Essays On the Optimal Interplay of Early and Late Education Subsidies and Taxation

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To Iga

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1 Introduction

A benevolent government that aims to increase welfare can employ various means in order to eliminate or mitigate market failures. An important example is the use of progressive taxation in order to reduce negative consequences of uninsurable, idiosyncratic income risk. Such interventions in the market, however, may lead to other inefficiences, since they distort household decisions. Progressive taxation, for instance, distorts the labor supply. And in addition, it distorts the incentive to reach for higher education, as it implies a relative decrease of the net income for higher skilled jobs. Addressing this trade-off, Krueger and Ludwig (2013) and Krueger and Ludwig (2016) develop a quantitative model that shows that college subsidies are a valuable tool to counteract the distorting effect of progressive taxation on educational decisions. Thus their papers are to be seen in the intersection of two strands of literature - on the one hand the literature on optimal income taxation and on the other hand a previously rather theoretical literature, dealing with the optimal combination of progressive income taxes and educational subsidies in models that abstract from idiosyncratic risk.

The extension of the work of Krueger and Ludwig, which is made in this thesis, links it additionally to the literature on human capital production of young people.[1] This literature emphasizes the differences in the formability of skills in the various stages of ability development across the life-cycle, and draws attention especially to the early years. Due to the dynamic-complementary nature of the human capital process, it is hardly possible to compensate for missed early investments in education at later stages. Hence, when this foundation is not built when children are young, for example, a subsequent college education will hardly be possible, regardless of its costs.[2] The key contribution of this thesis is thus to expand the model of Krueger and Ludwig by analyzing the endogenous formation of the human capital that people

[1]See Cunha and Heckman (2010) for a comprehensive summary of the literature on the production of skills of young people.

[2]Cunha, Heckman and Schennach (2010) dissect the skill formation process of children and estimate a multi-stage human capital production function, taking into account the empirical facts of the literature summarized in Cunha and Heckman (2010).

already have when they start into their adult life. This analysis takes into account parents' investments in primary and secondary education in response to college subsidies. These investments, in turn, have a major impact on the effectiveness of the subsidies. Additionally, the college process is further refined in this thesis. Students achieve a degree with a success probability that is dependent on their human capital. Hence, with college dropouts a third qualification status is introduced and the tertiary education process is modeled more realistically.

While early investments in human capital are very important, they differ across socioeconomic groups. In this context, the literature on human capital shows a high level of persistence of income over generations, which in addition to the inheritance of skills comes in particular from better access to education for children of better-earning households.[3] In this thesis, the government will have the opportunity to choose over not only college subsidies but also its investments in primary and secondary education. Although intergenerational immobility is not the focus of this thesis per se, it points to another market failure, when human capital can not flourish because of low innate abilities, but because of the lack of financial resources of parents.[4] As a result, I will show that the effectiveness of college subsidies and non-tertiary investments are dependent on each other, and by incorporating both instruments, the government can optimally balance the interplay of early and late education subsidies.

This thesis is structured as follows. In Chapter 2, I develop a model that, building on the work of Krueger and Ludwig, involves the human capital process during primary and secondary education. In doing so, I will analyze which mechanisms from the human capital literature are relevant to the problem at hand and where they find themselves in the model. In particular,

[3]Solon (2002) provides a cross-country survey of intergenerational mobility. Restuccia and Urrutia (2004) dissect the source of intergenerational persistence and stress the importance of early education investments. Blankenau and Youderian (2015) discuss how intergenerational persistence in earnings can be explained by government spending in early education.

[4]In a quantitative analysis Caucutt and Lochner (2017) show that ignoring the earlier investment responses may lead to a significant under-estimation of the impact of college subsidies and emphasize the importance of borrowing constraints of families at early stages. Findeisen and Sachs (2016) show that governmental education loans to young households combined with income-contingent repayment can be designed in a Pareto optimal way. In recent work, Lee and Seshadri (2019) underline the importance of financial frictions in early years for the persistence of economic status.

self-productivity and dynamic complementarity are key elements and I will show that these concepts are incorporated in all stages of human capital development in this thesis, i.e. primary, secondary and tertiary education.

Chapter 3 deals with the solution algorithm of the model. First, I show that the combination of endogenous grid method and level search over the value function used in Krueger and Ludwig (2013) leads to inaccuracies. In this thesis, an alternative solution method is developed, which delivers accurate results. This is illustrated using a two-period model, which is first solved analytically and then quantitatively under both solution methods. By comparing household decisions based on different cash-on-hand levels, the solution method in Krueger and Ludwig (2013) does not address the trade-off between consumption, savings and investments correctly and thereby underestimates the investment choice. In addition, the region in which households are borrowing constraint is not identified precisely, when situations occur in which savings are zero but human capital investments are positive.[5] In a second step, I extend the model of Krueger and Ludwig (2013) by introducing taste shocks to avoid kinks in the value function and jumps in first order conditions that would otherwise be caused by the discrete college decision. Next, I build a bridge to Chapter 2 by showing that the model's policy functions are consistent with the mechanisms of the human capital literature. I conclude this part with a quick look at the quantitative computation, in particular the parallelization of the solution algorithm, which was necessary for this complex model to be solved in a timely manner.

In Chapter 4, the model is calibrated to the core moments of the underlying problem. In addition to education-specific targets such as college attendance and college wage premium, this will be about a realistic representation of the government's budget and its expenditures in tertiary and non-tertiary education. Similar to the previous part, I conclude this chapter by showing that the life-cycle profiles resulting from the calibrated benchmark model are consistent with the findings from the human capital literature.

In Chapter 5, three policy experiments are conducted in partial equilibrium, in which wages

[5]This applies for both the solution of optimal vivos transfers and investments in primary and secondary education. While only the former are included in the paper of Krueger and Ludwig, in this thesis, this issue would spill over to the human capital investment periods of parents as well.

do not react to shifts in the labor market. Initially, the government can optimize either college subsidies or investments in non-tertiary education, while the other policy instrument remains at the level of the benchmark model. In both experiments, the welfare optimum is above the current status quo. However, these two univariate experiments show, that the two policy instruments work through different channels. College subsidies lead to a higher aggregate human capital as parents respond with an endogenous increase in their primary and secondary education investments. But this does not lead to a more equal distribution of income and consumption, since the human capital of children from education and income-poor households increases the least by this policy measure. By contrast, higher governmental investments in non-tertiary education lead to an increase in human capital across all income and education groups and, in addition to an increase in aggregate production and consumption, to a more equal distribution. Still in partial equilibrium, I then conduct a bivariate experiment, in which the government can determine both investments in non-tertiary education and college subsidies. It becomes apparent, that the effectiveness of college subsidies depends on the level of non-tertiary education investments - and vice versa. In a nutshell, this is because the benefits of college subsidies can only be claimed if the young adults have the skills to successfully complete college. Early investments lead to a high human capital level, but this potential remains unused and is not translated into higher wages, if college education can only be afforded by a small fraction of households. If investments are kept to a minimum, for example, one could come to the misleading conclusion that college subsidies are not an effective tool in increasing college attendance and welfare. In the bivariate optimum of partial equilibrium, both policy instruments are above their respective values of the benchmark model. Consequently, aggregate levels of production and consumption are higher while their distributions are more equal than in the status quo.

In Chapter 6, I will perform the same experiments in a small open economy setup, in which wages do react to changes in the labor market, but the interest rate remains constant.[6] This

[6]This intermediate step allows to disentangle the effects from the labor market and the capital market. In addition, computationally, the small open economy experiments converge much faster than the general equilibrium experiments. This was crucial in order to perform bivariate experiments due to restrictions of high performance computing resources.

will allow the college wage premium to adapt to shifts in the labor market. As a result, the differences with respect to the impact of the two policy instruments on the distribution of the economy are less sharp than in partial equilibrium, but they continue to work thorough different channels. Subsidies accomplish equality by a reduction in the college wage premium, while investments in non-tertiary education not only shift aggregate human capital to a higher level, they also accomplish a denser distribution of skills. The other results, however, are transferred from the partial equilibrium to the small open economy: (i) the effectiveness of one policy instrument depends on the level of the other, (ii) therefore univariate experiments can lead to misleading results and (iii) the optimal interplay of the two policy instruments is accomplished at a higher level for both college subsidies and non-tertiary education investments compared to their respective values from the benchmark model, financed by a higher labor tax rate.

In Chapter 7, I will summarize the results and give an outlook on other aspects that should be considered in future work related to this thesis.

Brief Summary of the Model Structure

The model allows the government to set up a progressive tax scheme and to use revenues to grant college subsidies as well as to invest in early human capital. In order to analyze the respective interactions, a life-cycle model with endogenous educational choices, labor supply and consumption-savings decisions in presence of risky labor productivity and borrowing constraints is developed. Educational investments of parents into their children take place at all stages of the child's life-cycle. When children reach adulthood, they decide whether or not to attend college, which relies on four key aspects. First, college education is costly, both in terms of time and monetary resources. Second, parents can transfer resources to their children which may relax potential financial constraints. Third, average wages of college graduates exceed those of non-college workers. Fourth, the success probability in college and the distribution of wages around mean wages over the life-cycle depends on the acquired human capital at the time of the college decision.

A benevolent government may affect all these margins through the choice of the three policies measures mentioned above: early education subsidies (primary and secondary), college subsidies (tertiary education) and progressive income taxes. The latter provide insurance against idiosyncratic labor productivity risk and redistribute across different types of workers, who can differ in education and their respective productivity. However, by intervening in the tax system, the government distorts labor supply and, given that the return on investments in human capital is expressed in higher net wages, the distortion also affects educational decisions. The government may mitigate the latter through education subsidies. But then the question arises, how to optimally design an educational system. Are education subsidies sufficient to improve the distribution of resources? How exactly do early and late education subsidies and progressive income taxes interact?

In order to address these questions, the life-cycle model is further embedded into a macroeconomic framework through which general equilibrium repercussions are acknowledged, that play an important role. For instance, increasing subsidies to tertiary education will increase the number of college workers relative to non-college workers. In a small open economy or general equilibrium, this increased abundance of college workers reduces the college wage premium which has welfare enhancing redistributional implications.

Related Literature

This thesis finds itself in the intersection of different strands of the literature and can be seen as an extension of the work of Krueger and Ludwig (2013) and Krueger and Ludwig (2016). Krueger and Ludwig (2013) characterize the optimal policy mix of capital income taxes, progressive labor income taxes and education subsidies in a model with income risk, borrowing constraints and endogenous human capital formation. They conclude that both the degree of tax progressivity as well as education subsidies for college education should be higher than in the current U.S. status quo. Krueger and Ludwig (2016) extend this earlier work by accounting for the feedback from an increase of the share of workers with a college degree on the college wage premium. Through this general equilibrium feedback education subsidies are a powerful instrument to achieve redistributional objectives. To mitigate distortions, tax progressivity should be reduced, especially along the economy's transition to a new steady state. The optimal policy is therefore characterized by higher education subsidies and lower progressivity than under the current status quo.

Their work is located in the intersection of two bodies of literature. First of all the literature on optimal income taxation, which examines the optimal income tax code a Ramsey type government[7] should implement in a quantitative OLG model,[8] when uninsurable idiosyncratic income risk is present[9]. In addition, their work is connected to a previously rather theoretical literature, studying the optimal combination of progressive income taxes and educational subsidies in models that abstract from idiosyncratic risk. By building up on the work of Krueger and Ludwig this thesis also incorporates both these strands of literature and the references they make.

By adding to the model the entire human capital process of primary and secondary education, this work additionally opens up to the literature on the production of skills of young people. Cunha and Heckmann (2010) provide a detailed overview of empirically established facts and the mechanisms models need to incorporate in order to be able to acknowledge them.[10] They emphasize the self-productive and dynamic complementary nature of skill production, implying that higher investments at early stages increase the return on investments in all following stages of human capital formation. Against the background of these results, Cunha, Heckman, and Schennach (2010) estimate a multistage human capital production function capable of taking all of these facts into account.

These properties of human capital development pave the way for intergenerational persistence of earnings.[11] One measure for this persistence frequently used in the literature is the

[7] See Chamley (1986) and Judd (1985).

[8] See Auerbach and Kotlikoff (1987).

[9] See Bewley (1986), Huggett (1993, 1997) and Aiyagari (1994).

[10] These these facts and findings will be discussed in more detail below, when I describe how the educational process is designed in this thesis.

[11] Solon (1999) starts with a very memorable illustration to highlight the importance of intergenerational persistence when considering equality in a society. In a cross-sectional analysis two countries with the same Gini

slope coefficient received by regressing log earnings of children (when they become adults) on log earnings of parents. A general finding is a high intergenerational persistence of earnings in the US. Following Stokey (1998) and Solon (1999) roughly 40% of the relative earnings position is passed from parents to their children.[12] At first glance, the intergenerational persistence is more of a philosophical aspect to this thesis and the underlying research question, since my welfare evaluation does not take intergenerational persistence into account. However, on closer inspection it points to a second inefficiency - despite the distorting effects of a progressive tax code - the government may address. In order to identify the reasons for intergenerational persistence and cross-sectional inequality of earnings, Restuccia and Urrutia (2004) set up a model where they allow three different sources to be their main drivers: innate ability, early education and college education. In a quantitative analysis they find that the half of the 40% of intergenerational persistence is accounted for by differences in early education.[13,14]

Looking at these results together, the question arises as to what the differences between the primary and secondary education of children are due to. Kaushal, Magnuson and Waldfogel (2011) arrive at the conclusion that parents in the top 20% spend 9% of their income on educational enrichment items whereas parents in the bottom income quintile only spend 3% of their income. In a similar context Caucutt and Lochner (2017) investigate the importance of borrowing constraints of families at early stages. They find that a $10,000 increase in discounted annual income of parents when children are at the age of zero to eleven reduces high school drop out rates by 2.5 percentage points, while it increases college attendance by 4.6 percentage points. Regarding the same increase at age 12 to 23 the effect is not only much smaller but also

coefficient would be considered as equal unequal. But adding the intergenerational information that in one of these countries children end up at the exact same position in the income distribution as their parents with certainty, whereas in the other country this is completely random, the second society would be considered as providing more equal opportunities.

[12] Canada' slope coefficient is by about 23% and Finland's is about 22% (Solon 2002). Wiegand (1997) performs a similar regression for Germany and arrives at a persistence of 34%.

[13] The other half is explained by intergenerational persistence of innate ability, whereas college education drives the extent of cross-sectional disparity, but does not explain its origin.

[14] In a more recent study Blankenau and Youderian (2015) discuss how intergenerational persistence in earnings can be reduced by government spending in early education, where they use the estimates of Cunha, Heckman and Schennach (2010) to set up their human capital production function.

statistically insignificant.
Caucutt and Lochner refine this result, highlighting the importance of endogenizing the human capital process to analyze the impact of college subsidies. In a quantitative analysis they show that ignoring the earlier investment responses may lead to a significant under-estimation of the total wage impact of college-age investment subsidies by around 60%.[15]

In summary, the literature on optimal income taxation shows that the government can implement a progressive tax code to compensate inefficiencies caused by uninsurable idiosyncratic income risk. Krueger and Ludwig have shown that college subsidies are a valuable tool to counteract the distorting effects of progressive tax codes on labor and education related decisions. However, the literature on human capital suggests that a change in college subsidies will lead to endogenous adjustments that in turn affect their effectiveness. In addition, evidence from this literature suggests that policies aimed at early education are the more powerful tool in shaping skills and thereby counter intergenerational persistence, which is not due to the inheritance of skills, but to access to education in childhood.

The contribution of this thesis to this literature is a large-scale OLG model that takes into account all core mechanisms of the human capital literature and allows for endogenous responses within the whole human capital process of primary, secondary and tertiary education to changes in college subsidies and non-tertiary education investments by the government. By embedding this in a large-scale OLG environment, I am able to compare and quantify the different paths of impact of the two policy instruments in a realistic framework. By contrasting the results of the univariate with the bivariate policy experiments in both partial equilibrium and a small open economy setup, the differences of the two policy measures could be highlighted very clearly. I quantify how the interplay of the policy instruments affects distributional aspects of the economy, human capital formation, and ultimately welfare. In addition, inaccuracies of the solution method used in Krueger and Ludwig (2013) were identified and corrected.

[15] Among others, in earlier work Bohacek and Kapicka (2008) and Bohacek and Kapicka (2012) allow for endogenous human capital accumulation in models with education subsidies.

2 A Model for the Interplay of Early and Late Education Subsidies and Taxation

Everything that follows is based on collaborative work of Dirk Krueger, Alexander Ludwig and myself.[16,17]

Large parts of the model we now develop are based on the work of Krueger and Ludwig (2013) and Krueger and Ludwig (2016). The key difference is, that they consider human capital at the age of the college decision to be exogenous. In our model, the formation of human capital is a core element and is shaped by the investments of government and parents during primary and secondary education. In addition, by introducing a stochastic college outcome, which is dependent on the human capital of the student, we refine the tertiary education process. Therefore, this chapter is organized as follows:
We start with a rather brief description of the parts of the model that are borrowed from Krueger and Ludwig. We will then describe the extensions we make in great detail in Section 2.2. In particular, it will be about demonstrating how our model approach does justice to the literature on human capital production.

2.1 Environment

Demographics

Population grows at the exogenous rate χ. We assume that parents give birth to children at the age of j_f and denote the fertility rate of households by f, both assumed to be

[16]We thank the participants of *CMR Lunch Seminar* and the *Money and Macro Brown Bag Seminar* at Goethe University Frankfurt for great feedback in the early phase of this project. All stages of the project were supported by great comments of the participants of the *Reading Group on Quantitative Macroeconomics* at Goethe University Frankfurt. We also thank the participants of the CEPR Workshop *Financing Human Capital* and the participants of the conference *Human Capital and Financial Frictions* at the Georgetown University.

[17]I gratefully acknowledge financial support of the Land North Rhine Westphalia.

the same across education groups.[18] Notice that f is also the number of children per household. Further, agents live with certainty until age J. The population dynamics are then given by

$$N_{t+1,0} = f \cdot N_{t,j_f}, \qquad N_{t+1,j+1} = N_{t,j}, \forall j = 0, \ldots, J-1. \tag{1}$$

Observe that the population growth rate is given by

$$\chi = f^{\frac{1}{1+j_f}} - 1. \tag{2}$$

Furthermore, we denote by $j_a < j_f$ the age of adulthood (i.e., the age when children leave the household, form an own adult household and make the college attendance decision) and by $j_r > j_f$ the retirement age ($j_r - 1$ is the last working age before retirement).

Technology We distinguish between workers according to their qualification $q \in \{n, c, d\}$, where $q = n$ denotes *n*on-college workers (without any attendance at college), $q = c$ denotes workers who completed *c*ollege and $q = d$ workers who attended but *d*ropped out of college. We assume that non-college workers and dropouts are perfect substitutes in production, whereas these two types of labor are imperfectly substitutable in production with respect to college workers (see Katz and Murphy (1992) and Borjas (2003)). Within each qualification-group labor is perfectly substitutable across different ages. Let $L_{t,q}$ denote aggregate labor of qualification group q, measured in efficiency units and let K_t denote the capital stock. Total labor efficiency units at time t, aggregated across the three qualification groups, is then given by

$$L_t = \left((L_{t,n} + L_{t,d})^{\rho} + L_{t,c}^{\rho}\right)^{\frac{1}{\rho}} = \left(L_{t,nd}^{\rho} + L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}, \tag{3}$$

[18] Note that due to the endogeneity of the education decision in the model, if we were to allow differences in the age at which households with different education groups have children it would be hard to assume that the model has a stationary joint distribution over age and skills.

where $L_{t,nd} \equiv L_{t,n} + L_{t,d}$. Aggregate production follows a nested CES-Cobb-Douglas production function, reading as

$$Y_t = F(K_t, L_t) = K_t^{\alpha} (L_t)^{1-\alpha} = K_t^{\alpha} \left[\left(L_{t,nd}^{\rho} + L_{t,c}^{\rho} \right)^{\frac{1}{\rho}} \right]^{1-\alpha}. \tag{4}$$

Perfect competition among firms and constant returns to scale in the production function imply zero profits for all firms and an indeterminate size distribution of firms. Thus, there is no need to specify the ownership structure of firms in the household sector, and without loss of generality we can assume the existence of a single representative firm.

This representative firm rents capital and hires the two skill types of labor on competitive spot markets at prices $r_t + \delta$ and $w_{t,q}$, where r_t is the interest rate, δ the depreciation rate of capital and $w_{t,q}$ is the wage rate per unit of labor of qualification q. Furthermore, we denote by $k_t = \frac{K_t}{L_t}$ the "capital intensity" defined as the ratio of capital to the CES aggregate of labor. Profit maximization of firms implies the standard conditions

$$r_t = \alpha k_t^{\alpha-1} - \delta, \tag{5a}$$

$$w_{t,q} = (1-\alpha) k_t^{\alpha} \left(\frac{L_t}{L_{t,q}} \right)^{1-\rho} = \omega_t \left(\frac{L_t}{L_{t,q}} \right)^{1-\rho}, \tag{5b}$$

where $\omega_t = (1-\alpha)k_t^{\alpha}$ is the marginal product of total aggregate labor L_t. The college wage premium follows as

$$\frac{w_{t,c}}{w_{t,nd}} = \left(\frac{L_{t,nd}}{L_{t,c}} \right)^{1-\rho} \tag{6}$$

and depends on the relative supplies of non-college and college dropout to college labor (unless $\rho = 1$) and the elasticity of substitution between the two types of skills.

Household Preferences

Households are born at age $j = 0$ and form independent households at age j_a, standing in for age 18 in real time. Households give birth at age j_f and children live with adult households until they form their own households. Hence for ages $j = j_f, \ldots, j_f + j_a - 1$ children are present in the parental household. Parents derive utility from per capita consumption of all household members and leisure that are represented by a standard time-separable expected lifetime utility function

$$E_{j_a} \sum_{j=j_a}^{J} \beta^{j-j_a} u\left(\frac{C_j}{1+\mathbf{1}_{\mathcal{J}_s}\zeta f}, \ell_j\right), \tag{7}$$

where C_j is total consumption, ℓ_j is leisure, $0 \le \zeta \le 1$ is an adult equivalence parameter and $\mathbf{1}_{\mathcal{J}_s}$ is an indicator function taking the value one during the period when children are living in the respective household, that is, for $j \in \mathcal{J}_s = [j_f, j_f + j_a - 1]$, and zero otherwise. Expectations are taken with respect to the stochastic processes governing labor productivity risk.

We model an additional form of altruism of households towards their children. At parental age j_f, when children leave the house, the children's expected lifetime utility enters the parental lifetime utility function with a weight $\tilde{\nu}\beta^{j_f}$, where the parameter $\tilde{\nu} = \nu f$ measures the strength of parental altruism.[19]

2.2 Human Capital and College Education

We will now describe the individual components of the human capital process and their impact on wages in our model. In Appendix A.1 we discuss in detail the facts of the human capital literature and where they find themselves in our model. In particular, we will demonstrate why dynamic complementarity and self-productivity (Section 2.2.3) are the key mechanisms we need to incorporate, in order to address the underlying research question.

[19] Evidently the exact timing when children lifetime utility enters that of their parents is inconsequential.

2.2.1 Human Capital Process

Initial Endowments and Human Capital At birth of age $j = 0$ children draw their innate ability h_0 from

$$\log(h_0) = \rho \log(h_0^p) + \epsilon \qquad \text{with } \epsilon \sim N(0, \sigma_{h_0}^2), \tag{8}$$

where h_0^p is innate ability of the respective child's parents.[20] After four periods children reach adulthood at age j_a. In periods $j_0, \ldots, j_a - 1$ kids receive consumption units as well as parents' and government's education investments (i_j^p and i_j^g respectively) in their human capital. Education investments of the government are certain, known by parents and each child receives the same amount. Human capital is acquired given a human capital production function

$$h_{j+1} = f\left(h_j, i_j^p, i_j^g\right) \tag{9}$$

that is assumed to be concave in investments and twice differentiable in its arguments. Governmental and parental investments within a period are assumed to be perfect substitutes, implying we can express the function in terms of total investments within a period, i.e. $I_j = i_j^p + i_j^g$.

The human capital process according to equation (9) is completed at age ja and is expresses in acquired human capital h_{ja}. While it was considered exogenous in Krueger and Ludwig (2013) and Krueger and Ludwig (2016), it is now endogenously determined and a result of a process to which both parents and the government contribute.

College At age j_a children form an adult household and have to decide whether or not to attend college. This decision leads to one of three different qualification outcomes $q \in \{n, c, d\}$. Households deciding not to attend college directly join workforce at age j_a with non-college qualification status $q = n$. Households attending college are enrolled from period j_a to $j_a + 1$.

[20]This specification is borrowed from Restuccia and Urrutia.

We assume that they are hit by a college completion shock and succeed with likelihood $\pi_c(h_{ja})$, which is increasing in acquired human capital, i.e., $\frac{\partial \pi_c(h_{ja})}{\partial h_{ja}} \geq 0$. This changes their qualification status to either $q = c$ for college graduates or to $q = d$ for college dropouts.[21] By extending the model of Krueger and Ludwig (2013) by different outcomes of college attendance, we implement a direct link between human capital and the prospect of a college degree, which refines the tertiary education process.

During college, students have to spend part of their time endowment for studying according to some function $\xi(h_{j_a})$ with $\frac{\partial \xi_{h_{ja}}}{h_{ja}} \leq 0$, i.e., higher skilled students need to spend less time for their studies. College dropouts only get $\phi\xi(h_{j_a})$ deducted from their time endowment, where parameter $\phi < 1$ stands in for average college enrollment of college dropouts. Further, all students are able to work for non-college wages in order to finance consumption and college fees, whereas the latter are proportional to average wages of high-skilled workers and are given by $\kappa w_{t,c}$ (accordingly, dropouts only have to pay $\phi\kappa w_{t,c}$). The government can choose to cover a fraction θ_t of college costs by implementing college subsidies. In addition, a fraction θ_{pr} is borne by private subsidies, capturing the fact that, empirically, a significant share of university funding comes from alumni donations and support by private foundations.

2.2.2 Labor Productivity and Wages

Households with qualification q and age j earn wage

$$w_{t,q}\epsilon_{j,q}\gamma\eta,$$

where $\epsilon_{j,q}$ is a deterministic life-cycle earnings profile, γ stands in for the productivity type of the household and η is an idiosyncratic shock. Recall from Section 2.1 that non-college workers and dropouts are perfect substitutes in aggregate production so that $w_{t,n} = w_{t,d}$.

The deterministic component of life-cycle earnings $\epsilon_{j,q}$ will be determined from life-cycle

[21] Accordingly the probability to drop out of college is given by $1 - \pi_c(h_{j_a})$.

earnings data. With regard to the productivity component γ, we assume that the continuous variable of acquired human capital h_{j_a} is mapped into qualification specific productivity types $\gamma \in \Gamma_q = \{\gamma_q^h, \gamma_q^l\}$, with $\gamma_q^h > \gamma_q^l$.[22] Hence, this fixed effect spreads out wages within each education group and, once determined, is constant over the life-cycle. It is drawn at age j_a for non-college households and at age j_{a+1} for households going to college (i.e., both college graduates and college dropouts), when entering the labor market. The probability of drawing the high productivity type γ_q^h is given by $\pi_\gamma(h_{j_a})$ and increasing in acquired human capital h_{j_a}.

The stochastic component η is an idiosyncratic earnings shock which is mean-reverting and follows a qualification group specific Markov chain with states $\mathcal{E}_q = \{\eta_{q1,...,\eta_{qM}}\}$ and transitions $\pi_{\eta_q}(\eta'|\eta) > 0$. Prior to the college decision, at age j_a, η is drawn from Π_n. Both college graduates and college dropouts re-draw an initial η from Π_q after college at age j_{a+1}. Table 1 summarizes the wage processes for the different types of households at different stages of their life-cycle.

	non-college	college	dropout
q	n	c	d
wage at $j = j_a$	$w_{t,n}\epsilon_{j,n}\gamma\eta$	$w_{t,n}\epsilon_{j,c}\eta$	$w_{t,n}\epsilon_{j,d}\eta$
college costs at $j = j_a$	-	$\kappa w_{t,c}(1-\theta_t-\theta_{pr})$	$\phi\kappa w_{t,c}(1-\theta_t-\theta_{pr})$
time loss at $j = j_a$, $\xi(h_{j_a})$	-	$\xi(h_{j_a})$	$\phi\xi(h_{j_a})$
wage at $j_s, \ldots, j_r$	$w_{t,n}\epsilon_{j,n}\gamma\eta$	$w_{t,c}\epsilon_{j,c}\gamma\eta$	$w_{t,d}\epsilon_{j,d}\gamma\eta$

Table 1: Educational CVs

2.2.3 Dynamic Complementarity and Self-Productivity

The trio of *"Human Capital Process"*, *"College"* and *"Labor Productivity and Wages"* altogether describes both the process of human capital formation and the incentive for its production, namely the return on investment in the form of resulting wages. In order for us to develop a reliable model, this whole process has to be in line with the human capital literature, as summarized in Cunha and Heckman (2007) and (2010). Besides the empirical facts, they show that

[22]Mapping a continuous variable into a discrete variable with only two outcomes is convenient computationally.

a CES production function can capture the core mechanisms behind the empiricism sufficiently. We discuss these facts and their relation to the functional form of the human capital production in detail in Appendix A.1. Here we concentrate on the technical requirements.

Regarding equation (9), let us assume the following function:

$$h_{j+1} = f_j(h_j, i_j) = \left(\upsilon_j h_j^{\phi_j} + (1 - \upsilon_j)\left(\psi I_j\right)^{\phi_j}\right)^{\frac{1}{\phi_j}}, \tag{10}$$

with $\phi_j \neq 0$. Parental and governmental investments are assumed to be perfect substitutes and summed up in $I_j = i_j^p + i_j^g$. Using the recursive form of (10) and taking into account the four period structure of our model, human capital at age j_a, when young adults face the college decision, is received by substituting in $h_{ja-1}, \ldots, h_{ja-4}$ and we arrive at:

$$h_{ja} = m(h_0, I_0, \ldots, I_3), \tag{11}$$

where h_0 is the innate human capital the kid was born with. Equation (11) expresses human capital as a function of all investments during childhood ($j = 0, \ldots, 3$). The first key mechanism a proper human capital exhibit is **dynamic complementarity**, which is defined as:

$$\frac{\partial^2 f_j(h_j, I_j)}{\partial h_j \partial I_j} > 0.$$

This property ensures, that when skills acquired up to current age j are higher, investments in human capital within this period (I_t) yield higher returns. In addition, as h_j is strictly increasing in all past investments, that also creates a direct, positive relation between past investments to the return on investment at current age j. Dynamic complementarity should not be confused with decreasing marginal products, as we still have $\frac{\partial^2 f_j(h_j, I_j)}{\partial I_j \partial I_j} < 0$. It rather reflects the dynamic structure of human capital formation. An example of this would be two children, both of whom have just completed elementary school. Assuming these children are identical except for their IQ, then the kid with higher IQ would pull more out of secondary school than the kid with lower IQ (dynamic complementarity). On the other hand, endless learning in secondary school would not lead to an arbitrarily high level of human capital (decreasing marginal returns).

The second key mechanism is **self-productivity**. Intuitive speaking, higher stocks of skills in one period need to create higher stocks of skills in the next period. Self-productivity arises, when the function fulfills the following property:

$$\frac{\partial f_j(h_j, I_j)}{\partial h_j} > 0.$$

In the context of our model, we have to distinguish two phases. Primary and secondary education are taking place during the first four model periods, in which human capital is developed according to a function in the spirit of (10). Thus, we remain in the standard notion of the human capital literature and skill development exhibits both dynamic complementarity and self-productivity. However, it does get more complicated when we look at college education. In our design, it is rather complicated to work with derivatives the way Cunha and Heckman (2007) do. Nevertheless, the basic idea should remain, implying that investing in a possible college degree should follow the core mechanisms of human capital formation.

As opposed to primary and secondary education, a successful college degree is not expressed in a higher skill level h_{ja}. Instead, it translates into a higher wage after college. In addition, college completion and drawing the productivity type is risky in our model and we therefore work with expectations over both mechanisms. The preservation of self-productivity in tertiary education is straightforward: higher skills h_{ja} lead to a higher (expected) human capital level, which is reflected in a higher expectation of both college completion and drawing productivity type γ^h.

In order to be able to address dynamic complementarity, we first have to define the *return on education* during the college period. As mentioned, unlike in primary and secondary education, a higher skill level after college directly translates into higher wages. Thus, we denote as return for college attendance the *expected wage increase*, i.e. the expected difference between wages after attending college and the outside option of directly joining the labor market. The idea of dynamic complementarity is retained, if attending college yields to a higher expected wage increase for households with higher skill levels. We abstract from idiosyncratic shocks as well as the qualification specific age profiles ($\epsilon_{j,q}$). The former is independent from human capital and the latter will be normalized and only differ in shape for different qualifications. Further, we abstract from time costs $\zeta(h_{j_a})$ reflecting opportunity costs of studying, which effects the return on attending college and is dependent of acquired ability. However, it is monotone decreasing in h_{j_a} and therefore not in conflict with the unambiguity of (12). The expected wage increase per period of an agent with acquired ability level h_{j_a} is the following:

$$
\begin{aligned}
E\left[\triangle w(h_{j_a})\right] =& \pi_c(h_{j_a})\left[(1-\pi_\gamma(h_{j_a}))\gamma_c^l + \pi_\gamma(h_{j_a})\gamma_c^h\right] w_{t,c} + (1-\pi_c(h_{j_a}))\left[(1-\pi_\gamma(h_{j_a}))\gamma_n^l + \pi_\gamma(h_{j_a})\gamma_n^h\right] w_{t,n} \\
& - \left[(1-\pi_\gamma(h_{j_a}))\gamma_n^l + \pi_\gamma(h_{j_a})\gamma_n^h\right] w_{t,n} \\
=& \pi_c(h_{j_a})\left[\gamma_c^l + \pi_\gamma(h_{j_a})(\gamma_c^h - \gamma_c^l)\right] w_{t,c} - \pi_c(h_{j_a})\left[\gamma_n^l + \pi_\gamma(h_{j_a})(\gamma_n^h - \gamma_n^l)\right] w_{t,n} \\
=& \pi_c(h_{j_a}) \underbrace{\left[\left(\gamma_c^l + \pi_\gamma(h_{j_a})(\gamma_c^h - \gamma_c^l)\right) w_{t,c} - \left(\gamma_n^l + \pi_\gamma(h_{j_a})(\gamma_n^h - \gamma_n^l)\right) w_{t,n}\right]}_{:=\hat{w}(h_{j_a})}. \qquad (12)
\end{aligned}
$$

The first line shows the expected wage after college for an agent of type h_{j_a}, while the second line denotes the expected wage of non-college agents. Uncertainty has two sources[23] for students. Before starting their studies, they do not know whether they will finish college successfully. In addition, they are unaware of the productivity shock they will draw, which also applies for non-college workers. We were able to collapse the equation, as we assume the wages and productivity types for dropouts and non-college agents to be the same. Further, we will assume that $\pi_\gamma(h_{j_a})$ is following the same distribution for skilled and unskilled labor.[24]

Regarding equation (12), $\pi_c(h_{j_a})$ is clearly increasing in human capital. In order for the remaining piece of (12) to be unambiguously increasing in h_{j_a}, we need $\frac{\partial \triangle \hat{w}(h_{j_a})}{\partial h_{j_a}} > 0$, which we

[23]We are still abstracting from the idiosyncratic shock, which is not important for the expected wage increase.

[24]It is worth noticing that there will be higher overall wages for dropouts in the calibrated version of the model, which will be driven by higher abilities of dropouts compared to non-college agents and therefore higher average productivity realizations.

can express as:

$$\begin{aligned}\frac{\partial\triangle\hat{w}(h_{j_a})}{\partial h_{j_a}} &= \frac{\partial\pi_\gamma(h_{j_a})}{\partial h_{j_a}}(\gamma_c^h-\gamma_c^l)w_{t,c} - \frac{\partial\pi_\gamma(h_{j_a})}{\partial h_{j_a}}(\gamma_n^h-\gamma_n^l)w_{t,n} \\ &= \frac{\partial\pi_\gamma(h_{j_a})}{\partial h_{j_a}}\Big((\gamma_c^h-\gamma_c^l)w_{t,c} - (\gamma_n^h-\gamma_n^l)w_{t,n}\Big). \end{aligned} \tag{13}$$

Thus, for (13) to be positive for all h_{ja} types, it is necessary that the wage spread within a qualification group, weighted with the respective average wage is higher for college graduates, i.e. $(\gamma_c^h-\gamma_c^l)w_{t,c} > (\gamma_n^h-\gamma_n^l)w_{t,n}$. This condition will be established in the calibration. However, the empirical background is straightforward: $w_{t,c} > w_{t,n}$ reflects a positive wage premium $\frac{w_{t,c}}{w_{t,n}}$ of 1.8 in the data.[25]

The different sources of risk make it possible for a household with lower human capital to receive a larger wage than a household with higher human capital. This is intended and in line with reality. However, what is important to properly model the mechanisms of human capital production is an expected wage increase which is positively dependent on human capital - ex ante dynamic complementarity, if you will - which our model captures.

2.3 Market Structure

We assume that financial markets are incomplete in that there is no insurance available against idiosyncratic and productivity labor income shocks. Households can self-insure against this risk by accumulating a risk-free one-period bond that pays a real interest rate of r_t. In equilibrium the total net supply of this bond equals the capital stock K_t in the economy, plus the stock of outstanding government debt B_t.

Furthermore, we severely restrict the use of credit to self-insure against idiosyncratic labor productivity and thus income shocks by imposing a strict credit limit. The only borrowing we permit is to finance a college education. Households that borrow to pay for college tuition and consumption while in college face age-dependent borrowing limits of $\underline{A}_{j,t}$ (whose size depends

[25] In the calibrated benchmark model, equation (13) will take a value of 0.1848.

on the degree to which the government subsidizes education) and also face the constraint that their balance of outstanding student loans cannot increase after college completion. This assumption rules out that student loans are used for general consumption smoothing. College dropouts are only allowed to borrow up to $\phi\underline{A}_{j,t}$.

The constraints $\underline{A}_{j,t}$ are set such that student loans need to be fully repaid by retirement at age j_r, which also insures that households can never die in debt. Beyond student loans we rule out borrowing altogether. This, among other things, implies that non-college households can never borrow. As the calibration of the model will make clear, we think of the constraints $\underline{A}_{j,t}$ being determined by public student loan programs, and thus one may interpret the borrowing limits as government policy parameters that are being held fixed in our analysis.

2.4 Government Policies

The government needs to finance an exogenous stream G_t of non-education expenditures and an endogenous stream E_t of education expenditures, financing both early (non-tertiary) and late (tertiary) education. It can do so by issuing government debt B_t, by levying linear consumption taxes τ_c and income taxes $T_t(y_t)$ which are not restricted to be linear. The initial stock of government debt B_0 is given. We restrict attention to a tax system that discriminates between the sources of income (capital versus labor income), taxes capital income $r_t A_t$ at the constant rate $\tau_{k,t}$, but permits labor income taxes to be progressive or regressive. We take consumption and capital income tax rates $\tau_c, \tau_{k,t}$ as exogenously given, but optimize over labor income tax schedules within a simple parametric class.

Specifically, the total amount of labor income taxes paid takes the following simple linear form

$$T_t(y_t) = \max\left\{0, \tau_{l,t}\left(y_t - d_t\frac{Y_t}{N_t}\right)\right\} = \max\{0, \tau_{l,t}\,(y_t - Z_t)\}, \tag{14}$$

where y_t is household taxable labor income, $\frac{Y_t}{N_t}$ is per capita income in the economy and $Z_t = d_t\frac{Y_t}{N_t}$ measures the size of the labor income tax deduction. Therefore, for every period there are two policy parameters on the tax side $(\tau_{l,t}, d_t)$. Note that the tax system is potentially progressive (if $d_t > 0$) or regressive (if $d_t < 0$).

The government uses tax revenues to finance education subsidies to tertiary education, θ_t, to early childhood education, $i^g_{0,t}, \ldots, i^g_{j_a-1,t}$ and exogenous government spending

$$G_t = gy \cdot Y_t,$$

where the share of output $gy = \frac{G_t}{Y_t}$ commanded by the government is a parameter to be calibrated from the data.[26]

In addition, the government administers a pure pay-as-you-go social security system that collects payroll taxes $\tau_{ss,t}$ and pays benefits $p_{t,j}(\gamma, q)$, which depend on the wages a household has earned during her working years, and thus on her characteristics (γ, q) as well as on the time period in which the household retired (which, given today's date t can be inferred from the current age j of the household). In addition, the introduction of social security is helpful to obtain more realistic life-cycle saving profiles and an empirically more plausible wealth distribution.

Since the part of labor income that is paid by the employer as social security contribution is not subject to income taxes, taxable labor income equals $(1 - 0.5\tau_{ss,t})$ per dollar of labor income earned, i.e.

$$Y_t = (1 - 0.5\tau_{ss,t}) w_{t,q} \gamma \eta \epsilon_{j,q} \ell. \tag{15}$$

2.5 Time Line

(i) A child is **born** at age $j = 0$ with innate ability h_0, drawn from a distribution that is dependent on parental innate ability, h^p_0.

(ii) **Childhood** is described by ages $j_0, \ldots, j_a - 1$ during which the human capital accumu-

[26] Once we turn to the determination of optimal tax and subsidy policies we will treat G rather than gy as constant. A change in policy changes output Y_t and by holding G fixed we assume that the government does not respond to the change in tax revenues by adjusting government spending (if we held gy constant it would).

lation process takes place. Parents decide on their investments into the education of their children as well as the per capita consumption of the household.

(iii) Turning j_a-years old, children form an own adult household and make the **college decision**. Prior to the decision parents make inter-vivos transfers B, based on children's acquired ability h_{j_a} and innate ability h_0. After these transfers have been made, children draw η from $\Pi_n(\eta)$ and make their college decision based on the state variables $\{j_a, A = B/(1 + r(1 - \tau_k)), h_0, h_{j_a}, \eta\}$. After the decision is made non-college children (from now on households) draw the fixed effect γ and start working. Households attending college draw the college completion shock, which depends on acquired human capital h_{j_a}. College dropouts pay fewer college fees, face tighter borrowing limits and lower time losses from studying, all of which stands in for the shorter time period they attend college (which reflects the subperiod structure of the model at age j_a for households that attend college). Given time losses, both groups can work for non-college wages during their studies until age $j_a + 1$.

(iv) At age $j_a + 1$ college graduates and dropouts draw their productivity shock from a distribution contingent on qualification and acquired human capital and redraw their idiosyncratic income shock from a college specific distribution, $\eta \in \Pi_q(\eta)$. Ages between $j_a + 1$ and $j_f - 1$ can be summarized as **working without children**: qualification q, productivity type γ, age-productivity profile $\epsilon_{j,q}$, and idiosyncratic shock η determine wages, $w_{t,q}\gamma\eta\epsilon_{j,q}$, and households face a standard labor-leisure choice in every period. This will change between $j_f - 1$ and j_f when children enter the utility function.

(v) The age between $j_f, \dots, j_f + j_a - 1$ is referred to as **working as parents of children**. At the beginning of period j_f kids enter the household and draw their innate ability based on the innate ability of parents following (8). Parents then maximize over per capita consumption of the household, their labor supply and their investments into their children's human capital, i.e. $i^p_{j=0}, \dots, i^p_{j_a-1}$. Hence, a change of state variables takes place: parent's innate human capital is replaced by the innate human capital of their children and acquired human capital of the child is added.[27]

(vi) When parents are at the age of $j_f + j_a$ their **children become adults**. Observing the acquired human capital of their children parents transfer monetary resources B to their kids (inter-vivos transfer). Parents draw utility from the expected value function of their children at that age according to the altruism parameter $\tilde{\nu}$. After that children are no longer part of parents' value functions, implying the state space is reduced by h_0^c and $h_{j_a}^c$.

(vii) While **working as parents of adults** at ages $j_f + j_a + 1, \dots, j_r - 1$ the model reduces to a standard consumption, labor-leisure choice model.

(viii) During **retirement** at ages $j_r, \dots, J$ households receive income from savings and social security $p_{t,j}(q, \gamma)$, which is dependent on their labor characteristics.

The timeline of the model is summarized in Figure 1.

[27]The innate human capital of the child is still relevant, as its future children will draw their innate ability from it.

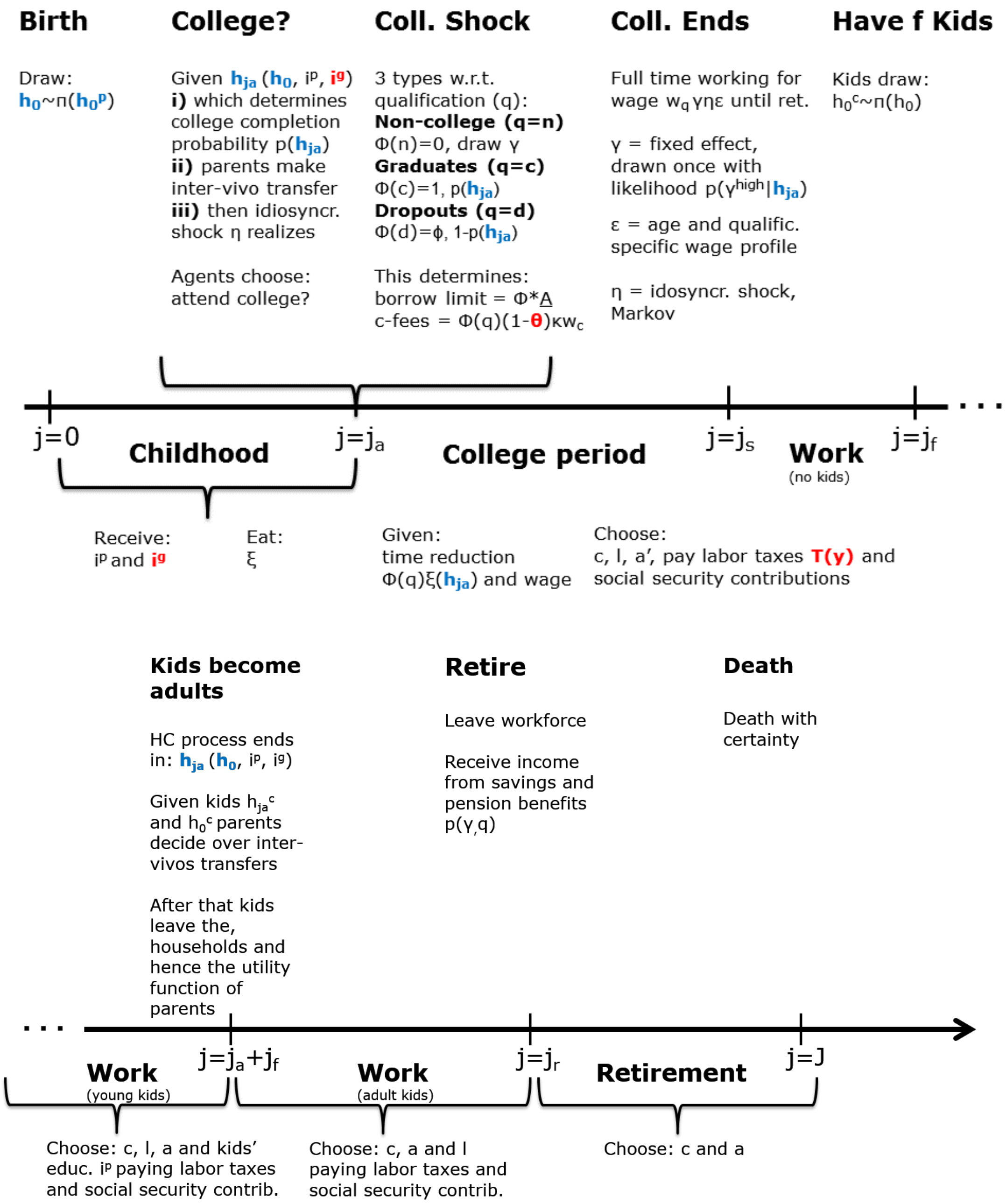


Figure 1: Time Line

3 Solving the Household Problem

In our model, households live through various stages and are facing different maximization problems, which we will now display recursively. Part of these periods can be solved by the standard endogenous grid method by Carroll (2005). This includes all ages in which households only have to deal with a labor-leisure and savings choice. Given the tax code (14) we use, there is different regions with respect to labor taxes. A detailed description of the household problem, first order conditions as well as the computational solution method that is being used can be found in Appendix B.1.

In period $j_f, \ldots, j_f + j_a - 1$ parents are shaping kids' human capital h through investments i^p_k into h', while in period j_t inter-vivos transfers B constitute kid's assets at the college decision. Therefore, in both cases the endogenous grid method by Carroll (2005) needs to be extended. In Section 3.2 we will show that a hybrid method combining (i) the endogenous grid method for the savings, labor and consumption decision with (ii) maximizing over the level of the value function for optimal investments or inter-vivos transfers is not feasible. We will develop an alternative that delivers accurate results.

In addition, in Section 3.3, we extend the model of Krueger and Ludwig (2013) by taste shocks in order to avoid kinks in the value function and jumps in first order conditions, which are caused by the discrete college decision.

3.1 Recursive Problem of Households

Childhood at $j = 0, \ldots, j_a - 1$**.** Children draw innate ability h_0 according to (8). During childhood kids consume and receive parental $(i^p_{j=0}, \ldots, i^p_{j_a-1})$ and governmental $(i^g_{j=0}, \ldots, i^g_{j_a-1})$ investments, which we sum up to $I_{j=0}, \ldots, I_{j_a-1}$.

College decision at j_a. Children become adults and form own households. Their acquired human capital is stored as the period j_a value of the endogenous state variable h. Before making the college decision, a child receives an inter-vivos transfer B from parents (leading to assets $A = B/(1 + r(1 - \tau_k)))$.[28] Next, children draw the idiosyncratic productivity shock η. Therefore, the state space at age j_a is given by $\{j_a, A, h_0, h, \eta\}$. Initial human capital, h_0, is part of the state space, because it is linked to the innate human capital of future children. In order to store the college decision, we denote by λ the following indicator function:

$$\lambda(j_a, A, h_0, h, \eta) = \begin{cases} 1 & \text{if } E_{q\in\{d,c\}|h}\left[V\left(j_a, A, h_0, h, \neg n, \eta\right)\right] > E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V\left(j_a, A, h_0, n, \eta\right)\right] \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Youngsters attending college are subject to the completion shock described by (32). Non-college households draw the productivity shock $\gamma \sim \pi(\gamma, n)$ already at age j_a, whereas college households ($q = c$ and $q = d$) draw it in period $j_a + 1$. Therefore, the expected value functions at age j_a write explicitly:

$$\begin{aligned} E_{q\in\{d,c\}|h}\left[V\left(j_a, A, h_0, h, \neg n, \eta\right)\right] = \pi_c(h)\left[V\left(j_a, A, h_0, h, c, \eta\right)\right] + \dots \\ \dots + (1 - \pi_c(h))\left[V\left(j_a, A, h_0, h, d, \eta\right)\right], \end{aligned}$$

and

$$\begin{aligned} E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V\left(j_a, A, h_0, n, \eta\right)\right] = \pi_\gamma(h)\left[V\left(j_a, A, h_0, h, n, \gamma^h, \eta\right)\right] + \dots \\ \dots + (1 - \pi_\gamma(h))\left[V\left(j_a, A, h_0, h, n, \gamma^l, \eta\right)\right]. \end{aligned}$$

Households with $\lambda(j_a, A, h_0, h, \eta) = 1$ will attend college and out of these households, a fraction $1 - \pi_c(h)$ will dropout from college.

[28]For all ages $j > j_a$ assets A brought into the period generate gross revenue $(1 + r(1 - \tau_k))A$. Given our timing assumption inter-vivos transfers B generate gross revenue of B. Thus the initial asset state of households of age j_a is $A = B/(1 + r(1 - \tau_k))$.

As described above, college graduates and college dropouts have to spend different amounts of time for studying, pay different fees and are subject to different borrowing constraints. To simplify notation we define by

$$\Psi(q) = \begin{cases} 1 & \text{if } q = c \\ \phi & \text{if } q = d \\ 0 & \text{if } q = n \end{cases}$$

implying that, depending on their qualification status, households spend time $\Psi(q)\xi(h)$ for studying, pay college fees $\Psi(q)(1-\theta_t-\theta_{pr})\kappa w_{t,c}$ and are allowed to borrow up to a limit $\Psi(q)\underline{A}_{j,t}$.

College period j_a until $j_a + 1$. Non-college households draw their productivity type γ and join the workforce. Their dynamic problem then reads as

$$V(j_a, A, h_0, n, \gamma, \eta) = \max_{\substack{C,\ell\in[0,1]\\ A'\geq 0}} \left\{u(C, 1-\ell) + \beta E_{\eta'|\eta}\left[V'(j_a+1, A', h_0, n, \gamma, \eta')\right]\right\},$$

subject to

$$(1+\tau_c)C + A' + T_t(Y_t) = R_t^n A + (1-\tau_{ss})w_{t,n}\gamma\eta\epsilon_{j,n}\ell,$$

where we defined $R_t^n = 1 + (1-\tau_{k,t})r_t$ and took Y_t from (15).

The problem of households in college reads as

$$V(j_a, A, h_0, h, q, \eta) = \max_{\substack{C,\ell\in[0,1-\Psi(q)\xi(h)]\\ A'\geq -\Psi(q)\underline{A}_{j,t}}} \{u(C, 1-\Psi(q)\xi(h)-\ell) +$$
$$\beta E_{\gamma|h,\eta'\in\Pi_q(\eta')}\left[V'(j_a, A', h_0, q, \gamma, \eta')\right]\Big\},$$

subject to

$$(1+\tau_c)C + A' + \Psi(q)(1-\theta_t-\theta_{pr})\kappa w_{t,c} + T_t(Y_t) = R_t^n A + (1-\tau_{ss})w_{t,n}\eta\epsilon_{j,q}\ell.$$

Expectations are formed with respect to γ the agent will draw given q and h as well as η redrawn from initial distribution $\Pi_q(\eta')$.

Working without children at $j_a+1,\ldots,j_f-2$. The state space of all households is now given by $\{j, A, h_0, q, \gamma, \eta\}$. The problem reads as:

$$V(j, A, h_0, q, \gamma, \eta) = \max_{\substack{C,\ell\in[0,1] \\ A'\geq -\Psi(q)\underline{A}_{j,t}}} \left\{u(C, 1-\ell) + \beta E_{\eta'|\eta}\left[V'\left(j+1, A', h_0, q, \gamma, \eta'\right)\right]\right\}$$

subject to

$$(1+\tau_c)C + A' + T_t(Y_t) = R_t^n A + (1-\tau_{ss})w_{t,q}\gamma\eta\epsilon_{j,q}\ell.$$

Preparing for parenthood at j_f-1. At this stage the *"soon-to-be" parents* have to form expectations about the initial human capital their children will be born with (h_0'):

$$V(j, A, h_0, q, \gamma, \eta) = \max_{\substack{C,\ell\in[0,1-\ell] \\ A'\geq -\Psi(q)\underline{A}_{j,t}}} \left\{u(C, 1-\ell) + \beta E_{\eta'|\eta, h_0'|h_0}\left[V'\left(j+1, A', h_0', h', q, \gamma, \eta'\right)\right]\right\}$$

subject to

$$(1+\tau_c)C + A' + T_t(Y_t) = R_t^n A + (1-\tau_{ss})w_{t,q}\gamma\eta\epsilon_{j,q}\ell.$$

As soon as parents' innate ability is mapped into innate ability of their children according to equation (8), h_0 is replaced by the human capital of their children, h_0'. The latter matters for two reasons: (i) it becomes part of the human capital process and (ii) it determines innate ability of future grandchildren. Also note that at birth acquired human capital equals innate human capital (implying $h'_0 = h'$ in V').

Working as parents of children at $j_f, \dots, j_f + j_a - 1$. In this period, children are living in the household and parents maximize utility from per capita consumption, own leisure and investments into their children. When the parent is of age j, the children living in the household are of age $j_k = j - j_f$. Now, the additional state variable relevant to the household is the acquired human capital of children at age j_k, which is stored in the current period state variable h. Furthermore, initial human capital of children, now denoted as h_0, continuous to be an element of the state space, because it determines the innate ability of grandchildren. The problem therefore reads as:

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C, \ell \in [0,1] \\ A' \geq -\Psi(q)\underline{A}_{j,t} \\ i_k^p \geq 0}} \left\{ u\left(\frac{C}{1 + \zeta f}, 1 - \ell\right) + \dots \right.$$
$$\left. \dots + \beta E_{\eta'|\eta}\left[V'(j+1, A', h_0, h', q, \gamma, \eta')\right] \right\},$$

subject to

$$(1 + \tau_c)C + A' + T_t(Y_t) + i_k^p f = R_t^n A + (1 - \tau_{ss}) w_{t,q} \gamma \eta \epsilon_{j,q} \ell,$$

where human capital accumulation follows (9).[29]

Children become adults at $j_f + j_a$. Before children leave the household they receive inter-vivos transfers B from their parents which determines their initial assets $A = B/(1 + r(1 - \tau_k))$. Parents can perfectly observe the innate ability h_0 of their children as well as their acquired human capital at the child's age j_a. The shock η' on children's earnings realizes after the inter-

[29] Human capital investments are made to each of the f children within a household, which is why i_k^p has to be scaled by f in the budget constraint.

vivos transfer decision has been made:

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C,\ell\in[0,1],B\geq 0\\ A'\geq -\Psi(q)\underline{A}_{j,t}}} \left\{u(C, 1-\ell) + \beta E_{\eta'|\eta}\left[V'(j+1, A', q, \gamma, \eta')\right]\right\}$$
$$+ \tilde{\nu} E_{\eta'\in\Pi_n(\eta)}\left[\max\left\{E_{q\in\{d,c\}|h}\left[V(j_a, A, h_0, h, \neg n, \eta)\right], E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V(j_a, A, h_0, n, \eta)\right]\right\}\right],$$

subject to

$$(1+\tau_c)C + A' + Bf + T_t(Y_t) = R_t^n A + (1-\tau_{ss}) w_{t,q}\gamma\eta\epsilon_{j,q}\ell.$$

Working as parents of adults at $j_a + j_f, \ldots, j_r - 1$. When children have left the household, the state space collapses to (j, A, q, γ, η). Despite the missing initial human capital, h_0, the problem is identical to the one at ages $j_a + 1, \ldots, j_f - 2$. Lifetime utility of the children is no longer part of parents' utility (which causes a discontinuity in the value function).

Retirement at $j_r, \ldots, J$. Households receive pension payments and income from savings. Debt from college is fully repaid. The maximization problem now reads as:

$$V(j, A, q, \gamma) = \max_{C, A'\geq 0}\left\{u(C, 1) + \beta V'(j+1, A', q, \gamma)\right\}$$

subject to

$$(1+\tau_c)C + A' = R_t^n A + p_{t,j}(q, \gamma).$$

Productivity type and qualification are still part of the state vector as they determine pension benefits.

3.2 Hybrid Solution Methods

In this chapter we will show, by comparing decisions based on different cash-on-hand levels, that the solution method used in Krueger and Ludwig (2013)[30] does not address the trade-off between consumption, savings and investments correctly. Thereby it systematically underestimates the investment choice. In addition, the region in which households are borrowing constraint is not identified correctly, when situations occur in which savings are zero but investments are positive.

At ages $j_f, \ldots, j_f + j_a - 1$ parents are nurturing kids' human capital h, shaping it through investments i_k^p into h', while in period j_t, inter-vivos transfers B constitute kids' assets at the college decision. Both cases make an extension of the endogenous grid method by Carroll (2005) necessary. Moreover, given the recursive structure of the model, our solution approach has to deal with the issue that both the value function and its derivative regarding inter-vivos transfers and investments are unknown at this point.

In Appendix B.2 and B.3 we describe two hybrid methods, searching for optimal decisions within these periods. The first one is borrowed from Krueger and Ludwig (2013) and is combining (i) the endogenous grid method for the consumption, leisure and savings decision with (ii) maximizing over the level of the value function in order to find optimal investments or vivos transfers (*HybLevEndo* hereafter). The second method, developed in thesis, also maximizes over the level of the value function for optimal investments and vivos transfers, but it replaces the endogenous grid method in step (i) by the exogenous grid method (*HybLevExog* hereafter).

Although essentially following the same idea, we will show that only *HybLevExog* delivers accurate results. By comparing decisions based on different cash-on-hand levels, *HybLevEndo* does not examine the trade-off between consumption, savings and investments correctly. In order to illustrate the deviating outcomes and their origin, in Section 3.2.1 we develop a simple

[30]Please note that as Krueger and Ludwig (2013) do not model human capital investments, this issue does only appear in the period of inter-vivos transfers in their model. However, in this thesis, the problem would also occur in periods in which parents invest in the human capital of their children.

two period model and derive its analytic solution. In Section 3.2.2 we then solve this model numerically under both methods and compare their respective results.

3.2.1 Analytical Solution of a Two-Period Model

We consider a two period model in which households receive utility from consumption in both periods and investments in the first period. The latter is weighted with altruism parameter $\nu < 1$. In period 0, households start with cash-on-hand $X \geq \underline{X}$, where $\underline{X} > 0$ is some minimum cash-on-hand level implied by the exogenous income process, while in period 1 they receive exogenous income $B \geq 0$. Households split their assets among the two periods in order to maximize the sum of utility from consumption and investments:

$$
\begin{aligned}
u &= \ln(C_0) + \ln(C_1) + \nu \ln(i_0 + \bar{i}) \qquad (17)\\
A_0' &= X - C_0 - i_0 \\
C_1 &= A_0' + B \\
A_0' &\geq 0 \\
i_0 &\geq 0.
\end{aligned}
$$

In the absence of interest rate and discount factor, combined with a concave utility function, households will try to choose $C_0 = C_1$. At the same time there will be a threshold at some $C_0 = C_1 > 0$ at which u_c becomes sufficiently small relative to u_i so that the household starts choosing $i_0 > 0$. Investments i_0 and savings A_0' have to be non-negative. Governmental investments $\bar{i}$ can be understood as some minimum level, provided by the state. That way we induce that optimal investment decisions might be zero, standing in for a situation in which parents are poor and children are rich (in terms of human capital), such that parents will choose to investment $i_0 = 0$. Apparently, the combination of X and B determines whether or not the household is borrowing constraint ($A_0' \geq 0$ is binding). In addition, the household will either choose to invest $i_0 \geq 0$ or be bound by this restriction (because the optimal unconstraint investment would be $i_0^* < 0$).

We start by examining the situation in which the constraint $A_0' \geq 0$ is not binding. By substituting $C_1 = A_0' + B = X - C_0 - i_0 + B$ we can rewrite the maximization problem as:

$$u = \ln(C_0) + \ln(X + B - (C_0 + i_0)) + \nu \ln(i_0 + \bar{i})$$
$$i_0 \geq 0.$$

The first-order condition for this interior solution with respect to consumption is

$$\frac{1}{C_0} = \frac{1}{X + B - (C_0 + i_0)}, \tag{18}$$

while the first-order condition with respect to investments reads as

$$\frac{1}{X + B - (C_0 + i_0)} = \nu \frac{1}{i_0 + \bar{i}}. \tag{19}$$

Now, by rewriting (18) and (19) we get:

$$C_0 = \frac{X + B - i_0}{2} \tag{20}$$
$$C_0 = X + B - \frac{1 + \nu}{\nu} i_0 - \frac{\bar{i}}{\nu}. \tag{21}$$

Equating (20) with (21) leads to:

$$i_0 = \frac{\nu(X + B) - 2\bar{i}}{2 + \nu}. \tag{22}$$

Having a closer look at the numerator, we can make the following case distinction:

(i) $X + B < 2\frac{\bar{i}}{\nu}$: In this case $i_0 \geq 0$ is binding and $i_0^\star = 0$, while, following (20), the interior solution for consumption would be $C_0 = C_1 = \frac{X+B}{2}$ and $A_0' = \frac{X-B}{2}$. Hence, we get the following sub-cases:

(a) $X < B$. Then $C_0 = X$, $C_1 = B$, $i_0 = 0$, $A_0' = 0$.

(b) $X \geq B$. Then $C_0 = C_1 = \frac{X+B}{2}$ and $A_0' = \frac{X-B}{2}$.

(ii) $X + B \geq 2\frac{\bar{i}}{\nu}$: Then $i_0^\star$ is given by (22). Using this in (20) we get

$$C_0 = C_1 = \frac{X + B + \bar{i}}{2 + \nu}. \tag{23}$$

and A_0' is given by

$$A_0' = \frac{X - (1 + \nu)B + \bar{i}}{2 + \nu}. \tag{24}$$

Again, the relation of X and B determines in which sub-case the household is:

(a) $X < (1 + \nu)B - \bar{i}$: The constraint household is forced to choose $A_0' = 0$ and $C_1 = B$. However, she will hold on to the intra-temporal Euler equation between consumption and investment which is given by:

$$\frac{1}{C_0} = \nu \frac{1}{i_0 + \bar{i}}$$

and therefore

$$C_0 = \frac{1}{\nu}\left(i_0 + \bar{i}\right). \tag{25}$$

Plugging this into the budget constraint $X = C_0 + i_0$, we get

$$\begin{aligned} X &= \frac{1}{\nu}\bar{i} + \frac{1 + \nu}{\nu} i_0 \\ \Leftrightarrow \quad i_0 &= \frac{\nu X - \bar{i}}{1 + \nu}. \end{aligned}$$

This leaves us with two new sub-cases:

1. $X < \frac{\bar{i}}{\nu}$: Then $i_0 = 0$ and $C_0 = X$.
2. $X \geq \frac{\bar{i}}{\nu}$: Then $i_0 = \frac{\nu X - \bar{i}}{1+\nu}$ and C_0 follows from the intra-temporal Euler equa-

tion (25):

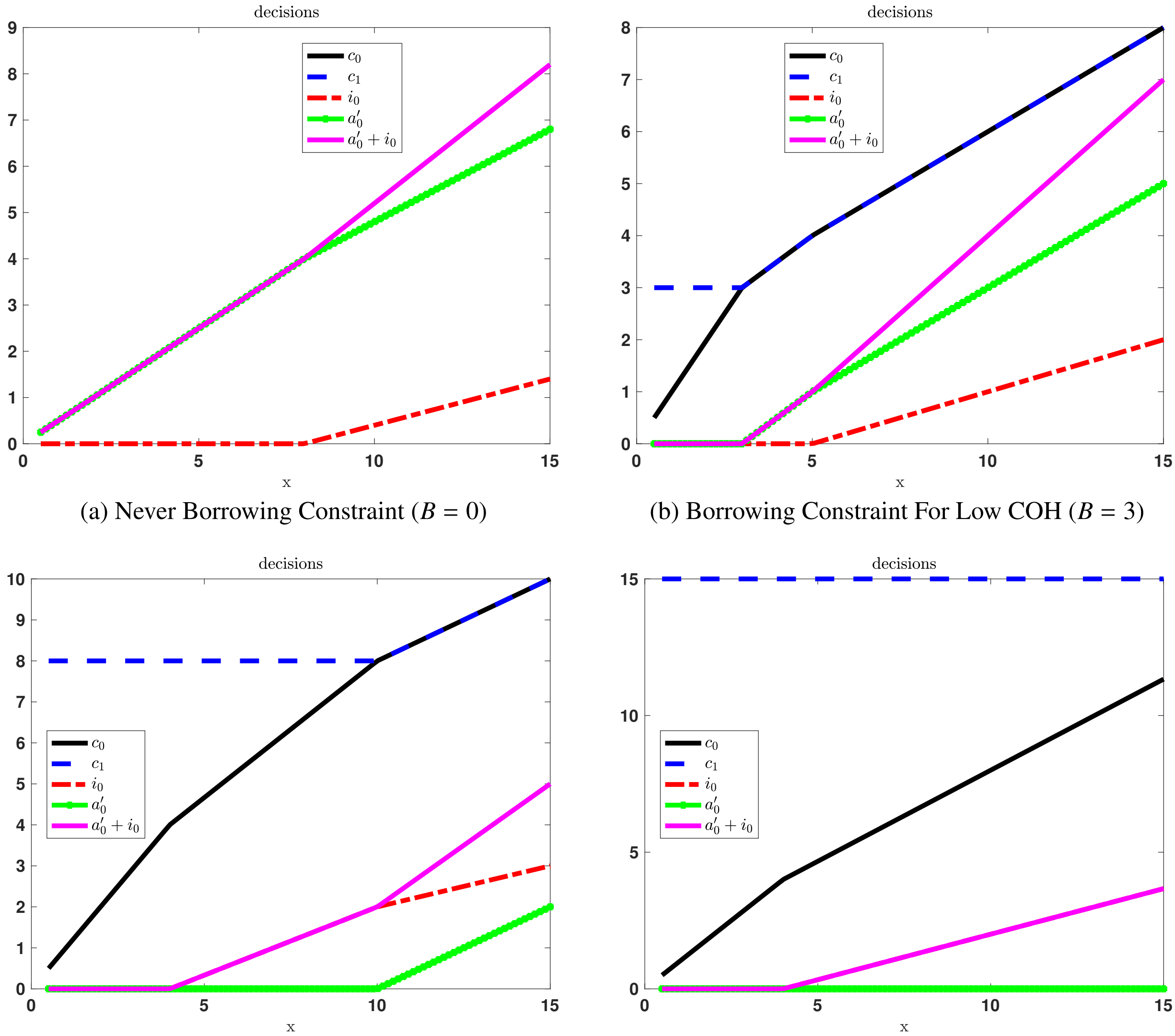
$$C_0 = \frac{X + \bar{i}}{1 + \nu}. \tag{26}$$

(b) $X \geq (1 + \nu)B - \bar{i}$: This could be referred to as the real interior solution. Optimal investments $i_0^\star$ are given by (22) and $C_0^\star$ as well as $C_1^\star$ are given by (23), while $A_0'^\star$ is described by (24).

Summing up, this simple model leaves us with five sub-cases. First, for low combinations of X and B, we are in a region in which u_c dominates u_i, so that households are not yet investing. However, the question is whether the household chooses $C_0 = C_1$ voluntarily (case (i.a)), which is the case for $X \geq B$, or whether she is borrowing constraint (case (i.b)) and forced to set $C_0 = X$ as well as $C_1 = B$. Second, there is cases in which the sum of X and B is high enough in order for u_c to loose its dominance over u_i, but first period resources X are relatively small compared to second period resources B, tying the household to the borrowing constraint. Whether or not (cases ii.a.1 and ii.a.2) the household invests in this situation is determined by the intra-temporal Euler equation. However, it is worth noticing that the latter is a situation in which the household is borrowing constraint and investing at the same time. Lastly, in case resources are high and X is relatively large compared to B, the household is completely in the interior solution (case (ii.b)). Figure 2 displays four different scenarios with respect to second period exogenous income B. Driven by first period resources X (x-axis), the various areas are traversed.

3.2.2 Endogenous vs. Exogenous Hybrid Method

The full algorithms of *HybLevExog* and *HybLevEndo* are described in Appendix B.3 and Appendix B.4 respectively. In the main part, we want to focus on the key difference between the two methods, which essentially boils down to two aspects: *HybLevEndo* does not address the trade-off between consumption, savings and investments correctly and thereby systematically



(a) Never Borrowing Constraint ($B = 0$)

(b) Borrowing Constraint For Low COH ($B = 3$)

(c) Borrowing Constraint Up to High COH ($B = 8$)

(d) Always Borrowing Constraint ($B = 15$)

Figure 2: Decisions as a Function of Cash-On-Hand (COH)

underestimates the investment choice. In addition, *HybLevEndo* has troubles to find the region in which households are borrowing constraint.

Before we can examine this in detail, we first have to briefly recall how endogenous grid method algorithms operate. An exogenous savings grid is generated and, given the recursive structure of the model, for each of these savings grid points, the part of the Euler equation related to the next period is automatically determined (see for example first-order-condition (67) given cash-on-hand definition (45)). Therefore, only consumption and leisure of the current period have to be adjusted accordingly, such that the first-order-condition is balanced and the respective savings grid point is in fact the optimal choice. The sum of decisions (consumption, leisure and savings) endogenously determines cash-on-hand via the budget constraint, which in turn pins down the corresponding asset level (which is important for the second difference explained below).

First-Order-Conditions

Now investments come into play, and thus a second variable, in addition to savings, affects the part of the Euler equation that is related to the next period. The question arises how both investments (or inter-vivos transfers respectively) and savings can be pinned down together and how a corresponding investment grid can be spanned. Krueger and Ludwig (2013) redefine the exogenous savings grid points as gross savings ($\bar{A}'_0 = A'_0 + i_0$), and each grid point is assigned a corresponding investment grid $\mathcal{G}^{i_0} = \{0, \ldots, \bar{i}_0\}$ with $\bar{i}_0 = \bar{A}$.[31] Thereby the savings grid point fixes the sum of savings and investments, for which the optimal combination is then searched.

As mentioned above, we also have to deal with the fact that we neither know the value function, nor its derivative with respect to inter-vivos transfers or human capital investments (see equations (60) and (64) for the respective vivos transfers of vivos and investments). To solve this problem, we guess the kid's value function and solve by iterating over the resulting fix point problem, which we describe in Appendix B.1.3. Thus, searching for the optimal solution,

[31] For the sake of simplicity we assume a borrowing constraint of zero for now. The detailed version can be found in Appendix B.4.2.

given a fixed exogenous gross savings grid point $\bar{A}'_0 = A'_0 + i_0$, works as follows:

We assume to be back in the two period model described above. Further, we abstract from the fix point problem (equation (17) is known). Given an exogenous income of $B = 0$ (Figure 2(a)) implies that the household cannot be borowing constraint. Let us suppose we search for the optimal choices for exogenous gross savings $\bar{A}'_0 = 10$. First of all, the corresponding grid $\mathcal{G}^{i_0} = \{0, \ldots, \bar{i}_0 = 10\}$ would be spanned. Given investments $\bar{i}_0$ consumption tomorrow is determined by $C_1 = A'_0 + B = \bar{A}'_0 - i_0 + 0$. By the first order condition between savings and consumption today, we further know that $C_0 = C_1$, implying $C_0 = A'_0$. *HybLevEndo* would now evaluate the resulting utility levels for all investment grid points in $\mathcal{G}^{i_0}$ as displayed in Table 2.

i_0	A	C_0	C_1	X	u	u_c	u_i
0.00	10.00	10.00	10.00	20.00	**4.95**	0.10	0.25
1.11	8.89	8.89	8.89	18.89	4.94	0.11	0.16
2.22	7.78	7.78	7.78	17.78	4.82	0.13	0.12
3.33	6.67	6.67	6.67	16.67	4.63	0.15	0.09
4.44	5.56	5.56	5.56	15.56	4.36	0.18	0.08
5.56	4.44	4.44	4.44	14.44	3.99	0.23	0.07
6.67	3.33	3.33	3.33	13.33	3.49	0.30	0.06
7.78	2.22	2.22	2.22	12.22	2.74	0.45	0.05
8.89	1.11	1.11	1.11	11.11	1.40	0.90	0.05
10.00	0.00	0.00	0.00	10.00	-	-	0.04

Table 2: *HybLevEndo* and $\bar{A} = 10$ ($B = 0, \bar{i} = 2, \nu = 0.5$)

Table 2 identifies the underlying problem: (i) *HybLevEndo* evaluates utilities based on different cash-on-hand levels X and (ii) the lower the investment, the higher the endogenous cash-on-hand level. The chain of events leading to this pattern is the following: given the gross savings definition, smaller investments lead to higher net savings (which are interchangeably with C_1 in this two period model). Via the first-order condition between savings (C_1) and consumption (C_0), this also leads to higher consumption in period 0. Thus, a decrease in i_0 increases both C_0 and C_1. In other words, the smaller the investment, the larger will be resources X that can be divided among C_0, C_1 and i_0. Thereby *HybLevEndo* overestimates the importance of consumption and savings relative to investments.

As can bee seen in Figure 2(a), roughly at a cash-on-hand level of $X = 8$, optimal investments start to be positive, while Table 2 shows that *HybLevEndo* would still return an optimal investment of zero for much higher values of $X = 10$. Thus, fist-order-conditions between u_c and u_i would be needed, in order for *HybLevEndo* to evaluated the situations correctly. A solver searching in the neighborhood of the falsely identified utility maximum cannot find the correct solution. Table 3 shows that *HybLevEndo* would only exhibits positive investments for very high exogenous gross savings grid points.

i_0	A	C_0	C_1	X	u	u_c	u_i
0.00	14.00	14.00	14.00	28.00	5.62	0.07	0.25
1.56	12.44	12.44	12.44	26.44	**5.68**	0.08	0.14
3.11	10.89	10.89	10.89	24.89	5.59	0.09	0.10
4.67	9.33	9.33	9.33	23.33	5.42	0.11	0.08
6.22	7.78	7.78	7.78	21.78	5.16	0.13	0.06
7.78	6.22	6.22	6.22	20.22	4.80	0.16	0.05
9.33	4.67	4.67	4.67	18.67	4.29	0.21	0.04
10.89	3.11	3.11	3.11	17.11	3.55	0.32	0.04
12.44	1.56	1.56	1.56	15.56	2.22	0.64	0.03

Table 3: *HybLevEndo* and $\bar{A} = 14$ ($B = 0, \bar{i} = 2, \nu = 0.5$)

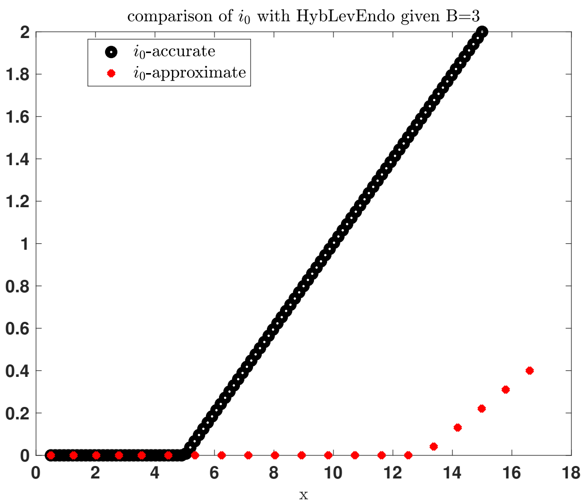
Having identified this problem of *HybLevEndo*, we developed the alternative solving algorithm *HybLevExog*. The starting point of each computation is today's resources X. In this way, the comparison of household choices is always based on the same cash-on-hand level. As a result, savings and investment are weighed on the same basis, and their respective first-order-conditons are taken into account implicitly. To illustrate the results in detail, we performed a computational exercise, in which the two period model is solved under the two different approaches. Figure 3 summarizes results for a scenario with exogenous income in period 1 of $B = 3$ (red dots show numerical results, black dots represent the analytic solution). The left hand side displays results for investments, savings and period 0 consumption under *HybLevEndo*, whereas the right hand side shows their respective counterpart under *HybLevExog*. While *HybLevEndo* underestimates the effect of investments and thereby chooses values for consumption and savings that are too high, *HybLevExog* matches the analytic solution derived in the previous section.

Borrowing Constraint

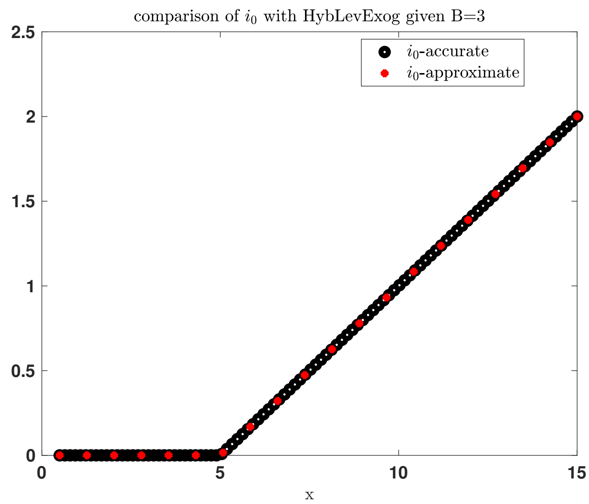
Now we get to the second issue. One crucial part for the accuracy of the endogenous grid method in light of borrowing constraints is to determine where they start to be binding for the household. In order to find this border, the lowest exogenous savings grid point is set to the borrowing constraint and the corresponding solution is then evaluated (this is shown in detail in Appendix B.2.1). The definition of gross savings $\bar{A}'_0 = A'_0 + i_0$ makes that harder, as we can see from Figure 2(d) that situations can occur, in which the household is borrowing constraint, but still chooses $i_0 > 0$. This problem does not exist with *HybLevExog*. The region in which the borrowing constraint is binding for the household is determined by the endogenous choice of savings, which is disconnected from the investment decision.

Nevertheless, the occurrence of the borrowing constraints leads to a special case, in which also *HybLevEndo* delivers accurate results. If the borrowing constraint is always binding ($B = 15$), then the inter-temporal first-order-condition becomes obsolete. Thus, *HybLevEndo* weighs up consumption against investments based on the same cash-on-hand level - basically as *HybLevExog* does - and identifies the right solution.

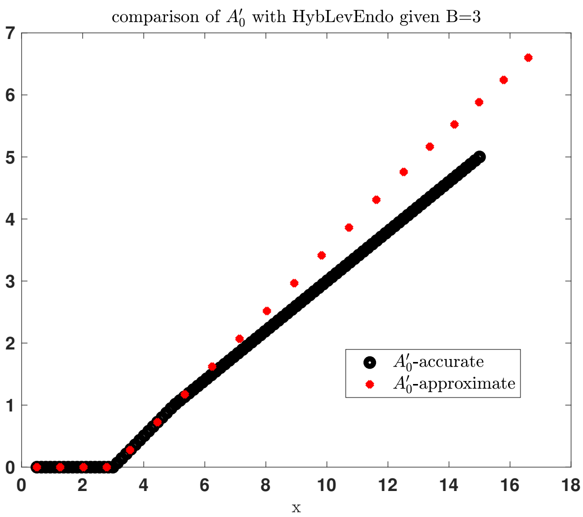
Figure 4 shows results under two special scenarios, i.e. always and never borrowing constraint, in order to disentangle the general inaccuracy from issues with the borrowing constraint. In summary, *HybLevEndo* only delivers accurate results when the household is always borrowing constraint (Figure 4 (a) and (b)), but it does not start working if we set off the borrowing constraint (Figure 4 (c) and (d)), which is due to the issues described in this chapter.



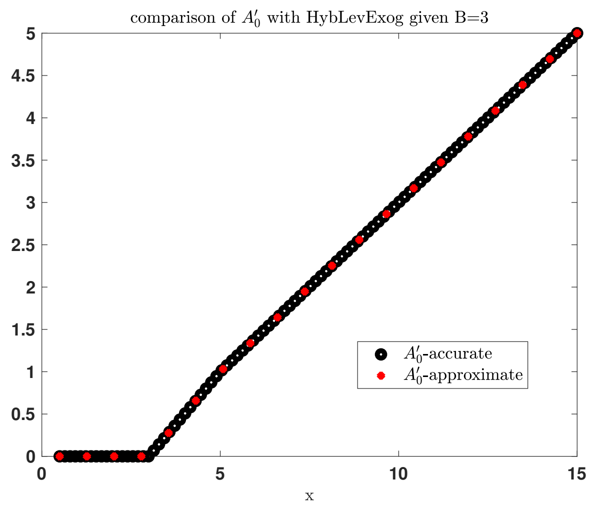
(a) Investments under *HybLevEndo*



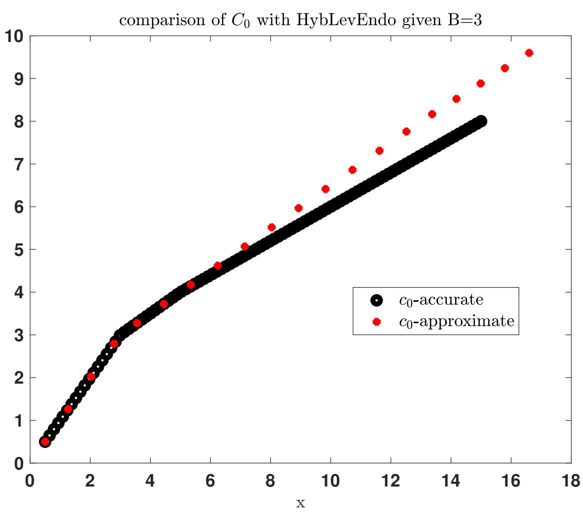
(b) Investments under *HybLevExog*



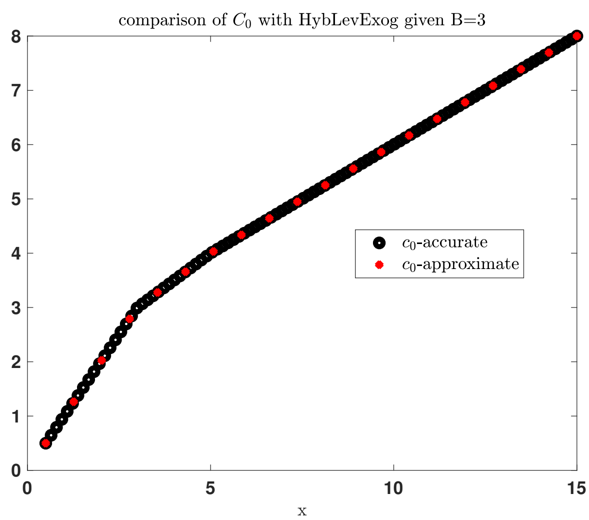
(c) Savings under *HybLevEndo*



(d) Savings under *HybLevExog*

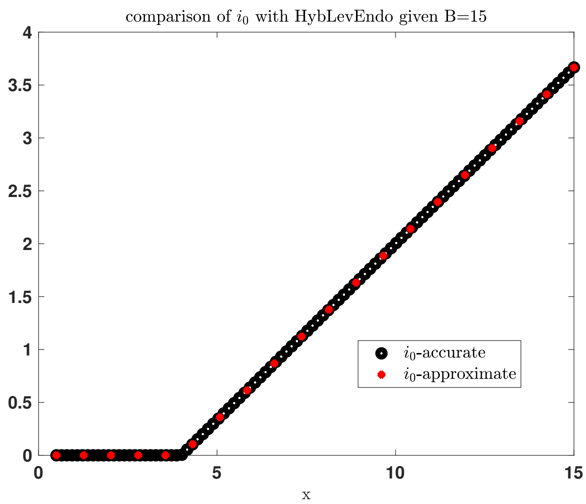


(e) Consumption period 0 under *HybLevEndo*

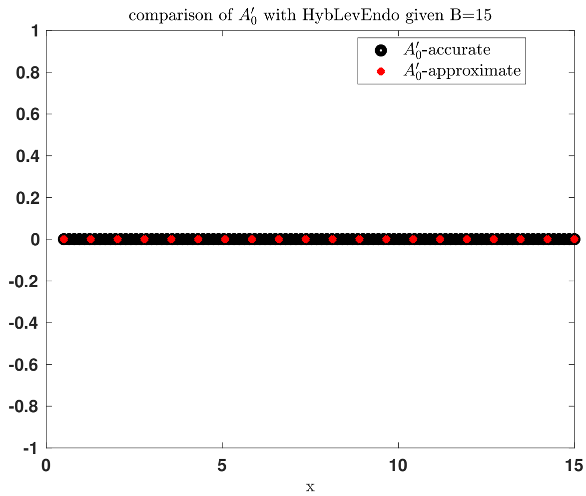


(f) Investments under *HybLevExog*

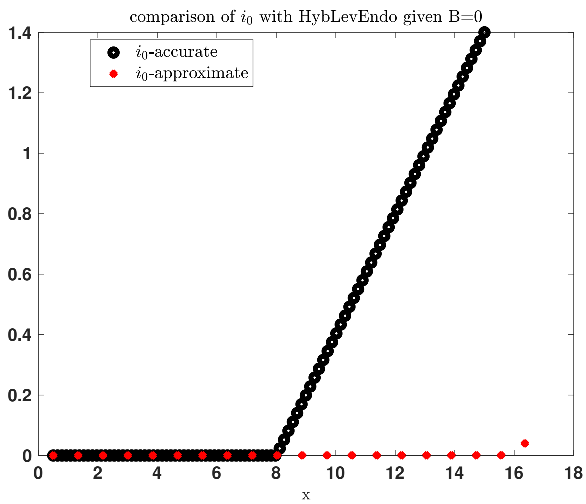
Figure 3: Savings and Investments Under Different Solution Methods



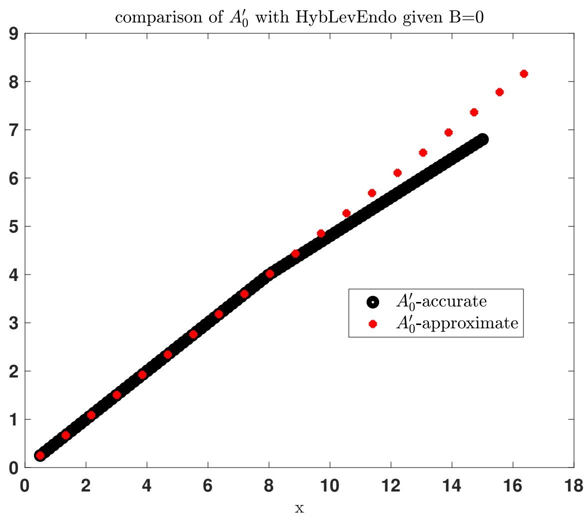
(a) Investments and always borrowing constraint



(b) Savings and always borrowing constraint



(c) Investments and never borrowing constraint

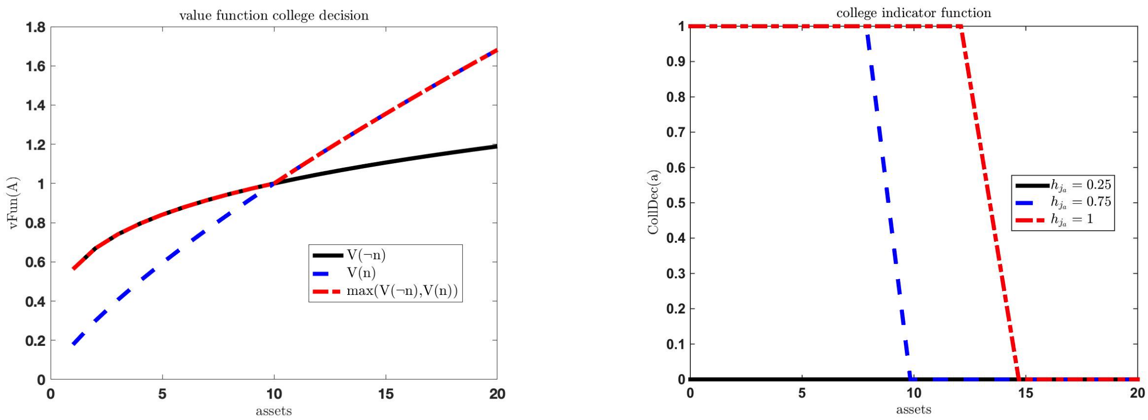


(d) Savings and never borrowing constraint

Figure 4: *HybLevEndo* in Special Scenarios

3.3 Taste Shocks at College Decision

The fist economic action of an household is the college choice. The young adult decides based on her innate and acquired human capital, her assets received as vivos transfers from her parents and the idiosyncratic shock she has drawn. Looking at an agent with a sufficiently high human capital, we can split her decision in two parts: low assets imply a more demanding working life, which makes an increased wage and thereby attending college more valuable. The richer she is in terms of assets, the lower the importance of labor income and at some asset level the value function of not attending college takes over. It is worth noticing that there is a special case, potentially leading to a third area, in which a minimum asset level is required for this agent to be able to afford attending university. We cover this in Section B.7.



(a) Value function

(b) College indicator function

Figure 5: Situation at College Without Taste Shocks

The left part of Figure 5 illustrates an example in which the two value functions of attending and not attending college are competing. At an asset level of about 10 not attending college becomes the better option and overtakes its adversary. The resulting value function of this agent is defined as the maximum of $V(n)$ and $V(\neg n)$. Figure 5(b) displays college indicator functions for different acquired human capital levels. Apparently, in this example, for agents with a human capital level $h_{ja} \leq 0.25$ the value function of not attending college dominates the college option for all asset levels. The model mechanics behind that are a high time deduction due to low human capital on the one hand and a low probability of succeeding in college on the other. In combination, this makes not attending college the better choice, even for low asset levels. In Figure 5(a) we can see that at the asset level splitting the two regions, there is a kink in the value function, which in turn leads to a jump in marginal products and first order conditions. In addition, these properties spill over to the maximization problem of parents in the period of inter-vivos transfers. As children are hit by the idiosyncratic income shock in between receiving these vivos transfers and making the college decision, parents need to form expectations over the value function of the kid:

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C, \ell \in [0,1], B \geq 0 \\ A' \geq -\Psi(q)\underline{A}_{j,t}}} \left\{ u(C, 1-\ell) + \beta E_{\eta'|\eta} \left[V'(j+1, A', q, \gamma, \eta') \right] \right\}$$

$$+ \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\max \left\{ E_{q \in \{d,c\}|h} \left[V(j_a, A, h_0, h, \neg n, \eta) \right], E_{\gamma \in \{\gamma^l, \gamma^h\}|h} \left[V(j_a, A, h_0, n, \eta) \right] \right\} \right],$$

where taking the maximum of $V(\neg n)$ and $V(n)$ is equivalent to $\max(V(\neg n), V(n))$ in Figure 5(a).

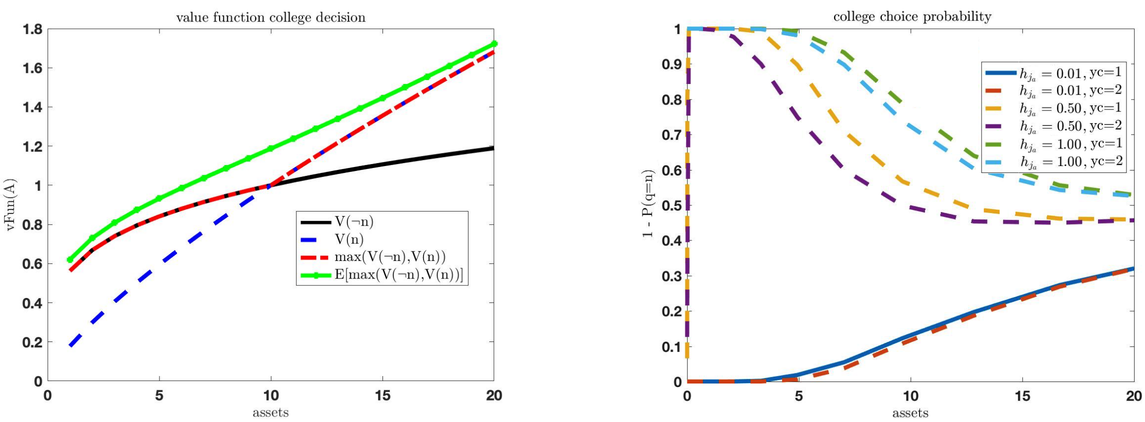
We introduce Gumbel iid taste shocks to our model, which enables us to create a value function that is smooth over the whole asset grid. This is a useful method in dealing with kinks in value functions and discontinuities in policy functions caused by discrete choices, which has been applied in recent work, e.g. Iskhakov et al. (2015) and Busch (2019). Conceptually, the idea is that given a group of households with the same characteristics, one part attends college and the other part does not, which is caused by taste shocks that are drawn randomly. Thereby, the college indicator function is transferred into a choice probability, which is determined by the distance in value functions. Regarding Figure 5(a), for low assets the likelihood of attending university is higher and decreasing in assets, while at about asset level 10 the likelihood of not attending university exceeds the 50% level.

The parental value function at age $j_f + j_a$ with taste shocks reads as

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C, l \in [0,1], B \geq 0 \\ A' \geq -\Psi(q)\underline{A}_{j,t}}} \{u(C, 1-l) + \beta E_t [V'(j+1, A', q, \gamma, \eta')]\} \tag{27}$$

$$+ \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\sigma \left(\frac{E_{\gamma \in \{\gamma^l, \gamma^h\}|h} [V_t(\cdot, n, \eta)]}{\sigma} + \log \left(1 + \exp \left[\frac{E_{q \in \{d,c\}|h} [V_t(\cdot, \neg n, \eta)] - E_{\gamma \in \{\gamma^l, \gamma^h\}|h} [V_t(\cdot, n, \eta)]}{\sigma} \right] \right) \right) \right], \tag{28}$$

which we develop in Section B.6. In addition, given our assumption of a continuum of households, the choice probabilities are also the fractions of agents with the respective characteristics actually choosing this option. Figure 6 shows the value function and the corresponding choice probabilities of attending college. By introducing taste shocks we are able to create smooth



(a) Value function

(b) College Choice Probabilities

Figure 6: Situation at College With Taste Shocks

value functions.[32] The extent of smoothing induced by the taste shocks is determined by the variance of the Gumbel distribution (σ).

[32]Please note that the expectation operator in $E[\max(V(\neg n), V(n))]$ comes from uncertainty with respect to the taste shock that will be drawn.

3.4 Policy Functions Education

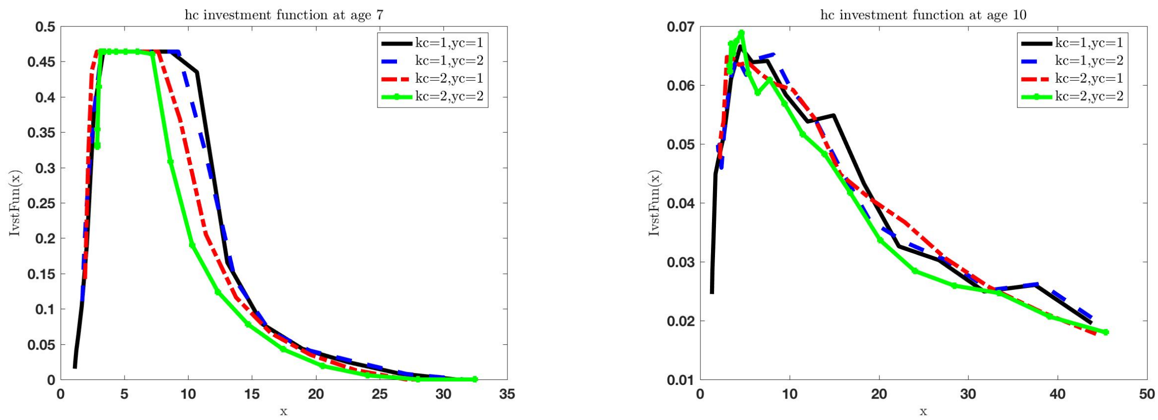
In this section we want to concentrate on policy functions connected to the educational mechanics in the model, in particular, we focus on the interplay of human capital investments, vivos transfers and the college decision. Human capital has an impact on several aspects in our model. Regardless of the college decision, it determines the likelihood for all households which productivity type they will draw (see equation (33)) in their respective qualification group. If the household decides to go to college, human capital also influences the likelihood of succeeding in graduating (see equation (31)). Finally, students have to spend time on studying, which is deducted from their time endowment during college (see equation (32)).

All functions are chosen such that it has no added value, should a human capital greater than one be achieved, which makes it the natural upper bound for h_{ja}. In addition, this implicitly sets an upper limit for parents' investments in human capital. Further, the substitution elasticity of the human capital function has a major impact on investment behavior. Following Cunha and Heckman (2007), we have chosen a function that exhibits a higher elasticity during primary education than during secondary education.

Furthermore, due to dynamic complementarity, high human capital can not be built up in a single period without investing in education before and after. If investments during primary education are insufficient, the resulting gaps in the subsequent secondary education phase can not be easily compensated by high investments. On the other hand, the fruits must be harvested during secondary education if they have been sown early on, since otherwise they will expire.[33] These are roughly the mechanisms of human capital formation and they are also reflected in the policy functions of our model.

Figure 7 shows human capital investments during primary and secondary education as a function of parents' cash-on-hand. While all other characteristics of parents and the human capital of their children are kept constant, parents differ with respect to their current idiosyn-

[33] In Appendix A.1 we give a more detailed overview of the six empirical facts of the literature on human capital and how they are linked to dynamic complementarity and self-productivity.



(a) Investments of parents in primary education

(b) Investments of parents in secondary education

Figure 7: Parental Investments in Primary and Secondary Edu.

Note: Age in both figures refers to model age of parents. Respective model age of kids is 2 and 14 in terms of real age.

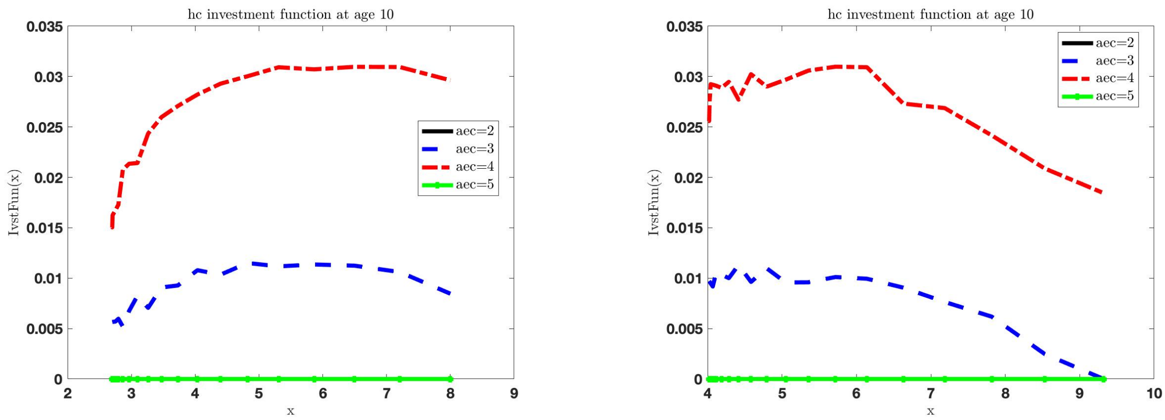
cratic shock (yc) and their productivity type (kc).[34] It is worth noticing, that the underlying investment level of the government is positive ($i_j^g > 0$ for all $j = 0, \ldots, 3$), so that a certain level of education is provided, even if parents do not invest additionally.

First of all, it is noticeable that given cash-on-hand investments in primary education are higher than in secondary. This is related to human capital being more malleable in this phase and coincides with both the data of human capital literature.[35] In both periods, parents already start to invest with minimum cash-on-hand. Thereafter, investments gradually increase until a certain point is reached. As indicated, this upper bound comes from the fact that investments leading to human capital above a certain level are not worthwhile, as they neither increase the likelihood of drawing the better productivity type, nor complete college, or make studying less time-consuming.

Two other aspects in Figure 7 are interesting: (i) investments are decreasing after a certain level of income is reached and (ii) this is happening sooner for households with better financial conditions (kc=2 and yc=2). We have to work this out in two steps.

[34] For both shocks "one" stands in for the negative and "two" for the positive realization

[35] In Section 4.11 we will see, that this is also reflected in the lifecycle plots of the calibrated model.



(a) Investments Low Productivity Parents in Secondary Edu.

(b) Investments High Productivity Parents in Secondary Edu.

Figure 8: Secondary Investments Given Abilities

Note: Age in both figures refers to model age of parents. Respective model age of kids is 14 in terms of real age.

Figure 8 represents parents' investments (again ceteris paribus) as a function of cash-on-hand, given four different human capital levels of children. The acquired human capital (aec) level of 5 represents the upper bound described above. It is defined as the level at which, even with zero parental investments (given the current, positive investment level of the government), in the next period the highest reasonable human capital will be reached. Thus, for $aec = 5$ we have zero investments for all cash-on-hand levels.[36] It should be said that this upper level is of a more theoretical nature and the fraction of households at this level will be close to zero in the calibrated model.

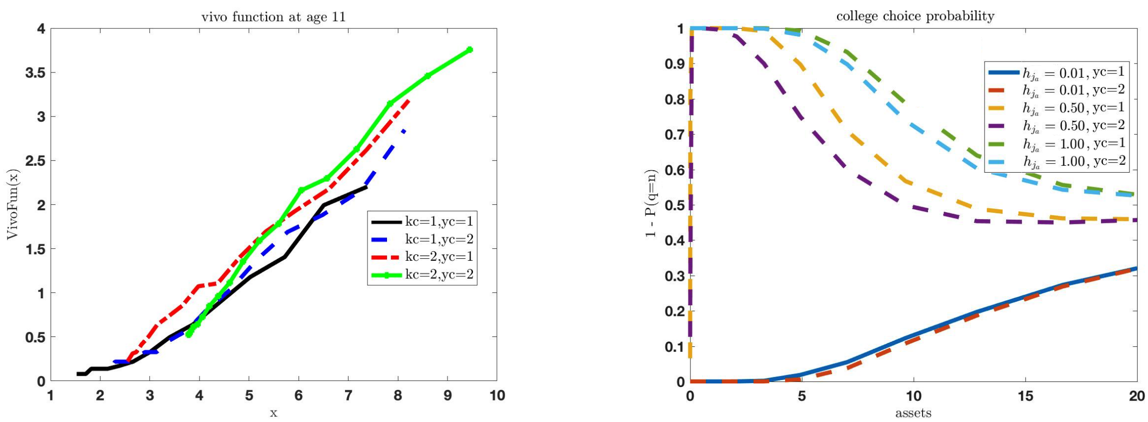
At the lower end of the human capital spectrum, parents' investments also stay at zero, but for a completely different reason. We are in the secondary education period: the elasticity of substitution is low, but we have to invest so that the sown fruits do not expire. In order to explain the missing investments of parents, the government comes into play, because the level it provides is already sufficient. Last but not least - and this is the most important area for the calibrated version of the model - it is noticeable that more is invested in kids with higher human

[36]This will help us in the computation of the model in defining reasonable investment grids, which is further described in Section B.3.3.

capital (aec = 4) than for the lower (aec = 3). It can be explained by dynamic complementarity:

$$\frac{\partial^2 f_j(h_j, I_j)}{\partial h_j \partial I_j} > 0.$$

As the return on investments is higher, when the child is more able (e.g. due to higher investments during primary education), the acquired human capital level has a positive impact on today's investments.



(a) Vivos transfers

(b) College Choice Probability

Figure 9: Vivos Transfers and the College Decision

Note: Age in both figures refers to model age of parents. Respective model age of kids is 18 in terms of real age.

Finally, we are able to bridge the gap between Figure 9 and Figure 7, which explains the hump-shaped course of human capital investments. Figure 9(a) shows parents' transfers as a function of their cash-on-hand. Figure 9(b) shows the college decision probability (which will be explained in detail in Section 3.3) as a function of assets, which at that point is determined by parents' vivos transfers. There are different phases to be distinguished: first of all, a certain asset level is necessary so that young households can generally afford to attend college. If this is exceeded, the college decision depends primarily on the skills of the respective household. But for extremely high cash-on-hand levels, the choice probability of attending college is decreasing, which has a rather theoretical background and will play only a very limited role for the calibrated model:

The incentive to reach for higher education in our model is purely driven by the prospect on higher wages. However, if the household receives extremely high transfer payments from parents, work, and in turn education, becomes less important. Therefore, in Figure 9(b), the likelihood of going to college drops off at some point, and this also explains the drop in human capital investment in Figures 7 and 8: extremely rich parents know that they will later make large transfer payments and therefore invest less in the human capital their children.

Again, the fraction of these special cases in the distribution of the calibrated model will be negligible. The purpose for this description was to analyze the different model mechanisms very precisely.

3.5 A Glance at the Numerical Computation

The research question we want to answer in this work is of purely economic nature. Nonetheless, due to the complex structure of the model, much of this work involved designing and programming a solution algorithm, as has already been shown in this section (and Appendix B). But that was not sufficient, because the "curse of dimensionality" has caught us with full force. That becomes clear, when we recap the problem and its state variables in periods $j_f, \dots, j_f + j_a - 1$, when children are part of the household:

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C,\ell \in [0,1] \\ A' \geq -\Psi(q)\underline{A}_{j,t} \\ i_k^p \geq 0}} \left\{ u\left(\frac{C}{1+\zeta f}, 1-\ell\right) + \dots \right.$$
$$\left. \dots + \beta E_{\eta'|\eta}\left[V'(j+1, A', h_0, h', q, \gamma, \eta')\right] \right\}.$$

Although we use relatively coarse grids, each with five human capital levels, we come to a total of $5 \cdot 5 \cdot 20 \cdot 3 \cdot 2 \cdot 2 = 6,000$ different combinations, with twenty asset grid points, three degrees of education, and two each of idiosyncratic shock and productivity type - per period.

However, there is the additional burden that we had to switch to the exogenous grid method due to the shortcomings of the hybrid method incorporating the endogenous grid method mentioned above. Thus, we lose the well known speed gains of the endogenous grid method by Carroll (2005) during $j_f, \ldots, j_f + j_a - 1$. In periods of human capital investments and transfer payments, the high dimensionality in addition with the necessity of the relatively slow exogenous grid method will further be combined with a level search over the value function (via Golden Search Algorithm), in order to arrive at optimal investments and transfer payments.

As a consequence, we would not have been able to solve the model without parallelizing the solution algorithm and running it on a high performance computer, simply because the computational time would have been too long. To outline the speed gain, we ran the same test program under the prerequisites listed in Table 4. Within this program, the household problem had to be solved given six different levels of college subsidies by the government. In each iteration the household problem had to be solved several times, in order for the fix point problem in the value function of children to converge for a tolerance level of e-07.

Architecture	Parallized	Threads	Time
2.50 GHz, 16GB RAM, 4 Cores	no	1	≈3000 mins
2.50 GHz, 16GB RAM, 4 Cores	yes	4	≈1500 mins
2.10 GHz, 192GB RAM, 40 Cores (CSC Skylake)	no	1	≈141 mins
2.10 GHz, 192GB RAM, 40 Cores (CSC Skylake)	yes	40	≈70 mins

Table 4: Comparison Computational Time

Following Amdahl's law, given we had the privilege to work on a 40-core high performance computer, we were able to parallize roughly 50% of the workload, given we had a time improvement of $\frac{140}{70} = 2$.[37] A brief overview of how we parallized the model can be found in Appendix B.8.

[37]The first two lines of Table 4 are rough approximations and are showing the computational time on a regular computer. They are supposed to express our gratitude to the Center for Scientific Computing (CSC) of the Goethe University Frankfurt, as well as the financial support by the state Hesse, without which this work would not have been possible. Without parallization and the use of the CSC cluster this work would not have been completed before February 2045, approximately.

4 Calibration

This section describes our assumptions on functional forms and parameters chosen. The calibration parameters are also summarized in Table 19, where we make explicit the distinction between first stage parameters (calibrated outside the model) and second stage parameters (calibrated by minimum distance methods using the model). Also, please note that as one period in the model lasts for four years, there is adjustments that have to be made, e.g. the capital output ratio $\frac{K}{Y}$ and the time discount factor β. The adjustments are described in C.9.

4.1 Demographics

The total fertility rate f in the economy is assumed to be $f = 1.14$, reflecting the fact that a mother on average has about $2f = 2.28$ children. This number also determines the population growth rate, cf. equation 1. Each period in the model has a length of four years. Children are born with age 0 and form households at biological age 18. We discard the first two years of childhood and accordingly set $j_a = \frac{18-2}{4} = 4$. Hence, children live as dependent members in adult households for ages $0, \ldots, 3$, which corresponds to the biological age bins $2-5, 6-9, 10-13, 14-17$. Households require 4 actual years to complete a college education and therefore exit college at model age $j_c = j_a = 4$. They have children at biological age 30, which is model age $j_f = 7$. Retirement occurs at biological age 66 (age bin $62-65$ is the last working period of life), hence $j_r = 16$. The maximum life span is 101 years, i.e., the last period households are alive is biological age bin $98-101$ and accordingly $J = 24$.

4.2 Preferences

The per period utility function u is specified as the usual nested Cobb-Douglas-CRRA specification

$$u(C, \ell) = \frac{\left(C^{\mu}\ell^{1-\mu}\right)^{1-\sigma} - 1}{1-\sigma}.$$

We a priori choose the inter-temporal elasticity of substitution (IES) $1/\sigma$ to equal $1/4$. We then calibrate the discount factor to match a capital output ratio of 3. The adult equivalence scale parameter for child consumption is set to $\zeta = 0.3$.

4.3 Human Capital Grid and Intergenerational Transmission

We discretize initial and acquired human capital. Initial human capital h_0 is assume to lie in $[0, \ldots, \bar{h}_0]$ with $\bar{h}_0 < 1$. As we argue below, our parametrization of the human capital accumulation process implies a natural upper bound on acquired human capital at age j_a of 1. Choosing the upper bound on initial human capital $\bar{h}_0 < 1$ then implies that also those kids born with the highest possible innate human capital benefit from investments by their parents (and / or the government).

To determine the transmission of innate human capital, we assume that the probability of drawing a given h_0 conditional on the human capital of parents h_0^p can be described by truncated triangular kernels with the following properties for all $i = 1, \ldots, n$ grid points of the innate human capital grid:

$$\pi_h(h_0 \mid h_0^p) = \bar{\pi}_h - \Delta_{\pi_{h_0^p}} \left| h_0 - h_0^p \right| \tag{29}$$
$$\sum_{i=1}^{n} \pi_h(h_{0,i} \mid h_0^p) = 1.$$

We take $n = 5$ grid points for the innate (and acquired) human capital grids. We set $\bar{h}_0 = 0.5$ and $\bar{\pi}_h = 0.25$. For each h_0^p the slope parameter $\Delta_{\pi_{h_0^p}}$ then follows from the requirement that probabilities sum to one.

4.4 The Human Capital Process

As pointed out by Cunha and Heckman (2007) as well as by Caucutt and Lochner (2017), both dynamic complementarity ($\frac{\partial^2 f_t(h_0, h_j, I_j)}{\partial h_j \partial I_j} > 0$) and self-productivity ($\frac{\partial f_t(h_0, h_j, I_j)}{\partial h_j} > 0$) should be incorporated in human capital production functions, which is satisfied by the following specification:

$$h_{j+1} = (1-\delta)h_j + \begin{cases} h_j^{\upsilon_j}\left(\psi i_j\right)^{(1-\upsilon_j)} & \text{for } \phi_j = 0 \\ \left(\upsilon_j h_j^{\phi_j} + (1-\upsilon_j)\left(\psi i_j\right)^{\phi_j}\right)^{\frac{1}{\phi_j}} & \text{otherwise.} \end{cases} \tag{30}$$

where $\sigma_j = \frac{1}{1-\phi_j}$ is the substitution elasticity across the two production factors.

To calibrate (30) we relate to the estimates of Cunha, Heckman and Schennach (2010). Accordingly, we restrict $\delta = 1$. With respect to σ_j and υ_j we assume that σ_j is decreasing in age (reflecting decreasing substitutability of the two production factors) whereas υ_j is increasing in age (reflecting increasing self-productivity of human capital). Specifically, we assume that $\{\sigma_j\}_{j=0}^{j_a-1} = \{1.5, 1.5, 0.5, 0.5\}$, which is directly based on the estimates in Cunha, Heckman and Schennach (2010) for their cognitive skill process, controlling for non-cognitive skills (Table 4 of their paper). We also base $\{\upsilon_j\}_{j=0}^{j_a-1}$ on these estimates which gives $\{\upsilon_j\}_{j=0}^{j_a-1} = \{\frac{0.48}{0.48+0.16} = 0.75, 0.75, \frac{0.83}{0.83+0.04} = 0.95, 095\}$. Finally, the investment productivity parameter ψ is to a value of 10.

As for the time requirement for college studies, we assume a simple truncated linear function as

$$\xi(h_{j_a}) = \max\left\{0, 1-\lambda, 1-\lambda h_{ja}\right\}, \tag{31}$$

where $1-\lambda$ only applies in cases where $\lambda < 1$. Together with the specification of the probabilities in (32) and (33), see below, this assumption ensures that the rational upper bound of h_{j_a} chosen by parents is about one.[38] This is convenient for constructing human capital grids in the computational implementation. We calibrate λ to match the fraction of households not attending college Φ_n. According to Restuccia and Urrutia (2004) this fraction is 54%, implying a fraction of college attendance of 46%. Also, they find a dropout rate of 50%, implying that the fraction of dropouts equals the fraction of college graduates and is $\Phi_c = \Phi_d = 0.46 \cdot 0.50 = 0.23$.

[38] In case $\lambda < 1$ and without $1-\lambda$ in (31) parents would have an incentive to invest such that their children's human capital at age j_a exceeds one.

Thus, by matching the fraction of non-college households and the dropout rate we meet all three moments. We will calibrate the latter by the success probability in college, which we assume is given by

$$\pi_c(h_{j_a}) = \min\left\{1 - \exp(-\mu_c), 1 - \exp(-\mu_c h_{j_a})\right\} \tag{32}$$

and $\mu > 0$ (see 4.10 for C.3 for details on calibration of both college attendance and dropout rate). Accordingly, the success probability will be equal to zero for $h_{j_a} = 0$ and cannot be larger than 1. Increasing μ_c increases the curvature of the function and we calibrate this curvature to match the mentioned college dropout rate of 50%.

In addition, we assume (exogenously) that dropouts spend 50% of their time in college and hence set $\phi = 0.5$, which is in line with data.[39]

4.5 Labor Productivity Process

Recall that a household of age j with education $q \in \{n, d, c\}$, fixed effect γ and idiosyncratic shock η earns a wage of

$$w_{t,q}\epsilon_{j,q}\gamma\eta,$$

where w_q is the qualification-specific wage per labor efficiency unit in period t. Also recall that $w_{t,n} = w_{t,d}$, because we assumed non-college workers and college dropouts to be perfect substitutes in final production. This assumption can be justified on the basis of the estimates by Altonji and Zimmerman (2017).

We calibrate the various components of the wage process as follows. First, the age- and qualification-specific component of labor productivity $\{\varepsilon_{j,q}\}$ is estimated from PSID data (cf. Ludwig, Schelkle and Vogel 2012), assuming that $\{\varepsilon_{j,d}\} = \{\varepsilon_{j,n}\}$. Average non-college and dropout wages are normalized to $w_{t,n} = w_{t,d} = 1$, while the average wage for college graduates $w_{t,c}$ is used to scale up the estimated wage profile to match an average college wage premium

[39] See Stinebrickner and Stinebrickner (2003) and Manski and Wise (1983).

Parameter	Interpretation	Value	Stage
	Population		
j_a	Age at HH form. (age 18)	4	1
j_c	Age, coll. compl. (age 21)	5	1
j_f	Fertility Age (age 30)	7	1
j_r	Retirement Age (age 66)	16	1
J	Max. Lifetime (age bin 98-101)	24	1
	Labor Productivity		
$\{\epsilon_{j,s}\}$	Age Profile	Estimates (PSID)	1
π_η	Transition probability of Markov process	Cf. Table 6	1
σ_η	Log-State of Markov process	Cf. Table 6	1
Δ_{γ_s}	Spread of γ_s	$[\Delta_n, \Delta_c] = [0.33, 0.24]$	1
μ_γ	Curvature parameter in drawing γ_h	0.2	1
	Preferences		
$1/\sigma$	Inter-temporal Elasticity of Substitution (IES)	0.25	1
ζ	Equivalence Scale	0.3	1
β	Time Discount Rate (per annum)	0.95	2
ν	Altruism Parameter (Avg. Transfers)	0.5	1
μ	Leisure Share (Fraction of h worked)	0.5	1
	Wages and Returns		
$w_{n,d}$	Mean Wages Dropouts and Non-College	1	1
w_c	Mean wages of college graduates	1.76	2
r	Rate of Return	3.3%	1
	Borrowing Constraints		
ϕ_{bc}	Tightness of Borrowing Constraint in College	0.75	1
	Ability and Education		
$\{\sigma_j\}$	Substitution Elasticity in Human Capital Production	$\{1.5, 1.5, 0.5, 0.5\}$	1
$\{\upsilon_j\}$	Productivity of Own Human Capital	$\{0.75, 0.75, 0.95, 0.95\}$	1
ψ	Investment Productivity of Human Capital Investments	10	1
μ_c	Curvature in probability for success in college $q = c$, $\pi(q = c \mid h)$	1.14	2
λ	Time Costs of College	1.81	2
$\bar{\pi}_h$	Probability to draw parental human capital	0.25	1
κ	Resource Cost of Coll.	0.203	1
ϕ	Dropout time in college	0.5	1
$\bar{h}_0$	Upper limit of initial human capital	0.5	1
	Government Policy		
θ	Public Tertiary Education Subsidy	38.8%	1
θ_p	Private Tertiary Education Subsidy	16.6%	1
τ_l	Labor Income Tax Rate	25.84%	2
d	Tax Deduction Rate	27.1%	1
τ_c	Consumption Tax Rate	5.0%	1
τ_k	Capital Income Tax Rate	28.3%	1
τ_{ss}	Social Security Payroll Tax	12.4%	1
ρ_{ss}	Pension Contribution Rate	7.4%	2
b	Debt to GDP Ratio	60%	1
gy	Government Consumption to GDP Ratio	17%	1
$\bar{i}^g/w_c$	Investment level in non-tertiary education	0.049	2
$\zeta^g_{e,t}$	Age profile of investments in non-tertiary education	0.52	1

Table 5: Calibration

Note: Table summarizes the parameter values for the benchmark economy, including the empirical targets the parameters are calibrated to. Stage: 1: first stage (calibrated outside the model), 2: second stage parameter.

in the model of 80% (see 4.10 and C.3 the detailed calibration strategy), in line with U.S. data for the later part of the 2000's (see, e.g., Heathcote et al. 2010).

Second, we choose the stochastic mean reverting component of wages η as a two state Markov chain with education-specific states for log-wages $\{-\sigma_s, \sigma_s\}$ and transition matrix

$$\Pi = \begin{pmatrix} \pi_s & 1-\pi_s \\ 1-\pi_s & \pi_s \end{pmatrix}.$$

In order to parameterize this Markov chain we first estimate the following process on the education-specific PSID samples selected by Karahan and Ozkan (2012):

$$\begin{aligned} \log w_t &= \alpha + z_t \\ z_t &= \varrho z_{t-1} + \eta_t, \end{aligned}$$

where α is an individual-specific fixed effect that is assumed to be normally distributed (with cross-sectional variance σ_α^2). The estimation results are summarized in the left part of table 6, where again college dropouts and non-college workers are treated the same.[40]

	Estimates			Markov Chain		
Group	ϱ	σ_η^2	σ_α^2	π_s	σ_s	$\mathcal{E}_s$
Non-College/Dropouts	0.928	0.0192	0.0644	0.871	0.250	$\{0.755, 1.244\}$
College	0.969	0.0100	0.0474	0.941	0.191	$\{0.811, 1.188\}$

Table 6: Estimates for Earnings Process and Markov Chain for Wages

Note: The left part of this table summarizes the estimates of the AR(1) stochastic earnings process from the PSID, separately for unskilled and skilled individuals. The right part summarizes the corresponding discretized earnings process used in the model economy.

For each education group we choose the two numbers (π_s, σ_s) such that the two-state Markov chain for wages we use has exactly the same persistence and conditional variance as the AR(1)

[40]For the details of the sample selection we refer the reader to Karahan and Ozkan (2012) and we thank the authors for providing us with the estimates for the process specified in the main text. In their paper they estimate a richer stochastic process (which, if implemented in our framework, would lead to at least one additional state variable).

process estimated above.[41] This yields parameter choices given in the right part of table 6.

After de-logging, the wage states are normalized so that the mean of the stochastic component of wages equals 1. We observe that college educated agents face somewhat smaller wage shocks, but that these shocks are slightly more persistent than for non-college educated households.

Third, the fixed component of wages γ_s. We again set $\gamma_n = \gamma_d$ and take a two-state specification for each. We normalize the mean of each γ_s to one and specify an education group specific spread Δ_{γ_s} such that $\gamma_s \in \{\gamma_{l,s} = 1 - \Delta_{\gamma_s}, \gamma_{h,s} = 1 + \Delta_{\gamma_s}\}$, which we calibrate to match the variance of the fixed effect estimates reported in Table 6.

Finally, we assume that the probability to draw the high realization, γ_s^h is given by

$$\pi_\gamma(h_{j_a}) = \min\left\{1, h_{ja}^{\mu_\gamma}\right\} \tag{33}$$

for some parameter $\mu_\gamma \in (0, 1)$. Together with our functional form assumption on the time requirement for studying in college described above, this functional form assumption implies that there is a rational upper bound of about one on the human capital at age j_a.

4.6 Education Costs and Subsidies

Education at a glance (OECD 2012, Table B3.2b) reports that the share of tertiary education expenditures borne by public and private subsidies is $\theta = 38.8\%$ and $\theta_{pr} = 16.6\%$. We borrow the resource costs for college education $\kappa = 0.203$ from Krueger and Ludwig (2016).

In order to reduce the dimensionality, we split non-tertiary education into two phases, i.e.

$$i_{j,t}^g = \bar{i}^g \zeta_{e,t}^g \text{ for } j = 0, 1 \tag{34}$$

$$i_{j,t}^g = \bar{i}^g \zeta_{l,t}^g \text{ for } j = 2, 3, \tag{35}$$

[41] The (unconditional) persistence of the AR(1) process is given by ϱ and the conditional variance by σ_η^2 whereas the corresponding statistics for the Markov chain read as $2\pi_s - 1$ and σ_s^2, respectively.

For a model where a period lasts 4 years and the AR(1) process is estimated on yearly data, the corresponding statistics are ϱ^4 and $(1 + \varrho^2 + \varrho^4 + \varrho^6)\sigma_\eta^2$.

with $\bar{i}^g > 0$, $\zeta^g_{e,t} \in [0,1]$ and $\zeta^g_{l,t} = 1 - \zeta^g_{e,t}$. The former stands in for the overall investment level of the government into non-tertiary education, while $\zeta^g_{e,t}$ determines how these investments are distributed into primary and secondary education. We calibrate $\zeta^g_{e,t}$ using data of OECD (2017) on public spending on education and receive $\zeta^g_{e,t} = 0.52$ (see 4.10 for the complete computation of $\zeta^g_{e,t}$.), implying higher investments at ages 0 and 1.

Regarding education subsidies, we want the relation of tertiary and non-tertiary education to be reasonable. Following OECD (2017), the ratio of non-tertiary to tertiary government education spending in the US is $\frac{E^e}{E^c} = 2.62$. In our model total expenditures on tertiary education are given by

$$E^c = \left(\Phi_{j,c} + \phi\Phi_{j,d}\right)\kappa w_{0,c}\theta_0, \tag{36}$$

where total expenditures on non-tertiary education are denoted by

$$E^e = \sum_{j=0}^{j_a-1} i^g_{j,t}\Phi_{j,0} = \bar{i}^g_t\left(\zeta^g_{e,t}\Phi_{j,0} + \zeta^g_{e,t}\Phi_{j,1} + \zeta^g_{l,t}\Phi_{j,2} + \zeta^g_{l,t}\Phi_{j,3}\right). \tag{37}$$

Thus, we use the non-tertiary investment level $\bar{i}^g$ in order for our model to hit the target of $\frac{E^e}{E^c} = 2.62$ (for details see 4.10 and C.2).

4.7 Borrowing Constraints

The borrowing constraints faced by agents pursuing a college degree allow such an agent to finance a fraction $\phi \in [0,1]$ of all tuition bills with credit. We specify a constant (minimum) payment rp such that at the age of retirement all college loans are repaid. Formally

$$\underline{A}_{j_a,t} = \phi(1 - \theta_t - \theta_{pr})\kappa w_{t,c}.$$

and for $j = j_a + 1, \ldots, j_r$:

$$\underline{A}_{j,t} = (1 + r_t)\underline{A}_{j-1,t-1} - rp$$

and rp is chosen such that the terminal condition $\underline{A}_{j_r,t} = 0$ is met.

The parameter ϕ to be calibrated determines how tight the borrowing constraint for college is. Note that in contrast rp is not a calibration parameter but an endogenously determined repayment amount that insures that households do not retire with outstanding student loans.

The maximum amount of publicly provided student loans for four years is given by \$27,000 for dependent undergraduate students and \$45,000 for independent undergraduate students (the more relevant number given that our students are independent households).[42] Relative to GDP per capita in 2008 of \$48,000, this given maximum debt constitutes 14% and 23.4% of GDP per capita. Compare that to the 31% of total costs computed above, this indicates that independent undergraduate students can borrow at most approximately 75% of the cost of college, and thus we set $\phi = 0.75$. The justification for our choice of ϕ again clarifies that it should best be thought of as an education policy parameter that is being held fixed in our optimal policy analysis.

4.8 Government

In the initial steady state the policy parameters to be chosen are $(\tau_k, \tau_l, \tau_c, \tau_p, d, b, gy)$. We pick $b = 0.6$ and $gy = 0.17$ to match a government debt to GDP ratio of 60% and government consumption (net of tertiary education expenditure) to GDP ratio of 17%. Consumption taxes can be estimated from NIPA data as in Mendoza, Razin and Tesar (1994) who find $\tau_c \approx 0.05$. For the capital income tax rate, we adopt Chari and Kehoe's (2006) estimate of $\tau_k = 28.3\%$ for the early 2000's. Given these assumptions on b and gy, the marginal tax rate on labor income τ_l is then endogenously calibrated to balance the government's budget accordingly (see 4.10 and for details)

The social security payroll tax is set to $\tau_{ss} = 12.4\%$ (excluding Medicare). We model social

[42]Note that about 66% of students finishing four year colleges have debt, and *conditional* on having debt the average amount is \$23,186 and the median amount is \$20,000.

security benefits $p_{t,j}(\gamma, q)$ as a linear function of average wages earned during a household's working life. Pensions of an agent with qualification q and productivity type γ are:

$$p_{t,j} = \rho_{ss}\gamma w_{t,q}.$$

Given the payroll tax τ_{ss} we calibrate ρ_{ss} to clear the equilibrium condition of the pension market (see 4.10 for details):

$$\begin{aligned}\tau_{ss}\sum_{q} w_{t,q}L_{t,q} &= \sum_{j=j_r}^{J} N_{t,j}\int p_{t,j}(\gamma, q)d\Phi_{t,j}\\ &= \rho_{ss}\sum_{j=j_r}^{J} N_{t,j}\int \gamma w_{t,q}d\Phi_{t,j}.\end{aligned} \tag{38}$$

Finally, we calibrate the labor income tax deduction $Z_t = d_t\frac{Y_t}{N_t}$ to match a deduction rate of $d_t = 27.1\%$, which is in line with Krueger and Ludwig (2016). The endogenous calibration is necessary, as we have to solve the model given a tax deduction Z_t, without knowledge of the resulting aggregate production Y_t (see 4.10 for C.1 for calibration strategy and computation respectively).

4.9 Prices

We start with a small open economy (partial equilibrium) variant of our model. We exogenously set relative prices for non-college agents and dropouts to $w_{t,n} = w_{t,d} = 1$. We calibrate college wages $w_{t,c}$ to match an average college wage premium in the model of 80% as stated above.

In addition, we choose an annual interest rate of 4.1%, which is consistent with a general equilibrium in a production economy featuring Cobb-Douglas production with a capital output ratio of 3, a capital elasticity parameter of $\alpha = 0.33$ and a depreciation rate of capital of $\delta = 0.07$. We calibrate time discount factor β to match $\frac{K}{Y} = 3$ (see 4.10 and C.1 for calibration strategy and computation respectively).

4.10 Calibration of Second Stage Parameters

The benchmark model is calibrated to hit the following five economic moments: fraction of non-college agents, college dropout rate, college wage premium, capital output ratio and the ratio of non-tertiary to tertiary education subsidies. The computation of the distributional moments takes place in the aggregation, which we describe in detail in Section C.8. Table 7 displays the real moments, their counterpart in the model and the parameters used to hit the respective targets.

Target	Data	Model	Parameter	Value
Fraction non-college	0.560	0.560	time costs college (λ)	1.81
Dropout rate	0.500	0.500	curv. coll. success (μ)	1.14
College wage premium	1.800	1.800	wage of graduates ($w_{t,c}$)	1.76
Capital output ratio	3.000	2.999	time discount rate (β)	0.95
Non-tertiary / tertiary edu	2.620	2.624	invest. level / wage (i/w)	0.049

Table 7: Summary Economic Second Stage Parameters

The model was able to match all five economic targets of interest very precisely. In addition, parameters τ_l, ρ_{ss} and Z were chosen to balance government budget, pension market and tax progressivity:

$$\left\| \tau_l^* - \frac{T_t^* - \tau_c C_t - \tau_k r_t K_t}{Y_t^d} \right\| = 0, \tag{39}$$

$$\left\| \rho_{ss}^* - \frac{\tau_{ss} \sum_q w_{t,q} L_{t,q}}{\sum_{j=j_r}^{J} N_{t,j} \int \gamma w_{t,q} d\Phi_{t,j}} \right\| = 0, \tag{40}$$

$$\left\| Z^* - d\frac{Y_t}{N_t} \right\| = 0. \tag{41}$$

For all parameters $p \in (\tau_l, \rho_{ss}, Z)$ the benchmark model matches the respective targets satisfying $f(p) = \|p^* - p\| < 0.001 \; \forall p$. A detailed computation of the five economic targets summarized in Table 7 and equations (39)-(41) can be found in Appendix C. Here we give a brief explanation of our calibration strategy.

Let Ψ be a vector of second stage parameters, $m(\Psi)$ a vector of simulated moments and m^T a vector of economic targets. In addition, let Γ be a vector of governmental instruments, $g(\Gamma)$ a vector of simulated moments and g^T a vector of respective targets. In a double nested fix point iteration, we operate as follows:

(i) $\tilde{m}^T = \omega \cdot m^T + (1-\omega)m(\Psi)$.

(ii) For all $\psi_i \in \Psi$ solve $||m_i(\psi_i) - \tilde{m}_i^T|| < \epsilon$.

(iii) Given ψ_i from (ii) solve the government budget until convergence:

  (a) Compute $\tilde{g}^T = \omega \cdot g^T + (1-\omega)g(\Gamma)$.

  (b) For all $\gamma_i \in \Gamma$ solve $||g_i(\gamma_i) - \tilde{g}_i^T|| < \epsilon$.

  Loop over (a) and (b) until $||\tilde{g}^T - g^T|| < \epsilon$ and compute $m(\Psi)$.

Loop over (i) to (iii) until $||\tilde{m}^T - m^T|| < \epsilon$.

This calibration strategy has two advantages: although we have a fixed point problem in eight dimensions, it delivers results in decent speed, while it simplifies splitting economic from other targets. This makes the execution of policy experiments now following straightforward.

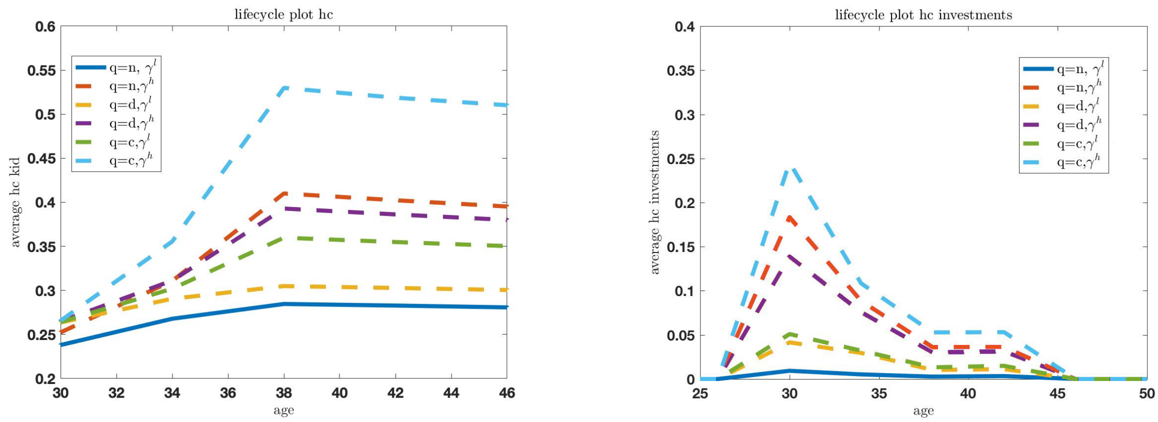
4.11 Lifecycle Plots of Benchmark Model

There is a consensus in human capital literature that different stages of the education process have different characteristics. As discussed in Section 2.2.3 and Appendix A.1, much of this can be explained by a human capital process that exhibits dynamic complementarity. This implies that a lack of investments in primary education has lasting effects, as it can hardly be compensated at later stages, which is closely related to the first empirical fact Cunha and Heckmann (2010) discuss: *"[. . .] ability gaps between individuals and across socioeconomic* *groups open up at early ages"*. One possible explanation would be that young families are often financially unable to provide sufficient primary education for their children, which then becomes a permanent disadvantage for children due to dynamic complementarity. This causal link is discussed very carefully in Caucutt and Lochner (2017), who quantify the importance of borrowing constraints for human capital investments, earnings, and intergenerational mobility. These mechanisms are of fundamental importance to the underlying question in this essay. Therefore, in this part we take a critical look at the crucial life-cycle profiles of our calibrated model.

Figure 10 (a) shows how children's human capital evolves in the benchmark model, given six different combinations of parental education and productivity type. The four investments take place at the parents' age 30, 34, 38 and 42, while the corresponding age of the children is 2, 6, 10, and 14 years, before they leave the household at the age of 18 and face the college decision. Parents differ in schooling (q=n stands in for non-college households, q=d for college dropouts and q=c for college graduates) and productivity type (γ^l stands in for low productivity and γ^h for high productivity). The first empirical fact is accurately reflected: after the first two investments of parents (ages 30 and 34), the resulting human capital gap reaches its maximum. The two subsequent investments during secondary education do not lead to further differences, but rather to maintaining the gap that has already been created.

In addition, there is a second, very interesting aspect. Having a closer look at the starting point - the innate human capital - differences are already apparent, but they develop differently. First of all, it should be noted that children whose parents did not attend college and are of the low productivity type, have the lowest initial human capital, and remain at the lower end of the ability distribution for the whole range of primary and secondary education. Almost the opposite is the case for children whose parents are college graduates and have drawn the high productivity type γ^h. However, on closer inspection, it can be seen that the average initial human capital of children of college graduates and dropouts hardly differs, but their subsequent development does.

This is explained by Figure 10 (b), which shows the average investments of the respective



(a) Human Capital Development Kids

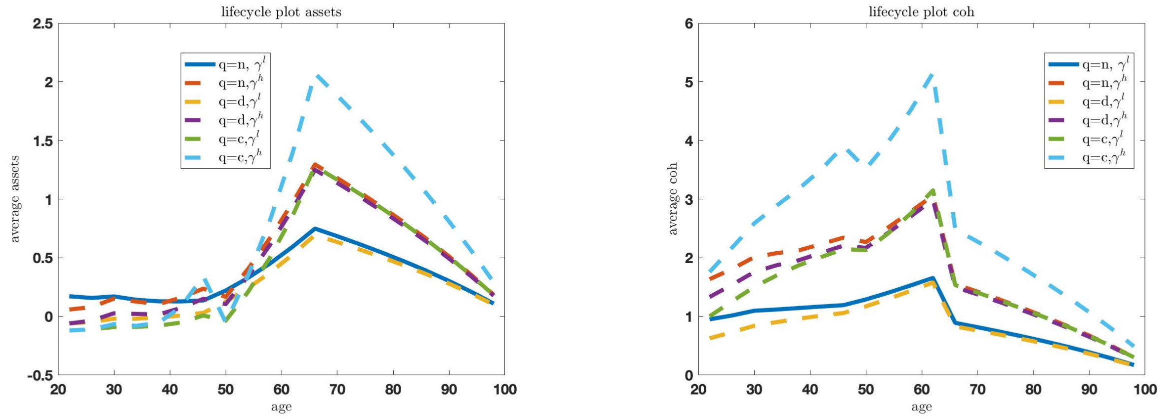
(b) Human Capital Development Parents

Figure 10: Ability Gaps Open Up Early

Note: qualification $q = \{n, d, c\}$, productivity $\gamma = \{\gamma^l, \gamma^h\}$ and age stands in for the type of parent, while the development of average human capital of children of the respective groups is plotted. Households become parents at the age of 28, implying corresponding ages of children are 2 to 18.

parents. When parents have drawn the low productivity type, they invest significantly less in comparison to parents with the same qualification but high productivity type. That even leads to children from non-college households with γ^h, ranked second to last in innate ability, rising to be the runner up in terms of human capital at adulthood.

The question arises as to what is behind the different investment behavior of parents. Figure 11, presenting their respective financial situation, provides the explanation. Parents with $q{=}n$ (non-college) and $q{=}d$ (dropouts) receive the same wages, if they are of the same productivity type γ. Further, parents who did not attend college were not subject to time deduction $\xi(h_{j_a})$ for studying and were thereby able to earn a higher income for one period. In addition, college dropouts still carry around negative assets caused by tuition fees. This results in a cash-on-hand ranking of parents in the phase of primary and secondary education investments (age 30-42 in Figure 11 (b)) that reflects the investment behavior and thereby highlights the intergenerational connection between the financial situation of parents and the human capital development of children, which is completely detached from the nature component (innate ability).



(a) Assets

(b) Cash-On-Hand

Figure 11: Life-Cycle Profiles

5 Partial Equilibrium Analysis

Starting from the benchmark model, we will perform two univariate and one bivariate policy experiment, in which the government can choose different (i) college subsidies, (ii) non-tertiary investments or (iii) combinations of both. We stay in partial equilibrium, which means that wages and the interest rate are not adjusting to the economic responses induced by the differing policies. However, the government budget as well as the pension market are cleared via τ_l and ρ_{ss} satisfying (39) and (40) respectively. The results in all three partial equilibrium experiments indicate that optimal policies require higher governmental investments in primary, secondary and tertiary education compared to the benchmark model.[43]

In the first experiment the government optimizes social welfare by setting college subsidies θ, reducing the burden of tuition fees ($\kappa w_c(1-\theta-\theta_{pr})$). The relevant range of college subsidies is between 0.0% and 350%,[44] why at all times the investment level in non-tertiary education of the benchmark model ($\bar{i}^g/w_c = 0.0486$) is maintained. In the second experiment college subsidies are fixed to the benchmark value ($\theta = 38.8\%$), while the government optimizes over the non-tertiary education investment level, which contributes to primary and secondary education. Governmental investments $\bar{i}^g/w_c$ will be displayed in the range of 0.017 and 0.340.[45]

Each set of policy parameters $\mathcal{T} = \{\theta, \bar{i}^g, \tau_l, \rho_{ss}\}$ corresponds to another partial equilibrium, which will be assessed by the following Utilitarian social welfare function:

$$SWF(\mathcal{T}) = \sum_j N_{t,j} \int V(j, A, h_0, h, q, \gamma, \eta)\, \Phi_{t,j}. \tag{42}$$

[43]The complete results can be found in the tables in Appendix D. In the main part we only focus on the most important aspects.

[44]Please note that for $\theta > 100\%$, the government not only covers all tuition fees, it also pays parts of the living expenses for students.

[45]In the univariate experiments we evaluate welfare for six different values of college subsidies and non-tertiary education investments respectively. In the bivariate experiment we allow for both measures to deviate from the banchmarkl values and evaluate welfare for all 36 combinations of college subsidies and non-tertiary education investments.

Regarding social welfare, the mechanisms behind potential increases or decreases are what interests us most. Besides policies leading to an overall increase in output and consumption, creating a more equal distribution can also enhance welfare, which is due to the concavity of utility functions. We can observe that college subsidies affect social welfare more through the channel of overall wealth, whereas an increase in non-tertiary investments additionally causes a more equal income and consumption distribution.

Lastly, after we analyzed both instruments separately, we allow for various combinations of both college subsidies and non-tertiary investments, which enables the government to take advantage of the described interplay of the different phases within the education process. To give a first impression, Table 8 summarizes the borders as well as the optimal policy parameters of the three experiments.

Experiment	Range θ	Range $\bar{i}^g/w_c$	Result
College subsidies	0.0% – 350%	0.0486	$\theta^* = 175\%$
Non-tertiary investments	38.8%	0.017 – 0.340	$\bar{i}^{g*}/w_c = 0.2594$
Both	0.0% – 350%	0.017 – 0.340	$\theta^* = 263\%, \bar{i}^{g*}/w_c = 0.2594$

Table 8: Results Policy Experiments Partial Equilibrium

Note: welfare is evaluated based on grids of the policy parameters. Thus, if welfare is maximized for the same policy parameter in different experiments, that should not be confused with a general best policy.

5.1 College Subsidies

In the first experiment non-tertiary investments of the government are fixed to the value of the benchmark model, while the government can set different college subsidies θ.[46] Tuition fees are given by $\kappa w_c(1 - \theta - \theta_{pr})$. During their studies, students can work part time for average non-college wages, given a reduced time endowment of $1 - \zeta(h_{j_a})$. Another financial option is to borrow up to $\underline{A}_{j,t}$ and stretch out repaying tuition fees over the life-cycle. Additionally, agents

[46]Table 20 in Appendix D shows a detailed list of the results for various economic moments.

might have received vivos transfers from their parents, also relaxing the financial bottleneck in a situation of relatively low wage income. In any case, subsidies make college attendance easier to afford, in particular in partial equilibrium, in which prices do not respond to a higher fraction of college attendance and a constant college wage premium.[47] However, since we allow for subsidies larger than 100%, the government has the option not only to cover tuition fees but also to provide students with positive assets to cover living expenses. Figure 12 displays social welfare as a function of college subsidies. The optimal policy calls for a much higher college subsidy (175%) than in the benchmark model. Nevertheless, there is a turning point, implying that there are different mechanisms working against each other.

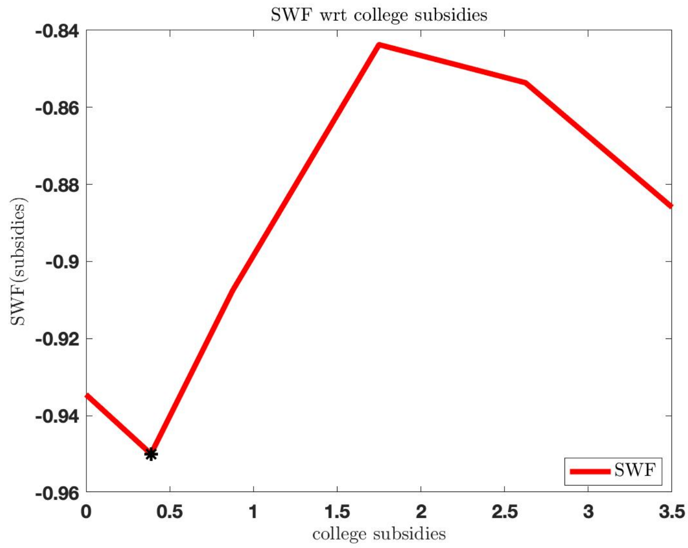


Figure 12: Experiment College Subsidy (*Benchmark Model)

Table 9 summarizes outcomes of the policy experiment that are related to production. Higher college subsidies combined with a higher college attendance rate lead to an increase in government expenditures $\kappa w_c \theta(\Phi_c + \phi\Phi_d)$. As a result, the required labor tax rate that clears the

[47]Please note that the average college wage premium of the respective wage groups changes a little bit, as there are shifts in human capital and thus in productivity types for different college subsidy levels (the definition of the college wage premium is described in equation (101). What remains unchanged in partial equilibrium is resource costs of labor w_{nd} and w_c.

government's budget is increasing throughout the whole experiment (it is worth noticing that this relation will not always be monotonic, which we will see in the bivariate experiment: when the positive effect of college subsidies on aggregate labor, and thereby the tax base, exceeds the increase in government expenditures the labor tax decreases when college subsidies go up). The development of aggregate labor supply with respect to the increase in college subsidies is hump-shaped. Hence, labor taxes start to rise only marginally, as the increase in government expenditures goes hand in hand with an increase in labor supply (and in consequence the tax base). From the point at which labor supply falls, while costs continue to rise, the increase in labor taxes are much higher. This is also the area in which the positive impact of subsidies is tipped and further increases lead to a reduction in welfare.

In addition, welfare is also rising relative to the benchmark model when college subsidies are set to zero. The bivariate experiment will show that this pattern only applies for low levels of human capital investments of the government. The cause lies in the effect of subsidies on equality in the economy, which increases (in partial equilibrium) for low levels of college subsidies.[48]

College Subsidy	0.0000	0.388*	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-0.9345	-0.9499	-0.9076	**-0.8437**	-0.8536	-0.8859
Labor Tax	0.2245	0.2584	0.2642	**0.2966**	0.3509	0.4160
Aggr. Labor	0.7731	0.8767	0.9005	**0.8807**	0.8507	0.8061
Aggr. Output	0.3994	0.4530	0.4653	**0.4550**	0.4396	0.4165

Table 9: Results Production

Note: * is the benchmark model. Bold letters mark policy measure with highest social welfare.

Table 10 shows the outcomes of the policy experiment related to education. Starting at the lower end, an increase in college subsidies leads to higher human capital investments by parents, which is linked to dynamic complementary. If college costs are decreased, children will be able to afford college, which in turn increases the return on investments in primary and secondary education. Thus, parents react with higher education investments, which leads to a rise in the

48 We will discuss this in more detail shortly. Note, however, that this mechanism does neither translate to the small open economy nor to the general equilibrium version of the experiment. In both cases, in the absence of college subsidies, the fraction of non-college households reaches its maximum, which further decreases w_{nd} relative to w_c and inequality rises as opposed to the partial equilibrium case we are examining here.

aggregate human capital level. This is perfectly in line with the findings of Caucutt and Lochner (2017), who highlight the importance of early investment responses to post-secondary subsidies. Further, it is worth noticing, that vivos transfers are also increased until the threshold $\theta > 100\%$ is passed and subsidies exceed tuition fees.

College Subsidy	0.0000	0.388*	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-0.9345	-0.9499	-0.9076	**-0.8437**	-0.8536	-0.8859
Aggr. HC Investments	0.0025	0.0030	0.0035	**0.0037**	0.0035	0.0033
Aver. Human Capital	0.3764	0.3899	0.4037	**0.4083**	0.3997	0.3964
Aggr. Vivos Transfers	0.0042	0.0047	0.0051	**0.0048**	0.0039	0.0028
Fraction Non-College	0.8097	0.5591	0.4125	**0.2879**	0.1152	0.0077
Fraction Graduates	0.1070	0.2205	0.2880	**0.3202**	0.3361	0.3352
Fraction Dropouts	0.0832	0.2205	0.2995	**0.3919**	0.5488	0.6571

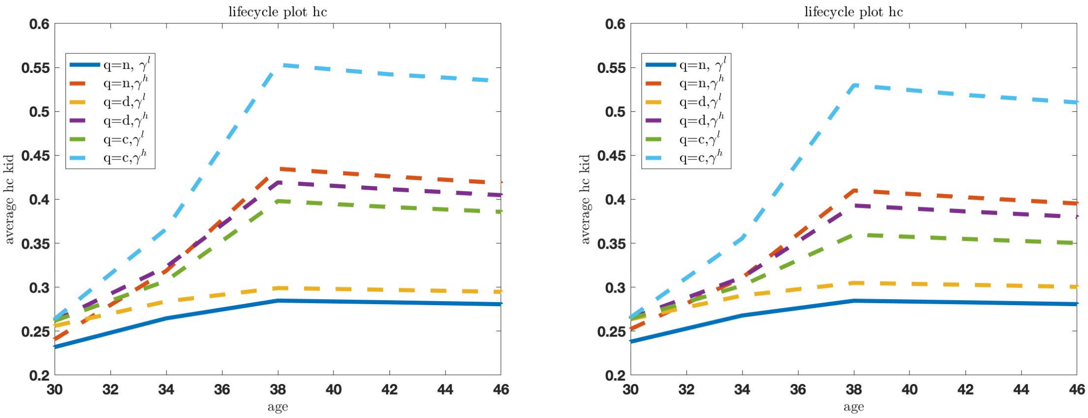
Table 10: Results Education

Note: * is the benchmark model. Bold letters mark policy measure with highest social welfare.

However, while the general human capital level is increased by higher college subsidies, this measure does not have a positive effect on children at the lower end of the income distribution. This can be seen in Figure 13, which shows the human capital development from the benchmark model and the social optimum of this policy experiment. By comparison, average human capital of children from parents who graduated college ($q = c$) and are high productive (γ^h) stands out further than before. The increase in the labor tax rate may have hit this group the hardest, but they also benefited most from negative tuition fees. Average human capital of children from the middle of the distribution is also increased by higher subsidies. It is only the two groups at the bottom of the distribution whose human capital the measure has not impacted positively. In fact, there is even a slight decrease for children of parents that dropped out of college ($q = d$) and drew the low productivity type (γ^l). As can be seen on the left end (age 30 of parents, i.e. age 2 of children), this is not due to lesser innate human capital. The gap opens up during primary education and remains constant afterwards, which is a reproduction of the first empirical fact stated in Cunha and Heckman (2010).[49] In summary, an increase in subsidies has yield higher

[49] See Appendix A.1 for a detailed discussion on the empirical literature on human capital development.

human capital on average, but did not produce a more equal distribution - it has rather led to the opposite.



(a) Social Optimum (θ = 175%) (b) Benchmark Model (θ = 39%)

Figure 13: Comparison Human Capital Development

Note: qualification $q = \{n, d, c\}$, productivity $\gamma = \{\gamma^l, \gamma^h\}$ and age stands in for the type of parent, while the development of average human capital of children of the respective groups is plotted. Households become parents at the age of 28, implying corresponding ages of children are 2 to 18.

These effects spill over to the distribution of income and consumption. Apart from endogenous effects, the group of non-college households does not benefit from college subsidies, but has to participate in the higher tax burden. In addition, comparing the outcome from the benchmark model and the social optimum, they suffer the biggest drop in average net wage income.[50] Apparently, part of theses changes are due to shifts from non-college households to either college graduates or dropouts, but it is worth noticing that average wages of non-college households increase in the non-tertiary investment experiment. The same applies for the Theil Index,[51] which increases for income[52] and consumption as college subsidies increase, and decreases, as investments in non-tertiary education are raised.

The different and partly opposing effects are clarified once more in the following special case. As can be seen in Figure 12, welfare falls when college subsidies are raised from zero

[50] It drops from 0.7684 to 0.6771 which is a decrease of 11.88%, while average net wage income of college graduates drops by only 5.03% (from 1.4951 to 1.4158).

[51] In Section D.1 we display the computation of the inequality measures.

[52] Please note that overall income stands in for net wage income plus net income from assets.

to the level of the benchmark model. Roughly speaking, redistributive measures are welfare-enhancing if they benefit a group with higher marginal utility than the group that was redistributed from. The described increase in subsidies improves households that were in college even under zero college subsidies, coming from rather high-income households. The group of non-college households with a rather low income has to take on the cost of college subsidies (in the form of higher labor taxes), but does not receive financial compensation. In this scenario, the negative effect on them outweighs the positive effect on students and the marginal households who go to college due to the implementation of college subsidies.[53]

College Subsidy	0.0000	0.388*	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-0.9345	-0.9499	-0.9076	**-0.8437**	-0.8536	-0.8859
Aver. Net Lab-Inc Non-College	0.8167	0.7684	0.7165	**0.6771**	0.6161	0.5263
Aver. Net Lab-Inc Dropouts	0.9182	0.8628	0.8646	**0.7991**	0.7086	0.6289
Aver. Net Lab-Inc Graduates	1.5894	1.4951	1.5013	**1.4158**	1.3195	1.2001
Aggr. Cons	0.4127	0.4223	0.4403	**0.4343**	0.4111	0.3793
Aver. Cons Non-College	0.3800	0.3574	0.3387	**0.3227**	0.2939	0.2899
Aver. Cons Dropouts	0.4106	0.3860	0.3923	**0.3720**	0.3356	0.3040
Aver. Cons College	0.6616	0.6230	0.6356	**0.6109**	0.5746	0.5289
Theil Index Net Wage Income	0.3746	0.3800	0.3881	**0.3940**	0.3971	0.3953
Theil Index Overall Net Income	0.2765	0.2844	0.2914	**0.2961**	0.3004	0.2990
Theil Index Consumption	0.1126	0.1188	0.1226	**0.1187**	0.1169	0.1146
Var LN Net Wage Income	1.3498	1.3318	1.3437	**1.3557**	2.0875	2.1505
Var LN Overall Income	0.8248	0.8327	0.8447	**0.8609**	1.6055	1.6850
Var LN Consumption	0.3145	0.3290	0.3384	**0.3328**	0.3319	0.3271

Table 11: Results Equality

Note: * is the benchmark model. Bold letters mark policy measure with highest social welfare.

In summary, increased welfare is triggered by higher overall productivity, driven by an increase in aggregate human capital. The latter is due to higher parental investments in primary and secondary education, which are an endogenous reaction to increased college subsidies. Production and aggregate consumption increase until the burden of taxation starts to outweigh these positive effects. At that point labor supply declines, which, in combination, also explains

[53]As already mentioned, this does not translate into the small open economy and the general equilibrium version of the model and occurs only in partial equilibrium when the level of non-tertiary human capital investments by the government is low.

the decline in human capital for very high level of college subsidies: in our model, higher future wages are the only reason for parents to invest into the human capital of their children. This incentive decreases with a higher tax progessivity and a lower future labor supply of the child. Last but not least, we can note that an increase in college subsidies does not lead to a more equal distribution of human capital and therefore not to a more equal distribution of wage income, overall income and consumption.

5.2 Non-Tertiary Investments

In this experiment, college subsidies θ are fixed to the status quo, while the government can set different levels of non-tertiary investments.[54] The social optimum requires higher non-tertiary education investments than the benchmark model (see Figure 14). The dynamics are different compared to the previous experiment of college subsidies: non-tertiary education investments have a larger impact on human capital than college subsidies, and, in particular, human capital of children from low income households is increased by this measure. Higher and more equal distributed human capital leads to both an increase in production and to a more equal distribution of income and consumption.

Ivst. Level Non-Tert. Edu.	0.0170	0.049*	0.0978	0.1786	**0.2594**	0.3401
Social Welfare	-1.0646	-0.9499	-0.9000	-0.8347	**-0.8020**	-0.8231
Labor Tax	0.2187	0.2584	0.2815	0.2946	**0.3081**	0.3390
Aggr. Labor	0.7615	0.8767	0.9187	0.9254	**0.9052**	0.8861
Aggr. Output	0.3935	0.4530	0.4747	0.4781	**0.4677**	0.4579

Table 12: Results Production

At first sight, at the macroeconomic level, an increase of investments in non-tertiary education leads to a similar outcome as an increase in college subsidies: labor, production and consumption increase in comparison to the benchmark model. At some point, there is a reversal, which is related to the increase of labor taxes making up for education expenditures.

[54] A detailed list of the results for various economic moments can be found in Table 21 in Appendix D.

The incentive to work diminishes, aggregate labor decreases and social welfare is declining if investments are further increased.

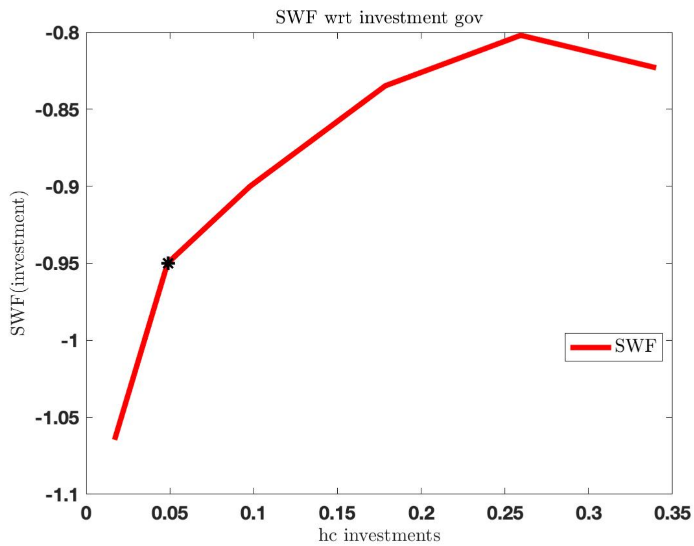


Figure 14: Experiment Non-Tertiary Investments (*Benchmark Model)

The reaction of parental investments into primary and secondary education with respect to governmental investments is hump-shaped. At the lower end, parents react with an increase of their education expenditures, which has two sources. On the one hand overall resources increase when education investments of the government are raised, and, in addition, this pattern fits the facts of human capital production. Due to dynamic complementarity and self-productivity, higher investments of the government in one period increase the return on investments of parents in all other periods. However, when public investments have exceeded a certain point, marginal returns of additional investments become very small and parental investments are crowded out.

The proportion of non-college households is developing very similar to the previous policy experiment. But as non-tertiary investments of the government lead to a stronger increase of human capital than the endogenous response of parents to higher college subsidies, more students graduate from college successfully.

Figure 15 displays the human capital development given governmental investments have been doubled compared to the benchmark model. Starting with innate human capital in the first

Ivst. Level Non-Tert. Edu.	0.0170	0.049*	0.0978	0.1786	**0.2594**	0.3401
Social Welfare	-1.0646	-0.9499	-0.9000	-0.8347	**-0.8020**	-0.8231
Aggr. HC Investments	0.0014	0.0030	0.0019	0.0008	**0.0004**	0.0001
Aver. Human Capital	0.1701	0.3899	0.5107	0.6720	**0.8027**	0.8744
Aggr. Vivos Transfers	0.0126	0.0047	0.0032	0.0022	**0.0010**	0.0000
Fraction Non-College	0.8122	0.5591	0.3920	0.2328	**0.1404**	0.0901
Fraction Graduates	0.1112	0.2205	0.3188	0.4494	**0.5392**	0.5844
Fraction Dropouts	0.0767	0.2205	0.2892	0.3178	**0.3204**	0.3255

Table 13: Results Education

four years, skills develop almost completely accordingly. In the second investment period, at the age of 6 (age 34 of parents), we can observe that human capital of children from households with high productivity types (γ^h) overtakes human capital of their respective counterparts with the same qualification status, implying that parents' income (and thereby investments) continue to be a factor, even though a much lesser one.

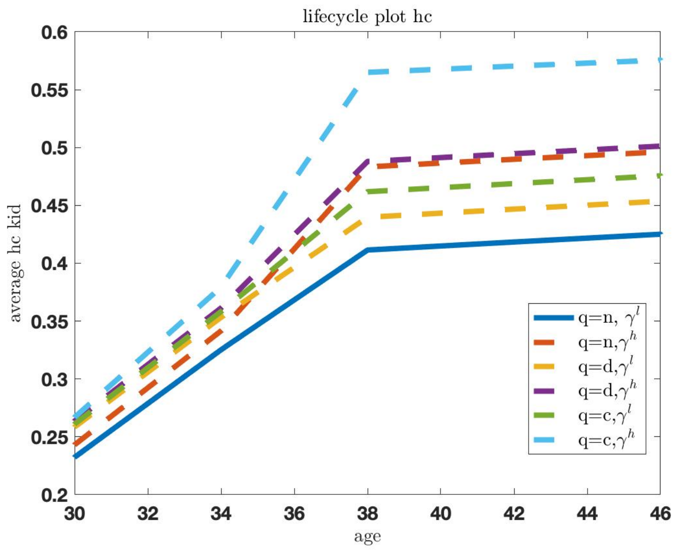


Figure 15: Human Capital Development Given $\bar{i}^g/w_c = 0.0978$

Nevertheless, the entire distribution is much denser. Accordingly, we find a more equal distribution of average net wage income and overall income (see Table 14). The average net income of college households is falling as they are most affected by the increase in progressive labor tax (in Table 21 in the appendix it can be seen that average gross wage income of college graduates is increasing). Both average net wage income and average consumption of non-college households are monotonically increasing in governmental non-tertiary investments, which is in sharp contrast to the effect of college subsidies. Accordingly, the Theil Index with respect to income and consumption reflects the same picture.

Ivst. Level Non-Tert. Edu.	0.0170	0.049*	0.0978	0.1786	**0.2594**	0.3401
Social Welfare	-1.0646	-0.9499	-0.9000	-0.8347	**-0.8020**	-0.8231
Aver. Net Lab-Inc Non-College	0.6757	0.7684	0.7830	0.7748	**0.7850**	0.8275
Aver. Net Lab-Inc Dropouts	0.9372	0.8628	0.8501	0.8553	**0.8546**	0.8313
Aver. Net Lab-Inc Graduates	1.6256	1.4951	1.4740	1.4821	**1.4753**	1.4349
Aggr. Cons	0.3738	0.4223	0.4467	0.4822	**0.5026**	0.4948
Aver. Cons Non-College	0.3280	0.3574	0.3623	0.3608	**0.3618**	0.3680
Aver. Cons Dropouts	0.4183	0.3860	0.3792	0.3821	**0.3806**	0.3655
Aver. Cons College	0.6778	0.6230	0.6118	0.6159	**0.6119**	0.5864
Theil Index Net Wage Income	0.3992	0.3800	0.3730	0.3671	**0.3576**	0.3459
Theil Index Overall Net Income	0.3028	0.2844	0.2786	0.2732	**0.2653**	0.2574
Theil Index Consumption	0.1338	0.1188	0.1137	0.1090	**0.1029**	0.0977
Var LN Net Wage Income	1.2898	1.3318	1.3175	1.3047	**1.2761**	1.2232
Var LN Overall Income	0.8294	0.8327	0.8201	0.8071	**0.7851**	0.7575
Var LN Consumption	0.3516	0.3290	0.3160	0.3032	**0.2850**	0.2679

Table 14: Results Equality

To sum up, while the effects on aggregate labor and production are similar, the two policies deviate regarding their distributional consequences. An increase in the non-tertiary education expenses of the government causes the level of human capital across all households to increase, while the tax burden continues to be borne especially by the highest-earning group. This results in a more equal income and consumption distribution. Moreover, the overall impact on human capital is higher, which causes not only the number of enrollments but also the fraction of graduates to increase substantially.

5.3 Tertiary and Non-Tertiary Education Measures

In the third experiment, the government has both instruments at its disposal and can thereby balance the interplay of college subsidies and investments in non-tertiary education.[55] The results indicate that considering only one of the two experiments separately could lead to wrong conclusions, since the effectiveness of college subsidies depend on investments - and vice versa. Figure 16 illustrates welfare as a function of college subsidies and investments in non-tertiary education. Welfare is maximized in a combination of college subsidies (θ=263%) and non-tertiary investments ($\bar{i}^{g*}/w_c$=0.2594). Thus, both policy measures are both above their respective values from the benchmark model. College subsidies even exceed their optimal value from the univariate experiment.

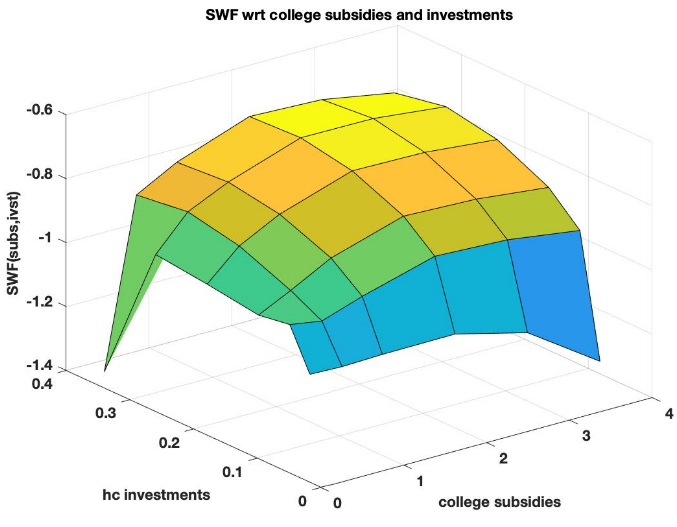


Figure 16: Experiment Tertiary and Non-Tertiary Education

We begin with a closer look at the bottom left corner of Figure 16, where both policy instruments exhibit their lowest value. Starting there, if the government were to perform the univariate

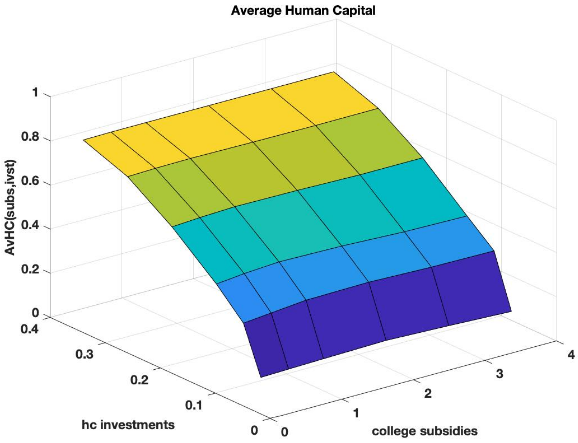
[55]For this experiment we combined the grids from the two univariate experiments, each consisting of six values for college subsidies and non-tertiary investments. This results in 36 different combinations that were evaluated in this experiment.

college subsidy experiment, it would find that initially this has a positive, but only weak effect, before welfare drops sharply through a further increase in subsidies.[56] The reason is, that this policy instrument is limited by the fact that affordable college education can only be a powerful tool, if the human capital level in the economy is high enough. Otherwise, the financial burden of tuition fees might be taken away, but a lack of human capital is the limiting component. This is underlined by Figure 17. The case just described (minimum investments and maximum subsidies) does not lead to a much higher human capital level (17(a)), but to higher college enrollment (17(b)). Since the human capital level has not increased, this drives up the dropout rate to its climax (17(c)) and the proportion of college graduates is almost unchanged (17(d)). Thus, the incentive appears not to be the prospect of a good degree (higher wage), but rather the financial benefit of negative tuition fees.

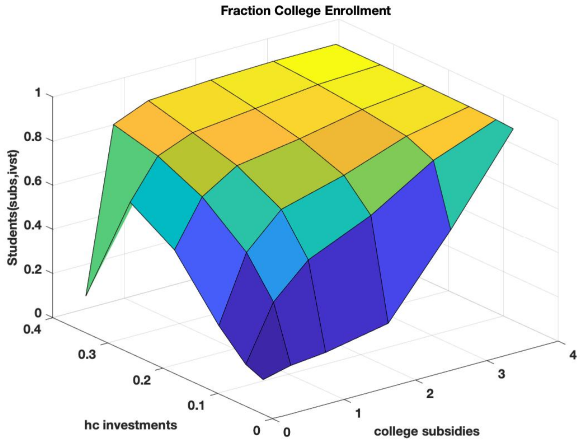
Performing the same thought experiment in the other direction (constant minimum subsidies and investments are increased) clearly shows that the effectiveness of non-tertiary education investments also depends on whether households can financially afford to attend college. This is an analogy to the fourth empirical fact of the human capital literature that Cunha and Heckman (2010) emphasize: *early investments have to be followed up by late investments*. In other words, fruits may have been sown by early investments and raised human capital, but they have to be harvested in the form of college education in order to pay off sustainably.

The importance of the interplay between subsidies and investment becomes particularly clear, when we consider Figure 17(a) and Figures 17(b) together: given maximum non-tertiary investment, aggregate human capital is always at a high level, regardless of college subsidies. However, if subsidies are set to zero, the college attendance rate drops to an extremely low value - despite the high human capital. The reason becomes apparent, when we have a closer look at Figure 18. Increased investments require a higher labor tax rate to balance the govern-

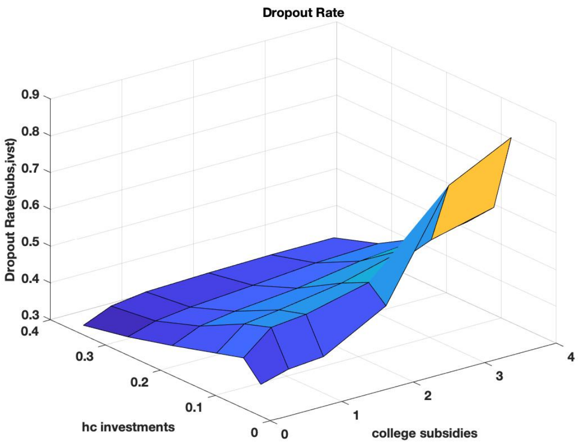
[56]Actually, welfare even decreases slightly when subsidies are raised from zero to the benchmark value of 38.8%, which follows the same dynamic as the special case described in the previous section.



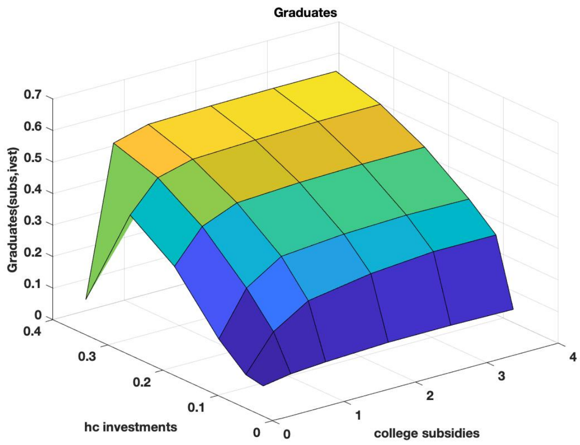
(a) Average Human Capital



(b) Fraction College Enrollment



(c) Dropout Rate



(d) College Graduates

Figure 17: Human Capital and College

ment's budget. This weakens the incentive for college education, which, given the absence of college subsidies, is on top of that very expensive in this scenario. As a result, enrollment drops although human capital is on a much higher level than in the benchmark model.

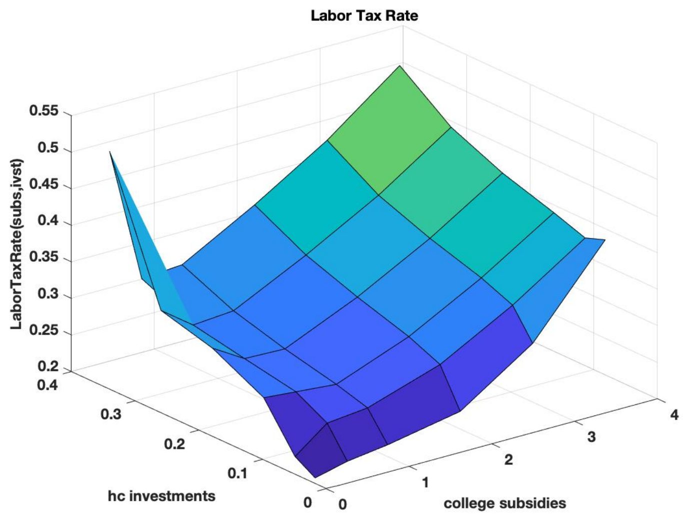


Figure 18: Labor Tax Rate

Figure 18 brings yet another finding to light. Higher college subsidies combined with a higher college attendance rate lead to an increase in government expenditures $\kappa w_c \theta(\Phi_c + \phi\Phi_d)$. However, the development of τ_l with respect to subsidies is U-shaped. This implies, that for low levels of college subsidies, an increase leads to a raise in revenues that exceed the additional college expenditures.[57]

5.4 Summary Partial Equilibrium Experiments

The experiments in partial equilibrium have shown that the two policy instruments affect welfare through different channels. College subsidies increase aggregate human capital via the mechanisms known from human capital literature: better study prospects increase the (expected)

[57] The sharp drop in the lower left corner is related to the two special cases discussed, in which welfare decreased when college subsidies were introduced. In these cases, labor taxes increase strongly with the introduction of college subsidies, while the subsidies only benefit a rather small group on the top of the income distribution.

return on non-tertiary investments, leading to an endogenous response of parents who increase their primary and secondary education expenditures. This results in higher aggregate production and consumption. However, college subsidies do not increase human capital of the kids from the lowest income group, which is why inequality in net labor income increases when subsidies are raised.

An increase in non-tertiary investments, on the other hand, affects children from all income groups, and the resulting human capital distribution is moving to a higher level as well as closer together. In addition to the increase in human capital, production and aggregated consumption, the investments in non-tertiary education thereby result in a more equal distribution of average net labor incomes and consumption, which has a positive effect on welfare, given concave utility functions and the social welfare evaluation we perform.

	BM	**OptSubs**	**OptIvst**	**BivOpt**
Ivst. Level Non-Tert. Edu.	0.0491	0.0491	0.2594	0.2594
College Subsidy	0.3880	1.7500	0.3880	2.6250
Social Welfare	-0.9499	-0.8437	-0.8020	-0.6672
Labor Tax	0.2584	0.2966	0.3081	0.4169
Aggr. HC Investments	0.0030	0.0037	0.0004	0.0003
Aver. Human Capital	0.3899	0.4083	0.8027	0.8003
Fraction Non-College	0.5591	0.2879	0.1404	0.0132
Fraction Graduates	0.2205	0.3202	0.5392	0.5798
Fraction Dropouts	0.2205	0.3919	0.3204	0.4070
Theil Index Net Wage Income	0.3800	0.3940	0.3576	0.3635
Theil Index Overall Income	0.2844	0.2961	0.2653	0.2714
Theil Index Consumption	0.1188	0.1187	0.1029	0.0931

Table 15: Comparison Univariate and Bivariate Optima

The bivariate experiment has shown that the effectiveness of early and late education subsidies are highly dependent on each other and therefore difficult to assess separately. College subsidies can do little if non-tertiary investments are not above a certain level. Otherwise average human capital is not sufficient for the low-cost college education to be taken advantage of. Conversely, non-tertiary investments increase human capital, however, this will not be unutilized if there is not a minimum of college subsidies provided and well-educated young adults can afford college education.

6 Small Open Economy

Since the economy was in partial equilibrium in the previous experiment, wages were fixed to the values of the benchmark model. Thus, although both policy instruments have increased college attendance and the proportion of college graduates, wages of labor types L_{nd} and L_c remained unchanged. In the small open economy, the interest rate remains constant,[58] but wages adapt to the firm's maximization problem,[59] and an increasing proportion of skilled relative to unskilled labor leads to a decline in w_c/w_{nd}.[60] As a result, college subsidies, unlike in partial equilibrium, will also cause a more equal income distribution, which is in line with the findings of Krueger and Ludwig (2016). Both measures result in a smaller college wage premium, which leads to a decrease in human capital expenditures of parents and a more equal distribution of skills. Non-tertiary investments overcompensate this effect and shift average human capital to a higher level. The best policy mix in the bivariate experiment implies an investment level that exceeds the optimal level from the univariate experiment, which highlights the importance of the interplay of primary, secondary and tertiary education.

In the partial equilibrium experiment, we have described the different mechanisms of college subsidies and non-tertiary investments very detailed. In this section, we focus on the differences between the effects of the political measures in partial equilibrium and the small open economy. Besides endogenous prices, the experiments are the same as in the previous chapter.[61,62] Figure 19 gives a first impression of the results of the two univariate experiments.

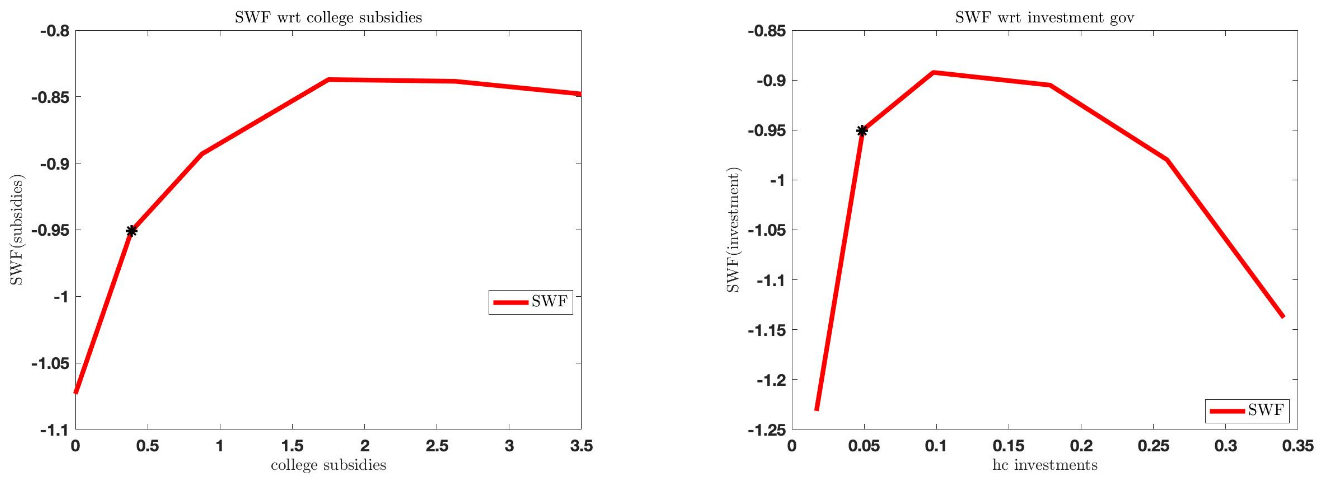
[58]This intermediate step allows us to disentangle the effects from the labor market and the capital market. An outlook on general equilibrium will be given in Section 6.4. Endogenous wages appear to be the more important mechanism for the question at hand. In addition, due to faster convergence than in general equilibrium experiments, the small open economy setup enables us to perform bivariate experiments in a timely manner.

[59]The computation of the firm's maximization problem is shown in Appendix C.5.

[60]In Appendix C.7, we show how the equilibrium in the small open economy is established.

[61]Please note that social welfare is still evaluated following equation (42).

[62]The tables in Appendix E show the results of this chapter in more detail.



(a) College Subsidies

(b) Non-Tertiary Investments

Figure 19: Results Univariate Experiments (SOE)

6.1 College Subsidies

The impact of college subsidies in the small open economy on aggregate output is very similar to partial equilibrium.[63] Wage adjustments, however, make the changes in aggregate labor somewhat weaker, which is why labor taxes fluctuate a little less than in partial equilibrium. Welfare in the benchmark model is higher than in the absence of college subsidies, unlike in partial equilibrium. This is due to the low wages of non-college households and dropouts, when college education is very expensive. It is worth noticing, that tuition fees $\kappa w_c(1 - \theta - \theta_{pr})$ are proportional to the factor price of skilled labor, w_c. Thus, in the small open economy, the increase in tuition fees is amplified when college subsidies are set to zero.

Wage adjustments affect educational choices of households. Subsidies in tertiary education continue to allow more young households to afford college. As describes above, due to dynamic complementarity, this increases the return on parental investments in primary and secondary education. When prices are endogenous, this positive effect on education expenditures has two adversaries. When subsidies are raised, the fraction of graduates increases, which leads

[63] An extension of the results in Table 16 can be found in Table 22 in Appendix E.

to decreasing wages of college graduates. In addition, the outside option (not attending college) becomes more attractive, since the wage for low-skilled labor goes up when L_{nd} becomes smaller.

College Subsidy	0.0000	0.388*	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-1.0732	-0.9509	-0.8930	**-0.8371**	-0.8384	-0.8479
Labor Tax	0.2523	0.2584	0.2651	**0.2970**	0.3396	0.3840
Aggr. Labor	0.8340	0.8767	0.8834	**0.8755**	0.8529	0.8298
Aggr. Output	0.4309	0.4530	0.4565	**0.4523**	0.4407	0.4287
Av-Wage Non-College	0.9255	1.0079	1.0178	**1.0233**	1.0855	1.0175
Av-Wage Dropouts	1.0700	1.1121	1.1662	**1.1921**	1.1367	1.1637
Av-Wage Graduates	2.3524	1.8646	1.7740	**1.6216**	1.5916	1.5444
Aggr. HC Investments	0.0038	0.0030	0.0028	**0.0025**	0.0024	0.0024
Aver. Human Capital	0.4008	0.3899	0.3840	**0.3747**	0.3729	0.3710
Fraction Non-College	0.7186	0.5591	0.4930	**0.3453**	0.2003	0.1177
Fraction Graduates	0.1581	0.2205	0.2469	**0.2888**	0.2988	0.3162
Fraction Dropouts	0.1233	0.2205	0.2601	**0.3660**	0.5010	0.5661
Av. College Wage Premium	2.4850	1.7974	1.6594	**1.4606**	1.4184	1.3564
Theil Index Net Wage Income	0.4219	0.3800	0.3720	**0.3642**	0.3633	0.3589
Aggr. Cons	0.4052	0.4223	0.4295	**0.4212**	0.4031	0.3836
Aver. Cons Non-College	0.3316	0.3574	0.3594	**0.3490**	0.3468	0.3068
Aver. Cons Dropouts	0.3682	0.3860	0.4040	**0.4007**	0.3663	0.3536
Aver. Cons College	0.7688	0.6230	0.5965	**0.5335**	0.5026	0.4657
Theil Index Consumption	0.1478	0.1188	0.1122	**0.1024**	0.0992	0.0962

Table 16: Results College Subsidy Experiment (SOE)

Note: * benchmark model. Bold letters policy with highest welfare. Beside factor prices w_q, average wages ("Av-Wage") take the productivity types γ of households into account.

These two negative effects on the incentive to invest in primary and secondary education dominate, which leads to a decline in aggregate human capital investments of parents. Compared to partial equilibrium (see Table 10), we can see that private aggregate human capital investments in the small open economy are only larger when college subsidies are zero, which underlines the importance of the college wage premium for parental education expenditures. Accordingly, the direction of development of the three qualification groups is the same, but stronger in comparison to partial equilibrium.

In the small open economy, college subsidies reduce both the inequality in income and consumption. For non-college households and dropouts, in the range between $0\% - 87.5\%$, average consumption even increases with college subsidies, which is in sharp contrast to the previous chapter (see Table 11). In addition to the adjustment of wages, there is a second component which impacts the distribution of wages significantly. Figure 20 shows the development of human capital, given welfare-maximizing college subsidies of $\theta = 175\%$. The effect on human capital of children from high-income households is reversing compared to partial equilibrium (see Figure 13): the decline in parental human capital investments, caused by the endogenous reduction in the college wage premium, affects them the strongest. They stay at the top, but the distribution moves closer together. In partial equilibrium, their human capital was increased, because parents endogenous reaction was only driven by better prospects on college education for their children, but not the counteracting effect of a decline in the college wage premium.

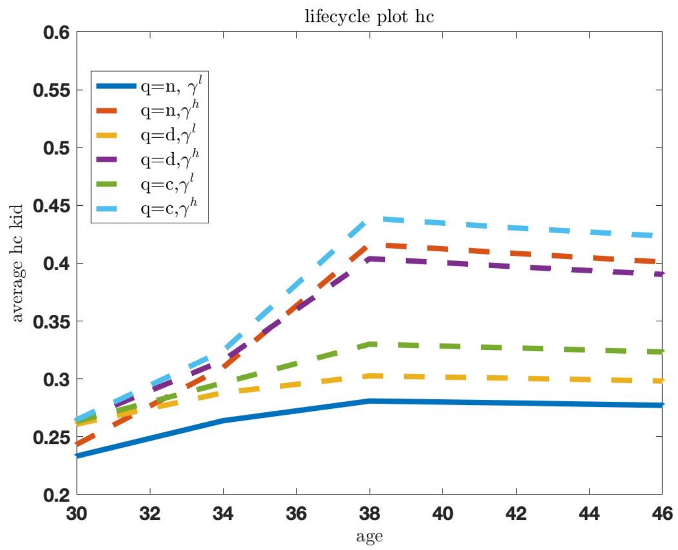


Figure 20: Human Capital Development ($\theta = 175\%$)

6.2 Non-Tertiary Investments

In the small open economy, the optimal non-tertiary investment level of the government[64] is smaller compared to partial equilibrium. The main difference is, that the effect on college at-

[64]Please note that we express non-tertiary investments of the government relative to college wages w_c, i.e. $\bar{i}/w_c$. Therefore, the investment levels displayed in this chapter deviate from the partial equilibrium analysis. However, the underlying investment level of the government $\bar{i}$ remains unchanged.

tendance is weaker, which is caused by a decrease in the endogenous college wage premium. Average productivity of the respective qualification groups is increased by governmental investments, but the effect on tax revenues is outweighed by additional education expenditures. As a result, labor taxes increase strongly. Combined with declining college wages, this even leads to a drop in college attendance for very high levels of primary and secondary education expenditures by the government. At that turning point, social welfare takes its highest value.[65]

Ivst. Level Non-Tert. Edu.	0.0131	0.049*	**0.1051**	0.2013	0.2972	0.3914
Social Welfare	-1.2314	-0.9499	**-0.8923**	-0.9050	-0.9797	-1.1379
Labor Tax	0.2333	0.2588	**0.2912**	0.3541	0.4234	0.5018
Aggr. Labor	0.7808	0.8767	**0.8917**	0.8935	0.8866	0.8715
Aggr. Output	0.4034	0.4530	**0.4607**	0.4617	0.4581	0.4503
Av-Wage Non-College	0.7512	1.0079	**1.1321**	1.2370	1.3072	1.3439
Av-Wage Dropouts	1.0198	1.1121	**1.1533**	1.1879	1.1997	1.2037
Av-Wage Graduates	2.5359	1.8646	**1.7185**	1.6229	1.5767	1.5599
Aggr. HC Investments	0.0024	0.0030	**0.0013**	0.0000	0.0000	0.0000
Aver. Human Capital	0.1922	0.3899	**0.4898**	0.6513	0.7946	0.8724
Fraction Non-College	0.7809	0.5591	**0.5206**	0.5350	0.5589	0.5723
Fraction Graduates	0.1310	0.2205	**0.2506**	0.2714	0.2793	0.2812
Fraction Dropouts	0.0880	0.2205	**0.2288**	0.1936	0.1617	0.1466
Av. College Wage Premium	3.2578	1.7974	**1.5094**	1.3260	1.2288	1.1860
Theil Index Net Wage Income	0.4845	0.3800	**0.3539**	0.3296	0.3088	0.2879
Aggr. Cons	0.3675	0.4223	**0.4259**	0.4043	0.3705	0.3252
Aver. Cons Non-College	0.2875	0.3574	**0.3809**	0.3783	0.3562	0.3166
Aver. Cons Dropouts	0.3625	0.3860	**0.3856**	0.3645	0.3305	0.2882
Aver. Cons College	0.8474	0.6230	**0.5562**	0.4839	0.4224	0.3621
Theil Index Consumption	0.1868	0.1188	**0.1018**	0.0895	0.0808	0.0752

Table 17: Results Non-Tertiary Investments Experiment (SOE)

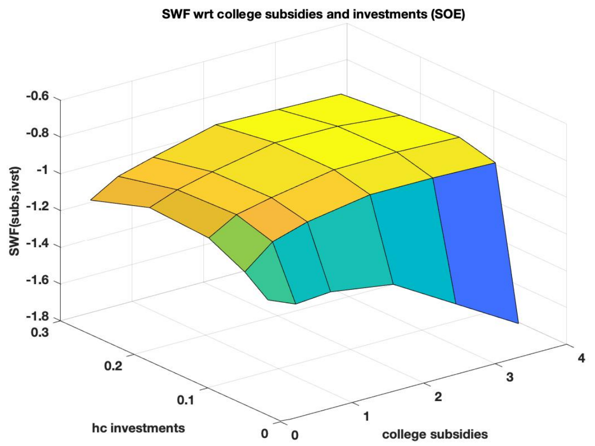
Note: * benchmark model. Bold letters policy with highest welfare. Beside factor prices w_q, average wages ("Av-Wage") take the productivity types γ of households into account.

6.3 Tertiary and Non-Tertiary Education Measures

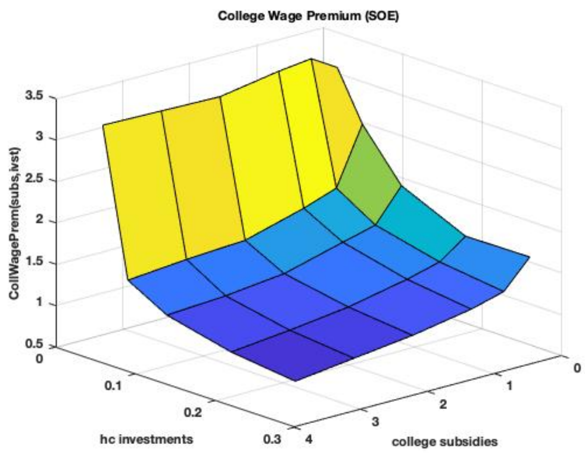
The best policy mix calls for non-tertiary investments of the government of $\bar{i}/w_c = 0.2013$ and college subsidies of $\theta = 175\%$. Thus, optimal primary and secondary investments of the

[65]Maximum college attendance is given by $1 - 0.5206 = 0.4794$.

government are higher when college subsidies exceed the benchmark value. Further, in Figure 21, the interdependence of the two policy instruments becomes clear once again.



(a) Social Welfare



(b) College Wage Premium

Figure 21: Results Bivariate Experiment (SOE)

For the representation of (b) the axes had to be sorted in descending order. Please note that due to convergence issues and time restrictions with respect to high performance computing resources, for three edge cases displayed in these figures, we had to approximate the results by extrapolation. However, given these points are not in the neighborhood of the best policy mix, that does not change the results (for the sake of completeness, these edge cases are: min. investments / min. subsidies, min. investments / max subsidies, max. investments / max subsidies).

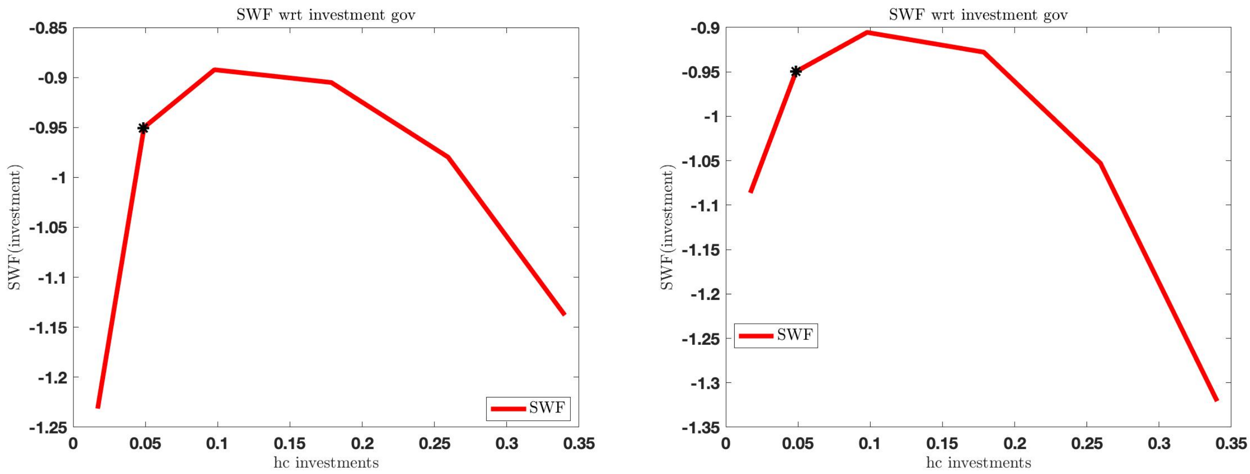
Given minimum primary and secondary education expenditures by the government, welfare decreases (almost) monotonically in college subsidies. Human capital is too small for college subsidies to make a significant difference in the fraction of college graduates. Hence, there is no reduction in the college wage premium and, the caused tax increase due to the college subsidies even leads to a redistribution from rather poor to rather rich households, which decreases welfare.[66]

The dependence works in both directions. The optimal investment level in non-tertiary education by the government, given college subsidies of either 38.8% or 87.50%, is given by $\bar{i}/w_c = 0.1051$, which is below the level of the best policy mix ($\bar{i}/w_c = 0.2013, \theta = 175\%$).

[66]See Figure 25 in Appendix E for more results on college enrollment, college graduation and the labor tax rate.

6.4 An Outlook on General Equilibrium

In contrast to the small open economy, in general equilibrium changes in supply and demand on the capital market will lead to adjustments of the interest rate.[67] Changes in the interest rate affect, among other things, the return on assets, which is another component that has an impact on the distribution of overall income. We performed the univariate experiments under general equilibrium in the same fashion as for the small open economy. The results of the two experiments can be found in Tables 24 and 25 as well as in Figure 26 in Appendix E.



(a) Small Open Economy (b) General Equilibrium

Figure 22: Comparison Results Non-Tertiary Investments

However, compared to the small open economy, the results change quantitatively only marginally, while the qualitative results remain unchanged. As an example, Figure 22 shows the results of the univariate non-tertiary investment experiments for both small open economy and general equilibrium.

This suggests that the endogenisation of wages is the more influential effect, compared to endogenous changes on the capital market, for the underlying research question. The incentive to reach for higher education in our model is to achieve a higher wage. By switching from

[67] See Appendix C.7 for a detailed description.

partial equilibrium to the small open economy setup, both policy measures had a direct impact on this incentive. In addition, these effects were very different for the two groups: an increase in high-skilled labor reduced college wages and increased wages for low-skilled labor. The different qualification groups have different asset holdings, but changes in the interest rate will most likely be less influential for educational decisions than the endogenization of the premium for higher education.

7 Conclusion

We have extended the model of Krueger and Ludwig by (i) linking the prospect of successful college completion to human capital, (ii) introducing taste shocks in order to deal with potential issues caused by the discrete college choice and (iii) incorporating the human capital process during primary and secondary education. In addition, we detected inaccuracies in the solution method used in Krueger and Ludwig (2013), caused by a miss-specification of the trade-off between consumption, savings and investments, which has led to an underestimation of inter-vivos transfers and human capital investment choices.[68] We developed a valid alternative. This results in a large-scale OLG model, that accounts for the core mechanisms of the human capital literature and allows for endogenous responses within the whole human capital process of primary, secondary and tertiary education, to changes in college subsidies and non-tertiary education investments by the government. By embedding this setup into a large-scale OLG environment, we were able to compare the different paths of impact of the policy measures in a realistic framework.

In partial equilibrium, non-tertiary education investments and college subsidies deviate regarding their distributional consequences. Both measures increase average human capital, which leads to higher aggregate production and consumption. However, while primary and secondary education expenses of the government increase the human capital level for children from all household types, college subsidies do not increase the human capital investments from education and income-poor parents, which leads to an increase of inequality. When wages respond to shifts in the labor market, both policy measures increase equality. This is accomplished by an endogenous decrease in the college wage premium, which lowers the return on parental human capital investments. The key difference between the two measures is, that while both policy instruments result in a more equal distribution of human capital, non-tertiary investments of the government compensate for the decreased human capital expenditures of parents and shift

[68]The latter only applies for this thesis, as Krueger and Ludwig (2013) do model inter-vivos transfers, but no human capital investment choices.

average human capital to a higher level.[69]

The bivariate experiments underline the interdependence of the two policy measures. Benefits of college subsidies can only be claimed if young adults have the skills to successfully complete college. Early human capital investments remain unused, if college education can only be afforded by a small fraction in the population. In all experiments we have performed, the best policy mix calls for an increase in primary, secondary and tertiary education investments by the government, financed by higher labor taxes relative to the current status quo.

Outlook on Further Potential Work

In a first step, the calibration could be expanded in order to relate parental education investments more closely to income and expenditures.[70] This can be accomplished by using the investment multiplier in the human capital production function as additional calibration parameter (see equation (30)). In addition, college attendance could also be tied to human capital, standing in for access restrictions of colleges. That could improve the subsidy experiment, by limiting enrollment for the sole sake of receiving negative tuition fees. In addition, it would further wash-out potential kinks in value functions and is rather straightforward to implement.

Financial constraints occur more often when families are young. In addition, primary education is extremely important for all consecutive stages of human capital development. Thus, another very interesting policy experiment would be to let the government not only choose the level of non-tertiary education, but also to allow for shifts between primary to secondary education. Given we used $\zeta_{e,t}^{g}$ in equations (34) and (35) in order to match primary relative to secondary education expenditures by the government, this experiment is easy to implement.

Investments are of purely monetary nature in the developed model. In addition, governmental and parental investments are considered to be perfect substitutes. This allows for situations, in which parental investments in primary and secondary education are fully crowded out by the

[69]This result is succinctly summarized by the following quote: "[...] an equity-efficiency trade-off exists for late investments, but not for early investments" (see Cunha and Heckman (2010), page 2).

[70]See, e.g., Kaushal, Magnuson and Waldfogel (2011).

government. That could be changed by introducing complementarity between governmental and parental investments. Apparently, this is also connected to the literature on time investments and a more complex extension would be to include that into the model. In the current version, higher efficiency of high-educated parents in shaping the human capital of their children is accounted for by a better access to financial resources. Distinguishing between time and money investments,[71] among other things, would add opportunity costs to this process. Whether or not this extension has a large impact on the results could be an interesting topic in itself, because we still know little about how sensitive these type of models react to the choice of the human capital production function.[72]

[71] See, e.g., Abbott (2019).

[72] Thanks to Lance Lochner for pointing that out to us.

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A Appendix to Chapter 2

This section contains supplementary material for Chapter 2.

A.1 Our Model in Light of the Human Capital Literature

Cunha and Heckman (2007) and (2010) (CH hereafter) provide a theoretical framework that summarizes, organizes and interprets a variety of empirical facts about the development of human capital. We want to pick up these facts and critically analyze if our model can reflect them by linking these facts to the mechanisms of human capital development. As mentioned above, these key mechanisms are dynamic complementarity

$$\frac{\partial^2 f_j(h_j, I_j)}{\partial h_j \partial I_j} > 0,$$

as well as self-productivity

$$\frac{\partial f_j(h_j, I_j)}{\partial h_j} > 0,$$

both of which are taken into account by our model. In the periods of primary and secondary education, we use a CES production function, such as Cunha and Heckman suggested and as was estimated in the extensive work of Cunha, Heckman and Schennach (2010) (CHS hereafter). Dynamic complementarity and self-productivity in the context of college attendance and productivity types has been described in Section 2.2.3. While dynamic complementarity comes from the positive impact of human capital on the expected wage spread, self-productivity is reflected by higher skills leading to a higher (expected) qualification, which is expressed in college completion and productivity type.

The first empirical fact CH highlight is that *differences in people's abilities form very early in life*, which is well documented in Cunha, Heckman, Lochner and Masterov (2006). They display the human capital development of children from different income percentiles, showing, on the one hand, that most of the divergence in skills of children is already existent at the age of eight - after that it develops almost parallel.[73] On the other hand, they point to a correlation between human capital development and income of parents. The interplay of dynamic complementarity and borrowing constraints is used to explain this empirical fact: if early investments fail to appear because of financial restrictions of parents, and at the same time, due to dynamic complementarity and (the lack of) self-productivity it is very difficult to compensate for this in later stages of life, this is exactly the pattern that arises. In addition, this is closely related to the second empirical fact: *the effects of credit constraints are age dependent*. In this context, Caucutt and Lochner (2017) (18) investigate the importance of borrowing constraints of families at early stages. They find that a $\$10,000$ increase in discounted annual income of parents when children are at the age of zero to eleven reduces high school drop out rates by 2.5 percentage points, while it increases college attendance by 4.6 percentage points. Both dynamic complementarity and borrowing constraints are present in our model and in the resulting life-cycle profiles this first empirical fact is represented (see Figure 10(a)).

The third fact that CH address is *deviating returns at different ages on investments targeted toward disadvantaged children*. On the one hand, this is pretty close to the definition of self-productivity. Assuming that early investments during primary education of disadvantaged children are very small, this would result in relatively low skills at the beginning of secondary education. In view of this, self-productivity implies a low return on investments within this period targeted at disadvantaged adolescents. Of course, this is also linked to dynamic complementarity, as it makes substituting missing investments from early periods much harder and thereby weakens the effect of investments in secondary education, given low investments in primary education. Thus, if a government had to choose how to distribute investments among primary and secondary education, this interplay is crucial, which is also emphasized by the fourth fact CH examine: *early investments have to be followed up by late investments*. Again, dynamic complementarity is the main mechanism behind this fact. Early investments are nec-

[73] They use PIAT Math scores as a proxy for human capital.

essary for later investments to be effective. However, if late investments fail to appear then the early sown fruits are not harvested. Given all these mechanisms are incorporated, our model is able to address facts three and four as well. In Section 3.4 we will show, how they are reflected in the policy functions of the model.

Caucutt and Lochner (2017) refine their above statement as follows: "*An important consequence of dynamic complementarity is that studying the impacts of a policy change exclusively in that period can be misleading. For example, a large literature considers the effects of college-age policies on schooling and labor market outcomes holding early investment and adolescent achievement levels fixed. [...] Our quantitative analysis highlights that ignoring these earlier investment responses can lead researchers to under-estimate the total wage impact of college-age investment subsidies by almost 60%.*" (see (18), page 4). This result essentially goes through all of the facts just summarized. Subsidizing college education increases the incentive for parents to invest in their children in early stages, and they will do so if they are financially capable. Through this endogenous response of parents, the average human capital of young adults in college will be higher, which in turn reinforces the effectiveness of college subsidization. This result underlines the importance of considering the whole human capital process in order to analyze the various policy instruments and their effectiveness. This is where this theses comes in, with both college subsidies and non-tertiary investments as instruments available to the government, and a complete endogenous human capital process within the family.

A.1.1 Adjacent Research

In our model we summarize abilities in scalar h. In this section, we will critically compare this to the alternative of modeling human capital as a vector of different types of skills and work out that it is the better choice for us to work with a skill scalar, given the research question we want to answer.

Two of the six empirical facts on human capital development analyzed in CH are still pending. The first one is, that there *are critical and sensitive periods in human capital production*. Critical periods arise when abilities can only be built in certain periods of life. An example is the ability to speak a foreign language without accent, which becomes almost impossible after a certain age. Examples for skills exhibiting sensitive periods are coordination and IQ, whose further development become increasingly difficult after the age of ten. Non-cognitive characteristics such as patience and emotional stability are generally considered more malleable during the age of 12 to 16. Technically, both of these properties would rather need a skill vector as opposed to the human capital scalar. An ability featuring a critical period would exhibit perfect complementary for this particular type of skill in all other periods except for the critical one, while other types of abilities are malleable in the whole development process.[74] In the same spirit, an ability type featuring a sensitive period would exhibit a very low elasticity in all other than the sensitive period.

The sixth and last empirical fact mentioned in CH is closely connected: *noncognitive skills foster cognitive skills and are an important product of successful families and successful interventions in disadvantaged families*. Here, too, a vector of at least two skill types would be necessary, if a distinction is important for the question under consideration. CHS estimate human capital functions for both one-skill and cognitive and non-cognitive skill models. Based on their estimates, they develop a theoretical model to answer the following question: If a government were to maximize aggregate schooling, in which children should it invest. They arrive at the conclusion, that a model with an underlying two-type human capital process would conclude that these investments should be distributed to young disadvantaged children, while a model with an underlying one-type technology would shift resources more to relatively advantaged children.[75]

The important difference to our model is the question at hand. CHS analyze which children

[74]Taking our function from above, i.e. $h_{ja} = m(h_0, I_0, \ldots, I_3)$, age 0 would be a critical period for h_{ja} if $\frac{\partial m(h_0, I_0, \ldots, I_3)}{\partial I_j} = 0$ for $j = 1, 2, 3$, but $\frac{\partial m(h_0, I_0, \ldots, I_3)}{\partial I_0} > 0$.

[75]As mentioned above, that comes from the fact that returns on investments are higher, given higher investments in the past and therefore a higher human capital stock in the period the investment is made.

should receive education subsidies, when the government wants to maximize human capital. Thus, in their model government measures are contingent on the characteristics of children. In contrast, our question is about the aggregate interaction of primary, secondary and tertiary education in a quantitative model, in which policy instruments are not made dependent on the characteristics of children. In addition, the government maximizes welfare instead of schooling, implying it is also about distributional aspects. Summing up, the subdivision into two skill types would be a further complication of our model, analytically and computationally, but would not make a contribution to the underlying question. The key mechanisms we need to adopt are dependencies and differences between the periods when human capital is shaped, which is captured by the interplay of dynamic complementarity and self-productivity.[76]

[76]We would especially like to thank Lance Lochner for the very helpful exchange on this topic.

B Appendix to Chapter 3

This chapter contains supplementary content for Chapter 3.

We solve the household problem in the different stages of the life-cycle of households by either the endogenous grid method or a hybrid method incorporating the exogenous grid method. Therefore, in both cases it is useful to express households' value functions in terms of cash-on-hand. We start this chapter we briefly introducing the cash-on-hand definition and highlight its interplay with labor given our tax code. In Section B.1.3 we apply this definition to the recursive household problem described in Chapter 3.1. The solution methods applied for the different stages are then displayed in Sections B.2 and B.3 respectively.

B.1 Reformulation of the Household Problem

B.1.1 Cash-On-Hand Definition and Asset Regions

In order to shorten notation we denote the net-capital return by:

$$R_t^n = 1 + (1 - \tau_{k,t})r_t. \tag{43}$$

In addition, we define age specific gross wages by

$$w_{t,j,q} = w_{t,q}\epsilon_{j,q}. \tag{44}$$

Further, we define cash-on-hand as available financial resources under maximum labor supply $\ell = 1$:

$$\begin{aligned} X' &= R_t^n A' + (1 - \tau_{ss}) w_{t+1,q} \gamma \eta' \epsilon_{j+1,q} \\ &= R_t^n A' + (1 - \tau_{ss}) w_{t+1,j+1,q} \gamma \eta'. \end{aligned} \tag{45}$$

This has the desirable effect making cash-on-hand tomorrow completely determined by choices from today, in particular, independent of the labor choice in the next period, which helps applying the endogenous grid method by Carroll (2005). In a similar fashion we make use of redefining gross savings as

$$\bar{A}' = A' + i_k^p f + Bf. \tag{46}$$

As this only effects periods parents either invest into human capital of their children or grant vivos transfers, so it is further explained below. Due to the tax free amount within our tax system, i.e. $T_t(Y_t) \max\{0, \tau_{\ell,t}(Y_t - Z_t)\}$, we will have agents paying $T_t(Y_t) = 0$ and agents paying $T_t(Y_t) > 0$. Thus, it is straightforward to compute the labor choice $\bar{l}$ splitting agents into these two groups. Setting $Y_t = (1 - 0.5\tau_{ss})w_{t,j,q}\gamma\eta\ell = Z_t$ we get:

$$\bar{\ell} = \frac{Z_t}{(1 - 0.5\tau_{ss})w_{t,j,q}\gamma\eta}. \tag{47}$$

So there will be asset rich households choosing to work below $\bar{\ell}$ avoiding and asset poor households working $\ell > \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ paying labor taxes. Figure 23 displays these different asset regions graphically. The expenditure side of the budget constraint differs accordingly:

$$\bar{A}' = \begin{cases} X - (1 + \tau_{c,t})C - T(Y_t) - (1 - \tau_{ss})w_{t,j,q}\gamma\eta(1 - \mathbf{1}\xi - \ell) & \text{if } l > \bar{\ell} \\ X - (1 + \tau_c)C - (1 - \tau_{ss})w_{t,j,q}\gamma\eta(1 - \mathbf{1}\xi - \ell) & \text{else.} \end{cases} \tag{48}$$

Plugging in $Y_t = (1 - 0.5\tau_{ss})w_{t,j,q}\gamma\eta\ell$ and rearranging the equation leads us to a more detailed version of (48) (equation (49) is developed in Appendix B.1.2):

$$\bar{A}' = \begin{cases} X - (1+\tau_c)C + \tau_{\ell,t}Z_t - (1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) \\ \qquad\qquad - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi) & \text{if } \ell > \bar{\ell} \\ X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) & \text{else.} \end{cases} \tag{49}$$

But for the sake of simplicity we will use (48) in the remainder of this section. Nevertheless, the following term is important for the first-order-conditions and we refer to it as net-wage:

$$w^n_{t,j,q} = \begin{cases} (1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta & \text{if } \ell > \bar{\ell} \\ (1-\tau_{ss})\, w_{t,j,q}\gamma\eta & \text{else.} \end{cases} \tag{50}$$

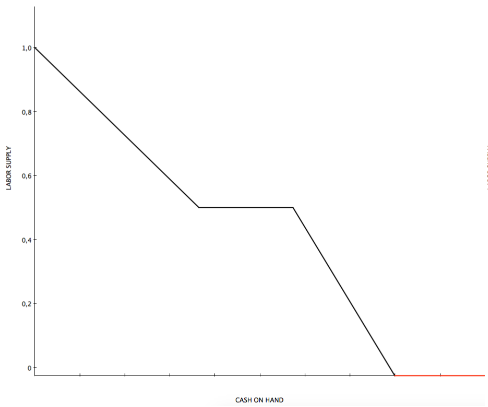
B.1.2 Savings Under New Cash-On-Hand Definition

Here we develop equation (49):

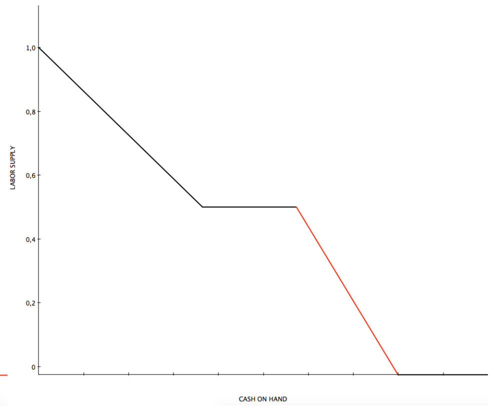
$$\bar{A}' = \begin{cases} X - (1+\tau_c)C + \tau_{\ell,t}Z_t - (1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta(1-\ell) - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta \\ X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell). \end{cases}$$

We start with the usual budget constraint:

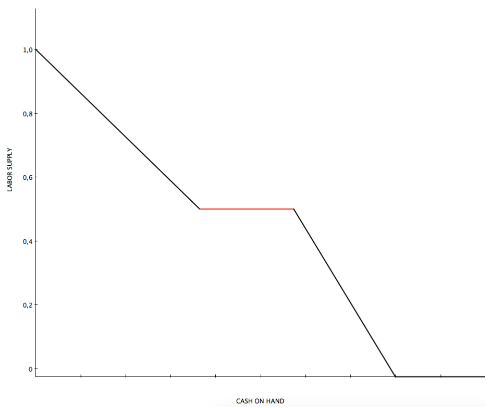
$$R^n_t A + (1-\tau_{ss})w_{t,j,q}\gamma\eta\ell = (1+\tau_c)C + \bar{A}' + T(Y_t).$$



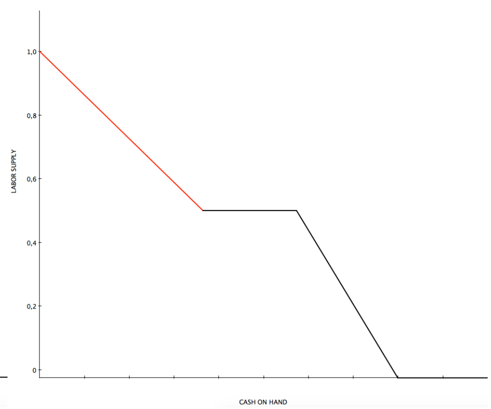
(a) $\ell = 0$ and $\mu_\ell > 0$ ($\ell \geq 0$ binding)



(b) $0 \leq \ell < \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ and $T(Y_t) = 0$



(c) $\ell = \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ and $T(Y_t) = 0$



(d) $\ell > \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ and $T(Y_t) > 0$

Figure 23: Labor as a function of cash-on-hand

Next, we incorporate our cash-on-hand definition $X = R^n_t A + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)$ where $\mathbf{1}\xi$ stands in for time deduction due to college:

$$\begin{aligned} R^n_t A &= (1+\tau_c)C + \bar{A}' + T(Y_t) - (1-\tau_{ss})w_{t,j,q}\gamma\eta\ell \quad | + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi) \\ \Leftrightarrow \quad X &= (1+\tau_c)C + \bar{A}' + T(Y_t) + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) \end{aligned}$$

which leads to:

$$\bar{A}' = \begin{cases} X - (1+\tau_c)C - T(Y_t) - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) & \text{if } \ell > \bar{\ell} \\ X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) & \text{else.} \end{cases}$$

With this notation we describe the budget constraint. But as households are maximizing with respect to leisure $1-\ell^n = 1-\mathbf{1}\xi-\ell$, some more steps are required. Plugging in $T(Y_t) = \tau_{\ell,t}((1-0.5\tau_{ss})w_{t,j,q}\gamma\eta\ell - Z_t)$ we get:

$$\bar{A}' = X - (1+\tau_c)C + \tau_{\ell,t}Z_t \underbrace{-\tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta\ell - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell)}_{ltx}.$$

and as

$$\begin{aligned} ltx &= -\tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta\ell - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) \\ &= -\tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta\ell - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) + \Big(\tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi) \\ &\qquad -\tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)\Big) \\ &= \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi) \\ &= -\left(1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss})\right) w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\ell) - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi) \end{aligned}$$

we get:

$$\bar{A}' = \begin{cases} X - (1+\tau_c)C + \tau_{\ell,t} Z_t - (1 - \tau_{ss} - \tau_{\ell,t}(1 - 0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta(1 - \mathbf{1}\xi - \ell) & \\ \qquad\qquad - \tau_{\ell,t}(1 - 0.5\tau_{ss}) w_{t,j,q}\gamma\eta(1 - \mathbf{1}\xi) & \text{if } \ell > \bar{\ell} \\ X - (1+\tau_c)C - (1 - \tau_{ss}) w_{t,j,q}\gamma\eta(1 - \mathbf{1}\xi - \ell) & \text{else.} \end{cases}$$

B.1.3 Recursive Problem of Households in Terms of Cash-On-Hand

Retirement at age $J, \ldots, j_r$

$$V(j, X, q, \gamma) = \max_{C, X'} \{u(C,1) + \beta V'(j+1, X', q, \gamma)\}$$

subject to

$$X' = R^n_{t+1} A' + p_{t+1,j+1}(q, \gamma)$$
$$\bar{A}' = A' = X - (1+\tau_c)C \geq 0.$$

We solve for optimal consumption building the Lagrange function

$$\mathcal{L}(C, \mu_{A'}) = u(C,1) + \beta V'\left(j+1, \underbrace{R^n_{t+1}(X - (1+\tau_c)C) + p_{t+1,j+1}(q,\gamma)}_{X'(C)}, q, \gamma\right) + \mu_{A'}(X - (1+\tau_c)C),$$

where $\mu_{A'}$ is the multiplier on the non-negativity constraint. First order and envelope conditions are:

$$\text{FOC}_C: \quad \frac{u_C(C,1)}{1+\tau_c} - \beta R^n_{t+1} V'_{X'}(j+1, X', q, \gamma) - \mu_{A'} = 0 \tag{51}$$

$$\text{EVLP}: \quad V_X(j, X, q, \gamma) = \beta R^n_{t+1} V'_{X'}(j+1, X', q, \gamma) + \mu_{A'}. \tag{52}$$

Given households choose $(1+\tau_c)C = X$ in the last period, we compute:

$$\frac{u_C(C,1)}{(1+\tau_c)} = V_X\left(j, X, q, \gamma\right). \tag{53}$$

Applying the endogenous grid method for solving periods $J-1,\ldots,j_r$ is straightforward. Given an exogenous savings grid point A' we receive $X' = R^n_{t+1}A' + p_{t+1,j+1}(q,\gamma)$. Now balance equation (51) to receive optimal consumption of the respective period and compute corresponding $V_X\left(j, X, q, \gamma\right)$ using (53). Next we can compute endogenous cash-on-hand today via $X = A' + (1+\tau_c)C$ and move on to the next period.

Working as parents of adults at $j_r-1,\ldots,j_a+j_f$

The problem reads as

$$V\left(j, X, q, \gamma, \eta\right) = \max_{C,\ell\in[0,1],X'} \left\{u(C, 1-\ell) + \beta E_{\eta'|\eta}\left[V'\left(j+1, X', q, \gamma, \eta'\right)\right]\right\} \tag{54}$$

subject to

$$X' = R^n_{t+1}A' + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\eta'$$
$$A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) - T_t(Y_t) \geq -\Psi(q)\underline{A}_{j,t}$$

as described in equation (48). In addition to the borrowing constraint on A', we also have $\ell \geq 0$ or $1-(1-\ell) \geq 0$. Thus, we solve for consumption and leisure with the Lagrange function

$$\begin{aligned}\mathcal{L}(C, 1-\ell, \mu_{A'}, \mu_\ell) = u(C, 1-\ell) &+ \beta E_{\eta'|\eta}\left[V'\left(j+1, X'(C, 1-\ell), q, \gamma, \eta'\right)\right] \\ &+ \mu_{A'}A'(C, 1-\ell) + \mu_\ell(1-(1-\ell)).\end{aligned}$$

The following first order and envelope conditions can be developed:

$$\text{FOC}_C: \quad \frac{u_C(C,1-\ell)}{1+\tau_c} - \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1,X',q,\gamma,\eta'\right)\right] - \mu_{A'} = 0 \tag{55}$$

$$\text{FOC}_{1-\ell}: \quad u_{1-\ell}(C,1-\ell) - w^n_{t,j,q}\left(\beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1,X',q,\gamma,\eta'\right)\right] + \mu_{A'}\right) - \mu_\ell = 0 \tag{56}$$

$$\text{EVLP}_X: \quad V_X\left(j,X,q,\gamma,\eta\right) = \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1,X',q,\gamma,\eta'\right)\right] + \mu_{A'}, \tag{57}$$

In order to solve for optimal consumption, leisure and savings we graze the asset regions and apply the endogenous grid method, which is described in B.2.

Children become adults at age $j_a + j_f$

Parents' optimization problem in the vivos transfer period is the following:

$$\begin{aligned} V\left(j,X,h_0,h,q,\gamma,\eta\right) = \max_{C,\ell\in[0,1],X',B\geq 0} &\left\{u\left(C,1-\ell\right) + \beta E_{\eta'|\eta}\left[V'\left(j+1,A',q,\gamma,\eta'\right)\right]\right\} \\ + \tilde{\nu} E_{\eta'\in\Pi_n(\eta)} &\left[\max\left\{E_{q\in\{d,c\}|h}\left[V^c\left(j_a,X^c,h_0,h,\neg n,\eta\right)\right], E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V^c\left(j_a,X^c,h_0,n,\eta\right)\right]\right\}\right], \end{aligned}$$

subject to[77]

$$\begin{aligned} X' &= R^n_{t+1}A' + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\eta' \\ A' &= X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) - T(Y_t) - Bf \geq -\Psi(q)\underline{A}_{j,t}. \end{aligned}$$

Given our timing assumption inter-vivos transfers B generate gross revenue of B, implying that children's assets are described by $A^c = B/R^k_t$ and $R^k_t A^c = B$. This leads to either $X^c = B + (1-\tau_{ss})w_{t,j_a,n}\tilde{\eta}(1-\Psi(q)\xi(h_{j_a}))$ in case the kid attends university or $X^c = B + (1-\tau_{ss})w_{t,j_a,n}\tilde{\eta}\tilde{\gamma}$ in case the kid decides not to attend university and joins workforce immediately. Due to our timing assumption, the actual realization of the kids η-shock and everything that follows does not translate into utility of the parent. It is just the expectation about it and the parent receives utility directly after transferring the money to the kid. Compared to the previous problem h_0

[77]Note that due to our fertility assumption one parent has f children and vivos are given to each child, it has to be adjusted in the parent's budget constraint.

and h are now part of the state space of V.

In addition to borrowing and time constraint we have $B \geq 0$ which determine kid's cash-on-hand X^c. The Lagrange function reads as:

$$\begin{aligned}\mathcal{L}(C, 1-\ell, B, \mu_{A'}, \mu_\ell, \mu_B) = u(C, 1-\ell) + \beta E_{\eta'|\eta}\left[V'\left(j+1, X'(C, 1-\ell), q, \gamma, \eta'\right)\right] \\ + \tilde{\nu} E_{\eta' \in \Pi_n(\eta)}\left[\max\left\{E_{q\in\{d,c\}|h}\left[V^c\left(j_a, X^c, h_0, h, \neg n, \eta\right)\right], E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V^c\left(j_a, X^c, h_0, n, \eta\right)\right]\right\}\right] \\ + \mu_{A'} A'(C, 1-\ell, B) + \mu_\ell(1-(1-\ell)) + \mu_B B.\end{aligned}$$

The first-order-conditions are:

$$\text{FOC}_C: \quad \frac{u_C(C, 1-\ell)}{1+\tau_c} - \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] - \mu_{A'} = 0 \tag{58}$$

$$\text{FOC}_{1-\ell}: \quad u_{1-\ell}(C, 1-\ell) - w^n_{t,j,q}\left(\beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] + \mu_{A'}\right) - \mu_\ell = 0 \tag{59}$$

$$\text{FOC}_B: \quad \tilde{\nu}\sum^{\eta} \pi_n(\eta) \frac{\partial\left(\max\left\{E_{\eta'|\eta}\left[V^c\left(\lambda^c_q(\cdot)=1\right)\right]; E_{\eta'|\eta}\left[V^c\left(\lambda^c_q(\cdot)=0\right)\right]\right\}\right)}{\partial B} - \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] - \mu_{A'} - \mu_B = 0 \tag{60}$$

$$\text{EVLP}_X: \quad V_X\left(j, X, q, \gamma, \eta, h^c_0, h^c_{j_a}\right) = \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] + \mu_{A'}, \tag{61}$$

where $w^n_{t,j,q}$ is described in (50). We solve for optimal vivos transfers by a hybrid method incorporating the exogenous grid method and describe that in Chapter B.3.2. However, there is another issue at this stage, namely we do not know $V^c(\cdot)$ (nor its derivative). But what we do know is that (given prices) value functions have to be the same across generations at the same stage in the lifecycle. Thus, whatever we assume for the value function of the kid at the age of j_a should coincide with the value function of parents at that age j_f years ago. This leaves us with a fixed point problem we will solve as follows.

Fix Point Problem

(i) Guess kids' value functions at age j_a as a function of cash-on-hand, i.e. guess: $E_{\eta' \in \Pi_n(\eta)} \left[\max \left\{E_{q \in \{d,c\}|h} \left[V\left(j_a, X, h_0, h, \neg n, \eta\right)\right], E_{\gamma \in \{\gamma^l, \gamma^h\}|h} \left[V\left(j_a, X, h_0, n, \eta\right)\right]\right\}\right].$

(ii) Given this guess solve for optimal vivos transfers as described in Chapter B.3.2

(iii) Going backwards in time solve all periods from $j_f + j_a - 1$ until arriving at parents age j_a

(iv) Compare the resulting value function with your current guess from step (i)

- **If** it is sufficiently close: STOP
- **Else** update guess from step (i) with the computed value function and start over

Working as parents of children at age $j_f + j_a - 1, \ldots, j_f$

The problem reads as

$$V\left(j, X, h_0, h, q, \gamma, \eta\right) = \max_{C, \ell \in [0,1], X', i_k^p \geq 0} \left\{u\left(\frac{C}{1+\zeta f}, 1-\ell\right) + \beta E_{\eta'|\eta}\left[V'\left(j+1, X', h_0, h', q, \gamma, \eta'\right)\right]\right\},$$

subject to[78]

$$\begin{aligned}
X' &= R_{t+1}^n A' + (1-\tau_{ss}) w_{t+1,j+1,q} \gamma \eta' \\
A' &= X - (1+\tau_c) C - (1-\tau_{ss}) w_{t,j,q} \gamma \eta (1-\ell) - T(Y_t) - i_k^p f \geq -\Psi(q) \underline{A}_{j,t} \\
h' &= f\left(h, i_k^p, i_k^g\right)
\end{aligned}$$

[78] For the same reason as for vivos transfers, in the parent's budget constraint human capital investments have to be scaled by the fertility rate.

further described in equation (30). The Lagrange function reads as:

$$\mathcal{L}(C, 1-\ell, i_k^p, \mu_{A'}, \mu_\ell, \mu_i) = u\left(\frac{C}{1+\zeta f}, 1-\ell\right) + \beta E_{\eta'|\eta}\left[V'\left(j+1, X', h_0, h', q, \gamma, \eta'\right)\right]$$
$$+ \mu_{A'} A'(C, 1-\ell, i_k^p) + \mu_\ell(1-(1-\ell)) + \mu_i i_k^p.$$

First-order-conditions and envelope conditions are:

$$\text{FOC}_C: \quad \frac{u_C(\frac{C}{1+\zeta f}, 1-\ell)}{(1+\tau_c)(1+\zeta f)} - \beta R_{t+1}^n E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] - \mu_{A'} = 0 \tag{62}$$

$$\text{FOC}_{1-\ell}: \quad u_{1-\ell}\left(\frac{C}{1+\zeta f}, 1-\ell\right) - w_{t,j,q}^n\left(\beta R_{t+1}^n E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] + \mu_{A'}\right) - \mu_\ell = 0 \tag{63}$$

$$\text{FOC}_{i_k^p}: \quad -\beta R_{t+1}^n E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] + \beta\frac{\partial h'}{\partial i_k^p} E_{\eta'|\eta}\left[V'_{h'}(\mathbf{Z}')\right] - \mu_{A'} + \mu_i = 0 \tag{64}$$

$$\text{EVLP}_X: \quad V_X(\mathbf{Z}) = \beta R_{t+1}^n E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right] + \mu_{A'} \tag{65}$$

$$\text{EVLP}_h: \quad V_h(\mathbf{Z}) = \beta\frac{\partial h'}{\partial h} E_{\eta'|\eta}\left[V'_{h'}(\mathbf{Z}')\right], \tag{66}$$

where we summarized the state space in $\mathbf{Z} = \{j, X, q, \gamma, \eta, h_0, h\}$. Kids are part of the household which is why per capita consumption is given by $\frac{C}{1+\zeta f}$. Human capital of the child today is denoted by h and it is fostered by investments leading to human capital tomorrow (h') following human capital production (30). We will solve for optimal investments in a similar fashion as for vivos transfers in the previous period. Nevertheless, it is more complicated, because in the vivos transfer period children only contributed to the value function via an additive term leaving first-order-conditions for consumption and leisure unaffected (see equations (58) and (59)). Regarding the current situation, investments into kids' human capital do effect $V'_{X'}(\mathbf{Z}')$ in (62) and (63), which requires multidimensional interpolation and we describe the solution method for the investment periods in Chapter B.3.3.

Preparing for parenthood at $j_f - 1$

$$V\left(j, X, h_0, q, \gamma, \eta\right) = \max_{C, \ell \in [0,1], X'} \left\{ u(C, 1-\ell) + \beta E_{\eta'|\eta, h_0'|h_0} \left[V'\left(j+1, X', q, \gamma, \eta', h_0', h'\right) \right] \right\}$$

subject to

$$\begin{aligned} X' &= R^n_{t+1} A' + (1-\tau_{ss}) w_{t+1, j+1, q} \gamma \eta' \\ A' &= X - (1+\tau_c) C - (1-\tau_{ss}) w_{t,j,q} \gamma \eta (1-\ell) - T(Y_t) \geq -\Psi(q) \underline{A}_{j,t}. \end{aligned}$$

Parents' h_0 is mapped into kids' innate human capital h_0' following (8) and for period j_f acquired and innate and acquired human capital of the kid are identical (i.e. $h_0' = h'$ in V'). The first-order-conditions read as:

$$\begin{aligned} &\mathrm{FOC}_C: && \frac{u_C(C, 1-\ell)}{1+\tau_c} - \beta R^n_{t+1} E_{\eta'|\eta, h_0'|h_0} \left[V'_{X'}\left(j+1, X', q, \gamma, \eta', h_0', h'\right) \right] - \mu_{A'} = 0 \\ &\mathrm{FOC}_{1-\ell}: && u_{1-\ell}(C, 1-\ell) - w^n_{t,j,q} \left(\beta R^n_{t+1} E_{\eta'|\eta, h_0'|h_0} \left[V'_{X'}\left(j+1, X', q, \gamma, \eta', h_0', h'\right) \right] + \mu_{A'} \right) - \mu_\ell = 0 \\ &\mathrm{EVLP}_X: && V_X\left(j, X, h_0, q, \gamma, \eta\right) = \beta R^n_{t+1} E_{\eta'|\eta, h_0'|h_0} \left[V'_{X'}\left(j+1, X', q, \gamma, \eta', h_0', h'\right) \right] + \mu_{A'}. \end{aligned}$$

The only difference at this stage is that parents have to take into account the upcoming drawing of h_0' given h_0. This makes the computation of $E_{\eta'|\eta, h_0'|h_0}\left[V'_{X'}\left(j+1, X', q, \gamma, \eta', h_0', h'\right)\right]$ a bit more complicated, but does not change the way we arrive at optimal decisions as described for periods *working as parents of adults*.

Working without children at $j_f - 2, \ldots, j_a + 1$

$$V\left(j, X, h_0, q, \gamma, \eta\right) = \max_{C, \ell \in [0,1], X'} \left\{ u(C, 1-\ell) + \beta E_{\eta'|\eta} \left[V'\left(j+1, X', h_0, q, \gamma, \eta'\right) \right] \right\}$$

subject to

$$X' = R^n_{t+1}A' + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\eta'$$
$$A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) - T(Y_t) \geq -\Psi(q)\underline{A}_{j,t}.$$

We are basically back to the problem at ages $j_r - 1, \ldots, j_a + j_f$, only with h_0 as additional state variable. First order and envelope conditions are:

$$\text{FOC}_C: \quad \frac{u_C(C, 1-\ell)}{1+\tau_c} - \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', h_0, q, \gamma, \eta'\right)\right] - \mu_{A'} = 0$$
$$\text{FOC}_{1-\ell}: \quad u_{1-\ell}(C, 1-\ell) - w^n_{t,j,q}\left(\beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', h_0, q, \gamma, \eta'\right)\right] + \mu_{A'}\right) - \mu_\ell = 0$$
$$\text{EVLP}_X: \quad V_X\left(j, X, h_0, q, \gamma, \eta\right) = \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', h_0, q, \gamma, \eta'\right)\right] + \mu_{A'}.$$

College period at $j_a + 1$ until j_a

Non-college agents draw their productivity type from $\pi(\gamma|n)$ which in turn determines cash-on-hand $X = B + (1-\tau_{ss})w_{t,j,n}\gamma\eta$. They face the exact same problem as agents in the stage *working without children*, i.e.

$$V\left(j, X, h_0, n, \gamma, \eta\right) = \max_{C,\ell\in[0,1],X'}\left\{u(C, 1-\ell) + \beta E_{\eta'|\eta}\left[V'\left(j+1, X', h_0, n, \gamma, \eta'\right)\right]\right\}$$

subject to

$$X' = R^n_{t+1}A' + (1-\tau_{ss})w_{t+1,j+1,n}\gamma\eta'$$
$$A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) - T(Y_t) \geq -\Psi(q)\underline{A}_{j,t}$$

and identical first-order-conditions. Students, on the other hand, start with $X = B + w_{t,n}\epsilon_{j,q}\tilde{\eta}(1-\tau_{ss})(1-\Psi(q)\xi(h_{j_a}))$ and solve:

$$V\left(j_a, X, h_0, h_{j_a}, q, \eta\right) = \max_{\substack{C,X' \\ \ell\in[0,1-\Psi(q)\xi(h_{j_a})]}}\left\{u(C, 1-\Psi(q)\xi(h_{j_a}) - l) + \beta E_{\gamma|h,\eta'\in\Pi_q(\eta')}\left[V'\left(j+1, X', h_0, q, \gamma, \eta'\right)\right]\right\},$$

subject to[79]

$$X' = R^n_{t+1}A' + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\eta'$$
$$A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,n}\eta(1-\Psi(q)\xi(h_{j_a}) - l) - T(Y_t) - \Psi(q)(1-\theta_t-\theta_{pr})\kappa w_{t,c} \geq -\Psi(q)\underline{A}_{j,t}.$$

First order and envelope conditions are:

$$\text{FOC}_C: \frac{u_C(C, 1-\Psi(q)\xi(h_{j_a}) - \ell)}{1+\tau_c} - \beta R^n_{t+1} E_{\gamma|h,\eta'\in\Pi_q(\eta')}\left[V'_{X'}\left(j+1, X', h_0, q, \gamma, \eta'\right)\right] - \mu_{A'} = 0$$
$$\text{FOC}_{1-\ell}: u_{1-\ell}(C, 1-\Psi(q)\xi(h_{j_a}) - \ell) - w^n_{t,j,n}\left(\beta R^n_{t+1} E_{\gamma|h,\eta'\in\Pi_q(\eta')}\left[V'_{X'}\left(j+1, X', h_0, q, \gamma, \eta'\right)\right] + \mu_{A'}\right) - \mu_\ell = 0$$
$$\text{EVLP}_X: V_X\left(j_a, X, h_0, h_{j_a}, q, \eta\right) = \beta R^n_{t+1} E_{\gamma|h,\eta'\in\Pi_q(\eta')}\left[V'_{X'}\left(j+1, X', h_0, q, \gamma, \eta'\right)\right] + \mu_{A'},$$

where students have to form expectation over both η and γ.

College decision j_a

The college indicator function is described by

$$\lambda(j_a, X, h_0, h, \eta) = \begin{cases} 1 & \text{if } E_{q\in\{d,c\}|h}\left[V\left(j_a, X, h_0, h, \neg n, \eta\right)\right] > E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V\left(j_a, X, h_0, n, \eta\right)\right] \\ 0 & \text{otherwise.} \end{cases}$$

The expected value functions at age j_a write as:

$$E_{q\in\{d,c\}|h}\left[V\left(j_a, X, h_0, h, \neg n, \eta\right)\right] = \pi_c(h)\left[V\left(j_a, X, h_0, h, c, \eta\right)\right] + \dots$$
$$\dots + (1-\pi_c(h))\left[V\left(j_a, X, h_0, h, d, \eta\right)\right],$$

[79] In terms of computation there is a special case which is covered in Section B.7. Summing up, there can be a minimum asset level required for certain h_{j_a} agents to be able to afford college in the first place. Low h_{j_a} means high time deduction $\xi(h_{j_a})$ and therefore low possible wage income. That can imply that even in case a student works all time she has left, it is still insufficient to cover tuition fees without violating the borrowing constraint.

and

$$E_{\gamma\in\{\gamma^l,\gamma^h\}|h}\left[V\left(j_a,X,h_0,n,\eta\right)\right]=\pi_\gamma(h)\left[V\left(j_a,X,h_0,h,n,\gamma^h,\eta\right)\right]+\dots$$
$$\dots+(1-\pi_\gamma(h))\left[V\left(j_a,X,h_0,h,n,\gamma^l,\eta\right)\right].$$

Cash-on-hand is given by $X = B + (1-\tau_{ss})w_{t,j,n}\gamma\eta$ in case the agent decides not to attend university and $X = B + (1-\tau_{ss})w_{t,j,n}\eta(1-\Psi(q)\xi(h_{j_a}))$ otherwise. Uncertainty of attending college is due to the completion shock given by (32), whereas the non-college option implies being subject to the productivity $\gamma-$shock described in (33) already at age j_a. Recall that we defined $\Psi(q)$ in order to shorten notation as follows:

$$\Psi(q)=\begin{cases}1 & \text{if } q=c\\ \phi & \text{if } q=d\\ 0 & \text{if } q=n.\end{cases}$$

B.2 Solving via Endogenous Grid Method

We demonstrate how we apply the Endogenous Grid Method by solving the household problem for parents of adults described in equation (54). First order and envelope conditions were given by:

$$\text{FOC}_C:\quad \frac{u_C(C,1-\ell)}{1+\tau_c}-\beta R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\left(j+1,X',q,\gamma,\eta'\right)\right]-\mu_{A'}=0 \tag{67}$$

$$\text{FOC}_{1-\ell}:\quad u_{1-\ell}(C,1-\ell)-w^n_{t,j,q}\left(\beta R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\left(j+1,X',q,\gamma,\eta'\right)\right]+\mu_{A'}\right)-\mu_\ell=0 \tag{68}$$

$$\text{EVLP}_X:\quad V_X\left(j,X,q,\gamma,\eta\right)=\beta R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\left(j+1,X',q,\gamma,\eta'\right)\right]+\mu_{A'}, \tag{69}$$

From equations (67) and (69) we get:

$$V_X\left(j, X, q, \gamma, \eta\right) = \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', q, \gamma, \eta'\right)\right] = \frac{u_C(C, 1-\ell)}{1+\tau_c}. \tag{70}$$

In order to solve for optimal consumption and leisure we now graze the asset regions described in figure (23) as follows:

(i) Check whether we are in the region where agents choose not to work (Figure 23.a): Labor $\ell = 0$ implies $w^n_{t,j,q}\gamma\eta = (1-\tau_{ss})\,w_{t,j,q}\gamma\eta$ (see (50)). Use $\ell = 0$ in (67) to obtain C. Next compute μ_ℓ via (68).

   (a) **If** $\mu_\ell > 0$ (even for maximum leisure $1-\ell = 1$ utility gain of $(1-\ell)\uparrow$ higher than corresponding utility drop of $X'\downarrow$, implying agent would like to choose $\ell < 0$) so we have $\ell^* = 0$. Next, rearrange (48) and cash-on-hand definition (45) to receive endogenous cash-on-hand as well as assets today:

$$\begin{aligned} X &= \bar{A}' + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) + (1+\tau_c)C \\ X &= R^n_t A + (1-\tau_{ss})w_{t,j,q}\gamma\eta \\ \Rightarrow \qquad A &= \frac{X - (1-\tau_{ss})w_{t,j,q}\gamma\eta}{R^k_t} \end{aligned}$$

   and compute V_X following (70).

   (b) **Else** proceed to next step.

(ii) Check whether we are in the region where agents choose $\ell \in \left(0, \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}\right]$ (Figure 23.b):
Given $w^n_{t,j,q} = (1-\tau_{ss})\,w_{t,j,q}\gamma\eta$, would the solution for optimal leisure imply $\ell^* \in \left(0, \bar{\ell}\right]$?
Two unknowns in equations (67) and (68), solving them gives us C and $1-\ell$.

(a) **If** $\ell < \bar{\ell}$ then

$$X = \bar{A}' + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) + (1+\tau_c)C$$
$$A = \frac{X - (1-\tau_{ss})w_{t,j,q}\gamma\eta}{R_t^k}$$

and compute V_X following (70).

(b) **Else** proceed to next step.

(iii) Check whether agent is at the border of being a (labor-) taxpayer (Figure 23.c):
Set $\ell = \bar{\ell} = \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ and compute optimal C from (67). Plug C and $(1-\bar{\ell})$ in (68) in order to compute μ_ℓ. At this point increasing ℓ implies becoming a taxpayer, so we need to compute μ_ℓ with $w^n_{t,j,q} = (1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta$.

(a) **If** $\mu_\ell > 0$ the agent would prefer to work less given this wage. But as we ruled out $\ell < \bar{\ell}$ in the previous step $\ell = \bar{\ell} = \frac{Z_t}{w_{t,j,q}\gamma\eta(1-0.5\tau_{ss})}$ it has to be. Once more we compute:

$$X = \bar{A}' + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) + (1+\tau_c)C$$
$$A = \frac{X - (1-\tau_{ss})w_{t,j,q}\gamma\eta}{R_t^k}$$

and compute V_X following (70).

(b) **Else** proceed to next step.

(iv) After eliminating all other possibilities, it has to be the interior solution with $T(Y_t), \tau_{\ell,t} > 0$ (Figure 23.d):
Solve equations (67) and (68) under net wage $w^n_{t,j,q} = (1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta$ in order to receive C and $1-\ell$. Back out

$$X = \bar{A}' + (1+\tau_c)C + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) + T_t(Y_t)$$
$$A = \frac{X - (1-\tau_{ss})w_{t,j,q}\gamma\eta}{R_t^k}$$

and compute V_X following (70). Note that $\ell \geq 1$ can be excluded (for borrowing unconstraint households) via Inada Conditions, in particular $\lim_{\ell \to 1} \frac{\partial u(C,1-\ell)}{\partial(1-\ell)} = \infty$.[80]

B.2.1 Smart Savings Grid Construction

The choice of the exogenous savings grid is crucial for the endogenous grid method. We want a high density of grid points where policy functions have high curvatures. We also want to avoid agents falling off the grids in the forward iteration. So before applying the solution methods described above we develop an agent-specific[81] savings grid $\mathcal{G}^S = \{s_1, s_2, \ldots, s_n\}$. In this section we abstract form vivos and transfers, because these periods have to be treated differently and we take care of that in Section B.3.

Determining the lowest savings grid point

We know that $A' = -\underline{A}_{j,t}$ can be either a situation in which the household chooses zero savings, i.e. she is on the edge of being borrowing constraint, or she is borrowing constraint and would like to choose $A' < -\underline{A}_{j,t}$. So let us denote the lowest possible savings grid point by A'_l, which is given by $A'_l = -\underline{A}_{j,t}$ in period t and compute $C(A'_l)$ as well as $l(A'_l)$, assuming we are in the interior solution. Cash-on-hand $X(A'_l)$ and asset level $A(A'_l)$ will then be determined by:

$$
\begin{aligned}
X &= A' + (1+\tau_c)C + (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) + T_t(Y_t) \\
A &= \frac{X - (1-\tau_{ss})w_{t,j,q}\gamma\eta}{R_t^n}.
\end{aligned}
$$

(i) $A(A'_l) < -\underline{A}_{j,t-1} - \epsilon$ for tolerance level ϵ. This violates the borrowing constraint making A'_l an invalid savings grid point. Further, this implies that the agent cannot possibly be borrowing constraint in period t, because entering the period with any valid asset level $A_{\text{valid}} \geq \underline{A}_{j,t-1} > A(A'_l)$ has to lead to savings $A'(A_{\text{valid}}) > A'_l = -\underline{A}_{j,t}$. Our task now is to

[80] We deal with borrowing constraint households in Section B.5.3.

[81] Note that agent specific stands in for a combination of age, γ- and η-shock, qualification, innate human capital and acquired human capital.

find the lowest valid savings grid point in period t which is the one leading to endogenous assets $A = \underline{A}_{j,t-1}$. We could do so by letting a solver search for $A(X(A')) + \underline{A}_{j,t-1} = 0$ under tolerance level ϵ, i.e. find

$$-\epsilon < A(X(A')) + \underline{A}_{j,t-1} < \epsilon$$

over A'. However, we could get a problem whenever we receive an asset level $0 < A(X(A') + \underline{A}_{j,t-1} < \epsilon$ and in the forward iteration for assets A_{fwd} we get $0 < A_{\text{fwd}} < A(X(A') + \underline{A}_{j,t-1}$. In this case we would have to extrapolate. We can avoid that by solving $A(X(A')) + \underline{A}_{j,t-1} + \epsilon = 0$ instead, i.e. find

$$-\epsilon < A(X(A')) + \underline{A}_{j,t-1} + \epsilon < \epsilon \tag{71}$$

implying $-2\epsilon < A(X(A')) + \underline{A}_{j,t-1} < 0$. These asset holdings violate the borrowing constraint theoretically, but they are numerically indistinguishable from $-\underline{A}_{j,t-1}$. So we can built the state contingent grid $\mathcal{G}^S = \{s_1, s_2, \ldots, s_n\}$ with $s_1 = A'$ received from (71).

(ii) $A(A'_l) \geq \underline{A}_{j,t-1} - \epsilon$. This tells us that an household entering period t with asset level $A(A'_l)$ would choose $A' = -\underline{A}_{j,t}$, implying it is a valid and interior solution. However, this also tells us that she is borrowing constraint in period t whenever she enters the period with an asset level A with $-\underline{A}_{j,t-1} \leq A < A(A'_l)$. So we set $s_2 = -\underline{A}_{j,t}$ and solve at grid points $\mathcal{G}^S = \{s_2, \ldots, s_n\}$ the way we are used to. In order to have a solution for the borrowing constraint situation (which is possible as opposed to CASE 1), we save the following solution on an extra grid point $s_1 = -\underline{A}_{j,t}$:
Set assets today to the minimum $A = -\underline{A}_{j,t-1}$. This leads to cash-on-hand today $\underline{X}(s_1) = R^n_t(-\underline{A}_{j,t-1}) + (1-\tau_{ss})w_{t,j,q}\gamma\eta$. Now solve for consumption and leisure via the intra-temporal Euler equation (87) and (91), which we describe in Section B.5.3. Note that usually we

should have $\underline{X}(s_1) < X(s_2)$ as

$$\underline{X}(s_1) = R_t^n(-\underline{A}_{j,t-1}) + (1-\tau_{ss})w_{t,j,q}\gamma\eta < R_t^n A(A_l') + (1-\tau_{ss})w_{t,j,q}\gamma\eta = X(s_2)$$

and our case distinction $A(A_l') \geq \underline{A}_{j,t-1} - \epsilon$. But, although unlikely, due to numerical inaccuracies we have $\underline{X}(s_1) < X(s_2)$ compute the solution at $x_1 = \omega x_2(s_2)$ for some ω close to one.

Determining the highest savings grid point

On the one hand s_n should be as high as possible, because we do not want to restrict the space our policy functions live in. On the other hand we want to prevent agents from falling of the grid. That could happen in case we choose a maximum savings grid point $s_n = \bar{A}'$ that might lead to cash-on-hand tomorrow of $X'(s_n) > X_n'$, where X_n' is the highest cash-on-hand level in the endogenous cash-on-hand-grid of $t+1$.

Cash-on-hand $t+1$ is given by $X' = R_{t+1}^k A' + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\eta'$. As wages tomorrow differ with the idiosyncratic income shock η' for each shock there is an asset level $\bar{A}'(\eta')$ satisfying:

$$R_{t+1}^n \bar{A}'(\eta') + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\eta' = X_n'.$$

And as better shocks imply higher wages, i.e. $w_{t+1,j+1,q}\gamma\bar{\eta}' > w_{t+1,j+1,q}\underline{\eta}'$, we have $\bar{A}'(\bar{\eta}') < \bar{A}'(\underline{\eta}')$. Therefore, choosing $s_n = \bar{A}'(\bar{\eta}')$ ensures $X'(s_n) = R_{t+1}^n s_2 + (1-\tau_{ss})w_{t+1,j+1,q}\gamma\bar{\eta}' \leq X_n'$. Formally speaking we choose s_n such that $s_n = \min_{\eta'}\{A'(\eta')\}$, or even more precisely, we set it to $s_n^* = 0.99 \cdot s_n$ in order to deal with computational inaccuracies.

B.3 Solving via Hybrid Exogenous Grid Method

In Chapter 3.2 we have shown that a hybrid method incorporating the exogenous grid method is feasible for periods with two endogenous state variables, which applies to the vivos transfer period as well as to the periods parents invest into the human capital of their children in our model. Before we start, we briefly demonstrate how the exogenous grid method works, given our definition of cash-on-hand and tax code.

B.3.1 Solving via Exogenous Grid Method

Again, we use the household problem for parents of adults described in equation (54) in order to demonstrate the exogenous grid method. First order and envelope conditions were given by:

$$\text{FOC}_C: \quad \frac{u_C(C, 1-\ell)}{1+\tau_c} - \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', q, \gamma, \eta'\right)\right] - \mu_{A'} = 0 \tag{72}$$

$$\text{FOC}_{1-\ell}: \quad u_{1-\ell}(C, 1-\ell) - w^n_{t,j,q}\left(\beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', q, \gamma, \eta'\right)\right] + \mu_{A'}\right) - \mu_\ell = 0 \tag{73}$$

$$\text{EVLP}_X: \quad V_X\left(j, X, q, \gamma, \eta\right) = \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', q, \gamma, \eta'\right)\right] + \mu_{A'}, \tag{74}$$

From equations (72) and (74) we get:

$$V_X\left(j, X, q, \gamma, \eta\right) = \beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\left(j+1, X', q, \gamma, \eta'\right)\right] = \frac{u_C(C, 1-\ell)}{1+\tau_c}. \tag{75}$$

Households' budgets read as:

$$X' = R^n_{t+1} A' + (1-\tau_{ss}) w_{t+1,j+1,q} \gamma \eta'$$
$$A' = X - (1+\tau_c)C - (1-\tau_{ss}) w_{t,j,q} \gamma \eta (1-\ell) - T_t(Y_t) \geq -\Psi(q) \underline{A}_{j,t}.$$

In contrast to the endogenous grid method, the exogenous grid method starts the solution with an exogenous asset level A which, given our cash-on-hand definition, also determines $X = R^n_t A + (1-\tau_{ss}) w_{t,j,q} \gamma \eta$. The asset grid $\mathcal{G}^A = \{A_1, A_2, \ldots, A_n\}$ is spanned such that the household cannot "fall off" and no extrapolation is required. For the lower end we simply set $A_1 = -\Psi(q)\underline{A}_{j,t}$. For the upper end we look at the asset grid for the respective household and identify the highest asset level $A'_{\max}$ a solution is stored for. Then we set A_n such that it is impossible for the household to save $A' > A'_{\max}$. In the same fashion as in the endogenous grid method we now graze the asset regions described in figure (23) as follows, given an asset level A_i from $\mathcal{G}^A$:

(i) Check whether we are in the region where agents choose not to work (see regions in Figure 23.a and an analytic description in Section B.5.2):
Given assets and labor $\ell = 0$ we know $X = R^n_t A + (1 - \tau_{ss})w_{t,j,q}\gamma\eta$ and net wage $w^n_{t,j,q}\gamma\eta = (1 - \tau_{ss})\, w_{t,j,q}\gamma\eta$. In contrast to the endogenous grid method we do not know A' and in turn $V'_{X'}(\cdot)$, so we need to balance equation (72) with a solver. However, each consumption level C pins down savings via $A' = X - (1 + \tau_c)C - (1 - \tau_{ss})w_{t,j,q}\gamma\eta$. As we know that the consumption level balancing (72) has to be between $\underline{C} = \epsilon$ and $\bar{C} = X$[82], we hand this bracket over the solver and receive $C(\ell = 0)$ as well as $A'(\ell = 0)$. Next, we compute μ_ℓ via (73).

   (a) **If** $\mu_\ell > 0$ optimal labor is $\ell = 0$:

      i. **If** savings $A' = X - (1 + \tau_c)C - (1 - \tau_{ss})w_{t,j,q}\gamma\eta \geq -\Psi(q)\underline{A}_{j,t}$ we have found a valid solution and can compute V_X following (75).

      ii. **Else** the household is borrowing constraint. The solution is found via the intra-temporal Euler equation as described in Section B.5.3.

   (b) **Else** proceed to next step.

(ii) Check whether we are in the region where agents choose $\ell \in \left(0, \bar{\ell} = \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}\right]$ (Figure 23.b):
Given net wage $w^n_{t,j,q} = (1 - \tau_{ss})\, w_{t,j,q}\gamma\eta$, would the solution for optimal leisure imply

[82] This is due to the Inada Condition. In case $\lim_{C\to 0} \frac{\partial u(C,1-\ell)}{\partial C} = \infty$ and (72) has to be larger than zero. On the contrary, if $C = X$ we not only have a small marginal utility of consumption today, it also minimizes potential savings and thereby drives up $V'_{X'}$, implying (72) has to be smaller than zero.

$\ell^* \in \left(0, \bar{\ell}\right]$? Given we are searching for an interior solution, we know that the intra-temporal Euler equation (87) between consumption and leisure has to hold. So we can *(i)* guess a consumption level C and *(ii)* back out the corresponding leisure level via the intra-temporal Euler equation. With consumption and leisure we *(iii)* also know savings $A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell)$ and *(iv)* are able to compute X' as well as *(v)* the right hand side of equation (72) in order to verify our choice of C. Again, given that optimal consumption has to be between $\underline{C} = \epsilon$ and $\bar{C} = X$, we let a solver perform steps *(i)*-*(v)* in this interval until it found optimal consumption and leisure given $w^n_{t,j,q} = (1-\tau_{ss})\, w_{t,j,q}\gamma\eta$.

(a) **If** $\ell < \bar{\ell}$ then

   i. **If** savings $A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) \geq -\Psi(q)\underline{A}_{j,t}$ we have found a valid solution and can compute V_X following (75).

   ii. **Else** the household is borrowing constraint. The solution is found via the intra-temporal Euler equation as described in Section B.5.3.

(b) **Else** proceed to next step.

(iii) Check whether agent is at the border of being a (labor-) taxpayer (Figure 23.c): Set $\ell = \bar{\ell} = \frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ and compute optimal C from (72) as described in step (i) under $\ell = 0$. Plug C and $(1-\bar{\ell})$ in (73) in order to compute μ_ℓ. Note that increasing ℓ implies becoming a taxpayer, so we need to compute μ_ℓ with $w^n_{t,j,q} = (1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta$.

   (a) **If** $\mu_\ell > 0$ the agent would prefer to work less given this wage. But as we ruled out $\ell < \bar{\ell}$ in the previous step, we found a solution in $\ell = \bar{\ell}$. Once more we perform the following check:

      i. **If** savings $A' = X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\bar{\ell}) \geq -\Psi(q)\underline{A}_{j,t}$ we have found a valid solution and can compute V_X following (75).

      ii. **Else** the household is borrowing constraint. The solution is found via the intra-temporal Euler equation as described in Section B.5.3.

(b) **Else** proceed to next step.

(iv) After eliminating all other possibilities, it has to be the interior solution with $T(Y_t), \tau_{\ell,t} > 0$ (Figure 23.d):

The solution method is identical to the one in step (ii) under $\ell \in \left(0, \bar{\ell}\right]$, with the difference being that now labor taxes have to be taken into account. Thus, we solve for optimal consumption, leisure and savings under net wage $w^n_{t,j,q} = (1 - \tau_{ss} - \tau_{\ell,t}(1 - 0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta$ as described above.

(a) **If** savings $A' = X - (1 + \tau_c)C - (1 - \tau_{ss})w_{t,j,q}\gamma\eta(1 - \ell) \geq -\Psi(q)\underline{A}_{j,t}$ we have found a valid solution and can compute V_X following (75).

(b) **Else** the household is borrowing constraint. The solution is found via the intra-temporal Euler equation as described in Section B.5.3.

Note that $\ell \geq 1$ can be excluded (for borrowing unconstraint households) via Inada Conditions, in particular $\lim_{\ell \to 1} \frac{\partial u(C, 1-\ell)}{\partial (1-\ell)} = \infty$.

B.3.2 Solving for Inter-Vivos Transfers

Solving for vivos transfers leaves us with two issues. First of all, going recursively, we do not know kids' value functions when parents are of age $j_a + j_f$. The resulting fix point problem is described in B.1.3 (Children become adults at age $j_a + j_f$) and considered solved in this chapter.

Thus, the goal is to solve the following household problem:

$$\begin{aligned} V(j, X, h_0, h, q, \gamma, \eta) = &\max_{C, \ell \in [0,1], X', B \geq 0} \left\{ u(C, 1 - \ell) + \beta E_{\eta'|\eta} \left[V'(j + 1, A', q, \gamma, \eta') \right] \right\} \\ &+ \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\max \left\{ E_{q \in \{d,c\}|h} \left[V^c(j_a, X^c, h_0, h, \neg n, \eta) \right], E_{\gamma \in \{\gamma^l, \gamma^h\}|h} \left[V^c(j_a, X^c, h_0, n, \eta) \right] \right\} \right]. \quad (76) \end{aligned}$$

Under the exogenous grid method we start the solution with an exogenous asset level A, which also determines cash-on-hand $X = R^n_t A + (1 - \tau_{ss})w_{t,j,q}\gamma\eta$. Usually we would now start to search

for the optimal trio of consumption, leisure and savings as just described. But in order to find optimal vivos transfers, we add an intermediate step:
Given the cash-on-hand level we know that optimal vivos transfers cannot be larger than X, as this would leave the household with negative resources for consumption and leisure. In fact, setting maximal vivos transfers to $\bar{B} = X + \Psi(q)\underline{A}_{j,t}$ would violate the borrowing constraint by definition:

$$\begin{aligned} A' &= X - (1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) - T(Y_t) - B \geq -\Psi(q)\underline{A}_{j,t} \quad | \, B = X + \Psi(q)\underline{A}_{j,t} \\ &= -(1+\tau_c)C - (1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell) - T(Y_t) \geq 0. \end{aligned}$$

Thus, for all household types $\{q, \gamma, \eta, h_0, h\}$ and each exogenous savings grid point in $\mathcal{G}^A = \{A_1, A_2, \ldots, A_n\}$ we act as follows:

(i) Given parents' assets A, span vivos transfer grid $\mathcal{G}^B(A) = \{B_1, B_2, \ldots, B_n\}$ from $B_1 = 0$ to $B_n = \bar{B} = X$.[83]

(ii) For all vivos transfer grid points in $\mathcal{G}^B(A)$ perform the following steps:

(a) Given vivos grid point B_i compute the parents' utility from the expected value function of the kid:[84]

$$\tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\max \left\{ E_{q \in \{d,c\}|h} \left[V^c\left(j_a, X^c(B_i), h_0, h, \neg n, \eta\right) \right], E_{\gamma \in \{\gamma^l, \gamma^h\}|h} \left[V^c\left(j_a, X^c(B_i), h_0, n, \eta\right) \right] \right\} \right].$$

(b) Adjust parents' cash-on-hand to *"net cash-on-hand"* to $X^n = X - B_i$. Then solve for consumption, leisure and savings as described in B.3.1, with the only difference in disposable X^n instead of X and therefore $\bar{C} = X^n$.

[83] In order to prevent numerical issues, we use $\bar{B} = X - \epsilon$ in order to have positive resources to split between consumption, leisure and savings.

[84] Note that due to our timing kids' assets are denoted by $A = \frac{B_i}{R_t^n}$ so they start the period with resources $AR_t^n = B_i$. The wage part in cash-on-hand is (i) depending on the idiosyncratic shock the kid draws after the vivos transfer has been made and (ii) her subsequent college decision.

(c) The results from (a) and (b) sum up to the value function (76) for the respective combination of A and B_i.

(iii) Evaluate at which point in $\mathcal{G}^B(A)$ the value function is the highest.

(iv) Hand this point and its neighborhood over to Golden Section Search algorithm and let it perform steps (a)-(c) in order to find optimal B^* and resulting choices C^*, ℓ^* and A'^*.

(a) **If** $A'^* \geq \Psi(q)\underline{A}_{j,t}$ then B^*, C^*, ℓ^* and A'^* are the optimal choice given A_i.

(b) **Else** the household is borrowing constraint under the respective asset level A_i.

The solution for borrowing constraint households is now straightforward. Spanning the vivos transfer grid works the same way as for unconstraint households. Evaluating parents' utility follows the intra-temporal Euler equations for consumption and leisure, which is described in Section B.5.3. It is worth noticing that borrowing constraint does not necessarily imply B^*, as shown in the two-period model in 3.2.1.

B.3.3 Solving for Human Capital Investments

First of all we construct a (i) reasonable and (ii) exogenous grid for kids' acquired human capital $\mathcal{G}^h_{j_a}$. Thereby we kill two birds with one stone: by (i) we again make sure not to store policy functions on irrelevant grid points, which makes our solution more precise. In addition, (ii) enables us to avoid issues coming along with interpolations on grids with two endogenous state variables as discussed in Ludwig and Schön (2014) (44), further explained below.

In order to do so we use our knowledge of human capital grid at age j_a. Due to our definition of college completion, time deduction and fixed effect probabilities, we know that every human capital level $h_{j_a} > \bar{h}_{j_a} = 1$ can be excluded as it does not gain any additional utility for the kid (see equations (31), (32) and (33)). Accordingly we can span the $\mathcal{G}^h_{j_a} = \left\{\underline{h}_{j_a}, \ldots, \bar{h}_{j_a}\right\}$ with $\underline{h}_{j_a} = \epsilon$ and $\bar{h}_{j_a} = 1$.

Given $\bar{h}_{j_a} = 1$ we can choose the **highest grid point** of $\mathcal{G}^h_{j_a-1}$, $\bar{h}_{j_a-1}$, such that $h_{j_a}(\bar{h}_{j_a-1}, i^p_k = 0, i^g_k) = \bar{h}_{j_a} = 1$, following human capital production function (30), because every human capital above would lead to $h_{j_a} > \bar{h}_{j_a}$ and therefore a waste of resources. Assuming $\delta = 1$ and $\phi_j \neq 0$ we get:

$$
\begin{aligned}
\bar{h}_{j+1} &= \left(\upsilon_j \bar{h}_j^{\phi_j} + (1-\upsilon_j)\left(\psi i_j\right)^{\phi_j}\right)^{\frac{1}{\phi_j}} \quad \mid \quad i_j = i^p_j + i^g_j = i^g_j \\
\upsilon_j \bar{h}_j^{\phi_j} &= \bar{h}_{j+1}^{\phi_j} - (1-\upsilon_j)\left(\psi i^g_j\right)^{\phi_j} \\
\bar{h}_j &= \left(\frac{\bar{h}_{j+1}^{\phi_j} - (1-\upsilon_j)\left(\psi i^g_j\right)^{\phi_j}}{\upsilon_j}\right)^{\frac{1}{\phi_j}}.
\end{aligned} \tag{77}
$$

Under $\phi_j = 0$ we get:

$$
\begin{aligned}
\bar{h}_{j+1} &= \bar{h}_j^{\upsilon_j}\left(\psi i_j\right)^{(1-\upsilon_j)} \quad \mid \quad i_j = i^p_j + i^g_j = i^g_j \\
\bar{h}_j^{\upsilon_j}\left(\psi i^g_j\right)^{(1-\upsilon_j)} &= \bar{h}_{j+1} \\
\bar{h}_j &= \left(\frac{\bar{h}_{j+1}}{\left(\psi i^g_j\right)^{(1-\upsilon_j)}}\right)^{\frac{1}{\upsilon_j}}.
\end{aligned} \tag{78}
$$

For cases with $\delta < 1$ we use a solver search for $\bar{h}_j$ by solving:

$$
h_{j+1}(\bar{h}_j, i^p_k = 0, i^g_k) - \bar{h}_{j+1} + \epsilon = 0, \tag{79}
$$

under tolerance level ϵ. That implies $-\epsilon < h_{j+1}(\bar{h}_j, i^p_k = 0, i^g_k) - \bar{h}_{j+1} + \epsilon < \epsilon$ and therefore $-2\epsilon + \bar{h}_{j+1} < h_{j+1}(\bar{h}_j, i^p_k = 0, i^g_k) < \bar{h}_{j+1}$.

A similar thought applies for the **lowest grid point**. We want prevent agents to fall off the grid. Thus, we choose $\underline{h}^c_{j_a-1}$ such that $h^c_{j_a}(\underline{h}^c_{j_a-1}, i^p = 0, i^g) = \underline{h}^c_{j_a} = \epsilon$. Under $\delta = 1$ and $\phi_j \neq 0$ we get it by plugging that into (77):

$$\underline{h}_j = \left(\frac{\epsilon^{\phi_j} - (1-\upsilon_j)\left(\psi i_j^g\right)^{\phi_j}}{\upsilon_j} \right)^{\frac{1}{\phi_j}}, \tag{80}$$

while for $\phi_j \neq 0$ we use (78) to receive:

$$\underline{h}_j = \left(\frac{\epsilon}{\left(\psi i_j^g\right)^{(1-\upsilon_j)}} \right)^{\frac{1}{\upsilon_j}}. \tag{81}$$

Again, for $\delta < 1$ a solver computes $\underline{h}_j$:

$$h_{j+1}(\underline{h}_j, i_k^p = 0, i_k^g) - \underline{h}_{j+1} - \epsilon = 0, \tag{82}$$

implying $-\epsilon < h_{j+1}(\underline{h}_j, i_k^p = 0, i_k^g) - \underline{h}_{j+1} - \epsilon < \epsilon$ and therefore $\underline{h}_{j+1} < h_{j+1}(\underline{h}_j, i_k^p = 0, i_k^g) < 2\epsilon + \underline{h}_{j+1}$. Once we know the upper and lower bound, we can span $\mathcal{G}^h_{j_a-1} = \left\{\underline{h}_{j_a-1}, \ldots, \bar{h}_{j_a-1}\right\}$. For all periods $j_f + j_a - 2, \ldots, j_f$ we can operate in the same manner to receive exogenous $\underline{h}_j$ and $\bar{h}_j$.

Building up on these thoughts, we can also put some structure on the grid for parents investments into human capital of children, $\mathcal{G}_j^{i_k^p}$, as well. Parents will never invest such that $h' > \bar{h}'$, so we define $i^p_{\bar{h}'}$ as the investment leading to $h' = \bar{h}'$. Therefore, the highest grid point in $\mathcal{G}_j^{i_k^p}$ is denoted by $\bar{i}^p_{k,j} = \min\{i_{\bar{h}'}; i^p = X\}$, where the latter distinction comes from the same thought as the maximum vivos transfer. Now we are all set to start solving for household types q, γ, η, h_0, h in $\mathcal{G}^A = \{A_1, A_2, \ldots, A_n\}$ by performing the following steps:

(i) Given kids' h and parents' A we know $\bar{i}^p_{k,j}$ and can span an investment grid $\mathcal{G}_j^{i_k^p} = \{i_1^p, \ldots, i_n^p\}$ with $i_1^p = 0$ and $i_n^p = \bar{i}^p_{k,j}$.

(ii) For all investment grid points in $\mathcal{G}_j^{i_k^p}$ go through the following steps:

(a) Given i_i^p and h compute h' following (30).

(b) Adjust parents' cash-on-hand to "net cash-on-hand" to $X^n = X - i_i^p$. Then solve for consumption, leisure and savings as described in B.3.1, with one more difference other than disposable X^n:
Constructing $E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right]$ now requires a separated interpolation method

(c) Compute parents' value function, again interpolating in both dimensions X' and h'

(iii) Evaluate under which investment grid point in $\mathcal{G}_j^{i_k^p}$ the value function has its maximum

(iv) Hand this point and its neighborhood over to Golden Section Search algorithm and let it perform steps (a)-(c) in order to find optimal i_p^{k*} and resulting choices C^*, ℓ^* and A'^*.

(a) **If** $A'^* \geq \Psi(q)\underline{A}_{j,t}$ then i_p^{k*}, C^*, ℓ^* and A'^* are the optimal choice given A_i.

(b) **Else** the household is borrowing constraint under the respective asset level A_i (the respective solution is described in B.5.3).

This solution method applies for all ages $j_f + j_a - 1, \ldots, j_f$.

B.4 An Incorrect Hybrid Solution Method

This part belongs to Chapter 3.2 in which we compare two hybrid methods, incorporating either the exogenous or the endogenous grid method. The latter is described here and does not deliver proper results, which is shown in Chapter 3.2.2.
This hybrid method is used in Krueger and Ludwig (2013) (40). However, as they took human capital of the child as exogenously given, in their work only one period - the vivos transfer period - was affected. In our model we endogenize the human capital of children, which would make this mistake spillover to ages $j_f, \ldots, j_f + j_a$ as well. To work out the source of the error as good as possible, and to set the record straight, we now describe the method in detail.

B.4.1 Solving for Inter-Vivos Transfers

The endogenous grid method acts in the spirit "*Let's assume the agent would transfer assets A' from t to $t+1$, how would her other choices today have to look like (given what we know about $t+1$) for these savings to be the optimal choice.*" Savings A' then define her cash-on-hand tomorrow, which in turn gives us all we need for the computation of the first-order-conditions (e.g. see the right hand side of equation (55) and (56)). However, this time it is slightly different, because today, besides consumption and labor, she also has to decide over optimal transfers B.

First of all an exogenous gross savings grid, $\mathcal{G}^{\bar{A}'} = \{\bar{A}'_1, \ldots, \bar{A}'_n\}$, is spanned, following our definition of gross savings (46). Net savings are given by $A' = \bar{A}' - Bf$, where f is the number of children within a household. Thereby, we hold gross savings exogenous and solve for the optimal combination of net savings and transfers. This leads to cash-on-hand tomorrow of

$$\begin{aligned} X' &= R^n_{t+1} A' + (1-\tau_{ss}) w_{t+1,j+1,q} \\ &= R^n_{t+1} (\bar{A}' - Bf) + (1-\tau_{ss}) w_{t+1,j+1,q}, \end{aligned}$$

enabling us to express $E_{\eta'|\eta}\left[V'_{t+1,X'}(j+1, X', q, \gamma, \eta')\right]$ as a function of B. Transfers will determine the value function of the child, while net savings will determine today's optimal choices via first-order-conditions as described in detail within the solution of periods $j_r - 1, \ldots, j_a + j_f$ in Section B.2. Putting it in words, the following question is posed: "*Let's assume the agent would invest a sum of $\bar{A}'$ into her savings and her children, how would her other choices today have to look like (given what we know about $t+1$) for these gross savings to be the optimal choice?*"

The first step is, given an exogenous gross savings level $\bar{A}_i$, to span a corresponding vivos transfer grid $\mathcal{G}^B(\bar{A}_i) = \{B_1, \ldots, B_n\}$. The lower end is set to $B'_1 = 0$, while the upper end follows from the gross savings definition and reads $B_n(\bar{A}_i) = \frac{\bar{A}_i+\Psi(q)\underline{A}_{j,t}}{f}$, leading to net savings of:

$$\begin{aligned}
A' &= \bar{A}_i - Bf \quad |B = B_n = \frac{\bar{A}_i + \Psi(q)\underline{A}_{j,t}}{f} \\
A' &= \bar{A}_i - (\bar{A}_i + \Psi(q)\underline{A}_{j,t}) \\
A' &= -\Psi(q)\underline{A}_{j,t}.
\end{aligned}$$

One issue of this method is at the upper end of the gross savings grid $\mathcal{G}^{\bar{A}'} = \{\bar{A}'_1, \ldots, \bar{A}'_n\}$. Usually $\bar{A}'_n$ is chosen such that it prevents agents to fall of the grid in period $t+1$ (see Section B.2.1), avoiding the necessity for inaccurate extrapolations and leading to a high density of grid point in the relevant area of the state space. Setting $\bar{A}'$ to this value and assuming that high savings will most likely be accompanied by positive vivos transfers B, the state space of policy functions cannot be filled that precise anymore in vivos transfer and investment periods.

The other issue arises on the lower end, i.e. $\bar{A}'_1 = -\Psi(q)\underline{A}_{j,t}$. As just shown, setting $B_i(\bar{A}_1) = \frac{-\Psi(q)\underline{A}_{j,t}+\Psi(q)\underline{A}_{j,t}}{f}$ implies zero vivos transfers. But as shown in Section 3.2.1, being borrowing constraint does not imply zero vivos transfers or human capital investments. Such an example is shown in Figure 2 (d), where the agent is always borrowing constraint, but does invest in the human capital of the child. Consequently, this also has an impact on finding the edge at which the household starts being borrowing constraint, demonstrated in B.2.1.

So a different method would have to be developed to accomplish both (i) make the value function $E_{\eta'|\eta}\left[V'_{t+1,X'}(j+1, X', q, \gamma, \eta')\right]$ expressible in terms of B and (ii) detach savings from vivos transfers. However, besides the issues arising in defining grids and dealing with borrowing constraints, the even bigger problems comes from imbalanced first-order-conditions further described below.

Thus, apart from these issues, in general for all household types $\{q, \gamma, \eta, h_0, h\}$ the following steps - for each exogenous gross savings grid point in $\mathcal{G}^{\bar{A}'} = \{\bar{A}'_1, \ldots, \bar{A}'_n\}$ - are executed:[85]

[85] How to solve the fixed point problem due to the unknown value function of kids is described in Section B.1.3.

(i) Given exogenous gross savings $\bar{A}_i'$, span the grid for vivos transfers $\mathcal{G}^B(\bar{A}_i') = \{B_1, \dots, B_n\}$, with $B_1 = 0$ and $B_n(\bar{A}_i) = \frac{\bar{A}_i + \Psi(q)\underline{A}_{j,t}}{f}$.

(ii) Compute parents' value functions for each vivos transfer in $\mathcal{G}^B$ and resulting net savings $A' = \bar{A}' - Bf$:

(a) Evaluate $E_{\eta' \in \Pi_n(\eta)}\left[\max\left\{E_{q \in \{d,c\}|h}\left[V\left(j_a, X, h_0, h, \neg n, \eta\right)\right], E_{\gamma \in \{\gamma^l, \gamma^h\}|h}\left[V\left(j_a, X, h_0, n, \eta\right)\right]\right\}\right]$ given B forming kid's assets $A = \frac{B}{R_t^k}$ and thereby defining cash-on-hand X.

(b) Given net savings A' compute parents choices for consumption and labor by the endogenous grid method as described in Section B.1.3.

(c) Use results from (a) and (b) to compute the parents' value function.

(iii) Evaluate at which vivos transfer in $\mathcal{G}^B(\bar{A}_i')$ the value function is the highest.

(iv) Hand this point and its neighborhood over to Golden Section Search algorithm and let it perform (a) and (b) until it found optimal B^*, C^* and ℓ^*.

B.4.2 Solving for Human Capital Investments

The issues coming along with two endogenous state variables are discussed in Ludwig and Schön (2014) (44). In particular, they show that multidimensional interpolation cannot be separated into several one-dimensional interpolations. The reason becomes apparent when we want to compute $V_{X'}'(X', h')$ within equation (62). Savings and investments today lead to $\tilde{X}'$ and $\tilde{h}'$ which are not located on the endogenous grids received from solving this period in the previous step. Therefore, we would have to find points bordering our imaginary $V_{X'}'\left(\tilde{X}', \tilde{h}'\right)$. In particular, separating the two-dimensional interpolation required into two one-dimensional interpolations would imply holding one dimension fixed while interpolating in the other and vice versa, i.e. fix h_1' close to $\tilde{h}'$ and jump to the left and to the right of $\tilde{X}'$ such that $X_1' < \tilde{X}' < X_2'$. But the problem is that moving around in the cash-on-hand dimension also changes the human capital dimension in case both are endogenous, e.g. the counterpart of $\hat{V}'_{X'}(X'_1, h'_1)$ cannot be found by $\hat{V}'_{X'}(X'_1, h'_2)$ and $h'_1 < \tilde{h}' < h'_2$, as the tuple (X'_1, h'_2) - most likely - will not exist on the grids. Summing up: We cannot hold one dimension constant while walking around in the other.

Ludwig and Schön (2014) (44) provide two possible solutions. One uses Delauny interpolation. The other is a hybrid interpolation method combining the endogenous and the exogenous grid method. The latter performs accurate and outperforms the Delauny method in terms of speed. The idea is to use an exogenous grid in the human capital dimension, while maintaining the endogenous grid method in the cash-on-hand dimension, enabling us to hold human capital constant, while jumping left and right in terms of the cash-on-hand of interest ($\tilde{X}'$). Equipped with this strategy, we will solve periods $j_f + j_a - 1, \ldots, j_f$ as follows.

First we need to construct an exogenous grid for kids' acquired human capital and investments as described in Section B.3.3. However, for the maximal investment the same issue as in the vivos transfer period arises. The upper bound for investments reads as $\bar{i}^p_k = \min\{i_{\bar{h}}, \frac{\bar{A}_i + \Psi(q)\underline{A}_{j,t}}{f}\}$, where $i_{\bar{h}}$ is the investment leading to reasonable upper bound $\bar{h}$ and the latter restriction comes from the gross savings definition, potentially ruling out optimal decisions. Such an example can be found in Figure 2 (d), in which a borrowing constraint household invests into the human capital of the child.

Again, as this is just a part of the problem, we start now describe the actual solution method in order to get the whole picture. First we span an exogenous savings grid as described in Section B.2.1 and can start solving for household type $q, \gamma, \eta, h_0, h, \bar{A}'$ with the following steps:

(i) Given h and $\bar{A}'$ we know $\bar{i}^p_{k,j}$ and can span an investment grid $\mathcal{G}^{i^p_k}_j = \{0, \ldots, \bar{i}^p_{k,j}\}$.

(ii) Compute parents' value function for each combination of net savings ($\bar{A}' = \bar{A} - i^p_k$) and investments:

(a) Compute h' following (30).

(b) Solve equations (62) and (63) by constructing $E_{\eta'|\eta}\left[V'_{X'}(\mathbf{Z}')\right]$ via the separated interpolation method in order to arrive at optimal consumption and leisure.

(c) Compute parents' value function, again interpolating in both dimensions X' and h'.

(iii) Evaluate under which investment grid point the value function has its highest value.

(iv) Hand this point (and its neighborhood) over to the golden section search algorithm in order to find optimal investment, consumption and leisure.

Perform these steps for all exogenous gross saving grid points, for all types q, γ, η, h_0, h and for all ages $j_f + j_a - 1, \ldots, j_f$.

The solution for borrowing constraint households in vivos transfer as well as in human capital investment periods could be handled by operating as described in Section B.5.3 under the exogenous grid method, but the issue remains to find that borrowing constraint asset level in the first place.
However, Figures 2 (c) and (d) reveal the even bigger problem of the *HybLevEndo* method. They display a situation in which the household is not borrowing constraint, but the results of the *HybLevEndo* method are still far off the actual ones. The reason is further explained in 3.2 and boils down to a unbalance in first-order-conditions between savings and investments or vivos transfers.

B.5 Solving for Consumption and Leisure

The following computations apply for both methods the endogenous and the exogenous grid method. Key of this task is equations (55)-(56). Per capita consumption as well as time endowment reduced by college effort also have to be taken into account. The utility function is given by:

$$u\left(\frac{C}{1+\zeta f}, 1-\mathbf{1}\zeta(h_{j_a})-\ell\right)=\frac{\left[\left(\frac{c}{1+\zeta f}\right)^{\mu}\left(1-\mathbf{1}\zeta(h_{j_a})-\ell\right)^{1-\mu}\right]^{1-\sigma}}{1-\sigma}.$$

Let's define $1-\ell^n = 1-\mathbf{1}\zeta(h_{j_a})-\ell$ in order to shorten notation for now. Derivatives are then given by:

$$\begin{aligned}
u_C &= \left[\left(\frac{c}{1+\zeta f}\right)^{\mu}(1-\ell^n)^{1-\mu}\right]^{-\sigma}\mu\left(\frac{c}{1+\zeta f}\right)^{\mu-1}\frac{1}{1+\zeta f}(1-\ell^n)^{1-\mu}\\
&= \mu\left(\frac{c}{1+\zeta f}\right)^{\mu-1-\sigma\mu}(1-\ell^n)^{1-\mu-\sigma+\sigma\mu}\frac{1}{1+\zeta f} \qquad (83)\\
u_{1-\ell^n} &= \left[\left(\frac{c}{1+\zeta f}\right)^{\mu}(1-\ell^n)^{1-\mu}\right]^{-\sigma}(1-\mu)\left(\frac{c}{1+\zeta f}\right)^{\mu}(1-\ell^n)^{-\mu}\\
&= (1-\mu)\left(\frac{c}{1+\zeta f}\right)^{\mu-\sigma\mu}(1-\ell^n)^{\sigma\mu-\mu-\sigma} \qquad (84)
\end{aligned}$$

Rearranging equations (55)-(56) slightly leads us to:

$$\text{FOC}_C: \quad \frac{u_C(C,1-\ell^n)}{1+\tau_c}=\beta R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]+\mu_{A'} \qquad (85)$$

$$\text{FOC}_{1-\ell^n}: \quad \frac{u_{1-\ell^n}(C,1-\ell^n)}{w^n_{t,j,q}}-\frac{\mu_\ell}{w^n_{t,j,q}}=\beta R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]+\mu_{A'}, \qquad (86)$$

where $w^n_{t,j,q}$ differs following (50), depending whether ℓ implies $Y_t \leq Z_t$ or $Y_t > Z_t$. The intra-temporal Euler equation reads as:

$$u_{1-\ell^n}(C,1-\ell^n)-w^n_{t,j,q}\frac{u_C(C,1-\ell^n)}{1+\tau_c}-\mu_\ell=0. \qquad (87)$$

B.5.1 Interior Solution of Consumption and Leisure

In case the household is not constraint, we start by setting (83) equal to (85) and rearrange:

$$
\begin{aligned}
& \mu\left(\frac{C}{1+\zeta f}\right)^{\mu-1-\sigma\mu}(1-\ell^n)^{1-\mu-\sigma+\sigma\mu}\frac{1}{1+\zeta f}=\beta(1+\tau_c)R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right] \\
\Leftrightarrow\quad & \left(\frac{C}{1+\zeta f}\right)^{\mu-1-\sigma\mu}(1-\ell^n)^{(1-\mu)(1-\sigma)}=\beta(1+\tau_c)R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]\frac{1+\zeta f}{\mu} \\
\Leftrightarrow\quad & 1-\ell^n=\left(\beta(1+\tau_c)R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]\frac{1+\zeta f}{\mu}\left(\frac{C}{1+\zeta f}\right)^{1+\sigma\mu-\mu}\right)^{\frac{1}{(1-\mu)(1-\sigma)}}
\end{aligned}
\tag{88}
$$

Setting (84) equal to (86) and rearranging leads us to:

$$
\begin{aligned}
& (1-\mu)\left(\frac{C}{1+\zeta f}\right)^{\mu-\sigma\mu}(1-\ell^n)^{\sigma\mu-\mu-\sigma}=\beta w^n_{t,j,q}R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right] \\
\Leftrightarrow\quad & (1-\ell^n)^{\sigma\mu-\mu-\sigma}=\beta w^n_{t,j,q}R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]\frac{1}{1-\mu}\left(\frac{C}{1+\zeta f}\right)^{\sigma\mu-\mu} \\
\Leftrightarrow\quad & 1-\ell^n=\left(\beta w^n_{t,j,q}R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]\frac{1}{1-\mu}\left(\frac{C}{1+\zeta f}\right)^{\mu(\sigma-1)}\right)^{\frac{1}{\sigma\mu-\mu-\sigma}}
\end{aligned}
\tag{89}
$$

Let's define $\mathbf{X}=\beta R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]$ in order to shorten notation. Now we can cancel out $1-\ell^n$ by setting (88) equal to (89):

$$
\begin{aligned}
& \left(\mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu}\left(\frac{C}{1+\zeta f}\right)^{\mu(\sigma-1)}\right)^{\frac{1}{\sigma\mu-\mu-\sigma}}=\left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\left(\frac{C}{1+\zeta f}\right)^{1+\sigma\mu-\mu}\right)^{\frac{1}{(1-\mu)(1-\sigma)}} \\
\Leftrightarrow\quad & \mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu}\left(\frac{C}{1+\zeta f}\right)^{\mu(\sigma-1)}=\left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\left(\frac{C}{1+\zeta f}\right)^{1+\sigma\mu-\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{(1-\mu)(1-\sigma)}} \\
\Leftrightarrow\quad & \mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu}\left(\frac{C}{1+\zeta f}\right)^{\mu(\sigma-1)}=\left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{(1-\mu)(1-\sigma)}}\left(\frac{C}{1+\zeta f}\right)^{\frac{(1+\sigma\mu-\mu)(\sigma\mu-\mu-\sigma)}{(1-\mu)(1-\sigma)}} \\
\Leftrightarrow\quad & \mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu}=\left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{(1-\mu)(1-\sigma)}}\left(\frac{C}{1+\zeta f}\right)^{\frac{(1+\sigma\mu-\mu)(\sigma\mu-\mu-\sigma)}{(1-\mu)(1-\sigma)}}\left(\frac{C}{1+\zeta f}\right)^{-\frac{(\mu(\sigma-1)(1-\mu)(1-\sigma))}{(1-\mu)(1-\sigma)}}.
\end{aligned}
$$

Now we rearrange the nominator of $\frac{(1+\sigma\mu-\mu)(\sigma\mu-\mu-\sigma)}{(1-\mu)(1-\sigma)} - \frac{\mu(\sigma-1)(1-\mu)(1-\sigma)}{(1-\mu)(1-\sigma)}$ as follows:

$$
\begin{aligned}
&\sigma\mu - \mu - \sigma + (\sigma\mu)^2 - \sigma\mu^2 - \sigma^2\mu - \sigma\mu^2 + \mu^2 + \sigma\mu - ((\mu\sigma - \mu)(1 - \sigma - \mu + \mu\sigma)) \\
=&\sigma\mu - \mu - \sigma + (\sigma\mu)^2 - \sigma\mu^2 - \sigma^2\mu - \sigma\mu^2 + \mu^2 + \sigma\mu - \left((\sigma\mu - \mu - \sigma^2\mu + \sigma\mu - \sigma\mu^2 + \mu^2 + (\sigma\mu)^2 - \sigma\mu^2\right) \\
=& - \sigma
\end{aligned}
$$

Plugging that in and we can further simplify:

$$
\begin{aligned}
& \mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu} = \left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{(1-\mu)(1-\sigma)}} \left(\frac{C}{1+\zeta f}\right)^{-\frac{\sigma}{(1-\mu)(1-\sigma)}} \\
\Leftrightarrow \quad & \left(\frac{C}{1+\zeta f}\right)^{\frac{\sigma}{(1-\mu)(1-\sigma)}} = \left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{(1-\mu)(1-\sigma)}} \left(\mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu}\right)^{-1} \\
\Leftrightarrow \quad & \frac{C}{1+\zeta f} = \left(\mathbf{X}(1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{\sigma}} \left(\mathbf{X}w^n_{t,j,q}\frac{1}{1-\mu}\right)^{-\frac{(1-\mu)(1-\sigma)}{\sigma}} \\
\Leftrightarrow \quad & \frac{C}{1+\zeta f} = \mathbf{X}^{\frac{(\sigma\mu-\mu-\sigma)-(1-\mu)(1-\sigma)}{\sigma}} \left((1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{\sigma}} \left(w^n_{t,j,q}\frac{1}{1-\mu}\right)^{-\frac{(1-\mu)(1-\sigma)}{\sigma}} .
\end{aligned}
$$

The exponent of $\mathbf{X}$ can be reduced to:

$$\sigma\mu - \mu - \sigma - (1-\mu)(1-\sigma) = \sigma\mu - \mu - \sigma - 1 + \sigma + \mu - \sigma\mu = -1,$$

which finally leads us to:

$$
\begin{aligned}
\Leftrightarrow \quad & \frac{C}{1+\zeta f} = \mathbf{X}^{-\frac{1}{\sigma}} \left((1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{\sigma}} \left(w^n_{t,j,q}\frac{1}{1-\mu}\right)^{-\frac{(1-\mu)(1-\sigma)}{\sigma}} \\
\Leftrightarrow \quad & \frac{C}{1+\zeta f} = \left(\beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\right]\right)^{-\frac{1}{\sigma}} \left((1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\frac{\sigma\mu-\mu-\sigma}{\sigma}} \left(w^n_{t,j,q}\frac{1}{1-\mu}\right)^{-\frac{(1-\mu)(1-\sigma)}{\sigma}} \\
\Leftrightarrow \quad & C^* = \left[\beta R^n_{t+1} E_{\eta'|\eta}\left[V'_{X'}\right] \left((1+\tau_c)\frac{1+\zeta f}{\mu}\right)^{\mu+\sigma(1-\mu)} \left(w^n_{t,j,q}\frac{1}{1-\mu}\right)^{(1-\mu)(1-\sigma)}\right]^{-\frac{1}{\sigma}} (1+\zeta f)
\end{aligned}
$$

Now we can get **optimal leisure** by simply plugging in optimal consumption into either equation (89) or (88).

B.5.2 Binding Leisure Constraint

In the first asset region we have $\ell^* = 0$. So we plug optimal leisure $1 - \mathbf{1}\zeta(h_{j_a}) - \ell^* = 1 - \mathbf{1}\zeta(h_{j_a})$ into equation (85) using (83) in order to arrive at optimal consumption:

$$\begin{aligned}
& \mu\left(\frac{C}{1+\zeta f}\right)^{\mu-1-\sigma\mu}(1-\ell^n)^{1-\mu-\sigma+\sigma\mu}\frac{1}{1+\zeta f} = \beta(1+\tau_c)R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right] \\
\Leftrightarrow \quad & \left(\frac{C}{1+\zeta f}\right)^{\mu-1-\sigma\mu} = \beta(1+\tau_c)R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]\frac{1+\zeta f}{\mu}(1-\ell^n)^{\mu+\sigma-\sigma\mu-1} \\
\Leftrightarrow \quad & C^* = \left(\beta(1+\tau_c)R^n_{t+1}E_{\eta'|\eta}\left[V'_{X'}\right]\frac{1+\zeta f}{\mu}(1-\ell^n)^{(\mu-1)(1-\sigma)}\right)^{\frac{1}{\mu-1-\sigma\mu}}(1+\zeta f). \qquad (90)
\end{aligned}$$

The same way we operate on the threshold value, in which we know that labor is $\frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ following equation (47). Plugging that into equation (90) leads us to consumption.

B.5.3 Binding Borrowing Constraint

In general the endogenous and the exogenous grid method do not differ when agents are borrowing constraint, because it is about the intra-temporal - not the inter-temporal - Euler equation in these cases. The only difference is how the asset level under which the agent is restricted (A_{bc}) is determined.

We defined $\bar{A}' = A' + i^p_k f + Bf$ as gross savings and as mentioned above, being borrowing constraint requires $A' = -\underline{A}_{j,t}$, but it does not necessarily imply $i^p = 0$ or $B = 0$. An example would be highly educated parents with high wage income, but low current assets due to tuition fees. Their gain from investments or vivos transfers can outweigh the marginal gain from savings.

In case the budget constraint is binding ($\mu_{A'} > 0$), we solve for consumption and leisure combining the intra-temporal Euler equation (87) received from the first-order-conditions, i.e.

$$u_{1-\ell^n}(C, 1-\ell^n) - w^n_{t,j,q}\frac{u_C(C, 1-\ell^n)}{1+\tau_c} - \mu_\ell = 0.$$

In order to shorten notation we work with $\mathbf{1}\zeta(h_{j_a}) = \mathbf{1}\zeta$ and $1-\ell^n = 1-\mathbf{1}\zeta-\ell$. Plugging in marginal products (83) and (84) we can develop the optimal leisure-consumption-ratio in case the non negativity constraint on labor is not binding ($\mu_\ell = 0$):

$$\begin{aligned}
(1-\mu)\left(\frac{C}{1+\zeta f}\right)^{\mu-\sigma\mu}(1-\ell^n)^{\sigma\mu-\mu-\sigma} &= \frac{w^n_{t,j,q}}{1+\tau_c}\mu\left(\frac{C}{1+\zeta f}\right)^{\mu-1-\sigma\mu}(1-\ell^n)^{1-\mu-\sigma+\sigma\mu}\frac{1}{1+\zeta f}\\
(1-\ell^n)^{\sigma\mu-\mu-\sigma-(1-\mu-\sigma+\sigma\mu)} &= \frac{\mu}{1-\mu}\frac{w^n_{t,j,q}}{1+\tau_c}\left(\frac{C}{1+\zeta f}\right)^{\mu-1-\sigma\mu-(\mu-\sigma\mu)}\frac{1}{1+\zeta f}\\
(1-\ell^n)^{-1} &= \frac{\mu}{1-\mu}\cdot\frac{w^n_{t,j,q}}{1+\tau_c}\left(\frac{C}{1+\zeta f}\right)^{-1}\frac{1}{1+\zeta f}\\
\frac{1-\ell^n}{C} &= \frac{1-\mu}{\mu}\cdot\frac{1+\tau_c}{w^n_{t,j,q}} = \text{lcr}. \qquad (91)
\end{aligned}$$

In case the household is borrowing constraint, we know that $\bar{A}' = A' + i^p_k f + Bf = -\underline{A}_{j,t} + i^p_k f + Bf$. All other resources, let's define them as[86]

$$\begin{aligned}
res = X - \bar{A}' &= A_{bc}R^n_t + (1-\tau_{ss})w_{t,j,q}\gamma\eta - \bar{A}'\\
&= A_{bc}R^n_t + (1-\tau_{ss})w_{t,j,q}\gamma\eta - (-\underline{A}_{j,t} + i^p_k f + Bf)\\
&= A_{bc}R^n_t + (1-\tau_{ss})w_{t,j,q}\gamma\eta + (\underline{A}_{j,t} - \exp^k f),
\end{aligned}$$

with costs per kid of $\exp^k = i^p_k + B$, are used for expenditures of consumption and leisure. Using

[86] There is a special case when agents are in college. In that case *res* has to be built differently, which we describe in Section B.7.

(49) and (50) we receive:

$$\bar{A}' = X - (1+\tau_c)C + \tau_{\ell,t}Z_t - w^n_{t,j,q}(1-\ell^n) - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)$$

$$1-\ell^n = \frac{res - (1+\tau_c)C + \tau_{\ell,t}Z_t - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)}{w^n_{t,j,q}} \tag{92}$$

$$C = \frac{res - w^n_{t,j,q}(1-\ell^n) + \tau_{\ell,t}Z_t - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)}{1+\tau_c}. \tag{93}$$

Now we operate the same way as described above, walking down the asset regions (see Section B.3.1 for vivos transfer and investment periods and Section B.2 for all other periods), where the outer steps (i)-(iii) are only relevant for vivos transfer and investment periods:

(i) Given assets A_{bc} we span a grid from $\exp^k_1 = 0$ to $\exp^k_n = \frac{X+\underline{A}_{j,t}}{f}$.[87] A corresponding grid $\mathcal{G}^{res}(A_{bc}) = \{res_0, \ldots, res_n\}$ directly follows with

$$res_0 = X - \left(-\underline{A}_{j,t} + \frac{X+\underline{A}_{j,t}}{f}f\right) = 0 \quad \text{and}$$
$$res_n = X - \bar{A}' = X + \underline{A}_{j,t}.$$

Next for all $\exp^k_i$ (and resulting res_i) we perform the following steps:

(a) Assume labor $\ell = 0$, $\tau_{\ell,t} = 0$ implying $w^n_{t,j,q} = (1-\tau_{ss})w_{t,j,q}\gamma\eta$ and consumption following from (93), i.e. $C = \frac{res-(1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)}{1+\tau_c}$. Next, compute μ_ℓ from (87). Given A_{bc} we can span a grid for resources

i. **If** $\mu_\ell > 0$ (agent would like to choose $\ell < 0$) so we have $\ell^* = 0$.

ii. **Else** proceed to next step.

(b) Assume $\ell \in \left(0, \frac{Z_t}{w_{t,j,q}}\right]$, $\tau_{\ell,t} = 0$ implying $w^n_{t,j,q} = (1-\tau_{ss})w_{t,j,q}\gamma\eta$. Plug (93) into (91)

[87] For numerical reasons we set the upper bound to $\exp^k_n = \frac{X+\underline{A}_{j,t}}{f} - \epsilon$, which leads to (almost) zero consumption and leisure and can never be optimal due to the Inada Conditions. In addition, the upper bound for investments might also be defined by $i^p_{\bar{h}'}$ as described in Section B.3.3, but we abstract from that for a moment.

in order to compute $1-\ell^n$:

$$\begin{aligned}
\frac{(1-\ell^n)(1+\tau_c)}{res-(1-\tau_{ss})\,w_{t,j,q}\gamma\eta(1-\ell^n)} &= \frac{1-\mu}{\mu}\cdot\frac{1+\tau_c}{(1-\tau_{ss})\,w_{t,j,q}\gamma\eta}\\
\frac{res-(1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\ell^n)}{1-\ell^n} &= \frac{\mu}{1-\mu}\cdot(1-\tau_{ss})w_{t,j,q}\gamma\eta\\
\frac{res}{1-\ell^n}-(1-\tau_{ss})w_{t,j,q}\gamma\eta &= \frac{\mu}{1-\mu}\cdot(1-\tau_{ss})w_{t,j,q}\gamma\eta\\
\frac{res}{1-\ell^n} &= \frac{1}{1-\mu}\cdot(1-\tau_{ss})w_{t,j,q}\gamma\eta\\
1-\ell^n &= \frac{res(1-\mu)}{(1-\tau_{ss})w_{t,j,q}\gamma\eta}
\end{aligned}$$

i. **If** $\ell<\bar{\ell}$ then $C=\frac{1-\ell^n}{\mathrm{lcr}}$.

ii. **Else** proceed to next step.

(c) Assume $\ell=\bar{\ell}=\frac{Z_t}{(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta}$ implying $C=\frac{res-(1-\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi-\bar{\ell})}{1+\tau_c}$, almost as in the first step. Next, compute μ_ℓ from (87), but now with $w^n_{t,j,q}=(1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))\,w_{t,j,q}\gamma\eta$, because at this point increasing ℓ implies becoming a taxpayer.

i. **If** $\mu_\ell>0$ the agent would prefer to work less given this wage. But as we ruled out $\ell<\bar{\ell}$ in the previous step, we have $\ell^*=\bar{\ell}$.

ii. **Else** proceed to next step

(d) After eliminating all other possibilities, it has to be the interior solution with $\tau_{\ell,t}>0$

and $w^n_{t,j,q} = (1 - \tau_{ss} - \tau_{\ell,t}(1 - 0.5\tau_{ss}))\, w_{t,j,q}\gamma\eta$. Plug (93) into (91) to compute $1 - \ell^n$:

$$\frac{(1+\tau_c)(1-\ell^n)}{\underbrace{res + \tau_{\ell,t}Z_t - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)}_{res^{\text{tax}}} - w^n_{t,j,q}(1-\ell^n)} = \frac{1-\mu}{\mu}\cdot\frac{1+\tau_c}{w^n_{t,j,q}}$$

$$\frac{res^{\text{tax}} - w^n_{t,j,q}(1-\ell^n)}{1-\ell^n} = \frac{\mu}{1-\mu}\cdot w^n_{t,j,q}$$

$$\frac{res^{\text{tax}}}{1-\ell^n} - w^n_{t,j,q} = \frac{\mu}{1-\mu}\cdot w^n_{t,j,q}$$

$$\frac{res^{\text{tax}}}{1-\ell^n} = \frac{1}{1-\mu}\cdot w^n_{t,j,q}$$

$$1-\ell^n = \frac{res^{\text{tax}}(1-\mu)}{w^n_{t,j,q}} = \frac{\left(res + \tau_{\ell,t}Z_t - \tau_{\ell,t}(1-0.5\tau_{ss})w_{t,j,q}\gamma\eta(1-\mathbf{1}\xi)\right)(1-\mu)}{w_{t,j,q}\,(1-\tau_{ss}-\tau_{\ell,t}(1-0.5\tau_{ss}))}$$

Get consumption via $C = \frac{1-\ell^n}{\text{lcr}}$. The possibility of $\ell \geq 1$ can be excluded (for borrowing unconstraint households) via the first Inada Condition for leisure.

(ii) Evaluate at which $\exp^k_i$ the value function is the highest.

(iii) Hand this point and its neighborhood over to Golden Section Search algorithm and let it perform steps (a)-(d) in order to find optimal $\exp^{k*}$ and resulting choices C^* and ℓ^*.

Again, please note that in periods without children being part of the household there is only one resource level given assets A_{bc} and the outer loop (i)-(iii) becomes superfluous.

B.6 Taste Shocks

At the college decision we have a discrete choice in our model, which leads to kinks in the value function, causing analytical and thereby computational issues. As done in previous work by Busch (2019) (15) or Iskhakov et al. (2015) (34), we add iid extreme value taste shocks to our model, in order to create a continuous and smooth value function.

The taste shocks are assumed to follow a Gumbel distribution with $E[\epsilon] = \mu + \sigma\gamma$, where γ is Euler's constant and σ denotes the scale parameter. By setting the location parameter to

$\mu = -\sigma\gamma$ we normalize the first moment to $E[\epsilon] = 0$. Given these taste shocks at the college decision of their children, the corresponding household problem of parents at the transfer period writes as:

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C, l \in [0,1], B \geq 0 \\ A' \geq -\Psi(q)\underline{A}_{j,t}}} \{u(C, 1-l) + \beta E_t [V'(j+1, A', q, \gamma, \eta')]\}$$
$$+ \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\max\left\{E_\epsilon \left[E_{q \in \{d,c\}|h} [V_t(j_a, A, h_0, h, \neg n, \eta)] + \epsilon\right], E_\epsilon \left[E_{\gamma \in \{\gamma^l, \gamma^h\}|h} [V_t(j_a, A, h_0, n, \eta)] + \epsilon\right]\right\}\right].$$

Concentrating on the part of interest and making it more readable, the **alt**ruistic part of the parents value can be written as:

$$\begin{aligned} V^{\mathbf{alt}} &= \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\max\left\{E_\epsilon \left[E_{q|h} [V_t(\cdot, \neg n, \eta)] + \epsilon\right], E_\epsilon \left[E_{\gamma|h} [V_t(\cdot, n, \eta)] + \epsilon\right]\right\}\right] \\ &= \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[E_\epsilon \left(\max\left\{E_{q \in \{d,c\}|h} [V_t(\cdot, \neg n, \eta)] + \epsilon, E_{\gamma|h} [V_t(\cdot, n, \eta)] + \epsilon\right\}\right)\right], \end{aligned}$$

where we integrated over the taste shocks. Next, we use a derivation of McFadden (1978) (46), who shows that the integral over the taste shocks can be analytically solved. This enables us to rewrite $V^{\mathbf{alt}}$ as follows:

$$\begin{aligned} V^{\mathbf{alt}} &= \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[E_\epsilon \left(\max\left\{E_{q|h} [V_t(\cdot, \neg n, \eta)] + \epsilon, E_{\gamma|h} [V_t(\cdot, n, \eta)] + \epsilon\right\}\right)\right] \\ &= \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\sigma \log\left(\exp\left[\frac{E_{q|h} [V_t(\cdot, \neg n, \eta)]}{\sigma}\right] + \exp\left[\frac{E_{\gamma|h} [V_t(\cdot, n, \eta)]}{\sigma}\right] + \frac{-\sigma\gamma}{\sigma} + \gamma\right)\right], \end{aligned}$$

and, by applying the log-sum formula, we finally receive:

$$V^{\mathbf{alt}} = \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\sigma \left(\frac{E_{\gamma|h} [V_t(\cdot, n, \eta)]}{\sigma} + \log\left(1 + \exp\left[\frac{E_{q|h} [V_t(\cdot, \neg n, \eta)] - E_{\gamma|h} [V_t(\cdot, n, \eta)]}{\sigma}\right]\right)\right)\right].$$

Thus, by adding taste shocks to our model, the household problem of parents at the transfer period reads as:

$$V(j, A, h_0, h, q, \gamma, \eta) = \max_{\substack{C, l \in [0,1], B \geq 0 \\ A' \geq -\Psi(q)\underline{A}_{j,t}}} \{u(C, 1-l) + \beta E_t [V'(j+1, A', q, \gamma, \eta')]\}$$
$$+ \tilde{\nu} E_{\eta' \in \Pi_n(\eta)} \left[\sigma \left(\frac{E_{\gamma \in \{\gamma^l, \gamma^h\}|h} [V_t(\cdot, n, \eta)]}{\sigma} + \log \left(1 + \exp \left[\frac{E_{q \in \{d,c\}|h} [V_t(\cdot, \neg n, \eta)] - E_{\gamma \in \{\gamma^l, \gamma^h\}|h} [V_t(\cdot, n, \eta)]}{\sigma} \right] \right) \right) \right].$$

The corresponding choice probability of not attending college is given by:

$$P(q = n \mid \cdot) = \frac{1}{1 + \exp\left(\frac{E_{q \in \{d,c\}|h} [V_t(\cdot, \neg n, \eta)] - E_{\gamma \in \{\gamma^l, \gamma^h\}|h} [V_t(\cdot, n, \eta)]}{\sigma} \right)}.$$

By the law of large numbers, $P(q = n \mid \cdot)$ is also the fraction in the population with characteristics j_a, A, h_0, h, η that decides not to attend college. An exemplary course of college attendance probabilities is displayed in Figure 24. The scale parameter σ determines the extent of the

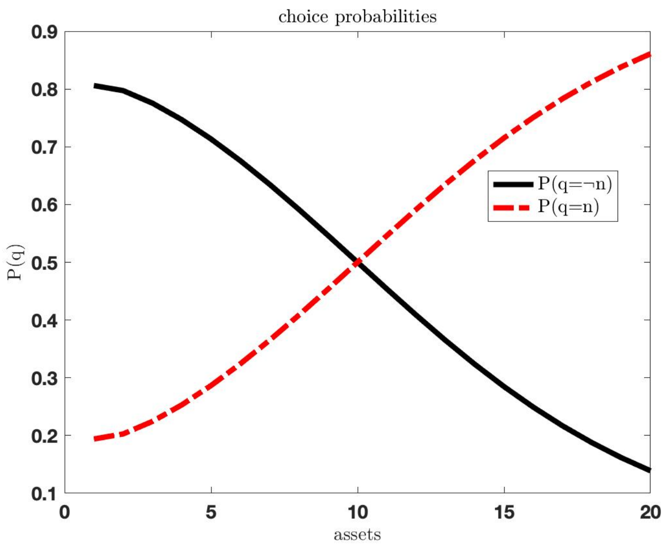


Figure 24: College Attendance Probability

smoothing induced by the taste shocks and we assume it to be $\sigma = 0.27$, which we borrow from Busch (2019) (15).

B.7 Budget in College

Cash-on-hand in college is given by

$$X = R_t^n A + w_{t,n}\epsilon_{j,q}\tilde{\eta}(1 - \tau_{ss})(1 - \Psi(q)\xi(h_{j_a})),$$

which we short for the purpose of illustration to

$$X = RA + \tilde{w}(1 - \tau_{ss})(1 - \Psi\xi).$$

Savings are demoted by

$$A' = X - (1 + \tau_c)C - w_{t,n}\epsilon_{j,q}\tilde{\eta}(1 - \tau_{ss})(1 - \Psi(q)\xi(h_{j_a}) - l) - T(Y_t) - \Psi(q)(1 - \theta_t - \theta_{pr})\kappa w_{t,c} \geq -\Psi(q)\underline{A}_{j,t},$$

which we boil down to

$$A' = X - C - \tilde{w}(1 - \tau_{ss})(1 - \xi - l) - T(Y_t) - \text{CollExp} \geq -\underline{A}_{j,t}$$

Bringing them together leads us to:

$$A' = RA + \tilde{w}(1 - \tau_{ss})(1\xi) - C - \tilde{w}(1 - \tau_{ss})(1 - \xi - l) - T(Y_t) - \text{CollExp} \geq -\Psi(q)\underline{A}_{j,t}$$

Now let us look at the situation of a borrowing constraint household. Assuming $\underline{A}_{j,t} = 0$ and received vivos of $B = 0$, we get:

$$A' = \tilde{w}(1 - \tau_{ss})(1 - \xi) - C - \tilde{w}(1 - \tau_{ss})(1 - \xi - l) - T(Y_t) - \text{CollExp} = 0 \tag{94}$$

The potential issue of setting *res* the way we usually do is (i) that the household cannot choose all values in $[0, 1 - \xi]$ and (ii) perhaps she even cannot afford to go to college even for $\ell = 1 - \xi$. This can also be true if we set *res* $\geq$ CollExp $+ \epsilon$.

The easiest way to illustrate that is to look at a guy who can only afford college in case he works full time, implying $l = 1 - \xi$, leisure equals zero and net wage income is given by:

$$\tilde{w}(1 - \tau_{ss})(1 - \xi^l) - T(Y_t) = \text{CollExp},$$

which, given (94), also implies $C = 0$. However, as net wage income increases monotonically in l, each $l < 1 - \xi$ would violate the borrowing constraint. Further, this shows that each household with $\xi^h > \xi^l$ could not afford to go to college in the first place.

Abstracting from our cash-on-hand definition, we have to make sure that the maximum a household can earn (and borrow) exceeds the costs she has to carry in this situation, for her to be able not to violate the budget constraint. For the college period that is:

$$\begin{aligned} AR^n_t + w_{t,j,q}(1 - \tau_{ss})(1 - \Psi(q)\xi(h_{j_a})) + \underline{A}_{j,t} &\geq \text{CollExp} + T(Y_t) \\ A &\geq \frac{\text{CollExp} + T(Y_t) - w_{t,j,q}(1 - \tau_{ss})(1 - \Psi(q)\xi(h_{j_a})) - \underline{A}_{j,t}}{R^n_t}. \end{aligned} \tag{95}$$

Usually that is taken care of by $w_{t,j,q}(1 - \tau_{ss})(1 - \Psi(q)\xi(h_{j_a})) > T(Y_t)$, even in case of $\underline{A}_{j,t} = 0$. However, in case of college and high time deduction $\xi(h_{j_a})$, there might be positive assets necessary for this student to afford college whenever $\text{CollExp} + T(Y_t) > w_{t,j,q}(1 - \tau_{ss})(1 - \Psi(q)\xi(h_{j_a})) + \underline{A}_{j,t}$.

Using the endogenous grid method that is not an issue for interior solutions, as the endogenous asset level can simply be checked. The question remains what are the lowest possible resources we can throw in at the borrowing constraint? Following (95), we can separate two cases:

$$A^{res} = \begin{cases} 0 & \text{if } w_{t,j,q}(1-\tau_{ss})(1-\Psi(q)\xi(h_{j_a})) + \underline{A}_{j,t} > \text{CollExp} + T(Y_t) \\ (95) + \epsilon & \text{otherwise.} \end{cases} \tag{96}$$

Resources are then given by $res = A^{res} R_t^n + w_{t,j,q}(1-\tau_{ss})(1-\Psi(q)\xi(h_{j_a})) - \bar{A}'$.

B.8 Parallelization

The model is completely coded in FORTRAN, due to its computational superiority. In addition, we parallized the code inside the household problem, using“Open Multi-Processing” (OpenMP), which is an open source API supporting multi-platform shared memory multiprocessing.

Here we will briefly discuss the fundamental challenges that parallelization has brought and how we have solved them. We were motivated to do so by the exchange with other colleagues and scientists, who turned out to be facing similar problems. In the enclosed code "ToyModel_Parallel" the following approaches and structures are presented. The potential issues such as the sop called "race condition", wrong placement of private variables as well as a small speed test can be performed by following the comments inside the code. In solving the household problem, we have used well known algorithms such as the "Golden Search" method and the "Brent Algorithm" (*zBrent*). Both are univariate solvers that find minima and roots of functions respectively. The following example illustrates a structural problem of parallelization and combined with the use of these solvers.

Architecture	Parallized	Threads	Time
2.10 GHz, 192GB RAM, 40 Cores (CSC Skylake)	no	1	$\approx$ 2.62sek
2.10 GHz, 192GB RAM, 40 Cores (CSC Skylake)	yes	40	$\approx$ 0.37sek

Table 18: Comparison Computational Time "ToyModel_Parallel"

Among other things, we have used *zBrent* to determine roots of the Euler equations. For this purpose, we have written a function only be dependent on consumption (it is called FEulerOfCons in both the code "ToyModel_Parallel" and in the solution algorithm of this work). However, different consumption leads to different savings, which in turn changes the next periods' cash-on-hand and thereby the right hand side of the Euler equation (see, for example, equation (55)).

This can be solved by the univariate function accessing another grid whenever *zBrent* calls it with a different value for consumption. Per se, this would not be an issue, but here the parallelization comes into play, through which the counters of the do-loops are private and therefore no longer cross-functional accessible. The solution was to use another declaration of the openMP toolbox: "THREADPRIVATE". Unlike for purely private variables, under THREADPRIVATE the entire thread (even nested functions) has access to these variables. In addition, the race condition is due to parallel working threads is still taken care off.

Another important aspect is to pay attention to the trade-off between parallelization on the one hand and overhead costs on the other hand. For example, it turned out to be an advantage for us to work with the OMP DO COLLAPSE command. This command collapses any number of nested do-loops to one large one. But OMP DO COLLAPSE has the disadvantage over the more popular OMP DO command, that nothing can be written between nested loops, if they should be to collapsed. This can be an issue, as this is where usually calculations are placed so that they are as far outside as possible and do not have to be recalculated unnecessarily in each iteration. In our particular application, however, the low overhead costs of the OMP DO COLLAPSE construct has emerged as the fastest option - despite this minor blemish. Table 18 shows the time for solving the "ToyModel_Parallel" with and without paralleization. The former is roughly 7.1 times faster.

C Appendix to Chapter 4

This chapter provides supplementary material for Section 4.

C.1 Households

Aggregate assets and the capital output ratio $\frac{K}{Y}$ have to be in line with the interest rate. Government debt over GDP is set to $\frac{B}{Y} = 0.6$ exogenously (see 4.8), while $\frac{K}{Y} = 3$ (see 4.9) is the simulated moment that we want to hit using second stage parameter β, which determines aggregate assets A. And from

$$\frac{A}{Y} = \frac{K+B}{Y} = \frac{K}{Y} + \frac{B}{Y}$$

we can extract our target $\frac{K}{Y} = \frac{A}{Y} - \frac{B}{Y} = \frac{A}{Y} - 0.6$. The interest rate can be developed via $r = \alpha \frac{Y}{K} - \delta$. Plugging in $\alpha = \frac{1}{3}$ and $\delta = 0.07$ we get

$$r = \frac{1}{3} \cdot \frac{1}{3} - 0.07 \approx 0.04, \tag{97}$$

and, given capital taxes of 28.3%, a net interest rate of $r^n = r(1 - \tau_k) \approx 0.03$. The following steps are required:

(i) Solve model given guess for β.

(ii) Compute aggregate A ass the sum of all asset holdings in the economy.

(iii) Compute aggregate L following (3) and $L_{t,q}$ the sum of all hours worked for the respective qualification type.

(iv) Compute aggregate Y following (4).

(v) Compute $\frac{K}{Y} = \frac{A}{Y} - 0.6$.

(vi) Adjust β accordingly.

Iterate on (i)-(vi) until $||\frac{K}{Y} - 3|| \leq \epsilon$. Please note that the capital output ratio and the time discount factor β need to be adjusted in the solving algorithm, as one period in the model lasts for four years (see C.9 for details).

C.2 Ability and Education

According to Restuccia and Urrutia (2004) (50) the fraction of non-college agents, Φ_n, is 0.54, implying a fraction of college attendance of 46%. Also, they find a dropout rate of 50%. Therefore, the fraction of dropouts equals the fraction of college graduates and is $\Phi_c = \Phi_d = 0.46 \cdot 0.50 = 0.23$. We match Φ_n with the time cost parameter λ in (32), by performing the following steps:

(i) Solve model given a guess for λ.

(ii) Compute $\Phi_n - 0.54$.

(iii) Adjust λ accordingly.

Iterate on (i)-(iii) until $||\Phi_n - 0.54|| \leq \epsilon$. In the same way we use the curvature parameter μ_c in the college completion probability (31) to hit the target of a 50% dropout rate:

(i) Solve model given a guess for μ_c.

(ii) Compute $\frac{\Phi_d}{1-\Phi_n}0.5$.

(iii) Adjust μ_c accordingly.

Iterate on (i)-(iii) until $||\frac{\Phi_d}{1-\Phi_n} - 0.5|| \leq \epsilon$.

In order to match the ratio of non-tertiary to tertiary government education spending in the US of $\frac{E^e}{E^c} = 2.62$, we use the investment level in non-tertiary education $\bar{i}^g$. In the initial steady state of our model, total expenditures on tertiary education are given by

$$E^c = \left(\Phi_{j,c} + \phi\Phi_{j,d}\right)\kappa w_{0,c}\theta_0, \tag{98}$$

whereas total expenditures on non-tertiary education are, using our simplifying assumption,

$$E^e = \sum_{j=0}^{j_a-1} i^g_{j,t}\Phi_{j,0} = \bar{i}^g_t \left(\zeta^g_{e,t}\Phi_{j,0} + \zeta^g_{e,t}\Phi_{j,1} + \zeta^g_{l,t}\Phi_{j,2} + \zeta^g_{l,t}\Phi_{j,3}\right)$$
$$= \bar{i}^g_t\Phi_{j,g}\left(2\zeta^g_{e,t} + 2(1-\zeta^g_{e,t})\right) = 2\bar{i}^g_t\Phi_{j,g}, \tag{99}$$

with $\Phi_{j,0} = \Phi_{j,1} = \cdots = \Phi_{j,g}$ being the constant mass of all generations (we do not incorporate survival risk in the model). Now, given κ, θ_0, ϕ and the endogenously determined objects $\Phi_c, \Phi_d, w_{0,c}, \Phi_{j,0}$, we can simplify that to

$$\frac{2\bar{i}^g_t\Phi_{j,g}}{(\Phi_c + \phi\Phi_d)\,\kappa w_{0,c}\theta_0} = 2.62$$
$$\Leftrightarrow \quad \bar{i}^{g*} = 2.62\frac{\left(\Phi_{j,c} + \phi\Phi_{j,d}\right)\kappa w_{0,c}\theta_0}{2\Phi_{j,g}}. \tag{100}$$

So we search for the investment level $\bar{i}^{g*}$ leading to $\frac{E^e}{E^c} = 2.62$ by performing the following steps:

(i) Solve model given a guess for $\bar{i}^g$.

(ii) Compute $\bar{i}^{g*} - \bar{i}^g$ following (100).

(iii) Adjust $\bar{i}^g$ accordingly.

Iterate on (i)-(iii) until $||\bar{i}^{g*} - \bar{i}^g|| \leq \epsilon$.

C.3 Labor Productivity and Wages

Wages are given by $w_{t,q}\epsilon_{j,q}\gamma\eta$. In the benchmark model we assume the average college wage premium to be 80%, which is in line with U.S. data. By normalizing non-graduate wages to $w_{t,n} = w_{t,d} = 1$, the parameter that is left to match this wage premium $\hat{w}_{t,c}$ is the average college wage $w_{t,c}$. We define $\hat{w}_{t,c}$ as:

$$\hat{w}_{t,c} = \left(w_{t,c}\sum_{j=jc}^{jr-1}\frac{\epsilon_{j,c}\int\gamma_i\eta_i\Phi_c}{\epsilon_{j,n}\int\gamma_i\eta_i\Phi_n + \epsilon_{j,d}\int\gamma_i\eta_i\Phi_d}\right)\frac{\Phi_n + \Phi_d}{\Phi_c}. \tag{101}$$

Summing up, the following steps are required:

(i) Solve model given a guess for $w_{t,c}$.

(ii) Compute $\hat{w}_{t,c} - 1.8$.

(iii) Adjust $w_{t,c}$ accordingly.

Iterate on (i)-(iii) until $||\hat{w}_{t,c} - 1.8|| \leq \epsilon$.

C.4 Government and Pension Budget

Given all policy instruments, i.e. college subsidies, early education subsidies and tax deduction, labor taxes τ_ℓ have to clear the government budget in accordance to our assumptions $b = \frac{t-e}{r-g^y} = 0.6$ and $\frac{G}{Y} = 0.17$. In addition, given social payroll taxes $\tau_s s$, the pension budget has to be cleared by ρ_{ss}.

Government Budget

The budget constraint of the government is composed of interest payments $B_t R_t$, expenditures E_t, tax income T_t and end of period debt B_{t+1}:

$$B_{t+1} = B_t R_t + E_t - T_t.$$

The government's expenditures (E_t) consist of exogenous government consumption G and endogenous educational expenditures:

$$E_t = G + \text{edu.exp}.$$

The latter are the sum of college subsidies and early education subsidies:

$$\text{edu.exp}_t = (\Phi_c + \phi\Phi_d)\,\kappa w_{t,c}(\theta_t + \theta_{pr}) + \sum_{j=0}^{3} i^g_{j,t}\Phi_{j,t}. \tag{102}$$

The government receives taxes from τ_c, τ_ℓ and τ_k:

$$T_t = \tau_c C_t + \sum_j N_{t,j} \int T_t(y_t) d\Phi_{t,j} + \tau_k r_t A_t. \tag{103}$$

In the steady state government debt relative to GDP is assumed to be constant at 0.6:

$$\begin{aligned} & B_{t+1} = B_t R_t + E_t - T_t \\ \Leftrightarrow \quad & \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} = b(1+r) + (e-t) \quad | \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} = b(1+g^y) \\ \Leftrightarrow \quad & b(r - g^y) = t - e \\ \Leftrightarrow \quad & b = \frac{t-e}{r-g^y} = 0.6, \end{aligned} \tag{104}$$

with $\frac{t-e}{r-g^y} = \frac{t-e}{r}$ as $g^y = 0$ in our model. Exogenous government consumption relative to GDP is assumed to be $gy = \frac{G}{Y} = 0.17$. In order for (104) to hold, labor taxes τ_ℓ have to balance T_t, edu.exp_t and Y_t such that:

$$\begin{aligned} & \frac{B}{Y} r = t - e = \frac{T_t - (\text{edu.exp}_t + G_t)}{Y} = \frac{T_t - \text{edu.exp}_t}{Y} - \frac{G}{Y} \\ \Leftrightarrow \quad & \frac{T_t - \text{edu.exp}_t}{Y} = \frac{B}{Y} r + \frac{G}{Y} \\ \Leftrightarrow \quad & T_t^* = \left(\frac{B}{Y} r + \frac{G}{Y} \right) Y + \text{edu.exp}_t \end{aligned} \tag{105}$$

Pension System

Pensions of an agent with qualification q and productivity type γ are given by:

$$p_{t,j} = \rho_{ss} \gamma w_{t,q}.$$

The pension system is financed by a fixed pension contribution rate τ_{ss}, implying that the pension market equilibrium condition reads as:

$$\tau_{ss} \sum_q w_{t,q} L_{t,q} = \sum_{j=j_r}^{J} N_{t,j} \int p_{t,j}(\gamma, q) d\Phi_{t,j}$$
$$= \rho_{ss} \sum_{j=j_r}^{J} N_{t,j} \int \gamma w_{t,q} d\Phi_{t,j}. \tag{106}$$

Clearing Budgets

Taking care of a the tax progressivity, while clearing both government budget and pension system requires and interplay of tax d_t, τ_ℓ and ρ_{ss}. They are solved jointly in the following way:

Overall taxes T_t are given by (103) and we define tax level $T_t^* = \left(\frac{B}{Y} r + \frac{G}{Y}\right) Y + \text{edu.exp}_t$ as the one satisfying (105). Labor taxes $\sum_j N_{t,j} \int T_t(y_t) d\Phi_{t,j}$ can be defined as $\tau_\ell Y_t^d$, where Y_t^d is aggregate income exceeding tax exemption limit d_t, implying $Y_t^d = \frac{\sum_j N_{t,j} \int T_t(y_t) d\Phi_{t,j}}{\tau_\ell}$. Using that within (103) and rewriting it leaves us with:

$$\tau_\ell^* = \frac{T_t^* - \tau_c C_t - \tau_k r_t K_t}{Y_t^d} = \frac{\left(\frac{B}{Y} r + \frac{G}{Y}\right) Y + \text{edu.exp}_t - \tau_c C_t - \tau_k r_t A_t}{\sum_j N_{t,j} \int T_t(y_t) d\Phi_{t,j}}. \tag{107}$$

Further, using (107) and defining the following function enables us to solve for τ_ℓ^* via both fix point and root-finding algorithms:

$$f_1(\tau_\ell) = \tau_\ell^* - \tau_\ell = 0. \tag{108}$$

Regarding pensions, we solve the model given a guess for ρ_{ss} and observe both overall social security contributions $\tau_{ss} \sum_q w_{t,q} L_{t,q}$ and pensions distributed $\rho_{ss} \sum_{j=j_r}^{J} N_{t,j} \int \gamma w_{t,q} d\Phi_{t,j}$.

We define:

$$\rho^*_{ss} = \frac{\tau_{ss} \sum_q w_{t,q} L_{t,q}}{\sum_{j=j_r}^{J} N_{t,j} \int \gamma w_{t,q} d\Phi_{t,j}}$$

and in the spirit of (108):

$$f_2(\rho_{ss}) = \rho^*_{ss} - \rho_{ss} = 0. \tag{109}$$

The tax free amount Z is set by the government and - in steady state - supposed to be in line with average income following $Z = d\frac{Y_t}{N_t}$. Again, we solve the model under Z, compute $Z^*(Z) = d\frac{Y_t}{N_t}$ and define

$$f_3(z) = Z^* - Z = 0 \tag{110}$$

in order to solve for the tax free amount in the steady state. Once we solve $f = [f_1, f_2, f_3] = 0$ jointly, governmental and pension budget are cleared and we have found $\Psi^* = [\tau^*_\ell, \rho^*, z^*]$.

C.5 GE Interpretation of PE Model

While we develop the solution in partial equilibrium, we have an underlying general equilibrium interpretation. Aggregate labor is given by:

$$L_t = \left((L_{t,n} + L_{t,d})^\rho + \Upsilon_2 L^\rho_{t,c}\right)^{\frac{1}{\rho}} = \left(L^\rho_{t,nd} + \Upsilon_2 L^\rho_{t,c}\right)^{\frac{1}{\rho}}, \tag{111}$$

Aggregate production is given by a nested CES-Cobb-Douglas production function:

$$Y_t = F(K_t, L_t) = K_t^\alpha (\Upsilon_1 L_t)^{1-\alpha} = K_t^\alpha \left[\Upsilon_1 \left(L^\rho_{t,nd} + \Upsilon_2 L^\rho_{t,c}\right)^{\frac{1}{\rho}}\right]^{1-\alpha}. \tag{112}$$

We can develop prices from profit maximization:

$$\Pi_t = K_t^{\alpha}\left[\Upsilon_1\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\alpha} - rK - w_{t,nd}L_{t,nd} - w_{t,c}L_{t,c} + \lambda\left(K_{t+1} - \delta K_t \ldots\right)$$

$$\frac{\partial \Pi_t}{\partial K} = \alpha K_t^{\alpha-1}\left[\Upsilon_1\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\alpha} - r - \delta = 0 \tag{113}$$

$$\frac{\partial \Pi_t}{\partial L_{t,nd}} = K_t^{\alpha}(1-\alpha)\Upsilon_1^{1-\alpha}\left[\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{-\alpha}\left[\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1-\rho}{\rho}}\right]L_{t,nd}^{\rho-1} - w_{t,nd} = 0 \tag{114}$$

$$\frac{\partial \Pi_t}{\partial L_{t,nd}} = K_t^{\alpha}(1-\alpha)\Upsilon_1^{1-\alpha}\left[\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{-\alpha}\left[\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1-\rho}{\rho}}\right]\Upsilon_2 L_{t,c}^{\rho-1} - w_{t,c} = 0. \tag{115}$$

From (113) we get:

$$r = \alpha K_t^{\alpha-1}\left[\Upsilon_1\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\alpha} - \delta, \tag{116}$$

which we can further reduce to:

$$r = \alpha\left(\frac{K_t}{\Upsilon_1\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}}\right)^{\alpha-1} - \delta$$

$$r = \alpha k_t^{\alpha-1} - \delta \quad \Leftrightarrow \quad k_t = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \tag{117}$$

with $k = \frac{K}{\Upsilon_1 L}$. Please note, that this could also be rearranged to

$$r = \alpha K_t^{\alpha-1}\left[\Upsilon_1\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\alpha} - \delta$$

$$r = \alpha K_t^{-1} K_t^{\alpha}\left[\Upsilon_1\left(L_{t,nd}^{\rho} + \Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\alpha} - \delta$$

$$r = \alpha\frac{Y_t}{K_t} - \delta. \tag{118}$$

Rearranging (114) leads to:

$$w_{t,nd} = K_t^{\alpha}(1-\alpha)\Upsilon_1^{1-\alpha}\left[\left(L_{t,nd}^{\rho}+\Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1}{\rho}}\right]^{-\alpha}\left[\left(L_{t,nd}^{\rho}+\Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1-\rho}{\rho}}\right]L_{t,nd}^{\rho-1}$$

$$w_{t,nd} = (1-\alpha)k_t^{\alpha}\left[\Upsilon_1\left(L_{t,nd}^{\rho}+\Upsilon_2 L_{t,c}^{\rho}\right)^{\frac{1-\rho}{\rho}}\right]L_{t,nd}^{\rho-1} \tag{119}$$

$$w_{t,nd} = (1-\alpha)k_t^{\alpha}\Upsilon_1\left(\frac{L_t}{L_{t,nd}}\right)^{1-\rho} \tag{120}$$

and, accordingly:

$$w_{t,c} = (1-\alpha)k_t^{\alpha}\Upsilon_1\Upsilon_2\left(\frac{L_t}{L_{t,c}}\right)^{1-\rho}, \tag{121}$$

as well as:

$$\frac{w_{t,c}}{w_{t,nd}} = \Upsilon_2\left(\frac{L_{t,nd}}{L_{t,c}}\right)^{1-\rho}. \tag{122}$$

C.6 Calibration in Partial Equilibrium

In the partial equilibrium variant of our model, the interest rate is set to $r = 0.04$. As can be verified by equation (118) this is consistent with a general equilibrium model with a capital output ratio of $\frac{K}{Y} = 3$, a depreciation rate of capital of $\delta = 0.07$ and a capital elasticity of production of $\alpha = 0.33$.

We normalize $w_{t,nd} = 1$, whereas $w_{t,c}$ is a result from hitting the average college wage premium. Rewriting (120) using (117) we observe that the normalization of $w_{t,nd} = 1$ is equivalent to a normalization of the technology parameter Υ_1:

$$\begin{aligned}
\Upsilon_1 &= \frac{w_{t,nd}}{(1-\alpha)} k_t^{-\alpha} \left(\frac{L_{t,nd}}{L_t}\right)^{1-\rho} \\
&= \frac{w_{t,nd}}{(1-\alpha)} \left(\left(\frac{\alpha}{r+\delta}\right)^{1-\alpha}\right)^{-\alpha} \left(\frac{L_{t,nd}}{L_t}\right)^{1-\rho} \\
&= \frac{w_{t,nd}}{(1-\alpha)} \left(\frac{r+\delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_{t,nd}}{L_t}\right)^{1-\rho} \\
&= \frac{1}{(1-\alpha)} \left(\frac{r+\delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{L_{t,nd}}{L_t}\right)^{1-\rho}.
\end{aligned} \tag{123}$$

Also, from (122) we see that targeting the college wage premium $w_{t,c}/w_{t,nd} = w_{t,c}$ is equivalent to a normalization of Υ_2:

$$\frac{w_{t,c}}{w_{t,nd}} = w_{t,c} = \Upsilon_2 \left(\frac{L_{t,nd}}{L_{t,c}}\right)^{1-\rho} \tag{124}$$

$$\Leftrightarrow \quad \Upsilon_2 = w_{t,c} \left(\frac{L_{t,c}}{L_{t,nd}}\right)^{1-\rho}. \tag{125}$$

We can use Υ_2 to compute Υ_1 and finally receive K from (117):

$$K_t = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \Upsilon_1 L_t, \tag{126}$$

as well as Y_t from (112) and the resulting capital output ratio $\frac{K_t}{Y_t}$.

C.7 Prices in General Equilibrium and Small Open Economy

In general equilibrium the three factor prices of production - college and non-college wages as well as the interest rate - have to be in line with the first order conditions of production (113)-(115). We solve for these prices as follows:

1. Guess prices $\tilde{r}$, $\tilde{w}_{nd}$ and $\tilde{w}_c$.

2. Solve the household given guessed prices and compute aggregate assets $\tilde{A}_t(\tilde{r}, \tilde{w}_{nd}, \tilde{w}_c)$ as well as aggregate labor types $\tilde{L}_{t,nd}(\tilde{r}, \tilde{w}_{nd}, \tilde{w}_c)$ and $\tilde{L}_{t,c}(\tilde{r}, \tilde{w}_{nd}, \tilde{w}_c)$.

3. Solve the firm's problem:

   (i) Compute aggregate labor from (111):

   $$\tilde{L}_t(\tilde{r}, \tilde{w}_{nd}, \tilde{w}_c) = \left(\tilde{L}_{t,nd}^{\rho} + \Upsilon_2 \tilde{L}_{t,c}^{\rho}\right)^{\frac{1}{\rho}},$$

   with production parameters Υ_1 and Υ_2 from the calibrated benchmark model.

   (ii) Compute the implied capital stock via (126):

   $$\tilde{K}_t(\tilde{r}, \tilde{w}_{nd}, \tilde{w}_c) = \left(\frac{\alpha}{\tilde{r} + \delta}\right)^{\frac{1}{1-\alpha}} \Upsilon_1 \tilde{L}_t.$$

   (iii) Use $\tilde{K}_t$ and $\tilde{L}_t$ to compute $\tilde{Y}_t(\tilde{K}_t, \tilde{L}_t)$ via (112).

   (iv) Compute government debt B given government debt to GDP ratio of 0.6, i.e. $\tilde{B} = 0.6\tilde{Y}_t$.

   (v) Use the marketing clearing condition of the capital market $A = K + B$ to update the capital stock $K = \tilde{A} - \tilde{B}$.

   (vi) Use the updated capital stock K as well as $\tilde{L}_t, \tilde{L}_{t,nd}$ and $\tilde{L}_{t,c}$ in order to compute prices r, w_{nd} and w_c via the firm's first order conditions (116), (120) and (121).

4. **If** distances between all guessed and model prices are small enough, i.e.

$$
\begin{aligned}
&||r - \tilde{r}|| \leq \epsilon, \\
&||w_{nd} - \tilde{w}_{nd}|| \leq \epsilon, \\
&||w_c - \tilde{w}_c|| \leq \epsilon,
\end{aligned}
$$

then STOP.

Else update prices accordingly and start over with STEP 2.

Small Open Economy

In a small open economy variant of the model, the procedure is the same until step (v). Instead, implied of computing the capital stock from marketing clearing, the implied capital stock from step (ii), $\tilde{K}_t$, is used in order to compute prices w_{nd} and w_c via the firm's first order conditions. An equilibrium in the small open economy is established if both of the following conditions hold:

$$
\begin{aligned}
&||w_{nd} - \tilde{w}_{nd}|| \leq \epsilon \text{ and} \\
&||w_c - \tilde{w}_c|| \leq \epsilon.
\end{aligned}
$$

C.8 Aggregation

The distribution of innate abilities of generation t does only depend on the distribution of innate abilities within the respective cohort of parents, i.e. generation $t - j_f$. Thus, the invariant distribution $\Phi_0(h_0)$ can be computed by iterating on $\Phi_{h,i+1} = \Phi_{h,i}\Pi_h(h_0 \mid h_0^p)$ until $\Phi_{h,i+1} = \Phi_{h,i}$, where $\Pi_h(h_0 \mid h_0^p)$ is the transition matrix of innate abilities following (29). However, we do not know the households these are born in with respect to other states like assets, acquired human capital level of the parents etc. Therefore, we neither know the development of abilities during

childhood, nor the distribution of abilities and other state variables at age j_a, the point in time where a household makes her first economic decision.
But we do know that the distribution at the college decision has a counterpart, which it needs to be consistent with, i.e. the distribution of this generation's future children, themselves facing the college decision, right after receiving vivos transfers from their parents. This sums up our fixed point problem: we guess a distribution at age j_a and iterate until the distribution of their respective children at the same age (parents' age $j_f + j_a = j_t$) is consistent with the guess. We operate in the following manner:

- Compute the invariant distribution of innate abilities $\Phi_0(h_0)$ by iterating on $\Phi_{h,i+1} = \Phi_{h,i}\Pi_h(h_0 \mid h_0^p)$ until $\Phi_{h,i+1} = \Phi_{h,i}$.
- Ages j_0 to j_a: $\Phi(j, A = 0, h_0, h)$.
  Children do not possess assets. In order to keep track of their abilities, we would have to know who their parents are. This gap will be closed from the corresponding distributions at age $j + j_f$.

  Begin fix point problem:

  - College decision at age j_a: $\Phi(j_a, A = B/R^n, h_0, h_{j_a}, \eta)$.
    Given the distribution of innate abilities $\Phi_0(h_0)$, we assume a distribution $\Phi^i(j_a, A, h_0, h_{j_a}, \eta)$ of innate abilities, assets, acquired human capital and idiosyncratic shocks. Given (i) this distribution, (ii) college decision probabilities, (iii) college completion shocks and (vi) conditional probabilities of productivity shocks, we can compute $\Phi(j_a, A = B/R^n, h_0, h, q_{\in\{c,d\}}, \eta)$ and $\Phi(j_a, A = B/R^n, h_0, n, \gamma_{\in\{\gamma_h,\gamma_l\}}, \eta)$ respectively.
  - College period at age j_a:
    Moving to the next period is achieved by applying policy functions:
    1. College households: look-up $A'(j, A, h_0, h, q_{\in\{c,d\}}, \eta)$ and compute:
       $\Phi(j, A, h_0, h, q_{\in\{c,d\}}, \eta) \to \Phi'(j + 1, A', h_0, q, \gamma, \eta')$[88]

[88]Please note that college graduates and dropouts draw γ in period $j_a + 1$.

2. Non-college households: look-up $A'(j, a, h_0, n, \gamma_{\in\{\gamma_h,\gamma_l\}}, \eta)$ and compute:
   $\Phi(j, A, h_0, n, \gamma_{\in\{\gamma_h,\gamma_l\}}, \eta) \rightarrow \Phi'(j+1, A', h_0, n, \gamma, \eta')$.

- Working without children at ages $j_a + 1$ to $j_f - 1$: $\Phi(j, A, h_0, q, \gamma, \eta)$.
  Moving from one period to another is achieved by:
  Look-up $A'(j, A, h_0, q, \gamma, \eta)$
  Compute $\Phi(j, A, h_0, \gamma, \eta) \rightarrow \Phi'(j+1, A', h_0, q, \gamma, \eta')$

- Preparing for parenthood at age $j_f - 1$: States in $\Phi(j_f - 1, A, h_0, q, \gamma, \eta)$ become $\Phi(j_f, A, q, \gamma, \eta, h_0^c, h_j^c)$, as kids are joining the household.
  Map parents' into the innate ability of the child via $\Pi_h(h_0^c \mid h_0)$
  Look-up $A'(j, A, h_0, q, \gamma, \eta)$
  Compute $\Phi(j, A, h_0, q, \gamma, \eta) \rightarrow \Phi'(j+1, A', q, \gamma, \eta', h_0^c, h^c)$ with $h_0^c = h^c$.

- Working as parents at ages j_f to $j_t - 1$: $\Phi(j_f, A, q, \gamma, \eta, h_0^c, h_j^c)$.
  In addition to assets, human capital of children changes following the investment decision:
  Look-up $i^p(j, A, q, \gamma, \eta, h_0^c, h_j^c)$ in order to compute $h'(h, h_0, i^p)$
  Look-up $A'(j, A, q, \gamma, \eta, h_0^c, h_j^c)$
  Update $\Phi(j, A, \gamma, \eta, h_0^c, h^c) \rightarrow \Phi'(j', A', \gamma, \eta, h_0^c, h^{c'})$

- Vivos transfer period at age j_t: $\Phi(j_t, A, q, \gamma, \eta, h_0^c, h_{j_a}^c)$.
  Look-up $B(j, A, q, \gamma, \eta, h_0^c, h_{j_a}^c)$
  Update $\Phi(j, A = B/R^n, h_0, h_{j_a})$
  After applying the idiosyncratic shock, we receive $\Phi(j, A = B/R^n, h_0, h_{j_a}, \eta)$ which is the object we need to match with the one we assumed at age j_a:

  * **If** $\left\|\Phi(j, A = B/R^n, h_0, h_{j_a}, \eta) - \Phi^i(j, A = B/R^n, h_0, h_{j_a}, \eta)\right\| \leq \epsilon$: Stop.
  * **Else** set $\Phi^{i+1}(j, A = B/R^n, h_0, h_{j_a}, \eta) = \Phi(j, A = B/R^n, h_0, h_{j_a}, \eta)$ and start with next iteration.

End fix point problem.

- Working as parents of adults at ages $j_t + 1$ to $j_r - 1$: $\Phi = (j, A, q, \gamma, \eta)$.
  Look-up $A'(j, A, q, \gamma, \eta)$
  Compute $\Phi = (j, A, q, \gamma, \eta) \rightarrow \Phi' = (j + 1, A', q, \gamma, \eta')$

- Retirement at ages j_r to $j_J - 1$: $\Phi = (j, A, q, \gamma)$.
  Look-up $A'(j, A, q, \gamma)$
  Compute $\Phi = (j, A, q, \gamma) \rightarrow \Phi' = (j + 1, A', h_0, q, \gamma)$.

Lastly, the distribution of kids at ages j_0 to j_a, $\Phi(j, A = 0, h_0, h)$, can be extracted from the distribution of parents at ages j_f to j_t.

We calibrate the model in order to match targets, which are partly connected to the distributions computed in the aggregation. On the one hand there is targets directly linked to the distribution, e.g. the fraction of non-college agents and college dropouts. On the other hand there is targets that are linked indirectly to the outcome of the aggregation, such as the ratio of non-tertiary to tertiary government education spending $\frac{E^e}{E^c}$. Therefore, the aggregation is nested in the outer loop of the calibration.

C.9 Transformation in 4-Year Model

Because in our model one period stands in for four years, we need to adjust some parameters for the solution algorithm. A summary can be found in 19.

Starting with the household side, β in the model expresses the discount factor for four years and therefore calculates with $\hat{\beta} = \beta^4$. The value for the interest rate in a 1-period model was developed in Section C.1. In the 4-period world, capital is a stock and contributes over the whole period for output Y, which is accumulated over time. Therefore, the capital output ratio needs to be adjusted to $\frac{\hat{K}}{Y} = \frac{K/Y}{4} = \frac{3}{4}$ and the marginal product of capital is transformed to $\hat{\text{mpk}} = \alpha \cdot \frac{\hat{Y}}{K} = \frac{1}{3} \cdot \frac{4}{3} = 0.44$. In the same way as the time discount factor, the adjusted interest rate is denoted by $\hat{r} = (1+r)^4 = 0.1699$, which is also used to compute the matching depreciation rate, i.e. $\hat{\delta} = \widehat{\text{mpk}} - \hat{r} = 0.27014$.

The capital market is cleared when $\frac{A}{Y} = \frac{K}{Y} + \frac{B}{Y}$ and, given our assumptions $\frac{B}{Y} = 0.6$ and $\frac{K}{Y} = 3$, this is the case when $\frac{A}{Y} = \frac{K}{Y} + \frac{B}{Y} = 3 + 0.6 = 3.6$. Transforming this condition the the 4-period model - again distinguishing between accumulated and stock values - the equilibrium is established when $\frac{\hat{A}}{Y} = \frac{\hat{K}}{Y} + \frac{\hat{B}}{Y} = 0.9$.

Parameter	Source	1-Period	4-Period
Time Discount	Calibration	$\beta = 0.96$	$\hat{\beta} = \beta^4 = 0.83$
Capital Output Ratio	Assumption	$\frac{K}{Y} = 3$	$\frac{\hat{K}}{Y} = \frac{K/Y}{4} = \frac{3}{4}$
MP Capital	$\text{mpk} = \alpha \cdot \frac{Y}{K}$	$\text{mpk} = \frac{1}{3} \cdot \frac{1}{3} = 0.11$	$\widehat{\text{mpk}} = \alpha \cdot \frac{\hat{Y}}{K} = \frac{1}{3} \cdot \frac{4}{3} = 0.44$
Interest Rate	Assumption	$r = 0.04$	$\hat{r} = (1+r)^4 = 0.1699$
Depreciation	$\delta = \text{mpk} - r$	$\delta = 0.07$	$\hat{\delta} = \widehat{\text{mpk}} - \hat{r} = 0.27014$
Gov. Debt to GDP	Assumption	$\frac{B}{Y} = 0.6$	$\frac{\hat{B}}{Y} = \frac{B/Y}{4} = 0.15$
Capital Market	Equilibrium	$\frac{A}{Y} = \frac{K}{Y} + \frac{B}{Y} = 3.6$	$\frac{\hat{A}}{Y} = \frac{\hat{K}}{Y} + \frac{\hat{B}}{Y} = 0.9$
Gov. Cons. / GDP	Assumption	$\frac{G}{Y} = 0.17$	$\frac{G}{Y} = 0.17$ (unchanged)
Tax Free Amount	$Z = \bar{d}\frac{Y}{N}$	$Z = \bar{d}\frac{Y}{N} = \bar{d}Y$	$Z = \bar{d}Y$ (unchanged)
Gov. Budget	Equilibrium	$\frac{T_t - \text{edu.exp}_t}{Y} = \frac{B}{Y}r + \frac{G}{Y}$	$\frac{T_t - \text{edu.exp}_t}{Y} = \frac{\hat{B}}{Y}\hat{r} + \frac{G}{Y}$

Table 19: Transformation Production in 4-Period Model

One last parameter that needs to be adjusted is $\Upsilon_1 = \frac{1}{1-\alpha}\left(\frac{r+\delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{L_{t,nd}}{L_t}\right)^{1-\rho}$. It is computed by plugging in the adjusted values for both gross interest and depreciation rate: $\Upsilon_1 = \frac{1}{1-\alpha}\left(\frac{\hat{r}+\hat{\delta}}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{L_{t,nd}}{L_t}\right)^{1-\rho}$.

D Appendix to Chapter 5

This chapter provides supplementary material for Section 5. Table 20 provides detailed results of the partial equilibrium experiment of college subsidies and Table 21 its counterpart for investments in non-tertiary education. In Section D.1 the computation of the inequality measures is shown.

D.1 Inequality Measures

To evaluate the results in terms of their distribution, we use two inequality measures. Wage income is received by agents at ages $j_a, \ldots, j_r - 1$ and we define $J_w = (j_r - 1) - j_a + 1 = j_r - j_a$ as the number of working periods. Agents live for J periods and we normalized the mass across all generations living at a given period to one. Thus, each cohort is of mass $\frac{1}{J}$ and the working population of mass $m_w = \frac{J_w}{J}$. We apply the two measures for:

(i) Wage income

(ii) Capital income

(iii) Overall income (i) + (ii).

Theil Index

Mass population $m = 1$. Mass working population $m_w = \frac{J_w}{J}$. Given mean $\mu = \frac{1}{m_w} \int_0^{m_w} y_i d\Phi_w$, the Theil Index is given as:

$$
\begin{aligned}
T &= \int_0^{m_w} \frac{y_i}{\mu} \ln\left(\frac{y_i}{\mu}\right) d\Phi_w = \frac{1}{\mu} \int_0^{m_w} y_i \left(\ln(y_i) - \ln(\mu)\right) d\Phi_w \\
&= \frac{1}{\mu} \int_0^{m_w} y_i \ln(y_i)\, d\Phi_w - \frac{1}{\mu} \int_0^{m_w} y_i \ln(\mu)\, d\Phi_w = \frac{1}{\mu} \int_0^{m_w} y_i \ln(y_i)\, d\Phi_w - \frac{\ln(\mu)}{\mu} \int_0^{m_w} y_i d\Phi_w \\
&= \frac{1}{\mu} \int_0^{m_w} y_i \ln(y_i)\, d\Phi_w - \frac{\ln(\mu)}{\mu} m_w \mu = \frac{1}{\mu} \int_0^{m_w} y_i \ln(y_i)\, d\Phi_w - \ln(\mu)\, m_w.
\end{aligned}
$$

College Subsidy	0.0000	0.388*	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-0.9345	-0.9499	-0.9076	**-0.8437**	-0.8536	-0.8859
Labor Tax	0.2245	0.2584	0.2642	**0.2966**	0.3509	0.4160
Education						
Aver. Human Capital	0.3764	0.3899	0.4037	**0.4083**	0.3997	0.3964
Fraction Non-College	0.8097	0.5591	0.4125	**0.2879**	0.1152	0.0077
Fraction Graduates	0.1070	0.2205	0.2880	**0.3202**	0.3361	0.3352
Fraction Dropouts	0.0832	0.2205	0.2995	**0.3919**	0.5488	0.6571
Dropout Rate	0.4375	0.5000	0.5098	**0.5503**	0.6202	0.6622
Aggr. HC Investments	0.0025	0.0030	0.0035	**0.0037**	0.0035	0.0033
Aggr. Vivos Transfers	0.0042	0.0047	0.0051	**0.0048**	0.0039	0.0028
Production						
Aggr. Labor	0.7731	0.8767	0.9005	**0.8807**	0.8507	0.8061
Aggr. Output	0.3994	0.4530	0.4653	**0.4550**	0.4396	0.4165
Av. College Wage Premium	1.8311	1.7974	1.8560	**1.8356**	1.8441	1.8403
Av-Wage Non-College	1.0334	1.0079	0.9482	**0.9320**	0.9033	0.9053
Av-Wage Dropouts	1.1423	1.1121	1.1317	**1.0980**	1.0484	1.0257
Av-Wage Graduates	1.9109	1.8646	1.9032	**1.8865**	1.8871	1.8851
Income and Wealth						
Aver. Assets Non-College	0.3878	0.3740	0.3715	**0.3721**	0.3649	0.5030
Aver. Assets Dropouts	0.3692	0.3536	0.3794	**0.3714**	0.3478	0.3430
Aver. Assets Graduates	0.5626	0.5455	0.5912	**0.5570**	0.5201	0.4973
Aver. Grs. Lab-Inc Non-College	1.2049	1.1647	1.0973	**1.0622**	1.0066	0.8949
Aver. Grs. Lab-Inc Dropouts	1.3402	1.2958	1.3011	**1.2346**	1.1448	1.0701
Aver. Grs. Lab-Inc Graduates	2.1417	2.0709	2.0860	**2.0181**	1.9683	1.8964
Aver. Net Lab-Inc Non-College	0.8167	0.7684	0.7165	**0.6771**	0.6161	0.5263
Aver. Net Lab-Inc Dropouts	0.9182	0.8628	0.8646	**0.7991**	0.7086	0.6289
Aver. Net Lab-Inc Graduates	1.5894	1.4951	1.5013	**1.4158**	1.3195	1.2001
Diff. Av. Gross Wage Inc. CN	0.9367	0.9061	0.9887	**0.9559**	0.9618	1.0015
Diff. Av. Gross Wage Inc. CD	0.8015	0.7750	0.7850	**0.7835**	0.8236	0.8263
Diff. Av. Net Wage Inc. CN	0.7727	0.7267	0.7847	**0.7387**	0.7034	0.6739
Diff. Av. Net Wage Inc. CD	0.6712	0.6322	0.6367	**0.6167**	0.6110	0.5712
Consumption						
Aggr. Cons	0.4127	0.4223	0.4403	**0.4343**	0.4111	0.3793
Aver. Cons Non-College	0.3800	0.3574	0.3387	**0.3227**	0.2939	0.2899
Aver. Cons Dropouts	0.4106	0.3860	0.3923	**0.3720**	0.3356	0.3040
Aver. Cons College	0.6616	0.6230	0.6356	**0.6109**	0.5746	0.5289
Inequality						
Theil Index Consumption	0.1126	0.1188	0.1226	**0.1187**	0.1169	0.1146
Theil Index Leisure	0.0029	0.0032	0.0033	**0.0050**	0.0058	0.0071
Theil Index Net Wage Income	0.3746	0.3800	0.3881	**0.3940**	0.3971	0.3953
Theil Index Overall Net Income	0.2765	0.2844	0.2914	**0.2961**	0.3004	0.2990
Var LN Consumption	0.3145	0.3290	0.3384	**0.3328**	0.3319	0.3271
Var LN Leisure	0.0113	0.0121	0.0128	**0.0182**	0.0217	0.0270
Var LN Net Wage Income	1.3498	1.3318	1.3437	**1.3557**	2.0875	2.1505
Var LN Overall Income	0.8248	0.8327	0.8447	**0.8609**	1.6055	1.6850

Table 20: Full Results Partial Equilibrium Experiment Subsidies

Note: * benchmark model. Bold letters policy with highest welfare. “Diff. Av. Net Wage Inc. CN” is difference between average net wage income of graduates and non-college households (“CD” college vs. dropouts). Beside factor prices w_q, average wages (“Av-Wage”) take the productivity types γ of households into account.

Ivst. Level Non-Tert. Edu.	0.0170	0.049*	0.0978	0.1786	**0.2594**	0.3401
Social Welfare	-1.0646	-0.9499	-0.9000	-0.8347	**-0.8020**	-0.8231
Labor Tax	0.2187	0.2584	0.2815	0.2946	**0.3081**	0.3390
Education						
Aver. Human Capital	0.1701	0.3899	0.5107	0.6720	**0.8027**	0.8744
Fraction Non-College	0.8122	0.5591	0.3920	0.2328	**0.1404**	0.0901
Fraction Graduates	0.1112	0.2205	0.3188	0.4494	**0.5392**	0.5844
Fraction Dropouts	0.0767	0.2205	0.2892	0.3178	**0.3204**	0.3255
Dropout Rate	0.4081	0.5000	0.4756	0.4142	**0.3727**	0.3577
Aggr. HC Investments	0.0014	0.0030	0.0019	0.0008	**0.0004**	0.0001
Aggr. Vivos Transfers	0.0126	0.0047	0.0032	0.0022	**0.0010**	0.0000
Production						
Aggr. Labor	0.7615	0.8767	0.9187	0.9254	**0.9052**	0.8861
Aggr. Output	0.3935	0.4530	0.4747	0.4781	**0.4677**	0.4579
Av. College Wage Premium	2.1867	1.7974	1.7399	1.7337	**1.7076**	1.6699
Av-Wage Non-College	0.8607	1.0079	1.0538	1.0617	**1.0888**	1.1779
Av-Wage Dropouts	1.1552	1.1121	1.1224	1.1485	**1.1648**	1.1688
Av-Wage Graduates	1.9376	1.8646	1.8841	1.9276	**1.9494**	1.9550
Income and Wealth						
Aver. Assets Non-College	0.3645	0.3740	0.3804	0.3880	**0.3812**	0.3577
Aver. Assets Dropouts	0.3757	0.3536	0.3497	0.3573	**0.3558**	0.3366
Aver. Assets Graduates	0.6173	0.5455	0.5284	0.5257	**0.5165**	0.4848
Aver. Grs. Lab-Inc Non-College	1.0169	1.1647	1.2014	1.1995	**1.2257**	1.3188
Aver. Grs. Lab-Inc Dropouts	1.3589	1.2958	1.2983	1.3172	**1.3289**	1.3258
Aver. Grs. Lab-Inc Graduates	2.1781	2.0709	2.0783	2.1107	**2.1241**	2.1209
Aver. Net Lab-Inc Non-College	0.6757	0.7684	0.7830	0.7748	**0.7850**	0.8275
Aver. Net Lab-Inc Dropouts	0.9372	0.8628	0.8501	0.8553	**0.8546**	0.8313
Aver. Net Lab-Inc Graduates	1.6256	1.4951	1.4740	1.4821	**1.4753**	1.4349
Diff. Av. Gross Wage Inc. CN	1.1612	0.9061	0.8769	0.9112	**0.8984**	0.8021
Diff. Av. Gross Wage Inc. CD	0.8192	0.7750	0.7800	0.7935	**0.7952**	0.7951
Diff. Av. Net Wage Inc. CN	0.9500	0.7267	0.6910	0.7074	**0.6903**	0.6074
Diff. Av. Net Wage Inc. CD	0.6885	0.6322	0.6240	0.6268	**0.6207**	0.6036
Consumption						
Aggr. Cons	0.3738	0.4223	0.4467	0.4822	**0.5026**	0.4948
Aver. Cons Non-College	0.3280	0.3574	0.3623	0.3608	**0.3618**	0.3680
Aver. Cons Dropouts	0.4183	0.3860	0.3792	0.3821	**0.3806**	0.3655
Aver. Cons College	0.6778	0.6230	0.6118	0.6159	**0.6119**	0.5864
Inequality						
Theil Index Consumption	0.1338	0.1188	0.1137	0.1090	**0.1029**	0.0977
Theil Index Leisure	0.0044	0.0032	0.0029	0.0027	**0.0023**	0.0020
Theil Index Net Wage Income	0.3992	0.3800	0.3730	0.3671	**0.3576**	0.3459
Theil Index Overall Net Income	0.3028	0.2844	0.2786	0.2732	**0.2653**	0.2574
Var LN Consumption	0.3516	0.3290	0.3160	0.3032	**0.2850**	0.2679
Var LN Leisure	0.0166	0.0121	0.0111	0.0104	**0.0093**	0.0082
Var LN Net Wage Income	1.2898	1.3318	1.3175	1.3047	**1.2761**	1.2232
Var LN Overall Income	0.8294	0.8327	0.8201	0.8071	**0.7851**	0.7575

Table 21: Full Results Partial Equilibrium Experiment Non-Tertiary Investments

Note: * benchmark model. Bold letters policy with highest welfare. “Diff. Av. Net Wage Inc. CN” is difference between average net wage income of graduates and non-college households (“CD” college vs. dropouts). Beside factor prices w_q, average wages (“Av-Wage”) take the productivity types γ of households into account.

Variance in Log Earnings

Accordingly, log-mean is $\mu_{\ln} = \frac{1}{m_w}\int_0^{m_w} \ln(y_i)\, d\Phi_w$. In addition, we define

$$E\left((\ln y)^2\right) = \frac{1}{m_w}\int_0^{m_w} (\ln(y_i))^2\, d\Phi_w.$$

Using

$$\operatorname{var}(X) = E\,(X - E\,(X)) = E\left(X^2\right) - E\,(X)^2\,,$$

we can express the variance of log earnings as:

$$\operatorname{var}(\ln y) = E\left((\ln y)^2\right) - \mu_{\ln}^2.$$

E Appendix to Chapter 6

This chapter provides supplementary material for Section 6. Table 22 shows detailed results of the small open economy experiment of college subsidies and Table 23 for investments in non-tertiary education.

College Subsidy	0.0000	0.388*	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-1.0732	-0.9509	-0.8930	**-0.8371**	-0.8384	-0.8479
Labor Tax	0.2523	0.2584	0.2651	**0.2970**	0.3396	0.3840
Education						
Aver. Human Capital	0.4008	0.3899	0.3840	**0.3747**	0.3729	0.3710
Fraction Non-College	0.7186	0.5591	0.4930	**0.3453**	0.2003	0.1177
Fraction Graduates	0.1581	0.2205	0.2469	**0.2888**	0.2988	0.3162
Fraction Dropouts	0.1233	0.2205	0.2601	**0.3660**	0.5010	0.5661
Dropout Rate	0.4381	0.5000	0.5130	**0.5590**	0.6264	0.6416
Aggr. HC Investments	0.0038	0.0030	0.0028	**0.0025**	0.0024	0.0024
Aggr. Vivos Transfers	0.0050	0.0047	0.0047	**0.0043**	0.0034	0.0028
Production						
Aggr. Labor	0.8340	0.8767	0.8834	**0.8755**	0.8529	0.8298
Aggr. Output	0.4309	0.4530	0.4565	**0.4523**	0.4407	0.4287
Av. College Wage Premium	2.4850	1.7974	1.6594	**1.4606**	1.4184	1.3564
Av-Wage Non-College	0.9255	1.0079	1.0178	**1.0233**	1.0855	1.0175
Av-Wage Dropouts	1.0700	1.1121	1.1662	**1.1921**	1.1367	1.1637
Av-Wage Graduates	2.3524	1.8646	1.7740	**1.6216**	1.5916	1.5444
Income and Wealth						
Aver. Assets Non-College	0.3459	0.3740	0.3788	**0.3768**	0.3738	0.3370
Aver. Assets Dropouts	0.3186	0.3536	0.3894	**0.3917**	0.3665	0.3622
Aver. Assets Graduates	0.6689	0.5455	0.5490	**0.4780**	0.4549	0.4371
Aver. Grs. Lab-Inc Non-College	1.5151	1.1647	1.1695	**1.1573**	1.2050	1.1236
Aver. Grs. Lab-Inc Dropouts	1.7735	1.2958	1.3335	**1.3282**	1.2394	1.2341
Aver. Grs. Lab-Inc Graduates	3.7617	2.0709	1.9527	**1.7543**	1.6831	1.5890
Aver. Net Lab-Inc Non-College	1.0492	0.7684	0.7714	**0.7498**	0.7605	0.6853
Aver. Net Lab-Inc Dropouts	1.2397	0.8628	0.8908	**0.8715**	0.7857	0.7576
Aver. Net Lab-Inc Graduates	2.8293	1.4951	1.3935	**1.2087**	1.1176	1.0122
Diff. Av. Gross Wage Inc. CN	2.2466	0.9061	0.7832	**0.5970**	0.4782	0.4655
Diff. Av. Gross Wage Inc. CD	1.9882	0.7750	0.6192	**0.4261**	0.4437	0.3549
Diff. Av. Net Wage Inc. CN	1.7802	0.7267	0.6222	**0.4588**	0.3571	0.3269
Diff. Av. Net Wage Inc. CD	1.5897	0.6322	0.5028	**0.3372**	0.3320	0.2547
Consumption						
Aggr. Cons	0.4052	0.4223	0.4295	**0.4212**	0.4031	0.3836
Aver. Cons Non-College	0.3316	0.3574	0.3594	**0.3490**	0.3468	0.3068
Aver. Cons Dropouts	0.3682	0.3860	0.4040	**0.4007**	0.3663	0.3536
Aver. Cons College	0.7688	0.6230	0.5965	**0.5335**	0.5026	0.4657
Inequality						
Theil Index Consumption	0.1478	0.1188	0.1122	**0.1024**	0.0992	0.0962
Theil Index Leisure	0.0036	0.0032	0.0031	**0.0040**	0.0047	0.0054
Theil Index Net Wage Income	0.4219	0.3800	0.3720	**0.3642**	0.3633	0.3589
Theil Index Overall Income	0.3233	0.2844	0.2766	**0.2695**	0.2698	0.2671
Var LN Consumption	0.3762	0.3290	0.3148	**0.2931**	0.2876	0.2811
Var LN Leisure	0.0135	0.0121	0.0119	**0.0150**	0.0176	0.0205
Var LN Net Wage Income	1.4009	1.3318	1.3098	**1.2757**	1.7640	1.7492
Var LN Overall Income	0.8912	0.8327	0.8151	**0.7975**	1.2712	1.2819

Table 22: Full Results Small Open Economy Experiment Subsidies

Note: * benchmark model. Bold letters policy with highest welfare. “Diff. Av. Net Wage Inc. CN” is difference between average net wage income of graduates and non-college households (“CD” college vs. dropouts). Beside factor prices w_q, average wages (“Av-Wage”) take the productivity types γ of households into account.

Ivst. Level Non-Tert. Edu.	0.0131	0.049*	**0.1051**	0.2013	0.2972	0.3914
Social Welfare	-1.2314	-0.9499	**-0.8923**	-0.9050	-0.9797	-1.1379
Labor Tax	0.2333	0.2588	**0.2912**	0.3541	0.4234	0.5018
Education						
Aver. Human Capital	0.1922	0.3899	**0.4898**	0.6513	0.7946	0.8724
Fraction Non-College	0.7809	0.5591	**0.5206**	0.5350	0.5589	0.5723
Fraction Graduates	0.1310	0.2205	**0.2506**	0.2714	0.2793	0.2812
Fraction Dropouts	0.0880	0.2205	**0.2288**	0.1936	0.1617	0.1466
Dropout Rate	0.4019	0.5000	**0.4773**	0.4163	0.3667	0.3427
Aggr. HC Investments	0.0024	0.0030	**0.0013**	0.0000	0.0000	0.0000
Aggr. Vivos Transfers	0.0165	0.0047	**0.0027**	0.0013	0.0005	0.0000
Production						
Aggr. Labor	0.7808	0.8767	**0.8917**	0.8935	0.8866	0.8715
Aggr. Output	0.4034	0.4530	**0.4607**	0.4617	0.4581	0.4503
Av. College Wage Premium	3.2578	1.7974	**1.5094**	1.3260	1.2288	1.1860
Av-Wage Non-College	0.7512	1.0079	**1.1321**	1.2370	1.3072	1.3439
Av-Wage Dropouts	1.0198	1.1121	**1.1533**	1.1879	1.1997	1.2037
Av-Wage Graduates	2.5359	1.8646	**1.7185**	1.6229	1.5767	1.5599
Income and Wealth						
Aver. Assets Non-College	0.3587	0.3740	**0.3852**	0.3705	0.3344	0.2795
Aver. Assets Dropouts	0.3245	0.3536	**0.3549**	0.3307	0.2886	0.2352
Aver. Assets Graduates	0.8300	0.5455	**0.4718**	0.3925	0.3240	0.2560
Aver. Grs. Lab-Inc Non-College	0.9015	1.1647	**1.2797**	1.3666	0.8334	1.4236
Aver. Grs. Lab-Inc Dropouts	1.2197	1.2958	**1.3250**	1.3421	0.7939	1.3110
Aver. Grs. Lab-Inc Graduates	2.8043	2.0709	**1.9066**	1.7911	0.9921	1.6805
Aver. Net Lab-Inc Non-College	0.5761	0.7684	**0.8363**	0.8556	0.4590	0.7796
Aver. Net Lab-Inc Dropouts	0.8121	0.8628	**0.8665**	0.8380	0.4348	0.7200
Aver. Net Lab-Inc Graduates	2.1271	1.4951	**1.3293**	1.1788	0.6003	0.9695
Diff. Av. Gross Wage Inc. CN	1.9028	0.9061	**0.6269**	0.4245	0.1587	0.2569
Diff. Av. Gross Wage Inc. CD	1.5846	0.7750	**0.5816**	0.4490	0.1982	0.3696
Diff. Av. Net Wage Inc. CN	1.5510	0.7267	**0.4931**	0.3232	0.1413	0.1898
Diff. Av. Net Wage Inc. CD	1.3150	0.6322	**0.4628**	0.3408	0.1655	0.2495
Consumption						
Aggr. Cons	0.3675	0.4223	**0.4259**	0.4043	0.3705	0.3252
Aver. Cons Non-College	0.2875	0.3574	**0.3809**	0.3783	0.3562	0.3166
Aver. Cons Dropouts	0.3625	0.3860	**0.3856**	0.3645	0.3305	0.2882
Aver. Cons College	0.8474	0.6230	**0.5562**	0.4839	0.4224	0.3621
Inequality						
Theil Index Consumption	0.1868	0.1188	**0.1018**	0.0895	0.0808	0.0752
Theil Index Leisure	0.0061	0.0032	**0.0026**	0.0023	0.0021	0.0020
Theil Index Net Wage Income	0.4845	0.3800	**0.3539**	0.3296	0.3088	0.2879
Theil Index Overall Income	0.3787	0.2844	**0.2614**	0.2425	0.2286	0.2171
Var LN Consumption	0.4371	0.3290	**0.2888**	0.2551	0.2290	0.2104
Var LN Leisure	0.0222	0.0121	**0.0103**	0.0093	0.0085	0.0081
Var LN Net Wage Income	1.4130	1.3318	**1.2730**	1.1877	1.0959	0.9883
Var LN Overall Income	0.9316	0.8327	**0.7846**	0.7347	0.6900	0.6456

Table 23: Full Results Small Open Economy Experiment Non-Tertiary Investments

Note: * benchmark model. Bold letters policy with highest welfare. “Diff. Av. Net Wage Inc. CN” is difference between average net wage income of graduates and non-college households (“CD” college vs. dropouts). Beside factor prices w_q, average wages (“Av-Wage”) take the productivity types γ of households into account.

Government						
College Subsidy	0.0000	0.3880	0.8750	**1.7500**	2.6250	3.5000
Social Welfare	-1.0601	-0.9509	-0.8935	**-0.8362**	-0.8401	-0.8475
Labor Tax	0.2528	0.2584	0.2689	**0.2983**	0.3396	0.3833
Education						
Aver. Human Capital	0.4019	0.3899	0.3849	**0.3749**	0.3725	0.3708
Fraction Non-College	0.7159	0.5591	0.4933	**0.3454**	0.2019	0.1180
Fraction Graduates	0.1596	0.2205	0.2469	**0.2887**	0.2981	0.3160
Fraction Dropouts	0.1246	0.2205	0.2599	**0.3659**	0.5000	0.5660
Dropout Rate	0.4384	0.5000	0.5128	**0.5590**	0.6265	0.6417
Aggr. HC Investments	0.0038	0.0030	0.0028	**0.0025**	0.0024	0.0024
Aggr. Vivo Transfers	0.0049	0.0047	0.0045	**0.0042**	0.0035	0.0028
Production						
Aggr. Capital	0.3284	0.3397	0.3507	**0.3431**	0.3295	0.3206
Aggr. Labor	0.8354	0.8767	0.8833	**0.8755**	0.8527	0.8299
Aggr. Output	0.4337	0.4530	0.4601	**0.4540**	0.4401	0.4284
Av. College Wage Premium	2.4566	1.7974	1.6522	**1.4548**	1.4162	1.3529
Av-Wage Non-College	0.9348	1.0079	1.0271	**1.0290**	1.0850	1.0178
Av-Wage Dropouts	1.0798	1.1121	1.1753	**1.1977**	1.1351	1.1637
Av-Wage Graduates	2.3492	1.8646	1.7814	**1.6232**	1.5871	1.5403
Gross Interest Rate	0.1657	0.1699	0.1627	**0.1666**	0.1707	0.1707
Income and Wealth						
Aver. Assets Non-College	0.3464	0.3740	0.3755	**0.3763**	0.3736	0.3374
Aver. Assets Dropouts	0.3188	0.3536	0.3843	**0.3899**	0.3664	0.3628
Aver. Assets Graduates	0.6620	0.5455	0.5422	**0.4753**	0.4541	0.4369
Aver. Grs. Lab-Inc Non-College	1.1033	1.1647	1.1780	**1.1624**	1.2047	1.1241
Aver. Grs. Lab-Inc Dropouts	1.2801	1.2958	1.3425	**1.3337**	1.2379	1.2343
Aver. Grs. Lab-Inc Graduates	2.5956	2.0709	1.9592	**1.7556**	1.6788	1.5854
Aver. Net Lab-Inc Non-College	0.7196	0.7684	0.7762	**0.7532**	0.7602	0.6860
Aver. Net Lab-Inc Dropouts	0.8478	0.8628	0.8954	**0.8749**	0.7845	0.7581
Aver. Net Lab-Inc Graduates	1.9229	1.4951	1.3947	**1.2084**	1.1143	1.0101
Diff. Av. Gross Wage Inc. CN	1.4923	0.9061	0.7811	**0.5931**	0.4741	0.4612
Diff. Av. Gross Wage Inc. CD	1.3155	0.7750	0.6166	**0.4218**	0.4409	0.3511
Diff. Av. Net Wage Inc. CN	1.2034	0.7267	0.6186	**0.4552**	0.3541	0.3242
Diff. Av. Net Wage Inc. CD	1.0752	0.6322	0.4993	**0.3335**	0.3298	0.2520
Consumption						
Aggr. Cons	0.4072	0.4223	0.4283	**0.4210**	0.4025	0.3838
Aver. Cons Non-College	0.3336	0.3574	0.3589	**0.3494**	0.3468	0.3073
Aver. Cons Dropouts	0.3705	0.3860	0.4029	**0.4008**	0.3660	0.3542
Aver. Cons College	0.7661	0.6230	0.5935	**0.5321**	0.5015	0.4653
Inequality						
Theil Index Consumption	0.1475	0.1188	0.1136	**0.1030**	0.0990	0.0958
Theil Index Leisure	0.0036	0.0032	0.0032	**0.0040**	0.0046	0.0054
Theil Index Net Wage Income	0.4198	0.3800	0.3710	**0.3636**	0.3630	0.3586
Theil Index Overall Income	0.3237	0.2844	0.2797	**0.2708**	0.2692	0.2664
Var LN Consumption	0.3775	0.3290	0.3195	**0.2950**	0.2868	0.2801
Var LN Leisure	0.0135	0.0121	0.0121	**0.0151**	0.0175	0.0205
Var LN Net Wage Income	1.3964	1.3318	1.3048	**1.2718**	1.7640	1.7484
Var LN Overall Income	0.8977	0.8327	0.8287	**0.8032**	1.2700	1.2800

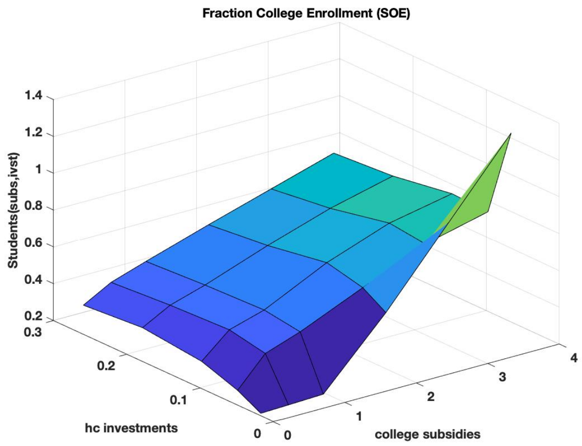
Table 24: Full Results General Equilibrium Experiment Subsidies

Note: * benchmark model. Bold letters policy with highest welfare. “Diff. Av. Net Wage Inc. CN.” is difference between average net wage income of graduates and non-college households (“CD” college vs. dropouts). Beside factor prices w_q, average wages (“Av-Wage”) take the productivity types γ of households into account.

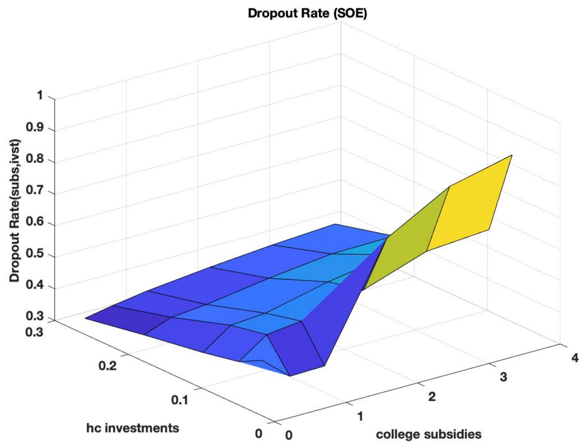
Government						
Ivst. Level Non-Tert. Edu.	0.0134	0.0491	**0.1061**	0.2068	0.3141	0.4290
Social Welfare	-1.0862	-0.9499	**-0.9055**	-0.9278	-1.0529	-1.3207
Labor Tax	0.2472	0.2588	**0.2897**	0.3486	0.4228	0.5168
Education						
Aver. Human Capital	0.2319	0.3899	**0.4871**	0.6507	0.7946	0.8724
Fraction Non-College	0.7404	0.5591	**0.5239**	0.5407	0.5698	0.5876
Fraction Graduates	0.1588	0.2205	**0.2487**	0.2680	0.2730	0.2730
Fraction Dropouts	0.1008	0.2205	**0.2274**	0.1913	0.1572	0.1394
Dropout Rate	0.3881	0.5000	**0.4777**	0.4165	0.3654	0.3380
Aggr. HC Investments	0.0045	0.0030	**0.0012**	0.0000	0.0000	0.0000
Aggr. Vivo Transfers	0.0137	0.0047	**0.0027**	0.0014	0.0005	0.0000
Production						
Aggr. Capital	0.4034	0.3397	**0.3338**	0.3144	0.2806	0.2352
Aggr. Labor	0.7997	0.8767	**0.8904**	0.8903	0.8768	0.8473
Aggr. Output	0.4507	0.4530	**0.4551**	0.4461	0.4253	0.3921
Av. College Wage Premium	2.7615	1.7974	**1.5174**	1.3332	1.2441	1.2102
Av-Wage Non-College	0.8624	1.0079	**1.1146**	1.1958	1.2164	1.1953
Av-Wage Dropouts	1.1766	1.1121	**1.1360**	1.1457	1.1129	1.0707
Av-Wage Graduates	2.4855	1.8646	**1.7011**	1.5768	1.4855	1.4176
Gross Interest Rate	0.0986	0.1699	**0.1797**	0.1981	0.2301	0.2801
Income and Wealth						
Aggr. Assets Non-College	0.3301	0.2091	**0.2021**	0.2078	0.2053	0.1894
Aggr. Assets Dropouts	0.0319	0.0780	**0.0812**	0.0655	0.0478	0.0359
Aggr. Assets Graduates	0.1106	0.1203	**0.1180**	0.1073	0.0908	0.0720
Aver. Assets Non-College	0.4458	0.3740	**0.3858**	0.3843	0.3604	0.3223
Aver. Assets Dropouts	0.3170	0.3536	**0.3571**	0.3425	0.3040	0.2573
Aver. Assets Graduates	0.6960	0.5455	**0.4744**	0.4002	0.3324	0.2637
Aver. Grs. Lab-Inc Non-College	0.9818	1.1647	**1.2631**	1.3251	1.2739	1.2510
Aver. Grs. Lab-Inc Dropouts	1.3711	1.2958	**1.3078**	1.2993	1.3066	1.1638
Aver. Grs. Lab-Inc Graduates	2.7332	2.0709	**1.8900**	1.7465	1.6241	1.5343
Aver. Net Lab-Inc Non-College	0.6430	0.7684	**0.8241**	0.8289	0.7362	0.6650
Aver. Net Lab-Inc Dropouts	0.9272	0.8628	**0.8540**	0.8103	0.7679	0.6199
Aver. Net Lab-Inc Graduates	2.0460	1.4951	**1.3175**	1.1507	0.9963	0.8633
Diff. Av. Gross Wage Inc. CN	1.7514	0.9061	**0.6269**	0.4215	0.3503	0.2833
Diff. Av. Gross Wage Inc. CD	1.3621	0.7750	**0.5822**	0.4472	0.3176	0.3705
Diff. Av. Net Wage Inc. CN	1.4030	0.7267	**0.4935**	0.3218	0.2601	0.1984
Diff. Av. Net Wage Inc. CD	1.1188	0.6322	**0.4636**	0.3404	0.2284	0.2435
Consumption						
Aggr. Cons	0.3986	0.4223	**0.4235**	0.4028	0.3632	0.3104
Aver. Cons Non-College	0.3156	0.3574	**0.3786**	0.3774	0.3499	0.3037
Aver. Cons Dropouts	0.3920	0.3860	**0.3834**	0.3628	0.3229	0.2741
Aver. Cons College	0.7894	0.6230	**0.5549**	0.4826	0.4142	0.3433
Inequality						
Theil Index Consumption	0.1824	0.1188	**0.0999**	0.0828	0.0676	0.0529
Theil Index Leisure	0.0088	0.0032	**0.0025**	0.0021	0.0019	0.0019
Theil Index Net Wage Income	0.4648	0.3800	**0.3543**	0.3306	0.3092	0.2863
Theil Index Overall Income	0.3935	0.2844	**0.2566**	0.2284	0.1995	0.1667
Var LN Consumption	0.4737	0.3290	**0.2826**	0.2345	0.1889	0.1441
Var LN Leisure	0.0311	0.0121	**0.0100**	0.0085	0.0076	0.0075
Var LN Net Wage Income	1.4007	1.3318	**1.2752**	1.1962	1.1018	0.9824
Var LN Overall Income	1.0699	0.8327	**0.7647**	0.6785	0.5779	0.4618

Table 25: Full Results General Equilibrium Experiment Non-Tertiary Investments

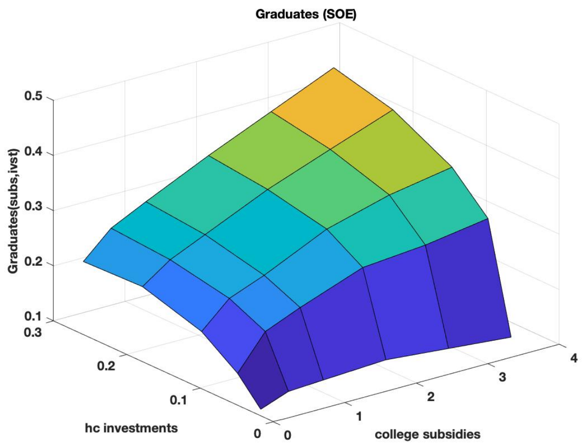
Note: * benchmark model. Bold letters policy with highest welfare. “Diff. Av. Net Wage Inc. CN.” is difference between average net wage income of graduates and non-college households (“CD” college vs. dropouts). Beside factor prices w_q, average wages (“Av-Wage”) take the productivity types γ of households into account.



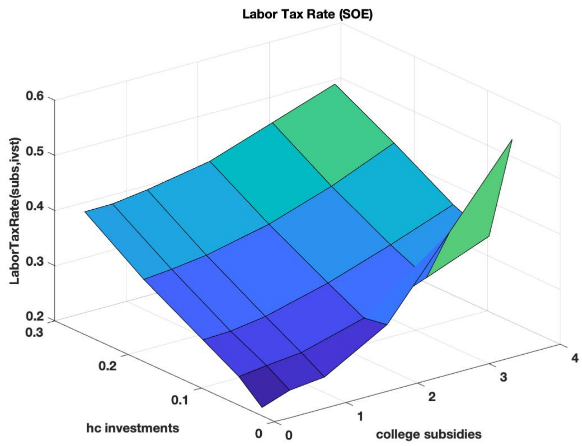
(a) College Enrollment



(b) Dropout Rate



(c) College Graduates



(d) Labor Taxe Rate

Figure 25: Results Bivariate Experiment (SOE)

Please note that due to convergence issues and time restrictions with respect to high performance computing resources, for three edge cases displayed in these figures, we had to approximate the results by extrapolation. However, given these points are not in the neighborhood of the best policy mix, that does not change the results (for the sake of completeness, these edge cases are: min. investments and min. subsidies, min. investments and max subsidies, max. investments and max subsidies).

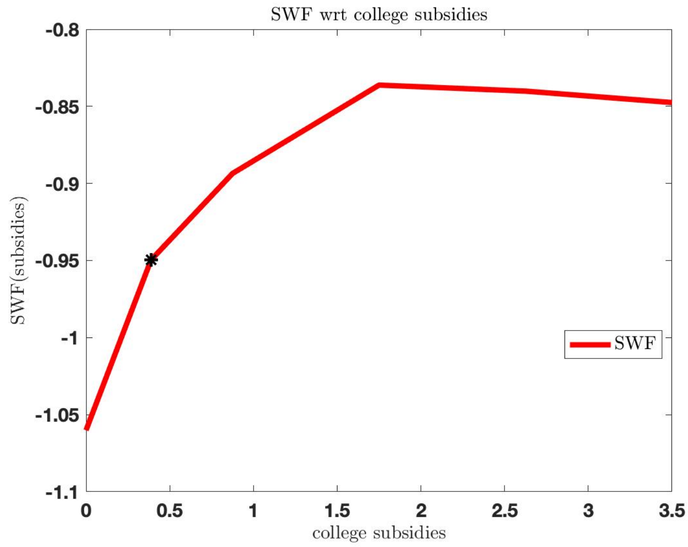


Figure 26: Experiment College Subsidy General Equilibrium (*Benchmark Model)

Eidesstattliche Erklärung

nach § 6 der Promotionsordnung vom 16. Januar 2008

“Hiermit erkläre ich an Eidesstatt, dass ich die vorgelegte Arbeit ohne Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe. Die aus anderen Quellen direkt oder indirekt übernommenen Aussagen, Daten und Konzepte sind unter Angabe der Quelle gekennzeichnet. Bei der Auswahl und Auswertung folgenden Materials haben mir die nachstehend aufgeführten Personen in der jeweils beschriebenen Weise entgeltlich/ unentgeltlich geholfen:

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Ort, Datum ______________________ Unterschrift ________________

Lebenslauf

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