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Abstract

Time-varying VAR models represent fundamental tools for the anticipation and analysis of economic crises. Yet they remain subject to a number of limitations. The conventional random walk assumption used for the dynamic parameters appears excessively restrictive, and the existing estimation procedures are largely inefficient. This paper improves on the existing methodologies in four directions:

i) it introduces a general time-varying VAR model which relaxes the standard random walk assumption and defines the dynamic parameters as general autoregressive processes with equation-specific mean values and autoregressive coefficients.
ii) it develops an efficient estimation algorithm for the model which proceeds equation by equation and combines the traditional Kalman filter approach with the recent precision sampler methodology.
iii) it develops extensions to estimate endogenously the mean values and autoregressive coefficients associated with each dynamic process.
iv) through a case study of the Great Recession in four major economies (Canada, the Euro Area, Japan and the United States), it establishes that forecast accuracy can be significantly improved by using the proposed general time-varying model and its extensions in place of the traditional random walk specification.

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1 Introduction

Vector autoregressive models have become the cornerstone of applied macroeconomics. Since the seminal work of Sims (1980), they have been used extensively by financial and economic institutions to perform routine policy analysis and forecasts. While convenient, VAR models with static coefficients and volatility often turn out to be excessively restrictive in capturing the dynamics of time-series, which typically exhibit some form of non-linearity in their behaviours. This motivated the introduction of time-varying coefficients (Canova (1993), Stock and Watson (1996), Cogley (2001), Ciccarelli and Rebucci (2003)) and stochastic volatility (Harvey et al. (1994), Jacquier et al. (1995), Uhlig (1997), Chib et al. (2006)), to account for potential shifts in the transmission mechanism and variance of the underlying disturbances. More recently, the two features have been combined (Cogley and Sargent (2005), Primiceri (2005)) to produce a class of fully time-varying VAR models.

With the events of the Great Recession, time-varying VARs have attracted renewed attention. Part of the literature has focused on the heteroskedasticity of the exogenous shocks (Stock and Watson (2012), Doh and Connolly (2013), Bijsterbosch and Falagiarda (2014), Gambetti and Musso (2017)), primarily interpreting the Great Recession as an episode of sharp volatility of the disturbances affecting the economy. Rather, other works have emphasized the changes in the transmission mechanism (Baumeister and Benati (2010), Benati and Lubik (2014), Ellington et al. (2017)), and view the Great Recession as a period of altered response of macroeconomic variables to economic policy. In either case, there is strong evidence that accounting for time variation is crucial to the accuracy of policy analysis and forecasts in a context of crisis. Consequently, time-varying VARs should represent a benchmark tool to predict economic downturns and their evolutions.

However, these models remain subject to limitations of both theoretical and methodological order. On the theoretical side, the literature has widely adopted the random walk specification for the laws of motion of the different dynamic parameters. Denoting for instance by θ_t any one of the dynamic parameters of the VAR model and by ϵ_t the shock on the process, a random walk law of motion expresses as:

$$\theta_t = \theta_{t-1} + \epsilon_t \qquad \text{or, equivalently} \qquad \theta_t = \sum_{j=0}^{\infty} \epsilon_{t-j} \tag{1}$$

While convenient and parsimonious, a random walk specification like (1) may be inadequate for several reasons. First, it implies that the range of values taken by θ_t increases over time and becomes eventually unbounded, resulting in explosive behaviour in the limit. This is at odd with both empirical observations and economic theory which suggest that in the long run, the behaviour of an economy should reach some equilibrium. In other words, a formulation like (1) is unlikely to constitute a good representation of the underlying data generating process. Second, and perhaps more importantly, a random walk specification may prove inadequate for forecasting purposes. As made clear by the right-hand side of (1), the random walk grants equal weight to all past shocks. Yet, time-varying models are typically intended for capturing the latest developments on some dynamic process. This supposes to account primarily for the effects of the most recent disturbances on the process, while granting less weight to past shocks.

A number of attempts have been made to restrict the undesirable properties of the random walk (Ciccarelli and Rebucci (2002), Koop and Korobilis (2010), Nakajima and West (2015), Eisenstat et al. (2016)). These approaches typically address the first issue, but not the second one. They also involve the estimation of a large number of additional parameters, which may generate parsimony issues and substantially complicate the estimation procedure. For these reasons, it is preferable to simply replace (1) by a stationary formulation of the kind:

$$\theta_t = (1-\gamma)\bar{\theta} + \gamma\theta_{t-1} + \epsilon_t \qquad \text{or, equivalently} \qquad \theta_t = \bar{\theta} + \sum_{j=0}^{\infty} \gamma^j \epsilon_{t-j} \tag{2}$$

with $0 \leq \gamma \leq 1$ and $\bar{\theta}$ respectively denoting the autoregressive coefficient and mean terms of the process. Formulation (2) addresses both issues related to the random walk, while remaining conceptually simple. A final issue related with the random walk specification is the homogeneity assumption it sets de facto, as it implies that all the dynamic parameters follow a similar unit-root process. There is yet no legitimate reason to assume that the dynamic parameters of different variables evolve homogeneously. In fact, it is quite likely that different economic variables are characterised by different behaviours of their dynamic coefficients and residual volatilities. For this reason, an equation-by-equation approach seems preferable for the dynamic processes. This leads to reformulate (2) as:

$$\theta_{i,t} = (1-\gamma_i)\bar{\theta}_i + \gamma_i\theta_{i,t-1} + \epsilon_{i,t} \qquad \text{or, equivalently} \qquad \theta_{i,t} = \bar{\theta}_i + \sum_{j=0}^{\infty} \gamma_i^j \epsilon_{i,t-j} \qquad i = 1, \ldots, n \tag{3}$$

where $\theta_{i,t}$ represents the component of θ_t for equation i of the model, and γ_i and $\bar{\theta}_i$ respectively denote the equation-specific autoregressive coefficient and mean terms. This specification is sufficiently flexible to capture the essential specificities of the behaviours of the different variables included in the model. It creates however new challenges as it becomes crucial to determine the values of γ_i and $\bar{\theta}_i$ properly. This question has attracted considerable attention in the univariate ARCH literature (Jacquier et al. (1994), Kim et al. (1998), Chib et al. (2002), Jacquier et al. (2004)), while contributions on the multivariate side have been more limited. Ciccarelli and Rebucci (2003), Prado and West (2010) and Lubik and Matthes (2015) propose a general stationary formulation for the law of motion of their time-varying VAR models, but retain the random walk for estimation. Clark and Ravazzolo (2015) test for a stationary stochastic volatility specification with inconclusive results compared to the random walk. Another option consists in estimating the parameters endogenously. Yet limited work has been done in this direction. In a first attempt to determine the mean of the structural shock volatility, Uhlig (1997) relies on a set of Beta prior distributions. Primiceri (2005) questions the random walk assumption and tests for exogenous estimation of the autoregressive coefficients on the dynamic processes. He concludes that no relevant differences exist compared to the homogeneous random walk specification. Mumtaz and Zanetti (2013) endogenously estimate the autoregressive coefficients on stochastic volatility, and obtain coefficients close to the random walk.

On the methodological side, time-varying VAR models have been criticised for their inefficiency in terms of estimation. Aside from a limited number of contributions relying on non-Bayesian methods (Delle Monache and Petrella (2016), Kapetanios et al. (2017), Gorgi et al. (2017)) and quasi-Bayesian methods (Petrova (2018)), the Bayesian methodology has been widely adopted by the literature for its flexibility. So far the benchmark methodology relies on the state-space formulation proposed by Primiceri (2005) and amended by Del Negro and Primiceri (2015). This approach builds on the algorithm developed by Carter and Kohn (1994), which uses a two-pass procedure based on the Kalman filter. The procedure is rather sophisticated and unintuitive. Also, the multiple loops through time and the building of the states in a recursive fashion may considerably slow down the estimation. This is especially true for large models for which the Primiceri (2005) approach becomes very inefficient. Yet the recent literature has emphasized the importance of large information sets (Banbura et al. (2010), Carriero et al. (2015), Giannone et al. (2015), Kalli and Griffin (2018)), establishing that large systems perform better than smaller systems in forecasting and structural analysis.

Different strategies have been adopted to overcome this inefficiency issue. Carriero et al. (2016) propose to estimate their large Bayesian VAR model equation by equation rather than jointly. Doing so considerably reduces the computational complexity of the estimation algorithm, rendering the estimation of large VARs feasible. Nevertheless, their model is only partially time-varying as it involves stochastic volatility but leaves the residual covariance and VAR coefficient parts of the model static. Hence, it is not yet established how much efficiency gains can be obtained from their methodology once applied to a fully time-varying model of the kind of Primiceri (2005). An alternative strategy has been proposed by Chan and Eisenstat (2018). The authors develop a precision sampler which replaces the traditional Carter and Kohn (1994) algorithm with a full sample formulation relying on sparse matrices. Significant efficiency gains are reported (of the order of 15-30%). Nevertheless, the few papers using the methodology so far (Chan and Jeliazkov (2009), Chan (2013)) have been limited to small dimensional parameters, and it is yet uncertain how well the precision sampler performs in larger dimensions. As the two approaches are not mutually exclusive, a natural strategy suggests to combine them in the hope of optimising the efficiency of the estimation procedure.

Based on these considerations, this paper contributes to the literature in four directions. First, it introduces a general, fully time-varying VAR model which is formulated on an equation by equation basis. For each dynamic parameter, the random walk assumption is relaxed and replaced with a general autoregressive process with equation-specific mean values and autoregressive coefficients. Second, it proposes an optimal sampling algorithm for the model which combines the equation by equation estimation procedure of Carriero et al. (2016) with the precision sampler of Chan and Eisenstat (2018) and the traditional Carter and Kohn (1994) methodology[1]. It shows that the procedure provides considerable efficiency gains, even on large models. Third, it proposes extensions to endogenously estimate the mean terms and autoregressive coefficients associated with the laws of motion of each dynamic parameter. The employed priors are informative and aim at getting closer to the underlying data generating process. Finally, the paper conducts a case study on the Great Recession. The exercise is realised on a large time-varying VAR comprising 12 variables, estimated for four major economies (Canada, the European Union, Japan and the United States). It establishes that the random walk is unambiguously rejected as a suitable formulation for forecasting purposes, and further shows that the extensions outper-

[1]Note that the equation by equation formulation of the model on the one hand, and the equation by equation estimation procedures of Carriero et al. (2016) on the other hand represent two complementary but distinct aspects of the model. The former is related to the specification of the model and concerns forecast accuracy, while the latter is related to the computational efficiency of the estimation algorithm. It is possible to include one of these features in the model, but not the other. For instance, the dynamic parameters could be formulated equation by equation but estimated jointly. Conversely, the dynamic parameters could be formulated homogeneously but estimated equation by equation, as done in Carriero et al. (2016).

form the base stationary formulation in terms of forecast accuracy. Following, it suggests that the crisis could have been better predicted with a proper use of time-varying VAR models.

The remaining of the paper is organised as follows: section 2 introduces the general time-varying model and provides the details of the estimation procedures; section 3 discusses the efficiency of different competing methodologies and introduce the optimal sampling algorithm; section 4 develops the extensions allowing for endogenous estimation of the autoregressive coefficients (random inertia) and mean terms (random mean) of the dynamic parameters; section 5 presents the results of the case study on the Great Recession and discusses the benefits of the general time-varying model and its extensions in terms of forecast accuracy; section 6 concludes.

2 A general time-varying model

2.1 The model

Consider the general time-varying model:

$$y_t = C_t z_t + A_{1,t} y_{t-1} + \cdots + A_{p,t} y_{t-p} + \varepsilon_t \qquad t = 1, \cdots, T \qquad , \qquad \varepsilon_t \sim \mathcal{N}(0, \Sigma_t) \tag{4}$$

y_t is a $n \times 1$ vector of observed endogenous variables, z_t is a $m \times 1$ vector of observed exogenous variables such as constant or trends, and ε_t is a $n \times 1$ vector of reduced-form residuals. The residuals are heteroskedastic disturbances following a normal distribution with variance-covariance matrix Σ_t. $C_t, A_{1,t}, \cdots, A_{p,t}$ are matrices of time-varying VAR coefficients comfortable with z_t and the lagged values of y_t. Stacking in a vector β_t the set of VAR coefficients, (4) rewrites:

$$y_t = X_t \beta_t + \varepsilon_t \tag{5}$$

with:

$$X_t = I_n \otimes x_t \ , \ x_t = \begin{pmatrix} z_t' & y_{t-1}' & \cdots & y_{t-p}' \end{pmatrix} \ , \ \beta_t = vec(B_t) \ , \ B_t = \begin{pmatrix} C_t & A_{1,t} & \cdots & A_{p,t} \end{pmatrix}' \tag{6}$$

Considering specifically row i of (5), the equation for variable i of the model rewrites:

$$y_{i,t} = x_t \beta_{i,t} + \varepsilon_{i,t} \tag{7}$$

where $\beta_{i,t}$ is the $k \times 1$ vector obtained from column i of B_t. Stacking (7) over the T sample periods yields a full sample formulation for equation i:

$$y_i = X \beta_i + \varepsilon_i \tag{8}$$

with:

$$y_i = \begin{pmatrix} y_{i,1} \\ y_{i,2} \\ \vdots \\ y_{i,T} \end{pmatrix} \quad , \quad X = \begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & x_T \end{pmatrix} \quad , \quad \beta_i = \begin{pmatrix} \beta_{i,1} \\ \beta_{i,2} \\ \vdots \\ \beta_{i,T} \end{pmatrix} \quad , \quad \varepsilon_i = \begin{pmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \\ \vdots \\ \varepsilon_{i,T} \end{pmatrix} \tag{9}$$

The variance-covariance matrix Σ_t for the reduced form residuals is decomposed into:

$$\Delta_t \Sigma_t \Delta_t' = \Lambda_t \qquad \Leftrightarrow \qquad \Sigma_t = \Delta_t^{-1} \Lambda_t \Delta_t^{-1\prime} \tag{10}$$

Δ_t (and Δ_t^{-1}) are unit lower triangular matrix, while Λ_t is a diagonal matrix with positive diagonal entries, taking the form:

$$\Delta_t = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \delta_{21,t} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \delta_{n1,t} & \cdots & \delta_{n(n-1),t} & 1 \end{pmatrix}, \ \Lambda_t = \begin{pmatrix} s_1 \ exp(\lambda_{1,t}) & 0 & \cdots & 0 \\ 0 & s_2 \ exp(\lambda_{2,t}) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & s_n \ exp(\lambda_{n,t}) \end{pmatrix} \tag{11}$$

The triangular decomposition of the variance-covariance matrix Σ_t implemented in (10) is common in time-series models.[2] Λ_t represents the volatility components of Σ_t, each s_i being a positive scaling term which represents the equilibrium value of the residual variance of equation i of the model. On the other hand, Δ_t can be interpreted as the (inverse) covariance component of Σ_t. Denoting by $\delta_{i,t}$ the vector of non-zero and non-one terms in row i of Δ_t so that $\delta_{i,t} = (\delta_{i1,t} \ \ \cdots \ \ \delta_{i(i-1),t})'$, $\delta_{i,t}$ then represents the covariance between the residual of equation i of the model and the other shocks.

The dynamics of the model time-varying parameters is specified as follows:

$$\begin{aligned}
&\beta_{i,t} = (1-\rho_i)b_i + \rho_i\beta_{i,t-1} + \xi_{i,t} && t = 2,3,\ldots,T && \xi_{i,t} \sim \mathcal{N}(0,\Omega_i) \\
&\beta_{i,1} = b_i + \xi_{i,1} && t = 1 && \xi_{i,1} \sim \mathcal{N}(0,\tau\Omega_i) \\
&\lambda_{i,t} = \gamma_i\lambda_{i,t-1} + \nu_{i,t} && t = 2,3,\ldots,T && \nu_{i,t} \sim \mathcal{N}(0,\phi_i) \\
&\lambda_{i,1} = \nu_{i,1} && t = 1 && \nu_{i,1} \sim \mathcal{N}(0,\mu\phi_i) \\
&\delta_{i,t} = (1-\alpha_i)d_i + \alpha_i\delta_{i,t-1} + \eta_{i,t} && t = 2,3,\ldots,T && \eta_{i,t} \sim \mathcal{N}(0,\Psi_i) \\
&\delta_{i,1} = d_i + \eta_{i,1} && t = 1 && \eta_{i,1} \sim \mathcal{N}(0,\epsilon\ \Psi_i)
\end{aligned} \tag{12}$$

ρ_i, γ_i and α_i represent equation-specific autoregressive coefficients while b_i, s_i and d_i represent the equation-specific mean values of the processes. These are treated for now as exogenously set hyperparameters, but the assumption will be relaxed in section 4. Clearly, each law of motion nests the usual random walk as a special case setting the autoregressive coefficient to 1. For each process, the initial period is formulated consistently with the overall dynamics of the parameters. The mean corresponds to the unconditional expectation of the process, while the variance is scaled by the hyperparameters $\tau, \mu, \epsilon > 1$ to account for the greater uncertainty associated with the initial period. All the innovations in the model are assumed to be jointly normally distributed with the following assumptions on the variance covariance matrix:

$$Var\begin{pmatrix} \varepsilon_t \\ \xi_{i,t} \\ \nu_{i,t} \\ \eta_{i,t} \end{pmatrix} = \begin{pmatrix} \Sigma_t & 0 & 0 & 0 \\ 0 & \Omega_i & 0 & 0 \\ 0 & 0 & \phi_i & 0 \\ 0 & 0 & 0 & \Psi_i \end{pmatrix} \tag{13}$$

This concludes the description of the model. For $i = 1,\ldots,n$, the parameters of interest to be estimated are: the dynamic VAR coefficients β_i ; the dynamic volatility terms λ_i; the dynamic covariance terms δ_i; and the associated variance-covariance parameters Ω_i, ϕ_i and Ψ_i. To these six base parameters must be added the parameter $r_{i,t}$ whose role will be clarified shortly.

[2] As discussed by Carriero et al. (2016), the triangular decomposition employed in (10) is used only as an estimation device and does not imply any structural identification. The triangular factorisation does however imply that the ordering of the variables may affect the joint posterior distribution of the model. This is not a specificity of this particular model, but rather a feature which is inherent to all models using the factorisation (10).

2.2 Bayes rule

Following most of the literature, Bayesian methods are used to evaluate the posterior distributions of the parameters of interest. Given the model, Bayes rule is given by:

$$\pi(\beta,\Omega,\lambda,\phi,\delta,\Psi,r|y) \propto f(y|\beta,\lambda,\delta,r)$$
$$\times \left(\prod_{i=1}^{n}\pi(\beta_i|\Omega_i)\pi(\Omega_i)\right)\left(\prod_{i=1}^{n}\pi(\lambda_i|\phi_i)\pi(\phi_i)\right)\left(\prod_{i=2}^{n}\pi(\delta_i|\Psi_i)\pi(\Psi_i)\right)\left(\prod_{i=1}^{n}\prod_{t=1}^{T}\pi(r_{i,t})\right) \tag{14}$$

2.3 Likelihood function

Starting from (5) and rearranging, a first formulation of the likelihood function obtains as:

$$f(y|\beta,\lambda,\delta,r) = (2\pi)^{-nT/2}\left(\prod_{i=1}^{n} s_i^{-T/2}\right)$$
$$\times\ exp\left(-\frac{1}{2}\sum_{i=1}^{n}\left\{\lambda_i' 1_T + (y_i - X\beta_i + \mathcal{E}_i\,\delta_i)'\ s_i^{-1}\tilde{\Lambda}_i\ (y_i - X\beta_i + \mathcal{E}_i\,\delta_i)\right\}\right) \tag{15}$$

with:

$$\lambda_i = \begin{pmatrix}\lambda_{i,1}\\ \lambda_{i,2}\\ \vdots\\ \lambda_{i,T}\end{pmatrix} \qquad 1_T = \begin{pmatrix}1\\1\\ \vdots\\ 1\end{pmatrix} \qquad \tilde{\Lambda}_i = diag(\tilde{\lambda}_i) \qquad \tilde{\lambda}_i = \begin{pmatrix}exp(-\lambda_{i,1})\\ exp(-\lambda_{i,2})\\ \vdots\\ exp(-\lambda_{i,T})\end{pmatrix}$$
$$\mathcal{E}_i = \begin{pmatrix}\varepsilon_{-i,1}' & 0 & \cdots & 0\\ 0 & \varepsilon_{-i,2}' & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & \varepsilon_{-i,T}'\end{pmatrix} \qquad \varepsilon_{-i,t} = \begin{pmatrix}\varepsilon_{1,t}\\ \varepsilon_{2,t}\\ \vdots\\ \varepsilon_{i-1,t}\end{pmatrix} \qquad \delta_i = \begin{pmatrix}\delta_{i,1}\\ \delta_{i,2}\\ \vdots\\ \delta_{i,T}\end{pmatrix} \tag{16}$$

(15) proves convenient for the estimation of β_i and δ_i, but does not provide any conjugacy for λ_i due to the presence of the exponential term $\tilde{\Lambda}_i$. This is a well-known issue of models with stochastic volatility and the most efficient solution is the so-called normal offset mixture representation proposed by Kim et al. (1998). The procedure consists in reformulating the likelihood function in terms of the transformed shock $e_t = (\Delta_t^{-1}\Lambda_t^{1/2})^{-1}\varepsilon_t$. It is trivially shown that e_t is a vector of structural shock with $e_t \sim \mathcal{N}(0, I_n)$. Considering specifically the shock $e_{i,t}$ in the vector, squaring, taking logs and rearranging eventually yields:

$$\hat{e}_{i,t} = \log(e_{i,t}^2) = \hat{y}_{i,t} - \lambda_{i,t} \qquad \hat{y}_{i,t} = \log(s_i^{-1}(\varepsilon_{i,t} + \delta_{i,t}'\varepsilon_{-i,t})^2) \tag{17}$$

$\hat{e}_{i,t}$ follows a log chi-squared distribution which does not grant any conjugacy. Kim et al. (1998) thus propose to approximate the shock as an offset mixture of normal distributions. The approximation is given by:

$$\hat{e}_{i,t} \approx \sum_{j=1}^{7}\mathbb{1}(r_{i,t}=j)\ z_j \quad , \quad z_j \sim \mathcal{N}(m_j, v_j) \quad , \quad Pr(r_{i,t}=j) = q_j \tag{18}$$

The values for m_j, v_j and q_j can be found in Table 4 of Kim et al. (1998). The constants m_j and v_j respectively represent the mean and variance components of the normally distributed random variable z_j. $r_{i,t}$ is a categorical random variable taking discrete values $j = 1, \dots, 7$, the probability of obtaining each value being equal to q_j. Finally, $\mathbb{1}(r_{i,t} = j)$ is an indicator function taking a value of 1 if $r_{i,t} = j$, and a value of 0 otherwise. To draw from the log chi-squared distribution, the mixture first randomly draws a value for $r_{i,t}$ from its categorical distribution; once $r_{i,t}$ is known, its value determines which component z_j of the mixture is selected. $\hat{e}_{i,t}$ then turns into a regular normal random variable with mean m_j and variance v_j. Given (17) and the offset mixture (18), an approximation of the likelihood function obtains as:

$$f(y|\beta, \lambda, \delta, r) = \prod_{i=1}^{n} \prod_{t=1}^{T} \sum_{j=1}^{7} \mathbb{1}(r_{i,t} = j) \left\{ (2\pi v_j)^{-1/2} \exp\left(-\frac{1}{2} \frac{(\hat{y}_{i,t} - \lambda_{i,t} - m_j)^2}{v_j} \right) \right\} \tag{19}$$

For the estimation of λ_i, a more convenient joint formulation can be adopted. Defining $r_i = (r_{i,1} \quad \dots \quad r_{i,T})'$, denoting by J any possible value for r_i, by m_J and v_J the resulting mean and variance vectors, and defining $V_J = diag(v_J)$, the likelihood function rewrites as a mixture of multivariate normal distributions:

$$\begin{aligned} & f(y|\beta, \lambda, \delta, r) \\ = & \prod_{i=1}^{n} \sum^{J} \mathbb{1}(r_i = J) \left\{ (2\pi)^{-T/2} |V_J|^{-1/2} exp\left(-\frac{1}{2} (\hat{y}_i - \lambda_i - m_J)' V_J^{-1} (\hat{y}_i - \lambda_i - m_J) \right) \right\} \end{aligned} \tag{20}$$

with:

$$\hat{y}_i = \begin{pmatrix} \hat{y}_{i,1} & \hat{y}_{i,2} & \dots & \hat{y}_{i,T} \end{pmatrix}' = log(s_i^{-1} \mathcal{Q}_i) \qquad \mathcal{Q}_i = (\varepsilon_i + \mathcal{E}_i \, \delta_i)^2 \tag{21}$$

2.4 Priors

The priors for the dynamic parameters β_i, λ_i and δ_i follow the precision sampler formulation of Chan and Eisenstat (2018).[3] Consider first the VAR coefficients β_i. Starting from (12), the law of motion can be expressed in compact form as:

$$\begin{pmatrix} I_k & 0 & \cdots & 0 \\ -\rho_i I_k & I_k & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & -\rho_i I_k & I_k \end{pmatrix} \begin{pmatrix} \beta_{i,1} \\ \beta_{i,2} \\ \vdots \\ \beta_{i,T} \end{pmatrix} = \begin{pmatrix} b_i \\ (1-\rho_i) b_i \\ \vdots \\ (1-\rho_i) b_i \end{pmatrix} + \begin{pmatrix} \xi_{i,1} \\ \xi_{i,2} \\ \vdots \\ \xi_{i,T} \end{pmatrix} \tag{22}$$

or:

$$(F_i \otimes I_k) \; \beta_i = \bar{b}_i + \xi_i \qquad F_i = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ -\rho_i & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & -\rho_i & 1 \end{pmatrix} \qquad \bar{b}_i = \begin{pmatrix} b_i \\ (1-\rho_i) b_i \\ \vdots \\ (1-\rho_i) b_i \end{pmatrix} \qquad \xi_i = \begin{pmatrix} \xi_{i,1} \\ \xi_{i,2} \\ \vdots \\ \xi_{i,T} \end{pmatrix} \tag{23}$$

[3]The alternative methodology of Carter and Kohn (1994) does not require an explicit derivation of the priors.

Also:

$$Var(\xi_i) = \begin{pmatrix} \tau\Omega_i & 0 & \cdots & 0 \\ 0 & \Omega_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Omega_i \end{pmatrix} = I_\tau \otimes \Omega_i \qquad I_\tau = \begin{pmatrix} \tau & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix} \tag{24}$$

(23) and (24) respectively imply $\beta_i = (F_i \otimes I_k)^{-1}\bar{b}_i + (F_i \otimes I_k)^{-1}\xi_i$ and $\xi_i \sim \mathcal{N}(0, I_\tau \otimes \Omega_i)$. From this and rearranging, the prior distribution eventually obtains as:

$$\pi(\beta_i|\Omega_i) \sim \mathcal{N}\left(\beta_{i0}\ ,\Omega_{i0}\right) \qquad \beta_{i0} = 1_T \otimes b_i \qquad \Omega_{i0} = (F_i'\ I_\tau^{-1}\ F_i \otimes \Omega_i^{-1})^{-1} \tag{25}$$

Using for λ_i and δ_i equivalent procedures and notations, it is straightforward to obtain:

$$\begin{aligned} &\pi(\lambda_i|\phi_i) \sim \mathcal{N}\left(0, \Phi_{i0}\right) && \Phi_{i0} = \phi_i(G_i' I_\mu^{-1} G_i)^{-1} \\ &\pi(\delta_i|\Psi_i) \sim \mathcal{N}\left(\delta_{i0}\ , \Psi_{i0}\right) && \delta_{i0} = 1_T \otimes d_i \qquad \Psi_{i0} = (H_i'\ I_\epsilon^{-1}\ H_i \otimes \Psi_i^{-1})^{-1} \end{aligned} \tag{26}$$

For the priors of the variance-covariance parameters Ω_i, ϕ_i and Ψ_i, the choice is that of standard inverse Wishart and inverse Gamma distributions. Precisely:

$$\pi(\Omega_i) \sim IW\left(\zeta_0, \Upsilon_0\right) \qquad \pi(\phi_i) \sim IG\left(\frac{\kappa_0}{2}, \frac{\omega_0}{2}\right) \qquad \pi(\Psi_i) \sim IW\left(\varphi_0, \Theta_0\right) \tag{27}$$

Finally, from (18), it is immediate that the prior distribution for $r_{i,t}$ is categorical:

$$\pi(r_{i,t}) \sim Cat(q_1, \ldots, q_7) \tag{28}$$

2.5 Posteriors for the dynamic parameters

The joint posterior obtained from (14) is analytically intractable. Following standard practices, the marginal posteriors are estimated from a Gibbs sampling algorithm relying on conditional distributions. The conditional posteriors of the dynamic parameters are first derived in the context of the precision sampler of Chan and Eisenstat (2018).

For β_i, Bayes rule (14) implies $\pi(\beta_i|y, \backslash\beta_i) \propto f(y|\beta, \lambda, \delta, r)\pi(\beta_i|\Omega_i)$.[4] From the likelihood (15), the prior (25) and rearranging, it follows that:

$$\begin{aligned} &\pi(\beta_i|y, \backslash\beta_i) \sim \mathcal{N}(\bar{\beta}_i, \bar{\Omega}_i) \qquad \text{with:} \\ &\bar{\Omega}_i = (s_i^{-1} X'\tilde{\Lambda}_i X + F_i'\ I_\tau^{-1}\ F_i \otimes \Omega_i^{-1})^{-1} \\ &\bar{\beta}_i = \bar{\Omega}_i(s_i^{-1} X'\tilde{\Lambda}_i[y_i + \mathcal{E}_i\,\delta_i] + F_i' I_\tau^{-1} F_i 1_T \otimes \Omega_i^{-1} b_i) \end{aligned} \tag{29}$$

For λ_i, Bayes rule (14) implies $\pi(\lambda_i|y, \backslash\lambda_i) \propto f(y|\beta, \lambda, \delta, r)\pi(\lambda_i|\phi_i)$. From the approximate likelihood (20), the prior (26) and rearranging, it follows that:

$$\begin{aligned} &\pi(\lambda_i|y, \backslash\lambda_i) \sim \mathcal{N}(\bar{\lambda}_i, \bar{\Phi}_i) \qquad \text{with:} \\ &\bar{\Phi}_i = (V_J^{-1} + \phi_i^{-1} G_i' I_\mu^{-1} G_i)^{-1} \qquad \bar{\lambda}_i = \bar{\Phi}_i(V_J^{-1}[\hat{y}_i - m_J]) \end{aligned} \tag{30}$$

[4] For θ_i any parameter, $\pi(\theta_i|\backslash\theta_i)$ is used to denote the density of θ_i conditional on all the model parameters except θ_i.

For δ_i, Bayes rule (14) implies $\pi(\delta_i|y,\backslash\delta_i) \propto f(y|\beta,\lambda,\delta,r)\pi(\delta_i|\Psi_i)$. From the likelihood (15), the prior (26) and rearranging, it follows that:

$$\pi(\delta_i|y,\backslash\delta_i) \sim \mathcal{N}(\bar{\delta}_i,\bar{\Psi}_i) \qquad \text{with:}$$
$$\bar{\Psi}_i = (s_i^{-1}\,\mathcal{E}_i'\,\tilde{\Lambda}_i\,\mathcal{E}_i + H_i'\;I_{-\epsilon}\;H_i \otimes \Psi_i^{-1})^{-1} \qquad \bar{\delta}_i = \bar{\Psi}_i(-s_i^{-1}\,\mathcal{E}_i'\,\tilde{\Lambda}_i\varepsilon_i + H_i'I_{-\epsilon}H_i 1_T \otimes \Psi_i^{-1}d_i) \quad (31)$$

For incoming developments, it is worth mentioning that as an alternative to the precision sampler, the conditional posteriors for the dynamic parameters can be derived from the algorithm of Carter and Kohn (1994). The algorithm is standard and the details are deferred to Appendix A in order to save space.

2.6 Posteriors for the other parameters

For Ω_i, Bayes rule (14) implies $\pi(\Omega_i|y,\backslash\Omega_i) \propto \pi(\beta_i|\Omega_i)\pi(\Omega_i)$. From the priors (25) and (27) then rearranging, it follows that:

$$\pi(\Omega_i|y,\backslash\Omega_i) \sim IW(\bar{\zeta},\bar{\Upsilon}_i) \qquad \text{with:} \qquad \bar{\zeta} = T + \zeta_0 \qquad \bar{\Upsilon}_i = \tilde{B}_i + \Upsilon_0$$
$$\tilde{B}_i = (B_i - 1_T' \otimes b_i)\;(F_i'\;I_\tau^{-1}\;F_i)(B_i - 1_T' \otimes b_i)' \qquad B_i = (\beta_{i,1} \quad \beta_{i,2} \quad \cdots \quad \beta_{i,T}) \quad (32)$$

For ϕ_i, Bayes rule (14) implies $\pi(\phi_i|y,\backslash\phi_i) \propto \pi(\lambda_i|\phi_i)\pi(\phi_i)$. From the priors (26) and (27) then rearranging, it follows that:

$$\pi(\phi_i|y,\backslash\phi_i) \sim IG(\bar{\kappa},\bar{\omega}_i) \qquad \text{with:} \qquad \bar{\kappa} = \frac{T+\kappa_0}{2} \qquad \bar{\omega}_i = \frac{\lambda_i'(G_i'I_\mu^{-1}G_i)\lambda_i + \omega_0}{2} \quad (33)$$

For Ψ_i, Bayes rule (14) implies $\pi(\Psi_i|y,\backslash\Psi_i) \propto \pi(\delta_i|\Psi_i)\pi(\Psi_i)$. From the priors (26) and (27) then rearranging, it follows that:

$$\pi(\Psi_i|y,\backslash\Psi_i) \sim IW(\bar{\varphi},\bar{\Theta}_i) \qquad \text{with:} \qquad \bar{\varphi} = T + \varphi_0 \qquad \bar{\Theta}_i = \tilde{D}_i + \Theta_0$$
$$\tilde{D}_i = (D_i - 1_T' \otimes d_i)(H_i'\;I_{-\epsilon}\;H_i)(D_i - 1_T' \otimes d_i)' \qquad D_i = (\delta_{i,1} \quad \delta_{i,2} \quad \cdots \quad \delta_{i,T}) \quad (34)$$

Finally, for $r_{i,t}$, Bayes rule (14) implies $\pi(r_{i,t}|y,\backslash r_{i,t}) \propto f(y|\beta,\lambda,\delta,r)\pi(r_{i,t})$. From the approximate likelihood (19) and the prior (28), it follows immediately that:

$$\pi(r_{i,t}|y,\backslash r_{i,t}) \sim Cat(\bar{q}_1,\ldots,\bar{q}_7) \quad \text{with:} \quad \bar{q}_j = (2\pi v_j)^{-1/2}\exp\left(-\frac{1}{2}\frac{(\hat{y}_{i,t}-\lambda_{i,t}-m_j)^2}{v_j}\right)q_j \quad (35)$$

2.7 MCMC algorithm

A preliminary, naive version of the MCMC algorithm for the general time-varying model is now introduced. This version fully relies on the precision sampler procedure, and its performance is discussed in the incoming section. The algorithm consists in a 7-step procedure, as follows:

Algorithm 1: MCMC algorithm for the general time-varying model:

1. For $i = 1,\ldots,n$, sample λ_i equation by equation from: $\pi(\lambda_i|y,\backslash\lambda_i) \sim \mathcal{N}(\bar{\lambda}_i,\bar{\Phi}_i)$.

2. For $i = 1,\ldots,n$, sample β_i equation by equation from: $\pi(\beta_i|y,\backslash\beta_i) \sim \mathcal{N}(\bar{\beta}_i,\bar{\Omega}_i)$.

3. For $i = 2,\ldots,n$, sample δ_i equation by equation from: $\pi(\delta_i|y,\backslash\delta_i) \sim \mathcal{N}(\bar{\delta}_i,\bar{\Psi}_i)$.

4. For $i = 1, \ldots, n$, sample Ω_i equation by equation from: $\pi(\Omega_i|y, \backslash\Omega_i) \sim IW(\bar{\zeta}, \bar{\Upsilon}_i)$.

5. For $i = 1, \ldots, n$, sample ϕ_i equation by equation from: $\pi(\phi_i|y, \backslash\phi_i) \sim IG(\bar{\kappa}, \bar{\omega}_i)$.

6. For $i = 2, \ldots, n$, sample Ψ_i equation by equation from: $\pi(\Psi_i|y, \backslash\Psi_i) \sim IW(\bar{\varphi}, \bar{\Theta}_i)$.

7. For $i = 1, \ldots, n$ and $t = 1, \ldots, T$, sample $r_{i,t}$ from: $\pi(r_{i,t}|y, \backslash r_{i,t}) \sim Cat(\bar{q}_1, \ldots, \bar{q}_7)$.

Observe that the ordering of the steps in the algorithm differs from the one used for the presentation of the model. It introduces λ_i first, then the other model parameters, and eventually the offset mixture parameters $r_{i,t}$. This specific ordering is necessary to recover the correct posterior distribution whenever the normal offset mixture is used to provide an approximation of the likelihood function. See Del Negro and Primiceri (2015) for details.

3 Efficiency

3.1 Preliminary comparison

As a preliminary exercise, this section discusses the computational efficiency of the MCMC algorithm developed in the previous section for the general time-varying model against a number of competing methodologies and different scales of models. The exercise is based on a quarterly macroeconomic model for the US economy, the details of which are introduced in section 5[5]. Three versions of the model are considered. The first is a "small" version of the model which corresponds to the small US economy model of Primiceri (2005) and includes three variables, two lags and a constant. The second "medium" model comprises six variables and three lags. The final "large" model expands the setting to twelve variables and four lags[6]. The three models are estimated on a quarterly sample of size $T = 160$.

The exercise compares four competing estimation methodologies. The benchmark methodology, labelled as method 1, consists in the general time-varying model introduced in the previous section, and estimated with Algorithm 1. Again, this procedure combines the equation by equation approach of Carriero et al. (2016) with the precision sampler of Chan and Eisenstat (2018). Following, a natural candidate for comparison consists in a similar model, but estimated jointly rather than equation by equation, and relying on the standard Kalman filter approach rather than on the precision sampler. This corresponds to the standard Primiceri (2005) approach, labelled as method 4. Two other in-between methodologies are considered for the sake of highlighting the respective contributions of the precision sampler and equation by equation approaches. Similarly to Carriero et al. (2016), method 2 adopts an equation by equation procedure but relies on the Kalman filter approach rather than on the precision sampler. At the opposite, in line with Chan and Eisenstat (2018), method 3 uses the precision sampler but estimates the model jointly rather

[5]The reader is thus referred to section 5 for a complete presentation of the model, its calibration and a description of the set of variables included along with their transformations.

[6]Note that a fully time-varying VAR with 12 variables effectively represents a large model. The literature has produced a number of arguably larger time-varying settings, for instance Banbura et al. (2010) (26 variables) or Carriero et al. (2016) (125 variables). These models however include considerably smaller time-varying components: 3 and 125 parameters per sample period respectively for Banbura et al. (2010) and Carriero et al. (2016), against 666 coefficients per sample period for the large model developed here. Other large time-varying models like Koop and Korobilis (2013) (25 variables) or Koop et al. (2018) (129 variables) rely on some simplifying device to keep estimation feasible, and are thus not comparable with the present approach.

than equation by equation. The results for 10000 repetitions of the algorithm are reported in Table 1[7].

	Method 1 equation by equation precision sampler	Method 2 (Carriero et al.) equation by equation Kalman filter	Method 3 (Chan et al.) jointly estimated precision sampler	Method 4 (Primiceri) jointly estimated Kalman filter
Small model	**1m 2s**	3m 52s (× 3.74)	3m 34s (× 3.45)	4m 47s (× 4.62)
Medium model	**14m**	**14m**	1h 12m 50s (× 5.20)	33m 40s (× 2.40)
Large model	2h 37m (× 2.08)	**1h 16m**	80d 1h (× 1523)	1d 1h (× 20.09)

Bold entry: best methodology; multipliers between brackets are computed respective to the best methodology.
Model variables: Small: UR, HICP, STR; Medium: UR, HICP, STR, GDP, LTR, REER; Large: all variables

Table 1: Estimation performances for the different methodologies (for 10000 iterations)

Two main conclusions derive from Table 1. First, estimation of the model equation by equation does improve significantly the computational performance. The gains are variable across estimation methodologies and model dimensions, but are always sizable. Smaller dimensions seem to produce the smallest computational benefits, with a bit more than 20% gain in the case of the small model (comparing methods 4 and 2) and around 70% gain in the case of the precision sampler (comparing methods 3 and 1). This is because small models maintain the dimensions of the dynamic parameters low anyway, even when they are estimated jointly. As a consequence, the dimensional issues traditionally arising with the selected algorithms are not too marked and the benefits remain moderate. At high dimensions however, the conclusions are quite different. For the large model the gains become very large, reaching 95% when comparing methods 2 and 4 and exceeding 99.8% when comparing methods 1 and 3. This confirms the lower relative computational efficiency of the estimation algorithms at high dimensions, and hence the relevance of the Carriero et al. (2016) approach to reduce the dimensionality of the dynamic parameters in the estimation process.

The second conclusion is that, perhaps surprisingly, the precision sampler of Chan and Eisenstat (2018) does not necessarily improve the computational efficiency of the procedure. Its efficiency seems to be in fact highly related to the dimension of the model. At small dimensions the precision sampler is fully efficient. In the case of the small model, it always represents the best option and is associated with considerable computational gains (almost 75% comparing methods 1 and 2, still more than 25% comparing methods 3 and 4). At medium dimensions the precision sampler plays at par with the Kalman filter when using the equation by equation approach (methods 1 and 2), but is already dominated with a joint estimation approach. At the highest dimension, the precision sampler becomes strictly dominated by the Kalman filter approach, and in the case of a joint estimation (methods 3 and 4) completely breaks down: a simple run of 10000 iterations with the Chan and Eisenstat (2018) methodology would take more than 80 days, rendering the estimation practically infeasible.

[7] All the estimations were conducted on a computer equipped with a 2 GHz Intel Core processor and 4 Go of RAM, for a Windows performance rating of 5.1/10, i.e., a fairly average computer. While the absolute numerical performances depend on the technical capacities of every machine, the ratio of the relative performance on estimating different models remains invariant to the computer used.

3.2 Optimal sampling algorithm

Following the conclusions of the previous section it is worth investigating further the properties of the precision sampler, which requires some understanding of the computational details. As discussed in Chan (2013), obtaining a draw from the precision sampler essentially consists in estimating the Cholesky factor of a sparse and banded precision matrix, and then run a backward/forward substitution with this Cholesky factor. What Chan (2013) fails to notice is that for a bandwidth of h in the precision matrix, the number of operations involved for the Cholesky factorisation and the backward/forward substitutions is of the order of $\mathcal{O}(h^2T)$ (Boyd and Vandenberghe (2004), p510). When h is very small, the computational cost is essentially determined by the sample size T. But as h increases, the flop count becomes quickly dominated by the bandwidth h, and the number of computations can escalate at a very fast rate[8]. For the general time-varying VAR model developed in section 2, the bandwidth of the matrices involved in the precision sampler methodology corresponds to the dimension of the dynamic parameters at each sample period. This is equal to k for the VAR coefficients β_i, and to a maximum of $n-1$ for the residual covariances δ_i. Because these values can be large, the precision sampler may become inefficient.

To make this point formal, the respective performances of the precision sampler and the Kalman filter are tested for different dimensions of the parameters β_i and δ_i. λ_i being always scalar-valued, the comparison is only realised for dimension one. The results are reported in Figure 1:

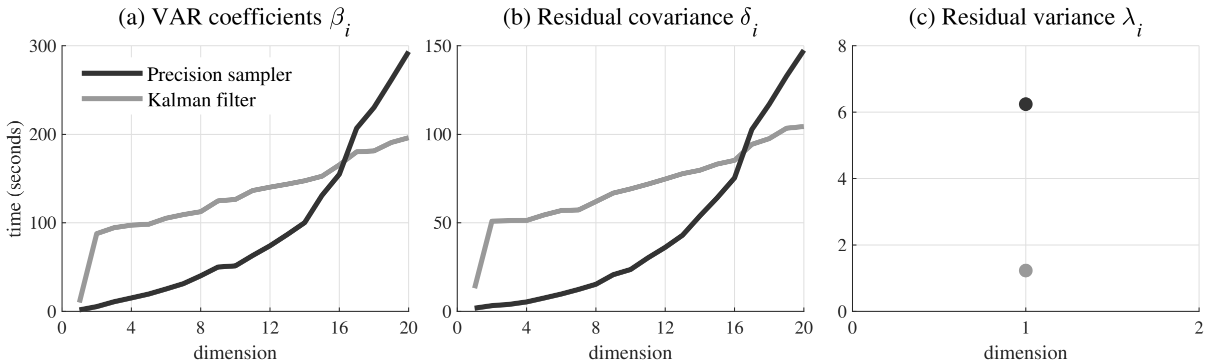

Figure 1: Approximate estimation time (in seconds) for 10000 repetitions of the MCMC algorithm at different dimensions

The characteristic quadratic shapes of the precision sampler curves in panels (a) and (b) confirm that the computational cost of the precision sampler grows at some quadratic rate. By contrast, the computational cost of the Kalman filter methodology looks more linear. At small dimensions, the comparison is clearly in favor of the precision sampler. At dimension 2 for instance, the precision sampler proves more than 15 times faster than the Kalman filter. The difference gets smaller as the dimension increases, the quadratic inefficiency of the precision sampler eventually outweighing the linear cost of the Kalman filter. A very important result is that the breaking

[8] Chan (2013) considers a pure stochastic volatility model where $h = 1$. In this case, the computational cost of the precision sampler becomes purely linear in T and involves only $\mathcal{O}(T)$ operations, as correctly reported by the author. The problem comes from the fact that the subsequent papers written by the author based on the precision sampler methodology, in particular Chan and Eisenstat (2018) neglect the bandwidth h, even though the parameters are not restricted anymore to the special case $h = 1$.

point occurs for both β_i and δ_i at dimension 16. At any dimension smaller than or equal to this value, the precision sampler remains more efficient than the Kalman filter though the gains may vary considerably. At any value above 16 the Kalman filter becomes strictly more efficient, and the precision sampler gets inefficient at a fast rate.

Panel (c) looks surprising. Even though λ_i is of dimension 1, the most efficient procedure to sample it happens to be the Kalman filter and not the precision sampler. The difference is neat, the Kalman filter procedure being more than five times faster than its precision sampler counterpart. There are two explanations for this puzzling result. First, panels (a) and (b) clearly show that dimension 1 constitutes a special case for which the difference between the Kalman filter and the precision sampler is considerably less than for other small dimensions. Second, λ_i represents a special case in the sense than its state-space formulation in the Kalman filter (see Appendix A, Table 6) is considerably simpler than that of β_i and δ_i. As the complexity of the formulation represents the main source of inefficiency in the Kalman filter procedure, simplifying the formulation results in considerable efficiency gains. Some gains also apply to the precision sampler, but they are much less given that the underlying state-space formulation is already efficiently vectorised in the procedure.

Based on these considerations, it is possible to propose the following optimal sampling algorithm:

Algorithm 2: Optimal sampling algorithm for the general time-varying model:

1. For $i = 1, \ldots, n$, sample λ_i equation by equation, using the Kalman filter procedure.

2. For $i = 1, \ldots, n$, sample β_i equation by equation:
   If $k \leq 16$, use the precision sampler and sample from $\pi(\beta_i|y, \backslash\beta_i) \sim \mathcal{N}(\bar{\beta}_i, \bar{\Omega}_i)$.
   If $k > 16$, use the Kalman filter procedure.

3. For $i = 2, \ldots, n$, sample δ_i equation by equation:
   If $n - 1 \leq 16$, use only the precision sampler and sample from $\pi(\delta_i|y, \backslash\delta_i) \sim \mathcal{N}(\bar{\delta}_i, \bar{\Psi}_i)$.
   If $n - 1 > 16$, use first the precision sampler for $i = 1, \cdots, 17$ and sample from $\pi(\delta_i|y, \backslash\delta_i) \sim \mathcal{N}(\bar{\delta}_i, \bar{\Psi}_i)$; then use the Kalman filter for $i = 18, \cdots, n$.

4. For $i = 1, \ldots, n$, sample Ω_i equation by equation from: $\pi(\Omega_i|y, \backslash\Omega_i) \sim IW(\bar{\zeta}, \bar{\Upsilon}_i)$.

5. For $i = 1, \ldots, n$, sample ϕ_i equation by equation from: $\pi(\phi_i|y, \backslash\phi_i) \sim IG(\bar{\kappa}, \bar{\omega}_i)$.

6. For $i = 2, \ldots, n$, sample Ψ_i equation by equation from: $\pi(\Psi_i|y, \backslash\Psi_i) \sim IW(\bar{\varphi}, \bar{\Theta}_i)$.

7. For $i = 1, \ldots, n$ and $t = 1, \ldots, T$, sample $r_{i,t}$ from: $\pi(r_{i,t}|y, \backslash r_{i,t}) \sim Cat(\bar{q}_1, \ldots, \bar{q}_7)$.

The performance of the optimal sampling algorithm is now compared with the competing methodologies. The results are presented in Table 2 (method 3 by Chan and Eisenstat (2018) is omitted to save space as it never constitutes the best methodology).

	Method 1 equation by equation precision sampler	Method 2 (Carriero et al.) equation by equation Kalman filter	Method 4 (Primiceri) jointly estimated Kalman filter	Method 5 (optimal sampling algorithm)
Small model	1m 2s (× 1.03)	3m 52s (× 3.86)	4m 47s (× 4.78)	**1m**
Medium model	14m (× 1.31)	14m (× 1.31)	33m 40s (× 3.15)	**10m 40s**
Large model	2h 37m (× 2.52)	1h 16m (× 1.21)	1d 1h (× 24.39)	**1h 2m**

Bold entry: best methodology; multipliers between brackets are computed respective to the best methodology.
Model variables: Small: UR, HICP, STR; Medium: UR, HICP, STR, GDP, LTR, REER; Large: all variables

Table 2: Estimation performances for the different methodologies (for 10000 iterations)

As expected, the optimal sampling algorithm represents the most efficient methodology in all cases. The gains are minimal at the smallest dimension and hardly reach 3% compared to method 1. This is because the two methodologies are very similar, the only difference residing in the fact that method 5 replaces the precision sampler by the Kalman filter to sample λ_i. The gains become more sizable for the medium and large model where they respectively exceed 30% and 20% compared to the best alternative. In the case of a large model with many iterations, the benefit of the optimal sampling algorithm becomes considerable, even compared to the efficient equation by equation methodology of Carriero et al. (2016) (method 2). For instance, producing 100000 iterations of the MCMC algorithm for the large model (a fairly common number of iterations for a time-varying model) would take 2 hours and 20 minutes less with the optimal sampling algorithm than with the methodology of Carriero et al. (2016). This is because the optimal sampling algorithm uses the precision sampler to draw the low-dimensional δ_i parameters, when Carriero et al. (2016) indiscriminately use the Kalman filter for all the parameters. Eventually, the optimal sampling algorithm qualifies both the approach of Chan and Eisenstat (2018) and Carriero et al. (2016). The former fail to notice that the precision sampler can become very inefficient at high dimensions, while the latter neglect the substantial gains it can generate at low dimensions. The optimal sampling algorithm, by contrast, ensures that the most suitable methodology is always applied. Finally, it is also worth noting that at high dimensions the optimal sampling algorithm is more than 24 times faster than the Primiceri (2005) methodology, which remains widely used.

4 Extensions

In the base version of the general time-varying model, the autoregressive coefficients ρ_i, γ_i and α_i and the mean terms b_i, s_i and d_i associated with the dynamic processes in (12) are treated as exogenous hyperparameters. These hyperparameters are key determinants of the model as they determine the posteriors and hence the quality of the forecasts. The traditional choice in the literature consist in setting $\rho_i = \gamma_i = \alpha_i = 1$ while ignoring b_i, s_i and d_i, which corresponds to the random walk assumption. As a first improvement, it is possible to propose a simple calibration. For instance, one may set $\rho_i = \gamma_i = \alpha_i = 0.9$ and determine b_i, s_i and d_i from their static OLS counterparts $\hat{b}_i$, $\hat{s}_i$ and $\hat{d}_i$. While this choice is reasonable, it is not necessarily optimal. For this reason, this section proposes simple procedures to estimate endogenously the autoregressive coefficients ρ_i, γ_i and α_i and the mean terms b_i, s_i and d_i.

4.1 Random inertia

Random inertia consists in estimating endogenously the autoregressive coefficients ρ_i, γ_i and α_i. Regarding the prior, the Beta distribution has sometimes been favoured by the literature for its support producing values between zero and one (Kim et al. (1998)). The Beta is however not conjugate with the normal distribution, which leads to an inefficient Metropolis-Hastings step in the estimation. On the multivariate side, a simpler alternative has consisted in using normal distributions (Primiceri (2005), Mumtaz and Zanetti (2013)). A diffuse prior is used to let the data speak and produce posteriors centered on OLS estimates. While simple, this strategy is unadvisable for two reasons. First, as the support of the normal distribution is unrestricted, part of the posterior distribution may lie outside of the zero-one interval, which is not meaningful from an economic point of view. Second, the use of a diffuse prior is suboptimal as relevant information can be introduced at the prior stage. For these reasons, the prior is chosen here to be a truncated normal distributions with informative hyperparameters. Considering for instance ρ_i in (12), the prior distribution is a normal distribution with mean ρ_{i0} and variance π_{i0}, truncated over the $[0,1]$ interval:

$$\pi(\rho_i) \sim \mathcal{N}_{[0,1]}(\rho_{i0}, \pi_{i0}) \tag{36}$$

An informative prior belief consists in assuming that with 95% probability an autoregressive coefficient value should be comprised between 0.6 and 1. This is obtained by setting a mean value of $\rho_{i0} = 0.8$ and a standard deviation of 0.1, yielding a variance of $\pi_{i0} = 0.01$. This way, the prior is sufficiently loose to allow for significant differences in the posterior distributions of the different ρ_i's, but also sufficiently restrictive to avoid posteriors that would be too far away from the prior and implausible. Finally, the truncation operated at the prior stage ensures that the posterior distribution is restricted over the same range $[0,1]$, thus ruling out irrelevant parts of the support. A similar strategy is applied to the other autoregressive coefficients in (12):

$$\pi(\gamma_i) \sim \mathcal{N}_{[0,1]}(\gamma_{i0}, \varsigma_{i0}) \qquad \pi(\alpha_i) \sim \mathcal{N}_{[0,1]}(\alpha_{i0}, \iota_{i0}) \tag{37}$$

The mean and variance parameters are set to $\gamma_{i0} = \alpha_{i0} = 0.8$ and $\varsigma_{i0} = \iota_{i0} = 0.01$. To account for the additional parameters, Bayes rule must be slightly amended:

$$\begin{aligned} \pi(\beta, \Omega, \rho, \lambda, \phi, \gamma, \delta, \Psi, \alpha, r|y) \propto f(y|\beta, \lambda, \delta, r) \left(\prod_{i=1}^{n} \pi(\beta_i|\Omega_i, \rho_i)\pi(\Omega_i)\pi(\rho_i) \right) \\ \times \left(\prod_{i=1}^{n} \pi(\lambda_i|\phi_i, \gamma_i)\pi(\phi_i)\pi(\gamma_i) \right) \left(\prod_{i=2}^{n} \pi(\delta_i|\Psi_i, \alpha_i)\pi(\Psi_i)\pi(\alpha_i) \right) \left(\prod_{i=1}^{n}\prod_{t=1}^{T} \pi(r_{i,t}) \right) \end{aligned} \tag{38}$$

Consider the posteriors. For ρ_i, Bayes rule (38) implies $\pi(\rho_i|y, \backslash\rho_i) \propto \pi(\beta_i|\Omega_i, \rho_i)\pi(\rho_i)$. From the priors (25) and (36) and some rearrangement, it follows that:

$$\begin{aligned} &\pi(\rho_i|y, \backslash\rho_i) \sim \mathcal{N}_{[0,1]}(\bar{\rho}_i, \bar{\pi}_i) && \text{with:} \\ &\bar{\pi}_i = (\ddot{\beta}'_{i,t-1}\ddot{\beta}_{i,t-1} + \pi_{i0}^{-1})^{-1} && \bar{\rho}_i = \bar{\pi}_i(\ddot{\beta}'_{i,t-1}\ddot{\beta}_{i,t} + \pi_{i0}^{-1}\rho_{i0}) \\ &\ddot{\beta}_{i,t} = vec\,(\Omega_i^{-1/2\prime}(\beta_{i,2} - b_i \quad \cdots \quad \beta_{i,T} - b_i)) \end{aligned} \tag{39}$$

For γ_i, Bayes rule (38) implies $\pi(\gamma_i|y, \backslash\gamma_i) \propto \pi(\lambda_i|\phi_i, \gamma_i)\pi(\gamma_i)$. From the priors (26) and (37) and some rearrangement, it follows that:

$$\pi(\gamma_i|y,\backslash\gamma_i) \sim \mathcal{N}_{[0,1]}(\bar{\gamma}_i, \bar{\varsigma}_i) \qquad \text{with:}$$
$$\bar{\varsigma}_i = (\ddot{\lambda}'_{i,t-1}\ddot{\lambda}_{i,t-1} + \varsigma_{i0}^{-1})^{-1} \qquad \bar{\gamma}_i = \bar{\varsigma}_i(\ddot{\lambda}'_{i,t-1}\ddot{\lambda}_{i,t} + \varsigma_{i0}^{-1}\gamma_{i0})$$
$$\ddot{\lambda}_{i,t} = \phi_i^{-1/2}(\lambda_{i,2} \quad \dots \quad \lambda_{i,T})' \tag{40}$$

Finally for α_i, Bayes rule (38) implies $\pi(\alpha_i|y,\backslash\alpha_i) \propto \pi(\delta_i|\Psi_i,\alpha_i)\pi(\alpha_i)$. From the priors (26) and (37) and some rearrangement, it follows that:

$$\pi(\alpha_i|y,\backslash\alpha_i) \sim \mathcal{N}_{[0,1]}(\bar{\alpha}_i, \bar{\iota}_i) \qquad \text{with:}$$
$$\bar{\iota}_i = (\ddot{\delta}_{i,t-1}{}'\ddot{\delta}_{i,t-1} + \iota_{i0}^{-1})^{-1} \qquad \bar{\alpha}_i = \bar{\iota}_i(\ddot{\delta}_{i,t-1}{}'\ddot{\delta}_{i,t} + \iota_{i0}^{-1}\alpha_{i0})$$
$$\ddot{\delta}_{i,t} = vec(\Psi_i^{-1/2\prime}(\delta_{i,2} - d_i \quad \cdots \quad \delta_{i,T} - d_i)) \tag{41}$$

The MCMC algorithm for the model with random inertia is similar to Algorithm 2, except that 3 additional steps must be inserted between steps 6 and 7:

Algorithm 3: additional steps of the MCMC algorithm for the model with random inertia:

1. For $i = 1, \dots, n$, sample ρ_i equation by equation, from $\pi(\rho_i|y,\backslash\rho_i) \sim \mathcal{N}_{[0,1]}(\bar{\rho}_i, \bar{\pi}_i)$.
2. For $i = 1, \dots, n$, sample γ_i equation by equation, from $\pi(\gamma_i|y,\backslash\gamma_i) \sim \mathcal{N}_{[0,1]}(\bar{\gamma}_i, \bar{\varsigma}_i)$.
3. For $i = 2, \dots, n$, sample α_i equation by equation, from $\pi(\alpha_i|y,\backslash\alpha_i) \sim \mathcal{N}_{[0,1]}(\bar{\alpha}_i, \bar{\iota}_i)$.

4.2 Random mean

The base version of the general time-varying model treats the mean parameters b_i, s_i and d_i in (11) and (12) as exogenously supplied hyperparameters. Though convenient, this assumption may be overly restrictive. For instance, the parameter s_i represents the long-run value of the residual volatility. As such, it determines the share of data variation endorsed by the noise component of the model, and the share explained by the time-varying responses. Determining s_i correctly is thus of paramount importance, and endogenous estimation comes as a natural extension. While the univariate ARCH literature has paid some attention to this question in the context of stochastic volatility processes (Jacquier et al. (1994), Kim et al. (1998)), the subject has been almost completely neglected in multivariate models. One notable exception is the contribution of Chiu et al. (2015) who integrate a (period-specific) mean component to the dynamic variance of the residuals. This section fills the gap by proposing simple estimation procedures for the mean components of the dynamic processes.

Consider first the priors. For b_i, the choice is that of a simple multivariate normal distribution with mean b_{i0} and variance-covariance matrix Ξ_{i0}:

$$\pi(b_i) \sim \mathcal{N}(b_{i0}, \Xi_{i0}) \tag{42}$$

Because the static OLS estimate $\hat{\beta}_i$ represents a reasonable starting point for b_i, the prior mean b_{i0} is set to $\hat{\beta}_i$ while the prior standard deviation is set to a fraction ϖ_i of this value, resulting in $\Xi_{i0} = diag((\varpi_i\hat{\beta}_i)^2)$. Small values of ϖ_i generate a tight and hence informative prior around

$\hat{\beta}_i$ while larger values can be used to achieve diffuse and uninformative priors. Given the lack of economic theory concerning the equilibrium value of the time-varying coefficients, the prior is set to be informative but somewhat looser than usual in order to leave sufficient weight to the data. This is achieved by setting $\varpi_i = 0.25$, implying that b_i lies within 50% of $\hat{\beta}_i$ with 95% confidence.

Similar strategies are applied for s_i and d_i. For the s_i which are positive scaling terms, the inverse Gamma represents a natural candidate. Specifically, the prior for each s_i is inverse Gamma with shape χ_{i0} and scale ϑ_{i0}:

$$\pi(s_i) \sim IG\left(\frac{\chi_{i0}}{2}, \frac{\vartheta_{i0}}{2}\right) \tag{43}$$

The hyperparameter values χ_{i0} and ϑ_{i0} are then chosen to imply a prior mean of $\hat{s}_i$, the OLS estimate used for the general time-varying model, and a prior standard deviation equal to a fraction ψ_i of this value[9]. As a base case, ψ_i is set to 0.25 in order to generate, again, an informative but sufficiently loose prior.

Finally, the prior for each d_i is multivariate normal with mean d_{i0} and variance-covariance matrix Z_{i0}:

$$\pi(d_i) \sim \mathcal{N}(d_{i0}, Z_{i0}) \tag{44}$$

The prior mean is set as $d_{i0} = \hat{d}_i$, with $\hat{d}_i$ the static OLS estimate. The prior standard deviation is set to a fraction ϱ_i of this value, resulting in $Z_{i0} = \ diag((\varrho_i \hat{d}_i)^2)$. An informative but loose prior is achieved by setting $\varrho_i = 0.25$. With random mean, Bayes rule becomes:

$$\begin{aligned} &\pi(\beta, \Omega, b, \lambda, \phi, s, \delta, \Psi, d, r|y) \propto f(y|\beta, \lambda, s, \delta, r) \left(\prod_{i=1}^{n} \pi(\beta_i|\Omega_i, b_i)\pi(\Omega_i)\pi(b_i)\right) \\ \times \; &\left(\prod_{i=1}^{n} \pi(\lambda_i|\phi_i)\pi(\phi_i)\right) \left(\prod_{i=1}^{n} \pi(s_i)\right) \left(\prod_{i=2}^{n} \pi(\delta_i|\Psi_i, d_i)\pi(\Psi_i)\pi(d_i)\right) \left(\prod_{i=1}^{n}\prod_{t=1}^{T} \pi(r_{i,t})\right) \end{aligned} \tag{45}$$

For b_i, Bayes rule (45) implies $\pi(b_i|y, \backslash b_i) \propto \pi(\beta_i|\Omega_i, \rho_i)\pi(b_i)$. From the priors (25) and (42) and some rearrangement, it follows that:

$$\begin{aligned} &\pi(b_i|y, \backslash b_i) \sim \mathcal{N}(\bar{b}_i, \bar{\Xi}_i) && \text{with:} \\ &\bar{\Xi}_i = \left(\tilde{\tau}_i \Omega_i^{-1} + \Xi_{i0}^{-1}\right)^{-1} && \bar{b}_i = \bar{\Xi}_i \left(\Omega_i^{-1}(\tilde{\rho}_i \otimes I_k)\beta_i + \Xi_{i0}^{-1} b_{i0}\right) \\ &\tilde{\tau}_i = \tau^{-1} + (1-\rho_i)^2(T-1) && \tilde{\rho}_i = \left(\tau^{-1} - (1-\rho_i)\rho_i \quad (1-\rho_i)^2 \quad \cdots \quad (1-\rho_i)^2 \quad (1-\rho_i)\right) \end{aligned} \tag{46}$$

For s_i, Bayes rule (45) implies $\pi(s_i|y, \backslash s_i) \propto f(y|\beta, \lambda, s, \delta, r)\pi(s_i)$. From the likelihood function (15), the prior (43) and some rearrangement, it follows that:

$$\begin{aligned} &\pi(s_i|y, \backslash s_i) \sim IG(\bar{\chi}_i, \bar{\vartheta}_i) && \text{with:} \\ &\bar{\chi}_i = \frac{T + \chi_{i0}}{2} && \bar{\vartheta}_i = \frac{\tilde{\lambda}_i' \ \mathcal{Q}_i + \vartheta_{i0}}{2} \end{aligned} \tag{47}$$

[9]This is conveniently achieved by exploiting the fact that the inverse Gamma distribution defines a unique correspondence between any pair of mean/variance values and shape/scale parameters.

Finally for d_i, Bayes rule (45) implies $\pi(d_i|y,\backslash d_i) \propto \pi(\delta_i|\Psi_i, d_i)\pi(d_i)$. From the priors (26) and (44) and some rearrangement, it follows that:

$$\begin{aligned}
&\pi(d_i|y,\backslash d_i) \sim \mathcal{N}(\bar{d}_i, \bar{Z}_i) \quad \text{with:} \\
&\bar{Z}_i = \left(\tilde{\epsilon}_i \Psi_i^{-1} + Z_{i0}^{-1}\right)^{-1} \qquad \bar{d}_i = \bar{Z}_i \left(\Psi_i^{-1}(\tilde{\alpha}_i \otimes I_{i-1})\delta_i + Z_{i0}^{-1} d_{i0}\right) \\
&\tilde{\epsilon}_i = \epsilon^{-1} + (1-\alpha_i)^2(T-1) \qquad \tilde{\alpha}_i = \left(\epsilon^{-1} - (1-\alpha_i)\alpha_i \quad (1-\alpha_i)^2 \quad \cdots \quad (1-\alpha_i)^2 \quad (1-\alpha_i)\right)
\end{aligned} \tag{48}$$

The MCMC algorithm for the model with random mean is similar to Algorithm 2, except that 3 additional steps must be inserted between steps 6 and 7:

Algorithm 4: additional steps of the MCMC algorithm for the model with random mean:

1. For $i = 1, \ldots, n$, sample b_i equation by equation, from $\pi(b_i|y,\backslash b_i) \sim \mathcal{N}(\bar{b}_i, \bar{\Xi}_i)$.

2. For $i = 1, \ldots, n$, sample s_i equation by equation, from $\pi(s_i|y,\backslash s_i) \sim IG(\bar{\chi}_i, \bar{\vartheta}_i)$.

3. For $i = 2, \ldots, n$, sample d_i equation by equation, from $\pi(d_i|y,\backslash d_i) \sim \mathcal{N}(\bar{d}_i, \bar{Z}_i)$.

5 A case study on the Great Recession

5.1 Setup

To conclude this work, a short case study on the Great Recession is proposed. The study focuses on four major economies which have been severely impacted by the crisis: Canada, the Euro area, Japan, and the United States. The experiment is conducted on a large 12-variable macroeconomic model comprising four blocks of variables: a general macroeconomic block with real gross domestic product (GDP), unemployment rate (UR) and consumer price index (HICP); a monetary policy block with short-term interest rate (STR), long-term interest rate (LTR) and real effective exchange rate (REER); a production block with industrial production (IP), capacity utilization (CU) and total industry employment (TIE); and, for the needs of the exercise, a crisis block with housing starts (HS), a financial stock index (FSI) and the OECD leading composite indicator (LCI) which acts as an overall business cycle indicator. Any series displaying persistence is turned to growth rate to obtain stationarity. The data is quarterly, the sample depending on data availability for each country. It respectively starts in 1971q1 for Canada, 1981q1 for the Euro Area, 1975q2 for Japan and 1971q1 for the United States. The full dataset ends at the end of 2018, but the estimation samples are typically shorter (see below). The data comes primarily from the OECD for Canada, Japan and the United States. For the Euro Area, it is obtained from the Area Wide Model Database of Fagan et al. (2001) which has become the standard for academic research. Financial stock index series come from Bloomberg[10].

The aim of the exercise consists in assessing the forecast performances of different models for key phases of the crisis. Figure 2 displays the growth rate of GDP for the four economies over the Great Recession periods. For each country, two critical periods of the crisis are considered. The first is the recession period, the period at which the country enters into negative growth.

[10] A complete description of the series, their transformations and their sources along with the dates of the estimation samples can be found in Appendix B.

For Canada, the Euro area, Japan and the United States, this respectively occurs in 2009q1, 2008q4, 2008q2 and 2008q4. The second period considered is the recovery period. It represents the period at which GDP growth starts increasing again, after having reached its minimum. This respectively happens in 2009q4, 2009q2, 2009q2 and 2009q3. These two periods are of special importance for policy makers as they correspond to the beginning of the phases where the crisis initiates and reverts. It is crucial to anticipate them correctly in order to provide an adequate answer to the rapidly changing economic conditions.

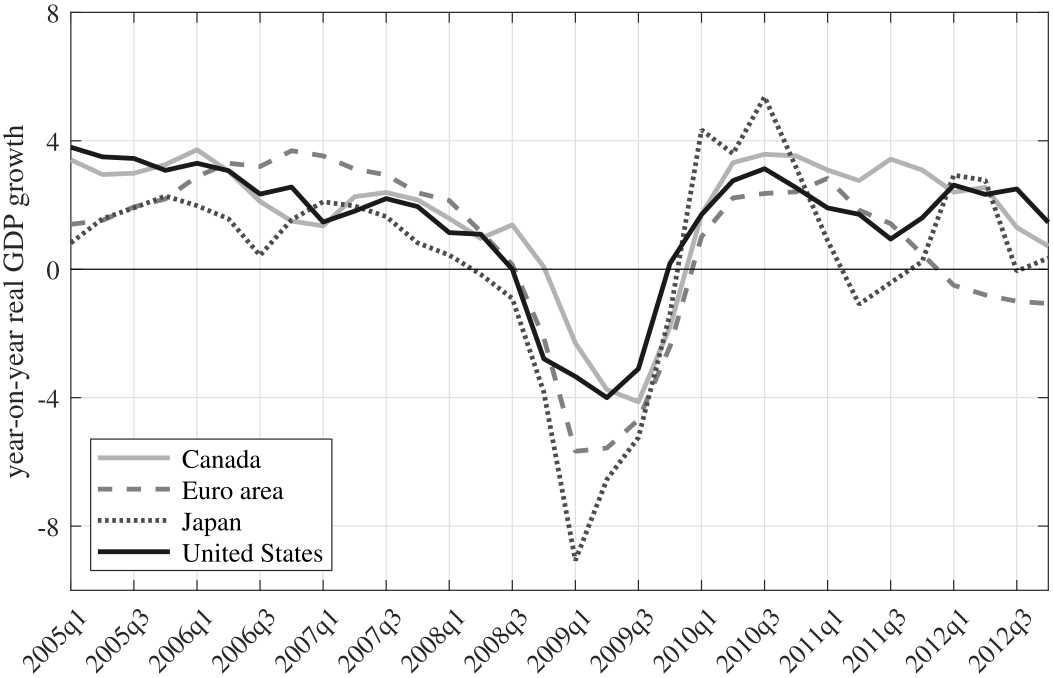

Figure 2: Year-on-year GDP growth for the four major economies

The forecasting exercise focuses on predictions from one to eight periods ahead. It is performed in pseudo real time, that is, it does not use information which is not available at the time the forecast is made[11]. For this reason, for each country and each considered period of the crisis the model is estimated up to the period preceding the beginning of the forecast exercise. To evaluate the performance, two criteria are considered. The first criterion is the classical Root Mean Squared Error (RMSE) which considers the accuracy of point forecasts. Denoting by $\tilde{y}_{t+h}$ the h-step ahead prediction and by y_{t+h} the realised value, it is defined as:

$$RMSE_{t+h} = \sqrt{\frac{1}{h}\sum_{i=1}^{h}(\tilde{y}_{t+h} - y_{t+h})^2} \tag{49}$$

The second criterion is the Continuous Ranked Probability Score (CRPS) of Gneiting and Raftery (2007) which evaluates density forecasts. As pointed by those authors, this criterion presents advantages over alternative density scores such as the log score as it rewards more density points close to the realised value and is less sensitive to outliers. Denoting by F the cumulative

[11] Ideally such a forecast exercise should use vintage data, that is, data as it was available at the period for which the forecast is realised. This is not the case for the present experiment due to the scarcity of the available data, in particular for the Euro Area.

distribution function of the h-step ahead forecast density and by $\hat{y}_{t+h}$ and $\hat{y}'_{t+h}$ independent random draws from this density, the CRPS is defined as:

$$CRPS_{t+h} = \int_{-\infty}^{\infty} (F(x) - \mathbb{1}(x \geq y_{t+h}))^2 dx = \mathbb{E}\,|\hat{y}_{t+h} - y_{t+h}| - \frac{1}{2}\,\mathbb{E}\,|\hat{y}_{t+h} - \hat{y}'_{t+h}| \tag{50}$$

For both criteria, a lower score indicates a better performance.

5.2 Calibration

The forecast exercise considers five competing models. The benchmark is the general time-varying model introduced in section 2 specified with stationary autoregressive processes (Sar) for all the dynamic parameters. Precisely, the dynamic parameters are calibrated by setting $\rho_i = \gamma_i = \alpha_i = 0.9$ and by using static OLS estimates for the mean terms. The second model considered is the homogenous random walk (Hrw) specification of Primiceri (2005), which obtains from the general time-varying model by setting the autoregressive coefficients of the dynamic processes to one. The third and fourth models respectively consist in the general time-varying model augmented by the random inertia (Ri) and random mean (Rm) extensions developed in section 4. The final model combines the two extensions, thus adding both random inertia and random mean (Rim) to the general time-varying model.

Unlike Primiceri (2005), the priors are not calibrated from a training sample as this strategy wastes a considerable amount of sample information. Rather, simple values are used. For the inverse Wishart priors on the variance-covariance hyperparameters Ω_i and Ψ_i, the degrees of freedom are set to a small value of 5 additional to the parameter dimension, namely $\zeta_0 = k + 5$ and $\varphi_0 = (i - 1) + 5$. The scale parameters are set to $\Upsilon_0 = 0.01 I_k$ and $\Theta_0 = 0.01 I_{i-1}$. Similarly, the shape and scale parameters of the inverse Gamma prior distribution on ϕ_i are set to $\kappa_0 = 5$ and $\omega_0 = 0.01$. These priors are mildly informative, being sufficiently loose to allow for a significant degree of time variation in the dynamic parameters, but sufficiently restrictive to avoid implausible behaviours. Finally, the initial period variance scaling terms are set to $\tau = \mu = \epsilon = 5$ in order to obtain a variance over the initial periods which is roughly equivalent to that prevailing for the rest of the sample. Estimations are run from 10000 iterations of the MCMC algorithm, discarding the initial 5000 iterations as burn-in sample[12].

5.3 Results

With four countries, twelve variables, eight forecast periods and two crisis phases, the full forecasting exercise consists in 768 forecasts, each of them produced for five competing models. Tables 3 and 4 summarize the results of the experiment[13]. Table 3 displays the average RMSE and CRPS values for the forecast exercise, while table 4 reports the ratio of these criteria to the benchmark stationary autoregressive formulation. Additionally, Table 4 indicates whether a forecast evaluation criterion is statistically larger (+ entries, for the Hrw model) or smaller ($*$ entries, for the Ri, Rm and Rim models) than the benchmark Sar model. The forecast performance is analysed both overall and according to a number of sub-criteria (country, variable, forecast horizon and crisis phase).

[12] For the sake of illustration, Appendix C displays the stochastic volatility estimates and one-period ahead impulse response functions obtained for the United States with the stationary autoregressive model.

[13] The full set of tables for the raw results can be found in Appendix D.

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
					Overall (N=768)					
Overall	5.03	3.35	2.90	3.28	2.86	14.39	5.00	2.83	4.66	2.76
					By country (N=192)					
CA	4.00	3.04	2.31	2.87	2.24	8.83	3.78	1.93	2.96	1.92
EA	4.64	2.90	2.57	2.50	2.33	14.69	4.92	2.77	4.10	2.67
JP	6.64	4.23	3.39	4.24	3.45	16.75	5.13	2.99	4.98	3.10
US	4.83	3.22	3.34	3.50	3.41	17.27	6.19	3.63	6.60	3.36
					By variable (N=64)					
GDP	4.18	2.40	2.00	2.80	2.18	13.13	4.34	2.31	4.17	2.30
UR	1.44	0.78	0.71	0.71	0.71	7.84	2.07	0.95	1.96	0.99
HICP	3.46	2.80	2.68	2.41	2.42	8.02	3.33	2.12	2.78	2.03
STR	2.18	2.47	2.38	2.05	2.27	11.26	4.11	2.36	3.68	2.38
LTR	1.70	1.36	1.19	1.12	1.21	8.68	2.90	1.44	2.69	1.49
REER	3.47	3.18	2.37	2.63	2.23	9.91	3.69	1.94	3.08	1.90
IP	14.31	8.91	7.46	10.04	8.09	34.17	12.46	7.46	12.51	7.49
CU	5.41	3.52	3.04	4.11	3.26	15.21	5.15	2.88	5.29	3.07
TIE	3.84	2.51	2.36	2.36	2.26	12.37	4.56	2.55	4.16	2.43
HS	3.63	2.35	2.28	2.34	2.16	10.26	3.35	1.98	3.00	1.97
FSI	11.32	7.25	6.20	6.37	5.67	24.23	8.96	5.38	8.09	4.72
LCI	5.39	2.63	2.19	2.38	1.85	17.55	5.13	2.60	4.54	2.41
					By forecast horizon (N=96)					
1q ahead	1.68	1.48	1.39	1.48	1.40	1.40	1.11	1.04	1.13	1.05
2q ahead	2.66	2.17	2.06	2.29	2.12	2.50	1.99	1.79	2.03	1.84
3q ahead	3.53	2.86	2.58	2.93	2.67	3.89	2.69	2.25	2.73	2.32
4q ahead	4.35	3.40	3.07	3.45	3.12	6.36	3.88	2.95	3.81	2.99
5q ahead	5.01	3.76	3.32	3.72	3.29	9.98	4.94	3.23	4.59	3.07
6q ahead	5.90	4.06	3.52	3.95	3.37	16.40	6.36	3.55	5.67	3.31
7q ahead	7.25	4.35	3.62	4.09	3.39	27.45	8.13	3.70	7.34	3.45
8q ahead	9.84	4.70	3.67	4.31	3.49	47.10	10.94	4.13	10.00	4.07
					By crisis phase (N=384)					
Recession	4.49	3.19	2.90	3.10	2.82	10.58	4.09	2.55	3.98	2.48
Recovery	5.57	3.50	2.91	3.45	2.90	18.19	5.92	3.11	5.35	3.05

Table 3: Mean RMSE and CRPS for the five competing models

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Overall (N=768)										
Overall	1.50^{+++}	-	0.87^{***}	0.98	0.85^{***}	2.87^{+++}	-	0.57^{***}	0.93	0.55^{***}
By country (N=192)										
CA	1.31^{+++}	-	0.76^{***}	0.94	0.73^{***}	2.34^{+++}	-	0.51^{***}	0.78^{***}	0.51^{***}
EA	1.60^{+++}	-	0.89	0.86^{*}	0.81^{**}	2.98^{+++}	-	0.56^{***}	0.83^{**}	0.54^{***}
JP	1.57^{+++}	-	0.80^{**}	1.00	0.82	3.27^{+++}	-	0.58^{***}	0.97	0.61^{***}
US	1.50^{+++}	-	1.04	1.09	1.06	2.79^{+++}	-	0.59^{***}	1.07	0.54^{***}
By variable (N=64)										
GDP	1.74^{+++}	-	0.83^{**}	1.17	0.91	3.03^{+++}	-	0.53^{***}	0.96	0.53^{***}
UR	1.84^{+++}	-	0.91	0.90	0.90	3.79^{+++}	-	0.46^{***}	0.95	0.48^{***}
HICP	1.24^{++}	-	0.96	0.86^{*}	0.87^{*}	2.41^{+++}	-	0.64^{***}	0.84^{*}	0.61^{***}
STR	0.88	-	0.96	0.83^{*}	0.92	2.74^{+++}	-	0.57^{***}	0.90	0.58^{***}
LTR	1.25	-	0.87	0.82	0.89	2.99^{+++}	-	0.50^{***}	0.93	0.51^{***}
REER	1.09	-	0.75^{**}	0.83^{*}	0.70^{***}	2.68^{+++}	-	0.53^{***}	0.83^{*}	0.51^{***}
IP	1.61^{+++}	-	0.84^{*}	1.13	0.91	2.74^{+++}	-	0.60^{***}	1.00	0.60^{***}
CU	1.54^{+++}	-	0.87	1.17	0.93	2.96^{+++}	-	0.56^{***}	1.03	0.60^{***}
TIE	1.53^{+++}	-	0.94	0.94	0.90	2.71^{+++}	-	0.56^{***}	0.91	0.53^{***}
HS	1.54^{+++}	-	0.97	1.00	0.92	3.06^{+++}	-	0.59^{***}	0.89	0.59^{***}
FSI	1.56^{+++}	-	0.85^{*}	0.88^{*}	0.78^{***}	2.71^{+++}	-	0.60^{***}	0.90	0.53^{***}
LCI	2.05^{+++}	-	0.83^{*}	0.91	0.70^{***}	3.42^{+++}	-	0.51^{***}	0.89	0.47^{***}
By forecast horizon (N=96)										
1q ahead	1.14	-	0.94	1.00	0.95	1.26	-	0.93	1.02	0.95
2q ahead	1.23	-	0.95	1.06	0.98	1.26	-	0.90	1.02	0.92
3q ahead	1.23	-	0.90	1.03	0.93	1.45^{+++}	-	0.84^{*}	1.01	0.86
4q ahead	1.28^{+}	-	0.90	1.01	0.92	1.64^{+++}	-	0.76^{**}	0.98	0.77^{**}
5q ahead	1.33^{++}	-	0.88	0.99	0.87	2.02^{+++}	-	0.65^{***}	0.93	0.62^{***}
6q ahead	1.45^{+++}	-	0.87	0.97	0.83^{*}	2.58^{+++}	-	0.56^{***}	0.89	0.52^{***}
7q ahead	1.67^{+++}	-	0.83^{*}	0.94	0.78^{***}	3.38^{+++}	-	0.46^{***}	0.90	0.42^{***}
8q ahead	2.09^{+++}	-	0.78^{**}	0.92	0.74^{***}	4.31^{+++}	-	0.38^{***}	0.91	0.37^{***}
By crisis phase (N=384)										
Recession	1.41^{+++}	-	0.91^{*}	0.97	0.88^{**}	2.59^{+++}	-	0.62^{***}	0.97	0.61^{***}
Recovery	1.59^{+++}	-	0.83^{***}	0.99	0.83^{***}	3.07^{+++}	-	0.52^{***}	0.90^{*}	0.51^{***}

Note:
+, ++, +++ : mean RMSE (CRPS) > mean RMSE (CRPS) of Sar model at 10%, 5% and 1% significance level.
$*, **, ***$: mean RMSE (CRPS) < mean RMSE (CRPS) of Sar model at 10%, 5% and 1% significance level.

Table 4: RMSE and CRPS ratios: Hrw, Ri, Rm and Rim / Sar

A number of conclusions stand. First, the results unambiguously disqualify the homogenous random walk as the best formulation regarding forecast accuracy. Considering the exercise overall, the Hrw specification produces RMSE which are on average 1.5 times larger than the benchmark Sar model, and CRPS which are on average almost 3 times larger. This indicates that the Sar model represents a considerable improvement over the random walk formulation, the magnitude of the difference being quite large. The difference is also statistically significant at the 1% level, which clears any doubt about the solidity of the conclusion. As additional evidence, the results display similar magnitudes of difference for all the considered sub-specifications, and a vast majority of the differences are statistically significant at the 1% level. This is encouraging as it suggests that the forecast performance is not conditioned on some specific part of the dataset. Finally, note that the Hrw model performs worse in terms of CRPS than in terms of RMSE. This implies that while the random walk performs already poorly in terms of point forecasts, it performs even worse in terms of forecast distributions. In other words, it tends to produce credibility bands which are excessively large compared to the benchmark Sar model, an undesirable feature for any forecast exercise.

Second, the Ri and Rim extensions improve significantly the forecast performances compared to the benchmark Sar model. The achievements of the two extensions prove in fact very close. For the exercise considered overall, they produce a 15% improvement in terms of RMSE, and a 45% improvement in terms of CRPS. The difference is significant at the 1% significance level for both criteria. The gain compared to the benchmark is again quite substantial and provides strong support in favor of the Ri and Rim extensions. It clearly suggests that to optimise forecast performance, it is not sufficient to simply replace the random walk with a stationary specification. It is further necessary to approach the data generating process underlying the behaviour of the dynamic parameters, which is what is typically achieved by the extensions. Considering the sub-specifications, the Ri and Rim models remain significantly better than the Sar model in terms of CRPS, but not so often in terms of RMSE. Part of the explanation lies in the shorter samples of the sub-specifications, and part in the greater variance of the RMSE criteria. In other words, there remains some variability in the performance of the Ri and Rim in terms of point forecasts compared to the Sar model. In terms of forecast distributions however, these extensions perform consistently better than the Sar benchmark, implying that they typically produce tighter confidence bands.

Third, the Rm model does not perform significantly better than the Sar benchmark. Its performances both in terms of RMSE and CRPS look fairly equivalent to those of the Sar model, and prove actually worse on a number of occasions. Also, the difference with the Sar model is hardly ever significant. One likely explanation for this is parsimony. Estimating the mean parameters generates a trade-off between the additional flexibility granted by the extension and the loss of precision implied by the use of additional degrees of freedom. The number of parameters estimated by the random mean extension can be large (k parameters for each b_i, and $i-1$ parameters for each d_i), and it seems that consequently the benefit of the extensions remains moderate. By contrast, the random inertia extension is quite cheap in terms of degree of freedom (one additional parameter per equation only), which may explain its significantly better results.

Note that the mediocre performance of the Rm model does not question the use of the random mean methodology altogether. On the contrary, the results of the experiment clearly indicate that the best model overall is the one which combines the two extensions. It thus appears that when used together, the gain from approaching the true data generating process exceeds the increased imprecision due to the estimation of the additional parameters. To illustrate this point, Table 5 provides the posterior estimates of key random inertia and random mean parameters for the United States[14]:

	GDP	UR	HICP	STR	LTR	REER	IP	CU	TIE	HS	FSI	LCI
ρ_i	0.48	0.50	0.48	0.49	0.50	0.49	0.58	0.55	0.55	0.52	0.61	0.52
γ_i	0.99	0.89	0.98	0.99	0.99	0.99	0.88	0.79	0.83	0.99	0.79	0.87
α_i	-	0.78	0.79	0.84	0.77	0.77	0.78	0.73	0.78	0.78	0.84	0.71
s_i	0.30	0.02	0.20	0.28	0.07	0.23	0.41	0.01	0.47	0.33	1.16	0.03
$\hat{s}_i$	0.52	0.03	0.37	0.49	0.10	0.40	0.59	0.01	0.68	0.52	1.63	0.05

Table 5: Random inertia and random mean estimates for the United States

The first three rows of Table 5 report the posterior estimates for ρ_i, γ_i and α_i. It appears that the estimates for ρ_i and α_i are considerably smaller than one, about respectively 0.5 and 0.8. The γ_i estimates are more ambiguous, taking values comprised between 0.8 and 1. Clearly, a random walk proves inappropriate as it leads to considerably overestimate the amplitude of most autoregressive coefficients. The stationary autoregressive model using $\rho_i = \gamma_i = \alpha_i = 0.9$ represents some improvement, but still creates a considerable gap with the values supported by the data. The results also confirm the relevance of a formulation equation by equation as considerable differences arise between different variables. For instance, the γ_i coefficients on GDP, STR, LTR and REER are at 0.99 against much lower values of 0.79 for CU and FSI. The last two rows of Table 5 respectively present the random mean posterior estimates s_i, and for the sake of comparison the OLS estimates $\hat{s}_i$ used as hyperparameters in the stationary autoregressive model. Remember that these coefficients represent the long-run equilibrium value for the residual volatility of the model, and that a random walk formulation amounts to assuming $s_i = 1$. Again, it is not difficult to see that a random walk leads to considerably overestimate the amplitude of the stochastic volatility component. The OLS estimates used for the Sar model are already significantly lower and thus represent an improvement, but the s_i values endogenously estimated by random mean turn out to be even lower.

The fourth and final conclusion about the crisis experiment is that the best forecasting model between the Ri and Rim specifications depends on the forecast horizon. At short horizon (one to four quarters), the pure Ri model performs slightly but consistently better than its Rim counterpart. At longer horizon (five to eight quarters) the relation is reversed and the Rim model becomes the leading model. This comes as another illustration of the trade-off between greater flexibility and increased imprecision. At short forecast horizons, the Rim model is too costly in terms of parameters and the simple, more parsimonious Ri model performs better. At longer horizons, it becomes crucial to approach the true data generating process to produce accurate predictions and the more flexible Rim model dominates.

[14]The estimates for the other countries are very similar.

6 Conclusion

This paper introduces a new general time-varying VAR model which adopts a full equation-specific approach. It provides general autoregressive formulations for the dynamic parameters, in contrast to the random walk assumption used by the canonical approach of Primiceri (2005). On the methodological side, it shows that the efficiency of the precision sampler developed by Chan and Eisenstat (2018) crucially depends on the dimension of the dynamic parameter considered. Based on this conclusion, it proposes an optimal sampling algorithm which maximizes the efficiency of the estimation procedure.

From a case study on four countries during the Great Recession, overwhelming evidence is found against the homogenous random walk formulation of Primiceri (2005). In general, it is shown that forecast accuracy can be significantly improved by adopting the random inertia model (at short forecast horizons) or a combination of random inertia and random mean (at longer forecast horizons). This confirms that a dynamic model which approaches the true data generating process performs typically better than a model relying on excessively simple formulations.

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Appendix A The Carter-Kohn algorithm

This appendix derives the posterior distributions for the dynamic parameters from the methodology of Carter and Kohn (1994). A general linear Gaussian dynamic model can be written in state-space form as:

$$\begin{array}{lll} \text{Observation equation:} & y_t = A_t z_t + \upsilon_t & \upsilon_t \sim \mathcal{N}(0, \Upsilon_t) \\ \text{Transition equation:} & z_t = B_t w_t + C_t z_{t-1} + \kappa_t & \kappa_t \sim \mathcal{N}(0, K_t) \end{array} \tag{51}$$

where y_t denotes the observed variable, z_t the state or unobserved variable, and w_t the exogenous observed variable. A_t, B_t and C_t denote matrices of coefficients. υ_t and κ_t are shocks with respective variance-covariance matrices Υ_t and K_t. For the dynamic parameters β_i, λ_i and δ_i, it can be shown that the state-space representation is given by:

	y_t	z_t	w_t	A_t	B_t	C_t	Υ_t	K_t
β_i	$y_{i,t} + \delta'_{i,t}\varepsilon_{-i,t}$	$\beta_{i,t}$	$(1-\rho_i)b_i$	x_t	1	ρ_i	$s_i\ exp(\lambda_{i,t})$	Ω_i
λ_i	$\hat{y}_{i,t} - m_J$	$\lambda_{i,t}$	–	1	–	γ_i	v_J	ϕ_i
δ_i	$\varepsilon_{i,t}$	$\delta_{i,t}$	$(1-\alpha_i)d_i$	$-\varepsilon'_{i,t}$	1	α_i	$s_i\ exp(\lambda_{i,t})$	Ψ_i

Table 6: State-space representation for the dynamic parameters

Given this state-space representation, note that the joint posterior $\pi(z|y)$ can rewrite as:

$$\pi(z|y) = \pi(z_T|y_T) \prod_{t=1}^{T-1} \pi(z_t|z_{t+1}, y_t) \tag{52}$$

Therefore, if one can sample z_T from $\pi(z_T|y_T)$, it is then possible to draw the values $z_{T-1}, \ldots, z_1$ recursively from $\pi(z_t|z_{t+1}, y_t)$, assuming the densities are known. The problem thus consists in determining $\pi(z_T|y_T)$ and the series of densities $\pi(z_t|z_{t+1}, y_t)$. To do so, introduce first the following notations:

$y_{t|s} = \mathbb{E}(y_t|y_1, \cdots, y_s)$ $\quad z_{t|s} = \mathbb{E}(z_t|y_1, \cdots, y_s)$ $\quad \Upsilon_{t|s} = var(y_t|y_1, \cdots, y_s)$

$K_{t|s} = var(z_t|y_1, \cdots, y_s)$ $\quad \tilde{z}_{t|s} = \mathbb{E}(z_t|z_s, y_1, \cdots, y_t)$ $\quad \tilde{K}_{t|s} = var(z_t|z_s, y_1, \cdots, y_t)$

This implies that:

$\pi(z_t|y_1, \cdots, y_s) \sim \mathcal{N}(z_{t|s}, K_{t|s})$ $\quad \pi(z_t|z_s, y_1, \cdots, y_t) \sim \mathcal{N}(\tilde{z}_{t|s}, \tilde{K}_{t|s})$

Carter and Kohn (1994) then propose the following procedure:

Algorithm A.1: Carter-Kohn algorithm for general dynamic parameters:

1. For $t = 1, \cdots, T$, apply the Kalman filter recursively (forward pass)[15]:
   step 1. state, prediction: $z_{t|t-1} = B_t w_t + C_t z_{t-1|t-1}$
   step 2. state, prediction error: $K_{t|t-1} = C_t K_{t-1|t-1} C_t' + K_t$
   step 3. observed, prediction: $y_{t|t-1} = A_t z_{t|t-1}$
   step 4. observed, prediction error: $\Upsilon_{t|t-1} = A_t K_{t|t-1} A_t' + \Upsilon_t$
   step 5. state, correction: $z_{t|t} = z_{t|t-1} + \Phi_t (y_t - y_{t|t-1})$
   step 6. state, prediction error correction: $K_{t|t} = K_{t|t-1} - \Phi_t \Upsilon_{t|t-1} \Phi_t'$
   with: $\Phi_t = K_{t|t-1} A_t' \Upsilon_{t|t-1}^{-1}$

2. Sample z_T from $\pi(z_T | y_T) \sim \mathcal{N}(z_{T|T}, K_{T|T})$.

3. For $t = T-1, \cdots, 1$, apply the following steps recursively (backward pass):
   step 1. state, correction: $\tilde{z}_{t|t+1} = z_{t|t} + \Xi_t (z_{t+1} - z_{t+1|t})$
   step 2. state, prediction error correction: $\tilde{K}_{t|t+1} = K_{t|t} - \Xi_t C_t K_{t|t}$
   with: $\Xi_t = K_{t|t} C_t' K_{t+1|t}^{-1}$
   step 3. sampling: $\pi(z_t | z_{t+1}, y_t) \sim \mathcal{N}(\tilde{z}_{t|t+1}, \tilde{K}_{t|t+1})$.

This simple algorithm can then be used to sample β_i, λ_i and δ_i, using the state-space formulations defined in Table 6.

[15] For the initial period $t = 1$, the first two steps are slightly different; they become $z_{1|0} = B_1 w_1$ and $K_{1|0} = K_1$, respectively.

Appendix B Data sources and transformations

Code	Series	Transformation
GDP	real gross domestic product	$100\ ln(y_t/y_{t-4})$
UR	unemployment rate	none
HICP	harmonised consumer price index	$100\ ln(y_t/y_{t-4})$
STR	short-term interest rate	none
LTR	long-term interest rate	none
REER	real effective exchange rate	$20\ ln(y_t/y_{t-4})$
IP	industrial production	$100\ ln(y_t/y_{t-4})$
CU	capacity utilization	$100\ ln(y_t/y_{t-4})$
TIE	total industry employment	$100\ ln(y_t/y_{t-4})$
HS	housing starts	$10\ ln(y_t/y_{t-4})$
FSI	financial stock index	$100\ ln(y_t/y_{t-4})$
LCI	leading composite indicator	$100\ ln(y_t/y_{t-4})$

Table 7: Dataset description and transformations

Country	recession sample	recovery sample	full dataset
Ca	1971q1-2008q4 (T=149)	1971q1-2009q3 (T=152)	1971q1-2018q2 (T=187)
Ea	1981q1-2008q3 (T=108)	1981q1-2009q1 (T=110)	1981q1-2016q4 (T=141)
Jp	1975q2-2008q1 (T=129)	1975q2-2009q1 (T=133)	1975q2-2018q2 (T=170)
Us	1971q1-2008q3 (T=148)	1971q1-2009q2 (T=151)	1971q1-2018q2 (T=187)

Table 8: Estimation samples

Sources:[16]

GDP:
Ca: OECD, volume estimates, seasonally adjusted (millions of Canadian dollars).
Ea: Area Wide Model database, volume estimates, seasonally adjusted (millions of Euros).
Jp: OECD, volume estimates, seasonally adjusted (millions of Yens).
Us: OECD, volume estimates, seasonally adjusted (millions of US dollars).

UR:
Ca: OECD, harmonised unemployment rate, seasonally adjusted.
Ea: Area Wide Model database, harmonised unemployment rate, seasonally adjusted.
Jp: OECD, harmonised unemployment rate, seasonally adjusted.
Us: OECD, harmonised unemployment rate, seasonally adjusted.

[16]The author is grateful to Jonathan Benchimol from Bank of Israel for providing part of this dataset.

HICP:
Ca: OECD, consumer price index, all items, index 2015=100.
Ea: Area Wide Model database, harmonised consumer price index, all items, index 2015=100.
Jp: OECD, consumer price index, all items, index 2015=100.
Us: OECD, consumer price index, all items, index 2015=100.

STR:
Ca: OECD, 3 month interbank rate.
Ea: Area Wide Model database, short-term interest rate (Euribor 3-month).
Jp: OECD Economic Outlook, short-term interest rate.
Us: OECD, 3 month interbank rate.

LTR:
Ca: OECD, 10 year government bond yield.
Ea: OECD, 10 year government bond yield.
Jp: OECD Economic Outlook, long-term interest rate on government bonds.
Us: OECD, 10 year government bond yield.

REER:
Ca: OECD, real effective exchange rate, index 2015=100.
Ea: OECD, real effective exchange rate, index 2015=100.
Jp: OECD, real effective exchange rate, index 2015=100.
Us: OECD, real effective exchange rate, index 2015=100.

IP:
Ca: OECD, industrial production, seasonally adjusted, index 2015=100.
Ea: OECD, industrial production, seasonally adjusted, index 2015=100.
Jp: OECD, industrial production, seasonally adjusted, index 2015=100.
Us: OECD, industrial production, seasonally adjusted, index 2015=100.

CU:
Ca: Statistics Canada, industrial capacity utilization rate (archived and regular series), total industry.
Ea: Eurostats, current capacity utilization rate, seasonally adjusted.
Jp: Federal Reserve Bank of Saint Louis Database (FRED), capacity utilization rate, seasonally adjusted.
Us: Federal Reserve Bank of Saint Louis Database (FRED), capacity utilization rate, seasonally adjusted.

TIE:
Ca: OECD, total industry employment (including construction), thousands of persons.
Ea: OECD, total industry employment (including construction), thousands of persons.
Jp: OECD, total industry employment (including construction), thousands of persons.
Us: OECD, total industry employment (including construction), thousands of persons.

HS:
Ca: Federal Reserve Bank of Saint Louis Database (FRED), total dwellings and residential buildings by stage of construction started for Canada, number of permits, seasonally adjusted.
Ea: Federal Reserve Bank of Saint Louis Archival Database (ALFRED), dwellings and residential buildings permits issued for construction for the Euro Area.
Jp: Federal Reserve Bank of Saint Louis Database (FRED), total dwellings and residential buildings by stage of construction started for Japan, number of permits, seasonally adjusted.
Us: Federal Reserve Bank of Saint Louis Database (FRED), total new privately owned housing units started (thousands of units), Seasonally Adjusted.

FSI:
Ca: Bloomberg, S&P/TSX Composite Index.
Ea: Bloomberg, Deutsche Boerse AG German Stock Index DAX (until 1986) and EURO STOXX 50 (from 1986 on).
Jp: Bloomberg, Nikkei 225 Index.
Us: Bloomberg, S&P 500 Index.

LCI:
Ca: OECD, composite leading indicator, index 2015=100.
Ea: OECD, composite leading indicator, index 2015=100.
Jp: OECD, composite leading indicator, index 2015=100.
Us: OECD, composite leading indicator, index 2015=100.

Appendix C Model estimates for the United States

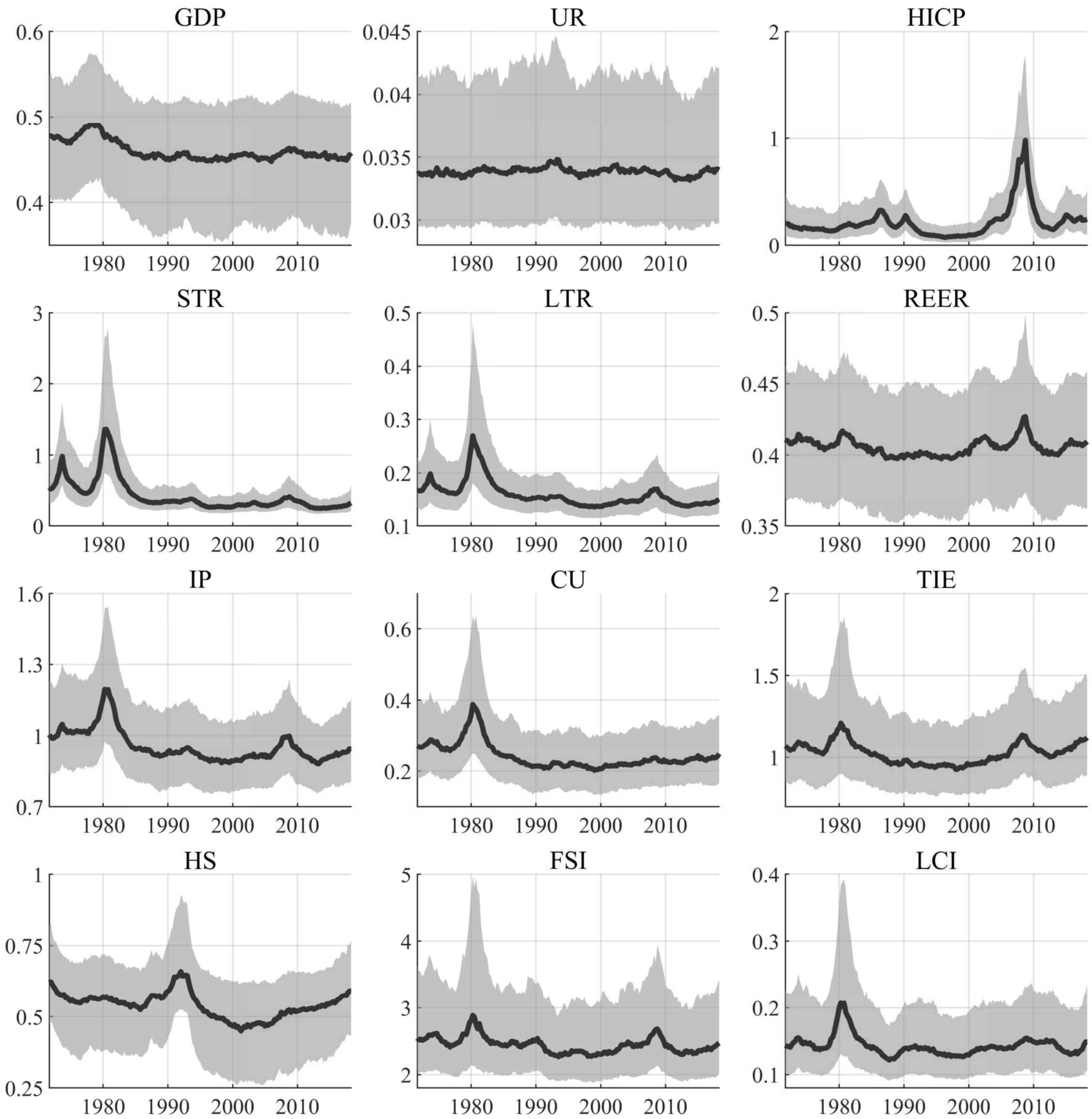

Figure 3: Stochastic volatility estimates with 70% credibility intervals,
United States, stationary autoregressive model, full dataset

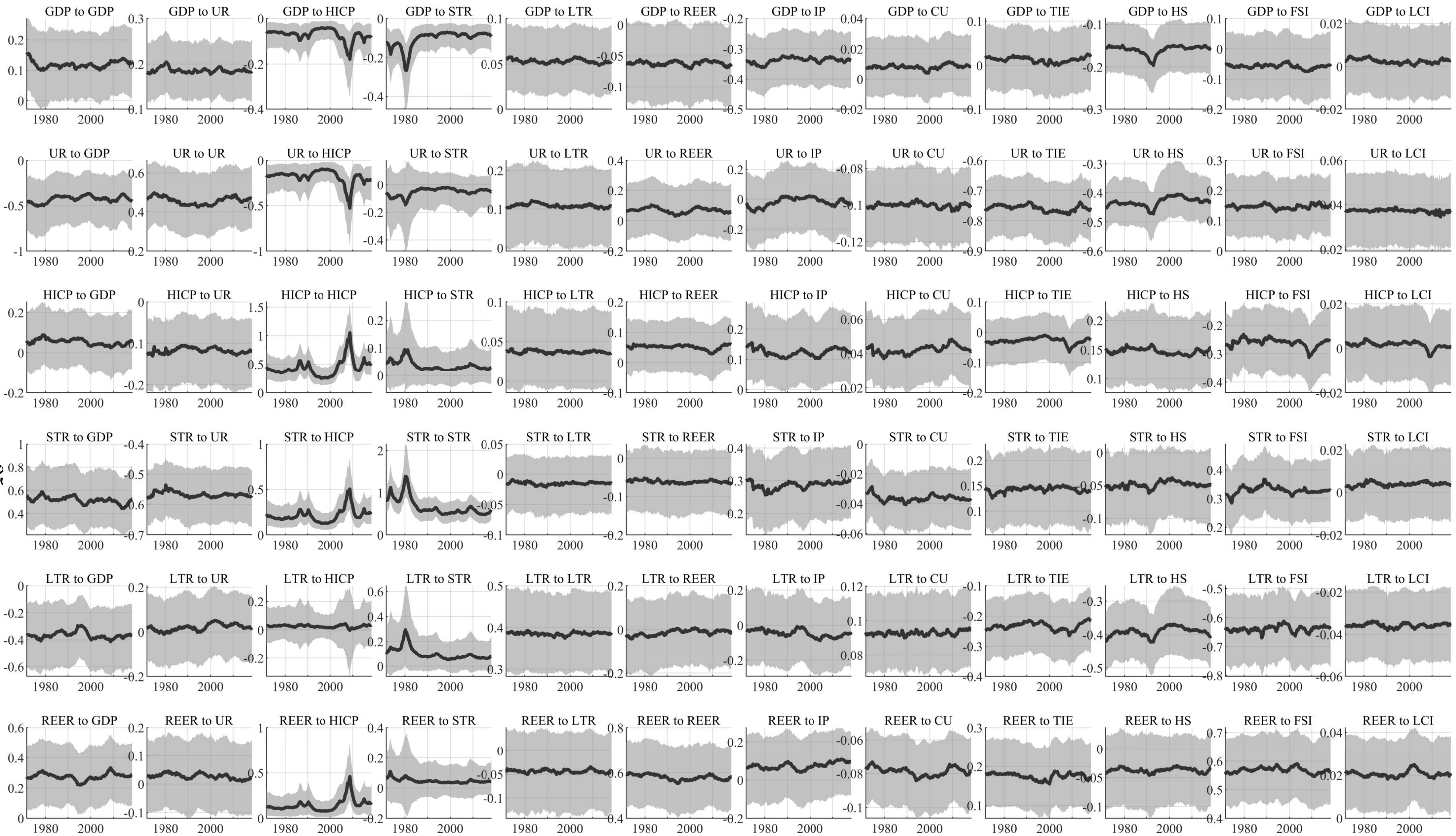

Figure 4: One-quarter ahead impulse response functions (Cholesky factorisation) with 70% credibility intervals, United States, stationary autoregressive model, full dataset

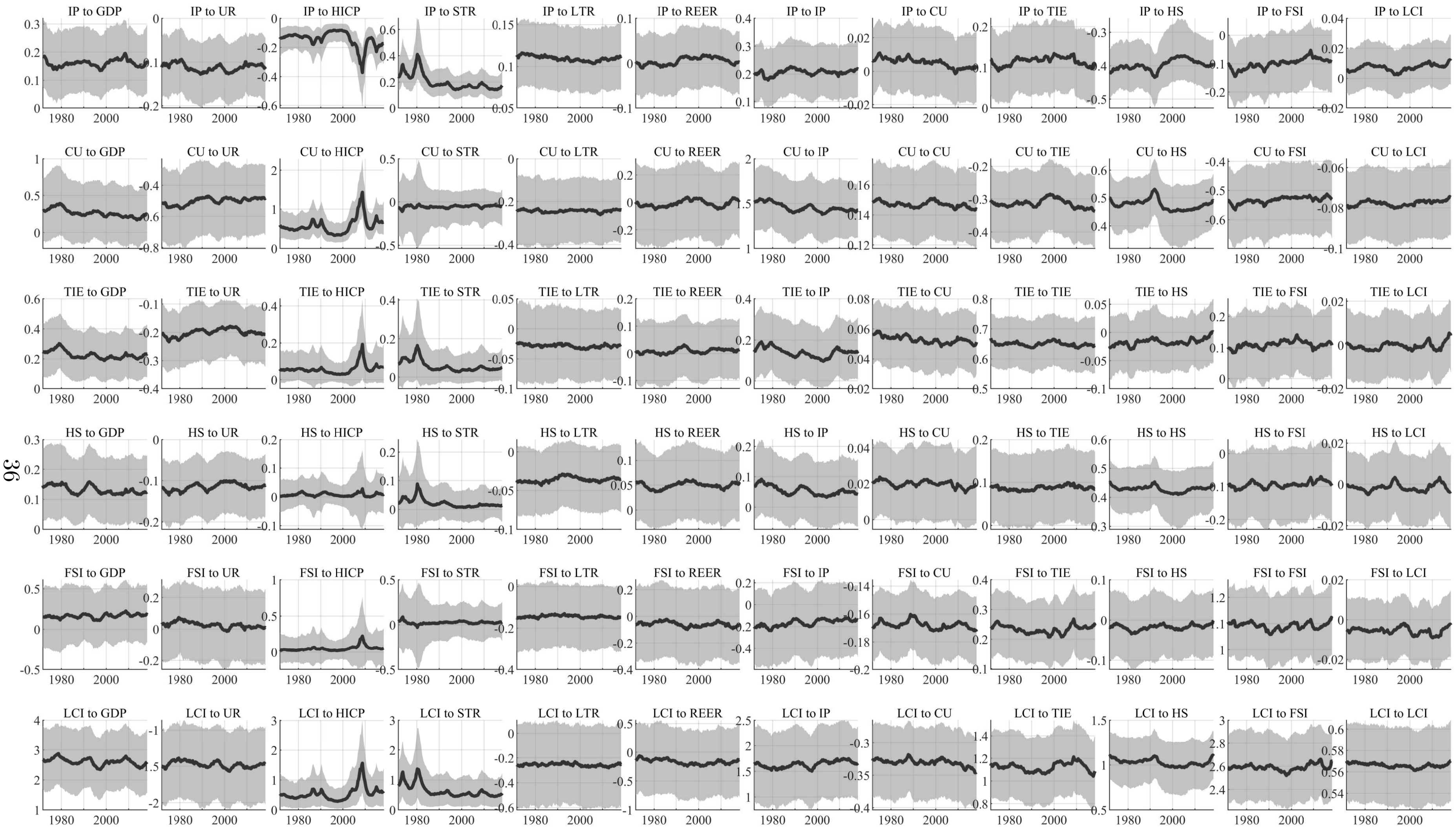

Figure 5: One-quarter ahead impulse response functions (Cholesky factorisation) with 70% credibility intervals, United States, stationary autoregressive model, full dataset

Appendix D Tables of results for the crisis experiment

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.48	1.11	1.19	1.06	1.18	0.42	0.67	0.80	0.62	0.78
Ca - UR	0.61	0.30	0.20	0.26	0.24	0.41	0.23	0.16	0.22	0.18
Ca - HICP	0.83	0.45	0.79	0.47	0.70	0.54	0.34	0.47	0.33	0.42
Ca - STR	0.46	0.52	0.61	0.39	0.45	0.44	0.34	0.36	0.34	0.29
Ca - LTR	0.66	0.16	0.13	0.03	0.05	0.45	0.23	0.14	0.22	0.15
Ca - REER	3.12	2.60	1.75	2.13	1.57	2.23	1.98	1.26	1.52	1.11
Ca - IP	1.98	2.45	1.91	2.14	1.83	1.16	1.56	1.22	1.32	1.13
Ca - CU	3.45	2.57	2.17	2.36	1.92	2.40	1.68	1.48	1.52	1.29
Ca - TIE	4.12	2.69	2.33	3.18	2.50	2.93	1.82	1.60	2.35	1.76
Ca - HS	3.02	3.81	3.70	3.47	3.60	1.95	2.78	2.97	2.53	2.81
Ca - FSI	3.92	3.19	2.67	2.79	2.61	2.52	2.12	1.78	1.79	1.73
Ca - LCI	0.78	0.59	0.41	0.69	0.49	0.52	0.36	0.25	0.41	0.30
Ea - GDP	0.65	0.69	0.68	0.77	0.76	0.39	0.42	0.41	0.46	0.46
Ea - UR	0.14	0.14	0.18	0.17	0.16	0.20	0.15	0.14	0.15	0.14
Ea - HICP	1.84	1.94	1.95	1.96	1.91	1.36	1.56	1.60	1.58	1.57
Ea - STR	0.30	0.25	0.17	0.24	0.20	0.26	0.22	0.17	0.21	0.18
Ea - LTR	0.07	0.18	0.16	0.15	0.14	0.22	0.19	0.17	0.19	0.16
Ea - REER	1.80	1.74	1.82	1.82	1.80	1.23	1.20	1.35	1.29	1.34
Ea - IP	3.56	3.72	3.64	3.83	3.60	2.75	2.96	3.00	3.10	2.94
Ea - CU	0.34	0.49	0.55	0.48	0.48	0.28	0.30	0.33	0.31	0.29
Ea - TIE	0.32	0.19	0.18	0.23	0.19	0.25	0.17	0.15	0.18	0.16
Ea - HS	1.09	0.65	0.45	0.18	0.27	0.65	0.42	0.32	0.28	0.29
Ea - FSI	2.94	2.93	2.66	3.82	3.18	1.88	1.91	1.77	2.63	2.05
Ea - LCI	0.07	0.15	0.23	0.19	0.19	0.23	0.17	0.17	0.19	0.17
Jp - GDP	0.47	0.12	0.21	0.01	0.02	0.41	0.23	0.22	0.26	0.23
Jp - UR	0.22	0.01	0.11	0.01	0.10	0.27	0.13	0.10	0.13	0.10
Jp - HICP	0.98	0.94	0.99	0.95	1.01	0.58	0.57	0.71	0.60	0.72
Jp - STR	0.16	0.21	0.19	0.13	0.18	0.28	0.20	0.14	0.18	0.13
Jp - LTR	0.28	0.11	0.12	0.09	0.11	0.30	0.18	0.12	0.16	0.13
Jp - REER	1.72	1.11	1.09	1.14	0.78	1.05	0.66	0.64	0.69	0.47
Jp - IP	0.79	1.54	1.83	1.53	1.74	0.66	0.92	1.12	0.91	1.08
Jp - CU	0.14	0.25	0.35	0.08	0.21	0.33	0.23	0.22	0.20	0.18
Jp - TIE	0.76	0.70	0.68	0.86	0.86	0.54	0.44	0.42	0.51	0.50
Jp - HS	0.10	0.16	0.08	0.01	0.20	0.46	0.36	0.31	0.27	0.26
Jp - FSI	1.53	1.76	1.79	1.26	1.38	1.00	1.03	1.05	0.77	0.85
Jp - LCI	0.05	0.04	0.04	0.00	0.02	0.32	0.17	0.11	0.16	0.12
Us - GDP	1.67	1.78	1.62	1.88	1.81	1.07	1.21	1.22	1.34	1.38
Us - UR	0.05	0.05	0.09	0.03	0.13	0.24	0.15	0.11	0.14	0.13
Us - HICP	4.38	3.94	3.86	3.77	3.73	3.68	3.40	3.44	3.27	3.29
Us - STR	0.64	1.03	1.02	1.08	1.12	0.45	0.61	0.65	0.64	0.70
Us - LTR	0.01	0.25	0.32	0.12	0.34	0.28	0.22	0.20	0.20	0.22
Us - REER	1.65	1.44	1.55	1.64	1.54	1.01	0.92	1.10	1.13	1.10
Us - IP	1.54	0.52	0.25	0.83	1.14	0.93	0.40	0.27	0.51	0.69
Us - CU	0.61	0.16	0.11	0.55	0.40	0.41	0.21	0.16	0.34	0.25
Us - TIE	4.08	3.42	3.19	3.48	3.23	3.07	2.65	2.60	2.77	2.65
Us - HS	2.04	1.68	1.81	1.88	1.99	1.32	1.09	1.39	1.30	1.47
Us - FSI	6.52	7.14	7.15	7.49	7.25	5.26	5.71	6.16	6.50	6.41
Us - LCI	0.95	1.27	1.50	1.24	1.61	0.58	0.89	1.19	0.83	1.25

Table 9: RMSE and CRPS: Recession phase, 1 quarter ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.87	1.37	1.49	1.20	1.39	1.11	0.97	1.08	0.83	0.95
Ca - UR	0.55	0.30	0.21	0.20	0.18	0.86	0.42	0.27	0.42	0.27
Ca - HICP	1.98	0.45	1.59	1.22	1.35	1.60	1.04	1.38	1.01	1.11
Ca - STR	0.33	0.52	0.43	0.33	0.42	1.03	0.56	0.35	0.61	0.40
Ca - LTR	0.63	0.16	0.32	0.32	0.38	0.93	0.51	0.34	0.52	0.40
Ca - REER	4.35	2.60	2.55	3.15	2.23	3.34	3.89	2.23	2.64	1.81
Ca - IP	4.33	2.45	4.01	4.30	4.03	3.50	4.38	3.89	3.83	3.79
Ca - CU	5.59	2.69	3.04	3.14	2.66	4.49	2.92	2.51	2.33	2.02
Ca - TIE	5.29	3.81	2.52	3.91	2.77	4.07	2.12	1.69	3.01	1.89
Ca - HS	2.35	3.81	2.88	2.54	2.79	1.30	1.00	1.03	0.82	0.96
Ca - FSI	8.01	3.19	4.98	5.44	4.73	7.07	6.15	4.68	4.85	4.32
Ca - LCI	2.07	0.59	1.25	1.88	1.45	1.93	1.45	1.01	1.54	1.20
Ea - GDP	1.85	1.94	1.90	2.09	2.05	1.54	1.73	1.88	1.93	1.98
Ea - UR	0.42	0.45	0.51	0.49	0.47	0.58	0.43	0.43	0.45	0.41
Ea - HICP	2.74	2.78	2.75	2.74	2.69	2.34	2.59	2.76	2.56	2.62
Ea - STR	1.21	1.12	0.97	1.20	0.97	1.04	0.92	0.84	0.97	0.82
Ea - LTR	0.59	0.28	0.27	0.29	0.30	0.74	0.48	0.37	0.46	0.38
Ea - REER	2.07	1.92	2.12	2.04	2.10	1.39	1.23	1.58	1.39	1.55
Ea - IP	7.12	7.65	7.67	7.72	7.46	6.65	7.97	8.52	7.96	8.11
Ea - CU	1.67	1.62	1.50	1.66	1.54	1.41	1.43	1.38	1.45	1.44
Ea - TIE	0.79	0.59	0.52	0.68	0.53	0.79	0.54	0.46	0.59	0.47
Ea - HS	1.09	0.54	0.34	0.28	0.26	0.85	0.58	0.42	0.53	0.47
Ea - FSI	2.21	2.24	1.98	3.68	2.68	1.51	1.26	1.04	2.07	1.47
Ea - LCI	0.66	0.41	0.19	0.27	0.22	1.01	0.65	0.45	0.62	0.54
Jp - GDP	1.34	0.95	0.96	0.82	0.70	1.18	0.81	0.80	0.74	0.61
Jp - UR	0.22	0.04	0.16	0.04	0.14	0.51	0.24	0.16	0.24	0.16
Jp - HICP	1.92	1.85	1.90	1.85	1.95	1.53	1.67	2.02	1.68	1.96
Jp - STR	0.46	0.54	0.61	0.45	0.62	0.64	0.50	0.49	0.43	0.51
Jp - LTR	0.37	0.29	0.31	0.24	0.19	0.54	0.35	0.27	0.32	0.23
Jp - REER	1.90	1.27	1.26	1.36	0.82	1.27	0.89	0.84	0.95	0.60
Jp - IP	2.90	1.75	1.54	2.08	1.51	2.48	1.24	0.84	1.49	0.86
Jp - CU	0.35	0.26	0.44	0.09	0.25	0.67	0.43	0.37	0.35	0.29
Jp - TIE	1.13	1.04	1.01	1.20	1.20	0.88	0.77	0.75	0.88	0.87
Jp - HS	3.42	3.10	3.03	3.24	3.00	3.39	3.30	3.40	3.72	3.47
Jp - FSI	1.59	1.28	1.28	1.50	1.16	1.48	0.90	0.75	1.13	0.83
Jp - LCI	0.58	0.54	0.59	0.50	0.46	1.07	0.61	0.52	0.57	0.45
Us - GDP	1.67	1.57	1.55	1.69	1.80	1.03	0.82	0.90	0.88	1.16
Us - UR	0.28	0.47	0.62	0.44	0.69	0.67	0.47	0.52	0.46	0.57
Us - HICP	4.89	4.73	4.58	4.49	4.49	3.97	4.26	4.37	4.12	4.28
Us - STR	0.46	0.93	0.81	0.98	0.84	0.89	0.73	0.46	0.73	0.49
Us - LTR	0.10	0.38	0.57	0.12	0.72	0.84	0.53	0.48	0.55	0.61
Us - REER	1.84	1.35	1.46	1.54	1.48	1.25	0.82	0.81	0.90	0.85
Us - IP	3.11	2.45	2.56	3.04	3.78	2.43	2.13	2.50	2.78	4.01
Us - CU	1.08	1.01	1.15	1.83	1.61	1.45	0.95	0.97	1.54	1.34
Us - TIE	5.24	4.86	4.56	4.56	4.96	3.82	4.24	4.28	3.45	4.40
Us - HS	3.23	2.92	3.41	2.88	3.63	2.46	2.55	3.50	2.27	3.56
Us - FSI	7.24	8.76	9.09	8.49	8.97	4.83	7.12	8.50	6.22	8.10
Us - LCI	1.39	2.31	2.94	2.01	3.07	1.37	1.86	2.93	1.50	2.95

Table 10: RMSE and CRPS: Recession phase, 2 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.95	1.21	1.38	1.00	1.20	2.12	1.11	0.77	1.00	0.70
Ca - UR	0.46	0.76	0.69	0.66	0.57	1.66	0.94	0.69	0.90	0.63
Ca - HICP	2.29	1.90	2.23	1.80	1.85	2.03	1.74	2.03	1.53	1.55
Ca - STR	0.48	0.88	1.10	1.07	1.15	2.09	1.26	1.08	1.32	1.16
Ca - LTR	0.72	0.64	0.85	0.75	0.88	1.75	1.00	0.85	0.97	0.87
Ca - REER	5.27	5.39	2.91	3.76	2.48	4.16	4.76	2.27	3.03	1.78
Ca - IP	5.41	5.28	4.26	4.55	4.27	4.65	3.56	2.86	3.01	2.79
Ca - CU	6.43	3.44	2.74	2.72	2.29	5.02	2.08	1.37	1.66	1.25
Ca - TIE	4.97	2.55	2.08	3.39	2.26	2.80	1.31	0.90	1.49	0.92
Ca - HS	1.97	2.42	2.35	2.21	2.28	2.31	1.27	0.75	1.40	0.80
Ca - FSI	11.82	9.62	6.74	7.69	6.41	10.37	9.56	6.56	7.11	5.93
Ca - LCI	3.29	3.09	1.99	3.15	2.37	4.08	2.82	1.76	2.86	2.12
Ea - GDP	1.64	1.76	1.77	2.04	1.94	1.44	1.05	0.95	1.24	1.09
Ea - UR	0.45	0.52	0.60	0.57	0.57	1.10	0.66	0.51	0.68	0.54
Ea - HICP	3.38	3.32	3.26	3.26	3.16	2.68	2.86	3.17	2.88	2.96
Ea - STR	1.19	1.04	0.92	1.19	0.90	1.49	1.01	0.68	1.03	0.71
Ea - LTR	1.16	0.69	0.65	0.63	0.65	1.58	1.00	0.74	0.91	0.76
Ea - REER	1.75	1.65	1.94	1.78	1.91	1.29	0.98	0.92	0.91	0.90
Ea - IP	6.63	7.40	7.64	7.66	7.38	4.11	4.03	4.73	4.48	4.38
Ea - CU	2.59	2.61	2.49	2.69	2.51	2.29	2.36	2.50	2.39	2.47
Ea - TIE	1.07	0.78	0.66	0.95	0.69	1.52	0.92	0.66	0.96	0.70
Ea - HS	1.59	0.98	0.84	0.30	0.45	1.73	1.20	0.91	0.88	0.79
Ea - FSI	3.00	2.47	2.33	3.04	2.24	3.46	2.52	2.10	2.04	1.95
Ea - LCI	2.19	1.62	1.11	1.33	1.14	2.88	1.89	1.29	1.67	1.42
Jp - GDP	3.29	2.56	2.49	2.37	2.18	3.18	2.64	3.00	2.53	2.58
Jp - UR	0.25	0.04	0.17	0.03	0.14	0.82	0.35	0.21	0.36	0.20
Jp - HICP	1.67	1.70	1.78	1.63	1.83	0.96	0.84	0.95	0.69	0.94
Jp - STR	0.65	0.79	0.84	0.62	0.92	1.01	0.77	0.72	0.63	0.80
Jp - LTR	0.43	0.29	0.35	0.27	0.18	0.90	0.49	0.32	0.44	0.30
Jp - REER	3.67	3.38	3.38	3.43	3.15	3.53	3.81	4.29	3.96	3.92
Jp - IP	8.19	6.45	5.89	7.23	5.92	8.44	7.49	7.50	8.78	7.69
Jp - CU	1.42	0.84	0.68	1.15	0.75	1.65	0.95	0.64	1.20	0.76
Jp - TIE	1.18	1.03	0.96	1.14	1.09	1.03	0.68	0.53	0.67	0.53
Jp - HS	3.17	2.70	2.60	2.87	2.58	1.63	0.97	0.82	1.16	0.82
Jp - FSI	5.84	5.34	5.37	6.32	5.89	5.86	6.14	6.79	7.75	7.65
Jp - LCI	1.68	1.65	1.89	1.68	1.69	2.27	1.71	2.08	1.69	1.82
Us - GDP	1.73	1.71	2.03	1.60	2.33	1.60	1.32	1.67	1.17	1.89
Us - UR	0.40	0.78	1.11	0.70	1.16	1.42	0.86	1.04	0.81	1.08
Us - HICP	5.23	5.11	4.91	4.99	4.82	3.68	4.09	4.36	4.24	4.26
Us - STR	0.42	1.14	0.80	1.47	0.70	2.16	1.44	0.86	1.73	0.96
Us - LTR	0.49	0.45	0.48	0.92	0.60	1.96	1.08	0.66	1.45	0.76
Us - REER	2.51	1.64	1.60	1.85	1.61	2.25	1.38	1.11	1.52	1.13
Us - IP	3.29	3.56	4.41	3.53	5.75	3.57	3.07	4.24	2.87	5.58
Us - CU	0.97	1.31	1.91	2.32	2.68	2.96	1.65	1.73	2.23	2.44
Us - TIE	5.52	5.61	5.47	4.59	5.59	4.43	4.03	4.34	3.23	3.97
Us - HS	3.41	3.61	4.36	3.28	4.38	2.62	2.83	4.28	2.37	3.88
Us - FSI	7.13	9.81	10.27	8.42	9.26	5.87	6.95	8.13	5.34	6.30
Us - LCI	1.19	2.77	3.98	1.95	3.81	2.76	2.19	3.58	1.66	3.19

Table 11: RMSE and CRPS: Recession phase, 3 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	1.04	1.43	1.31	1.37	1.29	3.72	1.98	0.99	1.89	1.17
Ca - UR	0.42	1.37	1.09	1.09	0.95	2.92	1.63	1.10	1.43	1.02
Ca - HICP	2.25	1.81	2.01	1.60	1.61	2.65	1.70	0.95	1.24	0.80
Ca - STR	0.62	1.73	2.05	1.79	2.01	3.61	2.27	2.19	2.14	2.12
Ca - LTR	1.21	0.92	1.20	0.88	1.18	2.99	1.50	1.18	1.30	1.12
Ca - REER	6.11	6.83	3.57	4.79	3.10	5.84	6.51	3.37	4.50	2.89
Ca - IP	5.23	4.60	3.69	3.95	3.70	6.39	2.93	1.55	2.56	1.74
Ca - CU	5.85	3.44	2.98	3.09	2.82	6.08	3.14	2.25	2.99	2.52
Ca - TIE	4.48	2.72	2.45	2.93	2.33	3.52	2.68	2.06	1.87	1.79
Ca - HS	1.81	2.93	2.19	2.84	2.28	3.93	2.65	1.28	2.70	1.53
Ca - FSI	16.25	13.65	9.09	10.79	8.65	15.83	14.69	10.03	11.07	9.16
Ca - LCI	4.46	4.06	2.34	4.17	2.91	7.04	3.91	1.99	3.94	2.52
Ea - GDP	1.50	1.54	1.54	1.79	1.70	2.73	1.56	1.00	1.46	1.18
Ea - UR	0.41	0.49	0.60	0.59	0.57	2.08	1.00	0.59	0.98	0.68
Ea - HICP	3.87	3.69	3.62	3.66	3.51	3.11	2.91	3.30	2.97	3.04
Ea - STR	1.03	0.91	0.81	1.04	0.78	2.75	1.66	0.94	1.49	1.02
Ea - LTR	1.60	1.06	0.90	0.84	0.90	2.74	1.61	1.08	1.40	1.13
Ea - REER	1.65	1.46	1.68	1.55	1.66	2.33	1.42	0.80	1.14	0.84
Ea - IP	5.90	6.42	6.73	6.70	6.50	6.91	3.96	2.67	3.79	3.17
Ea - CU	2.59	2.72	2.65	2.85	2.67	2.90	2.10	1.83	2.12	1.88
Ea - TIE	1.25	0.87	0.68	1.14	0.77	2.77	1.42	0.84	1.40	0.94
Ea - HS	1.67	1.19	1.04	0.29	0.52	2.76	1.77	1.15	1.31	1.11
Ea - FSI	5.93	4.59	4.20	2.88	2.90	7.47	5.20	4.54	3.21	3.42
Ea - LCI	4.34	3.13	2.41	2.61	2.15	5.99	3.69	2.64	3.21	2.61
Jp - GDP	5.77	5.08	5.09	5.05	4.84	6.16	6.59	7.71	6.87	7.37
Jp - UR	0.37	0.21	0.21	0.25	0.20	1.35	0.51	0.27	0.52	0.28
Jp - HICP	1.46	1.47	1.56	1.42	1.60	1.32	0.71	0.43	0.62	0.46
Jp - STR	0.62	0.82	0.86	0.61	0.97	1.53	0.89	0.58	0.69	0.67
Jp - LTR	0.39	0.33	0.45	0.31	0.22	1.41	0.72	0.47	0.61	0.44
Jp - REER	4.11	4.08	4.16	4.06	3.92	3.27	3.60	4.30	3.66	3.98
Jp - IP	19.72	17.64	17.14	18.62	17.30	26.74	28.04	29.41	30.41	29.76
Jp - CU	4.43	3.74	3.56	4.18	3.65	5.37	4.98	5.46	6.05	5.83
Jp - TIE	1.33	1.05	0.93	1.15	1.03	1.55	0.83	0.57	0.81	0.59
Jp - HS	2.76	2.48	2.45	2.60	2.48	1.43	1.02	1.17	0.92	1.35
Jp - FSI	6.66	6.31	6.47	7.33	7.09	5.57	5.38	6.36	6.45	7.18
Jp - LCI	2.62	2.79	3.08	2.75	2.87	3.58	2.83	3.64	2.86	3.37
Us - GDP	1.54	1.69	2.43	1.38	2.48	2.96	1.98	2.02	1.91	1.81
Us - UR	0.34	0.91	1.46	0.73	1.47	2.68	1.24	1.30	1.16	1.26
Us - HICP	5.39	5.13	4.88	5.19	4.83	3.69	3.20	3.37	3.66	3.26
Us - STR	0.81	1.57	0.87	2.00	0.82	4.24	2.54	1.33	2.79	1.45
Us - LTR	1.03	0.74	0.42	1.37	0.76	3.57	1.88	0.98	2.35	1.06
Us - REER	2.69	1.66	1.45	1.89	1.41	3.22	1.70	0.92	1.82	0.86
Us - IP	3.10	3.36	5.09	3.10	6.08	8.19	4.31	4.19	5.18	4.47
Us - CU	1.49	1.14	2.06	2.03	3.09	5.44	2.28	1.67	2.68	2.47
Us - TIE	5.00	6.00	6.22	4.26	5.62	7.41	4.90	4.85	4.46	3.52
Us - HS	3.04	3.69	4.62	3.13	4.23	3.89	2.56	3.39	2.31	2.33
Us - FSI	6.28	9.94	10.44	7.58	8.38	10.37	7.56	6.45	7.08	3.32
Us - LCI	1.80	2.77	4.44	1.71	3.82	6.04	2.92	3.30	2.75	2.38

Table 12: RMSE and CRPS: Recession phase, 4 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	2.21	2.44	1.79	2.36	1.97	6.50	3.54	1.92	3.16	2.21
Ca - UR	0.44	1.86	1.38	1.41	1.22	4.75	2.31	1.33	1.94	1.26
Ca - HICP	2.12	1.66	1.80	1.44	1.51	3.96	2.36	1.04	1.62	1.15
Ca - STR	0.70	2.44	2.76	2.17	2.68	5.86	3.38	2.85	2.75	2.66
Ca - LTR	1.47	1.17	1.53	0.91	1.41	4.66	2.10	1.49	1.63	1.34
Ca - REER	6.30	7.66	3.84	5.39	3.43	8.10	6.43	2.97	4.58	2.76
Ca - IP	4.76	4.94	4.07	4.47	4.04	10.00	5.31	3.47	4.73	3.49
Ca - CU	6.03	5.97	5.11	5.54	5.03	9.83	7.18	6.20	6.67	5.88
Ca - TIE	4.21	4.08	3.45	2.87	3.15	5.48	4.98	3.61	2.90	3.18
Ca - HS	3.62	4.22	2.61	4.16	2.89	7.27	4.59	2.32	4.49	2.74
Ca - FSI	18.90	15.38	9.37	11.88	8.95	19.62	12.95	6.66	9.71	6.28
Ca - LCI	6.16	4.32	2.17	4.63	2.92	11.29	4.62	1.68	4.39	2.23
Ea - GDP	2.37	1.85	1.65	1.73	1.59	5.20	2.71	1.67	2.25	1.77
Ea - UR	0.37	0.44	0.55	0.57	0.54	3.67	1.48	0.71	1.39	0.87
Ea - HICP	3.98	3.68	3.62	3.69	3.49	3.50	2.29	2.26	2.27	2.09
Ea - STR	1.13	1.13	0.98	0.96	0.85	5.03	2.82	1.53	2.31	1.60
Ea - LTR	2.03	1.40	1.11	1.08	1.04	4.57	2.39	1.46	2.07	1.51
Ea - REER	2.51	1.88	1.72	1.76	1.75	4.49	2.53	1.40	1.97	1.48
Ea - IP	8.31	6.66	6.39	6.44	6.06	14.37	7.35	4.24	6.23	4.78
Ea - CU	2.32	2.52	2.50	2.69	2.56	4.85	2.69	1.68	2.51	1.92
Ea - TIE	1.30	0.87	0.63	1.20	0.78	4.77	2.07	1.06	1.89	1.22
Ea - HS	1.50	1.14	1.02	0.31	0.47	4.80	2.48	1.32	1.90	1.43
Ea - FSI	10.76	7.67	7.13	4.45	5.02	13.97	9.16	8.66	5.77	5.99
Ea - LCI	6.83	4.42	3.46	3.76	2.85	10.43	5.40	3.71	4.71	3.49
Jp - GDP	5.87	5.35	5.38	5.37	5.15	4.62	3.82	4.61	4.08	4.41
Jp - UR	0.61	0.44	0.40	0.50	0.42	2.21	0.77	0.49	0.77	0.51
Jp - HICP	1.43	1.38	1.41	1.38	1.45	2.18	1.03	0.50	0.94	0.57
Jp - STR	0.56	0.76	0.80	0.55	0.92	2.37	1.09	0.54	0.85	0.62
Jp - LTR	0.38	0.35	0.51	0.33	0.24	2.29	0.97	0.55	0.76	0.52
Jp - REER	4.00	4.14	4.26	4.06	4.00	3.29	2.60	2.96	2.48	2.69
Jp - IP	22.47	20.17	19.76	21.36	19.97	19.10	20.82	23.75	23.85	24.30
Jp - CU	5.51	4.74	4.59	5.29	4.69	5.34	4.70	5.47	5.80	5.79
Jp - TIE	2.44	1.86	1.79	2.05	1.85	3.02	2.18	2.33	2.45	2.42
Jp - HS	2.65	2.63	2.71	2.70	2.73	2.41	1.94	2.48	1.89	2.68
Jp - FSI	6.31	6.24	6.53	7.17	7.14	6.17	3.70	4.20	3.79	4.65
Jp - LCI	2.72	3.18	3.50	3.11	3.29	4.75	2.72	3.19	2.56	2.97
Us - GDP	1.77	1.56	2.31	1.96	2.22	5.94	3.19	1.93	3.49	1.50
Us - UR	0.54	1.05	1.82	0.68	1.74	5.00	1.82	1.69	1.77	1.50
Us - HICP	4.97	4.61	4.38	4.75	4.36	4.72	2.06	0.97	2.05	1.24
Us - STR	1.87	2.07	1.02	2.47	1.33	7.81	3.85	1.82	4.15	2.06
Us - LTR	1.56	1.11	0.43	1.59	1.14	6.32	2.85	1.30	3.37	1.59
Us - REER	2.80	1.54	1.31	1.79	1.31	5.27	2.20	1.04	2.45	1.12
Us - IP	5.32	3.11	5.67	4.44	5.73	18.39	8.73	5.98	10.77	5.07
Us - CU	2.68	1.04	2.09	2.10	3.01	9.71	3.20	1.93	4.34	2.39
Us - TIE	4.50	6.23	7.10	3.95	5.44	12.87	6.85	5.85	6.79	3.49
Us - HS	2.80	3.60	4.58	2.87	3.91	6.87	3.02	2.63	3.09	1.58
Us - FSI	7.01	8.98	9.44	7.05	7.90	19.64	9.61	5.07	10.74	3.97
Us - LCI	4.26	2.49	4.32	2.49	3.42	11.94	4.63	2.93	5.07	1.97

Table 13: RMSE and CRPS: Recession phase, 5 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	3.81	3.03	1.92	2.91	2.18	10.67	4.55	1.85	3.84	2.16
Ca - UR	0.58	2.22	1.52	1.63	1.35	7.66	3.02	1.37	2.42	1.38
Ca - HICP	1.98	1.61	1.65	1.31	1.48	6.15	3.34	1.32	2.13	1.47
Ca - STR	1.07	3.12	3.39	2.37	3.24	9.27	4.69	3.36	3.40	3.19
Ca - LTR	1.43	1.37	1.80	0.89	1.59	7.29	2.79	1.70	2.01	1.54
Ca - REER	6.62	7.93	3.78	5.58	3.43	12.54	5.91	2.14	4.14	2.10
Ca - IP	6.61	6.03	4.95	5.84	4.92	16.69	8.01	4.93	7.28	5.06
Ca - CU	9.76	8.32	6.60	7.58	6.38	17.33	9.80	7.04	8.75	6.54
Ca - TIE	4.66	5.19	3.87	3.08	3.62	8.86	6.59	3.54	3.99	3.52
Ca - HS	5.00	4.76	2.65	4.77	3.04	10.99	4.93	1.98	4.74	2.34
Ca - FSI	20.72	15.21	8.63	11.62	8.29	27.73	10.05	2.94	7.55	3.18
Ca - LCI	8.36	4.06	2.15	4.64	2.67	17.62	5.42	2.13	4.65	2.06
Ea - GDP	4.48	2.83	2.50	2.36	1.84	9.54	4.38	3.04	3.55	2.56
Ea - UR	0.35	0.41	0.50	0.53	0.51	6.51	2.05	0.88	1.89	1.08
Ea - HICP	3.80	3.54	3.48	3.59	3.36	5.12	2.22	1.60	2.15	1.62
Ea - STR	1.57	1.60	1.31	1.02	1.05	8.85	4.24	2.16	3.35	2.24
Ea - LTR	2.76	1.79	1.34	1.32	1.09	7.89	3.46	1.87	2.86	1.93
Ea - REER	3.36	2.21	1.82	1.91	1.84	7.39	3.32	1.70	2.47	1.74
Ea - IP	15.93	9.40	8.18	8.55	7.32	28.13	12.75	8.66	11.15	8.07
Ea - CU	3.56	2.62	2.51	2.70	2.45	9.12	4.08	2.32	3.61	2.43
Ea - TIE	1.24	0.80	0.59	1.16	0.74	8.16	2.92	1.37	2.46	1.54
Ea - HS	1.40	1.16	1.10	0.34	0.43	8.58	3.54	1.79	2.52	1.85
Ea - FSI	15.30	10.11	9.10	5.93	6.49	21.29	11.44	9.74	7.83	7.06
Ea - LCI	9.54	5.14	3.95	4.57	3.05	17.11	6.75	3.91	5.89	3.82
Jp - GDP	5.73	5.28	5.34	5.36	5.11	5.93	3.08	3.27	3.22	3.20
Jp - UR	0.90	0.61	0.58	0.70	0.61	3.55	1.01	0.65	0.97	0.69
Jp - HICP	1.58	1.54	1.48	1.63	1.51	3.45	1.63	1.09	1.56	1.07
Jp - STR	0.51	0.70	0.73	0.51	0.87	3.78	1.42	0.60	1.03	0.70
Jp - LTR	0.39	0.38	0.56	0.35	0.24	3.65	1.26	0.64	0.96	0.59
Jp - REER	4.06	4.22	4.39	4.15	4.07	4.87	2.89	3.11	2.69	2.72
Jp - IP	22.35	20.17	19.85	21.35	20.10	14.55	12.53	15.61	14.78	16.04
Jp - CU	5.75	5.10	5.01	5.63	5.06	5.62	3.92	4.51	4.50	4.67
Jp - TIE	3.03	2.47	2.39	2.73	2.48	4.06	2.60	2.84	2.97	3.08
Jp - HS	2.67	2.92	3.05	2.93	3.07	3.79	2.49	3.17	2.46	3.37
Jp - FSI	5.78	5.75	6.06	6.58	6.62	8.91	2.81	1.93	2.31	1.94
Jp - LCI	2.49	2.99	3.36	2.94	3.15	7.22	2.29	1.55	1.73	1.42
Us - GDP	3.04	1.56	2.18	2.63	2.20	11.18	5.01	2.55	5.34	2.10
Us - UR	1.58	1.08	2.08	0.62	1.84	9.30	2.63	1.84	2.74	1.49
Us - HICP	4.60	4.22	4.01	4.35	3.99	8.53	3.16	1.34	2.78	1.38
Us - STR	3.28	2.77	1.19	2.93	1.52	14.35	5.64	2.37	6.17	2.43
Us - LTR	3.10	1.70	0.60	1.96	1.39	11.39	4.18	1.67	5.18	1.99
Us - REER	3.10	1.65	1.26	1.98	1.24	9.52	3.19	1.31	3.94	1.47
Us - IP	10.16	2.94	5.51	7.61	5.52	35.81	15.42	8.23	18.64	6.34
Us - CU	4.42	1.32	1.92	2.79	2.81	17.04	4.89	2.26	6.88	2.71
Us - TIE	6.85	5.78	7.16	3.87	4.98	23.88	9.09	5.40	10.44	3.46
Us - HS	3.81	3.29	4.21	2.73	3.58	12.60	3.91	1.80	4.88	1.55
Us - FSI	10.94	8.29	8.67	7.73	8.45	35.50	13.89	6.46	17.08	6.58
Us - LCI	8.17	2.52	3.99	3.78	3.44	22.28	7.22	3.26	7.99	2.86

Table 14: RMSE and CRPS: Recession phase, 6 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	5.89	3.18	1.78	3.17	2.06	17.18	5.45	1.76	4.39	1.89
Ca - UR	1.38	2.44	1.50	1.72	1.35	12.02	3.75	1.27	2.84	1.35
Ca - HICP	1.85	1.51	1.64	1.25	1.62	9.65	4.48	1.74	2.69	1.93
Ca - STR	1.45	3.68	3.84	2.47	3.66	14.64	6.13	3.62	4.11	3.53
Ca - LTR	1.47	1.45	1.89	0.83	1.65	11.38	3.54	1.60	2.44	1.51
Ca - REER	8.14	7.86	3.60	5.55	3.28	20.25	5.89	1.64	3.89	1.60
Ca - IP	8.95	6.32	4.82	6.58	4.94	26.17	9.46	3.75	8.21	4.14
Ca - CU	15.09	9.44	6.68	8.54	6.44	27.44	10.29	4.60	8.54	4.58
Ca - TIE	5.11	5.24	3.59	2.98	3.39	13.58	7.54	2.84	4.60	2.97
Ca - HS	6.49	4.82	2.45	4.89	2.82	17.15	5.23	1.63	4.49	1.76
Ca - FSI	23.32	14.55	8.07	11.14	7.69	42.44	10.34	3.25	7.61	3.16
Ca - LCI	12.45	3.76	2.78	4.53	2.68	28.68	6.94	3.28	5.31	2.62
Ea - GDP	6.63	3.51	2.95	3.01	2.02	16.07	5.61	3.34	4.68	3.01
Ea - UR	0.40	0.41	0.46	0.52	0.49	11.11	2.82	1.10	2.47	1.33
Ea - HICP	3.57	3.36	3.30	3.45	3.21	8.65	2.65	1.33	2.40	1.53
Ea - STR	2.18	2.03	1.72	1.21	1.23	15.53	5.81	2.76	4.59	2.80
Ea - LTR	4.01	2.09	1.42	1.50	1.06	13.68	4.69	2.14	3.74	2.32
Ea - REER	3.62	2.13	1.75	1.83	1.75	11.84	3.98	1.73	2.94	1.77
Ea - IP	23.62	11.55	9.42	10.52	7.97	46.35	16.22	9.44	14.19	8.96
Ea - CU	6.99	3.75	3.29	3.77	2.89	16.95	6.44	3.90	5.81	3.65
Ea - TIE	1.16	0.75	0.64	1.09	0.69	14.07	4.08	1.72	3.23	1.92
Ea - HS	1.58	1.07	1.03	0.33	0.57	14.85	4.76	2.08	3.37	2.32
Ea - FSI	18.93	11.45	9.69	6.62	6.75	31.57	12.27	7.61	9.06	6.47
Ea - LCI	12.80	5.57	3.92	5.03	2.93	28.46	8.24	3.60	7.25	4.18
Jp - GDP	5.32	4.91	4.97	4.99	4.76	8.66	2.43	1.09	1.80	1.07
Jp - UR	1.02	0.65	0.61	0.75	0.66	5.59	1.20	0.54	1.06	0.57
Jp - HICP	1.60	1.65	1.54	1.78	1.53	5.26	1.94	1.13	1.63	1.09
Jp - STR	0.56	0.65	0.68	0.47	0.82	6.11	1.89	0.71	1.27	0.81
Jp - LTR	0.51	0.38	0.59	0.36	0.22	5.71	1.58	0.71	1.17	0.66
Jp - REER	3.76	3.92	4.07	3.85	3.77	7.07	2.20	1.04	1.59	0.97
Jp - IP	20.71	18.75	18.48	19.86	18.71	15.88	5.22	3.24	3.96	3.20
Jp - CU	5.46	4.94	4.91	5.43	4.93	7.43	3.03	2.55	2.57	2.47
Jp - TIE	3.17	2.63	2.56	2.93	2.67	5.47	2.20	2.09	2.29	2.26
Jp - HS	2.48	2.79	2.93	2.78	2.97	5.55	1.47	1.28	1.17	1.35
Jp - FSI	6.65	5.81	5.94	6.57	6.44	14.80	4.42	3.11	4.15	3.09
Jp - LCI	2.97	2.81	3.12	2.75	2.93	11.84	2.76	1.03	1.89	1.04
Us - GDP	4.88	1.86	2.03	3.26	2.26	20.70	7.56	3.22	8.10	2.64
Us - UR	3.35	1.01	2.22	0.69	1.83	17.33	3.91	1.92	4.18	1.48
Us - HICP	4.45	3.91	3.72	4.04	3.77	15.52	4.59	1.71	4.13	2.03
Us - STR	7.31	4.00	1.38	4.20	1.58	27.44	8.47	2.96	10.01	2.88
Us - LTR	5.09	2.67	0.74	2.62	1.31	20.99	6.20	2.06	8.23	2.22
Us - REER	3.40	1.81	1.38	2.18	1.47	17.54	4.63	1.76	6.07	2.17
Us - IP	18.49	4.11	5.16	11.15	6.84	65.91	24.24	11.10	28.80	9.32
Us - CU	5.12	1.73	1.85	3.12	3.44	30.98	7.19	2.97	10.48	4.24
Us - TIE	13.64	5.65	6.84	5.26	4.62	45.69	13.38	5.55	17.10	4.34
Us - HS	5.68	3.05	3.93	2.75	3.33	23.15	5.80	2.24	7.90	2.01
Us - FSI	18.63	8.47	8.20	8.90	8.23	65.41	19.89	8.12	26.89	6.02
Us - LCI	13.77	2.85	3.70	4.62	3.54	40.04	10.36	4.06	11.90	3.55

Table 15: RMSE and CRPS: Recession phase, 7 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	9.30	3.17	1.78	3.35	1.93	27.88	6.65	2.10	5.09	2.02
Ca - UR	2.86	2.63	1.45	1.88	1.36	19.17	4.75	1.34	3.55	1.49
Ca - HICP	1.75	1.42	1.81	1.27	1.92	15.37	5.95	2.25	3.40	2.50
Ca - STR	3.06	4.03	4.08	2.51	3.88	23.74	7.75	3.62	4.87	3.64
Ca - LTR	1.45	1.55	1.95	0.77	1.67	18.40	4.54	1.68	3.03	1.61
Ca - REER	12.13	7.80	3.46	5.51	3.12	33.73	6.91	1.73	4.18	1.57
Ca - IP	11.84	6.16	4.52	6.99	4.65	41.39	11.73	3.72	9.36	3.94
Ca - CU	20.67	9.80	6.28	8.95	6.09	41.90	11.34	3.37	8.82	3.77
Ca - TIE	6.58	5.03	3.56	2.98	3.21	22.06	9.48	3.54	5.74	3.29
Ca - HS	9.42	4.75	2.37	4.84	2.67	28.22	6.23	1.99	4.94	2.00
Ca - FSI	31.35	13.94	7.69	10.93	7.20	70.14	12.56	3.71	9.28	3.29
Ca - LCI	19.71	3.53	3.32	4.54	2.78	47.88	8.79	3.76	6.38	2.95
Ea - GDP	9.34	3.99	3.10	3.47	2.09	27.49	7.19	3.36	5.97	3.57
Ea - UR	0.38	0.39	0.46	0.49	0.46	18.78	3.73	1.34	3.32	1.62
Ea - HICP	3.36	3.17	3.13	3.35	3.06	14.97	3.37	1.37	3.00	1.67
Ea - STR	3.84	2.51	2.07	1.42	1.34	26.61	7.74	3.34	6.11	3.27
Ea - LTR	5.72	2.25	1.41	1.58	0.99	23.37	6.19	2.40	4.95	2.73
Ea - REER	3.67	2.04	1.65	1.77	1.65	20.52	5.22	1.94	3.84	2.09
Ea - IP	32.87	12.85	9.61	11.62	7.98	76.81	19.91	8.47	17.09	9.38
Ea - CU	10.70	4.82	3.79	4.83	3.23	29.11	8.35	4.14	7.52	4.13
Ea - TIE	1.67	0.72	0.69	1.03	0.65	24.64	5.55	2.03	4.26	2.26
Ea - HS	1.62	1.01	0.96	0.51	0.54	25.99	6.43	2.47	4.41	2.74
Ea - FSI	21.73	12.23	9.62	6.75	6.78	51.38	14.46	6.56	11.17	7.35
Ea - LCI	16.19	5.78	3.69	5.45	6.78	48.48	10.35	3.71	9.39	4.97
Jp - GDP	5.39	4.88	4.91	4.93	4.71	14.44	3.76	2.73	2.95	2.71
Jp - UR	1.06	0.64	0.61	0.78	0.67	9.02	1.48	0.50	1.24	0.53
Jp - HICP	1.50	1.58	1.47	1.71	1.45	8.31	2.15	0.85	1.43	0.81
Jp - STR	0.68	0.61	0.64	0.44	0.79	9.81	2.44	0.83	1.57	0.94
Jp - LTR	1.00	0.36	0.58	0.35	0.21	9.43	2.02	0.74	1.46	0.75
Jp - REER	3.57	3.67	3.82	3.61	3.55	11.58	2.72	1.13	1.92	1.18
Jp - IP	21.24	19.08	18.70	19.99	18.91	28.13	12.99	14.55	13.53	15.19
Jp - CU	5.42	4.78	4.71	5.22	4.73	12.23	3.42	1.82	2.41	1.86
Jp - TIE	3.07	2.64	2.55	2.93	2.68	8.31	2.20	1.49	1.89	1.61
Jp - HS	2.55	2.61	2.75	2.60	2.78	8.94	1.47	0.61	1.03	0.52
Jp - FSI	8.76	6.32	6.29	6.85	6.66	24.31	6.20	5.09	5.46	4.96
Jp - LCI	4.27	2.94	3.13	2.84	2.97	19.57	3.75	1.96	2.66	1.95
Us - GDP	8.20	2.79	1.90	4.24	2.14	37.81	11.16	3.98	12.87	3.00
Us - UR	5.58	0.96	2.28	0.90	1.78	31.61	5.76	2.10	6.56	1.68
Us - HICP	4.64	3.67	3.48	3.79	3.66	28.46	6.82	2.18	6.41	2.56
Us - STR	13.33	5.44	1.61	5.99	1.56	51.57	12.79	3.68	16.22	3.54
Us - LTR	8.60	3.74	0.77	3.60	1.28	40.22	9.13	2.49	13.13	2.83
Us - REER	3.43	1.81	1.54	2.09	1.93	32.65	6.76	2.22	9.08	3.00
Us - IP	33.40	6.67	4.83	13.92	7.49	123.39	35.21	14.01	44.02	10.56
Us - CU	4.99	1.86	1.82	3.02	3.88	58.24	10.91	3.81	16.39	5.28
Us - TIE	25.57	6.76	6.45	7.77	4.33	87.22	19.94	6.49	27.87	5.37
Us - HS	9.96	2.90	3.72	2.78	3.21	43.67	8.54	2.88	12.41	2.70
Us - FSI	30.52	10.00	7.85	11.25	7.70	123.04	29.25	10.01	43.19	6.59
Us - LCI	21.39	3.75	3.46	5.29	3.32	74.46	15.07	5.12	18.35	3.87

Table 16: RMSE and CRPS: Recession phase, 8 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.16	0.18	0.16	0.16	0.14	0.40	0.37	0.28	0.35	0.28
Ca - UR	0.57	0.35	0.28	0.40	0.31	0.44	0.30	0.24	0.31	0.26
Ca - HICP	0.64	2.43	2.29	2.20	2.22	0.50	1.69	1.65	1.52	1.60
Ca - STR	0.19	0.03	0.25	0.12	0.16	0.46	0.34	0.29	0.37	0.29
Ca - LTR	0.45	0.11	0.06	0.24	0.09	0.44	0.31	0.24	0.30	0.24
Ca - REER	1.66	2.07	1.87	2.48	2.01	0.99	1.35	1.22	1.77	1.40
Ca - IP	1.86	1.96	1.70	0.95	1.41	1.10	1.17	1.01	0.70	0.84
Ca - CU	1.24	1.37	1.14	0.76	0.71	0.80	0.85	0.70	0.56	0.53
Ca - TIE	2.03	1.50	1.15	1.81	1.51	1.21	0.90	0.70	1.08	0.90
Ca - HS	0.39	0.61	0.68	0.46	1.38	0.59	0.59	0.50	0.55	0.82
Ca - FSI	4.98	3.38	3.01	3.67	3.09	3.40	2.15	1.89	2.47	2.05
Ca - LCI	0.39	0.32	0.09	0.60	0.26	0.44	0.31	0.23	0.40	0.26
Ea - GDP	2.38	2.10	1.93	2.07	1.80	1.48	1.37	1.33	1.36	1.18
Ea - UR	0.69	0.54	0.38	0.51	0.43	0.51	0.36	0.29	0.36	0.31
Ea - HICP	0.24	0.02	0.28	0.14	0.30	0.44	0.32	0.29	0.33	0.31
Ea - STR	1.70	1.68	1.57	1.40	1.60	1.05	1.06	1.02	0.86	1.03
Ea - LTR	0.49	0.34	0.22	0.31	0.24	0.48	0.35	0.28	0.34	0.29
Ea - REER	1.05	0.78	0.55	0.82	0.46	0.70	0.52	0.40	0.54	0.39
Ea - IP	4.43	4.68	3.80	3.91	3.32	3.04	3.58	2.86	2.80	2.27
Ea - CU	0.05	0.66	0.91	0.61	0.94	0.48	0.47	0.54	0.45	0.56
Ea - TIE	0.04	0.02	0.02	0.01	0.00	0.40	0.27	0.24	0.28	0.25
Ea - HS	2.40	1.42	1.38	1.07	0.97	1.46	0.85	0.83	0.66	0.60
Ea - FSI	2.93	1.79	1.61	1.66	1.07	1.76	1.11	1.00	1.04	0.80
Ea - LCI	0.32	0.00	0.15	0.12	0.16	0.47	0.30	0.25	0.30	0.27
Jp - GDP	7.50	4.37	3.00	4.38	2.73	5.58	3.00	1.98	3.03	1.69
Jp - UR	0.60	0.20	0.01	0.11	0.01	0.60	0.36	0.27	0.35	0.26
Jp - HICP	0.97	2.19	2.02	1.89	1.88	0.78	1.34	1.28	1.15	1.18
Jp - STR	0.82	0.57	0.72	0.26	0.52	0.68	0.51	0.46	0.44	0.40
Jp - LTR	0.09	0.14	0.11	0.17	0.11	0.59	0.42	0.31	0.42	0.32
Jp - REER	0.36	1.89	2.19	1.54	2.34	0.89	1.15	1.32	0.98	1.40
Jp - IP	21.71	17.71	14.49	18.62	16.14	18.36	15.33	12.52	16.09	13.66
Jp - CU	6.37	5.83	5.09	6.25	5.35	7.88	4.71	4.18	5.07	4.35
Jp - TIE	2.70	1.82	1.59	1.58	1.38	2.18	1.10	0.94	0.99	0.83
Jp - HS	1.50	0.52	1.50	1.40	1.64	2.38	0.67	0.91	0.97	1.03
Jp - FSI	0.01	1.84	2.11	2.06	2.81	4.05	1.23	1.28	1.33	1.66
Jp - LCI	0.20	0.28	0.41	0.01	0.12	3.04	0.44	0.36	0.43	0.34
Us - GDP	0.60	0.01	0.06	0.27	0.16	0.56	0.41	0.33	0.41	0.36
Us - UR	0.02	0.02	0.04	0.04	0.11	0.37	0.27	0.23	0.29	0.25
Us - HICP	4.39	1.03	1.03	0.91	0.65	3.25	0.61	0.62	0.58	0.46
Us - STR	0.90	0.86	1.49	0.75	1.40	0.66	0.60	0.90	0.54	0.83
Us - LTR	0.79	0.90	0.96	0.81	0.96	0.56	0.55	0.57	0.50	0.58
Us - REER	0.38	0.48	0.36	0.28	0.14	0.55	0.46	0.37	0.41	0.36
Us - IP	3.05	3.09	3.44	3.28	3.58	1.92	2.05	2.50	2.25	2.57
Us - CU	1.42	1.51	1.76	1.78	1.67	0.85	0.92	1.18	1.11	1.05
Us - TIE	0.37	0.76	1.08	1.12	1.65	0.68	0.59	0.67	0.72	0.98
Us - HS	1.59	1.69	1.62	1.68	1.71	0.95	0.99	1.01	1.02	1.07
Us - FSI	4.91	2.04	1.80	2.31	1.85	3.23	1.31	1.10	1.38	1.10
Us - LCI	1.16	0.37	0.36	0.91	0.45	0.75	0.36	0.31	0.56	0.36

Table 17: RMSE and CRPS: Recovery phase, 1 quarter ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.47	0.43	0.28	0.21	0.34	1.00	0.77	0.52	0.68	0.54
Ca - UR	0.98	0.59	0.54	0.58	0.59	1.01	0.65	0.48	0.64	0.52
Ca - HICP	0.45	3.07	2.74	2.51	2.66	0.81	2.29	2.10	1.68	1.95
Ca - STR	0.71	0.73	0.44	1.02	0.67	1.09	0.89	0.58	0.98	0.67
Ca - LTR	0.67	0.15	0.05	0.45	0.22	0.98	0.69	0.43	0.64	0.44
Ca - REER	2.40	2.46	2.00	3.21	2.32	1.79	1.68	1.28	2.41	1.58
Ca - IP	2.95	3.00	2.52	2.00	2.28	2.45	2.24	1.80	1.70	1.76
Ca - CU	2.76	2.95	2.42	2.32	1.88	2.35	2.33	1.94	1.95	1.57
Ca - TIE	2.74	1.80	1.35	2.09	1.79	2.01	1.38	1.01	1.44	1.21
Ca - HS	1.36	0.66	0.50	1.16	0.98	1.48	0.98	0.69	1.17	0.74
Ca - FSI	5.87	2.62	2.15	3.76	2.44	3.96	1.49	1.04	2.28	1.25
Ca - LCI	0.73	0.48	0.13	1.30	0.31	1.55	0.98	0.65	1.20	0.68
Ea - GDP	2.60	2.16	1.87	2.19	1.79	1.81	1.37	1.10	1.40	1.11
Ea - UR	1.24	0.98	0.77	0.98	0.81	1.34	0.89	0.66	0.87	0.72
Ea - HICP	0.46	0.28	0.27	0.13	0.26	1.15	0.78	0.54	0.75	0.57
Ea - STR	3.33	3.20	2.86	2.54	2.96	2.64	2.62	2.49	1.98	2.54
Ea - LTR	0.78	0.28	0.29	0.26	0.34	1.35	0.88	0.61	0.81	0.67
Ea - REER	1.25	0.56	0.41	0.65	0.36	1.50	0.95	0.66	0.86	0.73
Ea - IP	5.77	6.02	4.54	5.25	4.35	4.43	4.27	3.16	3.78	3.11
Ea - CU	1.26	0.51	0.73	0.48	0.77	1.49	0.81	0.60	0.82	0.66
Ea - TIE	0.11	0.19	0.16	0.08	0.09	1.24	0.78	0.56	0.75	0.60
Ea - HS	2.07	1.13	1.09	0.77	0.69	1.58	1.00	0.75	0.89	0.69
Ea - FSI	7.26	5.42	5.67	5.30	4.60	5.81	4.40	4.90	4.41	3.89
Ea - LCI	1.35	0.37	0.21	0.16	0.41	2.09	1.19	0.83	1.10	0.87
Jp - GDP	9.95	4.84	2.70	5.62	2.92	7.34	3.12	1.50	4.10	1.88
Jp - UR	1.35	0.36	0.05	0.31	0.05	1.81	0.92	0.54	0.88	0.53
Jp - HICP	1.87	2.65	2.11	2.15	2.01	2.05	1.89	1.30	1.58	1.33
Jp - STR	1.38	0.44	0.61	0.39	0.37	1.91	1.22	0.72	1.13	0.77
Jp - LTR	0.71	1.27	0.72	1.18	0.95	1.65	1.31	0.76	1.22	0.92
Jp - REER	0.95	5.52	5.27	4.00	4.89	2.37	4.65	4.85	3.28	4.19
Jp - IP	33.60	23.05	16.50	25.67	19.65	31.94	21.27	14.04	24.49	16.90
Jp - CU	9.68	7.57	5.69	8.39	6.38	7.88	6.09	4.44	7.29	5.25
Jp - TIE	2.56	1.38	1.41	1.23	1.22	2.18	1.21	0.97	1.19	0.97
Jp - HS	1.81	1.38	2.86	2.42	3.00	2.38	1.54	2.31	2.00	2.39
Jp - FSI	1.62	3.80	3.52	2.85	4.28	4.05	3.24	2.75	2.65	3.20
Jp - LCI	1.81	1.36	1.33	0.46	0.59	3.04	1.80	1.32	1.63	1.15
Us - GDP	1.97	1.66	1.81	2.16	2.15	1.70	1.44	1.55	1.87	1.86
Us - UR	0.05	0.18	0.06	0.03	0.08	0.97	0.66	0.47	0.70	0.50
Us - HICP	7.14	3.73	3.95	3.36	3.14	6.43	3.24	3.89	2.93	2.81
Us - STR	0.67	2.97	3.95	2.17	3.85	1.62	2.38	3.46	1.95	3.20
Us - LTR	1.41	1.85	1.89	1.32	2.10	1.50	1.49	1.48	1.24	1.66
Us - REER	0.27	0.94	0.73	0.53	0.35	1.11	1.07	0.79	0.85	0.71
Us - IP	5.29	4.57	5.23	5.71	5.61	4.10	3.52	4.46	4.80	4.92
Us - CU	2.26	2.10	2.64	3.33	2.59	2.27	1.72	1.99	2.63	1.96
Us - TIE	0.43	0.71	1.20	1.27	2.02	1.62	1.15	1.05	1.26	1.52
Us - HS	2.77	1.47	1.37	1.71	1.57	2.19	1.07	0.83	1.24	1.01
Us - FSI	8.54	5.46	4.90	6.18	5.32	7.15	4.48	4.18	5.34	4.70
Us - LCI	2.63	0.70	0.68	2.05	0.96	2.45	1.17	0.90	1.71	1.05

Table 18: RMSE and CRPS: Recovery phase, 2 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.39	0.39	0.40	0.27	0.32	1.96	1.28	0.82	1.15	0.75
Ca - UR	1.20	0.78	0.83	0.66	0.80	1.58	0.96	0.74	0.88	0.73
Ca - HICP	0.89	3.16	2.67	2.37	2.59	1.45	1.97	1.51	1.33	1.45
Ca - STR	1.07	1.29	0.98	1.62	1.30	1.86	1.55	1.09	1.55	1.24
Ca - LTR	0.79	0.23	0.09	0.53	0.34	1.54	1.05	0.54	0.86	0.57
Ca - REER	2.64	2.40	1.73	3.18	2.11	2.32	1.61	0.85	1.88	1.04
Ca - IP	4.31	3.85	2.96	3.71	3.23	4.56	3.31	2.34	3.52	2.71
Ca - CU	3.97	3.77	2.78	3.80	2.56	4.31	3.18	2.15	3.51	2.32
Ca - TIE	2.94	2.09	1.82	1.89	2.05	2.52	2.00	1.60	1.57	1.62
Ca - HS	1.59	0.85	0.41	2.01	0.83	2.54	1.70	1.08	2.04	1.16
Ca - FSI	5.85	2.35	2.75	3.37	2.18	4.35	2.45	2.36	2.37	1.72
Ca - LCI	0.99	0.49	0.42	1.87	0.26	3.13	1.89	1.17	1.99	1.12
Ea - GDP	2.24	2.06	1.82	2.34	1.93	3.25	2.01	1.46	2.08	1.63
Ea - UR	1.77	1.38	1.07	1.35	1.16	2.57	1.47	0.99	1.44	1.13
Ea - HICP	0.69	0.59	0.32	0.32	0.32	2.31	1.38	0.81	1.26	0.90
Ea - STR	4.50	4.72	4.06	3.53	4.22	4.27	3.94	3.69	2.94	3.73
Ea - LTR	1.08	0.23	0.75	0.47	0.92	2.74	1.60	1.12	1.51	1.25
Ea - REER	1.74	0.46	0.34	0.94	0.36	3.01	1.73	1.03	1.53	1.17
Ea - IP	5.96	6.99	5.54	6.74	6.03	9.20	6.51	4.94	6.64	5.30
Ea - CU	1.59	0.55	1.14	0.45	1.00	3.06	1.70	1.32	1.66	1.34
Ea - TIE	0.42	0.51	0.36	0.29	0.21	2.75	1.55	0.97	1.40	1.01
Ea - HS	2.07	1.17	1.08	1.69	1.28	2.94	1.80	1.21	1.92	1.43
Ea - FSI	12.22	9.53	10.32	9.51	8.93	11.11	8.87	10.22	8.77	8.53
Ea - LCI	3.36	1.39	0.27	0.29	0.40	5.33	2.79	1.70	2.40	1.61
Jp - GDP	11.88	4.83	2.44	6.63	3.33	9.41	3.77	1.95	5.12	2.78
Jp - UR	2.60	0.87	0.27	0.70	0.36	3.82	1.69	0.79	1.62	0.82
Jp - HICP	3.36	3.15	2.15	2.56	2.23	4.28	2.90	1.60	2.53	1.83
Jp - STR	2.26	0.60	0.50	0.78	0.55	3.92	2.23	1.12	2.07	1.31
Jp - LTR	1.79	2.47	1.52	2.11	1.53	3.54	2.60	1.49	2.25	1.54
Jp - REER	1.19	7.20	6.02	4.40	5.04	4.82	5.82	4.43	3.73	3.25
Jp - IP	43.94	24.20	15.33	29.36	20.25	36.15	16.00	7.63	23.39	13.17
Jp - CU	12.24	7.92	5.21	9.68	6.51	9.79	5.19	2.51	7.26	4.05
Jp - TIE	2.48	1.49	1.47	1.26	1.16	3.61	1.99	1.30	1.89	1.24
Jp - HS	3.14	2.30	3.80	3.17	3.74	4.73	2.61	3.14	2.83	2.91
Jp - FSI	4.83	3.92	2.92	2.39	3.58	9.31	4.63	2.75	4.01	2.90
Jp - LCI	4.87	2.91	2.47	1.36	0.93	7.95	4.08	2.64	3.70	2.19
Us - GDP	2.71	1.97	2.08	2.70	2.59	2.90	2.16	1.79	2.56	2.19
Us - UR	0.20	0.34	0.08	0.23	0.07	1.99	1.18	0.72	1.23	0.80
Us - HICP	7.50	4.85	5.42	4.32	4.16	4.94	3.92	5.03	3.46	3.37
Us - STR	0.97	4.29	5.50	2.76	5.45	4.27	4.07	4.72	3.65	4.66
Us - LTR	1.74	2.77	2.81	1.73	3.19	2.93	2.68	2.43	2.31	2.78
Us - REER	1.36	0.86	0.85	1.04	1.11	2.17	1.47	1.03	1.39	1.26
Us - IP	7.65	6.38	7.08	8.54	7.93	7.20	5.49	5.91	7.34	6.91
Us - CU	2.93	2.80	3.55	4.87	3.43	4.64	3.03	2.96	4.51	2.98
Us - TIE	2.69	1.40	2.05	2.34	3.32	4.70	2.93	2.44	3.11	3.25
Us - HS	3.86	1.31	1.16	1.97	1.40	3.54	1.66	1.09	1.82	1.29
Us - FSI	10.45	7.08	6.19	7.86	7.04	8.42	5.97	4.81	6.33	5.81
Us - LCI	3.42	0.87	0.68	2.89	1.19	4.52	2.29	1.51	2.79	1.72

Table 19: RMSE and CRPS: Recovery phase, 3 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	0.48	0.40	0.75	0.47	0.45	3.27	2.02	1.27	1.65	1.04
Ca - UR	1.25	0.94	1.12	0.66	1.01	2.29	1.27	1.05	1.10	0.94
Ca - HICP	0.99	3.23	2.65	2.30	2.57	2.06	2.26	1.58	1.56	1.52
Ca - STR	1.13	1.62	1.55	1.81	1.73	2.74	2.11	1.58	1.80	1.59
Ca - LTR	0.86	0.37	0.37	0.63	0.57	2.24	1.36	0.72	1.03	0.76
Ca - REER	2.47	2.15	1.52	2.87	1.84	3.01	1.59	0.82	1.23	0.74
Ca - IP	5.29	3.84	2.65	4.71	3.17	7.15	4.10	2.26	4.57	2.43
Ca - CU	4.94	3.81	2.52	4.85	2.59	6.64	3.74	2.01	4.48	2.45
Ca - TIE	2.83	2.67	3.06	1.70	2.72	3.66	3.04	3.12	2.15	2.52
Ca - HS	1.90	1.03	0.47	2.32	0.72	3.93	2.34	1.30	2.36	1.35
Ca - FSI	6.23	2.20	3.27	3.25	2.12	6.27	3.34	2.85	2.90	2.02
Ca - LCI	1.48	0.86	0.39	2.32	0.23	4.88	2.82	1.46	2.56	1.40
Ea - GDP	1.97	1.88	1.89	2.49	2.35	6.61	3.27	2.21	3.27	2.44
Ea - UR	2.00	1.57	1.26	1.59	1.47	4.39	2.00	1.22	1.97	1.45
Ea - HICP	0.69	0.86	0.37	0.35	0.34	3.86	2.08	1.05	1.83	1.18
Ea - STR	5.15	5.62	4.93	4.05	5.14	6.81	4.86	4.15	3.70	4.31
Ea - LTR	1.39	0.33	1.17	0.65	1.45	4.99	2.53	1.68	2.30	1.85
Ea - REER	2.10	0.76	0.42	1.06	0.40	5.05	2.66	1.45	2.08	1.61
Ea - IP	5.43	7.01	6.37	7.68	7.13	18.77	9.90	7.01	10.34	7.09
Ea - CU	1.66	0.48	1.01	0.53	0.91	6.39	3.26	1.94	3.11	2.00
Ea - TIE	0.89	1.01	0.62	0.56	0.33	5.31	2.57	1.47	2.22	1.43
Ea - HS	3.54	1.62	1.22	2.38	1.72	5.92	2.91	1.74	2.80	1.94
Ea - FSI	15.77	12.96	13.96	12.44	11.66	15.80	12.30	13.65	11.35	10.65
Ea - LCI	5.36	3.17	1.38	1.00	0.85	10.10	5.11	2.86	3.95	2.44
Jp - GDP	12.55	4.86	2.51	7.62	4.50	14.24	5.82	2.98	7.08	4.38
Jp - UR	3.75	1.08	0.28	1.02	0.46	6.71	2.41	0.94	2.39	1.03
Jp - HICP	5.65	3.38	2.06	2.88	2.29	7.72	3.80	1.77	3.43	2.20
Jp - STR	3.04	1.30	0.85	1.59	0.99	6.85	3.55	1.62	3.20	1.94
Jp - LTR	3.31	3.59	1.99	2.76	1.83	6.47	3.90	1.89	3.23	1.84
Jp - REER	3.57	8.67	6.15	4.65	4.81	9.19	7.47	3.99	4.93	3.09
Jp - IP	50.15	23.50	14.28	32.40	21.07	41.19	15.46	7.71	23.94	14.22
Jp - CU	13.66	7.62	4.63	10.46	6.53	13.07	5.70	2.63	7.64	4.11
Jp - TIE	2.76	1.34	1.29	1.14	1.03	6.04	2.60	1.37	2.54	4.11
Jp - HS	4.86	3.12	4.15	3.68	3.89	7.97	3.64	3.04	3.47	2.67
Jp - FSI	11.88	4.09	2.77	2.14	3.27	19.75	7.34	3.94	6.40	4.06
Jp - LCI	8.78	4.69	3.30	2.14	1.24	15.49	6.73	3.65	6.23	3.10
Us - GDP	3.05	1.97	1.97	2.91	2.64	4.88	3.45	2.45	3.81	2.73
Us - UR	0.27	0.39	0.12	0.36	0.08	3.61	1.77	0.97	1.89	1.06
Us - HICP	7.60	5.23	6.09	4.42	4.38	5.79	4.08	4.65	3.24	3.23
Us - STR	1.89	5.44	6.85	3.50	6.60	8.07	5.92	5.83	5.51	5.70
Us - LTR	1.82	3.47	3.58	2.08	3.94	4.77	3.76	3.22	3.37	3.43
Us - REER	2.41	1.37	1.58	1.90	1.99	3.86	2.24	1.75	2.27	2.11
Us - IP	9.56	8.02	8.35	10.39	9.75	13.03	8.54	7.07	9.86	8.32
Us - CU	3.31	3.20	4.02	5.82	3.99	7.79	4.29	3.37	5.77	3.54
Us - TIE	4.69	2.27	2.84	3.74	4.46	8.79	5.41	3.95	5.48	4.68
Us - HS	3.98	1.24	1.49	1.79	1.33	4.55	2.46	1.80	2.17	1.76
Us - FSI	11.23	7.59	6.36	8.02	7.47	10.93	7.55	5.24	6.95	5.58
Us - LCI	3.47	0.77	0.69	3.03	1.07	7.50	3.64	2.27	3.71	2.30

Table 20: RMSE and CRPS: Recovery phase, 4 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	1.40	0.98	0.73	1.19	0.42	5.09	2.80	1.43	2.31	1.19
Ca - UR	1.12	0.88	1.19	0.59	1.02	3.26	1.49	0.98	1.28	0.86
Ca - HICP	0.91	3.32	2.69	2.32	2.57	2.96	2.67	1.75	1.87	1.63
Ca - STR	1.01	1.82	1.93	1.86	1.97	3.92	2.67	1.90	2.04	1.77
Ca - LTR	0.87	0.38	0.49	0.66	0.70	3.17	1.63	0.81	1.21	0.83
Ca - REER	2.21	1.96	1.38	2.58	1.64	4.30	1.83	0.89	1.14	0.76
Ca - IP	6.80	4.03	2.43	5.92	3.09	10.87	5.68	2.79	5.92	2.73
Ca - CU	6.28	4.08	2.39	5.88	2.77	9.86	4.92	2.41	5.54	2.89
Ca - TIE	2.69	2.52	3.49	1.67	2.79	5.52	3.55	3.07	2.76	2.40
Ca - HS	1.99	1.59	0.48	2.42	0.80	5.68	2.91	1.39	2.42	1.49
Ca - FSI	6.51	2.19	2.93	3.59	1.99	8.74	3.96	2.04	3.56	1.94
Ca - LCI	2.10	1.89	0.94	2.71	0.78	7.07	3.74	1.78	2.92	1.64
Ea - GDP	2.31	1.81	1.69	2.34	2.22	11.74	4.98	2.80	4.54	2.66
Ea - UR	2.01	1.62	1.30	1.62	1.63	7.51	2.55	1.36	2.48	1.65
Ea - HICP	0.71	0.93	0.37	0.31	0.31	6.67	2.73	1.27	2.33	1.44
Ea - STR	5.02	5.97	5.38	4.09	5.48	11.19	5.81	4.19	4.50	4.35
Ea - LTR	1.76	0.31	1.26	0.69	1.64	8.71	3.73	2.08	3.18	2.28
Ea - REER	2.31	1.55	0.81	0.97	0.36	8.55	3.76	1.94	2.75	1.99
Ea - IP	4.96	6.27	5.79	7.53	6.56	33.60	14.43	7.89	13.96	7.47
Ea - CU	1.51	0.56	0.92	0.97	1.05	12.05	5.21	2.83	4.82	2.81
Ea - TIE	1.78	1.68	0.97	0.83	0.51	9.60	3.83	1.99	3.13	1.85
Ea - HS	7.17	3.01	2.15	3.43	2.60	11.42	4.85	2.88	4.16	3.03
Ea - FSI	17.22	14.81	15.15	13.14	12.01	21.30	12.99	11.57	10.79	8.27
Ea - LCI	6.51	4.89	2.84	1.62	2.11	16.27	7.57	4.27	5.41	3.57
Jp - GDP	11.26	4.38	2.26	7.10	4.36	23.59	7.79	3.35	7.99	3.97
Jp - UR	4.61	1.08	0.26	1.07	0.45	10.93	3.08	1.05	3.07	1.19
Jp - HICP	7.54	3.05	1.88	2.65	2.05	11.87	4.50	1.97	3.97	2.28
Jp - STR	3.70	2.29	1.35	2.21	1.42	11.54	5.09	2.16	4.28	2.47
Jp - LTR	4.72	4.25	2.11	3.21	1.88	10.73	4.84	1.99	4.17	1.95
Jp - REER	7.29	9.48	5.86	4.52	4.41	16.21	8.16	3.45	6.15	3.19
Jp - IP	46.08	21.82	14.30	29.49	18.85	53.65	19.15	10.48	18.71	9.20
Jp - CU	12.71	6.90	4.78	9.65	5.84	19.51	7.54	4.18	7.44	3.56
Jp - TIE	3.38	1.23	1.51	1.02	1.19	9.88	3.58	1.97	3.42	1.92
Jp - HS	6.77	3.82	4.03	4.00	3.73	12.54	4.48	2.23	4.00	2.17
Jp - FSI	22.34	4.94	2.57	3.44	2.94	34.99	10.48	4.75	9.66	4.79
Jp - LCI	11.77	5.61	3.59	2.97	1.47	25.02	8.70	4.01	8.49	3.77
Us - GDP	3.09	1.89	1.77	3.07	2.56	8.44	5.11	3.29	5.53	3.22
Us - UR	0.67	0.35	0.17	0.48	0.11	6.32	2.54	1.23	2.64	1.37
Us - HICP	7.24	5.41	6.43	4.23	4.47	8.04	4.97	4.69	3.70	3.50
Us - STR	2.67	6.21	8.06	4.76	7.35	13.51	7.53	7.05	7.80	6.24
Us - LTR	1.72	3.77	4.13	2.21	4.16	8.06	4.55	3.69	4.40	3.34
Us - REER	2.95	2.17	2.25	2.38	2.67	6.06	3.15	2.35	2.88	2.69
Us - IP	11.00	8.93	8.42	11.01	10.34	23.87	13.58	9.14	14.04	9.59
Us - CU	3.65	3.28	3.91	6.08	4.12	12.83	5.51	3.16	6.72	3.63
Us - TIE	5.55	2.70	3.15	4.80	4.78	14.39	8.11	5.12	7.69	5.05
Us - HS	3.85	1.59	2.49	1.71	1.81	7.16	3.55	2.96	2.88	2.46
Us - FSI	10.55	7.61	6.10	7.67	7.19	17.44	10.84	6.82	9.07	5.45
Us - LCI	3.60	0.69	1.27	2.83	0.96	13.03	5.52	3.40	5.21	3.14

Table 21: RMSE and CRPS: Recovery phase, 5 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	2.64	1.91	0.77	1.86	0.51	7.71	3.80	1.59	2.89	1.33
Ca - UR	1.05	0.80	1.19	0.54	0.97	4.77	1.91	0.96	1.50	0.88
Ca - HICP	1.06	3.41	2.78	2.41	2.61	4.48	3.27	2.04	2.24	1.79
Ca - STR	0.94	1.91	2.16	1.82	2.06	5.82	3.21	2.08	2.26	1.82
Ca - LTR	0.84	0.35	0.48	0.63	0.68	4.86	2.04	0.86	1.38	0.80
Ca - REER	2.05	1.81	1.27	2.36	1.50	6.45	2.13	0.90	1.37	0.81
Ca - IP	8.28	5.04	2.79	6.93	3.34	15.48	7.63	3.53	6.94	3.34
Ca - CU	7.65	4.08	2.86	6.74	3.56	13.94	6.46	3.22	6.35	3.80
Ca - TIE	4.19	2.52	3.19	2.83	2.57	8.96	4.82	2.45	4.07	2.15
Ca - HS	2.16	1.59	1.00	2.38	1.19	8.28	3.66	1.74	2.54	1.82
Ca - FSI	6.51	2.19	3.02	3.80	2.46	11.88	5.01	2.62	3.84	2.72
Ca - LCI	2.42	1.89	1.91	2.75	1.35	9.96	4.68	2.59	3.00	1.98
Ea - GDP	2.23	1.84	1.72	2.21	2.04	20.13	7.01	3.49	5.76	3.15
Ea - UR	1.91	1.54	1.28	1.63	1.68	13.04	3.38	1.60	3.17	1.82
Ea - HICP	1.69	0.92	0.36	0.39	0.46	11.50	3.59	1.46	3.00	1.75
Ea - STR	4.66	5.88	5.48	3.94	5.51	18.75	7.14	4.29	5.67	4.56
Ea - LTR	1.86	0.57	1.17	0.63	1.67	14.43	5.20	2.53	4.15	2.69
Ea - REER	3.32	1.77	0.85	1.13	0.72	14.75	5.06	2.29	3.73	2.46
Ea - IP	7.30	6.08	5.86	7.30	6.22	58.00	20.32	9.93	17.90	9.02
Ea - CU	1.63	1.32	1.19	1.05	1.01	21.25	7.35	3.67	6.43	3.35
Ea - TIE	2.94	2.19	1.38	0.96	0.75	16.50	5.27	2.54	4.13	2.27
Ea - HS	9.61	3.81	2.54	3.69	2.91	18.15	6.35	3.28	4.65	3.23
Ea - FSI	18.52	16.37	15.10	12.84	11.44	33.88	15.37	9.30	11.35	7.03
Ea - LCI	5.95	5.50	4.25	1.69	3.17	26.37	9.50	5.56	6.92	4.60
Jp - GDP	10.71	4.02	2.15	6.69	4.30	40.58	10.10	3.83	10.23	4.55
Jp - UR	5.48	1.04	0.24	1.05	0.43	17.33	3.82	1.15	3.88	1.36
Jp - HICP	8.72	3.07	2.24	2.47	1.98	17.50	6.01	2.78	5.05	2.66
Jp - STR	4.61	3.00	1.68	2.59	1.61	19.17	6.65	2.51	5.16	2.90
Jp - LTR	5.74	4.51	2.09	3.47	1.87	16.93	5.64	2.06	5.12	2.16
Jp - REER	9.21	8.99	5.35	4.13	4.04	25.62	8.12	3.20	7.48	3.42
Jp - IP	42.98	24.49	16.21	27.25	17.65	94.02	29.98	15.02	25.20	11.80
Jp - CU	11.67	7.78	5.78	8.84	5.50	33.88	11.43	6.14	9.89	4.63
Jp - TIE	4.15	1.54	1.90	1.00	1.30	16.56	5.00	2.57	4.63	2.27
Jp - HS	8.04	3.67	3.68	3.77	3.40	19.31	4.49	1.59	4.10	1.73
Jp - FSI	28.68	4.76	2.43	4.48	2.70	51.95	13.03	5.42	12.98	5.42
Jp - LCI	11.71	5.27	3.41	2.90	1.54	37.74	9.92	3.84	10.29	4.02
Us - GDP	3.29	1.73	1.70	3.10	2.34	15.35	7.17	4.14	7.70	3.70
Us - UR	1.35	0.32	0.31	0.50	0.16	10.91	3.64	1.62	3.80	1.76
Us - HICP	6.92	5.64	6.77	4.06	4.69	13.21	6.42	5.03	5.18	4.21
Us - STR	2.50	6.84	9.13	5.80	7.62	22.98	9.52	7.89	10.55	6.57
Us - LTR	1.60	4.06	4.79	2.43	4.23	13.67	5.88	4.35	6.18	3.66
Us - REER	3.07	2.80	2.83	2.62	3.14	10.07	4.30	2.81	3.61	2.95
Us - IP	12.73	9.60	8.14	11.36	10.21	43.63	21.07	12.70	20.41	11.62
Us - CU	5.11	3.38	3.64	5.95	4.05	21.96	7.31	3.47	6.72	4.12
Us - TIE	6.57	3.65	3.56	5.97	4.87	24.79	11.53	6.61	7.69	5.85
Us - HS	3.77	1.49	3.04	1.81	1.87	12.40	4.74	3.31	2.88	2.89
Us - FSI	10.24	8.10	6.12	7.44	6.90	31.44	15.84	8.96	9.07	6.20
Us - LCI	3.84	0.96	1.48	2.62	0.90	22.74	8.19	4.49	5.21	4.09

Table 22: RMSE and CRPS: Recovery phase, 6 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	3.92	2.97	1.33	2.21	0.84	11.52	4.99	2.07	3.16	1.57
Ca - UR	1.18	0.81	1.12	0.54	0.91	7.20	2.41	0.89	1.81	0.88
Ca - HICP	1.54	3.65	2.98	2.53	2.76	6.83	4.22	2.51	2.66	2.21
Ca - STR	1.28	1.86	2.22	1.78	2.03	8.94	3.94	2.05	2.62	1.83
Ca - LTR	0.79	0.35	0.47	0.64	0.66	7.36	2.57	0.97	1.67	0.89
Ca - REER	1.95	1.69	1.18	2.20	1.39	9.24	2.66	0.99	1.64	0.87
Ca - IP	9.30	6.25	3.51	7.17	3.57	21.90	9.58	4.39	6.80	3.59
Ca - CU	8.27	6.08	3.55	6.74	4.05	19.65	7.87	4.01	5.94	4.02
Ca - TIE	5.28	3.76	3.17	2.83	2.70	13.17	6.40	2.93	4.52	2.66
Ca - HS	2.41	2.98	1.83	2.35	1.58	12.13	4.61	2.53	3.04	2.19
Ca - FSI	6.38	3.65	3.29	3.67	2.72	17.09	6.05	3.07	4.12	2.79
Ca - LCI	2.54	3.49	2.49	2.60	1.55	14.84	5.51	2.87	3.33	1.98
Ea - GDP	3.91	1.75	2.04	2.23	2.00	34.71	9.63	4.46	7.65	3.90
Ea - UR	2.10	1.42	1.22	1.59	1.68	22.41	4.76	1.80	4.18	2.05
Ea - HICP	4.84	1.07	0.39	0.39	0.61	20.17	5.01	1.74	4.00	2.06
Ea - STR	4.96	5.62	5.39	3.74	5.38	32.62	9.34	4.58	7.34	5.01
Ea - LTR	1.80	0.53	1.10	0.59	1.71	24.70	7.07	2.96	5.44	3.22
Ea - REER	6.20	1.64	0.83	1.60	1.36	25.52	6.91	2.73	4.96	3.15
Ea - IP	23.92	5.69	6.49	7.70	6.27	104.41	27.79	12.08	23.42	10.60
Ea - CU	4.86	1.34	2.02	1.24	1.34	36.06	9.69	4.57	8.34	4.08
Ea - TIE	3.80	2.47	1.72	0.98	1.00	28.47	7.05	3.03	5.51	2.75
Ea - HS	10.54	4.20	2.75	3.60	3.14	28.83	8.14	3.81	5.64	3.61
Ea - FSI	21.13	17.71	14.68	12.21	10.65	57.45	19.24	8.46	14.33	7.07
Ea - LCI	7.54	5.12	5.22	1.58	3.71	47.52	12.09	6.17	8.90	5.11
Jp - GDP	9.93	3.74	1.99	6.40	4.01	67.65	12.53	4.04	12.69	4.42
Jp - UR	6.02	1.04	0.23	1.00	0.42	28.33	4.90	1.30	4.84	1.53
Jp - HICP	9.53	3.63	2.55	2.42	1.98	28.16	8.02	3.13	6.28	2.97
Jp - STR	5.36	3.34	1.79	2.73	1.64	31.47	8.31	2.74	6.33	3.21
Jp - LTR	6.20	4.34	1.94	3.44	1.80	27.40	6.46	2.10	5.97	2.32
Jp - REER	9.31	8.60	5.23	4.53	3.96	41.06	10.21	4.11	9.65	3.90
Jp - IP	42.30	25.78	16.78	25.89	17.09	159.51	35.38	14.73	32.98	13.75
Jp - CU	11.09	8.47	6.34	8.56	5.48	56.26	14.23	6.39	13.13	5.55
Jp - TIE	5.53	1.94	2.15	1.22	1.38	27.91	6.84	2.89	6.08	2.61
Jp - HS	8.24	3.40	3.54	3.51	3.18	31.40	5.48	2.10	4.94	2.03
Jp - FSI	28.68	5.27	3.16	4.17	2.60	78.75	16.97	6.28	15.53	6.05
Jp - LCI	11.94	5.56	3.23	2.90	1.44	61.72	12.76	4.00	12.25	4.13
Us - GDP	4.40	1.66	1.73	2.94	2.26	27.47	10.15	4.89	10.98	4.52
Us - UR	2.44	0.52	0.44	0.69	0.17	19.23	5.27	2.16	5.61	2.21
Us - HICP	7.35	6.06	7.11	4.07	5.07	23.09	8.72	5.69	7.64	5.31
Us - STR	2.61	7.51	9.85	6.27	7.88	39.47	13.01	8.44	14.19	7.59
Us - LTR	1.50	4.45	5.35	2.64	4.44	23.41	8.15	5.12	8.74	4.66
Us - REER	2.98	3.27	3.15	2.62	3.29	17.40	5.76	2.88	4.78	2.85
Us - IP	15.85	10.54	7.78	11.45	9.84	75.29	30.45	16.37	28.61	13.42
Us - CU	5.11	3.50	3.38	5.78	3.83	38.05	9.98	4.38	12.21	4.69
Us - TIE	6.57	4.22	3.64	6.58	4.79	43.48	16.03	7.98	14.54	6.87
Us - HS	3.77	1.43	3.22	2.10	1.77	22.94	6.99	3.72	6.23	3.93
Us - FSI	10.24	8.84	6.41	7.51	6.65	53.64	21.39	10.89	17.57	7.05
Us - LCI	3.84	1.55	1.45	2.44	0.91	39.93	11.76	5.70	10.54	4.97

Table 23: RMSE and CRPS: Recovery phase, 7 quarters ahead

	RMSE					CRPS				
	Hrw	Sar	Ri	Rm	Rim	Hrw	Sar	Ri	Rm	Rim
Ca - GDP	5.30	3.95	2.16	2.46	1.39	17.04	6.38	1.27	3.74	2.16
Ca - UR	1.49	0.90	1.05	0.52	0.85	11.10	3.04	1.05	2.13	0.91
Ca - HICP	1.83	3.90	3.10	2.50	2.84	10.25	5.18	1.58	2.86	2.25
Ca - STR	1.89	1.74	2.16	1.70	1.94	13.65	4.99	1.58	3.09	1.87
Ca - LTR	0.78	0.32	0.54	0.79	0.70	10.81	3.29	0.72	2.09	1.07
Ca - REER	1.97	1.59	1.10	2.12	1.30	13.63	3.42	0.82	1.99	0.95
Ca - IP	11.69	8.36	5.26	7.46	4.49	33.23	12.81	2.26	8.26	5.29
Ca - CU	9.33	7.33	4.92	7.09	4.93	29.39	10.20	2.01	6.84	5.41
Ca - TIE	6.22	4.94	3.71	3.57	3.01	19.16	8.15	3.12	4.84	3.20
Ca - HS	3.02	3.38	2.38	2.33	1.83	18.37	5.57	1.30	3.54	2.31
Ca - FSI	6.75	3.66	3.27	3.45	2.66	26.29	7.30	2.85	4.83	2.44
Ca - LCI	2.95	3.75	2.61	2.44	1.52	22.57	6.60	1.46	3.92	1.86
Ea - GDP	11.29	2.00	2.32	2.45	1.97	61.97	13.51	5.30	10.35	4.70
Ea - UR	3.29	1.33	1.14	1.54	1.62	39.45	6.71	2.12	5.56	2.41
Ea - HICP	7.56	1.33	0.43	0.38	0.72	34.49	6.91	2.08	5.36	2.33
Ea - STR	6.33	5.41	5.17	3.60	5.22	57.20	12.66	4.99	9.69	5.59
Ea - LTR	2.53	0.60	1.05	0.64	1.81	44.32	9.75	3.54	6.99	3.89
Ea - REER	8.53	1.84	1.09	2.04	1.95	44.12	9.30	3.31	6.51	3.88
Ea - IP	49.94	7.69	7.17	9.73	6.28	186.92	38.69	14.28	31.72	12.34
Ea - CU	12.75	1.73	2.53	2.32	1.56	64.68	13.42	5.19	11.45	4.74
Ea - TIE	4.08	2.58	2.00	1.00	1.23	49.25	9.52	3.58	7.23	3.23
Ea - HS	9.87	4.23	2.80	3.38	3.10	48.50	10.65	4.25	7.13	3.91
Ea - FSI	26.23	18.47	13.88	11.52	10.00	98.86	23.95	8.36	18.77	8.50
Ea - LCI	13.90	5.01	5.55	1.96	3.97	82.09	16.70	6.58	11.94	5.66
Jp - GDP	14.52	4.61	1.86	6.35	3.76	113.48	16.54	4.40	15.74	4.76
Jp - UR	7.35	1.16	0.22	0.96	0.47	47.15	6.34	1.50	6.06	1.74
Jp - HICP	9.52	3.64	2.67	2.38	1.94	46.17	10.18	3.32	7.94	3.30
Jp - STR	6.64	3.27	1.71	2.64	1.58	53.80	10.52	2.89	7.82	3.57
Jp - LTR	5.92	4.07	1.84	3.25	1.70	45.59	7.98	2.34	7.36	2.51
Jp - REER	9.14	9.66	5.41	5.54	3.97	68.42	14.27	4.84	12.08	4.32
Jp - IP	39.61	24.28	15.90	24.35	16.40	267.37	41.48	12.97	40.61	14.24
Jp - CU	11.71	7.99	6.04	8.05	5.22	92.17	16.69	5.41	16.01	5.61
Jp - TIE	5.38	2.08	2.15	1.26	1.38	45.71	8.56	2.92	7.78	2.85
Jp - HS	7.76	3.35	3.57	3.33	3.05	52.30	7.42	2.74	6.17	2.34
Jp - FSI	26.82	7.24	3.52	4.26	2.49	129.55	22.55	6.58	19.50	6.65
Jp - LCI	25.89	8.13	3.44	4.55	1.45	109.88	19.00	4.75	16.18	4.53
Us - GDP	5.48	1.65	1.77	2.75	2.21	46.87	14.39	6.00	15.55	5.56
Us - UR	4.70	0.62	0.56	0.80	0.32	33.87	7.57	2.73	8.14	2.76
Us - HICP	8.64	6.66	7.58	4.17	5.60	40.57	12.38	6.76	11.29	6.59
Us - STR	2.47	8.57	10.40	6.85	8.43	71.10	18.65	9.30	20.80	9.38
Us - LTR	1.61	5.07	5.73	2.73	4.65	42.13	11.75	5.65	12.57	5.48
Us - REER	2.89	3.45	3.14	2.46	3.19	30.43	7.72	2.71	6.91	2.76
Us - IP	22.56	11.66	7.38	11.47	9.20	129.78	42.05	19.80	39.65	15.28
Us - CU	9.97	3.54	3.16	5.41	9.20	65.89	14.43	5.68	17.82	5.74
Us - TIE	12.44	5.13	3.51	6.97	4.66	75.65	22.99	9.67	20.43	8.60
Us - HS	5.56	2.03	3.19	2.26	1.69	40.29	10.66	4.71	9.04	5.07
Us - FSI	10.84	9.44	6.78	7.56	6.39	91.74	29.14	12.70	25.39	8.44
Us - LCI	7.96	2.28	1.37	2.33	1.22	69.77	16.74	6.91	14.80	6.03

Table 24: RMSE and CRPS: Recovery phase, 8 quarters ahead