Turing's Genius – Defining an apt microcosm

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Alan Turing (1912–1954) is widely acknowledged as a genius. As well as codebreaking during World War II and taking a pioneering role in computer hardware design and software after the War, he also wrote three important foundational papers in the fields of theoretical computer science, artificial intelligence, and mathematical biology. He has been called the father of computer science, but he also admired by mathematicians, philosophers, and perhaps more surprisingly biologists, for his wide-ranging ideas. His influence stretches from scientific to cultural and even political impact. For all these reasons, he was a true polymath. This paper considers the genius of Turing from various angles, both scientific and artistic. The four authors provide position statements on how Turing has influenced and inspired their work, together with short biographies, as a starting point for a panel session and visual music performance.

Alan Turing. Artificial Intelligence. Computing. Developmental biology. Digital art. Genius. Morphogenesis. Visual music.

1. INTRODUCTION

"I had the good fortune to work closely with Alan Turing and to know him well for the last 12 years of his short life. It is a rare experience to meet an authentic genius. Those of us privileged to inhabit the world of scholarship are familiar with the intellectual stimulation furnished by talented colleagues. We can admire the ideas they share with us and are usually able to understand their source; we may even often believe that we ourselves could have created such concepts and originated such thoughts. However, the experience of sharing the intellectual life of a genius is entirely different; one realizes that one is in the presence of an intelligence, a sensitivity of such profundity and originality that one is filled with wonder and excitement."

- Peter Hilton (2017)

Alan Turing is generally acknowledged as a genius. He was selected as one of the top 43 scientists ever in a book of collected short biographies (Bowen 2012). It has been postulated that genius typically takes around ten years to develop (Robinson 2010). In Turing's case, he was studying and understanding Einstein's theory of relativity at the age of 15 (Copeland et al. 2017, p. 5). By the age of 24, he had published his first major paper on the nature of computability, using what became known as a "Turing machine" (Turing 1936), as a Fellow at King's College, Cambridge. This was before he had even started on his doctorate at Princeton University in the USA, completed in only 18 months (Turing 1938; Appel 2012). His 1936 paper is considered by many as the main foundational paper for the field of computer science (Bowen 2017; Dasgupta 2016).

Turing later provided a foundational philosophical paper on machine intelligence, later dubbed Artificial Intelligence (Turing 1950) and then a mathematical basis for morphogenesis (the "beginning of shape"), now his most citied paper (Turing 1952), foundational in mathematical biology. These two important papers were published in the last four years of his life, leading one to wonder what Turing could have gone on to do if his life had not been tragically cut short at the age of 41 (Hodges 2002). And this is in addition to his important codebreaking work at Bletchley Park during World War II and his later contributions to computer hardware design with the ACE computer at the National Physical Laboratory and software for the early Manchester Mark I computer at the University of Manchester.

Awareness of Alan Turing as a person was delayed until the 1970s, largely during to the lifting of the governmental secrecy surrounding the wartime codebreaking effort at Bletchley Park, although, interest in his Turing machine concept (Turing 1936) increased from the 1960s with the development of the field of computer science, for which the idea is foundational (see Figure 1). Interest in the Turing test (Turing 1950) rose in line with awareness of Alan Turing himself, but with a delay compared to the concept of Artificial Intelligence (dubbed "Machine Intelligence" by Turing), which gained great popularity in the late 1980s (see Figure 2) and is now seeing a resurgence again in recent years. Interest in morphogenesis (Turing 1952) has also risen, with a lesser peak in the early 1980s, but is of continued interest to mathematical biologists.

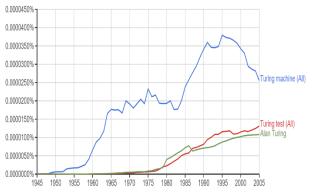


Figure 1: Plot of mentions of "Alan Turing", "Turing test", and "Turing machine" in books during 1945–2005 (Google Books Ngram Viewer, <u>https://books.google.com/ngrams</u>).

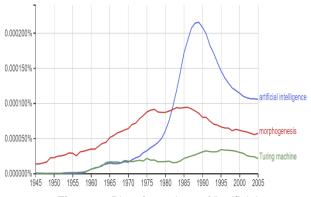


Figure 2: Plot of mentions of "artificial intelligence", "morphogenesis", and "Turing machine" in books during 1945–2005 (Google Books Ngram Viewer, <u>https://books.google.com/ngrams</u>).

Turing's remarkable genius has inspired this paper and its associated panel session. In the next section, the four authors provide position statements on how Turing has inspired their work in very varied ways. This forms the basis for a panel session discussion and visual music performance.

2. POSITION STATEMENTS

2.1 Jonathan Bowen

I first became aware of Alan Turing in the early 1980s as I became increasingly interested in computer science and its development, including Turing's role in it (Bowen 1995; 2003), having studied engineering science as a student in the mid-1970s. During a later visit to the USA, I spotted a copy of the brilliant and insightful Turing biography by Andrew Hodges, later reissued for the Turing centenary in 2012 (Hodges 1983/2012).

In the early 1990s, with the development of the web, I created the *Virtual Museum of Computing* (Bowen 1996), part of Tim Berners-Lee's *Virtual Library* online, and in parallel Andrew Hodges created a website (<u>http://www.turing.org.uk</u>), in association with his Turing biography (Bowen et al. 2005). We were both at Oxford University with ready free access to university web server facilities and interlinked our two resources at the time.



Figure 3: Artwork for "The Turing Guide" (Bowen 2016; Copeland et al. 2017).

Much later, I co-organised an event at Oxford University to celebrate Turing's centenary in 2012, at which Hodges was one of the speakers. We planned an associated proceedings, but joined forces with similar events at Bletchley Park and Cambridge. This led to significant delays, but a far more comprehensive and interdisciplinary volume of 42 chapters with 33 contributors, finally published in 2017 (see Figure 3 for the original cover artwork), including a foreword by Hodges.

The four main authors/editors of the book consisted of two philosophers, a mathematician, and me as a computer scientist, an interesting combination for anyone contemplating such an endeavour. No one of us could have produced the final result without the help of the others. This five-year effort has given me an enormous appreciation of Turing's genius and wide subsequent influence, not only scientifically but also culturally (Beckett 2012; Clements 2016) and even politically with the introduction of the "Alan Turing law" in the UK in 2017, formalising a pardon for convicted gay men.

The book includes eight parts on Turing's life and work. For example, Part IV *Biological Growth*, covers Turing's work on morphogenesis. Chapter 35 is by Bernard Richards (2017), covering his work on Radiolaria, protozoa with intricate mineral skeletons, as Turing's last master's student at Manchester University (Richards 1954). This includes illustrations of Radiolaria organisms (e.g., see Figure 4) together with computer solutions based on Turing's mathematical ideas that could produce similar shapes.

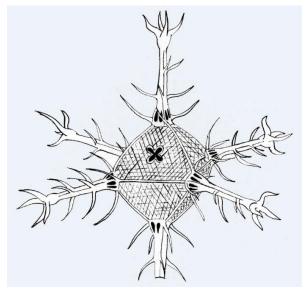


Figure 4: Circopurus octahedrus with six spines and eight faces (Richards 1954; 2005–6; 2017).

The cover of the book was inspired by Andy Warhol. I originally mocked up a version using an online facility for generating Warhol-like pictures using a photograph of Turing as a starting point (Bowen 2016). This was then redone in an improved form by a professional artist. Grey monochrome images as well as colourful images were deliberately included because Turing's life was not entirely happy. Finally, this artwork was transformed into the book cover itself (Figure 3) by the publisher, Oxford University Press.

Turing visited Bell Labs in the Greenwich Village, area of Manhattan, New York during World War II in the early 1940s (Giannini & Bowen 2017). Warhol was working in New York at his Manhattan studio, "The Factory", only two decades later in the 1960s, sadly after Turing's death. What a pity the two did not have a chance to meet, at a party at The Factory for example. I have a feeling the Warhol would have done his own version of a Turing screen print if they had.

In summary, I believe that Alan Turing is a leading scientist of all time (Bowen 2012) and that although recognition of this was delayed by the secrecy around his codebreaking work in World War II, his position in the pantheon of geniuses is now assured.

2.2 Terry Trickett

An initial fascination with Turing Patterns sparked off my interest in Alan Turing's morphogen theory which explained not only how tigers got their stripes but, also, gave the scientific explanation for the spontaneous miracle of development before birth. I celebrated this achievement in 'Turingalila' Visual Music on the Theme of Morphogenesis (Trickett 2016a). Here, I perturbed two Turing patterns to reveal processes of self-organisation reminiscent of those found in nature (see Figure 5). I have performed Turingalila at a few events such as EvoMUSART in Porto, Portugal (Trickett 2016a) and Balance-Unbalance in Colombia. In the process of creating Turingalila, my researches into Turing's lesser known discoveries made me acutely aware of both his extraordinary prescience and his new-found relevance in today's world. It was for this reason that I made special mention of Professor Jeremy Green's enthusiasm for Turing's morphogen theory in the knowledge that it has taken a long time for Turing's biological mathematics to gain general acceptance.

In reviewing Turingalila, Jeremy pointed out that, on first sight, Turing's disparate inspirational ideas may seem disconnected, one from another; what is the link between mathematical biology and artificial intelligence? But where we see separation, Turing would have seen only continuity. The overarching guiding principle of his work was directed towards modelling the human mind as a machine. For him, the way morphogens drive hundreds of simple steps that make one part of the embryo different from another was an integral part of his life-long search to establish the mathematical basis of how we think, how human intellectual abilities can be replicated using artificial neural networks. Little did Jeremy Green know, at the time, that it was his comments on Turingalila that set me off on a subsequent search into the astonishingly intricate architecture of the brain. The result – *Visual Music of the Brain and Mind* (Trickett 2018) – has led directly to the idea of promoting this panel discussion on Turing's genius – defining an apt microcosm.



Figure 5: "Turingalila" was inspired by Turing patterns and all the opportunities these offer for both artistic interpretation and scientific research. The first movement of the piece is based on perturbations of Turing's hexagonal stripe pattern.

In Visual Music of the Brain and Mind, I ask the simple question: if Alan Turing had lived longer would we now be further advanced in our knowledge of how the brain works and how we think? Turing's early papers, Intelligent Machinery (Turing 1948) and Computing Machinery and Intelligence (Turing 1950) addressed these subjects and provided the initial impetus that has led to the way we both understand and experience the world in which we live. It has taken a very considerable time for the full extent of his thinking on computer science and artificial intelligence to take hold (although he would recognise neither of these terms). It is true that a computer revolution is now happening largely because of him but our understanding of how the computer can replicate human intelligence has, as yet, made only tentative advances.

Andy Lomas, in his notes, mentions Turing's time at the National Physical Laboratory. It was here that his then employer, Sir Charles Darwin (grandson of the Charles Darwin) dismissed Turing's report. Intelligent Machinery (Turing 1948), which embraced thinking machines and modelling the human mind as a machine, as a 'schoolboy essay'. Turing's posthumous status as at least the equal of his ex-employer's grandfather is, I think, a nice form of retribution. In Computing Machinery and Intelligence (Turing 1950), Turing stated that the machine (i.e., the brain) should be of the sort where behaviour is in principle predictable by calculation. Had he lived. I feel certain, that this is a challenge that Turing would have taken up; in fact, his disciple, Robin Gandy, confirmed that, in the spring of 1954, Turing had spent some time inventing a new form of quantum mechanics "which showed him at his most lively and inventive" (Hodges 2002). This observation would equate with Turing's belief that what he needed to discover was some, as yet,

unknown mechanical law which accounted for the actions of human will (i.e., consciousness).

My paper/performance *Visual Music of the Brain and Mind* was given, last year, at Consciousness Reframed held in Beijing (see Figure 6). At EVA London 2018, I perform just the musical part of the piece.

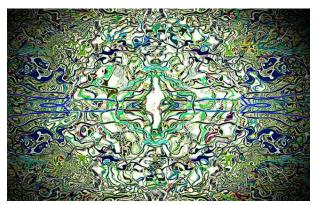


Figure 6: In "Visual Music of the Brain and Mind", a question is posed: what is the brain's geometry? The first section of the piece shows the proposed geometric pattern (an Islamic latticework), distorted by the brain's gyri (ridges) and sulci (valleys).

2.3 Jeremy Green

If you read the opening paragraphs of Alan Turing's landmark article The Chemical Basis of Morphogenesis (Turing 1952), his personality practically leaps off the page. There is brilliance and a little arrogance. There is also a fearlessness in tackling a profound intellectual problem. There is even humour: a sly in-joke about physicists approximating horses as spheres. Above all, there is an almost crystalline precision in articulating a bit of mathematics that perfectly straddles the abstract and real worlds. This is Turing's genius, not only here but also in the other fields. He was far from being the otherworldly Rain Man savant (a movie stereotype of autism rehashed for the otherwise excellent Turing biopic The Imitation Game) Turing was, rather, very practical and anchored in the real world. His bit of mathematics cleverly, and very simply, harnesses two disorder-generating processes of diffusion (think of an image blurring to the point of uniformity) to conjure up a crisp pattern of black and white, coming into focus out of almost nothing. He did not propose this solution to pattern generation in general abstract terms but as concrete, straightforward chemicals. His little spatial contrast-generator is enough to power a huge proportion of all the natural (and many of the artificial) forms and patterns we see around us.

For me, Turing's idea provided the missing piece in understanding how complex anatomy appears from the simplicity of a fertilised egg, a single cell.

Humans are made of trillions of cells, but we are not just big cellular snowballs. We have structure and anatomy: back and front; top and bottom; limbs, guts and brains. That anatomy comes from chemicals shifting around as the embryo develops. In the 1990s, I helped show what those chemicals are, and how cells become different from one another in response to them, but how they might be shifted was mysterious. Recently, my laboratory identified an anatomical structure - the transverse ridges on the roof of the mouth - that was simple enough to be tested for fit with Turing's hypothesis. We found two diffusing chemicals behaving exactly as his model predicted (Economou et al. 2012), a beautiful piece of simplicity in the midst of complicated biology (see Figure 7). Turing's ideas and his approach to simplification and idealisation continue to inform our research.

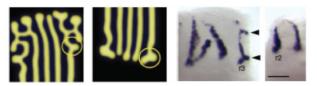


Figure 7: Computer simulations (left) and experimental results (right) showing bifurcating stripes of gene expression as predicted by the simplest of Turing "RD" models (Economou et al. 2012).

Working with Turing's model showed me something else. I initially ran a version of it (written by my later collaborator, Shigeru Kondo of Osaka University) and made a personal discovery. While I could adjust many different variables and set up any starting pattern, I struggled to produce a final pattern that was convincingly similar to my experimental results. The programme seemed to squirm away from where I wanted to go. This was far from the infinitely fudgeable simulator that many experimental scientists imagine computer models to be. But it is precisely the limitations on what a model can do that gives it value: if you cannot match your model to the data, one of them must be wrong and you have something to learn. The process of bridging that gap provides the value and the excitement of new discovery. I would like to think that this is a bit like the process of the artist: the canvas, the frame, the constraints of the medium (whatever it may be) challenge the artist. The wrestling process is the creative act that brings out novelty and insight.

2.4 Andy Lomas

Two works that I read 30 years ago as an impressionable mathematics undergraduate lit a fuse that has led to a sustained obsession with the emergent richness of morphogenetic processes: Alan Turing's seminal paper *The Chemical Basis of Morphogenesis* (Turing 1952) and D'Arcy Thompson's *On Growth and Form* (Thompson 1917/1942). I was particularly struck by Turing's realisation of how a computing machine could be used as a rich environment to simulate biological processes, such cell growth and neural systems, exploring how complex behaviour seen in the real world could potentially emerge from surprisingly simple processes.

My art work focuses on emergent complexity, in particular the type of forms and structure than can be created by simulation of growth processes. As well at the obvious influence of Turing's paper that directly considers reaction diffusion and morphogenesis, one thing that I found particularly inspiring reading Turing's work was how seamlessly he combines deep theoretical ideas, such as the fundamental limits of computability and the concept of a universal machine, with a practical focus on how machines could be used to solve genuinely interesting real-world problems.

When he was working on the design for the Automatic Computing Engine (ACE), Turing is reported as being "obsessed with the idea of speed on the machine" (Evans 1976). In Turing's lecture on the Automatic Computing Engine to the London Mathematical Society (Turing 1947), and his report on *Intelligent Machinery* (Turing 1948), he states that it is particularly important for a machine to have sufficient memory in order to be able to do interesting problems. As he states, "a large storage capacity is necessary if it is to be capable of anything more than rather trivial operations" (Turing 1947).

The specifications that Turing was looking for in the ACE included memory capable of storing 200,000 binary digits (Turing 1947). The reasons for such a demanding specification, effectively equivalent to many home computers in the 1980s, was to be able to tackle what he considered to be interesting problems rather than just being a demonstration of a stored program computing architecture. In particular, what he considered to be interesting problems were often inspired by biology, such as running "organised" and "unorganised" machines of neuron like processes to explore learning and how brains work (Turing 1948).

Alan Turing saw that the machines he was working on offered far more than just fast computation of numerical results: they are universal platforms that offer an incredible blank canvas for exploring processes. As Sir Charles Darwin, director of the National Physical Laboratory, said: Turing wanted to "extend his work on the machine [the ACE] still further towards the biological side", "hitherto the machine has been planned for work equivalent to that of the lower parts of the brain, and he wants to see how much a machine can do for the higher ones: for example, could a machine be made that could learn by experience?" (Darwin 1947).

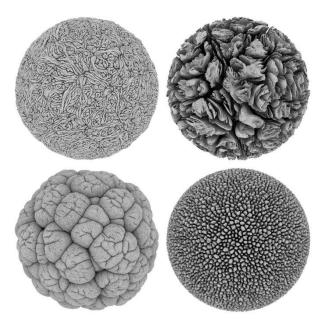


Figure 8: Examples of surface structures as morphogenetic art by Andy Lomas.

Turing can be seen as the founding father of conducting experiments "in silico" to complement the traditional methods of "in vivo" and "in vitro". Modern technology, such as GPUs with thousands of processors providing supercomputing level performance in desktop machines, are making Turing's dream of using computers to explore systems with complexity that can match biology an accessible reality. As well as providing a framework for testing ideas about real biological processes, these simulated environments enable us to explore the potential of what could be achieved by artificial variations.



Figure 9: "Morphogenetic Creations" exhibition in 2016 at Watermans, London, by Andy Lomas.

3. BIOGRAPHIES

3.1 Jonathan Bowen

Jonathan Bowen is Emeritus Professor of Computing at London South Bank University and Chair of Museophile Limited, a museum and IT consultancy company that he founded in 2002. He has been a visiting scholar/professor at a number of institutions including the Israel Institute for Advanced Studies (Jerusalem), the Pratt Institute (New York), the United Nations University (Macau), and King's College London. Previously he has held academic/research posts at Birmingham City University, University of Reading, Oxford University Computing Laboratory, and Imperial College London. He studied Engineering Science at Oxford University. Bowen's research interests are interdisciplinary and range from computer science, especially software engineering, through to the history of computing and museum informatics. He contributes to Wikipedia on cultural and computingrelated topics. He is a Fellow of the BCS and the Royal Society of Arts. In 2017, he co-authored The Turing Guide on the computing pioneer Alan Turing (Copeland et al. 2017; Cerf 2018). He has cochaired EVA London conferences on Electronic Visualisation and the Arts since 2006 (Bowen et al. 2016; 2017).

3.2 Terry Trickett

Working previously as an architect and designer, Terry Trickett has now become a digital artist performing visual music worldwide at various new media festivals and conferences. The subjects he chooses range far and wide, often taking him into unchartered territory - places where, sometimes, he invades the realm of science and, with the aid of music, brings the worlds of science and art closer together. In creating a piece of visual music, his aim is to share and communicate, an idea through a process that combines animated visual imagery with musical performance, usually on solo clarinet. As a participatory form of communication, he finds that visual music is effective because it succeeds in engaging with audiences at both an intellectual and emotional level. He has performed and presented pieces at EVA London conferences: Revealing the Colours of the Apocalypse through Visual Music (Trickett 2016b) and Ragatime: Glimpses of Akbar's Court at Fatehpur Sikri (Trickett 2017).

3.3 Jeremy Green

Jeremy Green is a developmental biologist who uses experiments to explore and test some of the big ideas that explain spatial organisation in biology. He is Professor of Developmental Biology at King's College London. After a PhD at Imperial College London on gene regulation, he discovered dosedependency thresholds and the ratchet effect for morphogen cell-type specification working with Dr (now Sir) Jim Smith. He was a Miller Fellow at the University of California Berkeley for two years before becoming a Principal Investigator at the Dana Farber Cancer Institute and Harvard Medical School Department of Genetics, where he focused on molecular signalling and cell polarity. He returned to London in 2005. Recent interests include apicobasal and planar cell polarity and Turing patterning systems as well as physical morphogenesis of mammalian tissue, especially in the face (Li et al. 2016; Mao & Green 2017).

3.4 Andy Lomas

Andy Lomas is a digital artist, mathematician, and Emmy award winning supervisor of computer generated effects. His art work explores how complex sculptural forms can be created emergently by simulating growth processes. Inspired by the work of Alan Turing, D'Arcy Thompson (1917/1942), and Ernst Haeckel (1873–76), it exists at the boundary between art and science. He has had work exhibited in over 70 joint and solo exhibitions, including at the Royal Society, SIGGRAPH, Japan Media Arts Festival, Ars Electronica Festival, Los Angeles Municipal Art Gallery, Los Angeles Center for Digital Art, Centro Andaluz de Arte Contemporaneo, Watermans (see Figure 9), the Science Museum and the ZKM. His work is in the collections at the Victoria & Albert Museum and the D'Arcy Thompson Art Fund Collection, and was selected by Saatchi Online to contribute to a special exhibition in the Zoo Art Fair at the Royal Academy of Arts. In 2014, his work Cellular Forms won The Lumen Prize Gold Award. He is a previous presenter and exhibitor at the EVA London conference (Lomas 2016; Papadimitriou et al. 2017).

4. CONCLUSION

"...the property of being digital should be of greater interest than that of being electronic." – Alan Turing (1947)

The genius of Alan Turing, although a mathematician and scientist, has had a great influence of cultural aspects as well as science. Some artists have responded directly to the inspiration of Turing (Olinick 2012). Writers and poets have responded to Turing's ideas (Beckett 2012; Clements 2016). As well as morphogenesis, there has been musical influence of Artificial Intelligence (AI), originally postulated as machine intelligence by Turing (1950), for example, for the automation of pop music (Wikipedia 2018).

We hope that the position statements in this paper provide a sample of very different ways in which Turing's genius has influenced each of the four authors. And we hope the reader may be similarly inspired by at least some of the many facets of Alan Turing's ideas.

Acknowledgement

Terry Trickett suggested the original idea for this paper that then developed into its current form.

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5. REFERENCES

Appel, A. W. (2012) *Alan Turing's Systems of Logic: The Princeton thesis.* Princeton University Press.

Beckett, C. (2012) The Turing Test. Elastic Press.

Bowen, J. P. (1995) A brief history of algebra and computing: An eclectic Oxonian view. *IMA Bulletin*, 31(1/2):6–9, January/February.

Bowen, J. P. (1996) Virtual Museum of Computing web site. *IEEE Annals of the History of Computing*, 18(4):67.

Bowen, J. P. (2003) Computer science. In J. L. Heilbron (ed.), *The Oxford Companion to the History of Modern Science*. Oxford University Press, pp. 171–174.

Bowen, J. P. (2012) Alan Turing. In A. Robinson (ed.), *The Scientists: An Epic of Discovery*. Thames and Hudson, pp. 270–275.

Bowen, J. P. (2016) Alan Turing: Virtuosity and visualisation. In Bowen et al. (2016), pp. 197–204. DOI: 10.14236/ewic/EVA2016.40

Bowen, J. P. (2017) Alan Turing: Founder of Computer Science. In J. P. Bowen, Z. Liu, and Z. Zhang (eds.), *Engineering Trustworthy Software Systems: Second International School, SETSS 2016*, Chongqing, China, March 28 – April 2, 2016. Springer, LNCS, vol. 10215, pp. 1–15. DOI: <u>10.1007/978-3-319-56841-6_1</u>

Bowen, J. P., Bennett, J., Borda, A., Hodges, A., Filippini-Fantoni, S., and Beler, A. (2005) The development of science museum websites: Case studies. In L. T. W. Hin and R. Subramaniam (eds.), *Elearning and Virtual Science Centers*, Section 3: Case Studies, Chapter XVIII, pp. 366–392. Idea Group Publishing.

Bowen, J. P., Diprose, G., and Lambert, N. (eds.) (2016) *EVA London 2016: Electronic Visualisation and the Arts.* BCS, Electronic Workshops in Computing (eWiC). <u>http://www.bcs.org/ewic/eva2016</u>

Bowen, J. P., Diprose, G., and Lambert, N. (eds.) (2017) *EVA London 2017: Electronic Visualisation and the Arts.* BCS, Electronic Workshops in Computing (eWiC). <u>http://www.bcs.org/ewic/eva2017</u>

Cerf, V. (2018) The man behind the machine. *Physics World*, pp. 38–39, January.

https://physicsworld.com/a/man-behind-machine/

Clements, W. (2016) Poetry beyond the Turing Test. In Bowen et al. (2016), pp. 213–219. DOI: <u>10.14236/ewic/EVA2016.42</u>

Copeland, B. J., Bowen, J. P., Sprevak, M., Wilson, R. J., et al. (2017) *The Turing Guide*. Oxford University Press. <u>https://en.wikipedia.org/wiki/The_Turing_Guide</u>

Darwin, C. (1947) Letter to Sir Edward V. Appleton, 23 July. Public Record Office, document reference DSIR 10/385). Also in The Turing Archive for the History of Computing. http://www.AlanTuring.net/darwin_appleton_23jul47

Dasgupta, S. (2016) *Computer Science: A Very Short Introduction.* Oxford University Press.

Economou, A. D., Ohazama, A., Porntaveetus, T., Sharpe, P. T., Kondo S., Basson, M. A., Gritli-Linde, A., Cobourne, M. T., and Green, J. B. A. (2012) Periodic stripe formation by a Turing-mechanism operating at growth zones in the mammalian palate. *Nature Genetics*, 44(3), pp. 348–351.

Evans, C. (1976) Interview with J. H. Wilkinson. *The Pioneers of Computing: An Oral History of Computing.* Science Museum, London.

Giannini, T. and Bowen J. P. (2017) Life in Code and Digits: When Shannon met Turing. In Bowen et al. (2017), pp. 51–58. DOI: <u>10.14236/ewic/EVA2017.9</u>

Haeckel, E. (1873–76) Report on the Radiolaria collected by H.M.S. Challenger, Zoology, Volume XVIII, *Report of the Scientific Results of the Voyage of H.M.S. Challenger During the Years* 1873–76. http://www.19thcenturyscience.org/HMSC/HMSC-Reports/Zool-40/

Hilton, P. (2017) Meeting a genius. In Copeland et al. (2017), chapter 3, pp. 31–34.

Hodges, A. (1983/2012) *Alan Turing: The Enigma*, Simon and Schuster / Princeton University Press.

Hodges, A. (2002). *What would Alan Turing have done after 1954?* Speaking notes of lecture given at Turing Day. Lausanne, Switzerland.

http://www.calculemus.org/Alg/Rob/biblioteka/hodges2002.html

Li, J., Chatzeli, L., Panousopoulou, E., Tucker, A., and Green, J. B. A. (2016) Epithelial stratification and placode invagination are separable functions in early morphogenesis of the molar tooth. *Development*, 143(4), pp. 670–681. DOI: 10.1242/dev.130187

Lomas, A. (2016) Species Explorer: An interface for artistic exploration of multi-dimensional parameter spaces. In Bowen et al. (2016), pp. 95–102. DOI: 10.14236/ewic/EVA2016.23

Mao, Y. and Green, J. B. A. (2017) Systems morphodynamics: Understanding the development of tissue hardware. *Philosophical Transactions of the Royal Society of London Series B: Biological Sciences*, 372(1720). DOI: <u>10.1098/rstb.2016.0505</u>

Olinick, M. (2012) Artists respond to Alan Turing. *Math Horizons*, 19(4), pp. 5–9, April. Mathematical Association of America. DOI: 10.4169/mathhorizons.19.4.5

Papadimitriou, I., Addis, J., Lomas, A., Dekker, L., Singporewaia, K., and Tresset, P. (2017) Digital Futures: Lumen Big Reveal. In Bowen et al. (2017), pp. 438–440. DOI: <u>10.14236/ewic/EVA2017.86</u>

Richards, B. (1954) *The Morphogenesis of Radiolaria*, MSc thesis, University of Manchester, UK.

Richards, B. (2005–6) Turing, Richards and morphogenesis. *The Rutherford Journal*, 1. http://www.rutherfordjournal.org/article010109.html

Richards, B. (2017) Radiolaria: Validating the Turing theory. In Copeland et al. (2017), chapter 35, pp. 383–388.

Robinson, A. (2010) *Sudden Genius?: The gradual path to creative breakthroughs.* Oxford University Press.

Thompson, D'A. W. (1917/1942) *On Growth and Form*, Cambridge University Press. <u>http://archive.org/details/ongrowthform1917thom</u> & <u>http://archive.org/stream/ongrowthform00thom</u>

Trickett, T. (2016a) 'Turingalila' Visual music on the theme of morphogenesis. *EvoMUSART 2016: International Conference on Evolutionary and Biologically Inspired Music and Art*, pp. 218–224. Springer. DOI: <u>10.1007/978-3-319-31008-4_15</u>

Trickett, T. (2016b) Revealing the colours of the apocalypse through visual music. In Bowen et al. (2016), pp. 5–9. DOI: <u>10.14236/ewic/EVA2016.2</u>

Trickett, T. (2017) Ragatime: Glimpses of Akbar's Court at Fatehpur Sikri. In Bowen et al. (2017), pp. 304–310. DOI: <u>10.14236/ewic/EVA2017.61</u>

Trickett, T. (2018) An astonishingly intricate Architecture: Visual music of the brain and mind. *Consciousness Reframed XX: Art and Consciousness in the Post-Biological Era*, 25–26 November 2017, China.

Turing, A. M. (1936) On computable numbers with an application to the Entscheidungsproblem, *Proceedings of the London Mathematical Society, Series* 2, 42(1), pp. 230–265. DOI: <u>10.1112/plms/s2-42.1.230</u>

Turing, A. M. (1938) *Systems of logic based on ordinals: a dissertation.* PhD thesis, Princeton University, USA.

Turing, A. M. (1947) *Lecture to the London Mathematical Society on 20 February 1947*. King's College Archive Centre, University of Cambridge.

Turing, A. M. (1948) *Intelligent Machinery: A Report.* National Physical Laboratory, UK, 8 July. Also in the Woodger Papers, The Turing Archive for the History of Computing.

http://www.AlanTuring.net/intelligent_machinery

Turing, A. M. (1950) Computing machinery and intelligence. *Mind*, 59(236), pp. 433–460, October. DOI: <u>10.1093/mind/LIX.236.433</u>

Turing, A. M. (1952) The chemical basis of morphogenesis. *Philosophical Transactions of the Royal Society of London*, 237(641), pp. 37–72. DOI: 10.1098/rstb.1952.0012

Wikipedia (2018) Pop music automation. *Wikipedia*. (Retrieved 10 January 2018.) <u>http://en.wikipedia.org/wiki/Pop music automation</u>