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# The Minkowski quantum vacuum does not gravitate

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**Abstract**

We show that a non-zero renormalised value of the zero-point energy in $\lambda\phi^4$-theory over Minkowski spacetime is in tension with the scalar-field equation at two-loop order in perturbation theory.
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## 1. Introduction

Quantum fields give rise to an infinite vacuum energy density [1–4], which arises from quantum-field fluctuations taking place even in the absence of matter. Yet, assuming that semi-classical quantum field theory is reliable only up to the Planck-energy scale, the cut-off estimate yields zero-point-energy density which is finite, though, but in a notorious tension with astrophysical observations.

In the presence of matter, however, there are quantum effects occurring in nature, which cannot be understood without quantum-field fluctuations. These are the spontaneous emission of a photon by excited atoms, the Lamb shift, the anomalous magnetic moment of the electron, and so forth [5]. This means quantum-field fluctuations do manifest themselves in nature and, hence, the zero-point energy poses a serious problem.

Lorentz symmetry implies that vacuum stress-energy tensor is proportional to the metric tensor [2]. As a consequence, the vacuum energy density must equal a quarter of the stress-tensor trace. In the case of Maxwell theory, the photon field cannot thus give a non-vanishing Lorentz-invariant vacuum stress tensor, due to conformal invariance of the theory. Still, its vacuum energy

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density is, rigorously speaking, infinite. This tension can be re-solved if the stress-energy tensor is (properly) regularised and then renormalised to zero. This procedure does not result in a fine-tuning problem, because that is actually required for a self-consistent description of the quantum vacuum. In view of this argument, the purpose of this article is to re-consider the zero-point energy in the context of interacting quantum fields by exploiting non-linear equations they satisfy.

Throughout, we use natural units with $c = G = \hbar = 1$, unless otherwise stated.

## 2. Zero-point energy

### *2.1. Quantum kinetic theory*

The vacuum expectation value of the stress-energy-tensor operator $\hat{T}^{\mu}_{\nu}(x)$ of a massive non-interacting quantum scalar field, $\hat{\phi}(x)$, in Minkowski spacetime reads

$$\langle 0|\hat{T}^{\mu}_{\nu}(x)|0\rangle = \frac{1}{2(2\pi)^3}\int \frac{d^3\mathbf{p}}{p_0}\, p^{\mu} p_{\nu}\,, \tag{1}$$

where $p_0 = (\mathbf{p}^2 + m^2)^{\frac{1}{2}}$ and $m$ is the scalar-field mass. The state $|0\rangle$ denotes the Minkowski vacuum of the non-interacting theory. This integral quartically diverges and, thereby, leads to the zero-point-energy problem [1–4].

Back in 1924, Bose suggested a phase-space description of photons constituting black-body radiation [6]. It seems that we can go one step further in this direction, by assuming that "virtual particles" also can be described by a distribution function. Namely, in kinetic theory, the stress-energy tensor is defined through a distribution function according to

$$T^{\mu}_{\nu}(x) = \int \frac{d^3\mathbf{p}}{p_0}\, f(x,p)\, p^{\mu} p_{\nu}\,. \tag{2}$$

The function $f(x,p)$ corresponds to a distribution of $p$ at each point $x$, in the sense that $f(x,p)\, d^3\mathbf{x}\, d^3\mathbf{p}$ gives the average number of particles in the volume element $d^3\mathbf{x}$ at $\mathbf{x}$ with momenta from the interval $(\mathbf{p}, \mathbf{p} + d\mathbf{p})$ [7]. Leaving aside the physical interpretation of $f(x,p)$, we wish *formally* to introduce a distribution function of quantum-field fluctuations in the Minkowski vacuum, i.e.

$$f_0(x,p) = \frac{1}{2(2\pi)^3}\,, \tag{3}$$

which, if inserted into (2), gives (1). This circumstance provides a motivation for the introduction of $f_0(x,p)$.

In general, a distribution function can be defined with the help of the covariant Wigner function

$$w_{\omega}(x,p) \equiv \int \frac{d^4y}{(2\pi)^4}\, e^{ipy}\, \langle\omega|\hat{\phi}(x+\tfrac{1}{2}y)\hat{\phi}(x-\tfrac{1}{2}y)|\omega\rangle\,, \tag{4}$$

where $|\omega\rangle$ is a given quantum state [7].[1] The second moment of the Wigner function corresponds to the stress-energy tensor:

[1] Note that the Wigner function is renormalised in [7] through the normal ordering. We refrain from doing that, as our goal is to study the vacuum energy.

$$\langle\omega|\hat{T}^{\mu}_{\nu}(x)|\omega\rangle = \int d^4p\, w_\omega(x,p)\, p^\mu p_\nu\,. \tag{5}$$

In particular, substituting the Wightman two-point function

$$\langle 0|\hat{\phi}(x)\hat{\phi}(y)|0\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2k^0}\, e^{-ik(x-y)}\,, \tag{6}$$

where $k^0 = (\mathbf{k}^2+m^2)^{\frac{1}{2}}$, into (4) with $|\omega\rangle = |0\rangle$, we find

$$w_0(x,p) = \frac{1}{(2\pi)^3}\,\theta(p_0)\,\delta\big(p^2-m^2\big) \equiv \frac{1}{p_0}\, f_0(x,p)\,\delta\big(p_0 - (\mathbf{p}^2+m^2)^{\frac{1}{2}}\big)\,, \tag{7}$$

where $f_0(x,p)$ agrees with (3).

The Wigner function provides the phase-space description of a quantum system. As a consequence, the distribution function can be of use to couple various observables in local quantum field theory. Bearing in mind the standard ultraviolet divergences in particle physics, it is tempting to conjecture that there is a relation between them and the vacuum-energy problem. We now wish to study how the Wigner distribution is related to elementary particle physics and its renormalisation procedure.

### 2.2. $\lambda\phi^4$-theory

The Standard Model of particle physics is a theory of interacting quantum fields [8]. To simplify our analysis, we intend to study the simplest non-linear field model, namely $\lambda\phi^4$-theory. This model is described by

$$S[g,\phi] = \int d^4x\sqrt{-g}\,\mathcal{L}(g,\phi)\,, \tag{8}$$

where $g$ is a determinant of the metric tensor $g_{\mu\nu}$ [9], which equals $\eta_{\mu\nu} = \text{diag}[+1,-1,-1,-1]$ in Minkowski spacetime, and

$$\mathcal{L}(g,\phi) = \frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\,m_0^2\phi^2 - \frac{\lambda_0}{4!}\,\phi^4 \tag{9}$$

with the bare mass $m_0$ and coupling constant $\lambda_0$. Note that a constant term can be always added to $\mathcal{L}(g,\phi)$ without changing the scalar-field dynamics. At quantum level, this shift of $\mathcal{L}(g,\phi)$ can be accounted for the ambiguity in working with local products of operator-valued distributions which represent quantum fields [10].[2]

For the same reason, radiative corrections to $m_0$ and $\lambda_0$ diverge. All divergences must be regularised and then absorbed into the non-physical parameters $m_0$, $\lambda_0$ and the field-strength renormalisation factor $Z$ which is defined as follows:

$$\hat{\phi}_r(x) \equiv Z^{-\frac{1}{2}}\hat{\phi}(x)\,, \tag{10}$$

where the index $r$ stands for "renormalised", and

[2] In this sense, the Wigner distribution is well-defined as based on the product of quantum fields placed at different space-time points, in contrast to its moments, due to the integration over momentum space.

$$\delta_Z \equiv Z - 1 = \sum_{n=1}^{\infty} \lambda^n \delta_Z^{(n)}\,, \tag{11a}$$

$$\delta_m \equiv m_0^2 Z - m^2 = \sum_{n=1}^{\infty} \lambda^n \delta_m^{(n)}\,, \tag{11b}$$

$$\delta_\lambda \equiv \lambda_0 Z^2 - \lambda = \sum_{n=2}^{\infty} \lambda^n \delta_\lambda^{(n)}\,, \tag{11c}$$

according to [8], where $m$ and $\lambda$ are, respectively, the physical mass and coupling constant.

*2.2.1. Vacuum Wigner function*

In the presence of non-linear terms in the scalar-field equation, we define the Wigner function of the Minkowski vacuum as follows:

$$w_\Omega(x,p) \equiv \int \frac{d^4y}{(2\pi)^4}\, e^{ipy}\, \langle\Omega|\hat{\phi}_r(x+\tfrac{1}{2}y)\hat{\phi}_r(x-\tfrac{1}{2}y)|\Omega\rangle\,, \tag{12}$$

where $|\Omega\rangle$ denotes the Minkowski state of the interacting theory. Since $|\Omega\rangle$ is invariant under the Poincaré group, the vacuum Wigner function cannot depend on the space-time point $x$ and, therefore, we shall denote $w_\Omega(x,p)$ by $w_\Omega(p)$ in what follows.

The distribution function can be used to compute the stress tensor in the Minkowski vacuum. Varying (8) with respect to $g_{\mu\nu}$ and then setting $g_{\mu\nu} = \eta_{\mu\nu}$, we find

$$\hat{T}_\nu^\mu = \partial^\mu\hat{\phi}\,\partial_\nu\hat{\phi} - \frac{1}{2}\,\delta_\nu^\mu\Big[(\partial\hat{\phi})^2 - m_0^2\hat{\phi}^2 - \frac{\lambda_0}{12}\,\hat{\phi}^4\Big]. \tag{13}$$

From this result and (12), we obtain

$$\langle\Omega|\hat{T}_\nu^\mu(x)|\Omega\rangle = Z\int d^4p\, w_\Omega(p)\, p^\mu p_\nu - \frac{1}{2}\delta_\nu^\mu\left[m^2\delta_Z - \delta_m - \frac{\lambda+\delta_\lambda}{4}\int d^4k\, w_\Omega(k)\right]\int d^4p\, w_\Omega(p)\,. \tag{14}$$

In the derivation of (14), we have taken into account that the Minkowski vacuum is Gaussian in $\lambda\phi^4$-theory, implying that $\langle\Omega|\hat{\phi}_r^4(x)|\Omega\rangle = 3(\langle\Omega|\hat{\phi}_r^2(x)|\Omega\rangle)^2$ generically holds, and

$$\langle\Omega|\hat{\phi}_r^2(x)|\Omega\rangle = \int d^4p\, w_\Omega(p)\,, \tag{15}$$

which directly follows from (12).

Next, we find from the scalar-field equation,

$$\left[Z\big(\Box_x + m_0^2\big) + \frac{\lambda_0 Z^2}{3!}\,\hat{\phi}_r^2(x)\right]\hat{\phi}_r(x) = 0\,, \tag{16}$$

that

$$\left[Z\big(p^2 - m_0^2\big) - \frac{\lambda_0 Z^2}{2}\int d^4k\, w_\Omega(k)\right] w_\Omega(p) = 0\,, \tag{17}$$

where we have taken into consideration that the state $|\Omega\rangle$ is Lorentz invariant and Gaussian, providing for $\langle\Omega|\hat{\phi}_r(y)\hat{\phi}_r^3(x)|\Omega\rangle = 3\langle\Omega|\hat{\phi}_r(y)\hat{\phi}_r(x)|\Omega\rangle\langle\Omega|\hat{\phi}_r^2(x)|\Omega\rangle$ (cf. (14) in Sec. III.2 of [7]). Integrating (17) over $p$ and then substituting into (14), we find

$$\langle\Omega|\hat{T}^{\mu}_{\mu}(x)|\Omega\rangle = \int d^4p\, w_\Omega(p)\big[2(p^2-m^2)Z + m_0^2 Z\big]\,. \tag{18}$$

Alternatively, taking the trace of (13) and then using the scalar-field equation to eliminate the quartic term, we obtain

$$\langle\Omega|\hat{T}^{\mu}_{\mu}(x)|\Omega\rangle = m_0^2 Z\langle\Omega|\hat{\phi}_r^2(x)|\Omega\rangle\,, \tag{19}$$

where we have employed $\langle\Omega|\partial\hat{\phi}_r\partial\hat{\phi}_r + \hat{\phi}_r\Box\hat{\phi}_r|\Omega\rangle = 0$, coming from $\partial\hat{\phi}_r = i[\hat{P},\hat{\phi}_r]$, where $\hat{P}$ is the space-time translation operator, and $\hat{P}|\Omega\rangle = P|\Omega\rangle$, where $P \in \mathbb{R}^4$. Comparing these traces with (15) taken into account, we find that $\big(p^2-m^2\big)w_\Omega(p) = 0$. This results in

$$w_\Omega(p) = \frac{1}{(2\pi)^3}\,\theta(p_0)\,\delta\big(p^2-m^2\big)\,\delta_w\,. \tag{20}$$

The as-yet-unknown parameter

$$\delta_w \equiv \sum_{n=0}^{\infty}\lambda^n\delta_w^{(n)} \tag{21}$$

needs to be determined, where, however, $\delta_w^{(0)} = 1$ must hold as it follows from (7).

Now, substituting $w_\Omega(p)$ in (17) and then integrating over $p$, we find the main equation of this section:

$$\big[m^2\delta_Z - \delta_m\big]\int d^4p\, w_\Omega(p) = \frac{\lambda+\delta_\lambda}{2}\left[\int d^4p\, w_\Omega(p)\right]^2. \tag{22}$$

Note, (22) does not exist if $\lambda = 0$. But, since $\lambda \neq 0$, we have two solutions of this equation, namely

$$\frac{1}{2}\int d^4p\, w_\Omega(p) = 0\,, \tag{23a}$$

$$\frac{1}{2}\int d^4p\, w_\Omega(p) = \frac{m^2\delta_Z - \delta_m}{\lambda+\delta_\lambda}\,. \tag{23b}$$

The trivial solution (23a) corresponds to the absence of the (regularised) zero-point energy, while the non-trivial solution (23b) implies that the vacuum energy is non-zero.

We find from (20) and (23b) that

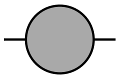
$$\delta_w^{(0)} = \frac{2(4\pi)^{\frac{d}{2}}}{(m^2)^{\frac{d}{2}-1}}\,\frac{m^2\delta_Z^{(1)} - \delta_m^{(1)}}{\Gamma\big(1-\frac{d}{2}\big)}\,, \tag{24a}$$

$$\delta_w^{(1)} = \frac{2(4\pi)^{\frac{d}{2}}}{(m^2)^{\frac{d}{2}-1}}\,\frac{m^2\delta_Z^{(2)} - \delta_m^{(2)} - \big[m^2\delta_Z^{(1)} - \delta_m^{(1)}\big]\delta_\lambda^{(2)}}{\Gamma\big(1-\frac{d}{2}\big)} \tag{24b}$$

and so forth, where we have used dimensional regularisation, $4 \to d < 4$, on the left-hand side of (23b). It follows from (24) that $\delta_w$ can be determined by computing $\delta_Z$, $\delta_m$ and $\delta_\lambda$. These come in turn from the renormalisation of self-energy and vertex diagrams [8].

*2.2.2. Self-energy renormalisation*

The Feynman propagator gets loop corrections which change its pole structure. We can represent this circumstance pictorially as follows:

 $$= \frac{i}{p^2 - m^2 - M^2(p)}\,, \tag{25}$$

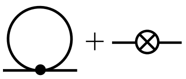
where the shaded circle denotes the sum of all possible (one-particle irreducible) self-energy diagrams. According to the standard renormalisation conditions [8], $M^2(p)$ satisfies

$$M^2(p)\big|_{p^2=m^2} = 0\,, \tag{26a}$$

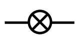
$$\frac{d}{dp^2}\,M^2(p)\big|_{p^2=m^2} = 0\,. \tag{26b}$$

These conditions turn out to be sufficient to compute $\delta_w$ up to the linear order in the coupling constant $\lambda$.

Specifically, at one-loop order, we have

$-iM^2(p) =$  , (27)

where the counter-term vertex

 $$= i\big(p^2\delta_Z - \delta_m\big)\,. \tag{28}$$

It is straightforward to compute the first diagram in (27), which gives from the renormalisation conditions (26) the following results (cf. (10.30) in [8]):

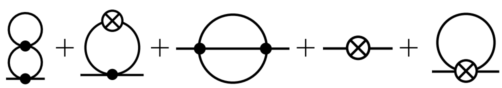
$$\delta_m^{(1)} = -\frac{1}{2}\,\frac{(m^2)^{\frac{d}{2}-1}}{(4\pi)^{\frac{d}{2}}}\,\Gamma\big(1-\tfrac{d}{2}\big)\,, \tag{29a}$$

$$\delta_Z^{(1)} = 0\,. \tag{29b}$$

Substituting (29) into (24a), we obtain

$$\delta_w^{(0)} = 1\,. \tag{30}$$

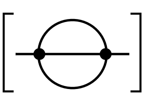
At two-loop order, however, we find

$-iM^2(p) =$  , (31)

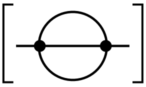
where the counter-term vertex

 $$= -i\delta_\lambda\,. \tag{32}$$

The first two diagrams in (31) cancel each other, whereas the last three diagrams give

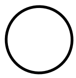
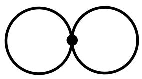
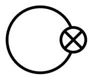
 $$= -i\lambda^2\Big(m^2\delta_Z^{(2)} - \delta_m^{(2)} + \delta_m^{(1)}\delta_\lambda^{(2)}\Big)\,, \tag{33}$$

where the renormalisation condition (26a) has been taken into account and the square brackets mean that the diagram is taken on the mass shell, i.e. $p^2 = m^2$. Making use of (29) and (33), we obtain from (24b) that

$$\delta_w^{(1)} = \frac{2(4\pi)^{\frac{d}{2}}}{(m^2)^{\frac{d}{2}-1}} \frac{i/\lambda^2}{\Gamma\big(1-\frac{d}{2}\big)} \Bigg[$$  $$\Bigg] \neq 0\,. \tag{34}$$

*2.2.3. Vacuum bubbles*

From another side, the parameter $\delta_w$ can be determined by considering vacuum-bubble diagrams. In general, we have

$$-i\,\langle\Omega|\hat{T}_0^0(x)|\Omega\rangle =$$  $+$  $+$  $$+\,\mathrm{O}\big(\lambda^2\big)\,. \tag{35}$$

Up to the linear order in the coupling constant $\lambda$, we find from (14) with (20) that

$$\begin{aligned}\langle\Omega|\hat{T}_0^0(x)|\Omega\rangle = {} & \frac{1}{d}\frac{(m^2)^{\frac{d}{2}}}{(4\pi)^{\frac{d}{2}}}\Gamma\big(1-\tfrac{d}{2}\big)\big(\delta_w^{(0)}+\lambda\delta_w^{(1)}\big) \\ & -\frac{\lambda}{8}\frac{(m^2)^{d-2}}{(4\pi)^d}\Gamma^2\big(1-\tfrac{d}{2}\big)\big(2-\delta_w^{(0)}\big)\,\delta_w^{(0)} + \mathrm{O}\big(\lambda^2\big)\,,\end{aligned} \tag{36}$$

where (29) have been used.

Computing the first diagram in (35), we find

$$\delta_w^{(0)} = 1\,, \tag{37}$$

which is consistent with the result (30). The second and third diagrams shown in (35) give

$$\delta_w^{(1)} = 0\,, \tag{38}$$

which does not agree with the result (34).

## 2.3. Discussion

The tension between (34) and (38) is not an artifact of dimensional regularisation. In fact, it is straightforward to show that the same problem also takes place at two-loop order in Pauli-Villars regularisation.

To summarise, it follows from the non-vanishing value of the vacuum energy density that the Wigner function of the Minkowski state, $w_\Omega(p)$, is non-zero. But, the substitution of $w_\Omega(p)$ into (22) does not solve this equation at two-loop order in perturbation theory. Thus, $w_\Omega(p) \neq 0$ violates the scalar-field equation. Since this is the main equation describing the quantum-field dynamics, its violation is unacceptable. According to (22), the only option left is to choose its trivial solution (23a). Consequently, $w_\Omega(x,p)$ defined in (12) has to be renormalised to zero. This implies in turn that the Minkowski vacuum has neither energy nor pressure.

Alternatively, we may consider $\mathcal{L}(g,\phi) \to \mathcal{L}(g,\phi) - \Lambda_0/8\pi$, where $\mathcal{L}(g,\phi)$ is defined in (9) and $\Lambda_0$ is a constant which cancels the sum of all vacuum-bubble diagrams in the scalar-field model. This procedure does not pose a fine-tuning problem, as that is a self-consistency condition.

As emphasised above, the crucial equation (22), which results in the tension between the renormalisation conditions (26) and the non-renormalised Wigner function, arises from the scalar-field equation. We have studied $\lambda\phi^4$-theory so far, but the same conclusion holds for other non-linear models. For example, the field equation in $g\phi^3$-theory gives

$$\frac{g_0 Z^{\frac{3}{2}}}{2} \int d^4 p \, w_\Omega(p) = 0 \,. \tag{39}$$

We are again forced to renormalise the Wigner function here, in such a way (39) is trivially satisfied.

The microscopic effects due to quantum-field fluctuations are observable only in the presence of matter. The fact that the renormalised vacuum Wigner function has to vanish does not lead to any tension with the up-to-date observations. It is because vacuum-bubble diagrams do not have external legs and, for this reason, these diagrams are not coupled through non-gravitational fields to elementary particles or atoms.

## 3. Conclusion

The Universe we observe is not Minkowski spacetime, although we have made this approximation as it is common in elementary particle physics [8]. In particular, the expectation value of (13) in a given quantum state represents a source term in the Einstein field equations, which distorts the spacetime. Still, the Universe locally looks as Minkowski spacetime, in accordance with the Equivalence Principle. Our computations are therefore valid in any local Lorentz frame. The conclusion is then that the Minkowski quantum vacuum in elementary particle physics is not a source of the gravitational field.

In curved spacetime, the vacuum expectation value of the stress-energy-tensor operator acquires, in general, curvature-dependent corrections. On dimensional grounds, the first non-trivial covariantly-conserved correction has the form

$$G^\mu_\nu(x) \langle \Omega | \hat{\phi}^2(x) | \Omega \rangle \,, \tag{40}$$

where $G^\mu_\nu(x)$ is the Einstein tensor and we have taken into account that $\langle \Omega | \hat{\phi}^2(x) | \Omega \rangle$ does not depend on $x$, due to local homogeneity of the Minkowski state. This correction quadratically diverges, while higher-order curvature corrections are at worst logarithmically divergent. Hence, (40) leads to an infinite shift of the inverse gravitational constant, $1/G$. This shift coincides up to a numerical factor with the result from effective-action computations (see, e.g., Sec. 6.2 in [9]). However, according to the self-consistency argument, the renormalised Wigner function is zero, which, according to (15), implies that (40) vanishes.

Yet, the very fact that the Minkowski vacuum does not gravitate in elementary particle physics does not solve the main cosmological constant problem. Indeed, the Higgs condensate contributes a negative energy density of order $(100\,\mathrm{GeV})^4$ to the cosmological constant [3]. The energy density of dark energy is, however, of order $(0.001\,\mathrm{eV})^4$. Besides, it seems that the vacuum in quantum chromodynamics makes a contribution of order $(0.3\,\mathrm{GeV})^4$ to the total vacuum energy of the Universe [4]. The former contribution may be got rid of by re-defining the Higgs potential, while the latter, probably, requires novel ideas, such as [10,11], to harmonise the Standard Model with astrophysical observations.

Admittedly, the physical meaning of the distribution function of "virtual particles" is not entirely clear. In this article, we have used this concept as a mathematical tool to study physics of the zero-point energy. We hope to come back to this question later.

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