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MEMORY IN SEA ICE FRICTION

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ABSTRACT

Friction between sea ice floes is a control on rafting, ridging, and in-plane sliding, and is therefore relevant to a range of engineering problems. Understanding sea ice friction is complicated, because the contact surfaces abrade and melt as sliding occurs. Currently most representations of sea ice friction (for example in discrete element modelling) use Amontons' law for friction, with a wide uncertainty over μ. In this paper we discuss recent work on incorporating memory into an empirical model of sea ice friction. We present a simple model with rate and state dependence derived from laboratory results, and show how this model predicts varying friction on a sliding surface under varying slip rate. We then use this model within a discrete element model to investigate the importance of friction modelling to the modelling of sea ice behaviour as aggregated across many floes.

INTRODUCTION

The friction properties of ice are of broad engineering and scientific interest. Manufacturers of winter tyres might wish to design treads with a more consistent grip on icy roads (Skouvaklis et al., 2012); winter sports equipment may be improved by reducing frictional drag (Poirier et al., 2011); and tectonic activity on other planets depends on ice-ice friction (Olgin et al., 2011). Many Arctic engineering projects need estimates of sea ice friction to predict ice-ice interactions and ice-structure interactions (Timco and Weeks, 2010).

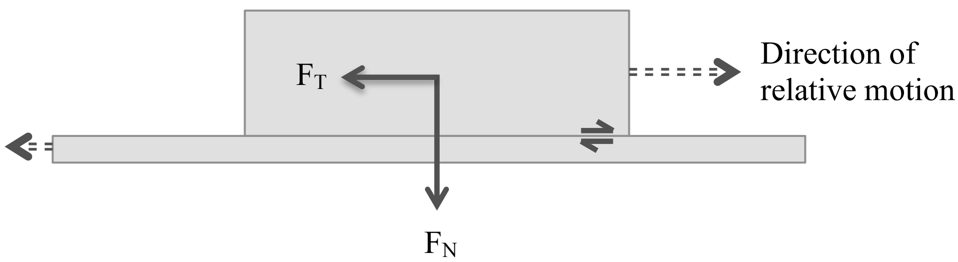

Figure 1: A simple frictional interaction.

Figure 1 shows two blocks sliding past each other. A model for predicting the friction was first noted by Leonardo da Vinci, published by Guillame Amontons, developed by Newton and Coulomb, and is taught in high school physics around the world:

$$F_T = \mu F_N \tag{1}$$

This model is simple, memorable, and widely used to predict the friction between dry solids. Under Amontons' law, the friction is independent of the contact area and of the sliding speed. The coefficient of friction, μ, is a property of the two sliding materials, and is determined

empirically (most easily by measuring the force needed for sliding under a known normal load F_N).
However, when ice slides against ice, experimental observations show that μ is not constant. Several physical processes may affect the forces between the ice blocks. For example:

1. Surface asperities come into contact and deform in compression until the effective contact area is sufficient to support the normal load F_N (Bowden and Tabor, 1986; Hatton et al., 2009).
2. Sliding will abrade the contact surface, and large scale features like serrations may break off, leading to a more planar sliding interface.
3. The energy released from frictional sliding (and possibly from pressure melting) will warm the interface, melting a small layer of water.
4. The meltwater released will form a lubricant, reducing the effective friction (Oksanen and Keinonen, 1982).
5. The lubricating water layer will be physically ejected from the sliding interface as the blocks move.
6. At zero- or very low slip rates, the lubricant may refreeze, causing an adhesive effect (Repetto-Llamazares et al., 2011; Schulson and Fortt, 2013).

Furthermore, in sea ice, abrasion and melting may lead to the opening of brine pockets, which will lead to changes in the salinity of the melt layer and thus complicate any thermodynamic balance based on numbers 3 and 4 in the list above. A complete description of sea ice friction is thus likely to be extremely complicated, and progress is made in small increments.
While we wait, however, we need to incorporate some form of friction modelling into larger models of the dynamics of sea ice. One obvious solution is simply to use Amonton's law as stated in equation (1), with some appropriate choice of μ. For example, Paavilainen and Tuhkuri (2012) run simulations of ice rubbling and find that varying the coefficient of friction from $\mu=0.1$ to $\mu=0.3$ has little effect on the outcomes of their simulations with thin ice. When they model the rubbling of thicker ice, however, they find that the friction coefficient has an important interplay with the plastic compressive strength of the ice. Ice-ice friction has been found to vary from less than 0.1 to over 1 in experiments (e.g. Lishman et al., 2011, Schulson and Fortt 2013). How should we choose a coefficient of friction? And how much does it matter?

A LAW FOR VARIABLE FRICTION

One answer to the first question is that we might look empirically at the factors which affect the friction coefficient, and produce a model of how ice-ice friction might vary during sliding. Such a model is proposed in Lishman et al., 2013. Sliding friction is known to vary with the *rate* of sliding, and with the *state* of the sliding interface. Further, the *state* of the sliding interface will itself change as sliding occurs (due to melting and abrasion). The sliding, therefore, has *memory*, since the current friction depends on not just the current sliding state but also on the sliding history. By analogy with rock mechanics, therefore, we have proposed a *rate and state* friction law (cf. Ruina, 1983):

$$\mu(t) = \mu_0 + \theta + A \ln\left(\frac{V}{V_*}\right) \quad (2a)$$

$$\frac{d\theta}{dt} = -\frac{1}{t_c}\left(\theta + B \ln\left(\frac{V}{V_*}\right)\right) \quad (2b)$$

(Lishman et al., 2013). In this model, θ is a state variable, which contributes to the overall friction $\mu(t)$, and also varies according to equation 2b. A, B and μ_0 are experimentally determined parameters which control the degree to which the friction depends on the rate and state: $A = 0.05$, $B=0.112$ and $\mu_0=0.872$ at -10°C. V_* is a scaling parameter for dimensional consistency, and we use $V_* = 10^{-5} ms^{-1}$. The critical slip time $t_c=3s$ is a measure of the rate at which sliding memory decays (Lishman et al., 2013). Note from eq. 2b that in equilibrium, $\theta=-B\ln(V/V_*)$, and so the steady-state rate-dependence of the model is controlled by $(A-B)(\ln(V/V_*)$ (from eq 2a). Overall, the rate and state model allows us to account for the variable nature of sea ice friction in a way which is simple to apply.
The model can be incorporated into, for example, discrete element models (DEMs) of the interactions of many ice blocks. Such models typically work by accounting for the forces acting on each ice block at time t_i, calculating the subsequent motion of the ice, and iterating. In models which currently allow only constant friction, the definition of μ can be replaced by equation 2. On initiation of a new sliding interface, we assume θ such that sliding is in equilibrium. Thereafter, θ is recalculated at each timestep for each sliding interface. We also note that equation 2 only applies at slip rates above 10^{-5}ms^{-1}, and below this we assume constant friction with $\mu=\mu_0$. This algorithm is summarised in figure 2.

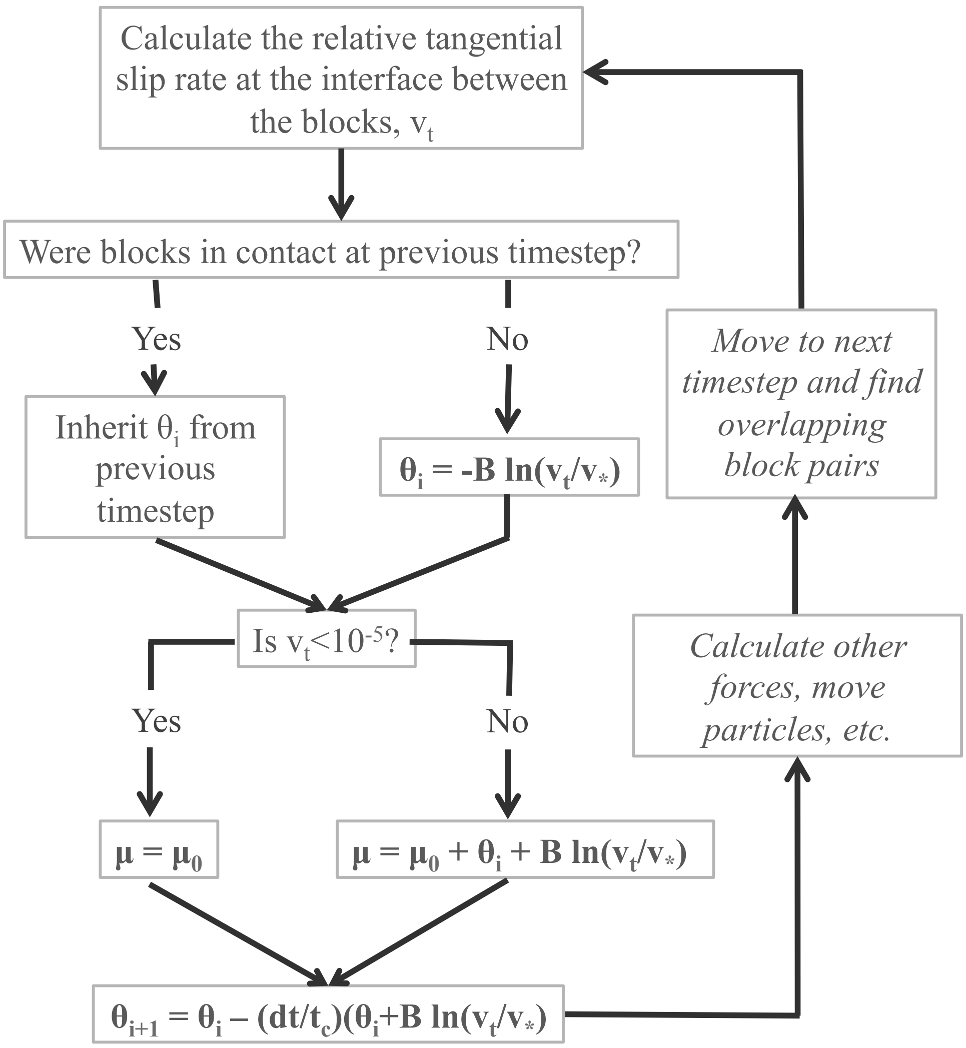

Figure 2: incorporating rate and state friction into multi-element simulations.

To get a sense of how friction behaves under the model of equation 2, we investigate how *μ(t)* varies across a single sliding interface under various simple patterns of slip rate. Graphs of the results of equation 2 are plotted in figure 3.

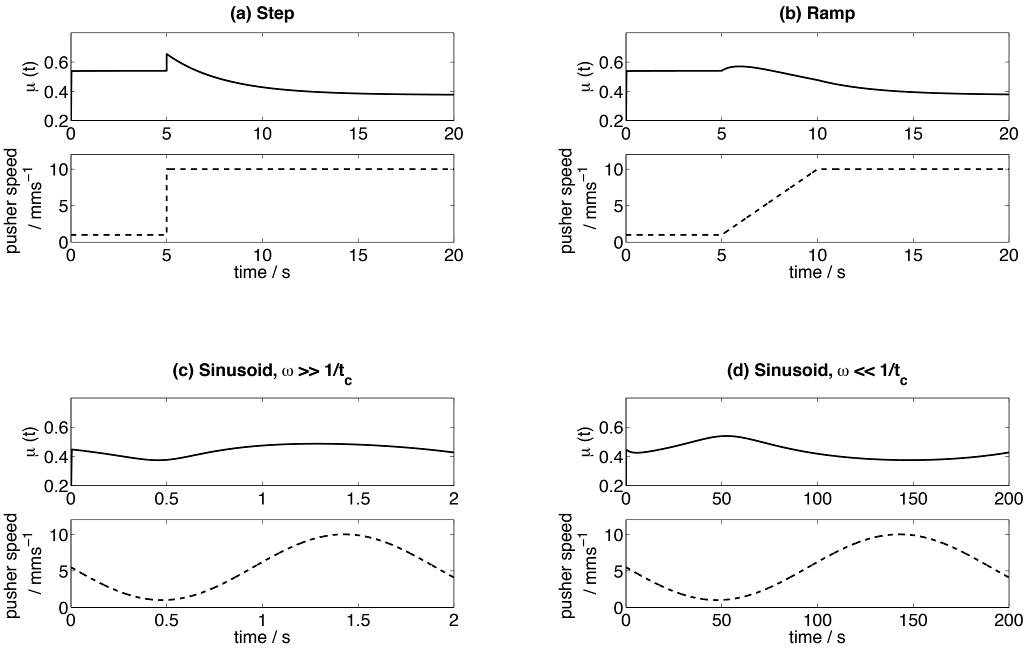

Figure 3: friction response to idealised variations in slip rate. For each of four slip rate profiles (shown as dashed lines: step, top left; ramp, top right; fast sinusoid, bottom left; slow sinusoid, bottom right) the friction responses (shown as solid lines) are plotted.

Figure 3 shows the how friction responds to various changes in slip rate: (a) a step, (b) a ramp, (c) a fast sinusoid, and (d) a slow sinusoid. The results of figure 3 can be seen as applying to the configuration of figure 1. The top block is moved, with a slip rate profile shown as a dashed line in each of the lower figures. The ratio of (F_T/F_N) is then plotted as a solid line in each of the upper figures. When a step increase is applied to the slip speed, the friction responds initially by rising, before decaying to its lower steady-state level. A similar result can be seen with a ramp change in block speed. The friction is seen to move in phase with the slip rate at during high frequency slip, and 180° out of phase at low frequency.

VARIABLE FRICTION WITHIN A SIMPLE DEM

Given this prediction of how friction behaves between two blocks of ice, we wish to test how the behaviour aggregates across a whole ensemble of ice blocks. To do this, we use a 2D discrete element model based on that laid out in Hopkins (1992). Ice blocks are modelled as viscoelastic units. At each time step in the simulation the overlap between ice blocks is determined; this allows us to determine the forces acting on each block, and hence to simulate the blocks' movement.

Table 1: ice properties used in discrete element simulation.

Symbol	Property	Value	
ρ_i	Ice density	920	kgm^{-3}
ρ_w	Water density	1010	kgm^{-3}
E	Young's modulus	2.5×10^8	Pa
k_e	Contact stiffness per meter of contact width	9.75×10^7	N/m

k_v	Contact viscosity per meter of contact width	3.16×10^4	Pa s
d	Ice thickness	0.3	m

Our simulation geometry is chosen to give a high level of in-plane sliding: very loosely, it can be thought of as ice rubble being forced past a static pillar. We simulate an array of 63 diamond-shaped rubble blocks, with long axis 1m and short axis 0.7m. These are pushed along a channel 5.6m wide. 2.7m across the channel (just off center to avoid complete symmetry) is a diamond-shaped pillar with both axes 1m. A rectangular block, with speed $0.1ms^{-1}$, pushes the free rubble blocks past the pillar. Figure 4 shows the evolution of the block positions over 20s of simulation, with constant friction, $\mu=0.5$.

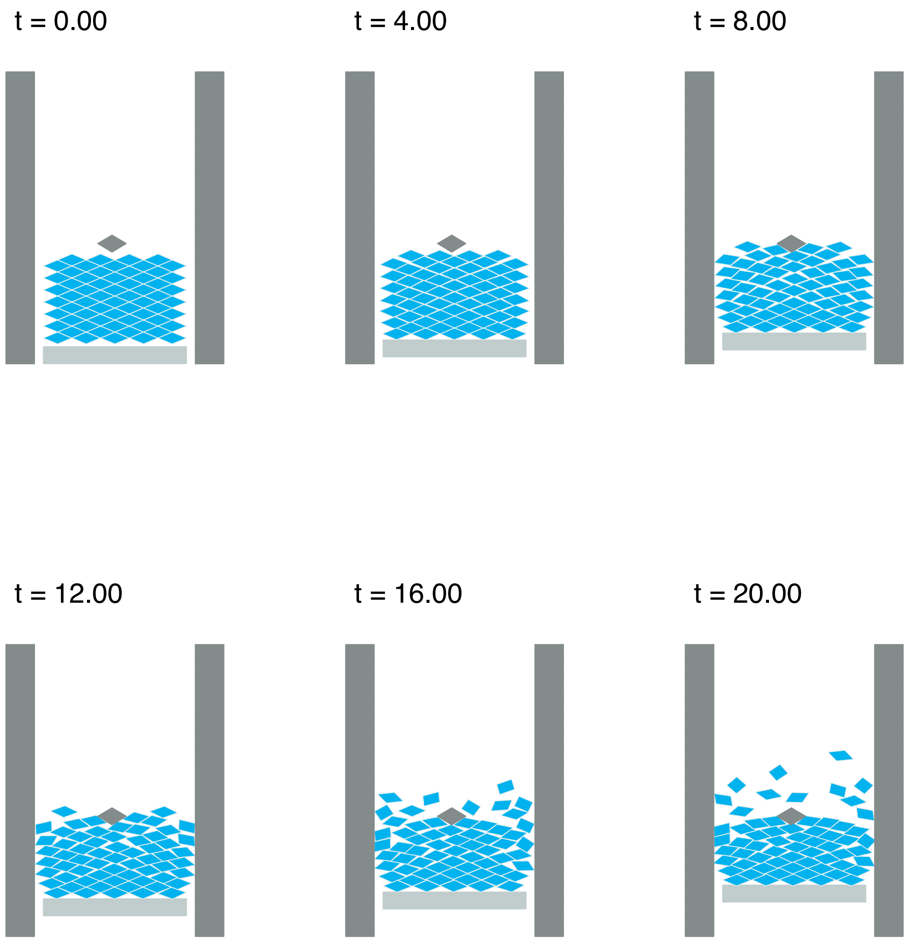

Figure 4: birds-eye view of simulated block positions, every four seconds, for a simulation with constant friction $\mu=0.5$. The outer walls and central grey pillar are fixed; the rectangle at the bottom of the images moves upwards at $0.1ms^{-1}$, and the remaining blocks are free to move, subject to water drag.

We run the same simulation with three values of constant friction, $\mu=0.1$, $\mu=0.5$, and $\mu=0.9$, and with the rate and state friction model discussed above. We include constant friction as high as $\mu=0.9$ since this is comparable to the low-speed limit of the rate and state model. Figure 5 shows the magnitude of the force acting on the central pillar, as a function of time, for each of these simulations, while figure 6 shows the frictional energy loss.

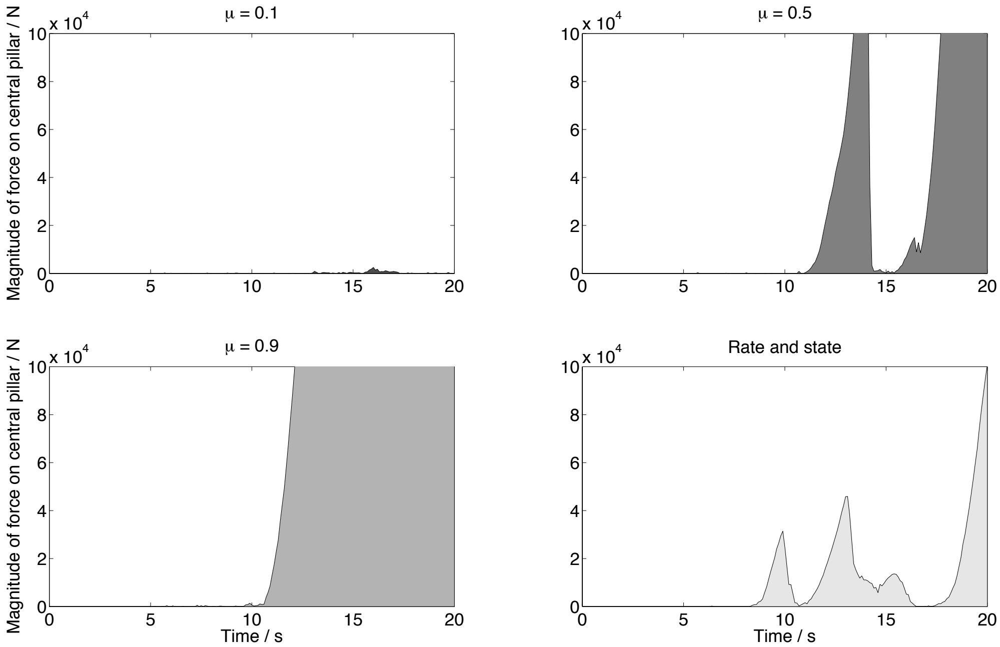

Figure 5: The magnitude of the force acting on the central pillar in our simulations (cf. figure 4), as a function of time, for: $\mu=0.1$ (top left), $\mu=0.5$ (top right), $\mu=0.9$ (bottom left), and a rate and state friction model as described above.

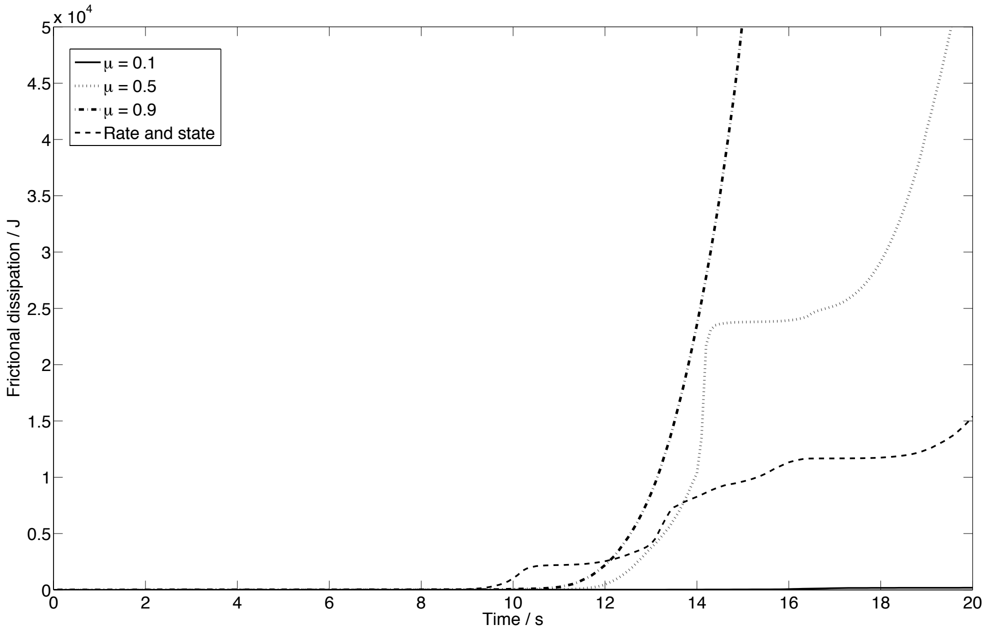

Figure 6: cumulative frictional energy dissipation for each friction model.

From figure 5 we see that with low friction, blocks slide easily past the pillar, and forces never get higher than 1kN, while with high friction the blocks jam against the pillar and the force rises dramatically. With intermediate friction, and with the rate and state model, the forces rise as jams are generated and then drop as blocks slide out of the way. Figure 6 shows the total energy lost to friction, as a function of time, for each simulation. Again, with low friction, forces and hence energy loss are small, and with large friction, the ice jam is never broken and the energy loss rises rapidly when the rubble meets the pillar. With intermediate friction ($\mu=0.5$) the energy loss seems to be concentrated in two individual events (at roughly 14s and 19s) whereas with rate and state friction the rate of increase of frictional loss is steadier.

DISCUSSION

The DEM presented here is clearly far from a real representation of ice-ice interactions: for example, out-of-plane failure and breakup of the ice blocks would dissipate the forces acting on the central pillar. However, we believe models of this type can be useful in learning how individual interactions can add up to complicated behaviour across an ensemble of ice blocks.

Friction is usually modelled in simulations as obeying Amontons' law. It is broadly agreed, however, that ice friction is complicated, and there is no single coefficient of friction for ice. Laboratory modelling has shown that a simple empirical model can bridge the gap between Amontons' law and a full thermomechanical description of friction (which does not yet exist). The empirical law, equivalent to rate and state laws in rock mechanics, accounts for the slip rate at the interface and incorporates memory of the state of the ice surfaces. This law can be implemented within current DEMs with only minor additions to the code, since it provides a direct replacement for the constant μ which is standard.

Initial testing with this friction law within a DEM gives results which are comparable to, but different from, results with constant friction. Intriguingly, the introduction of rate and state friction seems to produce results in which ice jams are released more easily, and hence frictional dissipation is less concentrated across a few discrete events. However, this is a tentative result since the forces and energies calculated by the DEM are highly sensitive to initial conditions. By running larger suites of experiments, across a wider range of friction values, it may be possible to clarify the specific effects of incorporating rate dependence, memory dependence, or both into sea ice friction models.

CONCLUSIONS

We have incorporated rate dependence and memory into sea ice friction modelling within a discrete element model, and we show how this can be done for general models of ice dynamics. Simulation results with this new friction model provide results which can be compared to constant friction results. Early tests with our discrete element model suggest that ice jams, with high frictional dissipation concentrated in a short period of time, may be less likely to occur under the new friction model.

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