Sensitivity Analysis in Zika Virus Dynamics and a Model Discrepancy Approach

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SENSITIVITY ANALYSIS IN ZIKA VIRUS DYNAMICS AND A MODEL DISCREPANCY APPROACH

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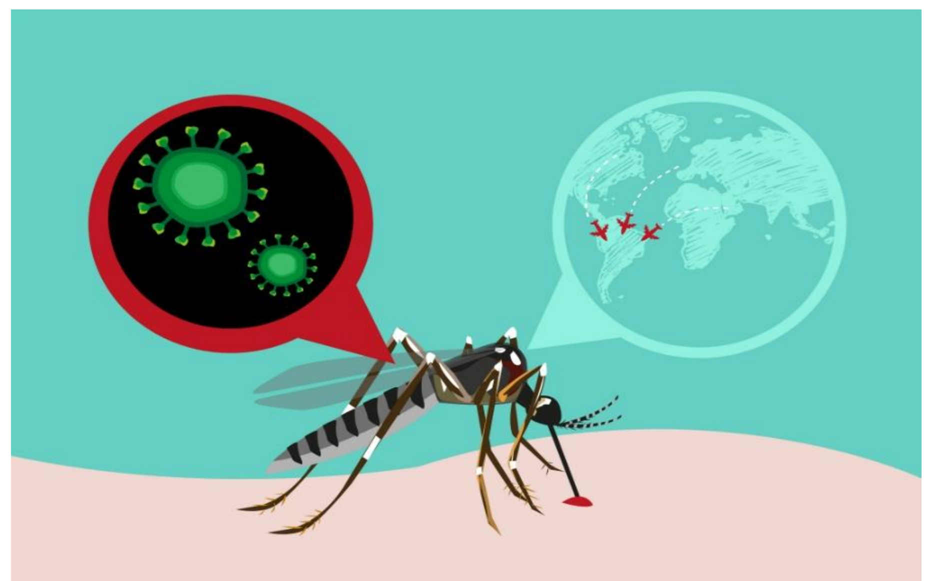
FAPESP

INTRODUCTION

- ➔Zika virus: global widespread and connection with congenital diseases;
- ➔ 2016: Zika becomes a public health emergency of international concern;
- ➔ Main vector: Aedes mosquitoes;
- ➔A validated model can reveal new characteristics of the disease;
- ➔ Relations of model parameters are also of interest;

FIGURE 1 – Zika transmission representation.

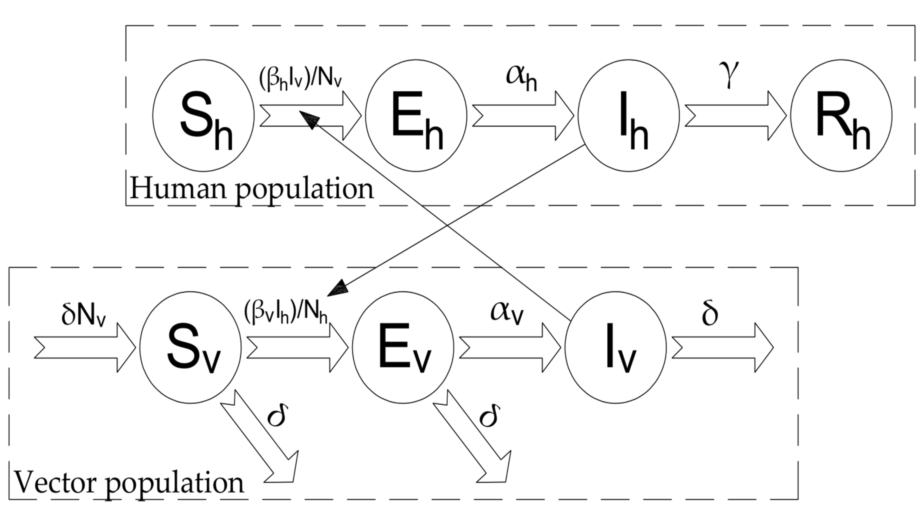


OBJECTIVE

- ◆Perform sensitivity analysis to compare the parameters' global effect under different scenarios;
- ◆ Develop a statistical framework using Bayesian Inference and Polynomial Chaos Expansion to quantify epidemic model discrepancies;

COMPUTATIONAL MODEL

FIGURE 2 – SEIR-SEI model schematic [1].



DYNAMICAL SYSTEM:

$$\frac{dS_h}{dt} = -\beta_h S_h \frac{I_v}{N_v}, \qquad \frac{dS_v}{dt} = \delta N_v - \beta_v S_v \frac{I_h}{N_h} - \delta S_v,$$

$$\frac{dE_h}{dt} = \beta_h S_h \frac{I_v}{N_v} - \alpha_h E_h, \qquad \frac{dE_v}{dt} = \beta_v S_v \frac{I_h}{N_h} - (\alpha_v + \delta) E_v,$$

$$\frac{dI_h}{dt} = \alpha_h E_h - \gamma I_h, \qquad \frac{dI_v}{dt} = \alpha_v E_v - \delta I_v,$$

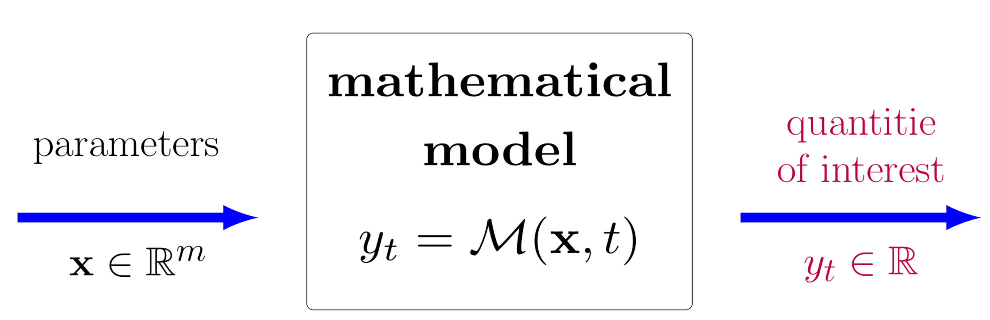
$$\frac{dR_h}{dt} = \gamma I_h, \qquad \frac{dC}{dt} = \alpha_h E_h.$$

+ Initial Conditions

QUANTITIES OF INTEREST:

- Cumulative cases of infectious: $C(t) = \int_{\tau=0}^{t} \alpha_h E_h(\tau)\, d\tau$
- New cases per week: $\mathcal{N}_w = C_w - C_{w-1}, \; w = 1 \ldots 52, \; \mathcal{N}_1 = C_1$

FIGURE 3 – Observation operator schematic [2].



SENSITIVITY ANALYSIS

The Hoeffding-Sobol' decomposition [3] for n iid $X_i \sim \mathcal{U}(0,1)$ gives

$$Y_t = \mathcal{M}_0 + \sum_{1 \le i \le n} \mathcal{M}_i(X_i) + \sum_{1 \le i < j \le n} \mathcal{M}_{ij}(X_i, X_j) + \cdots + \mathcal{M}_{1 \cdots n}(X_1 \cdots X_n),$$

$$\mathcal{M}_0 = \mathbb{E}[Y_t], \; \mathcal{M}_i(X_i) = \mathbb{E}[Y_t | X_i] - \mathcal{M}_0, \; \mathcal{M}_{ij}(X_i, X_j) = \mathbb{E}[Y_t | X_i, X_j] - \mathcal{M}_0 - \mathcal{M}_i - \mathcal{M}_j.$$

Sobol' Indices: interaction effect of inputs in $\mathbf{u}$

$$S_{\mathbf{u}} = \mathrm{Var}\left[\mathcal{M}_{\mathbf{u}}(X_{\mathbf{u}})\right] / \mathrm{Var}\left[\mathcal{M}(\mathbf{X})\right]$$

The Polynomial Chaos Expansion [2] of m $Y = \mathcal{M}(\mathbf{X})$, for a multivariate orthonormal polynomi $\Phi_{\boldsymbol{\alpha}}$ amily with $y_{\boldsymbol{\alpha}}$ coefficients ,

$$Y_t = \sum_{\boldsymbol{\alpha} \in \mathbb{N}^k} y_{\boldsymbol{\alpha}}(t)\, \Phi_{\boldsymbol{\alpha}}(\mathbf{X}),$$

enables analytic computation of Sobol Indices:

$$S_{\mathbf{u}} = \sum_{\boldsymbol{\alpha} \in \mathcal{A}_{\mathbf{u}}} y_{\boldsymbol{\alpha}}^2 \Big/ \sum_{\boldsymbol{\alpha} \in \mathcal{A} \setminus 0} y_{\boldsymbol{\alpha}}^2, \quad \mathcal{A}_{\mathbf{u}} = \{\boldsymbol{\alpha} \in \mathcal{A} : i \in \mathbf{u} \Longleftrightarrow \alpha_i \neq 0\}$$

FIGURE 4 – Sobol' indices of the model.

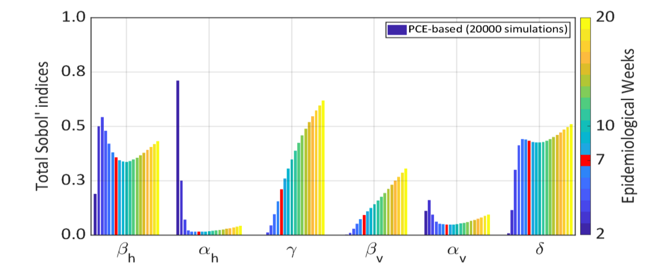


FIGURE 5 – Sobol' indices with α_v constant.

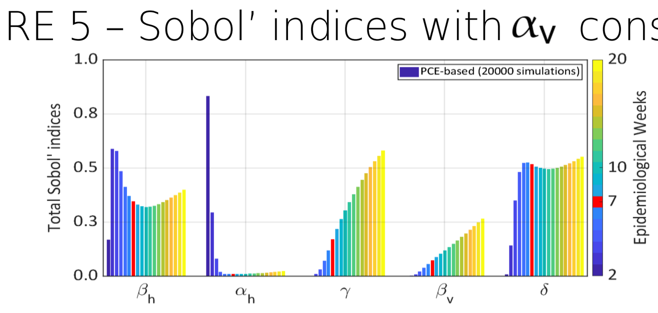


FIGURE 6 – Sobol' indices witho α_v

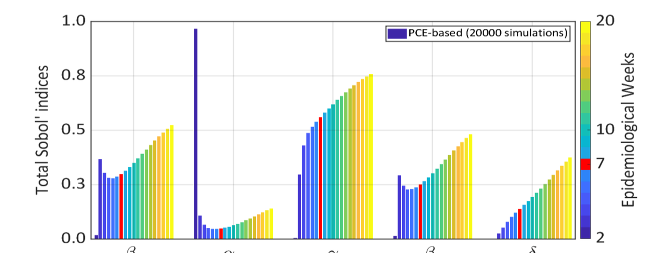
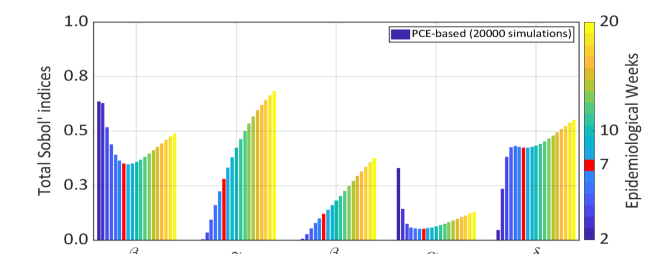


FIGURE 7 – Sobol' indices without α_h .



STATISTICAL INFERENCE (ONGOING RESEARCH)

DISCREPANCY CALCULATION:

Suppose a data se $\mathcal{D} = (t_1, y_1^{dat}), (t_2, y_2^{dat}), \ldots, (t_{N_d}, y_{N_d}^{dat})$ of measur y_t of th i . The -th observation is given by

$$\underbrace{y_i^{dat}}_{reality} = \underbrace{\mathcal{M}(\mathbf{x}, t_i)}_{model} + \underbrace{\varepsilon_i}_{error}.$$

Sargsyan, Najm and Ghanem's [4] novel approach to deal with the model discrepancies is to adopt a metamodel structure which lumps the error into the parameters

$$Y^{dat} \approx \mathcal{M}(\mathbf{X}_\varepsilon, t), \quad \mathbf{X}_\epsilon = \sum_{\boldsymbol{\alpha} \in \mathcal{I}} \mathbf{X}_{\boldsymbol{\alpha}}(t) \boldsymbol{\Psi}_{\boldsymbol{\alpha}}(\boldsymbol{\xi}),$$

where $\mathbf{X}_{\boldsymbol{\alpha}}$ coefficients are defined as random to be able to be identified by using Bayesian Inference.

BAYESIAN INFERENCE:

- Inference problem become use data information to update the prior probability density func-tion(PD $\mathbf{X}_{\boldsymbol{\alpha}}$ defined for . The solution corresponds posterior PDF;
- From Bayes' rule,

$$\rho(\mathbf{X}_{\boldsymbol{\alpha}} | \mathcal{D}) = \frac{\rho(\mathcal{D} | \mathbf{X}_{\boldsymbol{\alpha}})\, \rho(\mathbf{X}_{\boldsymbol{\alpha}})}{\rho(\mathcal{D})}.$$

→ $\rho(\mathbf{X}_{\boldsymbol{\alpha}} | \mathcal{D})$: posterior distribution → $\rho(\mathbf{X}_{\boldsymbol{\alpha}})$: prior distribution
→ $\rho(\mathcal{D} | \mathbf{X}_{\boldsymbol{\alpha}})$: likelihood function → $\rho(\mathcal{D})$: evidence

To define a good point of start, the Maximum Entropy Principle is applied to construct the most informative prior distribution.

FINAL REMARKS

- ✓ Comparative results of global Sobol' Indices show how the lack of some parameters can change the sensibility effect of the others;
- ✓ A framework for statistical inference exploring Polynomial Chaos to measure the model discrepancies was presented;
- ✓ In future works, the authors intend explore this new framework to quantify model discrepancy and then improve its predictions;

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