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Determining Alternator Reactances from Load Test Data

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DETERMINING ALTERNATOR REACTANCES FROM LOAD TEST DATA

BY

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DETERMINING ALTERNATOR REACTANCES

FROM LOAD TEST DATA

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Advisor Date

Head, Electrical Engineering Department Date

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R.K.J.

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CHAPTER I

INTRODUCTION

Background Synchronous Machine Theory

It is known that the induction motor requires a definite slip in order to produce the necessary rotor current for a given torque. However, if the rotor current necessary to produce the torque is supplied by a suitable external source, as in a polyphase commutator machine, then the desired torque can be produced with a slip of zero or even a negative value. In the first case, when the slip is zero, the rotor travels synchronously with the rotating field, and the rotor current supplied by the commutator machine must be a direct current, for it is only then that the stator and rotor mmfs can be stationary relative to one another; this condition must be fulfilled at all times.

Thus the synchronous machine represents a special case of the asynchronous machine; the rotor of the synchronous machine is supplied from a separate source of direct current. For this reason the synchronous machine has to operate at synchronous speed; in contrast with the induction machine, any other speed of operation is impossible for the synchronous machine.

In the treatment of the synchronous machine a distinction must be made between the SALIENT-POLE and the NONSALIENT-POLE CYLINDRICAL-ROTOR machine. In the salient-pole machine the d-c field winding consists of a number of concentrated coils while the field winding of the nonsalient-pole machine is set in slots in a cylindrical rotor. For the same reason as in the single-phase winding the winding of the non-salient-pole machine is distributed over about 2/3 of the pole pitch of the rotor. While the non-salient-pole machine has nearly constant magnetic reluctance along the whole armature surface, the reluctance of the salient-pole machine is variable because the salient poles are separated from each other by interpolar air spaces, which have a very high reluctance.[1]

Before the growth of the public utilities into their present enormous proportions with large generating stations and connecting tie lines, machine performance was largely judged in terms of the steady-state characteristics. The emergence of the stability problem gave rise to the analysis of the transient characteristics of machines and was largely responsible for our present knowledge of machine theory. A further contributing factor was the need for more accurate determination of short-circuit currents for the application of relays and circuit breakers.

The variable character of the air gap of the conventional salient-pole synchronous generator, motor, and condenser requires that the analysis follow a different line from that for machines such as induction machines, which have a uniform air gap and distributed windings. Blondel originally attacked this problem by resolving the armature mmf's and fluxes into two components, one in line with the axis of the poles and the other in quadrature thereto.[2]

Moreover, in the analysis of system stability, and in the calculation of the effect of short-circuits, the factors of interest to operating engineers are those which relate to the behavior of the machine as viewed from the armature terminals. The most significant of these factors are the armature reactances of machines to normal frequency current having any distribution between phases, any power-factor, and whether transient or sustained; also, in connection with transient components of current, their rates of decay, or decrements.

Aim of the Thesis

It is the purpose of this paper to establish equations, with some assumptions, for calculating the reactances of a synchronous generator, using load test data, and to analyze the results.

For this purpose a report on power angle measurements on Unit No. 8, at Station 16, (Waukegan) of The Commonwealth Edison Company, Chicago, Illinois, was used.

CHAPTER II

REACTANCES: DEFINITION AND DETERMINATION

Synchronous machine reactances can be considered in terms of:

(1). The distribution of current between phases.

Any distribution of armature current may be expressed as the superposed sum of three symmetrical components.

a. Balanced three-phase currents of normal phase rotation, or POSITIVE PHASE-SEQUENCE,

b. Balanced three-phase currents of reverse phase rotation, or NEGATIVE PHASE-SEQUENCE, and

c. Balanced three-phase currents of equal time phase, or ZERO-PHASE SEQUENCE.

(2). Method of application in time of positive phase-sequence currents.

a. Steadily applied, or sustained currents,

b. Suddenly applied, or transient currents.

(3). Position of the rotor with respect to axis of magnetization of positive phase-sequence currents.

When the rotor is moving synchronously, the positive phase-sequence current can be resolved into two components, one of which magnetizes along the axis of the poles, and the other in the inter-polar space. Accordingly, these components are referred to as

DIRECT-AXIS and QUADRATURE-AXIS components, and the corresponding reactances are:

a. Direct axis reactance, and

b. Quadrature axis reactance.[3]

DIRECT-AXIS SYNCHRONOUS REACTANCE X_d:

The standard I.E.E.E. definition is, in effect, as follows: Synchronous impedance is the ratio of field current required to circulate rated current on sustained three-phase short circuit to the field current which would produce rated voltage at no load if there were no saturation; referring to Fig. 1, $X_d = OA/OB$.

Another way of defining X_d which results in a clearer conception of the meaning of X_d, is: With a sustained three-phase short circuit, and the field current at any arbitrary value, X_d is the air-gap line voltage corresponding to this field current divided by the sustained short-circuit current. Thus, from Fig. 1, $X_d = OC/O'D$.[4]

Both the direct-axis synchronous reactance X_d and the quadrature-axis synchronous reactance X_q may be determined from a SLIP-TEST. Twenty to twenty-five per cent of the normal voltage is impressed on the armature winding producing a rotating mmf. The field winding is open and there is no damper winding. The rotor is driven by a motor in the direction of the rotating field at a speed differing but slightly from the speed of the rotating field. The field poles then will slip gradually through the rotating mmf

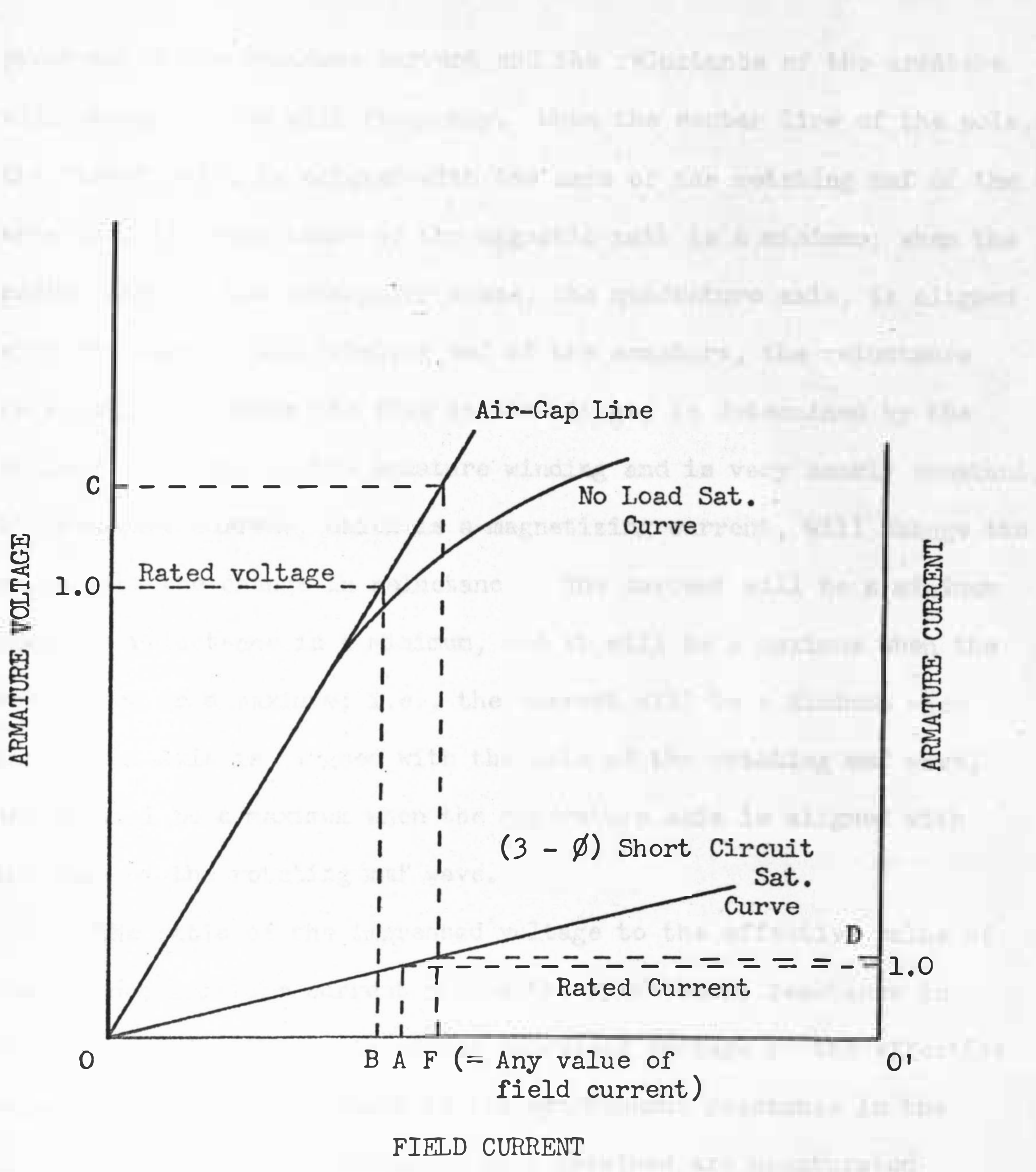

Fig. 1. Open Circuit and Short Circuit Characteristics of Synchronous Generator.

produced by the armature current and the reluctance of the armature will change at the slip frequency. When the center line of the pole, the direct axis, is aligned with the axis of the rotating mmf of the armature, the reluctance of the magnetic path is a minimum; when the center line of the interpolar space, the quadrature axis, is aligned with the axis of the rotating mmf of the armature, the reluctance is a maximum. Since the flux in the air gap is determined by the voltage impressed on the armature winding and is very nearly constant, the armature current, which is a magnetizing current, will change its value with the change in reluctance. The current will be a minimum when the reluctance is a minimum, and it will be a maximum when the reluctance is a maximum; i.e., the current will be a minimum when the direct axis is aligned with the axis of the rotating mmf wave, and it will be a maximum when the quadrature axis is aligned with the axis of the rotating mmf wave.

The ratio of the impressed voltage to the effective value of the minimum armature current yields the synchronous reactance in the direct axis; the ratio of the impressed voltage to the effective value of the maximum current is the synchronous reactance in the quadrature axis. The reactances thus obtained are unsaturated reactances.[1]

QUADRATURE-AXIS SYNCHRONOUS REACTANCE X_q:

The unsaturated value is obtained from the slip test at the same time that X_d is measured, as described previously. With the rotor magnetized in the quadrature axis, as evidenced by positive or negative peak voltage across the field, X_q is the ratio of applied voltage to line current.[4]

In a cylindrical rotor machine, the direct-axis path is very nearly the same as the quadrature-axis path. Hence X_d is very nearly equal to X_q.

POTIER REACTANCE X_p:

As seen in Fig. 2, DF = $I_a X_\ell$. This X_ℓ is called LEAKAGE OR POTIER REACTANCE X_p.

$$X_p = \frac{\text{Voltage drop DF per phase}}{\text{Zero power-factor armature current } I_a \text{ per phase}}$$

The geometrical construction for finding the triangle BCD is as follows: Take any terminal voltage such as OE. Select a point B on the zero-power-factor characteristic above the knee of the curve. Draw the horizontal line BC equal in length to the field current OA on short circuit. Through point C draw the straight line CD parallel to the air-gap line, intersecting the open-circuit characteristic at D. Draw the vertical line DF. The triangle BCD is commonly called the POTIER TRIANGLE after its inventor. The voltage represented by the length DF is known as the POTIER-REACTANCE voltage drop.[5,6]

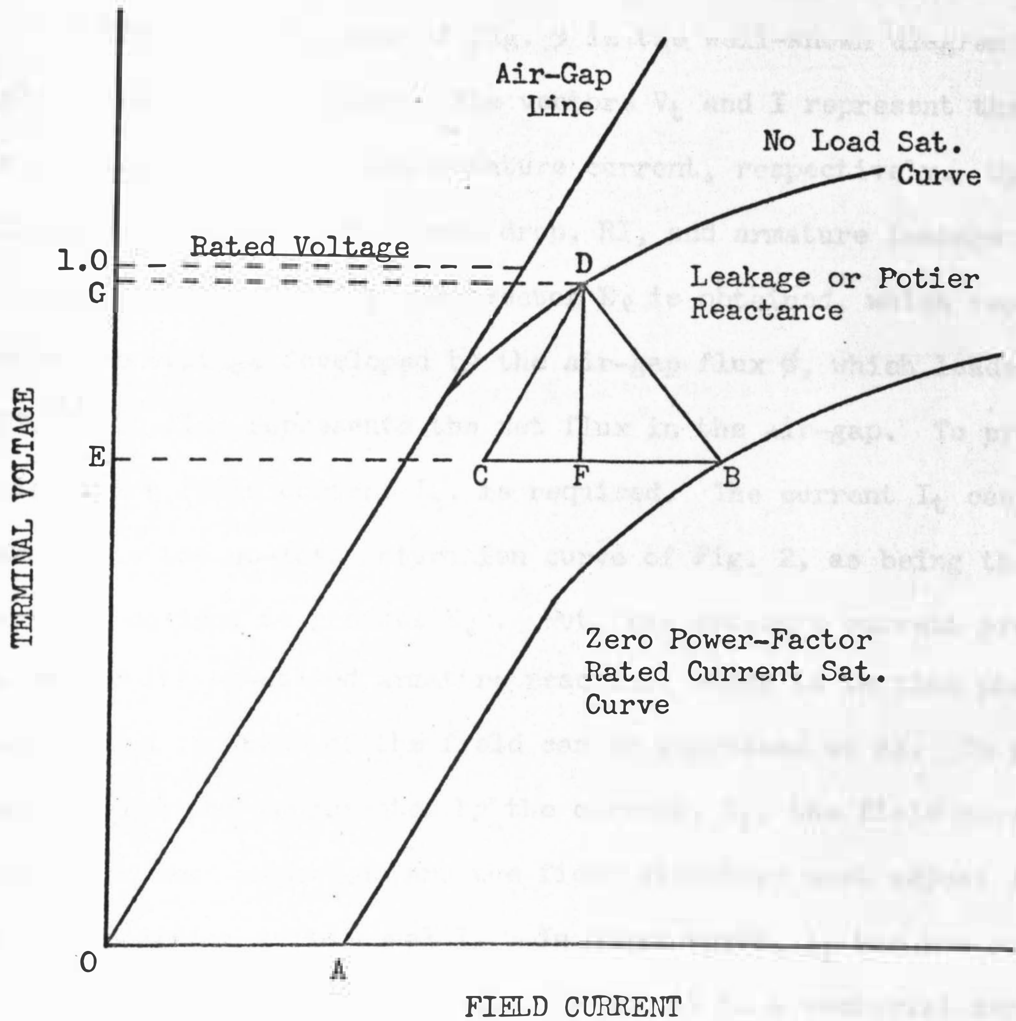

Fig. 2. No Load and Full Load Zero Power-Factor Characteristics of Synchronous Generator.

CHAPTER III

VECTOR REPRESENTATION

The vector diagram of Fig. 3 is the well-known diagram of a cylindrical-rotor machine. The vectors V_t and I represent the terminal voltage to neutral and armature current, respectively. Upon adding the armature resistance drop, RI, and armature leakage reactance drop, $X_\ell I$, to V_t, the vector E_ℓ is obtained, which represents the voltage developed by the air-gap flux ϕ, which leads E_ℓ by 90°. This flux represents the net flux in the air-gap. To produce this flux a field current I_t, is required. The current I_t can be taken from the no-load saturation curve of Fig. 2, as being the current required to produce E_ℓ . But, the armature current produces an mmf by its so-called armature reaction, which is in time phase with it and in terms of the field can be expressed as AI. To produce the net mmf represented by the current, I_t, the field current must be of such magnitude and the field structure must adjust itself to such position as to equal I_f. In other words, I_f has now such position and magnitude that adding I_f and AI in a vectorial sense equals I_t. The triangle OAB, formed by drawing AB perpendicular to I or AI and OB perpendicular to OC, is similar to the triangle ODC; OB has the same proportionality to OC and AB to AI as E_ℓ has to I_t. Neglecting saturation, OB, designated as E_d, is thus the open-circuit voltage corresponding to the field current I_f; it is the voltage taken from the air-gap line of the no-load saturation curve for the

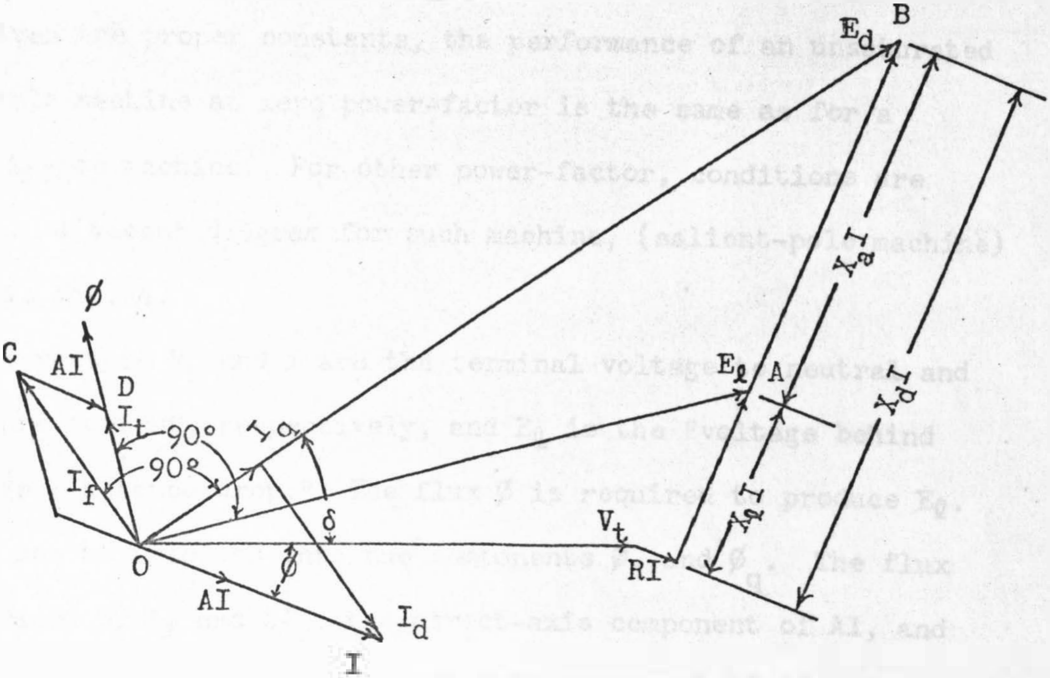

Fig. 3. Vector Diagram of Unsaturated Cylindrical-Rotor Machine.

abscissa corresponding to I_f. The side AB of the triangle, since it is proportional to AI and consequently proportional to the armature current, can be viewed as a fictitious reactance drop. It is called the drop of armature reaction and is designated X_aI. The drops $X_\ell I$ and X_aI can be combined into a single term called the synchronous reactance drop and there results $X_d = X_\ell + X_a$. It follows from the foregoing that the internal voltage, E_d, is equal to the vector sum of V_t, RI and jX_dI.

Given the proper constants, the performance of an unsaturated salient-pole machine at zero power-factor is the same as for a uniform air-gap machine. For other power-factor, conditions are different. A vector diagram for such machine, (salient-pole machine) is shown in Fig. 4.

The vectors V_t and I are the terminal voltage to neutral and the armature current, respectively, and E_ℓ is the "voltage behind the leakage reactance drop." The flux ϕ is required to produce E_ℓ. This flux can be resolved into two components ϕ_d and ϕ_q. The flux ϕ_d is produced by I_f and AI_d, the direct-axis component of AI, and ϕ_q is produced by AI_q, the quadrature-axis component of AI. Here the similarity ceases. Because of the saliency effect, the proportionality between the mmf's and their resultant fluxes are not the same in the two axes. When saturation effects are neglected, ϕ_d can be regarded being comprised of a component due to I_f acting alone and a component produced by AI_d. The component produced by I_f can be regarded as producing the internal voltage E_d. The mmf produced by

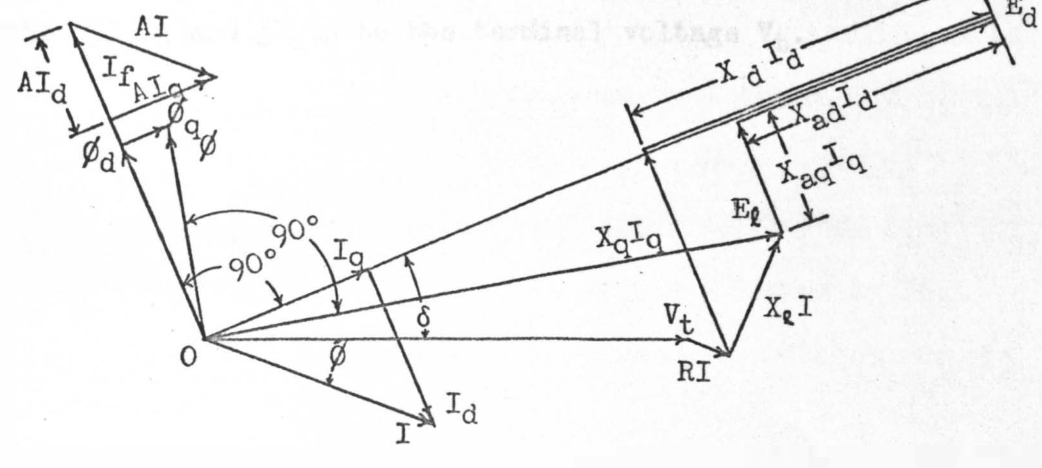

Fig. 4. Vector Diagram of Unsaturated Salient-Pole Machine.

AI_d has a general sinusoidal distribution in the direct axis. The resultant flux because of the variable reluctance of the air-gap has the distorted shape. It is the sinusoidal component of this flux that is effective in producing the $X_{ad}I_d$ drop. In the quadrature axis, the component of mmf is likewise sinusoidal in nature, and gives rise to the distorted flux form. The effect of this component is reflected in the $X_{aq}I_q$ drop of Fig. 4. In general, of course, X_{aq} is much smaller than X_{ad}.

The armature resistance and leakage reactance drops can also be resolved into two components in the two axes. When this is done the internal voltage E_d can be obtained by merely adding RI_d and RI_q and then jX_qI_q and jX_dI_d to the terminal voltage V_t.

CHAPTER IV

CALCULATIONS

Another form of the vector diagram of the machine is presented in Fig. 5, which shows much better the relation between those quantities that are most useful for calculation purposes. If from A the line AB of length IX_p is drawn perpendicular to I, then OB is the voltage, E_p, the Potier internal voltage or the voltage behind the Potier reactance drop. To include the effect of saturation, a length BC equal to the increment in field current for this voltage in excess of that required for no saturation, is added in phase with E_p, giving OC, E_{ps}. To E_{ps} is added $I(X_q - X_p)$, perpendicular to I, resulting in E_i. $I(X_q - X_p)$ is lengthened by a length $I(X_d - X_q)$; a perpendicular is dropped to E_i giving E_d[2].

Assuming $X_d = X_q$ for more simplification, Fig. 5, takes the form of Fig. 6. The angle δ is known as the power angle.

To get all of the parameters of this vector diagram of Fig. 6 into one equation, one can start from,

$$MN = E_d \sin (\phi + \delta),$$

it is also true that

$$MN = I (X_q - X_p) + KM;$$

therefore,

$$E_d \sin (\phi + \delta) = I (X_q - X_p) + KM \qquad (1)$$

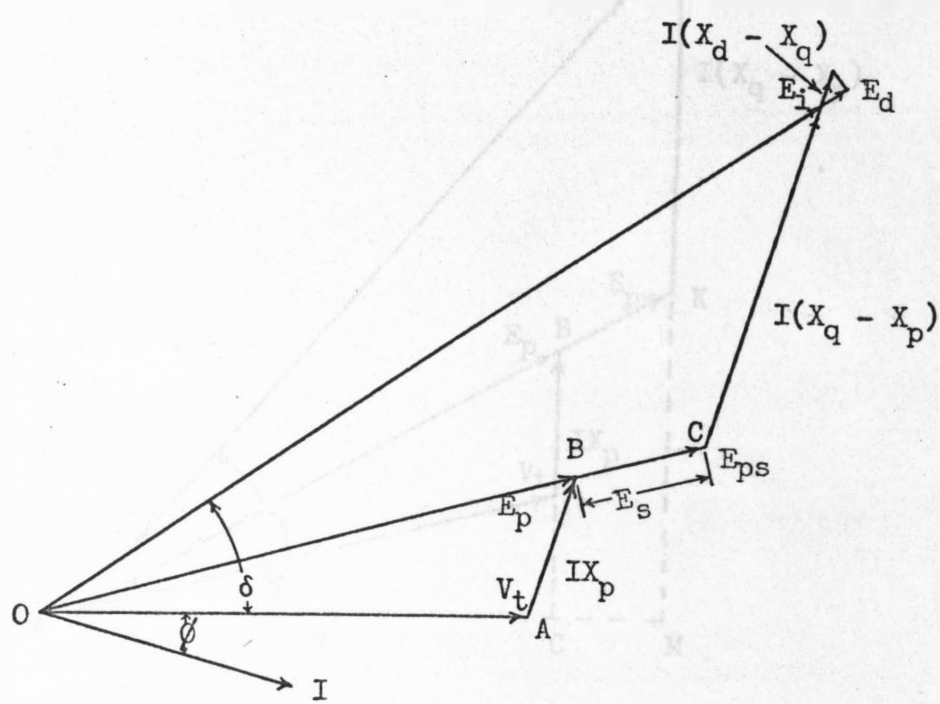

Fig. 5. Vector Diagram of Cylindrical-Rotor Machine with Saturation Included by Adding E_s.

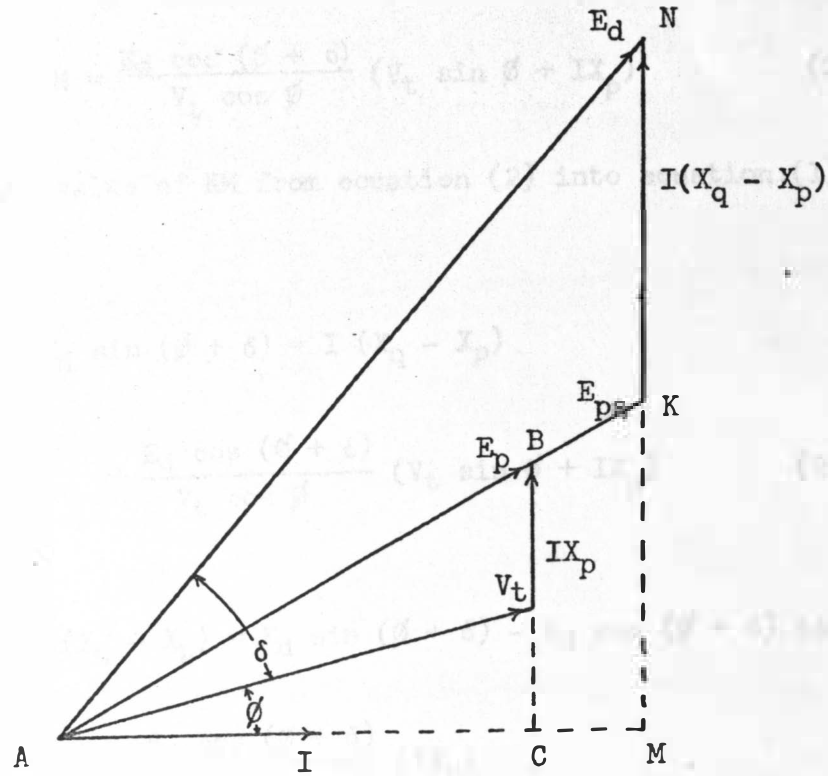

Fig. 6. Vector Diagram of Cylindrical-Rotor Machine with Saturation Included and $X_d = X_q$.

Comparison of triangles ABC and AKM gives,

$$\frac{AM}{AC} = \frac{KM}{BC}$$

$$KM = \frac{AM}{AC}(BC)$$

$$KM = \frac{E_d \cos(\phi + \delta)}{V_t \cos\phi}(V_t \sin\phi + IX_p) \qquad (2)$$

Substituting the value of KM from equation (2) into equation (1) gives,

$$E_d \sin(\phi + \delta) = I(X_q - X_p)$$

$$+ \frac{E_d \cos(\phi + \delta)}{V_t \cos\phi}(V_t \sin\phi + IX_p) \qquad (2a)$$

$$I(X_q - X_p) = E_d \sin(\phi + \delta) - E_d \cos(\phi + \delta) \tan\phi$$

$$- \frac{E_d \cos(\phi + \delta)}{V_t \cos\phi}(IX_p) \quad ,$$

and

$$E_d = \frac{I_f}{I_{fb}}$$

where

I_f = Field current in Amps.

I_{fb} = Base Field current in Amps.

$$I(X_q - X_p) = \frac{I_f}{I_{fb}} \left[\sin(\phi + \delta) - \cos(\phi + \delta) \tan\phi - \frac{IX_p \cos(\phi + \delta)}{V_t \cos\phi} \right]$$

$$I_{fb}I(X_q - X_p) = I_f \sin(\phi + \delta) - I_f \cos(\phi + \delta) \tan\phi - \frac{I_f I X_p \cos(\phi + \delta)}{V_t \cos\phi}$$

Let $a = I_f \sin(\phi + \delta)$,

$b = I_f \cos(\phi + \delta) \tan\phi$,

and $c = \dfrac{II_f \cos(\phi + \delta)}{V_t \cos\phi}$

Then,

$$I_{fb}I\,(X_q - X_p) = a - b - cX_p \tag{3}$$

This equation includes all parameters of the synchronous generator. Test data of Waukegan Unit No. 8 (H.P.) of The Commonwealth Edison Company, which is given in Appendix 1, was used to ascertain if a system of equations of the form of equation (3) could be used to determine machine reactances.

The simultaneous solution of two equations is developed below.

Reading No. 4:

KV	I_a	MW	MVAR	E_f	I_f	δ
17.2	3300	89	16	156	1080	29°

$P = 89 \text{ MW} = \sqrt{3}\ (17.2 \text{ KV})\ I \cos \phi$

$Q = 16 \text{ MVAR} = \sqrt{3}\ (17.2 \text{ KV})\ I \sin \phi$

$V_t = \frac{17.2}{18} = 0.956$ pu.

$\tan \phi = \frac{16}{89} = 0.1799$

$I = I_a = \frac{3300}{6400} = 0.497$ pu.

$\phi = 9.09°$

From equation (3),

$$a = I_f \sin(\phi + \delta) = 1080 \sin(9.09 + 29)$$
$$= 1080 \sin(38.09)$$
$$= 667$$

$$b = I_f \cos(\phi + \delta) \tan \phi = 1080 \cos(38.09) \tan(9.09)$$
$$= 1080\ (0.787)\ (0.1799)$$
$$= 152.8$$

$$c = \frac{I I_f \cos(\phi + \delta)}{V_t \cos \phi} = \frac{(0.497)(1080) \cos(38.09)}{(0.956) \cos(9.09)}$$

$$= \frac{(0.497)(1080)(0.787)}{(0.956)(0.987)}$$

$$= 448$$

$$I_{fb}\ (0.497)(X_q - X_p) = 667 - 152.8 - 448X_p \qquad (4)$$

Reading No. 12:

KV	I_a	MW	MVAR	E_f	I_f	δ
18	6780	192	80	291	1950	35°

$$P = 192 \text{ MW} = \sqrt{3}\ (18 \text{ KV})\ I \cos \phi$$

$$Q = 80 \text{ MVAR} = \sqrt{3}\ (18 \text{ KV})\ I \sin \phi \qquad V_t = \frac{18}{18} = 1.0 \text{ pu.}$$

$$\tan \phi = \frac{80}{192} = 0.417 \qquad I = I_a = \frac{6780}{6640} = 1.02 \text{ pu.}$$

$$\phi = 22.62°$$

From equation (3),

$$a = I_f \sin(\phi + \delta) = 1950 \sin(22.62 + 35)$$

$$= 1950 \sin(57.62)$$

$$= 1646$$

$$b = I_f \cos(\phi + \delta) \tan \phi = 1950 \cos(22.62 + 35) \tan(22.62)$$

$$= (1950)(0.535)(0.417)$$

$$= 435$$

$$c = \frac{I I_f \cos(\phi + \delta)}{V_t \cos \phi} = \frac{(1.02)(1950) \cos(57.62)}{(1.0) \cos(22.62)}$$

$$= \frac{(1.02)(1950)(0.535)}{(1.0)(0.9225)}$$

$$= 1137.5$$

$$I_{fb}\ (1.02)(X_q - X_p) = 1646 - 435 - 1137.5\ X_p \qquad (5)$$

$$0.497\ I_{fb}(X_q - X_p) + 448\ X_p = 514.2 \qquad (4)$$

$$1.02\ I_{fb}(X_q - X_p) + 1137.5\ X_p = 1211 \qquad (5)$$

Multiplication of equation (5) by 0.497 and equation (4) by 1.02 gives,

$$(0.497)(1.02\ I_{fb}(X_q - X_p)) + (0.497)(1137.5\ X_p) = (0.497)(1211) \qquad (5)$$

$$(1.02)(0.497\ I_{fb}(X_q - X_p)) + (1.02)(448\ X_p) = (1.02)(514.2) \qquad (4)$$

Subtraction gives,

$$565\ X_p = 604$$

$$-\ 457\ X_p = -\ 524.5$$

$$108\ X_p = 79.5$$

$$X_p = \frac{79.5}{108} = 0.736$$

Substitution of value of $X_p = 0.736$ in equation (5) gives,

$$1.02\ I_{fb}\ (X_q - X_p) + 1137.5\ X_p = 1211$$

$$(1.02)(751)(X_q - 0.736) + (1137.5)(0.736) = 1211$$

$$765\ X_q - 563 + 836 = 1211$$

$$765\ X_q = 938$$

$$X_q = 1.225$$

These values of X_p and X_q do not agree with the manufacturer's values of $X_p = 0.277$ and $X_q = 1.55$. It appears that some of the values used for equation (3) have not been read with sufficient accuracy.

However, at the same time one might question the two particular sets of readings used, so to get all possible values of X_p and X_q from all of the thirteen sets of readings, a computer program was written as shown in Appendix 2. Computed values are shown in Appendix 3, and differ greatly from the manufacturer's values.

When one looks at the computer program and computed values, a question arises about the mathematical problem. The equations for X_p and X_q are,

$$X_p = \frac{CA(I) - CA(J)}{CB(I) - CB(J)}$$

$$X_q = \frac{CA(I) - CA(J) + \left[\left[CA(J)\right]\left[CB(I)\right]\right] - \left[\left[CA(I)\right]\left[CB(J)\right]\right]}{CB(I) - CB(J)}$$

Now the values of CA(I) and CA(J), and CB(I) and CB(J) are so nearly equal in most cases, that subtraction gives very small numbers. Again division of such small numbers makes a big difference in results. Taking this problem in consideration, the computer program was re-written with double precision, which means that the computer carries twice as many significant figures. Results are as shown in Appendix 4.

Comparison of the results from Appendix 3 and Appendix 4 shows that there are no big differences in the values of X_p, but there are differences in the values of X_q. But still these values do not agree with the manufacturer's values. One question arises. Are the values of X_p and X_q computed in this manner extremely sensitive to small errors in various pieces of data? To try to answer this question, partial derivatives of X_p and X_q respect to E_d, V_t, I, ϕ and δ have been computed. The results are

Equation (2a):

$$E_d \sin(\phi + \delta) = I(X_q - X_p) + \frac{E_d \cos(\phi + \delta)}{V_t \cos \phi}(V_t \sin \phi + IX_p)$$

$$IX_p \left[1 - \frac{E_d \cos(\phi + \delta)}{V_t \cos \phi}\right] = IX_q - E_d \left[\sin(\phi + \delta) - \cos(\phi + \delta) \tan \phi\right]$$

$$X_p = \frac{V_t \cos \phi}{I} \left[\frac{IX_q - \left[E_d\left[\sin(\phi + \delta) - \cos(\phi + \delta) \tan \phi\right]\right]}{V_t \cos \phi - E_d \cos(\phi + \delta)}\right]$$

$$\frac{\partial X_p}{\partial E_d} = \frac{V_t \cos \phi}{I} \left[\frac{(\cos(\phi+\delta)\tan \phi - \sin(\phi+\delta))(V_t \cos \phi - E_d \cos(\phi+\delta))}{(V_t \cos \phi - E_d \cos(\phi + \delta))^2}\right.$$

$$\left. - \frac{(-\cos(\phi + \delta))(IX_q + E_d(\cos(\phi + \delta) \tan \phi - \sin(\phi + \delta)))}{(V_t \cos \phi - E_d \cos(\phi + \delta))^2}\right]$$

$$\frac{\partial X_p}{\partial V_t} = \left[\frac{(V_t \cos \phi - E_d \cos(\phi+\delta))(\cos \phi\, X_q - \frac{\cos \phi\, E_d}{I}(\sin(\phi+\delta) - \cos(\phi+ \delta)}{(V_t \cos \phi - E_d \cos(\phi + \delta))^2}\right.$$

$$\left.\tan \phi)\right] - \left[\frac{((V_t \cos\phi X_q) - \frac{V_t \cos\phi E_d}{I}(\sin(\phi+\delta) - \cos(\phi+\delta)\tan \phi)(\cos \phi))}{(V_t \cos \phi - E_d \cos(\phi + \delta))^2}\right]$$

$$\frac{\partial X_p}{\partial I} = V_t \cos \phi \left[\frac{(IV_t \cos \phi - IE_d \cos (\phi + \delta))(X_q)}{I^2(V_t \cos \phi - E_d \cos (\phi + \delta))^2} - \frac{(IX_q - E_d (\sin(\phi + \delta)}{} \right.$$

$$\left. \frac{- \cos (\phi + \delta) \tan \phi))(V_t \cos \phi - E_d \cos (\phi + \delta))}{(I^2(V_t \cos \phi - E_d \cos (\phi + \delta))^2} \right]$$

Rewriting (2a) gives,

$$X_p = \frac{V_t I \cos \phi X_q - V_t \cos \phi E_d \sin(\phi + \delta) + V_t \cos \phi E_d \cos (\phi + \delta) \tan \phi}{V_t I \cos \phi - IE_d \cos (\phi + \delta)}$$

$$X_p = \frac{V_t I \cos \phi X_q - V_t \cos \phi E_d \sin (\phi + \delta) + V_t E_d \cos (\phi + \delta) \sin \phi}{V_t I \cos \phi - IE_d \cos (\phi + \delta)}$$

$$\frac{\partial X_p}{\partial \phi} = \left[\frac{(V_t I \cos \phi - IE_d \cos(\phi + \delta))(-V_t I \sin \phi X_q + V_t E_d \sin \phi \sin(\phi + \delta)}{(V_t I \cos \phi - IE_d \cos (\phi + \delta))^2} \right.$$

$$\left. \frac{-V_t \cos \phi E_d \cos(\phi + \delta) + V_t \cos \phi E_d \cos(\phi + \delta) - V_t E_d \sin (\phi + \delta) \sin \phi)}{} \right]$$

$$- \left[\frac{(V_t I \cos \phi X_q - V_t \cos \phi E_d \sin(\phi + \delta) + V_t \sin \phi E_d \cos(\phi + \delta))(-V_t I \sin \phi}{(V_t I \cos \phi - IE_d \cos (\phi + \delta))^2} \right.$$

$$\left. \frac{+ IE_d \sin (\phi + \delta))}{} \right]$$

$$\frac{\partial X_p}{\partial \delta} = \left[\frac{(V_t I \cos\phi - IE_d \cos(\phi+\delta)(-V_t \cos\phi\, E_d \cos(\phi+\delta) - V_t \sin\phi\, E_d}{(V_t I \cos\phi - IE_d \cos(\phi+\delta))^2}\right.$$

$$\left.\frac{\sin(\phi+\delta))}{}\right] - \left[\frac{(V_t I \cos\phi\, X_q - V_t \cos\phi\, E_d \sin(\phi+\delta) + V_t \sin\phi}{(V_t I \cos\phi - IE_d \cos(\phi+\delta))^2}\right.$$

$$\left.\frac{E_d \cos(\phi+\delta))(IE_d \sin(\phi+\delta))}{}\right]$$

Equation (2a)

$$E_d \sin(\phi+\delta) = I(X_q - X_p) + \frac{E_d \cos(\phi+\delta)}{V_t \cos\phi}(V_t \sin\phi + IX_p)$$

$$IX_q = IX_p + E_d \sin(\phi+\delta) - \frac{E_d \cos(\phi+\delta)}{V_t \cos\phi}(V_t \sin\phi + IX_p)$$

$$X_q = X_p + \frac{E_d \sin(\phi+\delta)}{I} - \frac{E_d \cos(\phi+\delta)}{IV_t \cos\phi}(V_t \sin\phi + IX_p)$$

$$\frac{\partial X_q}{\partial E_d} = \frac{\sin(\phi+\delta)}{I} - \frac{\cos(\phi+\delta)}{IV_t \cos\phi}(V_t \sin\phi + IX_p)$$

$$\frac{\partial X_q}{\partial V_t} = -\left[-\frac{E_d \cos(\phi+\delta)\, X_p}{V_t^2 \cos\phi}\right]$$

$$\frac{\partial X_q}{\partial I} = -\frac{E_d \sin(\phi+\delta)}{I^2} + \frac{E_d \cos(\phi+\delta)\tan\phi}{I^2}$$

Rewriting equation (2a),

$$X_q = X_p + \frac{E_d \sin(\phi + \delta)}{I} - \frac{E_d \cos(\phi + \delta) \tan \phi}{I} - \frac{E_d \cos(\phi + \delta) X_p}{V_t \cos \phi}$$

$$\frac{\partial X_q}{\partial \phi} = \frac{E_d \cos(\phi + \delta)}{I} + \frac{E_d \sin(\phi + \delta) \tan \phi}{I} - \frac{E_d \cos(\phi + \delta) \sec^2 \phi}{I}$$

$$- X_p \left[\frac{(V_t \cos \phi)(-E_d \sin(\phi + \delta)) - (E_d \cos(\phi + \delta)(-V_t \sin \phi))}{(V_t \cos \phi)^2} \right]$$

$$\frac{\partial X_q}{\partial \delta} = \frac{E_d \cos(\phi + \delta)}{I} + \frac{E_d \sin(\phi + \delta) \tan \phi}{I} + \frac{E_d \sin(\phi + \delta) X_p}{V_t \cos \phi}$$

Reading No. 3, has been used to get numerical values of these partial derivatives.

KV	I_a	MW	MVAR	E_f	I_f	δ
18.1	5980	180	28	235	1600	38°

$P = 180 \text{ MW} = \sqrt{3}\ (18.1 \text{ KV})\ I \cos \phi$

$Q = 28 \text{ MVAR} = \sqrt{3}\ (18.1 \text{ KV})\ I \sin \phi$

$\tan \phi = \frac{Q}{P} = \frac{28}{180} = 0.1555$

$\phi = 8.83°$

$\cos \phi = 0.988$ $\qquad V_t = \frac{18.1}{18} = 1.005$ pu.

$\tan \phi = 0.1555$

$\cos(\phi + \delta) = \cos(46.83) = 0.684$

$\sin(\phi + \delta) = 0.729$

$X_q = 1.55$

$X_p = 0.277$

$E_d = \dfrac{I_f}{I_{fb}} = \dfrac{1600}{751} = 2.13$

$$\frac{\partial X_p}{\partial E_d} = 2.515$$

$$\frac{\partial X_p}{\partial V_t} = 0.377$$

$$\frac{\partial X_p}{\partial I} = -\ 3.0$$

$$\frac{\partial X_p}{\partial \phi} = -\ 1.738$$

$$\frac{\partial X_p}{\partial \delta} = 3.89$$

$$\frac{\partial X_q}{\partial E_d} = -\ 0.1849$$

$$\frac{\partial X_q}{\partial V_t} = 0.404$$

$$\frac{\partial X_q}{\partial I} = 0.1733$$

$$\frac{\partial X_q}{\partial \phi} = 2.62$$

$$\frac{\partial X_q}{\partial \delta} = 2.374$$

The values of the partial derivatives themselves give a clear idea about how sensitive X_p and X_q are to small error in data. In the case of X_q the partial derivatives are not as effective as for X_p. One which is very large is $\frac{\partial X_p}{\partial \delta}$. The $\frac{\partial X_q}{\partial \phi}$ is large, but not as large as $\frac{\partial X_p}{\partial \delta}$.

The angle δ is an angle between terminal voltage and internal voltage of synchronous generator. It is called "Load Angle" as well as "Displacement Angle." This angle is measured by some convenient method, on the rotor shaft outside of the synchronous generator, and is difficult to measure δ accurately.

CHAPTER V

CONCLUSIONS

It has been shown that partial derivatives of X_p and X_q taken with respect to all parameters are not extremely large, but at the same time, partial derivatives of X_p and X_q with respect to δ are bigger than any others, which shows that a little inaccuracy in reading of δ causes a big effect on the values of X_p and X_q. Unfortunately, the angle δ is the most difficult to read.

It also appears that the accuracy of ordinary switchboard instruments is not high enough to utilize the method described herein for computing X_p and X_q.

This work also depicts that it is not possible to get good values of reactances by just using readings from several load tests of synchronous generator. Higher accuracy instrumentation and an accurate method of determining the load angle δ would be absolutely necessary.

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APPENDIX 1

WAUKEGAN UNIT NO. 8 POWER ANGLE

MEASUREMENTS OF HIGH PRESSURE GENERATOR

Reading No.	Readings KV	I_a	MW	MVAR	E_f	I_f	ANGLE	Calculated I_f	ANGLE
1	18.1	6200	183	55	260	1760	35	1660	40
2	18.1	6400	182	80	290	1950	30	1850	35.3
3	18.1	5980	180	28	235	1600	38	1483	44.25
4	17.2	3300	89	16	156	1080	29	1021	31.4
5	17.2	3200	88	-21	126	900	35	820	39.65
6	17.4	4500	132	29	194	1330	34	1200	37.4
7	17.4	5100	136	74	235	1630	29	1458	31.4
8	17.6	5650	162	48	232	1600	36	1488	39.8
9	17.4	6380	167	91	278	1880	31	1681	34.0
10	17.4	5550	162	-11	192	1320	47	1128	52.15
11	18.0	6420	189	50	258	1750	39	1630	42.55
12	18.0	6780	192	80	291	1950	35	1842	38.05
13	18.0	6225	188	15	231	1570	44	1442	48.75

ELECTRICAL CHARACTERISTICS OF SYNCHRONOUS GENERATOR

Manufacturer: Westinghouse

Serial No.: 1563T350

Name Plate Rating:

KVA: 207,000

Volts: 18,000

P.F.: 0.85

RPM: 3600

Unsat.

Direct-axis Synchronous Reactance $X_d = 1.573$

Quadrature-axis Synchronous Reactance $X_q = 1.550$

Potier Reactance $X_p = 0.277$

Field Current Requirements:

I_f pu at Rated V_t 751 (air-gap) 825 (Sat.)

APPENDIX 2

THE COMPUTER PROGRAM USED FOR THE SOLUTION OF EQUATION (3),

FOR ALL OF THE THIRTEEN SETS OF READINGS

```
      DIMENSION CA(50),CB(50)
C
C     READ MACHINE RATINGS
C
      READ (11,100) RV,RKVA,BFI
      RAI = RKVA/(1.732*RV)
C
C     PRINT MACHINE DATA
C
      WRITE (12,101)
      WRITE (12,102) RV,RKVA,RAI,BFI
C
C     READ NUMBER OF SETS OF DATA
C
      READ (11,103) NT
C
C     READ TEST DATA
C
      WRITE (12,105)
      DO 15 I = 1,NT
      READ (11,104) N,AV,AI,P,Q,FI,DELD
      PHI = ATAN(Q/P)
      PHID = PHI*57.3
      V = AV/RV
      A = AI/RAI
      ED = FI/BFI
      DEL=DELD/57.3
      WRITE (12,106) N,PHID,V,A,ED
      ANG = PHI + DEL
      CA(N)=ED*(SIN(ANG)-COS(ANG)*SIN(PHI)/COS(PHI))/A
      CB(N)=ED*COS(ANG)/(V*COS(PHI))
   15 CONTINUE
C
C     CALCULATE AND PRINT VALUES FOR XP
C
      WRITE (12,127)
      NM = NT - 1
      DO 25 I = 1,NM
      NN = I+1
      DO 25 J = NN,NT
      XP = (CA(I)-CA(J))/(CB(I)-CB(J))
      XQ = (CA(I)-CA(J)+(CA(J)*CB(I))-(CA(I)*CB(J)))/(CB(I)-CB(J))
      WRITE (12,128) I,J,XP,CA(I),CA(J),CB(I),CB(J),XQ
```

```
 25 CONTINUE
100 FORMAT (3F10.1)
101 FORMAT ('1',10X,'RATED KV   RATED KVA   RATED ARM AMPS   BASE FLD A
    IMPS')
102 FORMAT('0',10X,F6.1,F13.0,F14.1,F16.0)
103 FORMAT (I10)
104 FORMAT (I10,6F10.1)
105 FORMAT('0',10X,'TEST   PHID   VPU   PU  ARM AMPS   PU FLD AMPS')
106 FORMAT ('0',10X,I3,F8.1,F7.2,F11.2,F15.2)
127 FORMAT ('1',10X,'TESTS USED  XP  CA(I)  CA(J)  CB(I)
   1   CB(J)     XQ')
128 FORMAT (' ',10X,I4,I5,6F10.3)
    END
```

APPENDIX 3

TESTS	USED	XP	CA(I)	CA(J)	CB(I)	CB(J)	XQ
1	2	0.207	1.589	1.613	1.259	1.377	1.535
1	3	0.616	1.589	1.548	1.259	1.194	1.429
1	4	0.790	1.589	1.448	1.259	1.081	1.384
1	5	0.499	1.589	1.486	1.259	1.054	1.459
1	6	0.736	1.589	1.466	1.259	1.093	1.398
1	7	0.784	1.589	1.499	1.259	1.145	1.385
1	8	0.396	1.589	1.554	1.259	1.173	1.486
1	9	2.334	1.589	1.483	1.259	1.214	0.983
1	10	1.156	1.589	91013.875	1.259	78700.813	1.289
1	11	0.384	1.589	1.570	1.259	1.211	1.489
1	12	-0.657	1.589	1.457	1.259	1.460	1.759
1	13	0.407	1.589	1.545	1.259	1.151	1.483
2	3	0.353	1.613	1.548	1.377	1.194	1.480
2	4	0.558	1.613	1.448	1.377	1.081	1.403
2	5	0.393	1.613	1.486	1.377	1.054	1.465
2	6	0.517	1.613	1.466	1.377	1.093	1.418
2	7	0.491	1.613	1.499	1.377	1.145	1.428
2	8	0.287	1.613	1.554	1.377	1.173	1.505
2	9	0.796	1.613	1.483	1.377	1.214	1.313
2	10	1.156	1.613	91013.875	1.377	78700.813	1.177
2	11	0.258	1.613	1.570	1.377	1.211	1.516
2	12	-1.884	1.613	1.458	1.377	1.460	2.324
2	13	0.303	1.613	1.545	1.377	1.151	1.499
3	4	0.892	1.548	1.448	1.194	1.081	1.375
3	5	0.444	1.548	1.486	1.194	1.054	1.462
3	6	0.814	1.548	1.466	1.194	1.093	1.391
3	7	1.011	1.548	1.499	1.194	1.145	1.353
3	8	-0.293	1.548	1.554	1.194	1.173	1.605
3	9	-3.149	1.548	1.483	1.194	1.214	2.158
3	10	1.156	1.548	91013.875	1.194	78700.813	1.324
3	11	1.273	1.548	1.570	1.194	1.211	1.302
3	12	-0.343	1.548	1.457	1.194	1.460	1.615
3	13	0.086	1.548	1.545	1.194	1.151	1.532
4	5	-1.398	1.448	1.486	1.081	1.054	1.561
4	6	1.580	1.448	1.466	1.081	1.093	1.320
4	7	0.802	1.448	1.499	1.081	1.145	1.383
4	8	1.165	1.448	1.554	1.081	1.173	1.353
4	9	0.266	1.448	1.483	1.081	1.214	1.426

TESTS	USED	XP	CA(I)	CA(J)	CB(I)	CB(J)	XQ
4	10	1.156	1.448	91013.875	1.081	78700.813	1.354
4	11	0.943	1.448	1.570	1.081	1.211	1.371
4	12	0.024	1.448	1.457	1.081	1.460	1.446
4	13	1.386	1.448	1.545	1.081	1.151	1.335
5	6	-0.518	1.486	1.466	1.054	1.093	1.514
5	7	0.142	1.486	1.499	1.054	1.145	1.478
5	8	0.575	1.486	1.554	1.054	1.173	1.455
5	9	-0.017	1.486	1.483	1.054	1.214	1.487
5	10	1.156	1.486	91013.875	1.054	78700.813	1.424
5	11	0.535	1.486	1.570	1.054	1.211	1.457
5	12	-0.072	1.486	1.457	1.054	1.460	1.490
5	13	0.602	1.486	1.545	1.054	1.151	1.454
6	7	0.632	1.466	1.499	1.093	1.145	1.408
6	8	1.105	1.466	1.554	1.093	1.173	1.364
6	9	0.143	1.466	1.483	1.093	1.214	1.453
6	10	1.156	1.466	91013.875	1.093	78700.813	1.359
6	11	0.881	1.466	1.570	1.093	1.211	1.384
6	12	-0.025	1.466	1.457	1.093	1.460	1.468
6	13	1.348	1.466	1.545	1.093	1.151	1.341
7	8	2.002	1.499	1.554	1.145	1.173	1.209
7	9	-0.227	1.499	1.483	1.145	1.214	1.532
7	10	1.156	1.499	91013.875	1.145	78700.813	1.331
7	11	1.079	1.499	1.570	1.145	1.211	1.343
7	12	-0.134	1.499	1.457	1.145	1.460	1.518
7	13	7.630	1.499	1.545	1.145	1.151	0.393
8	9	-1.708	1.554	1.483	1.173	1.214	1.849
8	10	1.156	1.554	91013.875	1.173	78700.813	1.355
8	11	0.411	1.554	1.570	1.173	1.211	1.483
8	12	-0.339	1.554	1.457	1.173	1.460	1.613
8	13	0.453	1.554	1.545	1.173	1.151	1.476
9	10	1.156	1.483	91013.875	1.214	78700.813	1.236
9	11	-25.146	1.483	1.570	1.214	1.211	6.870
9	12	-0.108	1.483	1.457	1.214	1.460	1.506
9	13	-0.968	1.483	1.545	1.214	1.151	1.691
10	11	1.156	91013.875	1.570	78700.813	1.211	1.326
10	12	1.156	91013.875	1.457	78700.813	1.460	0.925
10	13	1.156	91013.875	1.545	78700.813	1.151	1.370
11	12	-0.454	1.570	1.457	1.211	1.460	1.666
11	13	0.426	1.570	1.545	1.211	1.151	1.480
12	13	-0.284	1.457	1.545	1.460	1.151	1.587

APPENDIX 4

COMPUTED VALUES OF XP AND XQ

These were obtained using the computer program of Appendix 2 modified to provide double precision.

TESTS	USED	XP	CA(I)	CA(J)	CB(I)	CB(J)	XQ
1	2	0.207	1.589	1.613	1.259	1.377	1.535
1	3	0.615	1.589	1.548	1.259	1.194	1.429
1	4	0.790	1.589	1.448	1.259	1.081	1.384
1	5	0.499	1.589	1.486	1.259	1.054	1.459
1	6	0.736	1.589	1.466	1.259	1.093	1.398
1	7	0.784	1.589	1.499	1.259	1.145	1.385
1	8	0.396	1.589	1.554	1.259	1.173	1.486
1	9	2.334	1.589	1.483	1.259	1.214	0.983
1	10	1.156	1.589	96338.426	1.259	83304.934	1.289
1	11	0.383	1.589	1.570	1.259	1.211	1.489
1	12	-0.657	1.589	1.457	1.259	1.460	1.759
1	13	0.407	1.589	1.545	1.259	1.151	1.483
2	3	0.353	1.613	1.548	1.377	1.194	1.480
2	4	0.558	1.613	1.448	1.377	1.081	1.403
2	5	0.393	1.613	1.486	1.377	1.054	1.465
2	6	0.517	1.613	1.466	1.377	1.093	1.418
2	7	0.491	1.613	1.499	1.377	1.145	1.428
2	8	0.287	1.613	1.554	1.377	1.173	1.505
2	9	0.796	1.613	1.483	1.377	1.214	1.313
2	10	1.156	1.613	96338.426	1.377	83304.934	1.177
2	11	0.258	1.613	1.570	1.377	1.211	1.516
2	12	-1.884	1.613	1.457	1.377	1.460	2.324
2	13	0.303	1.613	1.545	1.377	1.151	1.499
3	4	0.892	1.548	1.448	1.194	1.081	1.375
3	5	0.444	1.548	1.486	1.194	1.054	1.462
3	6	0.814	1.548	1.466	1.194	1.093	1.391
3	7	1.011	1.548	1.499	1.194	1.145	1.353
3	8	-0.293	1.548	1.554	1.194	1.173	1.605
3	9	-3.149	1.548	1.483	1.194	1.214	2.158
3	10	1.156	1.548	96338.426	1.194	83304.934	1.324
3	11	1.273	1.548	1.570	1.194	1.211	1.302
3	12	-0.343	1.548	1.457	1.194	1.460	1.615
3	13	0.086	1.548	1.545	1.194	1.151	1.532
4	5	-1.398	1.448	1.486	1.081	1.054	1.561
4	6	1.580	1.448	1.466	1.081	1.093	1.320

TESTS	USED	XP	CA(I)	CA(J)	CB(I)	CB(J)	XQ
4	7	0.802	1.448	1.499	1.081	1.145	1.383
4	8	1.165	1.448	1.554	1.081	1.173	1.353
4	9	0.266	1,448	1.483	1.081	1.214	1.426
4	10	1.156	1.448	96338.426	1.081	83304.934	1.354
4	11	0.943	1.448	1.570	1.081	1.211	1.371
4	12	0.024	1.448	1.457	1.081	1.460	1.446
4	13	1.386	1.448	1.545	1.081	1.151	1.335
5	6	-0.518	1.486	1.466	1.054	1.093	1.514
5	7	0.142	1.486	1.499	1.054	1.145	1.478
5	8	0.575	1.486	1.554	1.054	1.173	1.455
5	9	-0.017	1.486	1.483	1.054	1.214	1.487
5	10	1.156	1.486	96338.426	1.054	83304.934	1.424
5	11	0.535	1.486	1.570	1.054	1.211	1.457
5	12	-0.072	1.486	1.457	1.054	1.460	1.490
5	13	0.602	1.486	1.545	1.054	1.151	1.454
6	7	0.632	1.466	1.499	1.093	1.145	1.408
6	8	1.105	1.466	1.554	1.093	1.173	1.364
6	9	0.143	1.466	1.483	1.093	1.214	1.453
6	10	1.156	1.466	96338.426	1.093	83304.934	1.359
6	11	0.881	1.466	1.570	1.093	1.211	1.384
6	12	-0.025	1.466	1.457	1.093	1.460	1.468
6	13	1.348	1.466	1.545	1.093	1.151	1.341
7	8	2.001	1.499	1.554	1.145	1.173	1.209
7	9	-0.227	1.499	1.483	1.145	1.214	1.532
7	10	1.156	1.499	96338.426	1.145	83304.934	1.331
7	11	1.079	1.499	1.570	1.145	1.211	1.343
7	12	-0.134	1.499	1.457	1.145	1.460	1.519
7	13	7.629	1.499	1.545	1.145	1.151	0.393
8	9	-1.708	1.554	1.483	1.173	1.214	1.849
8	10	1.156	1.554	96338.426	1.173	83304.934	1.355
8	11	0.411	1.554	1.570	1.173	1.211	1.483
8	12	-0.339	1.554	1.457	1.173	1.460	1.613
8	13	0.453	1.554	1.545	1.173	1.151	1.476
9	10	1.156	1.483	96338.426	1.214	83304.934	1.236
9	11	-25.138	1.483	1.570	1.214	1.211	6.868
9	12	-0.108	1.483	1.457	1.214	1.460	1.506
9	13	-0.968	1.483	1.545	1.214	1.151	1.691
10	11	1.156	96338.426	1.570	83304.934	1.211	1.326
10	12	1.156	96338.426	1.457	83304.934	1.460	0.925
10	13	1.156	96338.426	1.545	83304.934	1.151	1.370
11	12	-0.454	1.570	1.547	1.211	1.460	1.666
11	13	0.426	1.570	1.545	1.211	1.151	1.480
12	13	-0.284	1.457	1.545	1.460	1.151	1.587

APPENDIX 5

TABLE OF SYMBOLS

E_d = Voltage on the air-gap line

E_p = Voltage behind the Potier reactance drop

E_{ps} = Voltage behind the Potier reactance drop with saturation effects included

E_ℓ = Voltage behind leakage reactance drop

V_t = Terminal voltage to neutral

I = Armature current

I_f = Field current

I_t = Field current to produce air-gap flux ϕ

I_{fb} = Field current for rated voltage on air-gap line

X_d = Direct-axis synchronous reactance

X_q = Quadrature-axis synchronous reactance

X_p = Potier reactance

X_ℓ = Leakage reactance

ϕ = Power-factor angle

δ = Power angle