Dynamics of Vibratory Bowl Feeders

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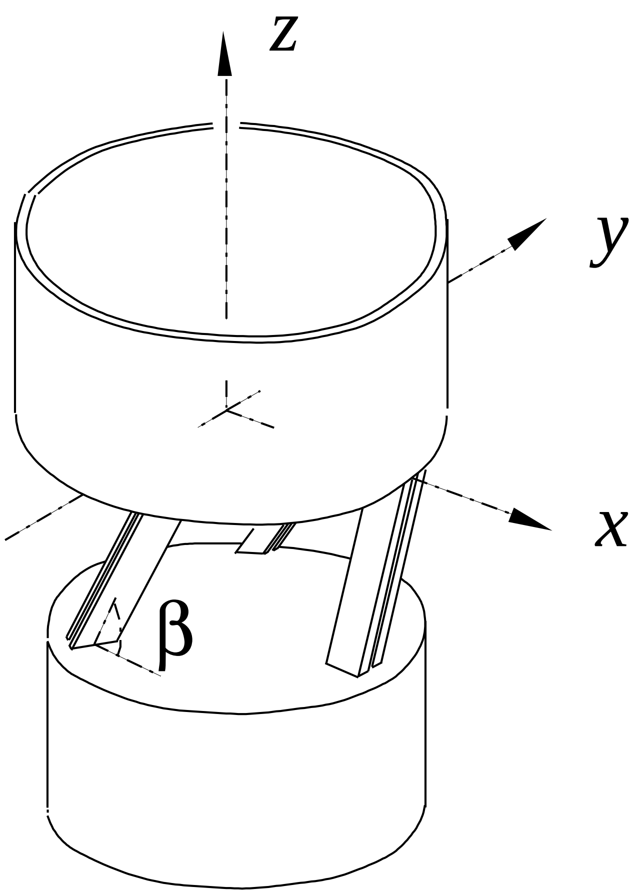
Introduction



- Bowl feeders are important industrial components.
- Most research on orienting of part, using gates and fences.
- Some work on hopping motion on vibrating plates.
- Here, look at the dynamics of the bowl itself.

The Stiffness matrix

Small displacement of the bowl described by a screw (twist),

$$\mathbf{s} = \begin{pmatrix} \boldsymbol{\delta\theta} \\ \boldsymbol{\delta}\mathbf{p} \end{pmatrix}$$

Hooke's law,

$$\mathcal{W} = K\mathbf{s}$$

K 6×6 stiffness matrix—leaf springs. Wrench $\mathcal{W}^T = (\mathbf{M}^T, \mathbf{F}^T)$ with moment $\mathbf{M}$ and force vector $\mathbf{F}$.

Equations of motion

Linearised equations for small motions.

$$N\ddot{\mathbf{s}} + K\mathbf{s} = \mathcal{W}$$

N 6×6 inertia matrix of bowl. $\mathcal{W}$ driving wrench.

Symmetry I

Assume bowl has cylindrical symmetry, symmetry axis along z-direction and origin at centre of mass then inertia matrix has form,

$$N = \begin{pmatrix} mk_1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & mk_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & mk^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & m \end{pmatrix}$$

m-mass, k_1, k-radii of gyration.

Symmetry II

Assume 3 or more leaf springs arranged symmetrically, then stiffness matrix has form,

$$K = \begin{pmatrix} \xi_{11} & 0 & 0 & \gamma_{11} & \gamma_{12} & 0 \\ 0 & \xi_{11} & 0 & -\gamma_{12} & \gamma_{11} & 0 \\ 0 & 0 & \xi_{33} & 0 & 0 & \gamma_{33} \\ \gamma_{11} & -\gamma_{12} & 0 & u_{11} & 0 & 0 \\ \gamma_{12} & \gamma_{11} & 0 & 0 & u_{11} & 0 \\ 0 & 0 & \gamma_{33} & 0 & 0 & u_{33} \end{pmatrix}$$

Elements ξ, γ and u can be related to material properties of the springs, see paper.

Equations of motion again

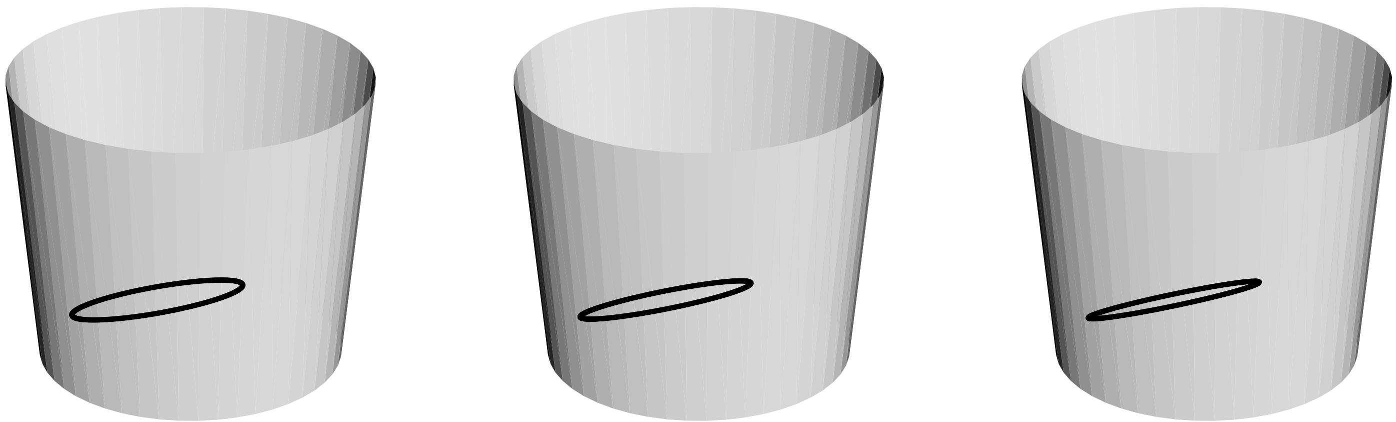
Assume these symmetries and driving wrench,

$$\mathcal{W}^T = (0, 0, pA, 0, 0, A)\cos\omega t$$

oscillating pitch p wrench with force amplitude A. Then equations of motion decouple to give,

$$\begin{pmatrix} mk^2 & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \ddot{\theta}_z \\ \ddot{z} \end{pmatrix} + \begin{pmatrix} \xi_{33} & \gamma_{33} \\ \gamma_{33} & u_{33} \end{pmatrix}\begin{pmatrix} \theta_z \\ z \end{pmatrix} = \begin{pmatrix} pA \\ A \end{pmatrix}\cos\omega t$$

Solutions



Sinusoidal solutions,

$$\begin{aligned} \theta_z &= A_1 \cos(\omega t + \phi_1) \\ z &= A_2 \cos(\omega t + \phi_2) \end{aligned}$$

Solutions are ‘elliptical’ motions—verified experimentally.

Resonance

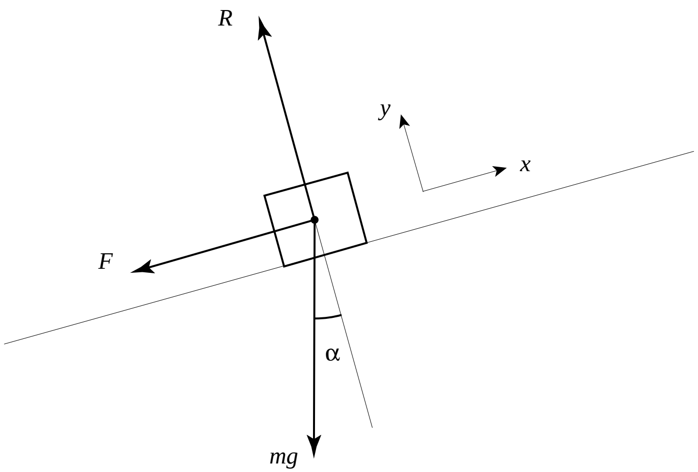
Common to talk about ‘resonance’ of the system. What does it mean here?

Possible interpretation—frequency for which area of ellipse is maximum.

Can find condition for this, see paper.

Note, can’t really alter the frequency in a bowl feeder, tied to electrical supply frequency.

Part on a Spiral Track



Assume elliptical motion, $\mathbf{q}(t) = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix}$.

Perpendicular to the plane,

$$R = m_0 g \cos\alpha - b\omega^2 \sin\omega t$$

m_0-mass of part, g-acceleration due to gravity.

Condition for Hopping

For no hopping the reaction R must be positive throughout motion,

$$b < \frac{m_0 g}{\omega^2} \cos \alpha$$

For part of a few grams and driving frequency of 50 Hz, b the width of the ellipse must be less than a few microns.

Conclusions

- Need to confirm experimentally whether parts hop or stick-slip.
- Consider off-axis modes, nothing is perfectly symmetrical.
- What about damping?
- Study dynamics of hopping on inclined planes.
- Optimise the design, spring sizes and tilt angles, and pitch of the spiral track.