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The Rate- and State- Dependence of Sea Ice Friction

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Abstract

Ice loading on offshore structures is often governed by the behaviour of granular ice flows around the structures. This behaviour is in turn governed by frictional interactions between ice floes. Observed stick-slip behaviour in ice friction suggests a dependence on slip history, which is not accounted for in current ice friction models. We propose, by analogy with results from rock friction, a rate- and state-model for ice friction.

In this paper we present the results of a series of metre-scale ice basin experiments, in which we determine this rate- and state- dependence. We then propose a simple parameterisation which gives a general rate- and state- model for sea ice friction. The model suggests that when the slip rate increases, the ice-ice friction reaches a peak before decaying to some steady-state value. To assess peak frictional loading, therefore, the slip history must be considered as well as the current slip velocity.

1. Introduction

The design of offshore structures and vessels in icy waters requires an understanding of sea ice dynamics. Recent work (e.g. Hopkins 1996) has focussed on the mesoscale interactions between ice floes. Frictional interactions between ice floes are typically modelled by Amonton's friction law:

$$F_t = \mu F_n \tag{1}$$

where F_t is the tangential or shear force, F_n is the normal force, and μ is a constant friction coefficient, typically in the range 0.2-0.8. However, observed stick slip behaviour (Sanderson, 1988; Sammonds et al, 1998) cannot be modelled by such a law, and suggests a useful analogy with rock mechanics, where stick-slip friction is also observed. Experiments in rock mechanics suggest that sliding is poorly modelled by a constant friction coefficient, and is in fact dependent upon both the slip velocity and the slip history. Modelling of ice floe interactions may be improved by a friction law which takes these further variables into account.

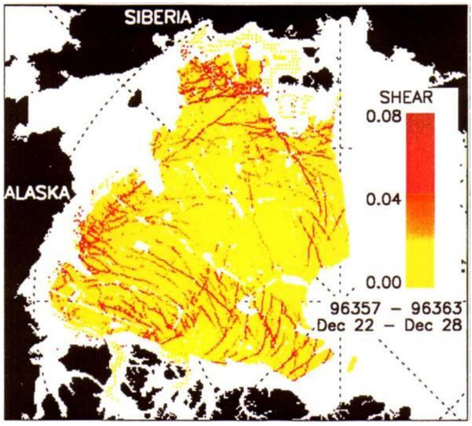

Figure 1. *RADARSAT image of the Arctic Ocean sea ice cover showing major shear faults in the sea ice on which slip occurs (after Kwok, 2001)*

Improved friction laws may also be important for basin-scale modelling. Figure 1 shows a RADARSAT image of the Arctic sea ice cover, which is dominated by basin-scale shear faults. By analogy with faulting in the earth's crust (Sammonds and Rist 2001), we believe that the overall dynamics of the Arctic sea ice cover are dominated by frictional sliding in these high-shear regions. We therefore propose to develop a model of sea ice friction which will allow the slip displacement over time to be accurately modelled.

Such models rely on empirical data. We have conducted a series of experiments in the Hamburgische Schiffbau-Versuchsanstalt (HSVA) ice tank in Hamburg, Germany, to provide this empirical data. These experiments are detailed in section 2 and follow a similar methodology to that used by Dieterich (1978) for determining rock friction. In section 3 we fit this empirical data to a one state variable constitutive law, as proposed by Ruina (1983) and developed by Gu et al (1984). In section 4 we discuss the advantages and limitations of this constitutive law; compare the proposed law to the literature on ice friction; and suggest potential avenues for further work.

2. Experimental programme

Figure 2. *HSVA Arctic Environmental Test Basin*

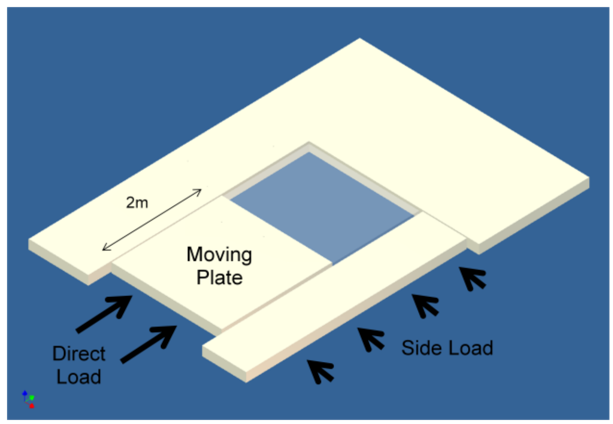

Figure 3. *Schematic of Experimental Setup*

To produce a quantitative constitutive description of ice friction, experimental data is needed. To provide such data we have undertaken a series of metre-scale experiments in the HSVA Arctic Environmental Test Basin. The basin is 30 m long, 6 m wide and 1.2 m deep. Air temperature can be regulated between -20°C and + 20°C, allowing the simulation of typical arctic ice conditions. A motorized carriage allows us to apply an in-plane force on the ice of up to 10kN along the length of the tank, while a series of piston-driven side-loading frames allow us to apply an in-plane force on the ice of up to 15kN across the width of the tank. Figure 2 shows a photograph of a typical experimental configuration.

Figure 3 illustrates schematically how ice-ice friction was measured. A 2m square floating block of 25cm thick ice was subjected to loading between two ice sheets. Typical normal (side) loads were 5kN, corresponding to a pressure of 10kPa. The block was then driven forwards by a pusher plate attached to the mechanical carriage, and the load required to move the block (which corresponds to the shear force due to friction) measured using two shear load cells attached to the pusher plate. The effective friction coefficient μ is given by the shear load divided by the normal load. The level ice was grown from saline water at -10°C over a period of two weeks, to a thickness of 0.25m. The ice was then cut manually into the required pieces using handsaws, and redundant ice cleared into a melting tank. Table 1 gives the relevant ice properties.

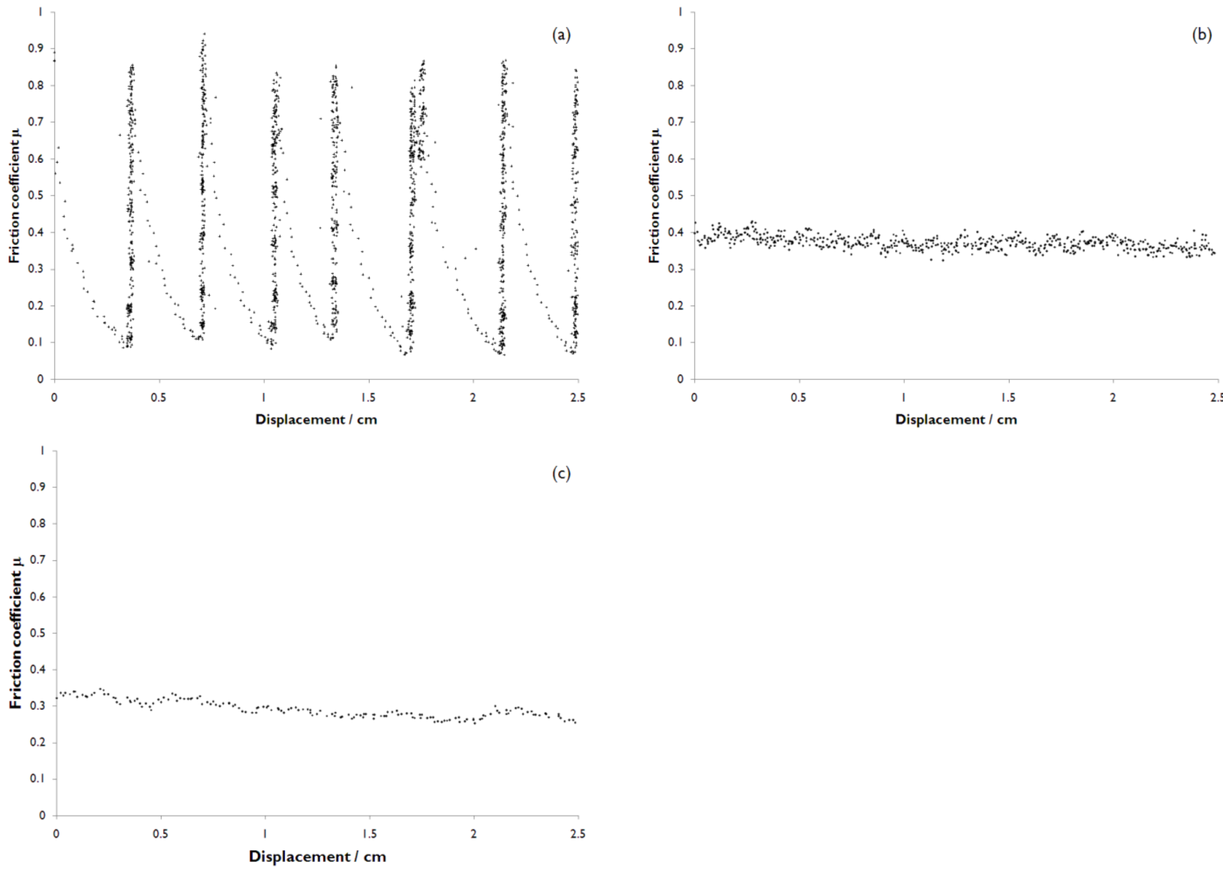

Figure 4. *steady state evolution of experimentally determined friction coefficient for three typical slip rates. Each figure shows the friction evolution over 2.5cm of displacement in steady state. 4a represents a slip rate of 0.28 cms^{-1}; 4b shows 0.80 cms $^{-1}$; and 4c shows 2.84 cms $^{-1}$. Stick-slip behaviour is clearly observed at low slip rates, with frictional buildup at fixed displacement followed by sudden movement and relaxation of slip.*

Table 1: Experimental Properties	
Ice thickness	0.25m
Temperature	-10°C
Water Salinity	33 ppt
Bulk Ice Salinity	7.3 ppt
Ice density	931 kg m^{-3}

Our first set of experiments was used to determine the rate-dependence of ice friction. Tests were run at three different carriage velocities: ~ 0.3 cm s^{-1}, ~ 0.8 cm s^{-1}, and ~3 cm s^{-1}. The velocities were selected by a relatively crude control on the carriage, but were measured by accurate displacement transducers. Typical evolutions for these three slip velocities are shown in figure 4. Note that at the lowest slip velocity (0.28 cm s^{-1}) stick-slip behaviour is observed, evidenced by repeated stress buildup (corresponding to increasing μ) at constant displacement, followed by periods of movement and rapid stress decrease. The time-averaged friction coefficients for the entire series of experiments are plotted in figure 5. The straight line plotted shows the LMS fit to the data:

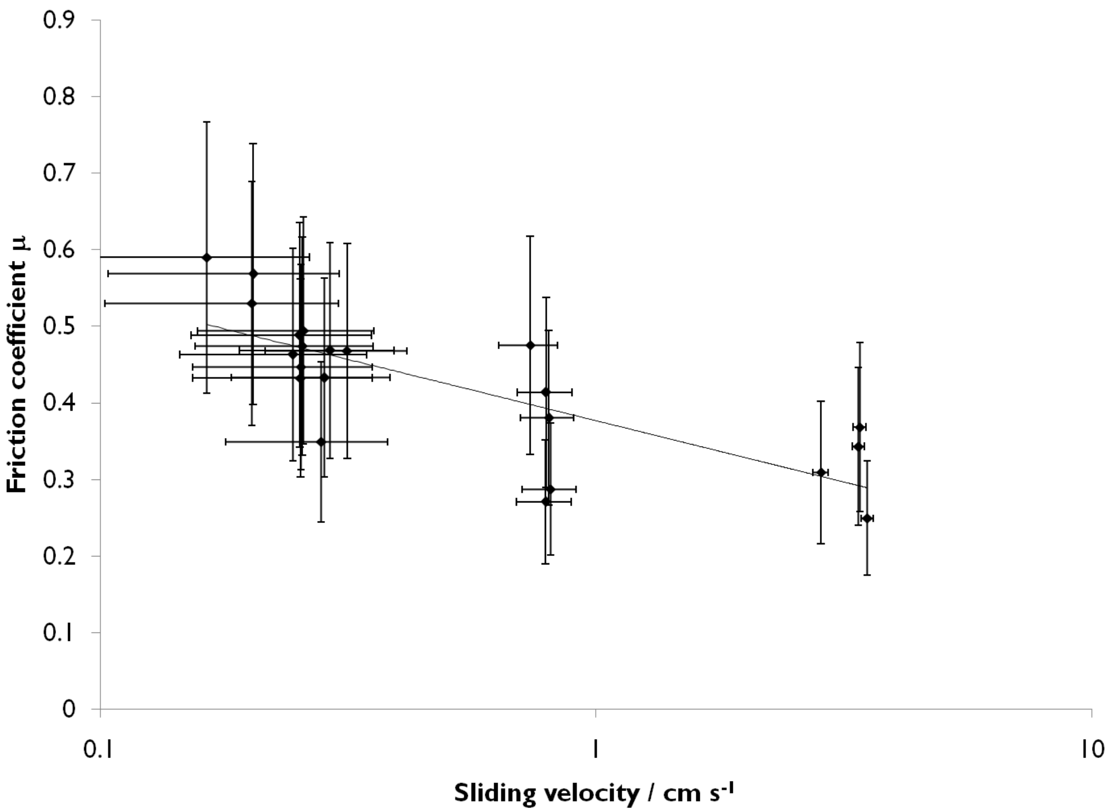

Figure 5. *The dependence of steady-state ice-ice sliding friction on slip rate. Each data point represents a separate experimental run. The line shows an LMS fit to the data, as described in the text. The vertical error bars represent the variation in load during the experiments, while the horizontal error bars show a fixed velocity error of 0.1 cms^{-1}, which accounts for the stick-slip variations from time-averaged velocity.*

$$\mu = \mu_0 + C_1 \ln\left(\frac{V}{V_*}\right) \tag{2}$$

where V_* is a characteristic velocity for dimensional consistency. We choose $V_* = 10^{-3}ms^{-1}$ and find that $\mu_0 = 0.537$ and $C_1 = -0.069$ (with a coefficient of determination $R^2 = 0.583$).

Having thus characterized the rate-dependence of sea ice friction, we went on to determine the state-dependence. Following the interpretation of Ruina (1983), after Dieterich (1978), the state-dependence can be determined from a series of experiments involving periods of motion interspersed with static "hold" times. Dieterich (1978) found that these static holds led to "spikes" in the friction on resumption of motion, and that in rock friction, μ_s, the maximum transient value of μ, obeys the relation

$$\mu_s = \mu_1 + C_2 \log_{10}(t) \tag{3}$$

We believe a similar relation holds in ice friction, subject to the caveats discussed in section 4. We use the experimental method described in Ruina (1983): the load point (in our case the pusher plate and carriage) moves steadily at constant speed; stops for a time (the "hold time"); and then moves again at the original speed. We used a constant speed of 0.8 cm s^{-1}, hold times of 10s, 100s, and 1000s, and measured the

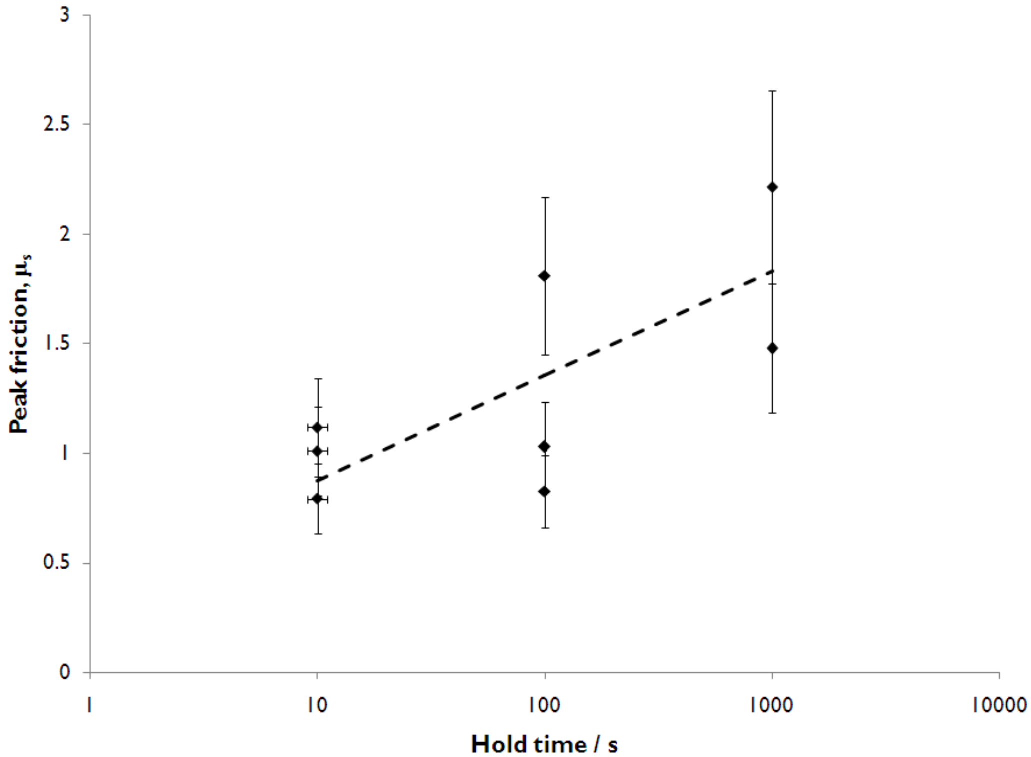

Figure 6: peak friction coefficient μ_s as a function of hold time. Each data point represents a separate experimental run. The line shows an LMS fit to the data, as described in the text. The error bars on the data points represent uncertainty in the friction coefficient due to variations in the applied side load.

maximum value of the friction coefficient. Results from this series of experiments are plotted in figure 6. The results show significant scatter (see discussion in section 4) but a clear trend of increasing μ_s with increasing hold time, as expected. Again, the straight line plotted shows the LMS fit to the data. We find that $\mu_1 = 0.394$, as predicted from the rate dependence, and that $C_2 = 0.48$ ($R^2 = 0.482$).

These separate quantifications of the rate- and state-dependence of sea ice friction can now be combined into a single constitutive friction law.

3. Constitutive relation with one state variable

We propose a single-state-variable constitutive law following Gu et al, 1984, in which a state variable θ is introduced. The evolution of this state variable accounts for the slip history. The model has the form:

$$\mu = \mu_0 + \theta + A \ln \frac{V}{V_*} \tag{4a}$$

$$\frac{d\theta}{dt} = -\frac{V}{L}\left(\theta + B \ln \frac{V}{V_*}\right) \tag{4b}$$

where L is a characteristic slip length found for our experiments to be 5 mm (see e.g. Ruina 1983), and μ_0, A and B are empirically determined constants. From our determination of rate dependence we observe that $\mu_0 = 0.537$ and the steady state rate dependence, (B-A) = $C_1 = 0.069$ (see Gu et al, 1984). To determine B we numerically model this friction law, combined with a spring-slider model of the pushing plate. The

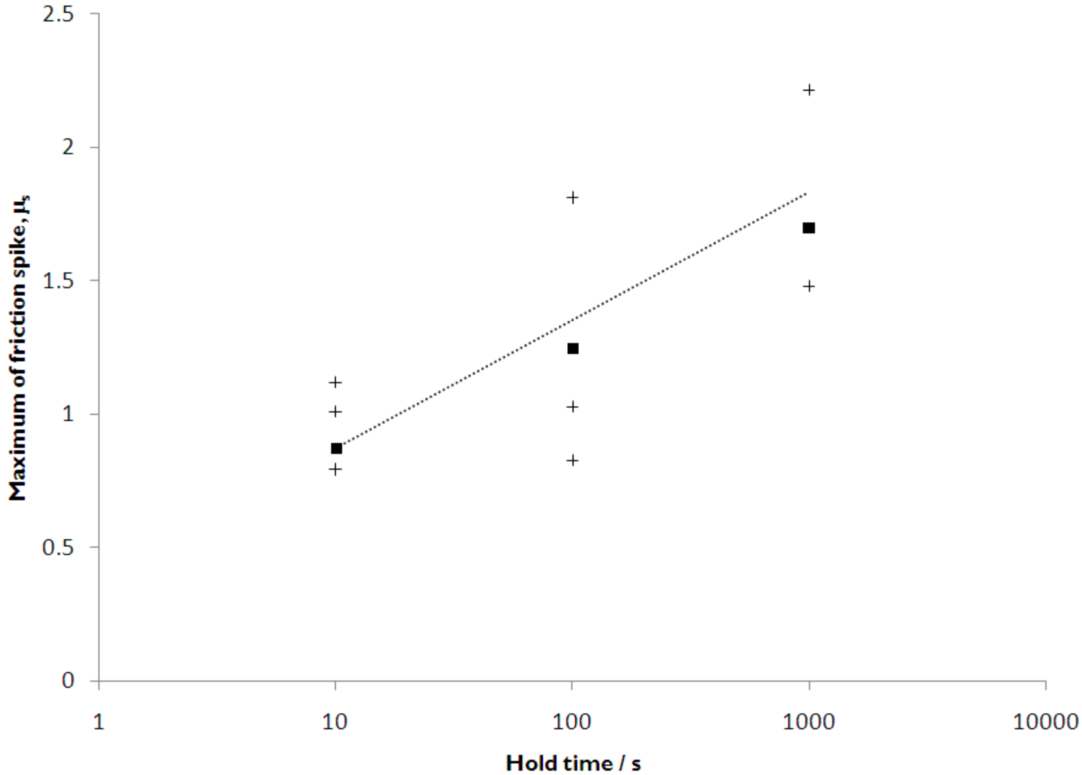

Figure 7: Comparison of state-dependence in model and experiment. The squares show the model-predicted friction peaks, while the crosses show experimental values and the dotted line is the best fit to these experimental values

pushing plate has two square steel supports, of side 10cm and length 0.6m: from first principles we find the spring constant to be 2 MPa (noting that the model is relatively insensitive to this stiffness). Note that the friction law above does not account for absolute static friction, but that the increases in μ_s are caused by small amounts of slip as the pusher plate returns to equilibrium while the carriage is still (Ruina, 1983). We find that our observed state dependence is well modeled by (4) when A = 0.61 and B = 0.679. Figure 7 shows the numerically calculated values alongside the experimental results. Since A and B have been specifically chosen to exactly match the LMS fit to the rate dependence, we do not replot figure 5.

We have proposed a constitutive model, with a single state variable, which accounts for frictional dependence on slip rate and slip history in sea ice. This model is based on a rock physics model which has been widely developed and applied for 25 years. In the next section we discuss advantages and limitations of the model as presented here, and potential improvements.

4. Discussion

This study presents a high-level model of ice-ice friction, with dependence only on the macroscopic properties of slip rate and slip history. This model offers the benefit that it can directly replace Amonton's law as the friction model of choice in analysis and simulation of ice-ice interactions. The parameterization presented is valid within the range of experiments conducted. In order to extend the range of validity of the parameterization, it is necessary to unite the model with the physical controls on friction: plastic and elastic deformation of asperities; hydrodynamic and other forms of lubrication; and refreezing and sintering of asperities.

The rate-dependence of sea ice friction has been investigated before (albeit without the inclusion of state-dependence), notably by Jones et al (1991) and Kennedy et al (2000). Kennedy et al. found a similar negative velocity dependence of sea-ice friction in the velocity range discussed here, but at a much lower friction coefficient (around 0.2). We believe the discrepancy here is due to differences in the nature of the experiments, and most significantly in the properties of the contact surface (see Gu et al, p 171, for a discussion of the importance of surface roughness in rock friction). Kennedy et al used a microtome to smooth the ice surface to 1μm precision, while the ice used in the present experiments was saw-cut and had visible asperities and notches on the millimetre scale. These experiments might therefore be seen as at opposite ends of a spectrum, and we believe there is the potential for the results of the two studies to be united by future work on the relevance of surface properties to ice friction. Ice contact surfaces in the Arctic are likely to initially be very rough (after fracture, for example) but may then be smoothed by abrasion and lubricated by ice gouge. Note that current studies use values of μ in the range 0.2-0.8 (e.g. Hopkins 1996), which closely matches our findings (which are in the range 0.2-0.6). The scatter visible in our data may be due in part to variations in the sliding surface from one experiment to the next. The experiments occurred up to four hours after the ice was cut, and so surface properties may have changed due to melting (although this was not directly observed, and variability in the data is not correlated with the time since cutting). Further work might extend the rate-dependence of our model beyond the velocity range discussed in this paper (3 mms^{-1} to 30 mms^{-1}).

The state-dependence of sea ice friction discussed here is, to the best of our knowledge, new, and suggests many avenues for further investigation. In a future paper, we plan to apply our new frictional model to a particle simulation of a granular ice flow (cf. Hopkins, 1996), which can be compared directly to experimental results. This will allow us to quantify the importance of friction models in predicting ice ensemble behaviour. Note also that this one-variable model is unable to predict the stick-slip behaviour seen in figure 4: Gu et al (1984) suggest that a two-state-variable model is more accurate in this regard. Further experimental work will allow us to determine such a model, as well as to increase the range of validity of our findings (for example, to include static contact times of a second or less). Previous work by this research group (Sammonds et al., 2005) investigated the mechanics of stick-slip behaviour in ice, showing that slip initiates in a nucleation zone and then propagates along the fault. One direction for future work is to reconcile this physical understanding with the high-level state-dependence discussed in the current paper.

We conduct ice tank experiments because it is not possible to control inputs to Arctic sea ice behaviour in the field. The experiments provide insights into sea ice behaviour: however, natural processes may be more complicated. In particular, the local polar environment may be more dynamic than the clamped holds which we investigate in the ice tank, and the effects of ocean waves and jostling of ice floes may lead to lower dependence on hold times than is observed experimentally. Local variations in temperature and salinity will also affect the quantitative results presented in this paper, although we believe the qualitative model discussed holds across a wide range of conditions.

It is important to note that the analogy between rock friction and ice friction is not perfect. The model presented here implicitly accounts for some refreezing of asperities during relatively short hold times, on the scale of minutes. However, for longer hold times, refreezing at the interface will dominate: indeed, we found during our experiments that after a hold time of 10000s (~3 hours) the loads required to move the ice were too high for our equipment. At some stage the assumption of a frictional contact is no longer valid and we expect that the contact stress asymptotically approaches the shear strength of level ice. We intend to conduct experiments to allow us to incorporate this consolidation effect into future work.

5. Conclusions

We have presented a new rate- and state- dependent constitutive law for ice friction. This law is derived from rock physics, where it has been adopted as the standard for rock friction. The major advantage of the new model is that it can account for the effects of variations in both slip rate and slip history, with only a slight increase in complexity from Amonton's law, which is the current standard.

We have undertaken a series of ice tank experiments in order to determine the parameters of the new friction model. These ice tank experiments suggest that friction decreases with sliding velocity, and that on movement following a period of static contact, the friction is initially high and then decays to steady state, all as expected. The model is valid in the range of sliding velocities and hold times considered in the experiments, and we have discussed potential further work to improve both the accuracy and range of validity of the model.

The potential advantages of this new model apply on a range of scales. On an engineering scale, ice-ice friction is crucial to predicting ice stresses and hence loading on offshore vessels and structures (see e.g. Hopkins and Hibler, 1989). On a larger scale, as GCM grid sizes shrink, the sea ice components of these models may move away from a continuum hypothesis and towards particle simulations in which the interactions between discrete ice floes are modeled. In this case, accurate models of ice friction will be important to climate modelers. Finally, we note that the overall behaviour of Arctic sea ice appears to be governed by several large shear zones, and so an improved model of ice friction, applied to these faults, could lead to rapid insights into Arctic behaviour.

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