

A Modulation Format Correction Formula for the Gaussian Noise Model in the Presence of Inter-Channel Stimulated Raman Scattering

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Abstract—A closed-form formula is derived, which corrects for the modulation format dependence of the Gaussian Noise (GN) model in the presence of inter-channel stimulated Raman scattering (ISRS). The analytical result enables a rapid estimate of the nonlinear interference (NLI) for arbitrary modulation formats and avoids the need for complex integral evaluations and split-step simulations. It is shown that the modulation format dependent NLI can be approximated by two contributions, one originating from a single span and one asymptotic contribution for a large number of spans. The asymptotic contribution is solved in closed-form for arbitrary links consisting of identical spans, making the result applicable for fiber systems using lumped, distributed or hybrid amplification schemes. The methodology is applied to the ISRS GN model and a modulation format correction formula in closed-form is derived which accounts for an arbitrary number of spans, inter-channel stimulated Raman scattering, arbitrary launch power distributions and wavelength dependent dispersion and attenuation. The proposed formula is validated by numerical simulations over the entire C+L band for multiple fiber types.

Index Terms—Optical fiber communications, Gaussian noise model, Nonlinear interference, Stimulated Raman Scattering, C+L band transmission, closed-form approximation, modulation format correction

I. INTRODUCTION

ANALYTICAL models experienced substantial popularity over the last decade, as they provide a low complexity alternative to split-step simulations with good agreement to experiments [1]–[7]. Perturbation models that estimate nonlinear interference (NLI) are key for rapid and efficient system design [8], achievable rate estimations of point-to-point links [9]–[11] and physical layer aware network optimization [12], [13].

The Gaussian Noise (GN) model [14]–[16] is extensively applied throughout research and industry, as it offers very low computational complexity while being reasonably accurate for high cardinality modulation formats. Additionally, the GN model offers approximations in closed-form for a variety of optical transmission scenarios [10], [14], [16]–[22]. Such closed-form approximations enable performance estimations in sub-milliseconds and are vital for real-time applications and on-the-fly optimizations.

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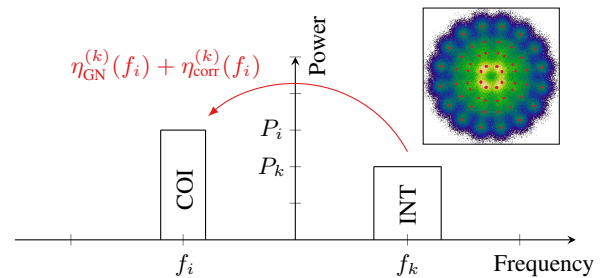


Fig. 1: Illustration of the NLI contribution of the Gaussian Noise (GN) model and the modulation format correction on the channel of interest (COI) originating from a single interferer (INT). The channels exhibit arbitrary modulation formats, power levels, bandwidths and center frequencies. The shown modulation format is taken from [23].

The major source of the inaccuracy of the GN model stems from its signal Gaussianity assumption, which states that the signal can be written as a Gaussian process at the transmitter. As a consequence, the model significantly overestimates the NLI for low cardinality modulation formats and in optical links with low accumulated dispersion. More complex models in integral form have been introduced to account for non-Gaussian modulation formats [24]–[30]. However, for those models to be applicable in a real-time environment and to avoid the high computational complexity of modulation format aware approaches, approximations in closed-form are necessary. A closed-form modulation format correction formula in the asymptotic limit of a large span number and optical bandwidths limited to C-band (5 THz) has been derived in [21]. Correcting the modulation format dependence by polynomial data fitting over large data sets (1000 entries) has been proposed in [31].

All of the aforementioned modeling approaches do not account for inter-channel stimulated Raman scattering (ISRS) and are therefore not applicable to ultra-wideband transmission systems that operate at optical bandwidths beyond C-band (5 THz). Extensions of the conventional GN model to account for ISRS, named ISRS GN model, have been recently proposed in integral form [32]–[37]. An approximation in closed-form has been first proposed in [32] which recently has been significantly generalized to include arbitrary launch power distributions, wavelength dependent dispersion and a more accurate description of ISRS [38], [39]. A comparable formula, based on the same approach and with a similar result, has been

reported in [40].

However, the works [32]–[40] assume signal Gaussianity and the impact of the modulation format has not been addressed analytically in the context of ultra-wideband transmission systems, where ISRS cannot be neglected. More importantly, a closed-form formula suitable for real-time and yet accurate performance estimations of arbitrary modulation formats has not been reported to date.

In this paper, a modulation format correction formula for the ISRS GN model is derived. It is shown that the modulation format correction can be, in general, approximated by two contributions. One contribution that accounts for the modulation format correction after one span and one accounting for the correction in the asymptotic limit of a large span number. The asymptotic contribution is solved for an arbitrary transmission system, described by a link function, yielding a generic asymptotic modulation format correction formula. Both contributions, yielding a formula for any number of spans, are solved particularly for the ISRS GN model. The proposed formula accounts for arbitrary launch power distributions, arbitrary number of spans, arbitrary modulation formats, lumped amplification and ultra-wideband effects such as inter-channel stimulated Raman scattering and wavelength dependent dispersion and attenuation. The finding is validated by numerical simulations for a low dispersive non-zero dispersion-shifted fiber (NZDSF) and a high dispersive standard single mode fiber (SMF) over 10 THz optical bandwidth.

The paper is organized as follows: In Section II, the formalism to analytically describe the impact of the modulation format on the NLI is reviewed and the assumptions necessary to solve it in closed-form are described. The methodology is then applied to the ISRS GN model in Section II-C to obtain a modulation format correction formula in closed-form, the key result of this work. The formula is validated by numerical simulations in Sections III and IV.

II. MODULATION FORMAT CORRECTION

After coherent detection, electronic dispersion compensation and neglecting the impact of transceiver noise, the signal-to-noise ratio (SNR) of the channel of interest (COI) i can be calculated as [14, Eq. (7)]

$$\text{SNR}_i \approx \frac{P_i}{P_{\text{ASE}} + \eta_n P_i^3}, \quad (1)$$

where P_i is the launch power of channel i at the transmitter and P_{ASE} is the accumulated amplified spontaneous emission (ASE) noise originating from optical amplifiers. ASE noise inflation as a result of gain equalization can be included by a channel dependent P_{ASE} . The nonlinear interference coefficient $\eta_n(f_i)$ after n fiber spans is dependent on the center frequency f_i of the COI.

Although modulation format independent models, such as the GN model, are in relatively good agreement with experiments using high cardinality modulation formats, the NLI has been shown to be dependent on the transmitted modulation format. This dependence is particularly high for low cardinality modulation formats and short to medium-haul transmission distances. Substantially more complex models

have been introduced in order to accurately model the impact of arbitrary modulation formats [24]–[30]. Most results show that the total NLI can be separated into two contributions, one ‘GN-like’ modulation format independent contribution and a correction term that accounts for the transmitted modulation format. The NLI coefficient can hence be written as [24, Eq. (70)]

$$\eta_n(f_i) = \eta_{\text{GN},n}(f_i) + \eta_{\text{corr},n}(f_i), \quad (2)$$

where $\eta_{\text{GN},n}$ is the GN model contribution and $\eta_{\text{corr},n}$ is the modulation format correction term. For Gaussian modulated signals, the correction term $\eta_{\text{corr},n}$ vanishes and one obtains the GN model description of NLI.

In [28], [29], a complete set of formulas in integral form has been derived to correct for arbitrary modulation formats for self-phase modulation (SPM/SCI), cross-phase modulation (XPM/XCI) and four-wave mixing (FWM/MCI) NLI contributions. In the approach taken in this work, only the most dominant contribution, the cross-phase modulation contribution, is taken into account. It has been shown that this approach is sufficiently accurate with only a minor loss in accuracy [21], [24]–[27]. The approach is particularly accurate in high dispersive fibers such as standard single mode fiber (SMF) and systems with high symbol rates or channel spacings. Two conditions that are prevailing in legacy systems and likely to be installed in next-generation optical transmission systems.

The XPM assumption, taking only XPM-like terms into account, evaluates the NLI within the channel of interest originating from a single interferer (INT). The XPM assumption is illustrated in Fig. 1. The total NLI is obtained by summing over all COI-INT pairs present in the transmitted signal. Mathematically, the total XPM modulation format correction is

$$\eta_{\text{corr},n}(f_i) = \sum_{k=1, k \neq i}^{N_{\text{ch}}} \eta_{\text{corr},n}^{(k)}(f_i), \quad (3)$$

where $\eta_{\text{corr},n}^{(k)}(f_i)$ is the XPM contribution of a single interfering channel k on channel i .

The dominant XPM modulation format correction term has been derived in [29, Eq. (17)], which can be written as a two dimensional integral (cf. [29, Appendix C])

$$\eta_{\text{corr},n}^{(k)}(f_i) = \frac{80}{81} \left(\frac{P_k}{P_i} \right)^2 \frac{\gamma^2 \Phi}{B_k^3} \int_{-\frac{B_i}{2}}^{\frac{B_i}{2}} df_1 \left| \int_{-\frac{B_k}{2}}^{\frac{B_k}{2}} \mu(f_1 + f_i, f_2 + f_i + \Delta f, f_i) \sum_{m=0}^{n-1} e^{jm f_1 (f_2 + \Delta f) \phi} df_2 \right|^2, \quad (4)$$

where B_i is the bandwidth of the COI i , $\Delta f = f_k - f_i$ is the channel frequency separation between COI and INT, γ is the nonlinearity coefficient and $\phi = -4\pi^2 [\beta_2 + \pi\beta_3(f_i + f_k)]L$ is a phase mismatch term accounting for coherent NLI accumulation, with span length L , the group velocity dispersion (GVD) parameter β_2 and its linear slope β_3 at the reference wavelength. In (4), it is assumed that the link consists of

TABLE I: Excess kurtosis of selected modulation formats.

Modulation format	Excess kurtosis Φ
uniform QPSK	-1
uniform 16-QAM	-0.6800
uniform 64-QAM	-0.6190
uniform 256-QAM	-0.6050
uniform ∞ -QAM	-0.6000
geom. shaped 64-QAM (12 dB SNR, GMI) [23]	-0.3403
proba. shaped 64-QAM (12 dB SNR, MI) [42]	-0.1871
Gaussian modulation	0

identical spans and the variation of the wavelength dependent dispersion is negligible over a single channel bandwidth. $\mu(f_1, f_2, f_i)$ is the link function of a single span, which is addressed in more detail in the following sections.

$\Phi = \frac{E[|X|^4]}{E^2[|X|^2]} - 2$ is the excess kurtosis of the transmitted modulation format. The excess kurtosis of a few selected modulation formats are listed in Table I. Most modulation formats exhibit a negative excess kurtosis resulting in a negative modulation format correction $\eta_{\text{corr},n}^{(k)}(f_i)$ and reduced nonlinear interference as a consequence. Loosely spoken, the modulation format correction is smaller (in absolute value) for more 'Gaussian-like' modulation formats. This property does not only extend to the coordinates of the individual symbols but also to their respective probability of occurrence [41].

The aim of the remainder of this paper is to find an approximation in closed-form of the modulation format correction in integral form (4). This gives the advantage of increase prediction accuracy for non-Gaussian modulation formats without sacrificing computation complexity and execution time.

A. Modulation format correction for generic transmission systems in closed-form

In this section, an approximation of the solution of Eq. (4) for a generic transmission system is derived. An optical fiber transmission system that is described by the nonlinear Schrödinger equation can be, to first-order, described by a link function $\mu(f_1, f_2, f_i)$. The link function describes the nonlinear perturbation (i.e. the NLI) of three interacting frequencies on the frequency f_i after propagation.

The approach in this work relies on two key assumptions. The first assumption is that the channel separation of two interfering channels (COI and INT) is much greater than the channel bandwidth, which allows to approximate $f_2 + \Delta f \approx \Delta f$ in (4). Mathematically, this coincides with the zeroth-order solution of the inner integral running over f_2 in (4). A detailed assessment of this assumption was carried out in [39, Appendix C], and negligible impact on the total NLI was found. Applying the assumption results in

$$\eta_{\text{corr},n}^{(k)}(f_i) \approx \tilde{\gamma} \int_{-\frac{B_i}{2}}^{\frac{B_i}{2}} df_1 |\mu(f_1 + f_i, f_k, f_i)|^2 \cdot \left| 1 + \sum_{m=1}^{n-1} \text{sinc}\left(m\phi f_1 \frac{B_k}{2}\right) e^{jm\phi f_1 \Delta f} \right|^2, \quad (5)$$

with $\tilde{\gamma} = \left(\frac{P_k}{P_i}\right)^2 \frac{80}{81} \frac{\gamma^2 \Phi}{B_k}$, which is introduced for notational brevity. A more detailed derivation of (5) starting from (4) can be found in Appendix A.

Eq. (5) does not seem to have an analytical solution. However, in order to find an approximate solution, we analyze its asymptotic behavior for a large number of fiber spans n . For a large number of spans, the oscillating terms in (5) can be approximated by a Dirac delta function $\delta(x)$ as

$$\lim_{n \rightarrow \infty} \left| 1 + \sum_{m=1}^{n-1} \text{sinc}\left(m\phi f_1 \frac{B_k}{2}\right) e^{jm\phi f_1 \Delta f} \right|^2 \approx 1 + \delta(f_1) \lim_{n \rightarrow \infty} C_n, \quad (6)$$

with normalization coefficient

$$C_n = \int_{-\infty}^{\infty} df_1 \left| \sum_{m=1}^{n-1} \text{sinc}\left(m\phi f_1 \frac{B_k}{2}\right) e^{jm\phi f_1 \Delta f} \right|^2. \quad (7)$$

The sinc function is defined as $\text{sinc}(x) = \frac{\sin(x)}{x}$. The normalization coefficient C_n is an immediate consequence of the property $\int_{-\infty}^{\infty} \delta(x) dx = 1$ of the Dirac delta function.

The approximation (6) is the second key assumption in this work and it can be motivated as follows: For $f_2 = 0$, both sides of (6) yield infinity, making the 1 negligible and as a result it can be pulled out of the absolute square term. For $f_2 \neq 0$, the oscillating terms add mostly out-of-phase (deconstructively) and they are further damped by the $\frac{1}{f_1}$ decay in the sinc(x) function. As a consequence, the sum over the oscillating terms can be approximated by a Dirac delta function, the oscillating terms are negligible with respect to 1 and the 1 can be, again, pulled out of the absolute square. The approximation (6) has negligible impact as shown via numerical simulations in Sec. III.

As a result of the asymptotic approximation (6), the NLI coefficient can be approximated by only two contributions. One modulation format correction originating in the first span and an asymptotic contribution originating in the limit of a large span number. The NLI coefficient can be thus written as

$$\eta_{\text{corr},n}^{(k)}(f_i) \approx \underbrace{\eta_{\text{corr},1}^{(k)}(f_i)}_{\text{1st span corr.}} + \tilde{n} \cdot \underbrace{\eta_{\text{corr},a}^{(k)}(f_i)}_{\text{asympt. corr.}}, \quad (8)$$

with

$$\eta_{\text{corr},1}^{(k)}(f_i) = \tilde{\gamma} \int_{-\frac{B_i}{2}}^{\frac{B_i}{2}} df_1 |\mu(f_1 + f_i, f_k, f_i)|^2, \quad (9)$$

$$\eta_{\text{corr},a}^{(k)}(f_i) = \tilde{\gamma} |\mu(f_i, f_k, f_i)|^2 \cdot \lim_{n \rightarrow \infty} \Delta_n C_n, \quad (10)$$

and

$$\tilde{n} = \begin{cases} 0, & \text{if } n = 1 \\ n, & \text{otherwise.} \end{cases} \quad (11)$$

The asymptotic contribution $\eta_{\text{corr},a}^{(k)}(f_i)$ is valid in the limit for a large number of spans. In order to calculate the modulation format correction for any number of spans, the asymptotic contribution is approximation by a Taylor series with respect to the number of spans and truncated to first-order. As the asymptotic contribution after one span must be zero, only its

slope (in the asymptotic limit) must be calculated as in (10). As the span number is an integer variable, the slope is computed using the backward difference operator, defined as $\Delta_n f = f(n) - f(n-1)$. The asymptotic slope can be computed exactly and the necessary identity is derived in Appendix B.

Using identity (18), derived in Appendix B, the asymptotic contribution can be written *exactly* in closed-form as

$$\eta_{\text{corr},a}^{(k)}(f_i) = \tilde{\gamma} |\mu(f_i, f_k, f_i)|^2 \cdot \frac{2\pi}{|\phi| B_k^2} \left[(2\Delta f - B_k) \ln \left(\frac{2\Delta f - B_k}{2\Delta f + B_k} \right) + 2B_k \right]. \quad (12)$$

Eq (12) represents a generic modulation format correction formula, valid in the asymptotic limit of a large span number. Remarkably, the result does not require any further integration and it, therefore, represents a closed-form correction formula for *any* optical fiber transmission system described by a link function $\mu(f_1, f_2, f_i)$ consisting of identical fiber spans. Eq. (12) can be applied to lumped as well as distributed amplification schemes and it is one of the main results in this paper.

To further increase the accuracy for arbitrary span numbers, the integral, describing the modulation format correction contribution after a single span (9), must be solved. However, this integral resembles the mathematical structure of the GN model contribution $\eta_{\text{GN},n}(f_i)$ (cf. [39, Eq. (14)]). In other words, if there exists a closed-form approximation for the GN model contribution after one span for an arbitrary transmission system, a modulation format correction formula in closed-form for any number of spans immediately follows with the results in this paper using (8)(9)(12).

B. The link function of the ISRS GN model

In this section, the link function of the GN model in the presence of inter-channel stimulated Raman scattering is reviewed. Equations (8)(9)(12) are then applied to the ISRS GN model link function in order to derive a modulation format correction formula for arbitrary span numbers. The ISRS GN model represents an extension of the conventional GN model to account for ultra-wideband effects such as the power transfer between propagating frequencies due to ISRS. The generic link function of the ISRS GN model is given by [35, Eq. (9)]

$$\mu(f_1, f_2, f_i) = \int_0^L d\zeta \frac{P_{\text{tot}} e^{-\alpha\zeta - P_{\text{tot}} C_r L_{\text{eff}} (f_1 + f_2 - f_i)}}{\int G_{\text{Tx}}(\nu) e^{-P_{\text{tot}} C_r L_{\text{eff}} \nu} d\nu} \cdot e^{j\tilde{\phi}(f_1, f_2, f_i, \zeta)}, \quad (13)$$

where P_{tot} is the total optical launch power, C_r is the slope of the linear regression of the Raman gain function, $L_{\text{eff}} = \frac{1 - e^{-\alpha\zeta}}{\alpha}$ and $\tilde{\phi} = -4\pi^2(f_1 - f_i)(f_2 - f_i) [\beta_2 + \pi\beta_3(f_1 + f_2)] \zeta$. Eq. (13) can be used to calculate the nonlinear perturbation on f_i after one span for any arbitrary frequency triplet (f_1, f_2, f_i) . Eq. (13) accounts for all occurring nonlinear mixing products, namely self-phase (SPM/SCI), cross-phase (XPM/XCI) and four-wave mixing (FWM/MCI) products. However, the proposed formulas (8)(9)(12), only correct for the dominant mixing products which are XPM

products. This restricts the frequency triplets to the XPM domain which is $(f_1 + f_i, f_k, f_i)$ with $f_1 \in [-\frac{B_i}{2}, \frac{B_i}{2}]$.

In our previous work [39, Eq. (18)], an approximation of (13) has been derived under the XPM assumption and a first-order description of ISRS. It was demonstrated that the ISRS GN model link function is well approximated by

$$\mu(f_1 + f_i, f_k, f_i) \approx -\frac{1 + \tilde{T}_k}{-\alpha + j\phi_{i,k} f_1} + \frac{\tilde{T}_k}{-A + j\phi_{i,k} f_1}, \quad (14)$$

where $\tilde{T}_k = -\frac{P_{\text{tot}} C_r}{\bar{\alpha}} f_k$, $\phi_{i,k} = -4\pi^2(f_k - f_i) [\beta_2 + \pi\beta_3(f_i + f_k)]$ and $A = \alpha + \bar{\alpha}$. If not specified otherwise, it holds that $\bar{\alpha} = \alpha$. The parameter $\bar{\alpha}$ is a generic attenuation parameter that is used to tailor the first-order power profile (14), and therefore the proposed formula, to more general cases. Such cases include improved accuracy for non-uniform (tilted) launch power distributions [35, Sec. IV], wavelength dependent attenuation and even the extension of the formula beyond 15 THz i.e. beyond the triangular region of the Raman gain spectrum. This is done by reinterpreting α , $\bar{\alpha}$ and C_r as channel *dependent* quantities. The parameters are then chosen such that (14) resembles the actual power profile of each channel and the proposed modulation format correction formula can be applied. The drawback of this strategy is larger complexity as the Raman equations must be solved numerically and additional regression operations are necessary in order to obtain the channel dependent α , $\bar{\alpha}$ and C_r .

In order to obtain the modulation format correction of the ISRS GN model in closed-form, the approximated link function (14) must be inserted in (9) and (12). As mentioned in Section II-A, the integral that needs to be executed in (9) resembles the GN model contribution after one span. This integral has been solved in our previous work [39]. Therefore, the modulation format correction for the ISRS GN model can be obtained using the results in this paper and the GN contribution derived in closed-form in [39].

C. Modulation format correction for the ISRS GN model in closed-form

Using the modulation correction formula for a generic system, derived in Section II-A, combined with the approximated link function (14) of the ISRS GN model, yields a modulation format correction formula for the ISRS GN model in closed-form as

$$\eta_{\text{corr},n}(f_i) \approx \frac{80}{81} \Phi \sum_{k=1, k \neq i}^{N_{\text{ch}}} \left(\frac{P_k}{P_i} \right)^2 \frac{\gamma^2}{B_k} \left\{ \frac{1}{\phi_{i,k} \bar{\alpha} (2\alpha + \bar{\alpha})} \cdot \left[\frac{T_k - \alpha^2}{\alpha} \text{atan} \left(\frac{\phi_{i,k} B_i}{\alpha} \right) + \frac{A^2 - T_k}{A} \text{atan} \left(\frac{\phi_{i,k} B_i}{A} \right) \right], \right. \\ \left. + \frac{2\pi \tilde{n} T_k}{|\phi| B_k^2 \alpha^2 A^2} \left[(2|\Delta f| - B_k) \log \left(\frac{2|\Delta f| - B_k}{2|\Delta f| + B_k} \right) + 2B_k \right] \right\}, \quad (15)$$

with $\Delta f = f_k - f_i$ and $T_k = (\alpha + \bar{\alpha} - P_{\text{tot}} C_r f_k)^2$. The formula is applicable for lumped-amplified links for optical

bandwidths of up to 15 THz, as the formula relies on a triangular Raman gain spectrum. For larger optical bandwidths, the variables α , $\bar{\alpha}$ and C_r can be matched to the actual power profile in the fiber and the formula can be applied (cf. Section II-A). The former summand in (15) corrects for the modulation format within a single span, while the latter summand corrects the modulation format contribution across multiple spans as described by (8). The sum in (15) corrects for all interfering channels within the transmitted WDM signal.

III. NUMERICAL VALIDATION

In this section the proposed closed-form correction formula (15) is validated by numerical simulations over the entire C+L band, covering 10.05 THz optical bandwidth. The validation is carried out for two fiber types, one high dispersive standard single mode fiber (SMF) and one low dispersive non-zero dispersion-shifted fiber (NZDSF).

A. Simulation Setup

The validation was carried out by numerically solving the Manakov equation using the well established split-step Fourier method (SSFM). Inter-channel stimulated Raman scattering was included in the SSFM by applying a frequency dependent loss at every linear step, so that the signal power profile in the presence of ISRS, is obtained.

A logarithmic step size distribution was implemented, where $0.25 \cdot 10^6$ simulation steps were found to be sufficient for the considered launch powers and parameters of both fiber types. The launch powers were chosen to maximize the performance of the central channel, assuming a 5 dB noise figure erbium-doped fiber amplifier (EDFA).

Gaussian symbols, drawn from a circular-symmetric Gaussian distribution and uniform QPSK symbols were used for transmission. In the case of QPSK modulation, the channels of interest, centered at $f_i = 0$ THz and $f_i = -4.0$ THz, exhibit Gaussian modulation while the rest of the channels are modulated with QPSK symbols. This was done for validation purposes and only in the results of this section III. This approach offers a more precise validation, as the proposed formula only corrects for modulation formats of interfering channels (XPM/XCI terms).

The receiver consisted of digital dispersion compensation, ideal root-raised-cosine (RCC) matched filtering and constellation rotation. The SNR was ideally estimated as the ratio between the variance of the transmitted symbols $E[|X|^2]$ and the variance of the noise σ^2 , where $\sigma^2 = E[|X - Y|^2]$ and Y represents the received symbols after digital signal processing. The nonlinear interference coefficient was then estimated via Eq. (1). To improve the simulation accuracy, four different data realizations were simulated and averaged for each transmission.

Ideal, noiseless amplifiers were considered to ease the NLI computation and for a fair comparison between numerical simulation and modeling results.

TABLE II: System parameters

Parameters	SMF	NZDSF
Loss (α) [dB/km]	0.2	0.19
Dispersion (D) [ps/nm/km]	17.0	4.5
Dispersion slope (S) [ps/nm ² /km]	0.067	0.05
NL coefficient (γ) [1/W/km]	1.2	1.3
Effective core area [μm^2]	80	72
Raman gain slope (C_r) [1/W/km/THz]	0.028	0.031
Raman gain ($C_r \cdot 14$ THz) [1/W/km]	0.39	0.44
Channel Launch Power (P_i) [dBm]	0	-2
Total Launch Power (P_{tot}) [dBm]	24	22
Symbol rate [GBd]	40	
Channel Bandwidth (B_i) [GHz]	40.004	
Channel spacing [GHz]	40.005	
Number of channels	251	
Optical bandwidth (B_{tot}) [THz]	10.05	
Reference Wavelength [nm]	1550	
Roll-off factor [%]	0.01	
Number of symbols [2^x]	17	
Simulation steps per span [10^6]	0.25	

B. Results

The nonlinear interference coefficient as a function of span number is shown in Fig. 2 and 3 for the channels with center frequencies $f_i = -4.0$ THz and $f_i = 0$ THz. The results are shown for both fiber types and for the case with ISRS and without ISRS. The (unphysical) case of not considering ISRS is shown for comparison.

Markers represent simulation results, while lines represent modelling results. In the case of Gaussian modulation, the ISRS GN model in integral form [35, Eq. (9)] [37, Eq. (2)] was used and is shown in dotted lines. In the case of QPSK modulation, the ISRS GN model in integral form was used with the modulation format correction in integral form (5) (solid lines), as well as the modulation format correction formula in closed form (15) (dashed lines).

The ISRS GN model has remarkable accuracy with numerical simulations exhibiting a negligible modeling error. In the case of QPSK modulation, the modulation format correction in integral form (5) models the impact of QPSK with good accuracy, despite one of the key assumptions made in this work ($\Delta f \gg \frac{B_k}{2}$). The average deviation between the modulation format correction in integral form and the numerical simulation is 0.26 dB throughout the shown results. The error mostly stems from the XPM assumption and assumptions inherited by Eq. (4). The small impact of the assumption $\Delta f \gg \frac{B_k}{2}$ has been mathematically shown in [35, Appendix C] and is, therefore, not surprising.

The modulation format correction formula in closed-form (15) shows good accuracy throughout all number of spans. Throughout Fig. 2 and 3, the average absolute error is 0.45 dB between closed-form and numerical simulation. The majority of the mismatch can be traced back to the asymptotic assumption (6) and its linear approximation of the asymptotic contribution (10) with respect to the span number (see Section II-A). The mismatch is smaller for an increasing number of spans. Particularly, the mismatch between integral and closed-form vanishes in the case of single span transmission and

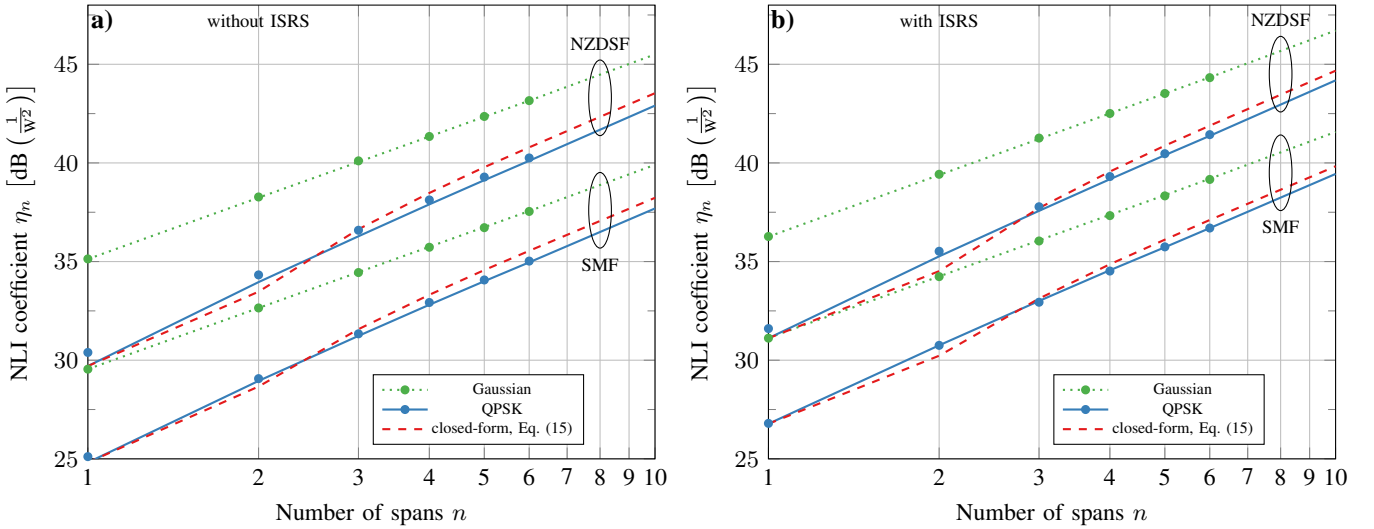


Fig. 2: The nonlinear interference coefficient as a function of fiber spans for the channel centered at $f_i = -4.0$ THz. The results were obtained by split-step simulations (markers) and the ISRS GN model [35, Eq. (9)] with the modulation format correction in integral form (5) (solid lines). The ISRS GN model with the modulation format correction in closed-form (15) is shown in dashed lines. In the case of QPSK, only the channel of interest exhibits Gaussian modulation for validation purposes.

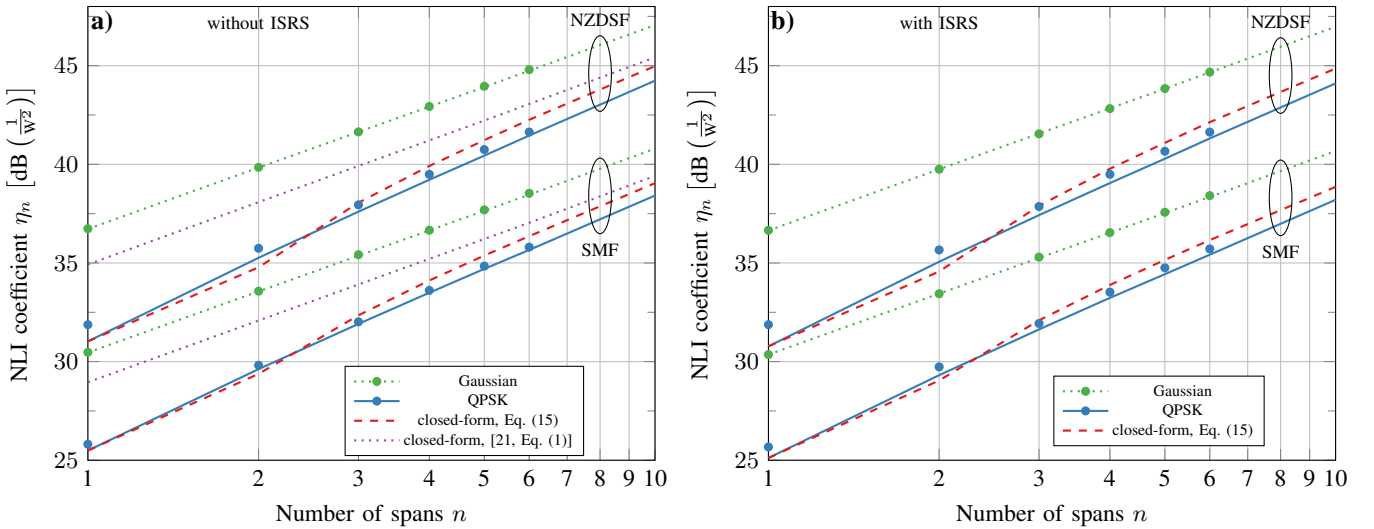


Fig. 3: The nonlinear interference coefficient as a function of fiber spans for the channel centered at $f_i = 0$ THz. The results were obtained by split-step simulations (markers) and the ISRS GN model [35, Eq. (9)] with the modulation format correction in integral form (5) (solid lines). The ISRS GN model with the modulation format correction in closed-form (15) is shown in dashed lines. In the case of QPSK, only the channel of interest exhibits Gaussian modulation for validation purposes. For comparison the result of [21, Eq. (1)] is shown, which proposed a modulation format correction formula in the absence of ISRS.

in the limit of infinite transmission spans. Not shown in the figures, the mismatch between modulation format correction in closed-form and integral form is < 0.1 dB after 100 spans. This shows that the approximation in (6) imposed, indeed, a negligible approximation error on the proposed formula.

The result in this paper is the first modulation format correction formula in the presence of ISRS. However, in the absence of ISRS (e.g. for optical bandwidths of at most 5 THz), modulation format correction formulas are already available in the literature. To compare our results to previously published works, [21, Eq. (1)] is shown in Fig. 3a), which proposed a modulation format correction formula in the absence of ISRS. As the result in [21] has been derived in the asymptotic limit of a large number of spans, it is rather

inaccurate for the first few spans. In the limit of a large span number and without ISRS, [21, Eq. (1)] is similar to the formula proposed in this paper. The comparison shows that (15) is not only capable of correcting the modulation format in ISRS impaired systems, but it also extends previously published results in the absence of ISRS for transmission over arbitrary span numbers.

IV. THE ISRS GN MODEL FOR ARBITRARY MODULATION FORMATS IN CLOSED-FORM

In this section, we combine the modulation format correction formula (15) with the ISRS GN model contribution in closed-form derived in [39]. The result is a closed-form formula, capable of predicting the total nonlinear performance

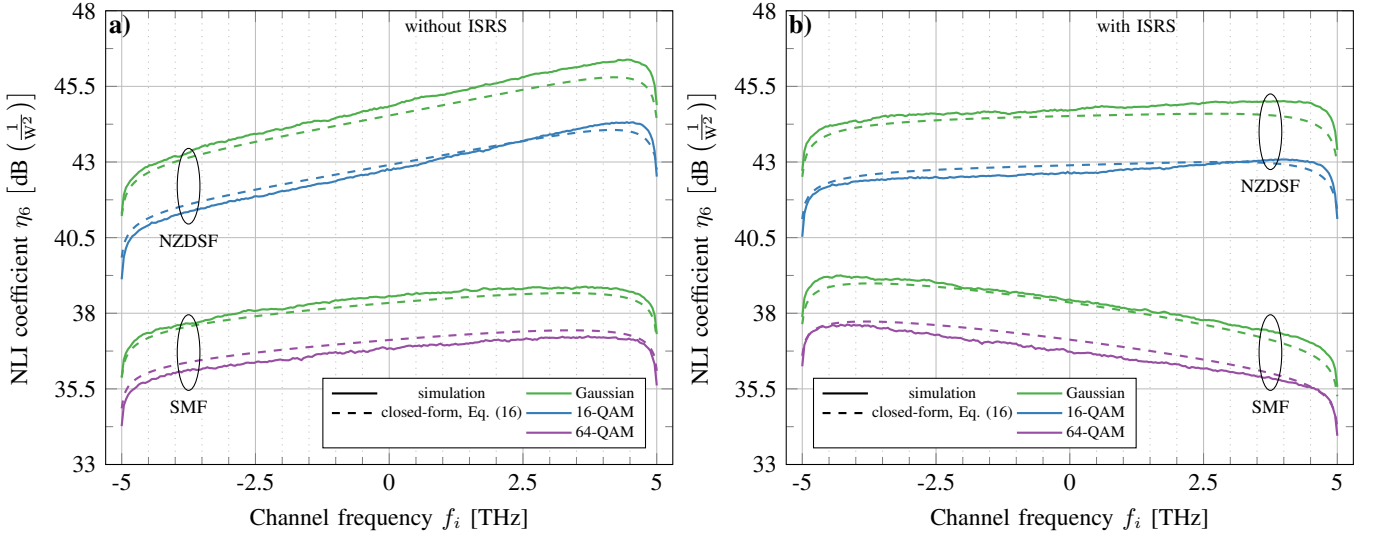


Fig. 4: The nonlinear interference coefficient as a function of channel frequency for different modulation formats and fiber types after 6x100 km spans. The results were obtained by split-step simulations and in closed-form using Eq. (16). Figures a) and b) show the case without and with considering inter-channel stimulated Raman scattering, respectively.

for arbitrary modulation formats. The proposed formula is once again validated by numerical simulations for the two fiber types with parameters listed in Table II. The formula for the *total* NLI coefficient as in (2) is

$$\begin{aligned} \eta_n(f_i) \approx & \frac{4}{9} \frac{\gamma^2}{B_i^2} \frac{\pi n^{1+\epsilon}}{\phi_i \bar{\alpha} (2\alpha + \bar{\alpha})} \\ & \cdot \left[\frac{T_i - \alpha^2}{a} \operatorname{asinh} \left(\frac{\phi_i B_i^2}{\pi a} \right) + \frac{A^2 - T_i}{A} \operatorname{asinh} \left(\frac{\phi_i B_i^2}{\pi A} \right) \right] \\ & + \frac{32}{27} \sum_{k=1, k \neq i}^{N_{\text{ch}}} \left(\frac{P_k}{P_i} \right)^2 \frac{\gamma^2}{B_k} \left\{ \frac{n + \frac{5}{6} \Phi}{\phi_{i,k} \bar{\alpha} (2\alpha + \bar{\alpha})} \right. \\ & \cdot \left[\frac{T_k - \alpha^2}{\alpha} \operatorname{atan} \left(\frac{\phi_{i,k} B_i}{\alpha} \right) + \frac{A^2 - T_k}{A} \operatorname{atan} \left(\frac{\phi_{i,k} B_i}{A} \right) \right], \\ & \left. + \frac{5}{3} \frac{\Phi \tilde{n} T_k}{|\phi| B_k^2 \alpha^2 A^2} \left[(2|\Delta f| - B_k) \log \left(\frac{2|\Delta f| - B_k}{2|\Delta f| + B_k} \right) + 2B_k \right] \right\} \end{aligned} \quad (16)$$

with $\phi_i = \frac{3}{2} \pi^2 (\beta_2 + 2\pi\beta_3 f_i)$, $T_k = (\alpha + \bar{\alpha} - P_{\text{tot}} C_r f_k)^2$, $\Delta f = f_k - f_i$, $\phi_{i,k} = -4\pi^2 (f_k - f_i) [\beta_2 + \pi\beta_3 (f_i + f_k)]$, $A = \alpha + \bar{\alpha}$ and coherence factor ϵ . The coherence factor can be obtained in closed-form from [16, Eq. (22)]. Eq. (16) models the GN model contribution of the SPM and XPM contributions, where the XPM terms are corrected for its modulation format dependence using (15). The reader is referred to [39] for more details about the GN model contribution.

The total NLI coefficient as a function of channel frequency with and without ISRS is shown in Fig. 4. The results are shown for SMF and NZDSF. All transmitted channels are entirely modulated with either Gaussian, 16-QAM or 64-QAM symbols. Overall, Eq. (16) shows very good agreement with the numerical results. In both cases (with and without ISRS), the average mismatch between the closed-form model (16) and numerical simulation is 0.3 dB and 0.2 dB for SMF and NZDSF, respectively.

The analysis shows that the derived closed-form approximation (16) is capable of predicting nonlinear performance of ultra-wideband optical transmission systems considering arbitrary modulation formats with sufficient accuracy. A key result in the real-time modeling of next-generation ultra-wideband transmission systems.

V. CONCLUSION

A methodology was presented to derive the modulation format correction in closed-form for arbitrary optical transmission systems that are described by a link function. The generic approach is applied to the Gaussian Noise model in the presence of inter-channel stimulated Raman scattering (ISRS GN model) to derive a closed-form formula that predicts the impact of modulation formats on the nonlinear interference in ultra-wideband transmission. The derived formula accounts for arbitrary span numbers, arbitrary launch power distributions, inter-channel stimulated Raman scattering and wavelength dependent dispersion and attenuation.

The analytical result was validated by numerical simulations over the entire C+L band (10 THz), with an average prediction error in nonlinear interference of 0.3 dB for standard single-mode fiber (SMF) and 0.2 dB for non-zero dispersion-shifted fiber (NZDSF).

The results in this paper are the first analytical investigation of the impact of modulation formats in ultra-wideband optical transmission and the first closed-form formula accounting for it. A significant result for the design of optical communication systems and the real-time performance modeling of ultra-wideband transmission systems.

APPENDIX A DERIVATION OF EQ. (5)

This section presents the key steps in the derivation of Eq. (5). For the sake of brevity, the derivation is carried out for

the *integrand* of (5). Starting from the integrand in (4) and assuming that $f_2 + \Delta f \approx \Delta f$ for the link function μ , we have that

$$\begin{aligned}
& \left| \int_{-\frac{B_k}{2}}^{\frac{B_k}{2}} \mu(f_1 + f_i, f_2 + f_i + \Delta f, f_i) \sum_{m=0}^{n-1} e^{jm f_1 (f_2 + \Delta f) \phi} df_2 \right|^2 \\
& \approx \left| \mu(f_1 + f_i, f_k, f_i) \int_{-\frac{B_k}{2}}^{\frac{B_k}{2}} \sum_{m=0}^{n-1} e^{jm f_1 (f_2 + \Delta f) \phi} df_2 \right|^2 \\
& = |\mu(f_1 + f_i, f_k, f_i)|^2 \\
& \cdot \left| B_k + \sum_{m=1}^{n-1} e^{jm f_1 \Delta f \phi} \int_{-\frac{B_k}{2}}^{\frac{B_k}{2}} e^{jm f_1 f_2 \phi} df_2 \right|^2 \\
& = B_k^2 |\mu(f_1 + f_i, f_k, f_i)|^2 \\
& \cdot \left| 1 + \sum_{m=1}^{n-1} \frac{e^{jm f_1 \Delta f \phi}}{jm f_1 \phi B_k} \left(e^{jm f_1 \frac{B_k}{2} \phi} - e^{-jm f_1 \frac{B_k}{2} \phi} \right) \right|^2 \\
& = B_k^2 |\mu(f_1 + f_i, f_k, f_i)|^2 \\
& \cdot \left| 1 + \sum_{m=1}^{n-1} \text{sinc} \left(m f_1 \frac{B_k}{2} \phi \right) e^{jm f_1 \Delta f \phi} \right|^2.
\end{aligned} \tag{17}$$

Eq. (17) is then used in order to derive (5).

APPENDIX B IDENTITY USED TO DERIVE EQ. (12)

In this section, an identity for $\lim_{n \rightarrow \infty} \Delta_n C_n$ is derived in order to obtain a closed-form formula for the asymptotic modulation format correction contribution (10). The asymptotic differential normalization coefficient can be calculated exactly and no approximations are needed.

For notational brevity, the identity is derived for arbitrary parameters a and b which reads

$$\begin{aligned}
C'_\infty &= \lim_{n \rightarrow \infty} \Delta_n \int_{-\infty}^{\infty} dx \left| \sum_{m=1}^n \text{sinc}(max) e^{jmbx} \right|^2 \\
&= \frac{\pi}{a^2} \left[(b-a) \ln \left(\frac{b-a}{a+b} \right) + 2a \right].
\end{aligned} \tag{18}$$

We start proving (18) by executing the backward differential operator with respect to the span number. For this purpose, the identity

$$\Delta_n \left(\sum_{m=1}^n f(m) \right)^2 = f^2(n) + 2 \sum_{m=1}^{n-1} f(n) f(m), \tag{19}$$

is used, which can be easily proven by induction. In the context of (18), we have that $\lim_{n \rightarrow \infty} f^2(n) = 0$ and $n-1 \approx n$ due

to large n . We can therefore solve the discrete derivative as

$$\begin{aligned}
C'_\infty &= \lim_{n \rightarrow \infty} \Delta_n \int_{-\infty}^{\infty} dx \left| \sum_{m=1}^n \text{sinc}(max) e^{jmbx} \right|^2 \\
&= \lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} dx \frac{1}{2a^2 n x^2} \sum_{m=1}^n \frac{1}{m} \{ \\
&\quad [\sin(mc_2 x) + \sin(mc_1 x)] [\sin(nc_2 x) + \sin(nc_1 x)] \\
&\quad + [\cos(mc_2 x) - \cos(mc_1 x)] [\cos(nc_2 x) - \cos(nc_1 x)] \} \\
&= \lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} dx \frac{1}{2a^2 n x^2} \sum_{m=1}^n \frac{1}{m} \{ \\
&\quad [\sin(mc_2 x) + \sin(mc_1 x)] [\sin(nc_2 x) + \sin(nc_1 x)] \\
&\quad + [\sin(mbx + nc_1 x) + \sin(mbx - nc_1 x)] \\
&\quad \cdot [\sin(max) - \sin(max)] \} ,
\end{aligned} \tag{20}$$

with $c_1 = a + b$ and $c_2 = a - b$, which are defined for the sake of a concise notation. The last step in (20) was obtained using trigonometric identities. Using the exact integral solution $\int_{-\infty}^{\infty} dx \frac{\sin(ax) \sin(bx)}{x^2} = \frac{\pi}{2} (|a+b| - |a-b|)$ yields

$$\begin{aligned}
C'_\infty &= \lim_{n \rightarrow \infty} \frac{\pi}{2a^2 n} \sum_{m=1}^n \frac{1}{m} \{ \\
&\quad - |mc_2 - nc_2| + |mc_1 + nc_2| + |mc_2 + nc_1| - |mc_1 - nc_1| \} .
\end{aligned} \tag{21}$$

To resolve the absolute value operations in (21), we identify that $b \geq 0$, $a \geq 0$ and $b \geq a$, resulting in $c_1 \geq 0$ and $c_2 \leq 0$. The conditions are met by default as the minimum feasible channel spacing is $\Delta f \geq \frac{B_k}{2}$ and the result is invariant to the sign of b . Applying the conditions results in

$$\begin{aligned}
C'_\infty &= \lim_{n \rightarrow \infty} \frac{\pi}{2a^2 n} \sum_{m=1}^n \frac{1}{m} \\
&\quad \{ b(m-n) + (n+m)a + |mc_1 - n|c_2| \} .
\end{aligned} \tag{22}$$

Eq. (22) can be written as two distinct series, which are solved separately in the following. The first series is exactly written and solved as

$$\begin{aligned}
&\lim_{n \rightarrow \infty} \frac{\pi}{2a^2 n} \sum_{m=1}^n \frac{1}{m} \{ b(m-n) + (n+m)a \} \\
&= \lim_{n \rightarrow \infty} \left[\frac{\pi}{2a^2} c_1 - |c_2| \text{Har}(n) \right],
\end{aligned} \tag{23}$$

with $\text{Har}(n)$ being the n 'th harmonic number. The second

series can be written and exactly solved as

$$\begin{aligned}
& \lim_{n \rightarrow \infty} \frac{\pi}{2a^2 n} \sum_{m=1}^n \frac{1}{m} |mc_1 - n|c_2|| \\
&= \lim_{n \rightarrow \infty} \frac{\pi}{2a^2} \left(\sum_{m=1}^{\lfloor \frac{n|c_2|}{c_1} \rfloor} \frac{|c_2|}{m} - \frac{c_1}{n} + \sum_{m=\lceil \frac{n|c_2|}{c_1} \rceil}^n \frac{c_1}{n} - \frac{|c_2|}{m} \right) \\
&= \lim_{n \rightarrow \infty} \frac{\pi}{2a^2} \left\{ |c_2| \text{Har} \left(\left\lfloor \frac{n|c_2|}{c_1} \right\rfloor \right) - |c_2| \text{Har}(n) \right. \\
&\quad \left. + |c_2| \text{Har} \left(\left\lceil \frac{n|c_2|}{c_1} \right\rceil \right) + \frac{c_1}{n} \left(n - \left\lfloor \frac{n|c_2|}{c_1} \right\rfloor - \left\lceil \frac{n|c_2|}{c_1} \right\rceil \right) \right\} \\
&= \lim_{n \rightarrow \infty} \frac{\pi}{2a^2} \left[2|c_2| \text{Har} \left(\frac{n|c_2|}{c_1} \right) - |c_2| \text{Har}(n) + c_1 - 2|c_2| \right], \tag{24}
\end{aligned}$$

where it was used that $\lim_{n \rightarrow \infty} \left\lfloor \frac{nc_2}{c_1} \right\rfloor = \lim_{n \rightarrow \infty} \left\lceil \frac{nc_2}{c_1} \right\rceil = \lim_{n \rightarrow \infty} \frac{nc_2}{c_1}$. Both series can be further simplified by recalling that $\lim_{n \rightarrow \infty} \text{Har}(n) = \lim_{n \rightarrow \infty} \log(n) + \gamma$, with γ being the Euler-Mascheroni constant.

Combining (23) and (24) yields the final result

$$\begin{aligned}
C'_\infty &= \lim_{n \rightarrow \infty} \frac{\pi}{a^2} \left\{ |c_2| \text{Har} \left(\frac{n|c_2|}{c_1} \right) - |c_2| \text{Har}(n) + 2a \right\} \\
&= \frac{\pi}{a^2} \left[|c_2| \ln \left(\frac{|c_2|}{c_1} \right) + 2a \right] \\
&= \frac{\pi}{a^2} \left[(b-a) \ln \left(\frac{b-a}{a+b} \right) + 2a \right], \tag{25}
\end{aligned}$$

which proves the identity in (18). Therefore, Eq. (18) can be used to write (10) in closed-form as (12), without imposing any additional assumptions.

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