

Measuring Similarity between Wavelet Function and Transient in a Signal with Symmetric Distance Coefficient

Nemuel Daniel Pah⁽¹⁾

⁽¹⁾Department Electrical Engineering, Universitas Surabaya, Surabaya, Indonesia
(Email: nemuelpah@ubaya.ac.id)

ABSTRACT

Wavelet transform has been developed and applied in various areas of science and engineering. It offers better signal processing capabilities due to the existence of a large number of wavelet functions. The superiority comes with a requirement of methods to select a proper wavelet function for each signal or application. Selection methods based on signal properties are not always applicable due to the lack of mathematical definition of most analyzed signals. Wavelet function can be selected based on its shape similarity to interested transient in input signal. The selection method is subjective since there is no quantified parameter to measure this similarity. This paper introduces a new parameter, symmetric distance coefficient (SDC) to measure similarity between wavelet function and transient in a signal. It is based on a fact that wavelet coefficients of a transient that has similar shape and similar time support to a wavelet function are always symmetric. The parameter measures similarity by measuring the degree of symmetry in wavelet coefficients. The paper also reports an experiment to demonstrate procedures to measure this similarity using SDC.

KEYWORDS

Wavelet; symmetric; transient; transformation.

I. INTRODUCTION

Wavelet Transform (WT) is a recent development in signal processing [1, 2]. Wavelet transform of a signal $f \in L^2(R)$ is defined as a correlation between the signal and a dilated-shifted wavelet function $\psi_{s,\sigma}$.

$$Wf(s, \sigma) = \langle f, \psi_{s,\sigma}^* \rangle = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\sigma}{s} \right) dt \quad (1)$$

The magnitude of wavelet coefficient, $|Wf(s, \sigma)|$, indicates how well they correlate around $t = \sigma$, and is determined by the matching between the

properties of wavelet function and the properties of the analyzed signal. WT offers a large option of wavelet functions. This flexibility comes with a need of methods to select wavelet function.

Common wavelet selection methods were based on the matching between the properties of wavelet function and the properties of the analyzed signal [1-8]. The methods require both, the wavelet function and the analyzed signal, to be defined mathematically. All wavelet functions are defined mathematically, but, most analyzed signals are natural signals with lack of mathematical expression. Other wavelet function selection method was based on visual shape similarity between interested transient in input signal and wavelet function [9-14]. The method, was not accurate since the similarity was only judged by visual inspection.

This paper introduces a new parameter to measure similarity between wavelet function and transient in a signal by measuring the degree of symmetry in wavelet coefficients. The new parameter is called 'symmetric distance coefficient' (SDC) [15]. This paper is organized into five sections. Following this introduction, Section II elaborates the issue of measuring shape similarity. Section III definition of SDC, while Section IV presents the procedure and experiments to measure shape similarity between a wavelet function and transients in a signal. Finally, Section V gives the concluding remark.

II. MEASURING SHAPE SIMILARITY

Shape similarity between a wavelet function and a transient in a signal can be observed on the symmetric pattern in wavelet coefficients in the neighborhood of the center of the transient, $t = t_c$ [16]. It is based on the fact that correlation

function of two similar shape functions is always symmetric (even-symmetric). Consequently, if a wavelet function has similar shape to a transient at $t = t_c$, the wavelet coefficients in the neighborhood of $\sigma = t_c$ are even-symmetric. This symmetry feature is not altered by signal amplification.

There is a difficulty in using the symmetric nature to measure shape similarity because in the real world, it is nearly impossible to find a wavelet function that has exactly a similar shape to any transients in input signal. One can only find a wavelet function that is almost similar. This condition will produce nearly even-symmetric wavelet coefficients. To measure this 'nearly similar' condition, it is necessary to have a parameter to measure symmetry in gradual levels. Unfortunately, symmetry is only defined as a discrete parameter (i.e. symmetry or asymmetry). The new parameter, introduced in this paper, was developed to measure symmetry in gradual levels. The parameter is applied to the case of wavelet function selection.

III. SYMMETRIC DISTANCE COEFFICIENT

A function, $f(t)$, is considered symmetric around $t = t_c$ if

$$\{f(t_c + \tau) = \pm f(t_c - \tau)\}_{\tau \in [-t_s, t_s]} \quad (2)$$

where τ is any value in a time support of $[-t_s, t_s]$. Equation (2) strictly distinguishes between symmetric and asymmetric function. SDC measures the degree of symmetry in gradual levels. It is inspired by the concept of symmetric distance in image analysis that measures the minimum effort required to modify pattern in an image into its symmetric shape [17-19].

SDC is the ratio between the energy of the difference between left and right side of a function around its center $t = t_c$, and the energy of the function within the same time support of $t = [t_c - t_s, t_c + t_s]$. It is illustrated in Fig. 1.

$$SDC(t_c, t_s) = \frac{\int_0^{t_s} (f(t_c - \tau) - f(t_c + \tau))^2 d\tau}{\int_{-t_s}^{t_s} f^2(t_c - \tau) d\tau} \quad (3)$$

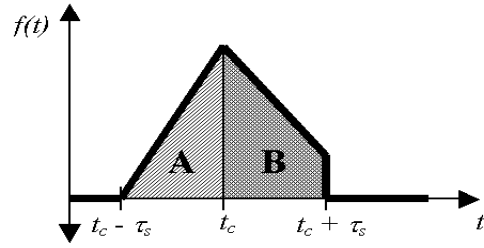


Figure 1. A nearly even-symmetric function at $t = t_c$ with time support $[t_c - t_s, t_c + t_s]$. The $SDC(t_c, t_s)$ is the ratio of the energy of (A-B) to the total energy of A and B.

The range of SDC in (3) is $[0, 2]$ which does not meet common sense. The equation is modified to shift its range to $[-1, 1]$ where -1 represents odd-symmetry, 0 represents asymmetry, and 1 represents even-symmetry as in (4).

$$SDC(t_c, t_s) = 1 - \frac{\int_0^{t_s} (f(t_c - \tau) - f(t_c + \tau))^2 d\tau}{\int_{-t_s}^{t_s} f^2(t_c - \tau) d\tau} \quad (4)$$

SDC is independent to input signal amplification. Suppose a signal, $f(t)$, is even-symmetry around $t = 0$ i.e. $\{f(t) = f(-t)\}_{t \in [-t_s, t_s]}$. Another signal, $g(t)$, is equal to $f(t)$ but asymmetric due to $\{e(t)\}_{t \in [0, t_s]}$:

$$g(t) = \begin{cases} f(t) & t \in [-t_s, 0] \\ f(t) + e(t) & t \in (0, t_s] \end{cases} \quad (5)$$

then SDC at $t = 0$ with support $[-t_s, t_s]$ is:

$$SDC(0, t_s) = 1 - \frac{\int_0^{t_s} e^2(\tau) d\tau}{\int_0^{t_s} 2f^2(\tau) + 2f(\tau)e(\tau) + e^2(\tau) d\tau} \quad (6)$$

If the signal is then amplified by a factor of A :

$$g(t) = \begin{cases} Af(t) & t \in [-t_s, 0] \\ Af(t) + Ae(t) & t \in (0, t_s] \end{cases} \quad (7)$$

the SDC becomes

$$SDC(0, t_s) = 1 - \frac{A^2 \int_0^{t_s} e^2(\tau) d\tau}{\int_0^{t_s} 2A^2 f^2(\tau) + 2Af(\tau)Ae(\tau) + A^2 e^2(\tau) d\tau} \quad (8)$$

By canceling out the amplification factors, A , equation (8) proves that SDC can measure the level of symmetry, independent to the strength of input signal or transient.

The ability to measure symmetry independent to signal strength may create problem if the input signal has too many unexpected small ripples (see Fig. 3). To avoid measuring SDC of unwanted small transients, a threshold level, th , is added to the equation to ignore transients with energy less than the threshold level.

$$SDC_{th}(t_c, t_s, th) = \begin{cases} SDC(t_c, t_s) & \text{if } \int_{-t_s}^{t_s} f^2(t_c - \tau) d\tau > th \\ 0 & \text{else} \end{cases} \quad (9)$$

Equation (8) and (9) are the mathematic definition of SDC.

Fig. 2 shows $SDC_{th}(t, 0.25, 0.1)$ of even-symmetric, nearly even-symmetric, odd-symmetric, nearly odd-symmetric and asymmetric transients located at $t = 0.75$ s, 1.75 s, 2.75 s, 3.75 s, and 4.75 s, respectively. The SDC of the first transient, $SDC_{th}(0.75, 0.25, 0.1)$, is 0.99 . It means that the transient is 99% even-symmetry. The second transient is only 90% even-symmetry, while the third to fifth transients are 99% odd-symmetry, 92% odd-symmetry, and 0% symmetry, respectively. The figure shows how SDC can measure various types and levels of symmetry.

Fig. 3 shows the necessity of using threshold. It shows SDC of a signal with a symmetric transient at $t = 0.475$ s and small noises. When threshold was set to zero (no threshold) the SDC of the investigated transient was obscured by a large number of SDCs of the noises (Fig. 3b). When a threshold value of 5 was used, the SDC of the noises were removed (Fig. 3c).

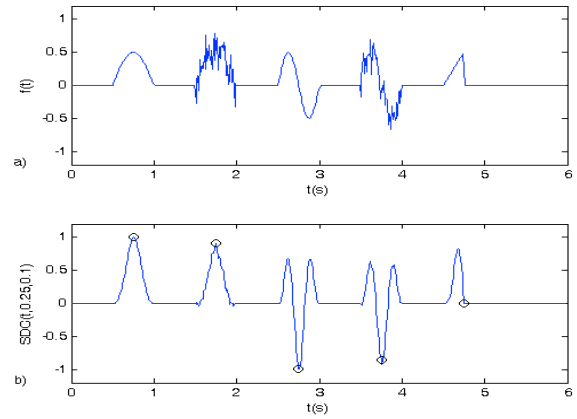


Figure 2. The SDC of various types and levels of symmetry.

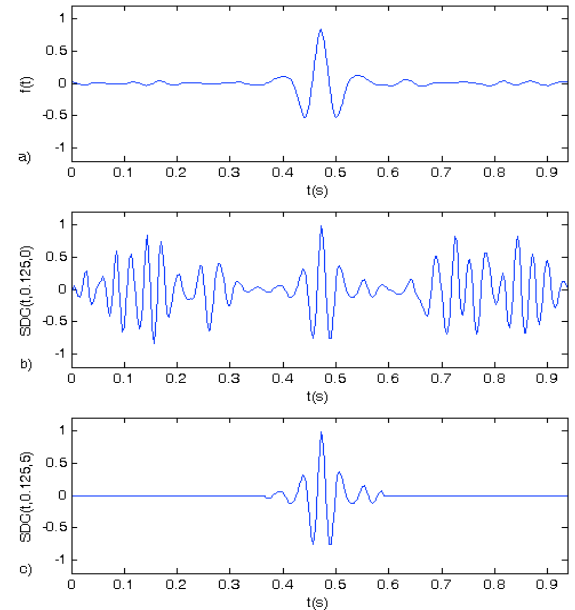


Figure 3. a) A signal with an even-symmetric transient added with noises. b) The $SDC_{th}(t, 0.125, 0)$, c) The $SDC_{th}(t, 0.125, 5)$.

IV. MEASURING PROCEDURES AND EXPERIMENT

The procedure to measure similarity between a wavelet function, $\psi_{s,\sigma}$, and a transient located at $t = t_c$ in an input signal, $f(t)$, is as follows:

1. Apply WT to $f(t)$ using the selected wavelet function, $\psi_{s,\sigma}$, to the whole time span of $f(t)$.
2. Calculate $SDC_{th}(t_c, t_s, th)$ of the wavelet coefficients at time location $\sigma = t_c$. The time support parameter, t_s , should be half of the selected wavelet function's time support or

half of the interested transient's time support. Use an appropriate threshold, th , to ignore small ripples in wavelet coefficients.

3. The value of $SDC_{th}(t_c, t_s, th)$ indicates the degree of shape similarity between the selected wavelet function and the transient. The closest the value to 1, the more similar the two functions.

The parameter and procedure was applied to electrocardiograph (ECG) signal [7, 20] to find a wavelet function that has the most similar shape to T-waves (marked with 'T' in Fig. 4a). An ECG signal, downloaded from MIT-BIH PhysioBank database record aami3a.dat (AAMI-EC13) [21] was used in this experiment (Fig. 4a). The signal was re-sampled with a sampling rate of 360 Hz. Wavelet coefficients were calculated using various wavelet functions. However, the results of only three wavelet functions are presented in this paper (i.e. *db2* at scale 28, *Gaus2* at scale 8, and *Mexh* at scale 12) due to their significant results. The SDC calculation was then applied to all wavelet coefficients with a half time support of 0.158 s and a threshold value of 0.1. The results (Fig. 4b-d) suggested that *db2* at scale 28 is the wavelet function with the most similar shape to T-waves. The average $SDC_{th}(t, 0.158, 0.1)$ of the four T-waves when calculated using *db2* wavelet at scale 28 was 0.915. The value was higher than that of *Gaus2* wavelet at scale 8 and *Mexh* wavelet at scale 12 (0.88, and 0.82, respectively). The experiment suggests that the parameter and the procedure can be used to find wavelet function that has the most similar shape to an interested transient.

V. CONCLUSION

This paper introduces a new parameter called 'symmetric distance coefficient' (SDC) and the procedure to measure shape similarity between a wavelet function and a transient in a signal. The parameter measures the similarity by measuring the symmetric pattern in wavelet coefficient. It is based on the fact that wavelet transform of a transient that has the same shape as a wavelet function is always symmetric. The parameter is useful as a guidance to select suitable wavelet

function for a specific transient in a signal or application.

The paper also presents an experiment to demonstrate the procedure and the parameter's ability to measure shape similarity between a wavelet function and transients in signal. The experiment demonstrates how the parameter can be used to find a wavelet function that has the most similar shape to T-waves in ECG signal.

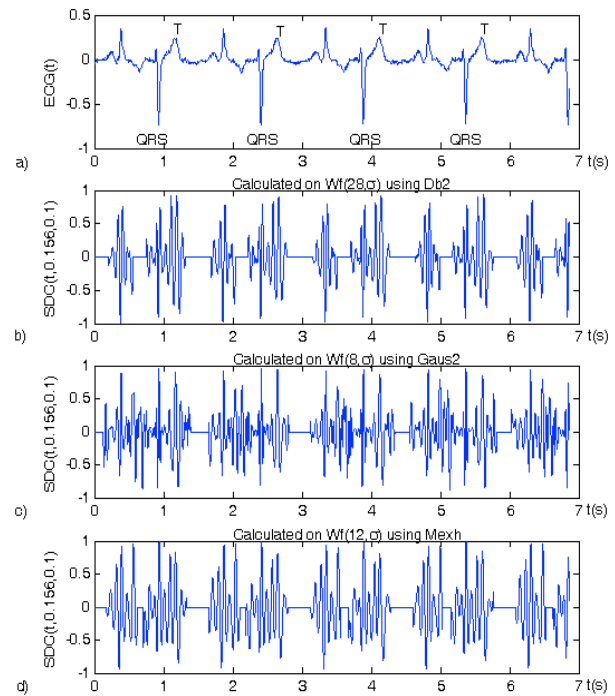


Figure 4. a) ECG signal (MIT-BIH aami3a.dat – AAMI EC13) with sampling rate 360 Hz, b) – d) The $SDC_{th}(t, 0.158, 0.1)$ calculated on wavelet coefficients using *Db2* at scale 28, *Gaus2* at scale 8, and *Mexh* at scale 12, respectively.

REFERENCES

- [1] I. Daubechies, *Ten Lectures on Wavelets*. Pennsylvania: SIAM, 1992.
- [2] S. Mallat, *A Wavelet Tour of Signal Processing*. London: Academic Press, 1999.
- [3] S. Mallat and W. L. Hwang, "Singularity Detection and Processing with Wavelets," *IEEE Transaction on Information Theory*, vol. 38, pp. 617-643, 1992.
- [4] N. D. Pah and D. K. Kumar, "Thresholding Wavelet Networks for Signal Classification," *International Journal of*

- Wavelets, Multiresolution, and Information Technology*, vol. 1, pp. 243-261, 2003.
- [5] D. K. Kumar and N. D. Pah, "Neural Network and Wavelet Decomposition for Classification of Surface Electromyography," *Electromyography and Clinical Neurophysiology*, vol. 40, 2000.
 - [6] J.-K. Zhang, T. N. Davidson, and K. M. Wong, "Efficient Design of Orthonormal Wavelet Bases for Signal Representation," *IEEE Transaction on Signal Processing*, vol. 52, pp. 1983-1996, 2004.
 - [7] C. Li, C. Zheng, and C. Tai, "Detection of ECG Characteristic Points Using Wavelet Transform," *IEEE Transaction on Biomedical Engineering*, vol. 42, pp. 21-28, 1995.
 - [8] S. Karlsson, J. Yu, and M. Akay, "Time-Frequency Analysis of Myoelectric Signals During Dynamic Contractions: A Comparative Study," *IEEE Transaction on Biomedical Engineering*, vol. 47, pp. 228-238, 2000.
 - [9] S. Kadambe and G. F. Boudreaux-Bartels, "Application of the Wavelet Transform for Pitch Detection of Speech Signals," *IEEE Transaction on Information Theory*, vol. 38, pp. 917-924, 1992.
 - [10] A. J. Hoffman and C. J. A. Tollig, "The Application of Classification Wavelet Networks to the Recognition of Transient Signals," *IEEE*, pp. 407-410, 1999.
 - [11] V. J. Samar, "Wavelet Analysis of Neuroelectric Waveforms: A Conceptual Tutorial," *Brain and Language*, pp. 7-60, 1999.
 - [12] R. Q. Quiroga, O. W. Sakowitz, E. Basar, and M. Schurmann, "Wavelet Transform in the Analysis of the Frequency Composition of Evoked Potentials," *Brain Research Protocols*, vol. 8, pp. 16-24, 2001.
 - [13] R. Zhang, G. McAllister, B. Scotney, S. McClean, and G. Houston, "Combining Wavelet Analysis and Bayesian Networks for Classification of Auditory Brainstem Response," *IEEE Transaction on Information Technology in Biomedicine*, vol. 10, pp. 458-466, 2006.
 - [14] P. Zhou and W. Z. Raymer, "Motor Unit Action Potential number Estimation in the Surface Electromyogram: Wavelet matching Method and Its Performance Boundary," presented at The 1st International IEEE EMBS Conference on Neural Engineering, Capri Island, Italy, 2003.
 - [15] N. D. Pah, "Measuring Wavelet Suitability with Symmetric Distance Coefficient," presented at Seminar Nasional Soft Computing, Intelligent System and Information Technology, Surabaya, Indonesia, 2005.
 - [16] P. Abry and P. Flandrin, "Multiresolution Transient Detection," presented at Proceedings of the IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis, 1994.
 - [17] Y. Keller and Y. Shkolnisky, "A Signal Processing Approach to Symmetry Detection," *IEEE Transaction on Image Processing*, vol. 15, pp. 2198-2206, 2006.
 - [18] I. Choi and S.-I. Chien, "A Generalized Symmetry Transform with Selective Attention Capability for Specific Corner Angles," *IEEE Signal Processing Letters*, vol. 11, pp. 255-257, 2004.
 - [19] A. Imiya, T. Ueno, and I. Fermin, "Symmetry Detection by Random Sampling and Voting Process for Motion Analysis," *International Journal of Pattern Recognition*, vol. 17, pp. 83-125, 2003.
 - [20] J. S. Sahambi, S. N. Tandon, and R. K. P. Bhatt, "Using Wavelet Transform for ECG Characterization," in *IEEE Engineering in Medicine and Biology*, 1997, pp. 77-83.
 - [21] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. C. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C. K. Peng, and H. E. Stanley, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals," *Circulation*, vol. 101, pp. e215-e220, 2000.

