Equity Risk Premiums and Risk Free Rates in Modelling and Prediction of Financial Markets

Mohammad Ghavami, *Senior Member, IEEE,* and Reza S Dilmaghani, *Member, IEEE*

Abstract—This paper proposes a novel adaptive framework for modelling financial markets using equity risk premiums, risk free rates and volatilities. The recorded economic factors are initially used to train four adaptive filters for a certain limited period of time in the past. Once the systems are trained, the adjusted coefficients are used for modelling and prediction of an important financial market index. Two different approaches based on least mean squares (LMS) and recursive least squares (RLS) algorithms are investigated. Performance analysis of each method in terms of the mean squared error (MSE) is presented and the results are discussed. Computer simulations carried out using recorded data show MSEs of 4% and 3.4% for the next month prediction using LMS and RLS adaptive algorithms, respectively. In terms of twelve months prediction RLS method shows a better tendency estimation compared to the LMS algorithm.

Keywords—Prediction of Financial Markets, Adaptive methods, MSE, LSE.

I. Introduction

Modelling and analysis of financial markets have become a major focus of attention and research in recent years. In particular, there have been several attempts to use adaptive filters and control theory principles to analyse and predict financial market indices. For example, [1] proposes an exponential moving average technique combined with adaptive filters to model price movements of stock values. Statistical surveying methods are discussed in [2] to find the turning points of stock markets. Some other approached, such as [3] and [4], show that incorporating subjective price beliefs into standard asset pricing models can temporarily de-correlate stock prices from their fundamental values and artificially increase their prices. Moreover, a correlation-based adaptive filter is discussed and implemented in [5].

The last ten years in financial markets has been different in many perspectives compared to any other periods in the history. First, the extraordinary monetary policy by major central banks around the world has pushed the interest rates to record low values. At the same time, the return on main financial indices such as S&P 500 has increased significantly during this period. In addition, the intervention of central banks has results in a low volatility environment for the markets. Moreover, the forward guidance of those central banks promises to keep the low interest rates in place for a foreseeable future [6], [7]. No prior research is therefore done to analyse, model and predict the markets from this particular perspective. This paper is an essential attempt to analyse and model the financial market data based on the unusually stimulative monetary policy of the main central banks.

The rest of this paper is organized as follows. Section II describes the basic relationship between the main market parameters affecting the required calculations. Section III introduces the volatility of the financial market as a potential parameter in the prediction process of share prices. Section IV explains briefly the main structure of the adaptive filter and two algorithms to perform modelling and prediction. Section V presents the computer simulation results and finally, Section VI concludes the paper.

M. Ghavami is with the School of Engineering, London South Bank University, London, SE1 0AA, UK, E-mail: ghavamim@lsbu.ac.uk.

R. S. Dilmaghani is a Member of IEEE, Email: rezad@ieee.org.

II. Expected Return

Expected return for a risky investment can be defined as the sum of the risk free rate plus an extra premium to compensate for the risk. This can be written mathematically as:

$$E_r = R_f + R_p \tag{1}$$

where E_r, R_f and R_p represent the expected return, risk free rate and the risk premium, respectively. There is no single definition for the calculation and estimation of R_f and R_p parameters. However, in this paper the ten year US treasury bond rate is considered as risk free because it is backed by the government.

There are several historical based methods to calculate R_p [8], [9]. However, we will be using implied equity risk premium method to calculate the risk premium. The implied equity risk premium method has two advantages over the historical based techniques. First, except for a short training period, it requires no historical data. Second, it relies on updated estimates of growth and cash flows for most financial markets. Hence, R_p can be estimated using the following relationship [8]:

$$\text{Current market level} = \frac{\text{Expected dividends}}{\text{Required return-expected growth}} \tag{2}$$

All above mentioned values and data are readily available to access and calculate for major world stock markets such as S&P 500.

III. Volatilities

The next parameter that can be used in the proposed modelling and analysis of financial markets is the 'volatility'. Volatility in its simple form means how unsure we are with respect to future values of an asset. The higher volatility therefore correlates to higher uncertainty. There are various techniques to calculate volatility. However, in this paper the method presented by the Chicago Board Options Exchange (CBOE) [10] will be used to introduce a parameter for market modelling and prediction.

The volatilities calculated using this approach are referred to as the VIX (volatility index) or the 'fear gauge'. This is the most widely used metric for the volatility.

IV. Modelling and prediction using adaptive filter structures

Figure 1 shows the main structure of the proposed adaptive prediction method of financial market data. The system comprises of four adaptive finite impulse response (FIR) filters $G(z)$, $F(z)$, $W(z)$ and $H(z)$, where z denotes the z-transform complex variable. The input discrete time sequences to these filters are risk premium rate $r_p(n)$, risk free rate $r_f(n)$, volatility $v(n)$, and the S&P 500 index $s_p(n)$, respectively. The parameter n shows the discrete time index.

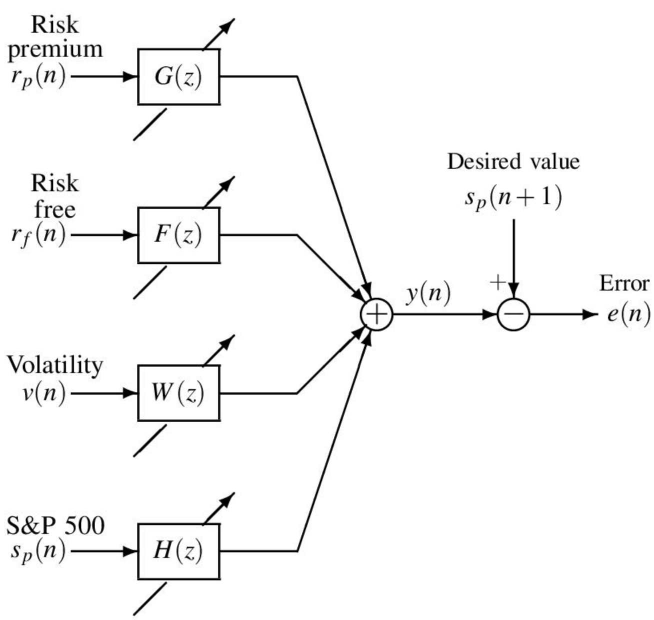

Fig. 1: Structure of the adaptive prediction method for financial market data.

As shown in Figure 1, the outputs of the four adaptive filters are added to produce the output signal $y(n)$ which is subtracted from the next month's value of the S&P 500 index $s_p(n+1)$ (desired value of the adaptation process) to generate the error signal $e(n)$. This error is used in the adjustment process of all adaptive coefficients within four filters.

A. LMS adaptation algorithm

The LMS algorithm [11] is used to direct the MSE of the prediction toward zero. In this algorithm the weights of four filters are updated as follows:

$$\mathbf{C}(n+1) = \mathbf{C}(n) + \mu\, e(n)\mathbf{X}(n) \tag{3}$$

where μ is the convergence factor controlling the speed and stability of the calculations. The vector $\mathbf{C}$ is generated by bringing together all adaptive coefficients of the filter as follows:

$$\mathbf{C} = [g_0, \ldots, g_{N-1}, f_0, \ldots, f_{N-1}, w_0, \ldots, w_{N-1}, h_0, \ldots, h_{N-1}]' \tag{4}$$

The input signal vector comprises of all relevant past market data

$$\begin{aligned}\mathbf{X} = [&r_p(n), \ldots, r_p(n-N+1), r_f(n), \ldots, r_f(n-N+1),\\ &v(n), \ldots, v(n-N+1), s_p(n), \ldots, s_p(n-N+1)]'\end{aligned} \tag{5}$$

where $'$ in (4) and (5) stands for the transpose. The length of all FIR filters is N and the coefficients are denoted by g, f, w and h for $G(z)$, $F(z)$, $W(z)$ and $H(z)$, respectively. The input signals to these filters are shown by $r_p(n)$, $r_f(n)$, $v(n)$ and $s_p(n)$, respectively.

In the training phase, filters are adjusted for a certain period of available past data. During this period $y(n)$ becomes very close to the next month's value of the S&P 500 index $s_p(n+1)$. After the training period, the filter coefficients are maintained to their latest (MSE optimal) values and the algorithm continuous to run for prediction of the new values of S&P 500 index for the coming months or year.

B. RLS adjustment technique

Compared to the LMS adaptive algorithm, the RLS algorithm [11] has a faster convergence speed and does not exhibit the eigenvalue spread problem. However, the RLS algorithm requires more computational resources than the LMS algorithm. The RLS algorithm is expressed in matrix form as:

$$\mathbf{C}(n+1) = \mathbf{C}(n) + \frac{\mathbf{P}(n)\mathbf{X}(n)}{\alpha + \mathbf{X}'(n)\mathbf{P}(n)\mathbf{X}(n)} e(n) \tag{6}$$

where α is the exponential weighting factor and $\mathbf{P}(n)$ is the inverse covariance matrix at step n. This matrix is also updated iteratively as follows:

$$\mathbf{P}(n+1) = \frac{1}{\alpha}\left\{\mathbf{P}(n) - \frac{\mathbf{P}(n)\mathbf{X}(n)\mathbf{X}'(n)\mathbf{P}(n)}{\alpha + \mathbf{X}'(n)\mathbf{P}(n)\mathbf{X}(n)}\right\} \tag{7}$$

This algorithm can be used for a real time implementation of the predictor and does not require calculation of the autocorrelation function of the input vector.

As in the LMS case, during the training phase of the RLS algorithm, the filters are adjusted for a certain period of available past data in which $y(n)$ approaches the next month's value of the S&P 500 index $s_p(n+1)$. When the training period is completed, the filter coefficients are kept constant and the algorithm runs to predict the new values of S&P 500 index for the future.

V. Simulation results and discussions

Figure 2 shows risk free rates and equity risk premiums on a monthly basis from September 2008 to March 2016. Two observations can be made:

- The average risk free rate within this period is 2.6%

Fig. 2: EPRs and risk free rates.

- The average equity risk premium is 5.7%

Moreover, it is very interesting to see that the expected year on year return over this period is close to 8%. This also shows that an increase in risk free rate results in reduction (in average) of equity risk premium and vice versa.

Figures 3 and 4 illustrate the VIX index and S&P 500 values on a monthly basis within the same period mentioned above. We assume that the training period of the adaptation uses 79 values of monthly data recorded from August 2008 to March 2015. The prediction of the S&P 500 index is done for the next 12 months from April 2015 to March 2016.

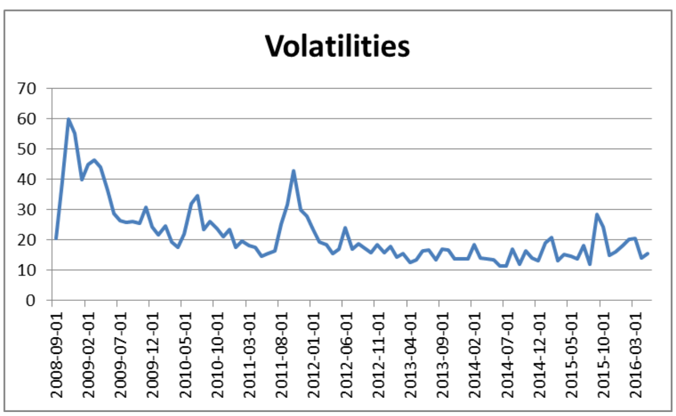

Fig. 3: Market volatility over the last 8 years.

Figure 3 demonstrates that there has been a significant volatility during the finical crisis of 2008 and the European debt crisis of 2011. Furthermore, comparing figures 2 and 3 results in following conclusions:

- There is a shift to risk free assets in times of higher uncertainty and a decrease in return of those assets;
- Investors demand a higher return during a high period of volatility.

The period between 2008 to 2016 is chosen for our modelling and analysis for the following reasons:

- No prior research is done to analyse the financial markets in low interest rate environments. In particular, the extraordinary monetary policy by major central banks around the world has pushed the interest rates and return on risk free assets to record lows. At the same time the return on main financial indices such as S&P 500 has increased significantly during this period. The return on S&P 500 for the period of our analysis is about 61%.

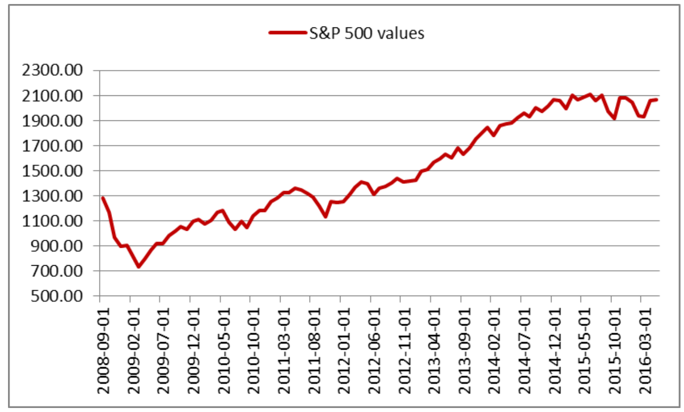

Fig. 4: S&P 500 index values for the period September 2008 to March 2016.

- The intervention of various central banks in financial markets has developed a lower volatility environment. This outcome coupled with a low interest rate environment has created a special situation that this paper tries to model and analyse.

In the first simulation, the LMS algorithm is used for both training and prediction of the data. Figure 5 illustrates the training process up to the 79th recorded value. The mean square error MSE is about 4.03%. Figure 6 shows the prediction of the S&P 500 values for the next 12 months.

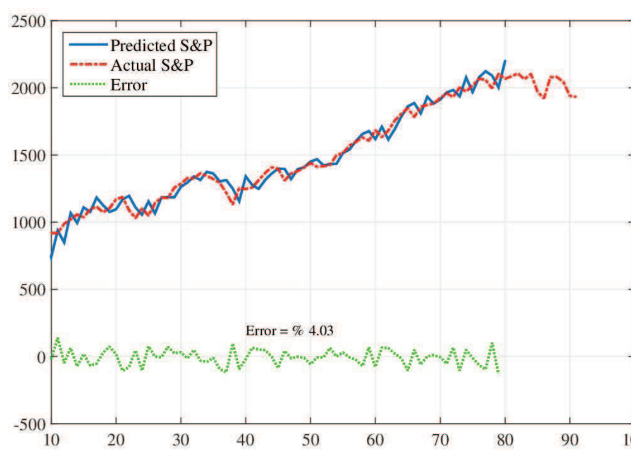

Fig. 5: LMS training process for the prediction of S&P 500 index during the first 79 values of the monthly recorded data.

In the second simulation, the RLS algorithm is used for both training and prediction of the data. Figure 7 illustrates the training process up to the 79^{th} value. The mean square error MSE is about 3.41%. Figure 8 shows the prediction of the S&P 500 values for the next 12 months.

Comparing Figures 6 and 8 show that both adaptive al-

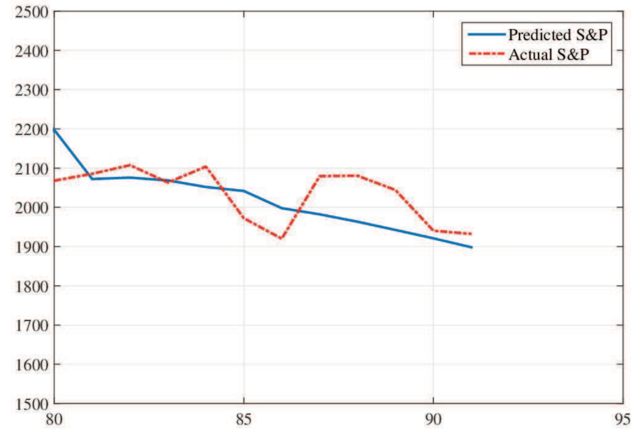

Fig. 6: LMS prediction for the next 12 months and comparison with the actual recorded data.

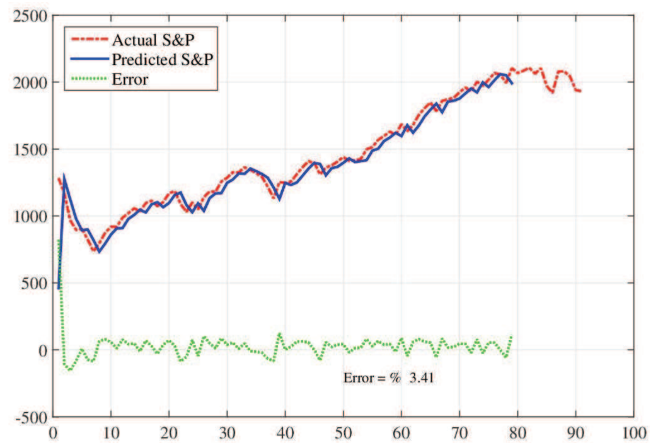

Fig. 7: RLS training process for the prediction of S&P 500 index during the first 79 values of the monthly recorded data.

gorithms are capable of detecting the future trend of the market for the next 12 months. However, RLS adjustment demonstrates a better resolution for long term predication and a lower MSE for one month modelling of the market data.

In terms of learning speed, both algorithms illustrate very quick convergence with a training length of less than 20 data samples. It should be mentioned that all four input signals, $r_p(n)$, $r_f(n)$, $v(n)$ and $s_p(n)$, have been normalized to the same maximum values of unity to maintain the balanced sensitivity across all filter coefficients.

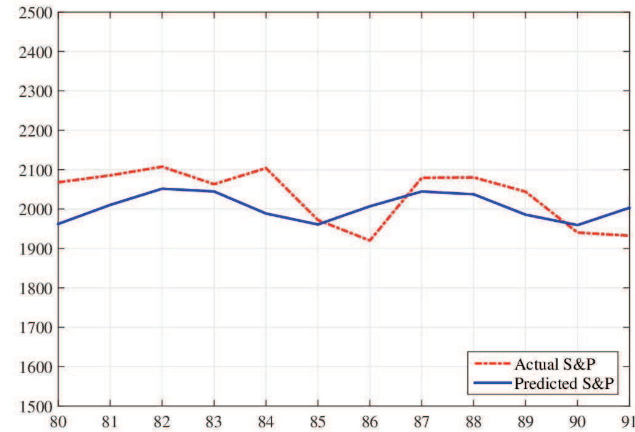

Fig. 8: RLS prediction for the next 12 months and comparison with the actual recorded data.

VI. Conclusions

In this paper a novel adaptive method is proposed to analyse and model financial markets. The proposed scheme has several advantages over the existing frameworks. First, equity risk premiums, free interest rates and volatility index are used to benchmark and train the predictor system. Second, unlike the existing methods, it does not purely rely on historical data to predict future values, but builds on future cash flows and estimated growths to predict future directions and values of the markets. The trained system using the economic factors is then used to model and analyse the financial markets. Finally, the results show that the performance obtained by the RLS adaptive algorithm is superior over LMS method, especially at lower risk free rates.

References

[1] X. Zheng and B. Chen, “Modelling and analysis of financial markets using system adaptation and frequency domain approach,” at the IEEE Int. Conf. Control and Automation, Christchurch, New Zealand, 2009, pp. 1068-1073.

[2] E. Andersson, “Turning point detection using non-parametric statistical surveillance: Influence of some turning point characteristics,” at the Int. Workshop on Intelligent Statistical Quality Control, Warsaw, Poland, 2004.

[3] K. Adam , J. Beutel and A. Marcet “Stock price booms and expected capital gains,” *CEPR Discussion Paper No.9988*, 2014.

[4] K. Adam and A. Marcet, “Internal Rationality, Imperfect Market Knowledge and Asset Prices,” *Journal of Economic Theory*, vol. 146, pp. 1224-1252, 2014.

[5] J. Wesen, V. Vermehren and H. M. de Oliveira, “Adaptive Filter Design for Stock Market Prediction Using a Correlation-based Criterion,” *arXiv preprint arXiv:1501.07504*, 2015.

[6] M. Labonte, “Monetary Policy and the Federal Reserve: Current Policy and Conditions,” *US Congressional Research Service*, Jan 2016.

[7] The ECB’s forward guidance, 2015.

[8] A. Damodaran, “Equity Risk Premiums (ERP): Determinants, Estimation and Implications,” *New York University, Stern Business School*, 2012.

[9] L. Booth, “Estimating the Equity Risk Premium and Equity Costs: New Way of Looking at Old Data,” *Journal of Applied Corporate Finance*, v12(1), pp. 100-112, 1999.

[10] CBOE, “The COBE Volatility Index VIX”, *white-paper*, available at CBOE website.

[11] B. Widrow and S. D. Stearns. *Adaptive Signal Processing*, Prentice-Hall, Englewood Cliffs, N.J., 1985.