistical significance at 10%, 5%, and 1%, respectively.

Appendix C. Value-at-Risk Backtesting Results

α		0.01				0.025				0.05			
Position		long		short		long		short		long		short	
Test		UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC
RW	1	0.0004	0.0010	0.0020	0.0054	0.0205	0.0593	0.0735	0.0567	0.2083	0.3288	0.6596	0.7190
	5	0.0095	0.0234	0.0009	0.0023	0.3801	0.3905	0.0205	0.0310	0.6596	0.2182	0.5628	0.4325
	10	0.0045	0.0032	0.0004	0.0015	0.1073	0.0867	0.0205	0.0310	0.0973	0.0813	0.3938	0.4113
	22	0.0020	0.0018	0.0000	0.0000	0.1073	0.0249	0.0003	0.0005	0.2083	0.0048	0.1272	0.0850
	44	0.0001	0.0000	0.0000	0.0001	0.0128	0.0001	0.0003	0.0005	0.1639	0.0051	0.2083	0.1949
	66	0.0001	0.0000	0.0001	0.0002	0.0015	0.0000	0.0005	0.0016	0.0048	0.0021	0.3228	0.5413
AR	1	0.0678	0.1017	0.0368	0.0814	0.1527	0.2514	0.1527	0.1162	0.4739	0.1014	0.9043	0.8877
	5	0.0368	0.0664	0.0020	0.0053	0.3801	0.3905	0.1527	0.2440	0.8715	0.1831	0.8715	0.1707
	10	0.0045	0.0032	0.0001	0.0006	0.1527	0.1014	0.0322	0.0830	0.4739	0.0111	0.6596	0.7014
	22	0.0045	0.0032	0.0001	0.0004	0.1527	0.0008	0.0027	0.0104	0.5628	0.0327	0.3938	0.2227
	44	0.0020	0.0018	0.0045	0.0113	0.1527	0.0008	0.0205	0.0583	0.3938	0.0125	0.6596	0.4278
	66	0.0191	0.0406	0.0678	0.1405	0.3801	0.0044	0.1527	0.3474	0.9043	0.0460	0.3233	0.4517
ARMA	1	0.0020	0.0053	0.0095	0.0235	0.0492	0.1168	0.0735	0.0567	0.3938	0.4254	0.8715	0.8368
	5	0.0095	0.0053	0.0045	0.0113	0.1073	0.2016	0.1073	0.1962	0.5628	0.2293	0.6596	0.2048
	10	0.0020	0.0018	0.0001	0.0002	0.2121	0.0286	0.0005	0.0003	0.3228	0.0138	0.4739	0.2220
	22	0.0004	0.0005	0.0020	0.0054	0.0735	0.0008	0.0078	0.0262	0.3938	0.0395	0.2611	0.2086
	44	0.0191	0.0406	0.0678	0.1405	0.2121	0.0007	0.6174	0.8301	0.7929	0.0096	0.4005	0.5414
	66	0.0678	0.1017	0.0368	0.0814	0.6174	0.0030	0.2875	0.5496	0.2561	0.0050	0.7929	0.2772
HAR	1	0.0678	0.1405	0.0095	0.0235	0.1527	0.1014	0.1527	0.1162	0.3938	0.1056	0.8715	0.9211
	5	0.0191	0.0083	0.0020	0.0053	0.3801	0.1282	0.1073	0.2622	0.8715	0.1831	0.9831	0.3564
	10	0.0020	0.0018	0.0009	0.0023	0.0492	0.0569	0.1527	0.2440	0.5628	0.0095	0.3228	0.2181
	22	0.0095	0.0053	0.0001	0.0002	0.0735	0.0008	0.0047	0.0168	0.4739	0.0364	0.3938	0.2227
	44	0.0009	0.0010	0.0009	0.0023	0.1073	0.0052	0.0205	0.0583	0.6596	0.0286	0.3938	0.2227
	66	0.0004	0.0005	0.0045	0.0115	0.2121	0.0007	0.0322	0.0416	0.4739	0.0111	0.7929	0.8546
HAR-J	1	0.1187	0.2279	0.0191	0.0445	0.1527	0.1014	0.1073	0.0824	0.3228	0.1073	0.8715	0.9211
	5	0.0191	0.0083	0.0020	0.0053	0.3801	0.1282	0.1073	0.2622	0.7630	0.2025	0.9043	0.3185
	10	0.0045	0.0032	0.0009	0.0023	0.0492	0.0569	0.1527	0.2440	0.5628	0.0095	0.3228	0.2181
	22	0.0095	0.0053	0.0001	0.0002	0.0735	0.0008	0.0047	0.0168	0.4739	0.0364	0.4739	0.2220
	44	0.0009	0.0010	0.0009	0.0023	0.1073	0.0052	0.0205	0.0583	0.6596	0.0286	0.4739	0.4268
	66	0.0009	0.0010	0.0045	0.0115	0.2121	0.0007	0.0322	0.0416	0.5628	0.0095	0.9043	0.8981
HAR-CJ	1	0.0678	0.1405	0.0095	0.0230	0.1527	0.2514	0.1527	0.1162	0.2611	0.1064	0.9831	0.9207
	5	0.0191	0.0083	0.0020	0.0053	0.4904	0.4184	0.1073	0.2622	0.6596	0.2182	0.5628	0.2159
	10	0.0020	0.0018	0.0020	0.0053	0.0492	0.0569	0.1527	0.2440	0.5628	0.0095	0.4739	0.2220
	22	0.0095	0.0053	0.0000	0.0001	0.1073	0.0249	0.0078	0.0262	0.3228	0.0418	0.3938	0.2227
	44	0.0009	0.0010	0.0045	0.0115	0.2121	0.0286	0.0128	0.0397	0.8715	0.0201	0.4739	0.4268
	66	0.0009	0.0010	0.0095	0.0235	0.2121	0.0007	0.1073	0.1962	0.6596	0.0080	0.6849	0.7930
HAR-PS	1	0.1187	0.2279	0.0191	0.0445	0.2121	0.1141	0.1527	0.1162	0.4739	0.1014	0.7630	0.7057
	5	0.0191	0.0083	0.0020	0.0053	0.4904	0.4184	0.1073	0.2622	0.7630	0.0770	0.8715	0.1707
	10	0.0045	0.0032	0.0009	0.0023	0.0735	0.0715	0.1073	0.1962	0.5628	0.0095	0.3228	0.2181
	22	0.0095	0.0053	0.0001	0.0002	0.1073	0.0052	0.0047	0.0168	0.5628	0.0327	0.3228	0.2181
	44	0.0009	0.0010	0.0020	0.0053	0.1073	0.0052	0.0205	0.0583	0.5628	0.0327	0.3228	0.2181
	66	0.0009	0.0010	0.0045	0.0115	0.2121	0.0007	0.0322	0.0416	0.3938	0.0125	0.7929	0.8546
LHAR-CJ	1	0.0368	0.0814	0.0095	0.0230	0.4904	0.7562	0.1527	0.1162	0.2611	0.2188	0.7630	0.7057
	5	0.0191	0.0083	0.0045	0.0113	0.4904	0.4184	0.1073	0.2622	0.4739	0.4427	0.8715	0.1707
	10	0.0045	0.0032	0.0045	0.0113	0.1073	0.0867	0.2121	0.2925	0.5628	0.0095	0.3228	0.1000
	22	0.0095	0.0053	0.0001	0.0004	0.0735	0.0049	0.0078	0.0262	0.4739	0.0364	0.3938	0.2227
	44	0.0009	0.0010	0.0045	0.0115	0.3801	0.0044	0.0078	0.0262	0.6596	0.0286	0.4739	0.4268
	66	0.0001	0.0000	0.0191	0.0453	0.2121	0.0001	0.0492	0.1143	0.7630	0.0065	0.5825	0.7170

Table C.37: Value-at-Risk backtesting results for *RV* for Corn Futures. Note that UC is the unconditional coverage test by Kupiec (1995), CC is the conditional coverage test by Christoffersen (1998), α is the Value-at-Risk level, Position corresponds with the trading position, which is either long (risk of rising prices) or short (risk of falling prices). The test results are presented as p-values from the UC and CC test for the forecast horizons 1-, 5-, 10-, 22-, 44-, and 66-days ahead.

α		0.01				0.025				0.05			
Position		long		short		long		short		long		short	
Test		UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC
RW	1	0.3078	0.4857	0.1942	0.3417	0.7472	0.8862	0.4809	0.3144	0.2648	0.5133	0.1175	0.2377
	5	0.4115	0.0980	0.8876	0.2532	0.0487	0.0124	0.0487	0.0775	0.0017	0.0004	0.0291	0.0778
	10	0.4115	0.0980	0.3078	0.2345	0.0149	0.0028	0.6379	0.2550	0.0048	0.0003	0.0613	0.0899
	22	0.6248	0.0057	0.3078	0.2345	0.0487	0.0124	0.6379	0.2550	0.0017	0.0016	0.1175	0.2730
	44	0.6583	0.2724	0.8661	0.8872	0.2741	0.0138	0.2741	0.1013	0.0009	0.0008	0.1573	0.0370
	66	0.6583	0.2724	0.4625	0.6409	0.2741	0.0138	0.6379	0.2550	0.0009	0.0002	0.5003	0.3870
AR	1	0.8661	0.8872	0.0663	0.0160	0.1273	0.2165	0.8984	0.4005	0.0125	0.0343	0.2062	0.1489
	5	0.6248	0.1542	0.8661	0.2104	0.0075	0.0011	0.1910	0.2889	0.0029	0.0029	0.0613	0.1549
	10	0.4115	0.0980	0.1942	0.1907	0.0149	0.0028	0.4995	0.1982	0.0009	0.0002	0.0291	0.0403
	22	0.8661	0.0100	0.4625	0.6409	0.0808	0.0233	0.4995	0.1982	0.0009	0.0008	0.3335	0.5992
	44	0.4625	0.2648	0.6583	0.7807	0.2741	0.0138	0.3772	0.1458	0.0009	0.0008	0.1573	0.0370
	66	0.3078	0.2345	0.3078	0.4857	0.7882	0.0631	0.7882	0.3113	0.0125	0.0048	0.7004	0.5268
ARMA	1	0.3078	0.4857	0.1942	0.3417	0.9448	0.8890	0.4809	0.3144	0.2062	0.4278	0.0613	0.1499
	5	0.6248	0.1542	0.6583	0.7807	0.0487	0.0124	0.6379	0.2550	0.0079	0.0006	0.0193	0.0525
	10	0.8661	0.2104	0.8876	0.2532	0.0808	0.0233	0.7882	0.3113	0.0029	0.0029	0.0613	0.0899
	22	0.6248	0.8141	0.6583	0.7807	0.6379	0.2550	0.9448	0.3617	0.0009	0.0008	0.0858	0.0530
	44	0.3078	0.2345	0.4625	0.6409	0.4995	0.0332	0.9448	0.3617	0.0291	0.0140	0.7004	0.5268
	66	0.6583	0.7807	0.0359	0.0651	0.6379	0.0471	0.0474	0.1135	0.1573	0.0004	0.3819	0.4198
HAR	1	0.0560	0.0082	0.8876	0.8723	0.0035	0.0049	0.2741	0.1013	0.0125	0.0158	0.1175	0.1766
	5	0.4115	0.0980	0.8876	0.2532	0.0035	0.0049	0.1273	0.1983	0.0029	0.0007	0.0613	0.1549
	10	0.4115	0.0980	0.8876	0.2532	0.0149	0.0028	0.4995	0.1982	0.0009	0.0002	0.0428	0.0612
	22	0.4115	0.0980	0.8876	0.8723	0.1910	0.0662	0.2741	0.1013	0.0009	0.0008	0.5003	0.6548
	44	0.6248	0.8141	0.8876	0.8723	0.1273	0.0406	0.7882	0.3113	0.0001	0.0001	0.2648	0.1975
	66	0.6583	0.2724	0.6583	0.7807	0.2741	0.0138	0.7882	0.0631	0.0029	0.0001	0.5003	0.1696
HAR-J	1	0.1262	0.0235	0.8876	0.8723	0.0015	0.0020	0.2741	0.1013	0.0125	0.0158	0.0613	0.0899
	5	0.4115	0.0980	0.8876	0.2532	0.0035	0.0049	0.1910	0.2889	0.0017	0.0004	0.0613	0.1549
	10	0.4115	0.0980	0.6583	0.2724	0.0149	0.0028	0.4995	0.1982	0.0005	0.0001	0.0428	0.0612
	22	0.2431	0.0529	0.8876	0.8723	0.1910	0.0662	0.2741	0.1013	0.0005	0.0004	0.5969	0.7392
	44	0.6248	0.8141	0.8876	0.8723	0.1273	0.0406	0.7882	0.3113	0.0003	0.0002	0.2648	0.1975
	66	0.6583	0.2724	0.6583	0.7807	0.2741	0.0138	0.7882	0.0631	0.0029	0.0001	0.4122	0.1329
HAR-CJ	1	0.1262	0.0235	0.8876	0.8723	0.0035	0.0049	0.3772	0.1458	0.0029	0.0078	0.0613	0.0899
	5	0.4115	0.0980	0.8661	0.2104	0.0075	0.0111	0.0487	0.0775	0.0009	0.0002	0.0428	0.1115
	10	0.4115	0.0980	0.6583	0.2724	0.0278	0.0061	0.2741	0.1013	0.0017	0.0004	0.0291	0.0403
	22	0.4115	0.0980	0.4625	0.2648	0.2741	0.1013	0.2741	0.1013	0.0029	0.0029	0.3335	0.5992
	44	0.8876	0.8723	0.8876	0.8723	0.1910	0.0080	0.8984	0.0966	0.0005	0.0004	0.5003	0.3870
	66	0.8876	0.2532	0.6583	0.7807	0.1910	0.0662	0.9448	0.0801	0.0017	0.0004	0.2062	0.0534
HAR-PS	1	0.0560	0.0082	0.6583	0.7807	0.0035	0.0049	0.3772	0.0221	0.0079	0.0093	0.0428	0.0612
	5	0.4115	0.0980	0.8876	0.2532	0.0035	0.0049	0.1910	0.0662	0.0029	0.0007	0.0858	0.2088
	10	0.4115	0.0980	0.8876	0.2532	0.0149	0.0028	0.4995	0.1982	0.0009	0.0002	0.0193	0.0256
	22	0.4115	0.0980	0.8876	0.8723	0.1910	0.0662	0.2741	0.1013	0.0005	0.0004	0.5003	0.6548
	44	0.4115	0.6663	0.8876	0.8723	0.1273	0.0406	0.9448	0.3617	0.0003	0.0002	0.2062	0.1489
	66	0.6583	0.2724	0.4625	0.6409	0.3772	0.0221	0.7882	0.0631	0.0017	0.0004	0.4122	0.1329
LHAR-CJ	1	0.1262	0.0235	0.4625	0.6409	0.0035	0.0049	0.2741	0.3969	0.0029	0.0078	0.0613	0.0899
	5	0.2431	0.0529	0.6248	0.8141	0.0075	0.0111	0.0278	0.0437	0.0005	0.0001	0.0193	0.0525
	10	0.2431	0.0529	0.8876	0.8723	0.0075	0.0011	0.0808	0.0233	0.0017	0.0004	0.0428	0.1115
	22	0.0560	0.0082	0.4625	0.2648	0.0808	0.1280	0.1273	0.0406	0.0009	0.0008	0.1175	0.2730
	44	0.6248	0.8141	0.6583	0.2724	0.1910	0.0662	0.4995	0.1982	0.0003	0.0002	0.0613	0.0899
	66	0.2431	0.0529	0.6583	0.7807	0.1273	0.0406	0.2741	0.1013	0.0017	0.0001	0.0291	0.0140

Table C.38: Value-at-Risk backtesting results for RV for Rice Futures. Note that UC is the unconditional coverage test by Kupiec (1995), CC is the conditional coverage test by Christoffersen (1998), α is the Value-at-Risk level, Position corresponds with the trading position, which is either long (risk of rising prices) or short (risk of falling prices). The test results are presented as p-values from the UC and CC test for the forecast horizons 1-, 5-, 10-, 22-, 44-, and 66-days ahead.

α		0.01				0.025				0.05			
Position		long		short		long		short		long		short	
Test		UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC
RW	1	0.0000	0.0000	0.0009	0.0025	0.0051	0.0182	0.0343	0.0981	0.1050	0.2198	0.3408	0.5877
	5	0.0000	0.0001	0.0200	0.0406	0.0523	0.0193	0.0523	0.1199	0.1752	0.3715	0.2767	0.3679
	10	0.0002	0.0003	0.0000	0.0000	0.0778	0.0742	0.0523	0.0563	0.3408	0.4103	0.2217	0.3327
	22	0.0001	0.0001	0.0009	0.0010	0.0220	0.0342	0.0001	0.0003	0.0230	0.0213	0.0796	0.0717
	44	0.0000	0.0000	0.0000	0.0000	0.0005	0.0004	0.0030	0.0033	0.0163	0.0087	0.0230	0.0364
	66	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0005	0.0001	0.0036	0.0038	0.1365	0.0883
AR	1	0.0001	0.0001	0.0047	0.0121	0.0138	0.0429	0.2216	0.4591	0.1752	0.3013	0.4141	0.6705
	5	0.0004	0.0001	0.0384	0.0657	0.0343	0.0458	0.0343	0.0875	0.2767	0.5025	0.4964	0.4335
	10	0.0009	0.0010	0.0009	0.0010	0.0523	0.0593	0.0220	0.0327	0.4141	0.4346	0.4964	0.4335
	22	0.0000	0.0001	0.0022	0.0018	0.0343	0.0458	0.0523	0.1199	0.0796	0.0368	0.2767	0.2143
	44	0.0000	0.0000	0.0001	0.0001	0.0017	0.0024	0.0220	0.0116	0.2767	0.2248	0.2767	0.2143
	66	0.0000	0.0000	0.0004	0.0011	0.0017	0.0024	0.0778	0.1590	0.3408	0.2344	0.4141	0.2262
ARMA	1	0.0000	0.0000	0.0009	0.0025	0.0085	0.0284	0.0220	0.0655	0.1050	0.2198	0.3408	0.5877
	5	0.0009	0.0010	0.0100	0.0236	0.0343	0.0458	0.0343	0.0875	0.2217	0.4367	0.2767	0.3679
	10	0.0000	0.0001	0.0004	0.0015	0.0343	0.0458	0.0343	0.0875	0.1365	0.0936	0.3408	0.5588
	22	0.0000	0.0000	0.0001	0.0001	0.0010	0.0006	0.0523	0.0177	0.1050	0.0850	0.2767	0.3679
	44	0.0001	0.0001	0.0100	0.0241	0.0220	0.0125	0.0778	0.1590	0.4141	0.2388	0.4964	0.2243
	66	0.0000	0.0001	0.0009	0.0034	0.0051	0.0016	0.0778	0.1590	0.3408	0.1090	0.4964	0.4335
HAR	1	0.0009	0.0034	0.0100	0.0246	0.0138	0.0247	0.1130	0.2036	0.1050	0.1513	0.4141	0.6705
	5	0.0002	0.0003	0.0384	0.0657	0.0343	0.0458	0.0523	0.1199	0.4141	0.6225	0.4964	0.6588
	10	0.0009	0.0010	0.0047	0.0031	0.0523	0.0593	0.0343	0.0875	0.3408	0.5656	0.5872	0.4368
	22	0.0000	0.0000	0.0047	0.0129	0.0138	0.0247	0.0523	0.1199	0.0440	0.0292	0.2767	0.4976
	44	0.0000	0.0000	0.0001	0.0001	0.0051	0.0051	0.0523	0.0563	0.1050	0.0850	0.2767	0.2143
	66	0.0000	0.0000	0.0002	0.0002	0.0010	0.0006	0.1602	0.2519	0.2767	0.2248	0.3408	0.2228
HAR-J	1	0.0009	0.0034	0.0100	0.0246	0.0138	0.0247	0.1130	0.2036	0.2217	0.2106	0.3408	0.5877
	5	0.0002	0.0003	0.0384	0.0657	0.0220	0.0342	0.0343	0.0875	0.3408	0.5656	0.4964	0.6588
	10	0.0009	0.0010	0.0047	0.0031	0.0343	0.0458	0.0343	0.0875	0.2767	0.5025	0.5872	0.4368
	22	0.0000	0.0000	0.0022	0.0018	0.0085	0.0173	0.0778	0.1590	0.0440	0.0292	0.2767	0.4976
	44	0.0000	0.0000	0.0001	0.0001	0.0051	0.0051	0.0523	0.0563	0.1050	0.0850	0.2767	0.2143
	66	0.0000	0.0000	0.0002	0.0002	0.0010	0.0006	0.1130	0.2036	0.2767	0.2248	0.3408	0.2228
HAR-CJ	1	0.0004	0.0005	0.0100	0.0246	0.0220	0.0342	0.1130	0.2036	0.1752	0.1929	0.6856	0.8711
	5	0.0002	0.0003	0.0384	0.0657	0.0343	0.0158	0.0523	0.1199	0.3408	0.5656	0.5872	0.6911
	10	0.0009	0.0010	0.0004	0.0005	0.0343	0.0458	0.0343	0.0875	0.3408	0.5656	0.4141	0.4200
	22	0.0000	0.0000	0.0009	0.0010	0.0343	0.0458	0.0523	0.1199	0.0440	0.0292	0.2767	0.4976
	44	0.0000	0.0000	0.0001	0.0001	0.0051	0.0051	0.0343	0.0436	0.1365	0.0936	0.1752	0.1849
	66	0.0000	0.0000	0.0022	0.0067	0.0010	0.0015	0.0778	0.1590	0.2217	0.1059	0.4141	0.2262
HAR-PS	1	0.0009	0.0034	0.0047	0.0121	0.0138	0.0247	0.1602	0.2519	0.1752	0.1929	0.4141	0.6705
	5	0.0004	0.0005	0.0384	0.0657	0.0523	0.0593	0.0220	0.0618	0.4141	0.6225	0.5872	0.6911
	10	0.0009	0.0010	0.0022	0.0018	0.0778	0.0742	0.0343	0.0875	0.2767	0.3786	0.5872	0.4368
	22	0.0000	0.0001	0.0047	0.0129	0.0138	0.0247	0.0343	0.0875	0.0440	0.0292	0.2767	0.4976
	44	0.0000	0.0000	0.0001	0.0001	0.0051	0.0051	0.0523	0.0563	0.1050	0.0850	0.2767	0.2143
	66	0.0000	0.0000	0.0002	0.0002	0.0010	0.0006	0.1602	0.2519	0.2767	0.2248	0.2767	0.2143
LHAR-CJ	1	0.0002	0.0003	0.0100	0.0246	0.0343	0.0892	0.1602	0.2519	0.2217	0.2106	0.4964	0.7483
	5	0.0002	0.0003	0.0384	0.0657	0.0523	0.0593	0.0778	0.1590	0.3408	0.5656	0.6856	0.4297
	10	0.0004	0.0005	0.0004	0.0005	0.0523	0.0593	0.0220	0.0618	0.2217	0.3415	0.4141	0.4200
	22	0.0000	0.0000	0.0022	0.0067	0.0085	0.0173	0.0343	0.0875	0.0796	0.0368	0.3408	0.5588
	44	0.0000	0.0000	0.0001	0.0001	0.0085	0.0071	0.0343	0.0436	0.1752	0.1007	0.1752	0.1849
	66	0.0000	0.0000	0.0004	0.0015	0.0051	0.0051	0.0778	0.1590	0.1365	0.0936	0.3408	0.2228

Table C.39: Value-at-Risk backtesting results for RV for Soy Futures. Note that UC is the unconditional coverage test by Kupiec (1995), CC is the conditional coverage test by Christoffersen (1998), α is the Value-at-Risk level, Position corresponds with the trading position, which is either long (risk of rising prices) or short (risk of falling prices). The test results are presented as p-values from the UC and CC test for the forecast horizons 1-, 5-, 10-, 22-, 44-, and 66-days ahead.

α		0.01				0.025				0.05			
Position		long		short		long		short		long		short	
Test		UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC
RW	1	0.0001	0.0002	0.0000	0.0000	0.0031	0.0020	0.0002	0.0008	0.1098	0.2260	0.0039	0.0019
	5	0.1251	0.2380	0.0000	0.0000	0.0053	0.0168	0.0002	0.0000	0.0173	0.0363	0.0173	0.0091
	10	0.0010	0.0026	0.0000	0.0001	0.0018	0.0073	0.0002	0.0004	0.0121	0.0401	0.0026	0.0016
	22	0.0103	0.0250	0.0000	0.0000	0.0006	0.0018	0.0000	0.0000	0.0026	0.0097	0.0003	0.0001
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0007	0.0027	0.0026	0.0016
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
AR	1	0.0010	0.0026	0.0000	0.0000	0.0018	0.0058	0.0000	0.0000	0.0026	0.0047	0.0001	0.0000
	5	0.0206	0.0476	0.0000	0.0000	0.0010	0.0033	0.0000	0.0001	0.0057	0.0113	0.0026	0.0016
	10	0.0004	0.0011	0.0001	0.0003	0.0010	0.0043	0.0000	0.0001	0.0057	0.0175	0.0011	0.0009
	22	0.0010	0.0035	0.0000	0.0000	0.0002	0.0005	0.0000	0.0000	0.0007	0.0027	0.0001	0.0000
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARMA	1	0.0002	0.0004	0.0000	0.0000	0.0031	0.0020	0.0002	0.0008	0.1098	0.2260	0.0039	0.0019
	5	0.0002	0.0004	0.0000	0.0000	0.0053	0.0168	0.0000	0.0000	0.0057	0.0050	0.0017	0.0006
	10	0.0103	0.0250	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0017	0.0065	0.0000	0.0000
	22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0003	0.0002
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0001
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR	1	0.0049	0.0123	0.0000	0.0000	0.0018	0.0058	0.0000	0.0001	0.0835	0.1750	0.0039	0.0019
	5	0.0206	0.0476	0.0000	0.0000	0.0010	0.0033	0.0000	0.0000	0.0173	0.0178	0.0011	0.0028
	10	0.0010	0.0026	0.0000	0.0000	0.0089	0.0274	0.0000	0.0000	0.0039	0.0118	0.0004	0.0003
	22	0.0103	0.0250	0.0000	0.0000	0.0001	0.0003	0.0000	0.0000	0.0017	0.0051	0.0000	0.0000
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0002	0.0001
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-J	1	0.0022	0.0058	0.0000	0.0000	0.0031	0.0100	0.0000	0.0001	0.0835	0.1750	0.0057	0.0050
	5	0.0206	0.0476	0.0000	0.0000	0.0010	0.0033	0.0000	0.0000	0.0084	0.0078	0.0017	0.0055
	10	0.0022	0.0058	0.0000	0.0000	0.0053	0.0168	0.0000	0.0000	0.0084	0.0255	0.0004	0.0001
	22	0.0206	0.0431	0.0000	0.0000	0.0001	0.0003	0.0000	0.0000	0.0017	0.0051	0.0000	0.0000
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000	0.0002	0.0001	0.0001
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-CJ	1	0.0103	0.0248	0.0000	0.0000	0.0018	0.0011	0.0001	0.0004	0.1098	0.1345	0.0039	0.0019
	5	0.0206	0.0476	0.0000	0.0000	0.0018	0.0058	0.0000	0.0002	0.0121	0.0119	0.0017	0.0039
	10	0.0010	0.0026	0.0000	0.0000	0.0031	0.0100	0.0000	0.0001	0.0338	0.0955	0.0003	0.0002
	22	0.0206	0.0431	0.0000	0.0000	0.0001	0.0003	0.0000	0.0000	0.0017	0.0051	0.0003	0.0002
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0004	0.0017	0.0001	0.0001
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-PS	1	0.0022	0.0058	0.0000	0.0000	0.0006	0.0025	0.0000	0.0002	0.0835	0.1750	0.0121	0.0149
	5	0.0394	0.0864	0.0000	0.0000	0.0031	0.0100	0.0000	0.0001	0.0121	0.0119	0.0017	0.0039
	10	0.0004	0.0011	0.0000	0.0000	0.0031	0.0100	0.0000	0.0000	0.0084	0.0255	0.0004	0.0003
	22	0.0103	0.0250	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0026	0.0078	0.0001	0.0000
	44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0003	0.0004
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LHAR-CJ	1	0.0049	0.0123	0.0000	0.0000	0.0010	0.0033	0.0000	0.0001	0.0244	0.0261	0.0057	0.0024
	5	0.1251	0.2380	0.0000	0.0000	0.0018	0.0058	0.0000	0.0000	0.0057	0.0050	0.0007	0.0003
	10	0.0004	0.0011	0.0000	0.0001	0.0230	0.0672	0.0000	0.0001	0.0057	0.0113	0.0002	0.0001
	22	0.0022	0.0071	0.0000	0.0000	0.0006	0.0018	0.0000	0.0000	0.0084	0.0078	0.0001	0.0001
	44	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0000	0.0000	0.0004	0.0017	0.0001	0.0001
	66	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000

Table C.40: Value-at-Risk backtesting results for *RV* for Sugar Futures. Note that UC is the unconditional coverage test by Kupiec (1995), CC is the conditional coverage test by Christoffersen (1998), α is the Value-at-Risk level, Position corresponds with the trading position, which is either long (risk of rising prices) or short (risk of falling prices). The test results are presented as p-values from the UC and CC test for the forecast horizons 1-, 5-, 10-, 22-, 44-, and 66-days ahead.

α		0.01				0.025				0.05			
Position		long		short		long		short		long		short	
Test		UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC
RW	1	0.0714	0.1469	0.0022	0.0018	0.0142	0.0430	0.0087	0.0170	0.1404	0.1740	0.2832	0.3725
	5	0.6813	0.7924	0.0002	0.0000	0.9375	0.4807	0.1633	0.0255	0.9775	0.0588	0.1082	0.0148
	10	0.3234	0.5028	0.0048	0.0004	0.3996	0.2683	0.1154	0.0236	0.8655	0.0646	0.0616	0.0316
	22	0.1243	0.2367	0.0022	0.0003	0.7852	0.4398	0.0031	0.0003	0.4223	0.4979	0.2272	0.1003
	44	0.0102	0.0246	0.0000	0.0000	0.1633	0.1191	0.0018	0.0000	0.2832	0.5081	0.0616	0.0000
	66	0.0391	0.0857	0.0000	0.0000	0.0352	0.0257	0.0005	0.0000	0.0822	0.1862	0.0455	0.0015
AR	1	0.3234	0.5028	0.0204	0.0412	0.1633	0.3649	0.0536	0.0041	0.8655	0.4850	0.5055	0.2251
	5	0.6813	0.7924	0.0022	0.0003	0.9375	0.4807	0.2255	0.0046	0.4573	0.0748	0.1798	0.0046
	10	0.6813	0.7924	0.0204	0.0079	0.3996	0.2683	0.1633	0.0255	0.6495	0.0728	0.1404	0.0150
	22	0.1243	0.2367	0.0022	0.0003	0.6422	0.3871	0.0052	0.0004	0.5055	0.5770	0.1082	0.0015
	44	0.0048	0.0121	0.0000	0.0000	0.0796	0.0588	0.0031	0.0001	0.3481	0.4206	0.1082	0.0004
	66	0.0714	0.1469	0.0000	0.0000	0.1633	0.1191	0.0018	0.0000	0.5055	0.5770	0.1082	0.0004
ARMA	1	0.1243	0.2367	0.0022	0.0018	0.0142	0.0430	0.0087	0.0068	0.1798	0.2237	0.4223	0.4234
	5	0.9122	0.8768	0.0048	0.0004	0.3996	0.2683	0.0796	0.0713	0.6495	0.5031	0.2832	0.1025
	10	0.3234	0.5028	0.0001	0.0000	0.7852	0.4398	0.0536	0.0041	0.6961	0.7258	0.0822	0.0050
	22	0.0714	0.1469	0.0000	0.0000	0.1633	0.3649	0.0005	0.0000	0.2832	0.5081	0.1404	0.0003
	44	0.1243	0.2367	0.0001	0.0000	0.5128	0.3282	0.0087	0.0001	0.2272	0.4442	0.0822	0.0004
	66	0.0102	0.0246	0.0000	0.0000	0.0226	0.0664	0.0010	0.0005	0.4223	0.4381	0.0239	0.0001
HAR	1	0.6813	0.7924	0.0048	0.0132	0.3996	0.2683	0.3039	0.0263	0.3481	0.1708	0.3481	0.2246
	5	0.4822	0.6569	0.0004	0.0001	0.4694	0.4528	0.1633	0.0988	0.9775	0.0588	0.2832	0.0039
	10	0.6813	0.7924	0.0204	0.0079	0.6422	0.3871	0.0796	0.0210	0.8655	0.0646	0.2272	0.0412
	22	0.0714	0.1469	0.0004	0.0001	0.7852	0.4398	0.0226	0.0031	0.5970	0.2971	0.1404	0.0049
	44	0.0714	0.1469	0.0000	0.0000	0.3996	0.2683	0.0031	0.0000	0.6961	0.8773	0.2272	0.0003
	66	0.0714	0.1469	0.0001	0.0000	0.0796	0.0588	0.0142	0.0001	0.5970	0.6543	0.1404	0.0001
HAR-J	1	0.6813	0.7924	0.0048	0.0132	0.3996	0.2683	0.3039	0.0263	0.5055	0.2523	0.3481	0.2246
	5	0.4822	0.6569	0.0004	0.0001	0.4694	0.4528	0.2255	0.1100	0.9775	0.0588	0.2832	0.0039
	10	0.6813	0.7924	0.0204	0.0079	0.6422	0.3871	0.0796	0.0210	0.8655	0.0646	0.2272	0.0412
	22	0.0714	0.1469	0.0004	0.0001	0.7852	0.4398	0.0226	0.0031	0.5055	0.5770	0.1798	0.0046
	44	0.0714	0.1469	0.0000	0.0000	0.5128	0.3282	0.0031	0.0000	0.8011	0.7872	0.1798	0.0003
	66	0.0714	0.1469	0.0000	0.0000	0.0796	0.0588	0.0087	0.0001	0.8011	0.7872	0.2272	0.0001
HAR-CJ	1	0.6813	0.7924	0.0010	0.0010	0.5128	0.3282	0.3039	0.0263	0.5055	0.2523	0.2832	0.1025
	5	0.6813	0.7924	0.0010	0.0002	0.3520	0.3989	0.2255	0.1100	0.9106	0.0524	0.2272	0.0011
	10	0.3234	0.5028	0.0391	0.0112	0.5128	0.3282	0.0796	0.0210	0.9775	0.0588	0.2272	0.0412
	22	0.1243	0.2367	0.0004	0.0001	0.5128	0.3282	0.0352	0.0036	0.2832	0.3479	0.1798	0.0149
	44	0.1243	0.2367	0.0001	0.0000	0.3039	0.2117	0.0142	0.0001	0.5055	0.7538	0.0822	0.0004
	66	0.1243	0.2367	0.0001	0.0000	0.3039	0.2117	0.0087	0.0004	0.6961	0.0389	0.2272	0.0011
HAR-PS	1	0.6813	0.7924	0.0048	0.0132	0.6422	0.3871	0.3039	0.0263	0.4223	0.2098	0.2272	0.2039
	5	0.4822	0.6569	0.0004	0.0001	0.6038	0.4907	0.2255	0.1100	0.9775	0.0588	0.3481	0.0034
	10	0.4822	0.6569	0.0391	0.0112	0.5128	0.3282	0.0796	0.0210	0.8655	0.0646	0.1082	0.0377
	22	0.1243	0.2367	0.0002	0.0000	0.6422	0.3871	0.0226	0.0031	0.5970	0.2971	0.2272	0.0043
	44	0.0714	0.1469	0.0000	0.0000	0.5128	0.3282	0.0031	0.0000	0.6961	0.8773	0.2272	0.0003
	66	0.0714	0.1469	0.0001	0.0000	0.0796	0.0588	0.0087	0.0000	0.6961	0.7258	0.1404	0.0001
LHAR-CJ	1	0.6813	0.7924	0.0022	0.0018	0.6422	0.3871	0.3039	0.0263	0.6961	0.3425	0.6961	0.2045
	5	0.9122	0.8768	0.0022	0.0003	0.2539	0.3348	0.2255	0.1100	0.9775	0.0588	0.4223	0.0112
	10	0.4822	0.6569	0.0391	0.0112	0.9059	0.5048	0.2255	0.0265	0.7554	0.0693	0.2272	0.0144
	22	0.1243	0.2367	0.0010	0.0002	0.9375	0.4807	0.0796	0.0210	0.6961	0.3425	0.2832	0.0404
	44	0.2058	0.3572	0.0002	0.0000	0.5128	0.3282	0.0352	0.0001	0.8011	0.7872	0.2832	0.0010
	66	0.3234	0.5028	0.0004	0.0001	0.9375	0.4807	0.0142	0.0005	0.8655	0.0646	0.2832	0.0010

Table C.41: Value-at-Risk backtesting results for *RV* for Wheat Futures. Note that UC is the unconditional coverage test by Kupiec (1995), CC is the conditional coverage test by Christoffersen (1998), α is the Value-at-Risk level, Position corresponds with the trading position, which is either long (risk of rising prices) or short (risk of falling prices). The test results are presented as p-values from the UC and CC test for the forecast horizons 1-, 5-, 10-, 22-, 44-, and 66-days ahead.